

Inverse modeling of pan heating in domestic cookers

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Abstract

The heating uniformity of the cooking vessels in domestic stoves depends on the type of heat source (induction, electric resistance, gas burner...) and of the way in which the power is transferred to the pan. The evaluation of the stoves functionalities is currently carried out by the manufacturers with costly experimental tests with real food, which are an important phase of the design process for the improvement of their performance in the food elaboration. In order to help to design the cookers and avoid the expensive tests, it is interesting to know how the heating power is distributed in each situation, so that the cookers can be adapted to obtain a more uniform heating. The contribution of this work is an inverse thermal model for the three aforementioned technologies of domestic cookers, which allows the calculation of the power distribution generated in the bottom of the pan from the measurement of the surface temperature. The results show that the proposed inverse model is of interest in many practical situations and can be used under diverse conditions.

Keywords: Domestic cooking; Thermal model; Inverse model; Power distribution.

1. Introduction

Domestic cooking appliances have significantly evolved during the last decades, not only those based in the traditional technology of gas burners, but also the newest cooking hobs with induction heating elements. The main

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21 improvements which have been introduced are related to the efficiency, the total heating power, and the number and
22 configuration of the burners, inductors, hotplates, etc. In order to study the performance of the technologies and
23 configurations of cookers, the manufacturers carry out different tests over their products, including cooking tests
24 with real food, emulating a normal operation at the user's home. However, these tests imply spending time and
25 money which could be saved with an appropriate modeling and simulation of the cooking processes. Moreover, with
26 theoretical studies of different cooking situations (contact frying, deep-frying, boiling, etc.), a deeper understanding
27 of the elaboration of the food could be reached, and the cooking stoves could be improved with better designs and
28 functionalities.

29 Some authors in the current literature have studied the behavior of cooking stoves and pots with theoretical
30 models or experimentation. Authors in [1] experimented with different technologies of stoves (gas, electric hotplate,
31 induction) studying the performance in heating speed and temperature distribution in a pan frying process. Also in
32 this context, the thermal efficiency of pots in a particular type of electric stove was calculated and measured in [2].
33 Further authors have developed different theoretical models of some cooking techniques, e.g. a thermal model of the
34 elaboration of frozen hamburger patties was proposed in [3] and authors in [4] presented a model of the contact
35 baking of thick pancakes. These models combine heat and mass transfer phenomena and include the food state
36 variation as a result of the temperature rise. They are centered in the behavior of the food, and take into
37 consideration that the heat source is well known and controlled, and that the temperature distribution in the pan is
38 uniform. However, this assumption is far from a real cooking situation in a domestic environment, where the power
39 distribution which determines the heating uniformity depends on the type of technology, the cooker design or the
40 cooking vessel. In order to develop theoretical models closer to real scenarios and analyze the effects of the non-
41 uniform heating on the elaboration of the food, it becomes of high interest to study the power distribution in the
42 cooking vessels depending on the type of heat source.

43 In the particular case of induction cooktops, which is still a recent technology in the field of home appliances,
44 there scarcely are previous works related to thermal modeling [5]. There can be found works in which the authors
45 propose different methods to calculate the induced power distribution, generally using FEM software, which
46 implicate elevated computational cost [6–8]. The power distribution transferred to the pans using gas range stoves or
47 electric hotplates barely appears in the literature, even though they are the traditional technologies used in

48 worldwide kitchens.

49 In this work we propose an inverse model which allows the calculation of the power distribution generated in the
50 bottom of the pan from the measurement of the top surface temperature. Different models are developed considering
51 the most common heating technologies used in domestic cooktops. The inverse method has been used by other
52 authors in thermal modeling for industrial applications [9–13], and it is particularly useful for the estimation of non-
53 measurable variables, such as heat fluxes or heat transfer coefficients. In this work, for the temperature measurement
54 we use an infrared camera, and the model is solved using a numerical method based in finite differences. In order to
55 be able to use the infrared camera the cooking vessel must be empty, because the emissivity of the measured surface
56 must be well known. Moreover the power distribution is considered independent of the content of the cooking
57 vessel, thus it is not included in the proposed model.

58 2. Theory

59 The temperature reached inside a cooking vessel when it is heated in a domestic cooker is a consequence of the
60 heating source and the way in which the power is transferred to the base of the pan. For example, in induction
61 cookers this power distribution depends on the geometry of the coils, number of turns, distance between turns, etc.
62 Thus, the design of the inductor can be optimized to obtain a uniform heating of the pans.

63 As stated in our previous work [5], the temperature evolution on each point of the base of a cooking vessel,
64 placed over a heating source, obeys the heat equation:

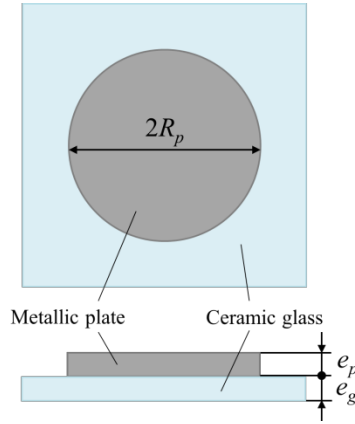
$$P + \lambda \nabla^2 T = \rho C_p \frac{\partial T}{\partial t} \quad (1)$$

65 where P is the volumetric power density generated by the heating source. The second term is the heat flux of
66 conduction, where $\nabla^2 T$ is the Laplacian of the temperature (T) and λ is the heat conductivity of the material. The
67 term on the right is the variation of the energy stored in the system with time, where the material has density ρ and
68 specific heat capacity C_p . The inverse modeling in this thermal problem consists in obtaining the value of the power
69 density distribution in each point of the domain and each instant, from the temperature distribution measured during
70 a heating process (for example with an infrared camera). The first step is to define the domain and state the
71 boundary conditions of the problem, which are different depending on the heating source employed.

72 The most common cooking vessels in the world's market have a round shape. Therefore, we propose a model for
 73 a cylindrical geometry using polar coordinates (r, ϕ, z) with origin in the center of the base of the pan. Thus, the
 74 heat equation (1) can be expressed as:

$$P = -\lambda \left(\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right) + \rho C_p \frac{\partial T}{\partial t} . \quad (2)$$

75 The inverse thermal model proposed allows the study of the power transfer with different configurations of
 76 inductors, and also for other common technologies, which are gas burners and electric stoves. It is valid for each one
 77 of the studied technologies. However, a different model with different domain and boundary conditions is
 78 considered for each case. The pan is modeled as a thin disk of homogeneous ferromagnetic material with known
 79 thermal properties and geometry, Fig. 1. The disk radius is R_p and its thickness is e_p . In the case of the induction
 80 hob and the electric stove, the steel disk is placed on a ceramic glass with thickness e_g , which also appears in the
 81 model. In the case of a gas cooker only the metallic disk is considered. Due to the complexity of the thermal
 82 problem, and the discrete nature of the input data (thermography), a numerical method based in finite differences is
 83 used to solve the inverse problem and compute the power distribution.



84 Fig. 1. Scheme of the plate which models the cooking vessel and the ceramic glass used in the induction hob and the electric stove to isolate the
 85 heating element from the user. In a gas cooker there is no glass beneath the disk.

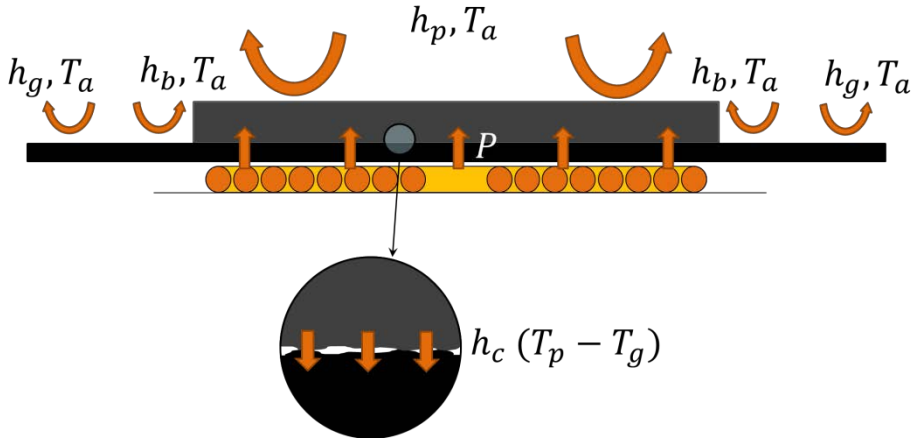
86 2.1. Induction hob

87 In an induction hob a planar coil generates a current density J inside the base of a cooking vessel, which is made

88 of ferromagnetic material. This current is dissipated because of the Joule effect and generates a volumetric power
 89 density distribution P which heats the material

$$P = \frac{J^2}{\sigma e_p} \quad (3)$$

90 where σ is the electrical conductivity of the material and e_p is the pan base thickness. There is also an additional
 91 heat generated by the hysteresis effect, which is usually negligible. In most induction hobs the pan is placed over a
 92 ceramic glass which electrically isolates the inductor from the user (Fig. 2). The heat generated in the pan is
 93 transferred by conduction to the glass through a thermal contact conductance h_c , which includes the imperfect
 94 contact between both surfaces due to the intrinsic rugosity of the materials. Losses to the environment in the top
 95 surface of the pan base, the disk border and the glass are due to convection and radiation (h_p, h_b, h_g).



96 Fig. 2. Scheme of the thermal model in an induction hob. The coil generates a magnetic field which induces an eddy current density which is
 97 dissipated in the metallic material, resulting in the power distribution P . The heat is transferred from the pan by conduction to the glass, through a
 98 contact conductance factor h_c . The temperature of the metallic plate and the glass are T_p and T_g . The losses to the environment at temperature T_a ,
 99 from the pan and the glass, are modeled with heat transfer coefficients h_p, h_g , which include convection and radiation. The losses in the
 100 perimeter of the pan base are modeled with a different heat transfer coefficient h_b , which can also model the effect of the walls of the pan.

101 As aforementioned, in the proposed model the pan base is modeled as a disk with radius R_p and thickness e_p of
 102 homogeneous material with known conductivity, density and heat capacity ($\lambda_p, \rho_p, C_{p_p}$). A cylindrical area of the

103 glass with radius R_g and thickness e_g , being $R_g \geq R_p$, is also considered, with known properties $(\lambda_g, \rho_g, C_{p_g})$. The
 104 boundary conditions imposed in this model are Neumann boundary conditions [14]. In the top surface of the plate it
 105 is considered that the heat flux in z direction is equal to the losses to the environment due to convection and
 106 radiation:

$$-\lambda_p \frac{\partial T_p}{\partial z} = h_p (T_p - T_a) \Big|_{z=e_p, 0 \leq r \leq R_p} \quad (4)$$

$$h_p = h_{p_{conv}} + h_{p_{rad}} = h_{p_{conv}} + \sigma \varepsilon (T_p^2 + T_a^2)(T_p + T_a) \quad (5)$$

107 where T_p is the plate temperature, T_a is the ambient temperature, $h_{p_{conv}}$ is the convection coefficient on the plate, σ
 108 is the Stefan Boltzmann constant and $\varepsilon = 1$ is the emissivity considering that the radiant surface behaves as a black
 109 body and that the surrounding surfaces are at the ambient temperature. A similar condition is set on the top surface
 110 of the glass, in the outer area which is not in contact with the disk:

$$-\lambda_g \frac{\partial T_g}{\partial z} = h_g (T_g - T_a) \Big|_{z=0, R_p \leq r \leq R_g} \quad (6)$$

$$h_g = h_{g_{conv}} + h_{g_{rad}} = h_{g_{conv}} + \sigma \varepsilon (T_g^2 + T_a^2)(T_g + T_a) \quad (7)$$

111 where T_g is the glass temperature and $h_{g_{conv}}$ is the convection coefficient on the glass. For simplicity, in this work
 112 we consider that the convection coefficients are constant with the temperature, as the range of temperature in a pan
 113 in a domestic cooker is relatively small. In the bottom surface of the plate the vertical heat flux depends on the
 114 contact conductance h_c and the temperature of the glass, and it is identical to the heat flux in the top surface of the
 115 glass:

$$-\lambda_p \frac{\partial T_p}{\partial z} = -\lambda_g \frac{\partial T_g}{\partial z} = h_c (T_g - T_p) \Big|_{z=0} \quad (8)$$

116 The vertical heat flux in the bottom surface of the glass is considered negligible for simplicity

$$-\lambda_g \frac{\partial T_g}{\partial z} = 0 \Big|_{z=-e_g} . \quad (9)$$

117 In the radial direction, it is considered that the heat flux in the center is null, in both the plate and the glass, due to
 118 the symmetry of the problem. In the border of the plate a heat losses coefficient h_b is defined, which includes
 119 convection and radiation:

$$-\lambda_p \frac{\partial T_p}{\partial r} = -\lambda_g \frac{\partial T_g}{\partial r} = 0 \Big|_{r=0} \quad (10)$$

$$-\lambda_p \frac{\partial T_p}{\partial r} = h_b (T_p - T_a) \Big|_{r=R_p} . \quad (11)$$

120 Finally, a Dirichlet boundary condition is considered in the border of the domain which models the glass

$$T_g = T_a \Big|_{r=R_g} . \quad (12)$$

121 In order to solve the model a finite differences method is proposed [15]. The disk and the cylindrical volume of
 122 glass are discretized with the two-dimensional polar grid shown in Fig. 3. The position of each element is defined
 123 with R_i, ϕ_j , and the size of each grid element is $\Delta R, \Delta \phi$. The domain is not discretized in z direction because the
 124 heat conduction inside the plate and the glass is considered unimportant in this dimension due to the small thickness.
 125 The size of the time step selected in the discretization of the temporal dimension is Δt . The proposed method is an
 126 implicit finite differences method, in which the second derivatives of the temperature are approximated with a
 127 central differences scheme, and the first derivatives are approximated with a regressive differences scheme. Thus,
 128 the stability and convergence of the method are ensured. The power distribution in each element $P_{i,j}$ for each instant
 129 is related with the disk temperature measured in each element $T_{p_{i,j}}$ and the glass temperature $T_{g_{i,j}}$ with the
 130 expressions:

$$(1 \leq i \leq n, \quad 1 \leq j \leq M)$$

$$P_{i,j} = a_{p_i} T_{p_{i,j}} + b_{p_i}^+ T_{p_{i+1,j}} + b_{p_i}^- T_{p_{i-1,j}} + c_{p_i} T_{p_{i,j+1}} + c_{p_i} T_{p_{i,j-1}} - \frac{\rho_p C_{p_p}}{\Delta t} T_{p_{i,j}}^\tau - a_{p_i}^* T_a - d_{p_i} T_{g_{i,j}} \quad (13)$$

$$(1 \leq i \leq N, 1 \leq j \leq M)$$

$$0 = a_{g_i} T_{g_{i,j}} + b_{g_i}^+ T_{g_{i+1,j}} + b_{g_i}^- T_{g_{i-1,j}} + c_{g_i} T_{g_{i,j+1}} + c_{g_i} T_{g_{i,j-1}} - \frac{\rho_g C_{p_g}}{\Delta t} T_{g_{i,j}}^\tau - a_{g_i}^* T_a - d_{g_i} T_{p_{i,j}} \quad (14)$$

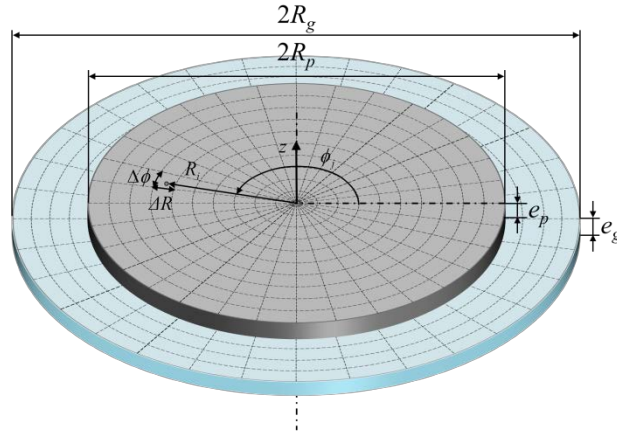


Fig. 3. Discretization grid used in the finite differences method. The grid is based in a polar coordinate system, in which the position of the element i, j is given by R_i and ϕ_j , and ΔR , $\Delta \phi$ are the size of the grid in radial and azimuthal directions.

where N, M are the number of nodes in the grid in radial and azimuthal directions, n is the index of the last element of the disk in radial direction, $T_{p_{i,j}}^\tau$ and $T_{g_{i,j}}^\tau$ are the temperatures of elements i, j of the disk and the glass in the previous instant. The coefficients $a_p, a_p^*, b_p, c_p, d_p$ are different in the boundaries and inside the grid, and are presented in Table 1. The coefficients $a_g, a_g^*, b_g, c_g, d_g$ are presented as well in Table 2.

The effect of the radiation increases with the temperature of the material, as seen in (5) and (7). Thus, the value of the losses factor in the disk and the glass is recalculated on each iteration, using the temperature values from the previous instant:

$$h_p = h_{p_{conv}} + h_{p_{rad}} = h_{p_{conv}} + \sigma \varepsilon (T_p^{\tau 2} + T_a^2) (T_p^\tau + T_a) \quad (15)$$

$$h_g = h_{g_{conv}} + h_{g_{rad}} = h_{g_{conv}} + \sigma \varepsilon (T_g^{\tau 2} + T_a^2)(T_g^{\tau} + T_a) . \quad (16)$$

140 The convection coefficients and the contact conductance between the disk and the glass are obtained from
 141 experimental results with an induction hob in stationary regime.

142 Table 1. Coefficients in the equation of the thermal model for the disk, in the case of induction, electric and gas cookers.

Node index	a_{p_i}	$a_{p_i}^*$	$b_{p_i}^+$	$b_{p_i}^-$	c_{p_i}	d_{p_i}
$i = 1$	$\frac{\lambda_p}{\Delta R^2} + \frac{\lambda_p}{R_1 \Delta R} + \frac{2\lambda_p}{R_1^2 \Delta \phi^2} + \frac{\rho_p C_{p_p}}{\Delta t} + \frac{h_p}{e_p} + \frac{h_c}{e_p}$	$\frac{h_p}{e_p}$	$-\left(\frac{\lambda_p}{\Delta R^2} + \frac{\lambda_p}{R_1 \Delta R}\right)$	0	$-\left(\frac{\lambda_p}{R_1^2 \Delta \phi^2}\right)$	$\frac{h_c}{e_p}$
$1 < i < n$	$\frac{2\lambda_p}{\Delta R^2} + \frac{\lambda_p}{R_i \Delta R} + \frac{2\lambda_p}{R_i^2 \Delta \phi^2} + \frac{\rho_p C_{p_p}}{\Delta t} + \frac{h_p}{e_p} + \frac{h_c}{e_p}$	$\frac{h_p}{e_p}$	$-\left(\frac{\lambda_p}{\Delta R^2} + \frac{\lambda_p}{R_i \Delta R}\right)$	$-\left(\frac{\lambda_p}{\Delta R^2}\right)$	$-\left(\frac{\lambda_p}{R_i^2 \Delta \phi^2}\right)$	$\frac{h_c}{e_p}$
$i = n$	$\frac{\lambda_p}{\Delta R^2} + \frac{2\lambda_p}{R_n^2 \Delta \phi^2} + \frac{\rho_p C_{p_p}}{\Delta t} + \frac{h_p}{e_p} + \frac{h_c}{e_p} + \frac{h_b}{\Delta R} + \frac{h_b}{R_n}$	$\frac{h_p}{e_p} + \frac{h_b}{\Delta R} + \frac{h_b}{R_n}$	0	$-\left(\frac{\lambda_p}{\Delta R^2}\right)$	$-\left(\frac{\lambda_p}{R_n^2 \Delta \phi^2}\right)$	$\frac{h_c}{e_p}$

143 Table 2. Coefficients in the equation of the thermal model for the glass, in the case of induction and electric cookers.

Node index	a_{g_i}	$a_{g_i}^*$	$b_{g_i}^+$	$b_{g_i}^-$	c_{g_i}	d_{g_i}
$i = 1$	$\frac{\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_1 \Delta R} + \frac{2\lambda_g}{R_1^2 \Delta \phi^2} + \frac{\rho_g C_{p_g}}{\Delta t} + \frac{h_c}{e_g}$	0	$-\left(\frac{\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_1 \Delta R}\right)$	0	$-\left(\frac{\lambda_g}{R_1^2 \Delta \phi^2}\right)$	$\frac{h_c}{e_g}$
$1 < i \leq n$	$\frac{2\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_i \Delta R} + \frac{2\lambda_g}{R_i^2 \Delta \phi^2} + \frac{\rho_g C_{p_g}}{\Delta t} + \frac{h_c}{e_g}$	0	$-\left(\frac{\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_i \Delta R}\right)$	$-\left(\frac{\lambda_g}{\Delta R^2}\right)$	$-\left(\frac{\lambda_g}{R_i^2 \Delta \phi^2}\right)$	$\frac{h_c}{e_g}$
$n < i < N$	$\frac{2\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_i \Delta R} + \frac{2\lambda_g}{R_i^2 \Delta \phi^2} + \frac{\rho_g C_{p_g}}{\Delta t} + \frac{h_g}{e_g}$	$\frac{h_g}{e_g}$	$-\left(\frac{\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_i \Delta R}\right)$	$-\left(\frac{\lambda_g}{\Delta R^2}\right)$	$-\left(\frac{\lambda_g}{R_i^2 \Delta \phi^2}\right)$	0
$i = N$	$\frac{2\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_N \Delta R} + \frac{2\lambda_g}{R_N^2 \Delta \phi^2} + \frac{\rho_g C_{p_g}}{\Delta t} + \frac{h_g}{e_g}$	$\frac{\lambda_g}{\Delta R^2} + \frac{\lambda_g}{R_N \Delta R} + \frac{h_g}{e_g}$	0	$-\left(\frac{\lambda_g}{\Delta R^2}\right)$	$-\left(\frac{\lambda_g}{R_N^2 \Delta \phi^2}\right)$	0

144 2.2. Electric stove

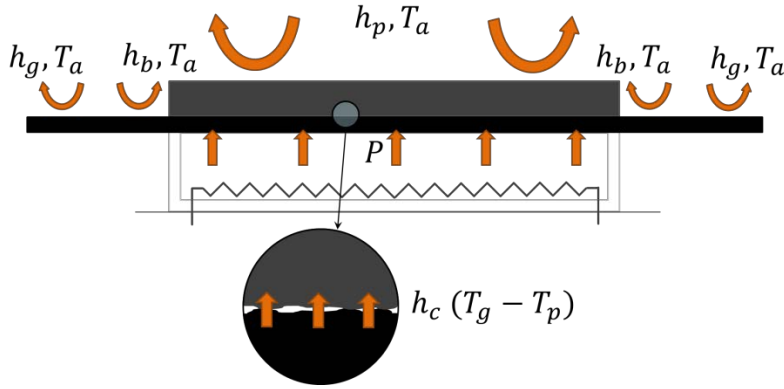
145 In the electric stove (Fig. 4) the power is transferred by radiation from a hot electric resistance to a ceramic glass,
 146 which is heated up to temperatures around 400°C, and transfers the heat to the pan placed on the glass by
 147 conduction. The power density distribution depends on the temperature of the resistance T_r , with:

$$P = \frac{\sigma \varepsilon}{e_g} (T_r^4 - T_g^4) . \quad (17)$$

148 In contrast with the direct heat generation in the pan which occurs in an induction hob, the heating source in the
 149 electric stove generates a power distribution in the glass. Hence, the heat flux in vertical direction flows in this case
 150 from the glass to the pan. This affects to the boundary condition (9) of the previous model, which is reformulated as:

$$-\lambda_g \frac{\partial T_g}{\partial z} \Big|_{z=-e_g} = \sigma \varepsilon (T_r^4 - T_g^4) = P e_g . \quad (18)$$

151 However, the rest of the boundary conditions are the same as those considered in the model of the induction cooker.



152 Fig. 4. Scheme of the thermal model in an electric stove. The resistor beneath the glass dissipates electric energy increasing its temperature and
 153 radiates heat to the surface of the glass, generating a power distribution P . Heat is transferred to the pan by conduction from the glass, through a
 154 contact conductance factor h_c . The losses to the environment from the pan and the glass are modeled with heat transfer coefficients h_p, h_g , which
 155 include convection and radiation. The losses in the perimeter of the pan base are modeled with a different heat transfer coefficient h_b .

156 Using the same numerical method with the discretization grid shown in Fig. 3 the power density distribution $P_{i,j}$
 157 can be obtained with the expressions

$$(1 \leq i \leq n, \quad 1 \leq j \leq M)$$

$$0 = a_{p_i} T_{p_{i,j}} + b_{p_i}^+ T_{p_{i+1,j}} + b_{p_i}^- T_{p_{i-1,j}} + c_{p_i} T_{p_{i,j+1}} + c_{p_i} T_{p_{i,j-1}} - \frac{\rho_p C_{pp}}{\Delta t} T_{p_{i,j}}^\tau - a_{p_i}^* T_a - d_{p_i} T_{g_{i,j}} \quad (19)$$

$$(1 \leq i \leq n, \quad 1 \leq j \leq M)$$

$$P_{i,j} = a_{g_i} T_{g_{i,j}} + b_{g_i}^+ T_{g_{i+1,j}} + b_{g_i}^- T_{g_{i-1,j}} + c_{g_i} T_{g_{i,j+1}} + c_{g_i} T_{g_{i,j-1}} - \frac{\rho_g C_{p,g}}{\Delta t} T_{g_{i,j}}^r - a_{g_i}^* T_a - d_{g_i} T_{p_{i,j}} \quad (20)$$

$$(n < i \leq N, 1 \leq j \leq M)$$

$$P_{i,j} = 0 \quad (21)$$

158 where it is considered that the disk diameter is equal or larger than the electric resistance diameter. The values of the
159 coefficients are the same as in the induction cooker model, which were presented in Table 1 and Table 2.

160 2.3. Gas burner

161 In the gas cooker (Fig. 5) there is a heat exchange between the gas produced in the combustion process and the
162 bottom surface of the pan. The relationship between the configuration of the burner, the flame and the power
163 distribution generated, can be studied with a computational fluid dynamics (CFD) analysis of the problem similar to
164 that carried out in [16], where the heat transfer mechanism is modeled as convection. If we considered a heat
165 transfer coefficient h_{gas} , it would be related to the power density distribution which heats the pan with

$$P = \frac{h_{gas}}{e_p} (T_f - T_p) \quad (22)$$

166 where T_f is the flame temperature.

167 The thermal model in this case is simpler, since only the base of the pan is considered and there is no glass
168 between the plate and the heating source. The boundary conditions in radial direction and in the top surface of the
169 plate are the same, and in the bottom surface the heat flux is given by the heat transmission from the flame:

$$-\lambda_p \frac{\partial T_p}{\partial z} \Big|_{z=0} = h_{gas} (T_f - T_p) = P e_p. \quad (23)$$

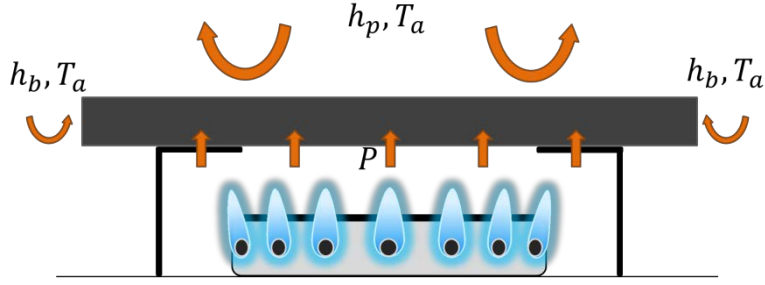


Fig. 5. Scheme of the thermal model in a gas cooker. The combustion gas heats the plate by convection, generating a power distribution P in the base of the pan. The losses to the environment are modeled with a heat transfer coefficient h_p which includes convection and radiation, and the losses in the disk border are modeled with a different heat transfer coefficient h_b .

Again, a numerical calculation method based in finite differences is used. In this case the domain considered only includes the disk. The power distribution in each element $P_{i,j}$ for each instant is obtained with the expression:

$$(1 \leq i \leq n, 1 \leq j \leq M)$$

$$P_{i,j} = a_{p_i} T_{p_{i,j}} + b_{p_i}^+ T_{p_{i+1,j}} + b_{p_i}^- T_{p_{i-1,j}} + c_{p_i} T_{p_{i,j+1}} + c_{p_i} T_{p_{i,j-1}} - \frac{\rho_p C_{p_p}}{\Delta t} T_{p_{i,j}}^\tau - a_{p_i}^* T_a \quad (24)$$

where the coefficients a, a^*, b, c are the same as presented in Table 1, considering $h_c = 0$.

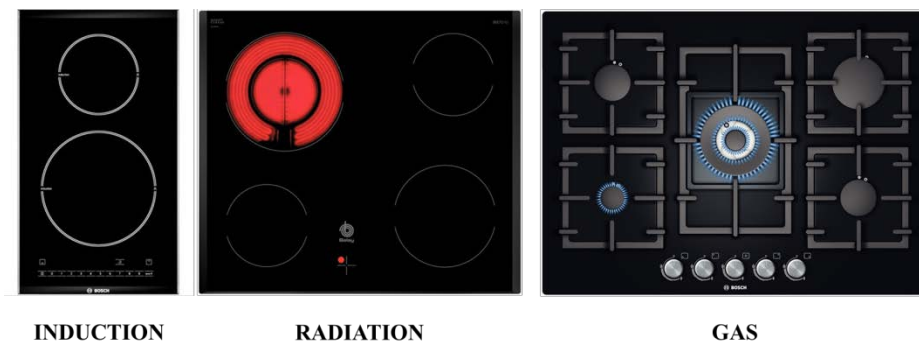
3. Materials and methods

The model presented in the previous section for an induction hob, an electric stove and a gas cooker requires knowing the temperature on the pan base during a heating process. In order to use the model for the study of real situations, some experiments with real stoves have been carried out, heating ferromagnetic steel disks and measuring the temperatures with an infrared camera.

3.1. Experimental setup

Three conventional cooktops are used to perform the experiments: an induction hob, an electric stove, and a gas cooker, Fig. 6. Different C45 ferromagnetic steel disks of 0.23 m of diameter and thickness from 3e-3 to 5e-3 m, emulating the base of conventional pans, are heated with the three cooktops. The thermal properties of the steel and of the ceramic glass which is present in the induction hob and the electric stove are shown in Table 3. The

186 measurement of the disk temperature has been carried out with an infrared camera FLIR A650 (Fig. 7), which
 187 records the temperature evolution at each point with resolution of 640x480 pixels and accuracy of ± 2 °C.



188 Fig. 6. Cooktops employed in the experiments, one of each type of heating source: induction hob (PIE375N14E), electric stove (3EE721LS) and
 189 gas stove (PPQ716B91E).



190 Fig. 7. Infrared camera FLIR A655sc used to measure the temperature distribution.

191 Table 3. Thermal properties of C45 ferromagnetic steel and of the conventional ceramic glass.

Property	Value	Units	Property	Value	Units
λ_p	46	$\frac{W}{m \cdot K}$	λ_g	2.6	$\frac{W}{m \cdot K}$
ρ_p	7850	$\frac{kg}{m^3}$	ρ_g	2600	$\frac{kg}{m^3}$
C_{p_p}	500	$\frac{J}{kg \cdot K}$	C_{p_g}	800	$\frac{J}{kg \cdot K}$

3.2. Thermography and temperature filtering

The thermographies recorded with the infrared camera in the experiments with the induction hob, the electric stove and the gas cooker, contain on each pixel the mean temperature measured in the corresponding pixel area. This information must be processed before computing the inverse model with two operations: a coordinate transformation and a noise-reduction filtering.

The original image gives the temperature data for each point in a rectangular grid, which must be converted to a polar grid in order to fit the proposed model. For this purpose a coordinate change is performed, using a cubic interpolation algorithm in Matlab. In the resultant image the data in the first row is the temperature in the center of the disk and the last row contains the temperatures in the circumference of the disk border. Thus the radial direction is given along the rows, and the azimuthal direction along the columns (Fig. 8).

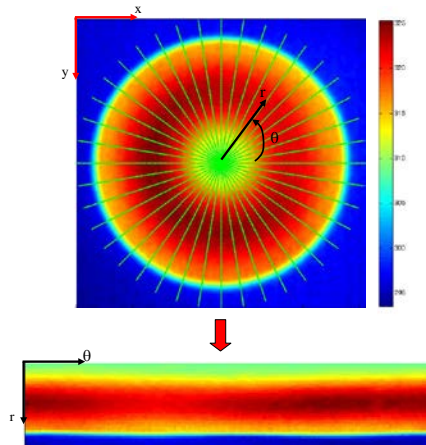
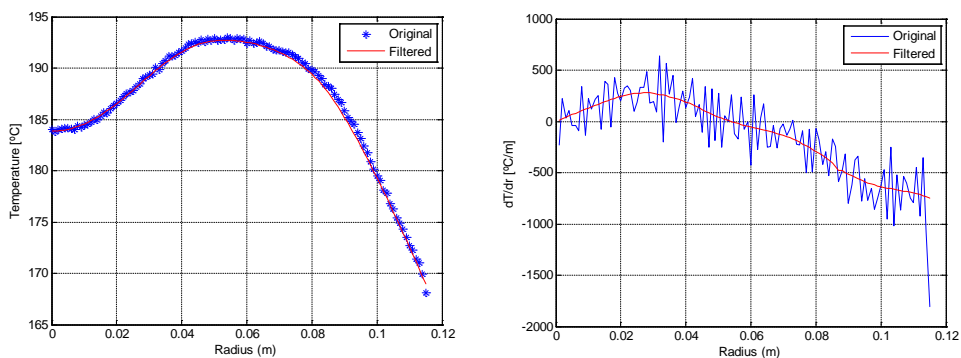


Fig. 8. Coordinates transformation of the thermographies recorded with the infrared camera.

The infrared camera provides the temperature measurement with a certain noise level, which is produced by the non-uniformity of the emissivity of the disk surface and the own signal noise of the infrared sensor. Moreover the discrete nature of the data in the images produces discontinuities in the derivatives after the coordinate transformation. Hence, in order to improve the calculation of temperature derivatives, the images must be filtered in a previous stage. In this work we propose to use a smooth filter proposed by Savitzky & Golay, [17], which is one of the most commonly used filters in image smoothing due to the improved results which can be obtained with respect to other filters such as mean or median filters. An example of the behavior of this filter is shown in Fig. 9, where

210 both the original and filtered temperature profiles along the disk radius are shown. The first derivative shows the
211 high noise suppression obtained.



212 Fig. 9. Smooth filtering with the Savitzky-Golay method. Radial profile and first derivative of the original and filtered data.

213 3.3. Parameter identification

214 Although the model takes into account the thermal properties of the materials, which are known from the
215 literature, there have been also considered other parameters such as convection coefficients or the contact
216 conductance, which are specific of each problem. In order to identify these parameters, an experimental procedure is
217 proposed in this section.

218 In order to simplify the identification of the parameters, it is only used the induction hob because the
219 measurement and control of the input power can be obtained with higher accuracy. A power analyzer (Yokogawa
220 PZ4000) is connected in the output of the power stage which feeds the inductor and the real heating power
221 dissipated in the disk is estimated as the 97% of the power measured, which is the typical efficiency of the inductor
222 [18]. Two different configurations are used in the experiments. First, the glass of $4\text{e-}3$ m is replaced with an
223 isolation blanket of the same thickness, which simplifies the model and allows considering null heat conduction
224 through the bottom surface of the disk, i.e. the contact conductance is negligible, $h_c = 0$. With the temperatures
225 measured in these experiments the losses coefficients h_p and h_b can be easily identified. Once the values of these
226 parameters are known, from different experiments under the same conditions using the glass instead of the blanket,
227 the contact conductance is obtained.

228 The C45 steel disks are heated with a low power level, such that the stationary regime is reached at a mean
 229 temperature around 250 °C. Thus, the parameters are calculated by means of the energy balance in the stationary
 230 regime, obtained from the measurement of the temperature and the input power. The first balance is obtained by
 231 integrating the heat equation in the whole disk, in which both the losses through the top surface and the disk border
 232 are taken into account. The resultant expression is

$$P_T - \iint_{0,0}^{R_p,2\pi} h_p(T_p - T_a) r dr d\phi - \iint_{0,0}^{e_p,2\pi} h_b(T_{p_b} - T_a) R_p d\phi dz = 0 \quad (25)$$

233 where P_T is the total mean power measured in the coil with the power analyzer, and T_{p_b} is the temperature in the
 234 border of the disk. The integral terms are numerically obtained using the same polar grid proposed for solving the
 235 theoretical model:

$$\iint_{0,0}^{R_p,2\pi} h_p(T_p - T_a) r dr d\phi = \sum_{i,j}^{n,M} \left(h_{p_{conv}}(T_{p_{i,j}} - T_a) + \sigma \varepsilon (T_{p_{i,j}}^4 - T_a^4) \right) \frac{(R_i^2 - R_{i-1}^2) \Delta \phi}{2} \quad (26)$$

$$\iint_{0,0}^{e_p,2\pi} h_b(T_{p_b} - T_a) R_p d\phi dz = \sum_j^M \left(h_b(T_{p_{n,j}} - T_a) \right) e_p R_p \Delta \phi \quad (27)$$

236 The second balance is stated in the disk border, where the boundary condition (11) is employed:

$$\iint_{0,0}^{e_p,2\pi} -\lambda_p \frac{\partial T_{p_b}}{\partial r} R_p d\phi dz = \iint_{0,0}^{e_p,2\pi} h_b(T_{p_b} - T_a) R_p d\phi dz \quad (28)$$

237 The solution of the system with two equations and two unknowns gives the value of h_p and h_b in the considered
 238 experiment. In order to obtain a better adjustment, the mean values from several experiments with different input
 239 powers have been calculated.

240 The losses coefficient in the glass surface h_g is obtained experimentally. In this case a round pan with water is
 241 placed on the induction hob with the glass beneath, and it is heated up to reach a uniform boiling. Then the induction
 242 cooker is switched off and the pan is removed, letting the infrared camera measure the glass temperature. The glass
 243 is cooled by effect of the losses coefficient, which can be obtained with an energy balance

$$\int_{t_1}^{t_2} \left(\iint_{0,0}^{R_p,2\pi} h_g(T_g - T_a) r dr d\phi + \iint_{0,0}^{e_g,2\pi} -\lambda_g \frac{\partial T_{gb}}{\partial r} R_p d\phi dz \right) dt = \int_{t_1}^{t_2} \rho_g C_{pg} \frac{\partial T_g}{\partial t} dt \quad (29)$$

where $[t_1, t_2]$ is the considered time interval.

Once the losses parameters are identified, the contact heat transfer h_c can be experimentally identified. Using the induction cooktop with a conventional glass, a disk is heated and its temperature recorded with the camera. From a power balance in the disk with the temperature evolution measured, the power transferred from the disk to the glass P_c can be calculated:

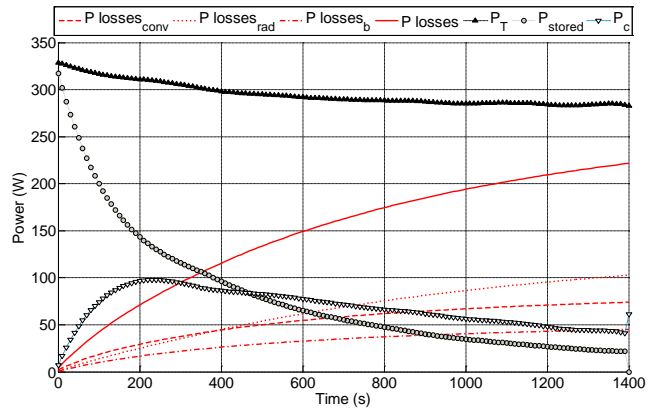
$$P_c = P_T - P_{losses} - P_{stored}$$

$$P_c = \iint_{0,0}^{R_p,2\pi} h_c(T_p - T_g) r dr d\phi$$

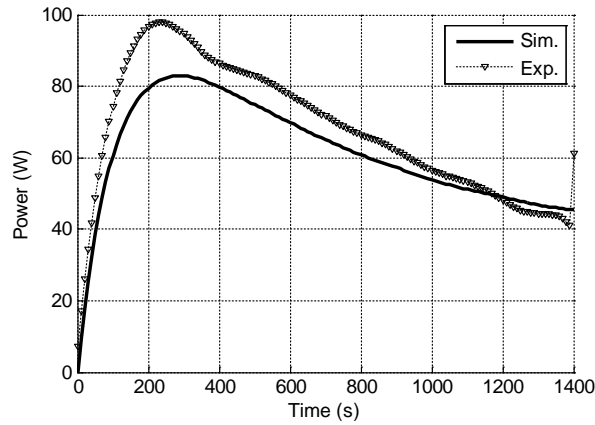
$$P_{losses} = \iint_{0,0}^{R_p,2\pi} h_p(T_p - T_a) r dr d\phi + \iint_{0,0}^{e_p,2\pi} h_b(T_{pb} - T_a) R_p d\phi dz + \iint_{R_p,0}^{R_g,2\pi} h_g(T_g - T_a) r dr d\phi \quad (30)$$

$$P_{stored} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \rho_p C_{pp} \frac{\partial T_p}{\partial t} dt .$$

An example of this calculation is shown in Fig. 10, where each contribution to the power balance is plotted separately. The heated disk has a diameter of 0.23 m and a thickness of 4e-3 m. The coil has a diameter of 0.21 m and the ceramic glass has a thickness of 4e-3 m. The values estimated for the losses coefficients in this case are $h_{p_{conv}} = 9.5 \frac{W}{m^2K}$ and $h_b = 75 \frac{W}{m^2K}$. In order to obtain the value of the contact conductance the temperature of the glass must be obtained, which at the same time depends on the contact conductance. Thus, an iterative algorithm is used to simulate the temperature of the glass, based on the equations of the theoretical model proposed, for different values of h_c . Then the power transferred from the disk to the glass is calculated from the simulations and compared with the obtained through (30). We select a value of the coefficient with which a best fitting is obtained. For the considered case the calculated value is $h_c = 50 \frac{W}{m^2K}$. The fitting between the curves of power transferred from disk to glass is presented in Fig. 11.



259 Fig. 10. Power balance obtained from the temperature measurement when heating a disk of diameter 0.23 m and thickness $4\text{e-}3$ m with a 0.21 m
 260 coil in an induction cooktop.



261 Fig. 11. Power transferred from disk to glass due to the contact conductance with the time. The experimental and simulated curves are shown.

262 **4. Results and discussion**

263 In this section, the power density distribution in different heating situations with the three technologies
 264 considered in this work is calculated with the proposed inverse model. The values of the parameters in the model are
 265 identified using the previous method with long duration experiments in which a stationary heating is reached.
 266 Nevertheless, in order to improve the quality of the results obtained with the inverse model, the power density is
 267 obtained from new experiments using higher power levels, considering the transient regime of the heating process.

268 The larger inductor of the induction hob with two cooking zones is used for heating a disk of 0.23 m of diameter

269 and 3e-3 m of thickness. The coil diameter is 0.21 m, with a non-equally spaced distribution of turns, as seen in Fig.
 270 12. The power stage which feeds the inductor is a half-bridge resonant inverter, and it is activated with constant
 271 frequency and duty cycle during the heating process, which ends when the maximum temperature reached in the
 272 disk is 200 °C. The temperature of the disk is measured with the infrared camera and the power density distribution
 273 is calculated for each instant with the proposed inverse model, using the parameters shown in Table 4. An example
 274 of the result is presented in Fig. 13, which shows a three-dimensional representation of the power distribution
 275 generated in the disk after 20 s, normalized to a total power of 1W. We also present the temperature map in color
 276 scale at the same instant. Fig. 14 shows the evolution of the mean power calculated by integration of the power
 277 density distribution, and the measured power in the output of the power stage. The result obtained from the model
 278 shows a good agreement with the experimental data.

279 Table 4. Parameter values identified for a disk of 0.23 m of diameter and 0.003 m of thickness, placed on a ceramic glass.

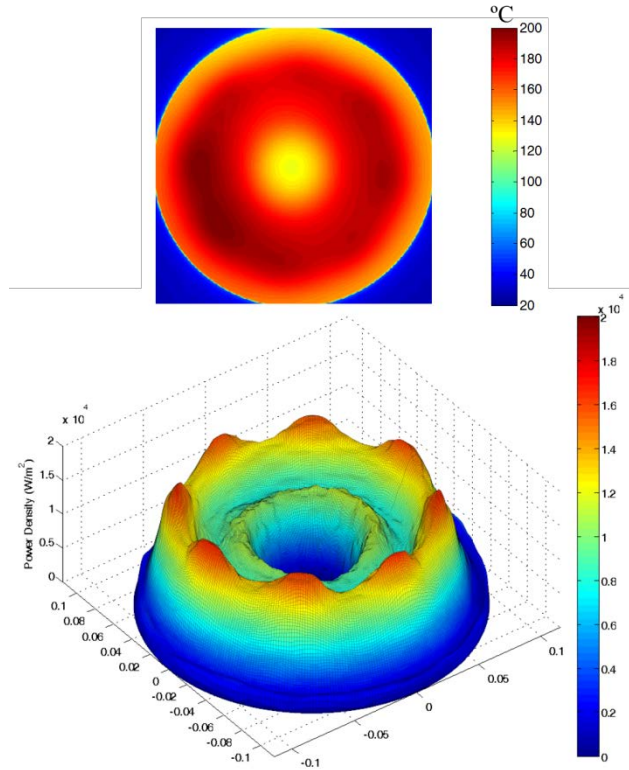
Parameter	Value	Units
$h_{p_{conv}}$	9.5	$\frac{W}{m^2 \cdot K}$
h_b	75	$\frac{W}{m^2 \cdot K}$
$h_{g_{conv}}$	15	$\frac{W}{m^2 \cdot K}$
h_c	50	$\frac{W}{m^2 \cdot K}$



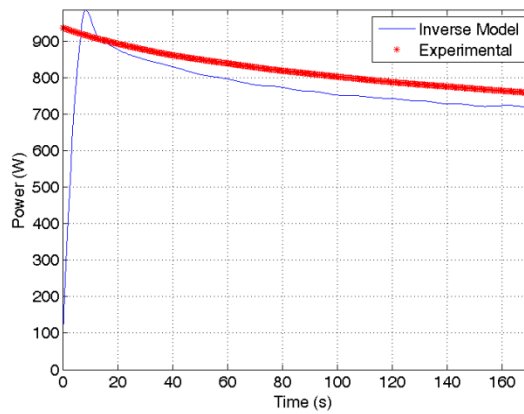
280 Fig. 12. Planar coil in the conventional induction hob with 0.21 m of diameter and optimized geometry for a uniform heating.

281 The same disk is heated with the double cooking zone of the electric stove, which consists of two resistances that
282 can be activated independently, Fig. 15. The diameter of the inner resistance is 0.12 m and the diameter of the outer
283 resistance is 0.21 m. The maximum power level of the cooking zone is selected, such that both heating elements are
284 fed constantly from mains with a mean consumption of 1990 W, measured in the input of the stove. The temperature
285 of the disk is measured with the infrared camera and the inverse model is calculated using the same values of the
286 parameters from the previous case. The temperature map in the instant in which the maximum temperature reaches
287 200 °C and the 3D representation of the normalized power distribution transmitted from the glass to the disk through
288 the contact conductance are shown in Fig. 16. This power distribution is comparable with the power distribution
289 generated inside the disk in the case of the induction hob, because it is the one which determines the temperature
290 distribution in the pan. Nevertheless the power distribution which heats the glass and the glass temperature can also
291 be obtained with the model. Integrating the power distribution in the disk volume for each instant, the mean power
292 transferred to the disk along the time is obtained, Fig. 17. The difference between this curve and the power measured
293 in the input of the electric hob is the energy employed to heat up the resistance and the glass, which involves
294 spending more than the 60% of the supplied energy.

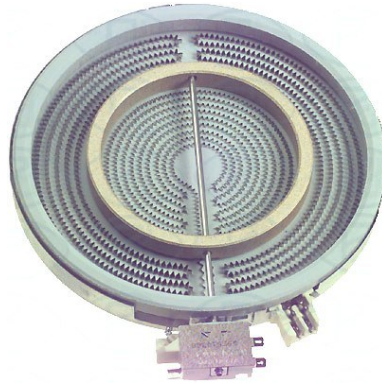
295 Finally, a similar experiment is carried out with the gas stove, burning butane gas in the medium-sized burner
296 with a diameter of 0.12 m. The maximum power supplied by this burner is 3000 W, according to the manufacturer,
297 and the gas faucet is opened approximately at the half of the maximum. The temperature of the disk is monitored
298 and the power distribution transferred through the bottom surface of the plate is calculated with the inverse model.
299 The obtained results are presented in Fig. 18. Again, the mean power transferred to the disk for each instant is
300 calculated by integrating the power density distribution in the disk volume, Fig. 19.



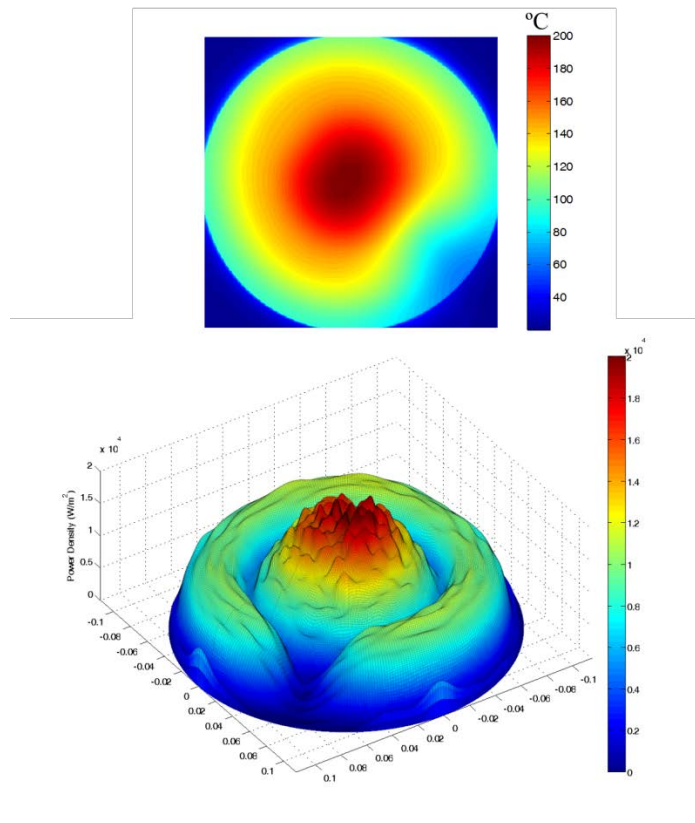
301 Fig. 13. Temperature map measured with the infrared camera and 3D representation of the normalized power density distribution obtained with
 302 the inverse model for the inductor of diameter 0.21 m after 20 s of heating.



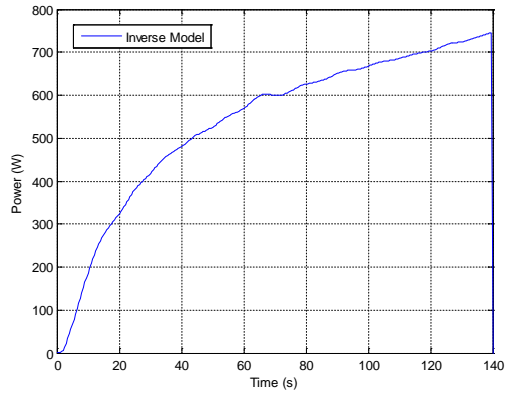
303 Fig. 14. Evolution of the mean power which heats the disk in the induction hob. The measured power and the calculated with the inverse model
 304 are in good agreement.



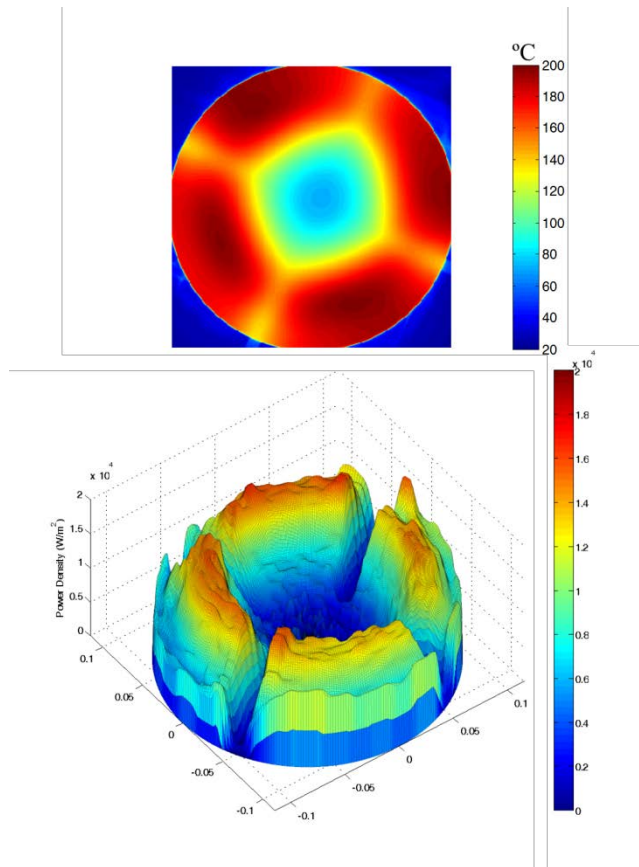
305 Fig. 15. Double resistance in the electric stove. The inner resistance has a diameter of 0.12 m and the outer resistance a diameter of 0.21 m.



306 Fig. 16. Temperature map measured with the infrared camera after 125 s of heating and 3D representation of the normalized power density
307 distribution transferred from the glass to the disk calculated with the inverse model for the double cooking zone of the electric stove.



308 Fig. 17. Evolution of the mean power which heats the disk, transferred through the bottom surface in the electric stove.



309 Fig. 18. Temperature map measured with the infrared camera after 72 seconds of heating and 3D representation of the normalized power density
 310 distribution, calculated with the inverse model, transferred to the base of the plate from the burnt gas.

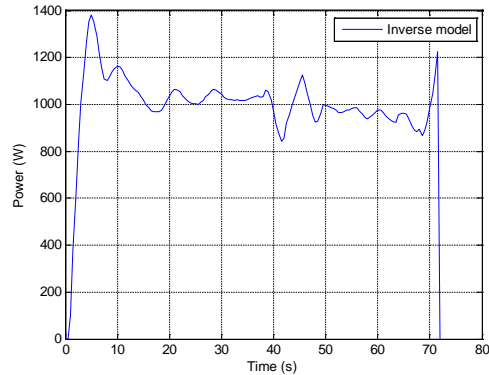


Fig. 19. Evolution of the mean power which heats the disk using the medium-size burner in the gas stove.

From the obtained results, the heating performance of the three considered technologies can be observed and compared. There are some differences in the power distributions which heat the disk, which are a consequence of the type of heating source.

In the case of the induction hob, the power distribution is null in the center of the pan and has two maximums in the radial direction, which are produced by the particular distribution of wire turns of the coil. It is also observed the effect of the 8 ferrite bars beneath the coil, which produce an amplification effect of the magnetic field over its position, increasing the efficiency of the coil. The evolution of the mean power transferred to the disk shows the effect of the temperature in the electrical conductivity of the material, which decreases as the material is heated, thus the induced current decreases and the dissipated power is lower. Nevertheless, in a conventional induction hob the power control implemented in the induction electronics automatically corrects the modulation parameters in order to obtain a constant power supply.

The power distribution transferred to the disk in the double cooking zone of the electric stove shows a peak in the central area of the plate, which is originated by the higher power density radiated by the internal resistance. It can be also observed the effect of the ring which isolates both resistances, producing a low-power area with circular shape. The mean power transferred from the glass to the disk is much lower than the power measured in the input of the stove. This reflects the behavior of this technology in a transient heating, in which the resistance and the glass take the major part of the energy to be heated up to the working temperature. As a result, the efficiency of the stove during the first minutes of a preheating process is low, seen from the point of view of the pan.

330 The power distribution generated in the gas cooker shows a maximum in the area in which the temperature of the
331 flame is higher, and a large central zone in which the burnt gas has a lower incidence. It can also be observed the
332 effect of the metallic structure which supports the disk above the burner, which isolates the plate surface from the
333 flame and generates gaps in the power distribution. The evolution of the mean power shows in this case that a high
334 power is transferred from the beginning of the heating process, and there is a fluctuation of the power transfer due to
335 the instability of the flame temperature and the gas flow.

336 **5. Conclusions**

337 In this work the thermal modeling in the most common cooking stoves, with different heating technologies, has
338 been presented. The inverse model developed in this paper allows the calculation of the power distribution heating
339 the base of the pan, from the measurement of the temperature distribution with an infrared camera.

340 The proposed model provides good results with the three considered technologies (induction, electric resistance
341 and gas), and can be used to establish comparisons between the different power distributions and achieve a deeper
342 understanding of the different behavior of the pan temperature during a cooking process in each situation. In the
343 particular cookers studied in this work, the induction hob provides a more uniform power distribution and efficiency
344 when using a cooking vessel of a similar size than the inductor. However, the gas cooker is suitable for a wider
345 range of pan sizes, because the power is more concentrated in the outer area of the pan base. The electric stove
346 showed the worst performance in the heating uniformity, which could be improved by increasing the power supplied
347 by the outer resistance in the double cooker.

348 The proposed inverse model is an alternative method for the calculation of the power distribution which can be
349 used in situations where a higher computational cost is required with other methods, e.g. with finite element
350 methods or CFD. The results obtained with the inverse model are of high interest and can be employed to simulate
351 heating processes with a direct thermal model in situations under different conditions, with different pans or
352 including the effect of the food.

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