Journal of Honai Math, 5(1), 61-74, April 2022
https:/doi.org/10.30862/jhm.v5i1.240

The impact of sending letters in improving teaching-learning process of natural number of pre-service teachers

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Received: 4 February 2022 | Revised: 24 February 2022 | Accepted: 12 March 2022 | Published 13 March 2022

Abstract

The first contact that a pre-service teacher has with didactics of mathematics is the notion of natural number, being still in the university classroom and not having started working a real classroom. Therefore, the main objective of this work is to relate the knowledge of a trainee teacher to the different difficulties developed by the children and to evaluate the learning processes in a real environment. The participants were 20 future teachers and 40 (9-year-old) children. During the experience, six letters closely related to the contents of the university subject were exchanged; as well as two socialisation letters; and four videos, including two presentation and two farewell videos. Among the results obtained, we highlight that the participating university students have been able to reinforce the knowledge learned in class through the analysis of the children's resolutions, specifically the following: a) counting, b) resolution of additive-concrete situations, c) resolution of multiplicative-concrete situations, d) resolution of a monetary problem, and e) being aware of the manipulation of Cuisenaire's rods.

Keywords: Natural Number, Pre-Service Teachers, Problem-Solving, Teaching-Learning

Introduction

Students of a Bachelor's Degree in Primary Education in Spain receive training in two places: at the university, where they spend most of their training, and at the school where they carry out the different work placements. The work placement at the school allows the trainee teacher to observe the reality of the classroom, as well as to put into practice some of the content acquired during their university training (Póveda et al., 2021). This connection between the contents learnt in the degree and their applicability in a school context is a latent concern for university teachers. In the classroom, a practical component is needed to be able to apply the knowledge acquired in real intervention scenarios (Valle & Manso, 2018). In particular, the teacher has to be able to design situations that are suitable for the student to mobilise the knowledge to be learnt. This can be a complex task since aspects such as the age and level of the students, as well as the content, have to be taken into consideration.

Based on this motivation to encourage trainee teachers to acquire skills in the design of tasks, we have developed an experience that has been implemented in the subject of Didactics of Arithmetic I. The choice of this subject is because it is the first contact of pre-service teachers with the didactics of mathematics in their university training. This subject develops the mathematical and didactic training of the future Primary Education teachers with regard to the natural number.

The main objective of this work has been to put into practice a situation-problem that the university students proposed to a group of primary school students. The analysis of the latter's answers helps trainee teachers to identify the difficulties that children can develop, evaluating the learning processes in a real environment. On the other hand, the situations posed by the university students are included as content within the subject they were studying, so that the experience also aims to help the trainee teachers to acquire the didactic contents associated with the subject.

Natural number and problem solving in primary education

The experiences in which number sense is developed in primary education are closely related to emotional aspects that bring about positive or negative attitudes towards this subject, which can be maintained throughout life (Bracho et al., 2011). In order for this first contact between the child and mathematics to develop satisfactorily, Alsina (2006) states that children need to: observe, in order to be able to interpret the environment; experience situations with their bodies and movement; manipulate, experiment, play, verbalise their observations; etc. Despite this need, the Spanish mathematics curricula, within which this study was carried out, are oriented towards the acquisition of content, focusing exclusively on the acquisition of symbols and techniques (Alsina, 2019), without developing mathematical competence through a variety of contexts and situations, which is essential to be able to understand, judge, do and use mathematics (Niss, 2020).

The world is full of mathematics and numbers, so it is normal for children to have regular contact with them (Treffers, 2008), thus children should be given the possibility to develop and use their own strategies to solve problems and develop good number sense. This is one of the reasons why problem-solving environments should be encouraged in a classroom (Chapman, 2011).

Teacher training in mathematics

Shulman (1986) established the knowledge that a teacher should have: content knowledge, pedagogical knowledge and curricular knowledge. For the particular case of mathematics, Ball et al. (2008) established the Mathematical Knowledge for Teaching model, which contains two domains: knowledge of the content and pedagogical knowledge of the content. Likewise, this model was adapted in Spain, giving rise to the Mathematics Teacher's Specialised Knowledge

(Carrillo et al., 2018), which provides categories for analysing this specialised knowledge, which is exclusive of teachers, and facilitates an understanding and interpretation of their actions and knowledge. This new model considers three domains: mathematical knowledge, didactic knowledge of mathematical content, and beliefs and conceptions.

Following this last model (Carrillo et al., 2018), it is determined that the process of learning to teach mathematics requires professional tasks that are part of the routine activities of initial training and understanding of mathematical content (Ma, 1999), placing them in the context of practice (Ball & Cohen, 1999).

Given that there is a gap between university students who are training to become teachers and the professional practice that they will develop, Goos (2014) proposes that the tasks that these trainee teachers carry out should be inspired by professional tasks, fostering the look that they will have to have when they are in the classroom.

For all of the above reasons, the university students have participated in the development of tasks for Primary Education students to work on. The contents worked on were as follows: counting, additive-concrete situations, multiplicative-concrete situations, monetary problems and manipulation of Cuisenaire's rods.

Methods

The work was carried out within a qualitative approach, in which its exploratory character has a descriptive purpose, allowing to interpret the written productions of future teachers and children (Elliot & Timulak, 2005; McMillan & Schumacher, 2005).

As described above, the tasks were implemented in the subject of Didactics of Arithmetic I of the 2nd year of the Degree in Teacher Training in Primary Education in a Spanish public university during the academic year 2020-2021. This course had certain special characteristics. Half of the university students attended in-person classes while the other half followed online lessons, with both groups alternating on a weekly basis. The primary school children attended all the classes face-to-face, but with bubble groups to reduce contagions. The course began after 4 months of distance learning at all levels and stages at national level, giving a great change to the educational reality, in favour of a continuous adaptation and a greater use of technology. Due to these special circumstances, there was some difficulty in transferring university students to the school, which is why it was decided to establish this experience.

The sample consisted of 20 trainee teachers in their 2nd year of the Faculty of Education at the University of Zaragoza, who participated on a voluntary basis, and 40 children in the 4th year of Primary Education (9 years old) from CEIP Cantos Altos in the Autonomous Community of Madrid, both in Spain. University of Zaragoza and CEIP Cantos Altos are the workplaces of the researchers who carried out this study. Moreover, the type of tasks designed is related to the content of the subject. Therefore, the trainees were able to apply the contents of the course in a real context.

The experience consisted of the exchange of six letters closely related to the contents of the subject, as well as two socialisation letters, and four videos, including two of presentation and two of farewell. The experience lasted four months and was monitored through the Moodle platform, and at all times they were supervised by the teacher of the subject at the University of Zaragoza and by the teacher responsible at the CEIP Cantos Altos, both belonging to the research team of the present study. In this study, we focused on the presentation of the six letters related to the contents of the subject. In the case of the future teachers, their productions were collected in the subject's Moodle, enabling the homework tool for their submission. At the end of the session, the children handed in each of their productions to their teacher, who subsequently digitised them.

Each trainee teacher was assigned two pupils in the 4th year of primary school. Throughout the whole experience, it was very important that the children knew how to explain what they were doing at all times, so they were grouped in pairs. Considering the contingency plan of the schools during the current school year, due to covid-19, the children had to respect the safety distance and these groupings could only be done in pairs at the most. In order to solve the proposed problems, the classroom was equipped with various manipulative materials: Cuisenaire rods, coins and banknotes, wool, paper, scissors and different colour pencils.

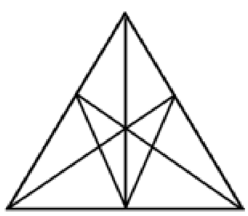
Results and Discussion

In this section we present the results obtained from the different tasks, according to the different moments in which an exchange of information is established. During this experience, we did not give importance to the number of successes or errors, but rather to how the children told the future teachers about the process followed to reach the result, the doubts that arose, whether they could work with the proposed activity (data, vocabulary, clues, handwriting, and so on), that is, their experiences in solving the problems.

Results of counting situation

The following task (Figure 1) has been worked on by future teachers at university in recent years, in the subject Didactics of Arithmetic I.

¿Cuántos triángulos pueden verse en el dibujo?
Los triángulos deben tener sus lados sobre las líneas del dibujo.



Translation: How many triangles can be seen in the drawing? The triangles must have their sides on the lines in the drawing.

Figure 1. Triangle counting problem

On this occasion, the same task was additionally posed to the primary school children. Thus, the trainee teachers were able to propose some indications to facilitate the resolution of the triangle counting problem. The university students were asked to provide the children with hints, thus facilitating the resolution of this problem (see Figure 2, Figure 3, and Figure 4).

Hola Alex y Sergio, para ayudaros y enfrentarnos juntos a estos nuevos retos aquí os dejo una serie de pistas para que podáis resolver los ejercicios de una manera mucho más sencilla y rápida. Espero que os sirvan de gran apoyo. ¡Sé que lo vais a hacer genial y vais a dejar a todos con la boca abierta! ¡Mucho ánimo!

EJERCICIO 1.

PISTA 1. Imaginaros que la figura se trata de un puzle, y tenéis que identificar todas las piezas en las que se puede dividir la figura.

PISTA 2. Colocar en cada parte/pieza una letra del abecedario para que así no os perdáis a la hora de contar los triángulos que hay.

PISTA 3. Juntar las letras en grupos de tal manera que formen triángulos y apuntar dichas letras. Además, es mucho más fácil si seguís un orden, de más grande a más pequeño o de más pequeño a más grande.

Translation: Hi Alex and Sergio, to help you and to face these new challenges together, here are some hints so that you can solve the exercises in a much easier and quicker way. I hope they will be of great help to you, I know you will do great and you will leave everyone with a hung jaw, so keep up the good work!

Hint 1. Imagine that the figure is a puzzle, and you have to identify all the pieces into which the figure can be divided.

Hint 2. Put a letter of the alphabet on each part/piece so that you don't get lost when counting the triangles.

Hint 3. Put the letters together in groups so that they form triangles and copy those letters on a paper. It is also much easier if you follow an order, from the biggest to the smallest or the smallest to the biggest.

Figure 2. Instructions from prospective teachers to children

PROBLEMA 1

- Se puede resolver de varias **maneras.**
- Pon un **número** diferente a cada triángulo.
- Ayúdate de **pinturas** y rotuladores, pero que no sea la única herramienta.

Translation:

- It can be solved in several ways.
- Give each triangle a different number.
- Use colour pencils and markers, but don't use them as the only tool.

Figure 3. Instructions from prospective teachers to children

Problema 1. ¿Cuántos triángulos pueden verse en el dibujo?

Pista 1: ¿Qué tal si probáis a marcar de alguna manera cada uno de los triángulos que componen el triángulo grande, para que así no os confundáis y no repitáis ni os olvidéis de contar alguno de ellos?

Pista 2: ¿Por qué no probáis a coger otro folio y escribir en él todas las combinaciones posibles de triángulos que vayáis encontrando?

Pista 3: Y si lo preferís, igual os gusta más la idea de colorear cada uno de los triángulos. ¡Tened mucho cuidado, tenéis que ser muy organizados y estar atentos a los triángulos que vais descubriendo para no olvidaros de ninguno!

Translation:
Problem 1: How many triangles can be seen in the drawing?
Hint 1. How about trying to mark in some way each of the triangles that make up the big triangle, so that you don't get confused and don't repeat or forget to count any of them?
Hint 2. Why don't you try taking another sheet of paper and writing on it all the possible combinations of triangles that you find?
Hint 3. And if you prefer, you might like the idea of colouring in each of the triangles. Be very careful, you have to be very organised and pay attention to the triangles you discover so that you don't forget any of them!

Figure 4. Instructions from prospective teachers to children

From the analysis of Figure 5, we observe different ways of naming the pieces to start counting. These strategies guided by the indications given by the trainee teachers favour the success of the task. We highlight the consideration of letters in comparison with a numerical code that can create confusion between the part that constitutes the triangle and the number of triangles counted, whereas this imprecision is resolved if a coding through letters is chosen.

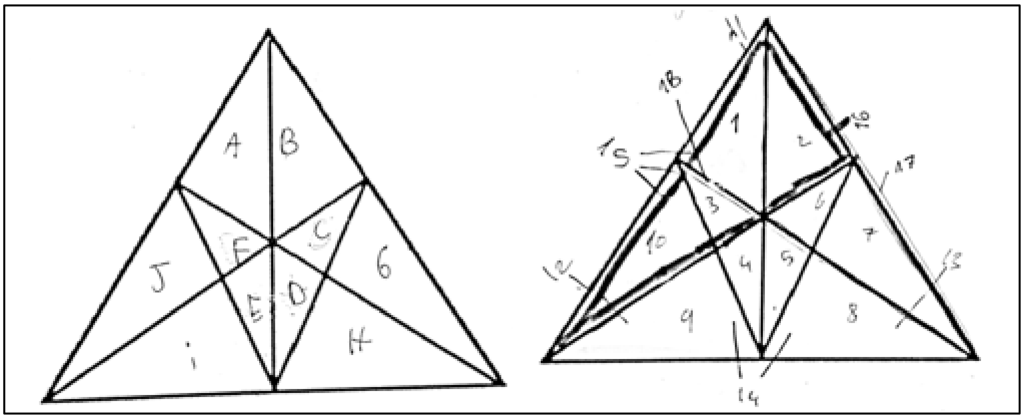
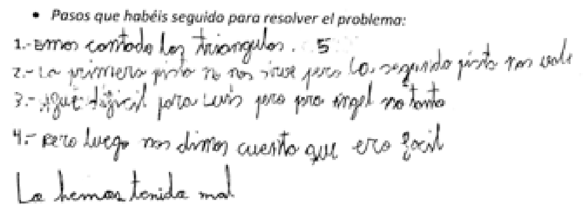


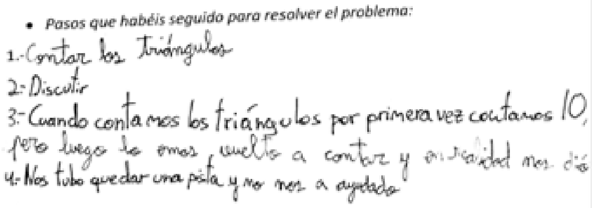
Figure 5. Children's resolutions to the proposed task

In addition, the children had to make a reflection on the procedure carried out and put it in writing (Figure 6) so that the future teachers could analyse it and propose the children some suggestions for improvement in the up-coming letters.



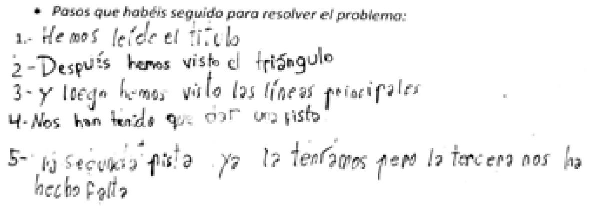
• Pasos que habéis seguido para resolver el problema:
1.- Emos contado los triangulos. 5
2.- La primera pista no nos sirve pero la segunda pista nos vale
3.- Que difícil para Luis pero pra Ángel no tanto
4.- pero luego nos dimos cuenta que era facil
La hemos tenido mal

Translation: Steps followed to solve the problem:
1. We have counted the triangles. 5.
2. The first clue doesn't work, but the second clue works.
3. It was difficult for Luis, but not so difficult for Ángel.
4. But then we realised that it was easy.
We've had it bad.



• Pasos que habéis seguido para resolver el problema:
1.-Contar los Triángulos
2-Discutir
3-Cuando contamos los triángulos por primera vez contamos 10 pero luego lo emos vuelto a contar y en realidad nos dió
4-Nos tubo quedar una pista y no nos a ayudado

1. Count the triangles
2. Discuss
3. The first time that we counted the triangles we got 10, but then we counted them again and it actually gave us (no number indicated).
4. He had to give us a clue and it didn't help us.



• Pasos que habéis seguido para resolver el problema:
1.- Hemos leído el título
2-Después hemos visto el triángulo
3-y luego hemos visto las líneas principales
4-Nos han tenido que dar una pista
5- La segunda pista ya la teníamos pero la tercera nos ha hecho falta

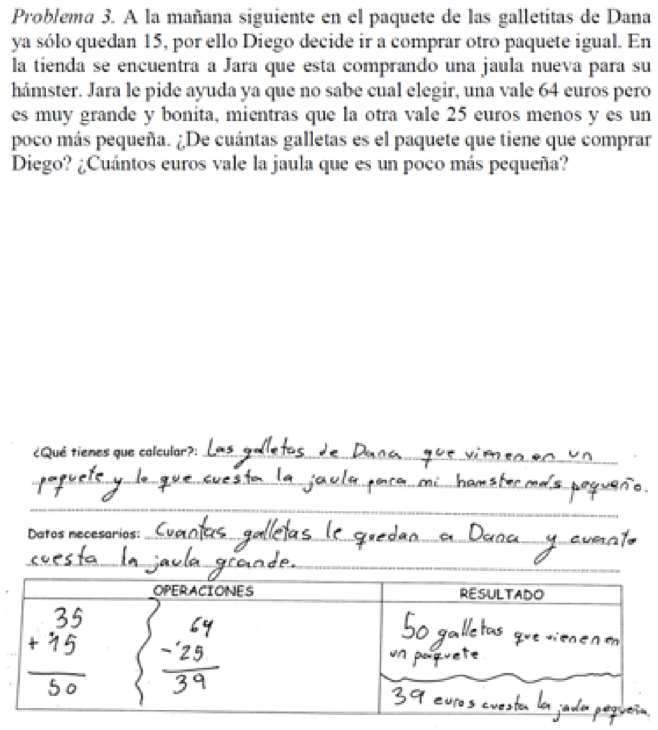
1. We have read the title
2. Then we saw the triangle
3. And then we have seen the main lines
4. They had to give us one clue
5. We already had the second clue, but we needed the third one.

Figure 6. Explanation of the procedure followed by the children

As can be seen from their responses (Figure 6), before asking for the clues provided by the prospective teachers, the students decided to count the number of triangles with their own strategies. Faced with the different difficulties encountered, it was time for the students to ask their classroom teacher for the clues that the prospective teachers assigned to them had provided in the letter.

Results of the additive situation and management of the monetary system

Each of the future teachers proposed three additive problems involving different semantic structures (Riley et al., 1983; Riley & Greeno, 1988; Polotskaia & Savard, 2018; Cañadas, Molina, & del Río, 2018; Polotskaia & Savard, 2021), previously worked on in the subject. From among all these semantic structures to be worked on, the future teachers had to choose the ones they considered most appropriate. On the other hand, the teacher responsible for the children prepared a resolution sheet for the children to provide the information as they were used to (see Figure 7 and Figure 8).



Problema 3. A la mañana siguiente en el paquete de las galletitas de Dana ya sólo quedan 15, por ello Diego decide ir a comprar otro paquete igual. En la tienda se encuentra a Jara que esta comprando una jaula nueva para su hámster. Jara le pide ayuda ya que no sabe cual elegir, una vale 64 euros pero es muy grande y bonita, mientras que la otra vale 25 euros menos y es un poco más pequeña. ¿De cuántas galletas es el paquete que tiene que comprar Diego? ¿Cuántos euros vale la jaula que es un poco más pequeña?

¿Qué tienes que calcular?: Las galletas de Dana que vienen en un paquete y lo que cuesta la jaula para mi hamster más pequeño.

Datos necesarios: Cuantas galletas le quedan a Dana y cuanto cuesta la jaula grande.

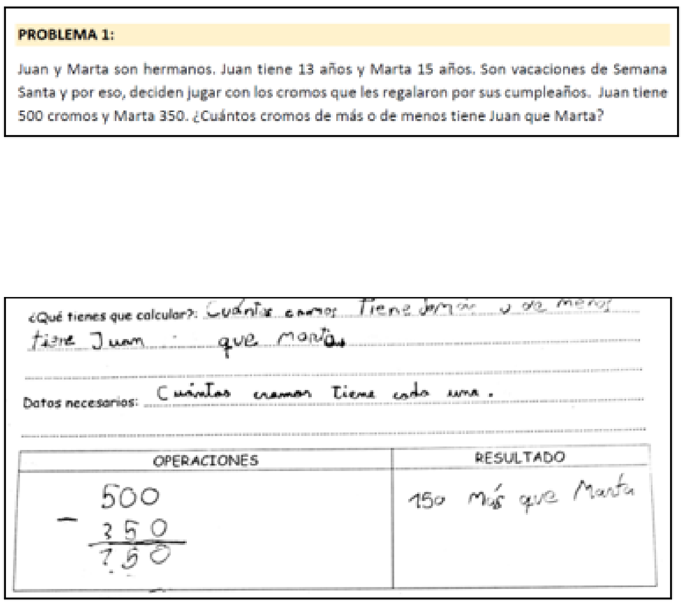
OPERACIONES	RESULTADO
35 + 15 = 50; 64 − 25 = 39	50 galletas que vienen en un paquete 39 euros cuesta la jaula pequeña

Translation: Problem 3. The next morning there are only 15 biscuits left in Dana's packet of biscuits, so Diego decides to buy another packet like Dana's one. In the shop he meets Jara who is buying a new cage for her hamster. Jara asks him for help, as she doesn't know which one to choose, one is worth 64 euros but is very big and beautiful, while the other is worth 25 euros less and is a bit smaller. How many biscuits will the packet that Diego has to buy have? How many euros is the cage that is a bit smaller?

What do you have to calculate? Dana's biscuits in a packet and the cost of the cage for my smallest hamster.

Data needed: How many biscuits Dana has left and how much the big cage costs.

Figure 7. Proposed and solved additive problem



PROBLEMA 1:

Juan y Marta son hermanos. Juan tiene 13 años y Marta 15 años. Son vacaciones de Semana Santa y por eso, deciden jugar con los cromos que les regalaron por sus cumpleaños. Juan tiene 500 cromos y Marta 350. ¿Cuántos cromos de más o de menos tiene Juan que Marta?

¿Qué tienes que calcular?: Cuántos cromos tiene Juan de más o de menos tiene Juan que Marta.

Datos necesarios: Cuántos cromos tiene cada uno.

OPERACIONES	RESULTADO
500 − 350 = 150	150 más que Marta

Translation: Juan and Marta are brother and sister. Juan is 13 years old and Marta is 15 years old. It's Easter holidays and so they decide to play with the stickers they were given for their birthdays. Juan has 500 stickers and Marta has 350 stickers. How many more or fewer stickers does Juan have than Marta?

What do you have to calculate? How many more or fewer stickers Juan has than Marta.

Data needed: How many cards each of them has.

Figure 8. Additive problem and problem solving

In addition, following the structure of the previous mailing, a common problem (Figure 9) was established in which the future teachers had to write down different aids for its resolution. The children were provided with manipulatives to solve the problem (Figure 10).

Una señora entra en una zapatería y compra unas zapatillas que cuestan 30 euros. La mujer paga al dependiente con un billete de 50 euros, pero el vendedor, que acaba de abrir la tienda, no tiene cambio para darle así que sale un momento a la tienda de al lado donde le cambian el billete de 50 por 5 billetes de 10. El vendedor entrega el cambio a la señora y esta se marcha de la tienda. Al poco rato, entra la dependienta de la tienda de al lado y le dice a nuestro vendedor que el billete que le ha entregado era falso. Cómo ya es imposible localizar a la mujer y por lo tanto no se puede recuperar el dinero ni los zapatos, el vendedor le cambia el billete de 50 euros falso por otro de curso legal y se disculpa. ¿Después de todo esto, cuánto dinero ha perdido el vendedor?

Translation: A woman goes into a shoe shop and buys some trainers costing 30 euros. The woman pays the shop assistant with a 50-euro note, but the shop assistant, who has just opened the shop, has no change to give her, so she goes to the next shop and exchanges the 50-euro note for five 10-euro notes. A short time later, the shop assistant from the next shop comes in and tells our sales assistant that the note he has given her was counterfeit.

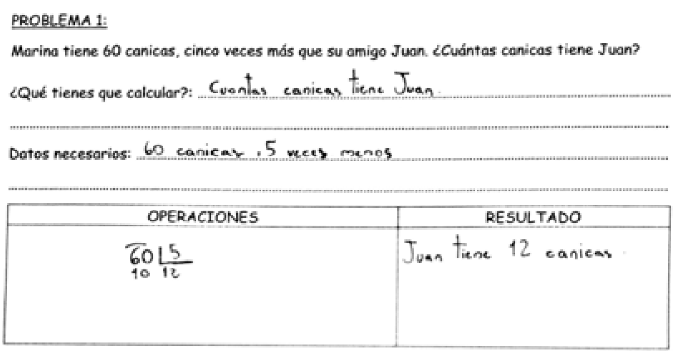
As it is impossible to locate the woman and therefore the money and the shoes cannot be recovered, the shop assistant exchanges the counterfeit 50 euro note for a legal tender note and apologies. After all this, how much money has the shop assistant lost?

Figure 9. Proposed problem involving the monetary system

Figure 10. Solving the monetary problem

Results of the multiplicative situation and manipulation of Cuisenaire's rods

Following the usual structure in which problems are presented in the classroom where the participating children were, the trainee teachers checked the homework of their assigned students, and each one proposed three multiplicative problems involving different semantic structures (Riley et al., 1983; Riley & Greeno, 1988; Cañadas, Molina, & del Río, 2018; Alghamdi, Jitendra, & Lein, 2020) previously provided in the subject. From among all these semantic structures to be worked on, the future teachers had to choose the ones they considered most appropriate. On this occasion, the pupils were given the model sheet that is used by their teacher at school, so that they could incorporate the problems themselves (Figure 11 and Figure 12). It should be noted that the future teachers could see both the original answers to the problem given by the students and the corrections from the teacher at school.

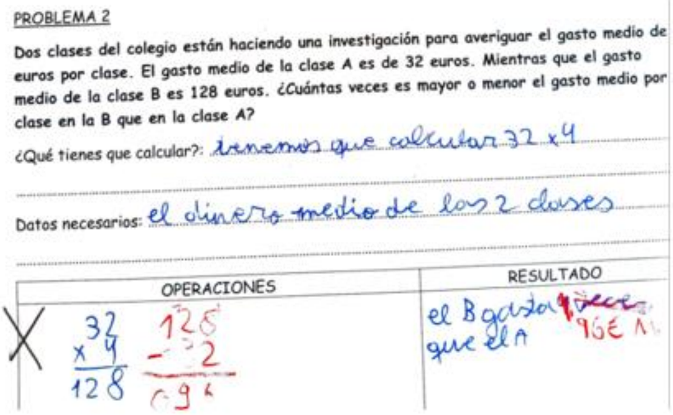

PROBLEMA 1:
Marina tiene 60 canicas, cinco veces más que su amigo Juan. ¿Cuántas canicas tiene Juan?
¿Qué tienes que calcular?: Cuantas canicas tiene Juan
Datos necesarios: 60 canicas, 5 veces menos

OPERACIONES	RESULTADO
60 |5 10 12	Juan tiene 12 canicas

Translation: Marina has 60 marbles, five times more than her friend Juan. How many marbles does Juan have?

What do you have to calculate? How many marbles does Juan have?
Facts needed: 60 marbles, 5 times less.

Figure 11. Problem and multiplicative problem solving


PROBLEMA 2
Dos clases del colegio están haciendo una investigación para averiguar el gasto medio de euros por clase. El gasto medio de la clase A es de 32 euros. Mientras que el gasto medio de la clase B es 128 euros. ¿Cuántas veces es mayor o menor el gasto medio por clase en la B que en la clase A?
¿Qué tienes que calcular?: tenemos que calcular 32 x 4
Datos necesarios: el dinero medio de las 2 clases

OPERACIONES	RESULTADO
32 x 4 = 128; 128 − 32 = 96	el B gasta 4 veces que el A; 96€

Translation: Two classes at the school are doing research to find out the average expenditure of euros per class. The average expenditure of class A is 32 euros while the average expenditure of class B is 128 euros. How many times higher or lower is the average expenditure per class in class B than in class A?

What do you have to calculate? We have to calculate 32x4.
Necessary data: The average money in the two classes.

Figure 12. Multiplicative problem and problem solving

In addition, they had to come up with a problem involving least common multiple, to be solved using Cuisenaire's rods and no algorithm, and whose context was that of spies (Figure 13).

Figure 13. Solving a problem with Cuisenaire rods

Results achieved during the Experience

The following results were obtained through observation presented in Table 1.

Table 1. Results achieved in children and pre-service teachers

Results Observed in Children	Results Observed in Pre-Service Teachers
Greater motivation in problem-solving exercises than in other courses and in other regular classroom tasks.	Occasionally, use of vocabulary or expressions that were difficult for children to understand.
They found that, thanks to teamwork (pairs), they reached the solution earlier due to the reflection and proposals they put forward.	Some activities were difficult for their level. They needed help from the teacher in charge of the classroom to understand them.
The written reflection on their own work made them aware of their knowledge and avoided impulsivity in their answers.	Some clues were very obvious, while others were difficult to understand.
They created a very good connection with the students at the university thanks to their videos and letters and, in particular, the way children addressed them.	When the pre-service teachers provided the final result of the problem, children experienced frustration in case they did not get it. They learned that the process is the most important part.
Due to the children's explanations, the work sent by the university students was gradually being adapted to the reality of the students at the school.	Some children did not understand the teachers' handwriting, so a computer transcription of the proposed activities had to be proposed.

Among the results obtained, we highlight that the participating university students have been able to reinforce the knowledge learned in class through the analysis of the primary school students' answers to the problems they were given, specifically the following: a) counting, b) additive-concrete situations solving, c) multiplicative-concrete situations solving, d) monetary problem solving, and e) be aware of the importance of the manipulation of Cuisenaire's rods. The pre-service teachers have managed to develop the didactics of natural number from professional tasks, as recommended by Goos (2014), placing them in a practical context (Ball & Cohen, 1999; Chorney & Bakos, 2021). Furthermore, we attribute the good results obtained by the children to the variety of contexts and situations posed, which are necessary to develop mathematical competence, as stated by Alsina (2019), and to foster the notions involved through problem solving (Chapman, 2011; English, 2020).

Conclusion

From this experience, the pre-service teachers have had the opportunity, on the one hand, to review the children's resolutions in relation to concepts that they had worked on the subject previously. On the other hand, they have connected with the critical perception of the way in which each of the tasks were presented that a 4th-grade (9 years-old) Primary School child has towards the subject of mathematics. One of the possible limitations of this study is the analysis of each of the tasks, as the conclusions are based on the descriptive results of the experience.

Therefore, future research will be focused on identifying a classification of resolutions through specific qualitative analysis with deductive categories.

We would like to highlight that the participating university students have been able to reinforce the knowledge learnt in class through the analysis of the primary school students' problem solving. In addition, to connect with the critical perception of the way in which each of the tasks were presented. Last but not least, to observe the motivation that a 4th-grade Primary School child has towards the subject of mathematics. Given that the pupils have valued very positively this type of activities implemented in the classroom, this proposal will be consolidated in the 2021-2022 academic year and will be continued in the rest of the subjects in the Didactics of Mathematics area in future years.

Acknowledgment

This experience is planned to be continued during the academic year 2021-2022 due to the approval of the PIIDUZ_1 180 project “Cartas matemáticas para la adquisición del número natural en el Grado en Magisterio en Educación Primaria” in the call for Proyectos de Innovación: Docencia, Tecnología, Orientación, Social y Transferencia (PI_DTOST) 2021/2022 of the University of Zaragoza.

Conflicts of Interest

The authors declare that no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely by the authors.

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