Appendix A

Details about the truncation

This appendix has been based on Eduardo Sánchez Burillo's master thesis [4], one of the advisor of this work, and it has been inserted here with the consent of the advisers.

The problem consists of minimising iteratively a function which is a sum of scalar products (2.26). During this section, we will consider the generic scalar product $\langle\xi|\chi\rangle$, whose MPS tensors are $A(\xi)$ and $A(\zeta)$ respectively. First of all, since the whole function is minimised with respect to the tensors linked to a given site, we are going to write a scalar product between MPS in the following way:

$$\langle\xi|\zeta\rangle = (v_n^\xi)^\dagger E v_n^\zeta, \tag{A.1}$$

where v_n^ξ and v_n^ζ are the vectorisations of $A(\xi)^n$ and $A(\zeta)^n$ respectively. Using the MPS representation, this scalar product is:

$$\langle\xi|\zeta\rangle = (A(\xi)_1^{i_1})^*_{k_1,k_2}(A(\xi)_2^{i_2})^*_{k_2,k_3}\dots(A(\xi)_L^{i_L})^*_{k_L,k_1}(A(\zeta)_1^{i_1})_{l_1,l_2}(A(\zeta)_2^{i_2})_{l_2,l_3}\dots(A(\zeta)_L^{i_L})_{l_L,l_1} \tag{A.2}$$

We rewrite it in the following way:

$$\langle\xi|\zeta\rangle = (A_L)_{k_1,l_1,k_n,l_n}(A(\xi)_n^{i_n})^*_{k_n,k_{n+1}}(A(\zeta)_n)^{i_n}_{l_n,l_{n+1}}(A_R)_{k_{n+1},l_{n+1},k_1,l_1}, \tag{A.3}$$

where A_L and A_R are:

$$(A_L)_{k_1,l_1,k_n,l_n} := (A(\xi)_1^{i_1})^*_{k_1,k_2}(A(\zeta)_1^{i_1})_{l_1,l_2}\dots(A(\xi)_{n-1}^{i_{n-1}})^*_{k_{n-1},k_n}(A(\zeta)_{n-1}^{i_{n-1}})_{l_{n-1},l_n} \tag{A.4}$$

$$(A_R)_{k_{n+1},l_{n+1},k_1,l_1} := (A(\xi)_{n+1}^{i_{n+1}})^*_{k_{n+1},k_{n+2}}(A(\zeta)_{n+1}^{i_{n+1}})_{l_{n+1},l_{n+2}}\dots(A(\xi)_L^{i_L})^*_{k_L,k_1}(A(\zeta)_L^{i_L})_{l_L,l_1} \tag{A.5}$$

Now, another tensor C is introduced:

$$C_{k_{n+1},l_{n+1},k_n,l_n} := (A_R)_{k_{n+1},l_{n+1},k_1,l_1}(A_L)_{k_1,l_1,k_n,l_n} \tag{A.6}$$

Then, the scalar product becomes:

$$\langle\xi|\zeta\rangle = (A(\xi)^{i_n})^*_{k_n,k_{n+1}}C_{k_{n+1},l_{n+1},k_n,l_n}A(\zeta)^{i_n}_{l_n,l_{n+1}} \tag{A.7}$$

We introduce a Kronecker delta because of convenience:

$$\langle\xi|\zeta\rangle = (A(\xi)^{i_n})^*_{k_n,k_{n+1}}C_{k_{n+1},l_{n+1},k_n,l_n}\delta_{i_n,j_n}A(\zeta)^{j_n}_{l_n,l_{n+1}} \tag{A.8}$$

Another tensor is defined:

$$D_{k_n,i_n,k_{n+1},l_n,j_n,l_{n+1}} := C_{k_{n+1},l_{n+1},k_n,l_n}\delta_{i_n,j_n} \tag{A.9}$$

Then:

$$\langle\xi|\zeta\rangle = (A(\xi)_n^{i_n})^*_{k_n,k_{n+1}}D_{k_n,i_n,k_{n+1},l_n,j_n,l_{n+1}}(A(\zeta)_n^{j_n})_{l_n,l_{n+1}} \tag{A.10}$$

Finally, we can define the vectorisations $A(\xi)_n$ by joining all the indices:

$$(v_n^\xi)_{[k_n,i_n,k_{n+1}]} := (A(\xi)_n^{i_n})^*_{k_n,k_{n+1}}, \tag{A.11}$$

and the same for v_n^ζ. Here, $[k_n, i_n, k_{n+1}]$ is again a single index, in such a way that we construct a vector from a tensor with three indices (the same for v_n^ζ). In the same way, we define a matrix E by joining the indices of the tensor D in two sets:

$$E_{[k_n,i_n,k_{n+1}],[,l_n,j_n,l_{n+1}]} := D_{k_n,i_n,k_{n+1},l_n,j_n,l_{n+1}} \tag{A.12}$$

Then, the scalar product is:

$$\langle\xi|\zeta\rangle = (v_n^\xi)^*_{[k_n,i_n,k_{n+1}]} E_{[k_n,i_n,k_{n+1}],[,l_n,j_n,l_{n+1}]} (v_n^\zeta)_{[,l_n,j_n,l_{n+1}]}, \tag{A.13}$$

or, in vectorial notation:

$$\langle\xi|\zeta\rangle = (v_n^\xi)^\dagger E v_n^\zeta \tag{A.14}$$

So, we achieve a result like (2.27), as we wanted to show, as well as the form in which the matrix E has to be constructed.