Application of virtual distances methodology to laser tracker verification with an indexed metrology platform.

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High range measuring equipment like laser trackers need big dimension calibrated reference artifacts in their calibration and verification procedures. In this paper, a new verification procedure for portable coordinate measuring instruments based on the generation and evaluation of virtual distances with an indexed metrology platform is developed. This methodology enables the definition of an unlimited number of reference distances without materializing them in a physical gauge to be used as a reference. The generation of the virtual points and reference lengths derived is linked to the concept of the indexed metrology platform and the knowledge of the relative position and orientation of its upper and lower platforms with high accuracy. It is the measuring instrument together with the indexed metrology platform the one that remains still, rotating the virtual mesh around them. As a first step, the virtual distances technique is applied to a laser tracker in this work. The experimental verification procedure of the laser tracker with virtual distances is simulated and further compared with the conventional verification procedure of the laser tracker with the indexed metrology platform. The results obtained in terms of volumetric performance of the laser tracker proved the suitability of the virtual distances methodology in calibration and verification procedures for portable coordinate measuring instruments, broadening and expanding the possibilities for definition of reference distances in these procedures.

1. Introduction

The first developments of measurements based on laser trackers date back to the 1980's applied to robots accuracy estimation and calibration [1]–[4]. Aeronautic and naval industries fostered [5] the usage of high range measuring instruments based on laser technology such as laser trackers and laser scanners, trying to improve the existing dimensional verification techniques for big volume parts but also used for machine tool calibration and verification [6]–[8] or deformation analysis [9]. The laser trackers [10], [11] show big advantages in comparison with other equipment because of their portable condition, reliability and application to any type of materials and surfaces together with their broad measuring range from 0 to 200 m according to the commercial available models. But these portable instruments need to be periodically calibrated or verified so as to obtain reliable measurements. The calibration procedure of a measuring instrument allows to obtain the correction models for the measurement results in comparison with the measurements obtained on a calibrated gauge. Nevertheless, a calibration procedure pursues to quantify the effects of the influence variables on the final measurement results, obtaining correction and uncertainty values as a result of the equipment calibration.

Verification and calibration procedures developed for laser trackers as described by the authors in [12] try to determine the alignment and angle encoder errors of a laser tracker together with their uncertainties by means of a set of fixed targets. Also with the same focus on the estimation of the errors in the horizontal angle encoder of a laser tracker, should be remarked the work presented by Muralikrishnan et al. in [13] where they use a stable but not calibrated length. Gassner and Ruland [14] developed a laser tracker horizontal angle calibration test stand based on a high precision rotary table. Different methodologies for calibrating the laser tracker's angle encoder errors such as the National Institute of Standandards and Technology (NIST) technique [13] , the NPL [12] and a precision angular indexing table techniques are compared in [15]. The authors in [16] examined different methodologies that required the laser tracker's probing system to be in continuous movement during the testing. In this case, physical geometries representing a plane, circle and line were used where a spherically mounted retroreflector (SMR) moves.

In relation to the reference artifacts to be used in calibration and verification procedures of laser trackers, the applicable standards for laser tracker evaluation ASME B89.4.19 – 2006 [17], VDI/VDE 2617- 2011 part 10 [18] and ISO/CD10360-10 [19] describe the main requirements for the reference artifacts and procedures to be implemented. Conventional gauges as calibrated gage blocks, step gauges, ball bars or other gauge types with spherical or parallel geometries are commonly used, being manufactured with low thermal expansion coefficient materials. The calibrated uncertainty of these reference artifacts must be lower than the maximum permissible error given by the laser tracker manufacturer. Nevertheless, high range measuring instruments like laser trackers need to be evaluated with big dimension reference artifacts, which sometimes could not be manufactured or could not maintain the accuracy required during the measuring process due to the gravity influence or to artifact fixation effects. Therefore, calibration and verification procedures for laser trackers use the concept of reference lengths, defined as the distance between two reference points. As a general rule, these reference lengths must be defined and measured previously with a calibrated and accurate measuring instrument. The reference points could be for example the centers of retroreflectors located on fixed structures creating the concept of retroreflectors nest. In order to determine the reference length, the center coordinates of the two retroreflectors which define the initial point and final point of the reference length, are measured with a calibrated interferometer or a second calibrated laser tracker equipment. An example of other procedures for defining reference lengths is included in [18] and consists on deflecting the laser beam emitted by the laser tracker with a swiveling mirror in a way that the deflected laser beam runs along a straight line through the two points whose distance is to be determined as the reference length. The distance between reflector and laser tracker changes when the reflector is moved from one point to the other. In addition ASME B89.4.19 [17] and ISO/CD10360-10 [19] include the concept of reference length defined with a linear laser guide. These guides have mobile fixtures to place the retroreflectors, being the measurements carried out with an interferometer independent of the laser tracker to be calibrated. Two retroreflectors are normally used, one to measure the displacement with the interferometer and the second one for the laser tracker to be evaluated. It is important to assure the correct alignment of the linear guide system in order to avoid Abbe error generation which would hinder the correct measurement of the interferometer and the laser tracker. A real testing configuration for a linear guide with laser tracker and interferometer used for a length measurement testing according to [17] is described in a work presented by the National Institute of Standards and Technology [20]. Unkuri et al. [21] describe the development of a 30 m linear guide with measurement by interferometer developed by the Finland National Institute of Metrology. The guide is composed of two parallel axis adjustable each meter and has a mobile platform where the retroreflectors are placed depending on the measurement to be carried out. It is reported to have an estimated expanded uncertainty of 2.6 μm for a 30 m linear displacement. The linear guide has Abbe correction and it is applied for calibration and verification of high range measuring instruments. Other reference artifacts used in laser tracker calibration procedures are described in the technical recommendation VDI/VDE 2617- part 10 [18].

The application of laser trackers for the evaluation of coordinate measuring machines (CMMs) geometrical errors has been covered in vast literature. In [22] is reported a high precision laser tracker for CMM calibration with a tracking system based on a hemisphere to reach measurement uncertainties below 0.3 μm. The use of a laser tracker for CMM calibration with a small residual and an standard deviation of 1 μm when comparing the parametric errors estimated by the laser tracking system to raw data measured by the ball plate method is also described in [23]. The authors in [24] map a CMM with a laser tracer obtaining uncertainties in the range of 1 μm. Laser trackers or laser tracers have been also used for articulated arm coordinate measuring machines (AACMM) calibration, as in [25] where a new calibration procedure for a AACMM with laser tracker multilateration is presented or in [26] showing a new approach for AACMM calibration with a laser tracer used as reference instrument for the accuracy assessment of the AACMM which is moved by an industrial robot.

This work presents a new verification procedure for portable coordinate measuring instruments based on the generation and evaluation of virtual distances by means of a capacitive sensor based indexed metrology platform (IMP). This methodology allows the definition of an unlimited number of reference distances without materializing them in a physical gauge to be used as a reference. The technique is especially useful for high range measuring equipment such as laser trackers where the calibrated reference distances to be measured have big dimensions, being the methodology able to generate virtual reference distances independently of their magnitude.

2. Verification procedure with an indexed metrology platform by virtual distances

2.1. The indexed metrology platform

In order to optimize calibration and verification procedures for laser trackers, it is analyzed in this work the use of an indexed metrology platform (IMP) [27] as an auxiliary instrument in these procedures, developing an alternative methodology to evaluate the volumetric accuracy and repeatability of a laser tracker, in comparison with the conventional procedures established in the standards [17], [18], [19] which will be used as a basis for this new verification procedure development. Verification and calibration procedures normally start with the definition and construction of the kinematic model of the measuring instrument, laser tracker and indexed metrology platform in this case, generating the geometric transformations, the reference system's location and the initial nominal geometric parameters. The kinematic model of the laser tracker developed in this work is based on the Denavit Hartenberg model (D-H) [28] which has been already applied to laser trackers as in [29], where the kinematic model of the laser tracker is elaborated and a new algorithm for the laser tracker parameter identification is presented improving the accuracy of the system. The integration of the laser tracker's kinematic model and the mathematical model of the platform enables to express a point captured with the laser tracker in the global platform coordinate reference system, which is located in the lower platform. By means of the mathematical model of the platform explained in [27], a homogenous transformation matrix (HTM) is found allowing the change of coordinate reference systems required.

The indexed metrology platform is composed of two hexagonal platforms, one fixed lower platform and a mobile upper platform which rotates every 60º defining six different rotation positions. It has high mechanical repeatability achieved through kinematic couplings configuration of spheres and cylinders and high mechanical position repeatability fact that allows to measure with high precision the orientation and position of the upper platform with respect to the lower platform with the capacitive sensors. The six capacitive sensors have nanometer resolution, a measuring range of 100 µm for an output voltage from 10 to -10 V and an operational range from 100 to 200 µm with their sensors and targets assembled in the upper and lower platforms respectively. The use of the IMP shows a clear testing time and man efforts reduction in comparison with conventional verification procedures. In this case, the laser tracker placed on the IMP, rotates jointly with the upper platform during the verification procedure, enabling a big coverage of the laser tracker's working volume and the definition of a broad number of testing positions but avoiding the movement of the calibrated gauge object during the verification. Also the space needed in the data capturing process is diminished since the number of physical testing positions of the gauge are minimized and this is a clear advantage for verification and calibration procedures of high range measuring instruments. With the laser tracker assembled on the indexed metrology platform, it is possible to express the coordinates of a captured point with the laser tracker in the fixed lower platform or global coordinate reference system during the verification procedure. In this work, the new verification procedure is applied to a laser tracker model API T3-15m assembled on the indexed metrology platform as it is shown in Figure 1.

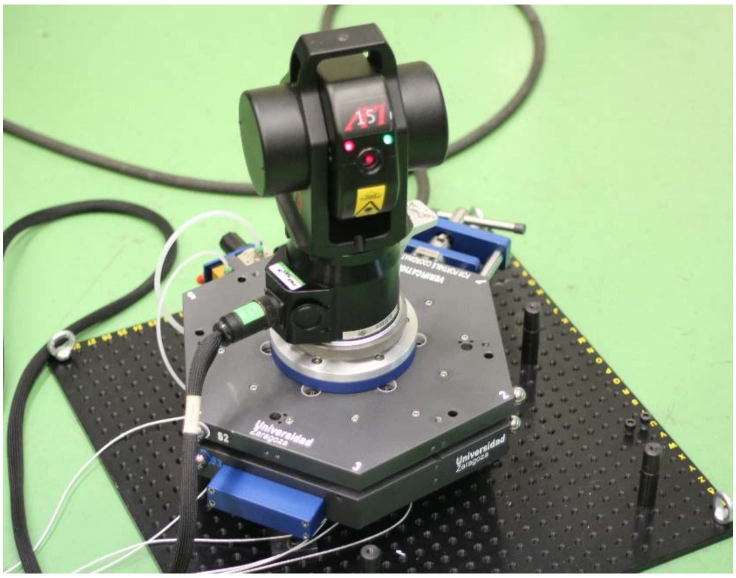


Figure 1. Laser tracker with the indexed metrology platform

An estimation of the indexed metrology platform's uncertainty using the Monte Carlo method, considering the complex mathematical model of the platform, was previously developed in order to validate the use of the indexed metrology platform in verification procedures for portable coordinate measuring machines. In a first step, the model's input variables which could affect the output variable were defined. The possible error sources that may influence the uncertainty of the indexed metrology platform were the calibration uncertainty of the platform, the capacitive sensors' error, the error of the portable measuring equipment that will be used with the platform, the temperature and the dynamic behavior of the platform during the measuring process. The Monte Carlo simulation was run for 10000 iterations. The mean, uncertainty and confidence interval values for the output variables were calculated out of the results of the simulation. The n-homogeneous transformation matrices (XYZABC) that allow the change of reference systems from the upper platform coordinate reference system to the lower platform or global coordinate reference system were considered as output variables of the IMP's mathematical model. The indexed metrology platform position and orientation uncertainty for a given platform position and point measured is presented in Table 1:

Table 1. Indexed metrology platform position and orientation uncertainty in homogeneous transformation matrices upper to lower platform, sphere 1, point 1, n-iterations 10000

${}^{RS\ Global}T_{RS\ UpperPlat}$ ***(Sphere 1 / Point 1 / Platform position 1)***			
	Nominal	**Mean**	**Uncertainty** (µm / º)
X (mm)	-0.13500	-0.13502	0.01996
Y (mm)	196.61710	196.61707	0.04489
Z (mm)	40.84180	40.84181	0.04965
A (º)	179.99880	179.99879	0.02057
B (º)	0.01940	0.01941	0.01677
C (º)	60.05620	60.05621	0.01159

Based on these n-homogeneous transformation matrices obtained in the Monte Carlo simulation and considering the possibility of expressing points in a global coordinate reference system located in the lower platform base, it is possible to estimate the IMP's uncertainty in a distance measurement between pairs of the n-sphere's centers simulated in the global platform coordinate reference system. For this purpose, two reference calibrated distances were defined, d12 = 100.80247 mm and d15= 399.96137 mm to calculate a distance error value as a difference between the distance obtained in the Monte Carlo simulation and the calibrated distance value. The IMP's uncertainty values obtained in a distance measurement are shown in Table 2 and could validate the correct operation of the IMP as an auxiliary instrument in verification procedures for portable coordinate measuring machines such as laser trackers considering the accuracy range needed.

Table 2**.** Indexed metrology platform uncertainty in a distance measurement, n-iterations 10000

	d12 (Sphere 1 - 2)	**d15 (Sphere 1 - 5)**
Mean distance error (mm)	0.058721	0.063400
Standard deviation (mm)	0.000245	0.000242

2.2. Verification procedure methodology

In the verification procedure developed in this work, it is used a big dimension measuring mesh of 6 x 6 x 6 m with 27 spherically mounted reflectors (SMR), whose area materialize part of the working volume of the measuring instrument [30]. The laser tracker model used in the experimental testing, API T3-15m, has an angular accuracy of 3.5 µm/m, ADM accuracy of ± 15 µm and IFM accuracy of ± 0.5 ppm. The 27 surface mounted reflectors distributed in the mesh are model Hallow 40M with 1.5" (38.1 mm) diameter and sphere roundness grade 50 ± 0.00005" (±1.3 µm). The positions of the retroreflectors are defined in terms of the height of the target on the wall and its distance to the center of the laser tracker's working volume, which is located at 1.5 m from the ground assembled on the indexed metrology platform, see Figure 1. In Figure 2 it could be observed the physical disposition of the retroreflectors in the mesh together with their distances to the laser tracker. All the retroreflectors in the mesh were measured with the laser tracker from the six rotating positions of the platform.

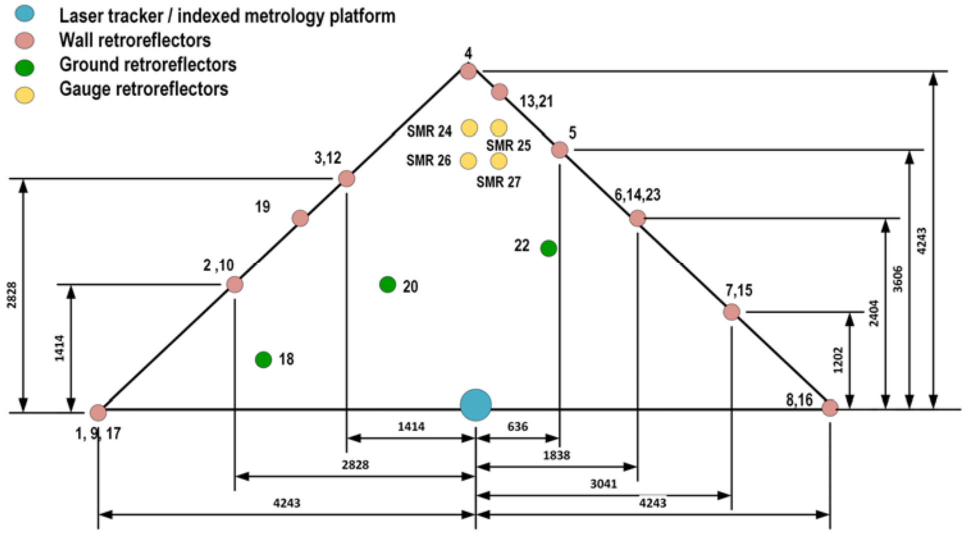


Figure 2. Measuring mesh with retroreflectors position

The generation of a virtual mesh is intrinsically linked to the concept of the indexed metrology platform. In the verification procedure developed in this work, the physical mesh shown in Figure 2 is measured with the laser tracker assembled on the indexed metrology platform from all the rotating positions of the platform (1-6). Each time the platform rotates 60º to a new position, the laser tracker measures the same physical mesh from a different position of the platform, and therefore a new working volume of the laser tracker is explored. This will be equivalent to measure six physical meshes located in different locations of the working volume of the laser tracker from the same position of the platform. The new verification procedure of the laser tracker with the indexed metrology platform enables to reduce the testing time, setups and space needed. The measurements of the same mesh carried out from the six positions of the platform will define the *measured points*, having as a result six *measured meshes*, and the Euclidean distances among the points measured will be named as *measured distances* and will be used as parameter in the volumetric performance evaluation of the laser tracker. In parallel to the measurement of the points, the values of the capacitive sensors assembled in the indexed metrology platform are captured for each measurement and position of the platform. These captures will be used to obtain a single homogenous transformation matrix per point measured according to the mathematical model of the indexed metrology platform [27]. This matrix allows us to make a coordinate reference system change from the upper platform coordinate reference system to the lower platform coordinate reference system or global coordinate system, being able in this way to express a point captured by the laser tracker in a global coordinate reference system located in the lower platform.

Taking as a reference the measurements of the measured mesh done with the laser tracker from a platform position and considering that we could know with high accuracy the position of the upper platform with respect to the lower platform, the generation of the virtual mesh through the IMP mathematical model is based on the concept of applying the known rotation angle of the platform to the reference artifact, measured mesh, being able in this way to generate virtual meshes rotated 60º / 120º / 180º / 240º / 300º from the measured mesh in the selected position of the platform. For example, considering as a reference the mesh measured in the platform position number 1, *measured mesh 1*, it is possible to generate by means of the mathematical model of the indexed metrology platform, a set of virtual points that will integrate the *virtual mesh 1* corresponding to the indexed metrology platform position number 1. This procedure is repeated successively for the six rotating positions of the platform creating the six virtual meshes as it is shown in Figure 3. The six virtual meshes generated will be affected by the 60º rotation of the platform from one position to the following. In this work, it will be taken as coordinate reference system, the one corresponding to the platform position 1 which will be named as laser tracker reference system 1 (RS $_{LT1}$). Therefore, a point measured from the laser tracker reference system 1 (RS $_{LT1}$) will have the same coordinates as its virtual equivalent point expressed in the laser tracker reference system 1 (RS $_{LT1}$).

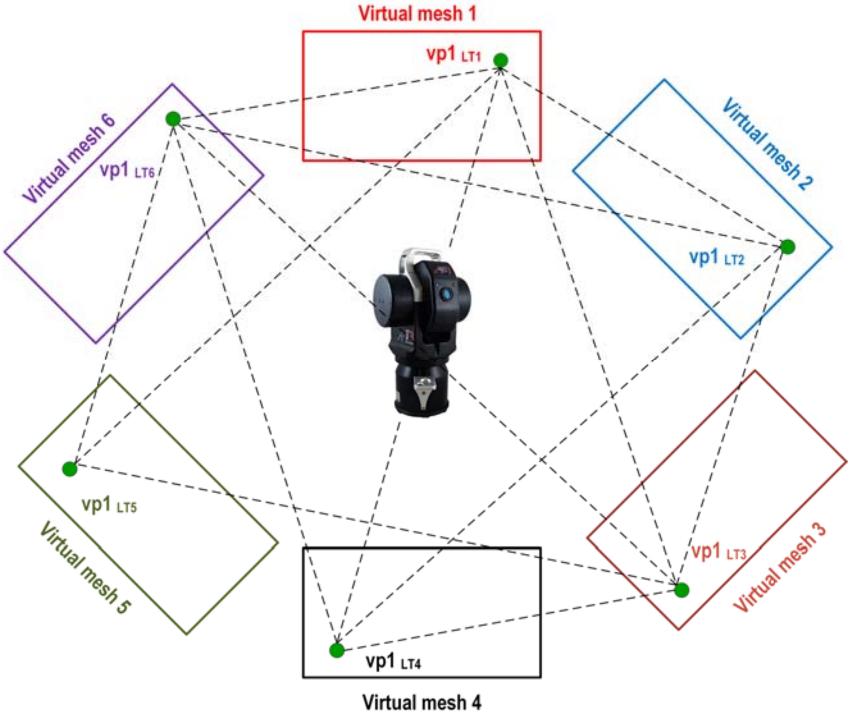


Figure 3. Virtual meshes (1-6)

The procedure developed for generating a virtual point through the indexed metrology platform mathematical model is following explained:

1. Given a point located in a mesh named as *point 1* with its coordinates measured with the laser tracker assembled on the indexed metrology platform in the platform position 1, see Figure 4, the associated coordinate reference system to express the center coordinates measured with the laser tracker will be the laser tracker reference system 1, RS $_{LT1}$.
2. We rotate the platform 60º from position 1 to position 2 with the laser tracker rotating jointly with the indexed metrology platform. The coordinates of a point measured from this platform position 2 will be expressed in the laser tracker reference system 2, RS$_{LT2}$.
3. We measure again from this platform position 2 the *point 1* expressing its coordinates in laser tracker reference system 2, RS $_{LT2}$.
4. This measurement will be equivalent to capture a virtual point, named as *point 1'* from platform position 1, see Figure 4, with its coordinates expressed in laser tracker reference system 1, RS $_{LT1}$. In this way, the virtual point will be affected by the rotation of the platform from platform position 1 to platform position 2.

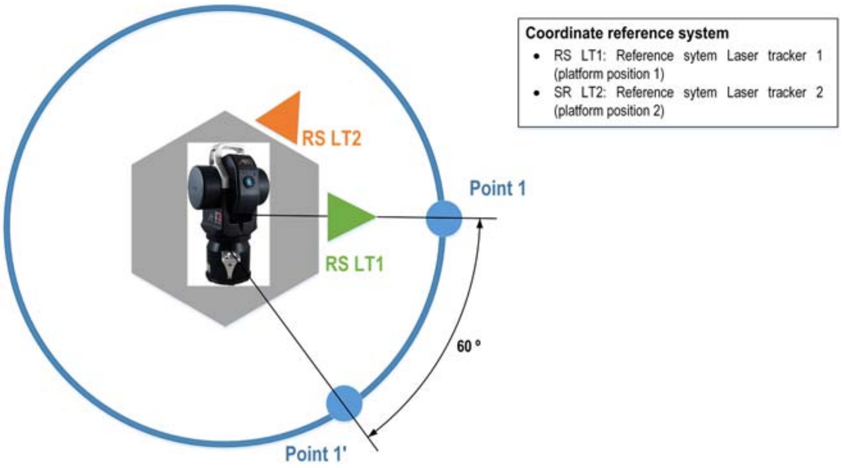


Figure 4. Virtual point concept with indexed metrology platform applied to laser tracker

Taking as a reference one point measured in the platform position 1, where measured and virtual point have the same coordinates in the virtual mesh 1 and expressing these coordinates in the laser tracker reference system 1 (RS $_{LT1}$), it is possible to generate a virtual point in a virtual mesh 2 through the indexed metrology platform mathematical model. In this calculation, the translation and rotation components of the homogeneous transformation matrix, which changes from platform position 1 to platform position 2 are taken into account, assuming that the coordinates of the virtual point will be expressed in the laser tracker reference system 1, see equation (1).

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}_{RS_{LT2}} = {}^{RS_{LT2}}T\,_{RS_{LT1}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}_{RS_{LT1}} \tag{1}$$

The T matrix is a homogeneous transformation matrix which provides a change of coordinates from the laser tracker reference system 1 (RS $_{LT1}$), corresponding to the virtual point in the virtual mesh 1, to the laser tracker reference system 2 (RS $_{LT2}$), where the new virtual point in the virtual mesh 2 will be created affected by the quantified rotation of the platform. The main difference in this case is the assumption that the new virtual point generated in the virtual mesh 2 will have its coordinates expressed in the laser tracker reference system 1 (RS $_{LT1}$), as if the laser tracker will be measuring from the position 1 of the platform.

The homogeneous transformation matrix T is expressed with the following equation (2) and it is explained in Figure 5.

$$^{RS_{LT2}}T_{RS_{LT1}} = (^{RS_{UpperPlat}}M_{RS_{LT2}})^{-1}(^{RS_{Global}}Mi,j_{RS_{UpperPlat}})^{-1}\,^{RS_{Global}}Mi,j_{RS_{UpperPlat}}\,^{RS_{UpperPlat}}M_{RS_{LT11}} \tag{2}$$

Denoting by:

– $^{RS\ UpperPlat}M_{RS\ LT2}$: laser tracker reference system 2 to upper platform reference system homogeneous transformation matrix .
– $^{RS\ UpperPlat}M_{RS\ LT1}$: laser tracker reference system 1 to upper platform reference system homogeneous transformation matrix .
– $^{RS\ Global}M_{i,j\ RS\ UpperPlat}$: Upper platform reference system to global or lower platform reference system homogeneous transformation matrix . This matrix is generated per each measured point out of the values of the capacitive sensors assembled in the indexed metrology platform.

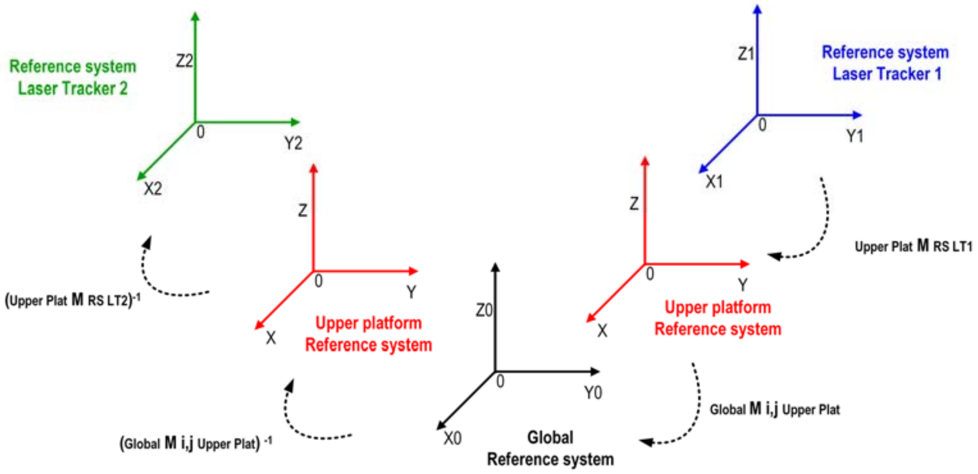


Figure 5. Change of coordinate reference system to obtain a virtual point

By means of this procedure, it is possible to obtain a set of virtual points in six virtual meshes linked to the rotating positions of the platform and generate an unlimited number of virtual distances calculated as the Euclidean distance between two virtual points positioned in the same or different virtual mesh. Taking as a reference the platform position 1, the concept of *measured distance* and *virtual distance* is shown in Figure 6. The *measured distance* is defined as the Euclidean distance between the point 1 measured from the position 2 of the platform but assuming its coordinates to be expressed in the laser tracker reference system 1 (RS_{LT1}) and its coordinates measured from the platform position 1 and expressed in the laser tracker reference system 1 (RS_{LT1}). The *virtual distance* will be defined as the Euclidean distance between the virtual point 1 generated in the virtual mesh 2 assuming its coordinates to be expressed in the laser tracker reference system 1 (RS_{LT1}) and the virtual point 1 with its coordinates expressed in the laser tracker reference system 1 (RS_{LT1}). The distance deviation D_i, is calculated as the difference between the virtual distance L_{Virtual} and the measured distance L_{Measured}, see equation (3).

$$D_i = L_{\text{Virtual}} - L_{\text{Measured}} \tag{3}$$

The error inherent to the laser tracker together with the error of the indexed metrology platform which is considered negligible in comparison to the laser tracker’s error, will be the causes of the deviation of the coordinates of the virtual point 1 in the virtual mesh 2 and the coordinates of the measured point 1 from the platform position 2, being both points expressed in the laser tracker reference system 1 (RS_{LT1}). The deviation of the coordinates of the point will generate the corresponding distance deviation between the virtual distance L_{Virtual} and the measured distance L_{Measured}. As a final result, three evaluation parameters will be obtained. First, the maximum distance deviation D_i among all the positions of the platform, the range of the deviations and a mean distance deviation. These error parameters will be used to evaluate the volumetric performance of the laser tracker in its working volume.

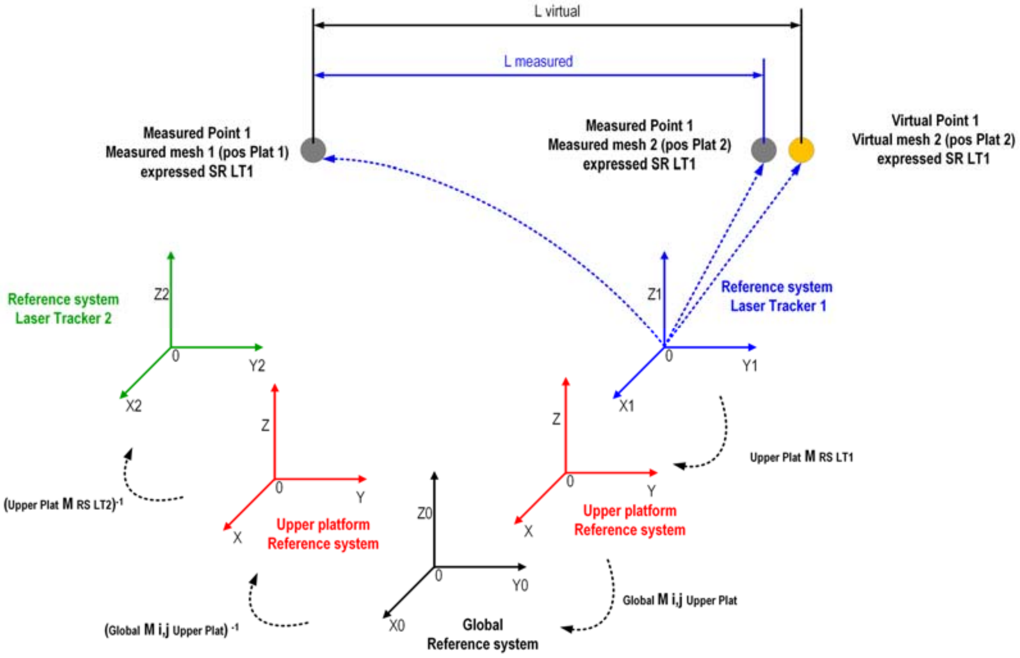
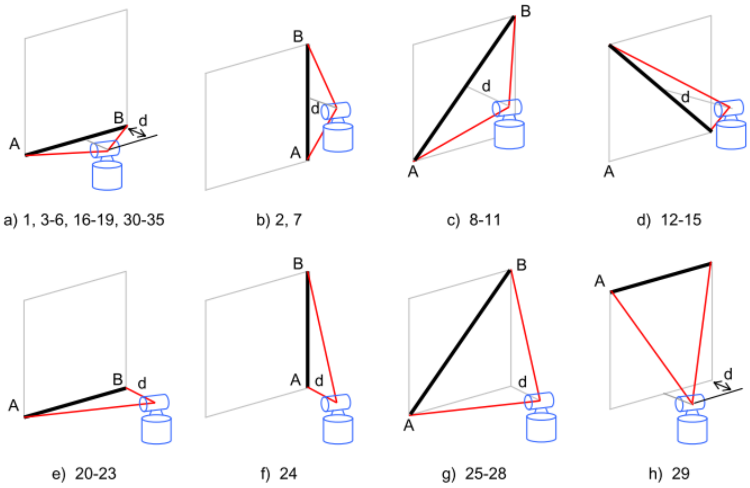


Figure 6. Measured distance and virtual distance concept

Two evaluation alternatives were proposed in order to estimate the azimuthal rotation, elevation and distance errors of the laser tracker maximizing the working volume of the instrument to be evaluated. The definition of the distances together with their distribution were done taking into consideration the evaluation standards applicable to laser trackers ASME B89.4.19 – 2006 [17], the technical recommendation VDI/VDE 2617- 2011 part 10 [18] and the draft of ISO/CD10360-10 [19]. According to the standards, the laser tracker could be repositioned for each reference length if this could be easier than relocating the measuring line, but in this case the laser tracker remains fixed assembled on the indexed metrology platform turning around the six platform's positions. The reference lengths or measuring lines will be defined by this three factors: the distance between the laser tracker and the ends of the reference length, the azimuthal rotation of the laser tracker derived from the rotation of the platform and the position of the reference length. In Figure 7 the examples of measuring lines with A and B ends of the reference length could be seen [18] and Figure 8 shows a possible arrangement of eight measurement lines according to [19].



a) 1, 3-6, 16-19, 30-35
b) 2, 7
c) 8-11
d) 12-15
e) 20-23
f) 24
g) 25-28
h) 29

Figure 7. Positions of the reference artifact for length measurement error test. Source: ISO/CD10360-10

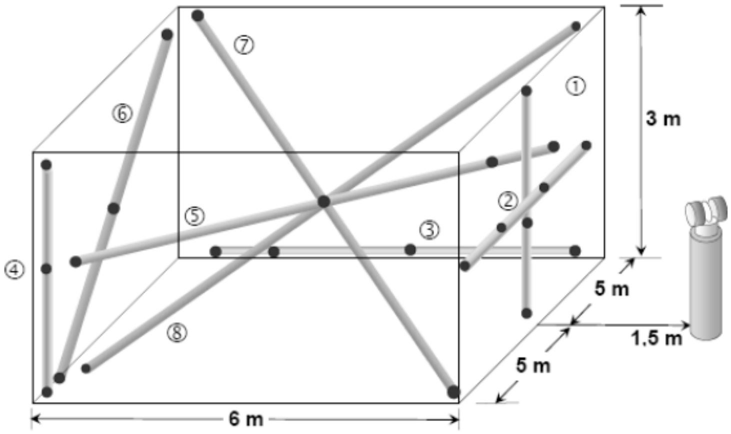


Figure 8. Arrangement example of the measurement lines for length measurement error test. Source: VDI/VDE 2617- part 10

The evaluation alternatives developed will be explained considering in all of them the laser tracker reference system 1 (RS $_{LT1}$) as the reference system of the procedure, assuming the platform in position 1 and being equal the coordinates of a measured and a virtual point in RS $_{LT1}$.

2.2.1. *Evaluation method 1: virtual distances among virtual points in mesh 1 and equivalent virtual points in meshes 2-6*

In this first evaluation method, the definition of a virtual distance is based on the distance calculations among the 27 virtual points coordinates in the virtual mesh 1 and their coordinates in the other virtual meshes (2-6). This method enables to generate virtual distances of different lengths bigger than the ones which could be defined in the single physical mesh, being this fact a remarkable advantage in evaluation procedures for high range measuring instruments. 135 virtual distances are calculated and the graphical concept explanation could be seen in Figure 9 taking as an example six virtual points 1, vp1$_{LT1}$ – vp1$_{LT6}$, generated in the corresponding meshes (1-6). The Euclidean distance between virtual point 1 located in the virtual mesh 1 as a reference, and the rest of virtual points generated in the virtual meshes 2 to 6 could be obtained following the equation (4):

$$D_{i,j} = \sqrt{\left(X_{i,j} - X_{1,j}\right)^2 + \left(Y_{i,j} - Y_{1,j}\right)^2 + \left(Z_{i,j} - Z_{1,j}\right)^2} \quad i = 2, \dots, 6; j = 1, \dots, 27 \tag{4}$$

Denoting $D_{i,j}$ the euclidean distance between the virtual point j in each of the i platform positions or virtual meshes, and the virtual point j in the virtual mesh 1 with coordinates expressed in laser tracker reference system 1.

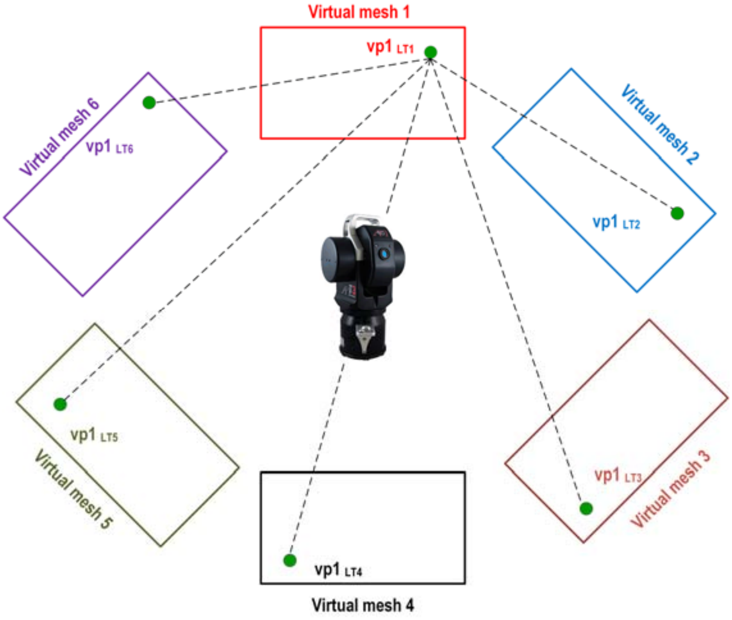


Figure 9. Evaluation method 1: virtual distance between virtual point in mesh 1 and equivalent virtual points in virtual meshes 2-6.

2.2.2. Evaluation method 2: virtual hexagon, evaluation through virtual distances among virtual points in consecutive meshes

The definition of the virtual distances in this method consist on the definition of virtual distances between the coordinates of equivalent virtual points located in consecutive meshes, defining virtual hexagons at different heights depending on the height of the virtual points in the mesh. In this way, 135 virtual distances following the scheme shown in Figure 10 are defined. As an example, six virtual points 1 $vp1_{LT1}$ – $vp1_{LT6}$ are generated in the six virtual consecutive meshes. The Euclidean distance between virtual points situated in virtual mesh i and the equivalent virtual points located in the consecutive virtual mesh according to the positions of the platform (1-6) has the expression shown in equation (5).

$$D_{i,j} = \sqrt{\left(X_{i+1,j} - X_{i,j}\right)^2 + \left(Y_{i+1,j} - Y_{i,j}\right)^2 + \left(Z_{i+1,j} - Z_{i,j}\right)^2} \quad i = 1, \dots, 6; j = 1, \dots, 27 \tag{5}$$

Denoting $D_{i,j}$ the euclidean distance between the virtual point j in each of the i platform positions or virtual meshes, and the virtual point j in the next virtual mesh i+1 with coordinates expressed in laser tracker reference system 1.

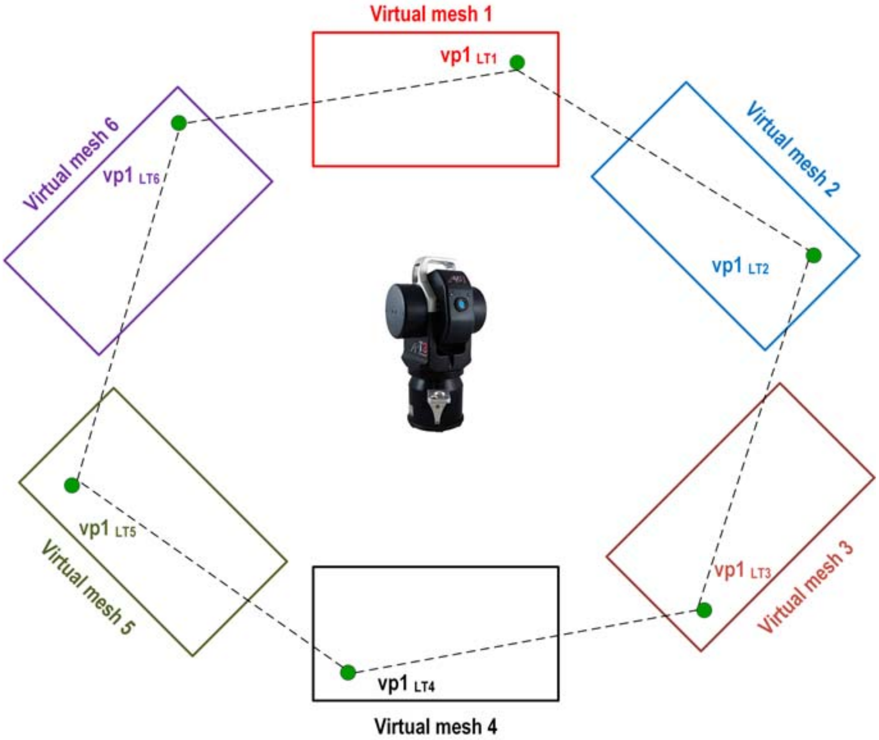


Figure 10. Evaluation method 2: virtual distance between virtual point in mesh 1 and their equivalent virtual points in consecutive meshes.

Figure 11 shows the virtual hexagon created from the Euclidean distances between virtual points in consecutive virtual meshes. As an example, virtual hexagons at 6 m, 2 m, 1 m and ground height are described. This evaluation technique allows to compare the error of the instrument at different heights and therefore with several elevation angles of the laser tracker head. The azimuthal error could be also evaluated due to the fact that the laser tracker rotates jointly with the indexed metrology platform in the six positions of the platform each 60º.

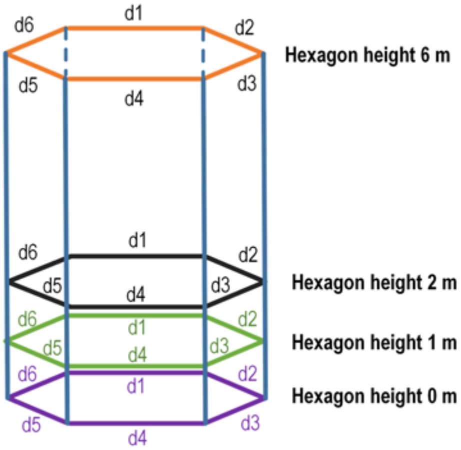


Figure 11. Virtual hexagons (height 0/1/2/6m)

3. Results of the laser tracker verification procedure with the virtual distances method

The virtual distances methodology allows an evaluation of the errors of the laser tracker and the indexed metrology platform in the following terms:

- θ error: azimuthal rotation error estimation linked to the platform and the laser tracker's head rotation.
- φ error: elevation error estimation due to the different elevation angles of laser tracker.
- Distance error: deviation between the virtual distance and the measured distance.

Making a comparison of the results obtained with the two evaluation methods explained in Section 2, it could be observed that the mean distance error values are similar in the two methods being the average value 0.0270 mm. The maximum distance error is 0.1031 mm corresponding to the first evaluation method and the range of deviations is 0.0912 mm. A summary of the complete evaluation results is shown in Table 3.

Table 3. Distance error results per virtual distances evaluation method

	Method 1	Method 2	Mean	Maximum
Mean distance error (mm)	0.0223	0.0317	0.0270	
Max distance error (mm)	0.1031	0.0811		0.1031
Range of distance error (mm)	0.1027	0.0797	0.0912	
Standard deviation (mm)	0.0186	0.0212		

One important aspect to analyze is the evolution of the distance error in relation to the magnitude of the reference length evaluated. All the evaluation methods showed an increasing distance error with the length. The biggest error values are given for the biggest values of the virtual distances. An example of this behavior in the evaluation method 1 could be seen in Figure 12 where a linear decreasing trend in the distance error could be observed with decreasing values of the virtual lengths defined between the virtual mesh 1 and virtual mesh 2.

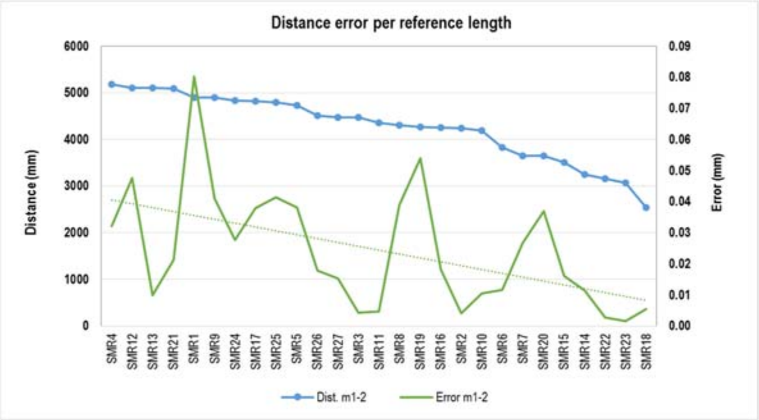


Figure 12. Distance error per virtual distance evaluated (evaluation method 1)

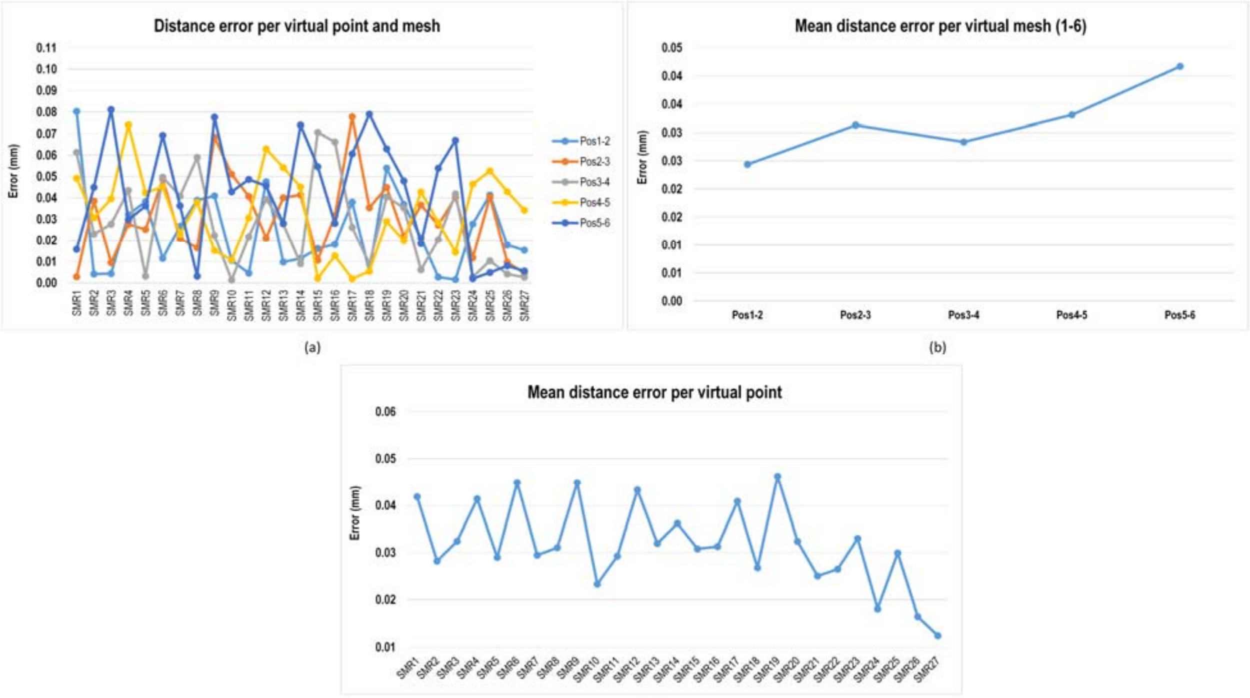
3.1. Evaluation method 1: virtual distances among virtual points in mesh 1 and equivalent virtual points in meshes 2-6.

Figure 13 (a) shows the virtual distance error among equivalent virtual points located in the pairs of virtual meshes 1-2,1-3,1-4,1-5,1-6. The biggest distance error values are obtained for the virtual point 9 (SMR 9) and the virtual point 17 (SMR 17) in the distances calculated between virtual mesh 1 and 5. The mean distance error per virtual mesh is described in Figure 13 (b) where it could be observed that the mean error decreases from platform’s position 2 to 4 and increases from position 4 to 6. It could be clearly noted, see Figure 13 (c), that the biggest mean distance error values occur for the virtual points 1, 9 and 17 located in the left corner of the retroreflector mesh where the laser tracker angle to the reflector is on its most unfavourable position generating a more complicated measurement of the retroreflector. The virtual point 19 which shows also high error value is located at 1 m height but it is also in the left corner of the measuring mesh.

Figure 13. Distance error values according to virtual distances evaluation method 1

3.2. Evaluation method 2: virtual hexagon, evaluation through virtual distances among virtual points in consecutive meshes.

The second evaluation method named as virtual hexagon, let us estimate the azimutal rotation error of the laser tracker. In Figure 14 (a) the distance error between equivalent virtual points in consecutive meshes is represented. Figure 14 (b) shows the evolution of the azimutal rotation error from one position to the next, increasing the error with the rotation angle of the platform in a linear trend.



(a) (b) (c)

Figure 14. Distance error values according to virtual distances evaluation method 2

Figure 14 (c) reveals the mean distance error per virtual point evaluated. Depending on the height of the virtual point in the mesh according to Table 4, an estimation of the laser tracker's error at different elevation angles φ could be calculated, being the laser tracker located at 1.5 m height. No clear trend of increasing error with the height could be concluded out of the error values obtained. The points located higher at 6 m, virtual points 1 to 8, did not show bigger error values.

Table 4. Heights of virtual points in the virtual mesh

Height	Virtual point
6 m	1 - 8
2 m	9 - 16
1 m	19, 21, 23
0 m	17, 18, 20, 22, 24-25 (gauge)

If we take into consideration the results obtained in former works carried out by the same authors to evaluate the suitability of a laser tracker verification procedure with an indexed metrology platform, a comparison between both verification methodologies with and without virtual distances in terms of volumetric performance of the laser tracker could be performed. In the verification procedure of the laser tracker with an indexed metrology platform without virtual distances, the distance error values were calculated as the difference between the distance measured on a small dimension gauge, SMRs 24-27 in Figure 2 with the laser tracker assembled on the indexed metrology platform and the calibrated distance measured on the same gauge with a coordinate measuring machine (CMM). The measurements of the gauge were made from all the positions of the platform with the laser tracker assembled on the indexed metrology platform located in the central position of the measuring mesh shown in Figure 2. The results obtained are listed in the Table 5.

Table 5. Distance error comparative results between laser tracker verification procedures with indexed metrology platform with or without virtual distances

	Method 1	Method 2	Laser tracker – Indexed metrology platform (CMM)
Mean distance error (mm)	0.0223	0.0317	0.0650
Max distance error (mm)	0.1031	0.0811	0.1768
Range of distance error (mm)	0.1027	0.0797	0.1595
Standard deviation (mm)	0.0186	0.0212	

The error values obtained with the conventional verification procedure for a laser tracker with the indexed metrology platform are higher than with the virtual distances method. For example the mean error value obtained with virtual distances is 0.0270 mm (see Table 3) and with the conventional procedure 0.0650 mm. Regarding the maximum error value, the result obtained with virtual distances is 0.1031 mm (see Table 3) and with the conventional procedure 0.1768 mm. Finally the range of distance deviations shows a mean value of 0.0912 mm with virtual distances and 0.1595 mm in the conventional procedure.

According to the results obtained, it could be concluded that both methodologies are valid for a laser tracker verification procedure with an indexed metrology platform, considering therefore the platform as a suitable instrument in calibration and verification procedures for laser tracker. In addition, the virtual distances methodology shows in the volumetric evaluation better results than the conventional procedure with the platform, and it is considered accurate enough for the measurement's accuracy required by the laser tracker.

4. Conclusions

This works presents a new methodology for portable measuring coordinate instruments verification procedures based on the use of an indexed metrology platform, with direct application to high range measuring instrument like laser trackers where big dimension gauges are necessary. One of the main advantages of the indexed metrology platform showed in this work is the capacity of generating virtual points located in a virtual gauge mesh. This fact allows the definition of an unlimited number of virtual points and virtual reference lengths eliminating the need of materializing the reference length in a physical gauge, and generating big dimension reference gauges covering the working volume of the laser tracker. The procedure lets us apply the rotation of the platform to a group of measured points with the laser tracker assembled on the indexed metrology platform, generating in this way a set of virtual points. This is possible due to the fact that the position and orientation of the upper platform with respect to the lower platform could be known with high accuracy. On these grounds, six virtual meshes were generated with 27 virtual points each. In order to validate the new laser tracker verification procedure developed, two evaluation methods were defined based on virtual distances creation according to the laser tracker applicable standards [17], [18] and [19]. The distance error value calculated as deviation between the virtual distance and the measured distance, allows to quantify the volumetric performance of the laser tracker. The comparative results among the two evaluation methods in terms of distance error show no significant differences, being the mean distance error 0.0270 mm. The maximum distance error is 0.1031 mm corresponding to the first evaluation method and the mean range of distance deviations value is 0.0912 mm.

One important factor which was taken into account was the influence of the magnitude of the reference length. It was stated in this work that all the evaluation methods show increasing distance error values with the length evaluated. In regard to the influence of the laser tracker's elevation angle φ in the measurement error, no clear trend of bigger error with increasing elevation angle was assessed. The effect of the azimuthal rotation of the laser tracker during the measuring procedure is also evaluated because the laser tracker rotates jointly with the indexed metrology platform. It could be stated that the distance error increases with the rotation of the platform, obtaining higher mean distance error values in the virtual mesh associated with the higher rotation degree of the platform. This could be seen in the evaluation according to method 2 with incremental mean error values in the consecutive virtual meshes.

Finally, the outcomes of the virtual distance verification procedure were also compared with the results obtained in a verification procedure for a laser tracker with the indexed metrology platform without using virtual distances, being the results with the virtual distances method better than with the conventional verification procedure for laser tracker with the indexed metrology platform.

The results obtained in this work show that the new laser tracker verification procedure developed with an indexed metrology platform based on virtual distances could be suitable for a laser tracker's evaluation. It has big advantages like the possibility to generate an unlimited number of reference lengths no matter their dimension and it could be considered in a further step for integration into the existing verification procedures carried out for high range measuring instruments.

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