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Log-growth rates of CO_2: An empirical analysis

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Abstract

We study the parametric distribution of log-growth rates of CO_2 emissions and CO_2 per capita emissions for 207 countries and territories taking data from 1994 to 2010. We define the log-growth rates for different duration periods, from one year apart to fifteen years apart. The considered densities have been the following: the normal (N), the asymmetric double Laplace normal (adLN), the exponential tails normal (ETN) and a mixture of two normal (2N) or three normal (3N) distributions. The main result is that the best density is different depending on the period considered, in such a way that there is not a systematically dominant function. Thus, the behaviour may change from one year to the next one, and possibly this is influenced by policy measures such as the Kyoto protocol or the Clean Development Mechanism. Moreover, a policy measure that can be derived from this paper is that some countries can still reduce their emissions of CO_2 compared with others, as seen by the non-uniformity of the preferred density for each period. We also model a stochastic differential equation whose associated Fokker–Planck equation has as a solution the observed time-dependent probability density function.

Keywords: CO_2 emissions, log-growth rates, density functions

1. Introduction

The study of the statistical distribution of CO_2 emissions is a relevant topic since with its knowledge the pollution costs can be easier allocated and identified for each country. This target can be better reached after analyzing in depth the density of the distribution of CO_2 emissions. However, the study of the statistical distribution of log-growth rates of CO_2 emissions has not been carried out yet. On levels, some previous work is [2]. Possibly the log-growth rates of these quantities have never been modelled before due to the skewness and, above all, very high kurtosis of the data. In this paper we perform the first attempt, to the best of our knowledge, to define the densities that fit best to the observed data. The distributions chosen are the exponentiated versions of that in [2], that is to say, the normal (N), the asymmetric double Laplace normal (adLN) (exponentiated version of the double Pareto lognormal, dPLN), the exponential tails normal (ETN) (exponentiated version of the Pareto tails lognormal, PTLN)[2] and the exponentiated version of two densities recently introduced in the literature of city sizes and strike size [28, 6, 22, 21, 48, 38, 39, 7], namely a mixture of two (2N) or three (3N) normal distributions (see also, e.g., [35]).

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[2][2] also estimate the lognormal upper-tail Pareto (LNUTP) but we do not consider it explicitly because is a particular case of the PTLN. The cited distributions (LN, dPLN and PTLN) are designed to be estimated with data on levels.

The need to manage CO_2 emissions has emerged in the global policy agenda [27]. The reason is that currently greenhouse effect gases are probably growing, and the climate of the earth is becoming warmer [46]. Thus, the analysis of the statistical distribution of log-growth rates of CO_2 emissions is worldwide relevant for economic policy and it is a useful tool for designing efficient international regulatory policies [2]. In contrast to other studies where the authors analyze the structural composition of CO_2 emissions among sectors within a country [5], this paper sheds further light on the distribution of CO_2 emissions among countries. The main conclusion is that there is not an uniformly outperforming distribution. On the contrary, we find a great variability, depending on the years considered, regarding the best density. This outcome is in line with the key message derived from [38, 39]. This reflects the effort made by most countries in the reduction of CO_2 emissions, with some countries with a higher effort than others, leading to year-to-year shifts in the statistical distribution of CO_2 emissions among nations.

The rest of the paper is organized as follows. The next Section describes the distributions we employ. Section 3 shows the data sets used in this study. The results are reported in Section 4. In Section 5 we discuss briefly the results. Finally, we offer some conclusions.

2. The distributions and generating mechanisms

2.1. The distributions

For CO_2 or CO_2 emissions per capita (CO_2pc), we have computed the log-growth rates of these quantities by using the well-known formula

$$g = \ln(x_{\text{final}}) - \ln(x_{\text{initial}}) \tag{1}$$

where x_{final} and x_{initial} are the values of the CO_2/CO_2pc corresponding emissions in the final and initial samples, respectively, of the period considered for computing the log-growth rate g.

As for the distributions, we will consider first the usual normal distribution (N), which will serve us as a baseline model, given by

$$f_{\text{N}}(g;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(g-\mu)^2}{2\sigma^2}\right), \tag{2}$$

where now $g, \mu \in (-\infty, \infty)$, $\sigma > 0$. The corresponding cumulative distribution function (CDF) is

$$\Phi(g;\mu,\sigma) = \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{g-\mu}{\sigma\sqrt{2}}\right). \tag{3}$$

where erf denotes the error function associated to the standard normal distribution.

The 2-mixture of normal distributions (2N), is [20, 21, 22, 28, 38, 39, 7]

$$f_{2\text{N}}(g;\mu_1,\sigma_1,\mu_2,\sigma_2,p_1) = p_1 f_{\text{N}}(g;\mu_1,\sigma_1) + (1-p_1) f_{\text{N}}(g;\mu_2,\sigma_2), \tag{4}$$

where $0 \le p_1 \le 1$, $0 \le 1 - p_1 \le 1$, and $g, \mu_i \in (-\infty, \infty), \sigma_i > 0$, $i = 1, 2.$ and the 3-mixture of normal distributions (3N) is

$$f_{3\text{N}}(g;\mu_1,\sigma_1,\mu_2,\sigma_2,\mu_3,\sigma_3,p_1,p_2) = p_1 f_{\text{N}}(g;\mu_1,\sigma_1) + p_2 f_{\text{N}}(g;\mu_2,\sigma_2) + (1-p_1-p_2) f_{\text{N}}(g;\mu_3,\sigma_3), \tag{5}$$

where $0 \le p_1, p_2 \le 1$, $0 \le 1 - p_1 - p_2 \le 1$, and $g, \mu_i \in (-\infty, \infty), \sigma_i > 0$, $i = 1, 2, 3$.

According to [2] and [41], the exponentiated version of the dPLN is the asymmetric double Laplace normal (adLN) introduced by [42, 43, 44] and later used, e.g., by [33]:

$$\begin{aligned} f_{\text{adLN}}(g;\alpha,\beta,\mu,\sigma) \quad = \quad & \frac{\alpha\beta}{(\alpha+\beta)}\left(\exp\left(-\alpha(g-\mu) + \frac{1}{2}\alpha^2\sigma^2\right)\Phi(g;\mu+\alpha\sigma^2,\sigma)\right. \\ & \left. + \exp\left(\beta(g-\mu) + \frac{1}{2}\beta^2\sigma^2\right)\Phi(-g;-\mu+\beta\sigma^2,\sigma)\right) \end{aligned} \tag{6}$$

where $\mu \in \mathbb{R}$, $\alpha, \beta, \sigma > 0$ are the four parameters of the distribution. It has the property that it approximates different exponential laws in each of its two tails: $f_{\text{adLN}}(g) \approx \exp(-\alpha g)$ when $g \to \infty$ and $f_{\text{adLN}}(g) \approx \exp(\beta g)$ when $g \to -\infty$.

The body is approximately normal, although it is not possible to delineate exactly the switch between the normal and the exponential behaviors since the adLN distribution is the convolution of an asymmetric double Laplace with a normal distribution.

Finally, we call "exponential tails normal" (ETN) to the exponentiated version of the PTLN of [31, 32, 2], and is as follows

$$f_{\mathrm{ETN}}(g;\alpha,\tau_l,\mu,\sigma,\tau_r,\beta)=\begin{cases} de\exp(\alpha g), & -\infty<g\le\tau_l\\ df_{\mathrm{N}}(g;\mu,\sigma), & \tau_l\le g\le\tau_r\\ dc\exp(-\beta g), & \tau_r\le g<\infty\end{cases} \tag{7}$$

where the continuity constants are $e=\dfrac{f_{\mathrm{N}}(\tau_l;\mu,\sigma)}{\exp(\tau_l\alpha)}$, $c=\dfrac{f_{\mathrm{N}}(\tau_r;\mu,\sigma)}{\exp(-\tau_r\beta)}$, and the normalization constant d is given by

$$d=\left(\frac{1}{\alpha}f_{\mathrm{N}}(\tau_l;\mu,\sigma)+\Phi(\tau_r;\mu,\sigma)-\Phi(\tau_l;\mu,\sigma)+\frac{1}{\beta}f_{\mathrm{N}}(\tau_r;\mu,\sigma)\right)^{-1}. \tag{8}$$

The ETN distribution has, by construction, exponential tails and normal body separated by two definite thresholds: τ_l (left tail-body) and τ_r (right tail-body). This distribution is very appropriate when modelling data that have exponential tails, which can originate straight lines in log-rank or log-corank plots as we will see later.

2.2. The generating mechanisms

The log-growth rates data we have at hand are collected at specific moments in time, i.e., on a yearly basis, and one could observe the probability density functions at that given times. If one uses a parametric description, the parameters of the corresponding distribution are estimated at those moments in time. If one interpolates in a smooth way (for example, by higher order polynomials) the values of the parameters for all times, one could construct a time-dependent probability density function $f(g,t)$ that approximates the true one that could be observed if we had log-growth rates data on an approximately continuous time basis.[3] Another possibility could be to estimate a three dimensional stochastic kernel $f(g,t)$ to approximate the true one (see, e.g., [40]). Then, let us derive a stochastic process whose associated $f(g,t)$ is one of the so constructed approximations.

In order to achieve this task, let our log-growth variable $g\in(-\infty,\infty)$, and its dependence of time by g_t. We assume that its evolution or dynamics is governed by the Itô differential equation (see, e.g., [36, 17])

$$dg_t=b(g_t,t)dt+\sqrt{a(g_t,t)}dB_t \tag{9}$$

where B_t is a standard Brownian motion (Wiener process) (see, e.g.,[25, 29] and references therein). The quantity $a(g_t,t)$ corresponds to the *diffusion term*, and $b(g_t,t)$ to the *drift term*. This process can be associated to the *forward Kolmogorov equation* or *Fokker-Planck equation* for the time-dependent probability density function (conditional on the initial data) $f(g,t)$ (see also [15, 16]):

$$\frac{\partial f(g,t)}{\partial t}=-\frac{\partial}{\partial g}(b(g,t)f(g,t))+\frac{1}{2}\frac{\partial^2}{\partial g^2}(a(g,t)f(g,t)). \tag{10}$$

Since the approximate probability density function $f(g,t)$ is evolving on time and perhaps there is no limiting stationary distribution, let us propose a way of solving (10) for the cited $f(g,t)$ by specifying the diffusion term and the drift term (see, e.g., [37] for another recent approach to time-dependent solutions of the Fokker–Planck equation). In fact, if we take $a(g,t)=s^2$, where $s>0$ is a real constant, then by choosing

$$b(g,t)=\frac{s^2}{2f(g,t)}\frac{\partial f(g,t)}{\partial g}-\frac{1}{f(g,t)}\frac{\partial\,\mathrm{cdf}(g,t)}{\partial t} \tag{11}$$

where $\mathrm{cdf}(g,t)$ is the CDF corresponding to $f(g,t)$, it is solved (10) for $f(g,t)$ [9, 10]. However, we remark that we do not claim that the solution of (10) with this choice of $a(g,t)$ and $b(g,t)$ is unique, only that $f(g,t)$ is a solution by

[3]It is not inconceivable that this could be possible in the near future.

construction. Also, $b(g,t)$ might have bounded discontinuities in the variable g, as in the case of the ETN, in a finite number of points in the domain [18]. And a third remark is that we may add to the expression of $b(g,t)$ in (11) a term of the form $h(t)/f(g,t)$, where $h(t)$ is an arbitrary function of t.

With this set-up in mind, the corresponding expressions for the parameterizations given by the adLN and the ETN are very long to be shown here, and seem not to provide a special insight;[4] however, for the normal (N), 2-mixture of normal distributions (2N) and 3-mixture of normal distributions (3N) the expressions take rather remarkable forms, which we show next.

For the normal (N) case, let us denote for the sake of brevity the corresponding time-dependent density function by

$$j_{\mathrm{N}}(g,t) = f_{\mathrm{N}}(g;\mu(t),\sigma(t))\,.$$

Also, let us denote for convenience the following expression:

$$k(g;\mu,\sigma,s) = \dot{\mu} + \frac{g-\mu}{2\sigma^2}\left(2\sigma\dot{\sigma} - s^2\right)$$

where μ is real and $\sigma > 0$ are supposed to depend smoothly on t (explicit dependence is omitted for notational simplicity), the dot means derivative with respect to t, and $s > 0$ is a real constant. Then, if we select $a(g,t) = s^2$ and $b(g,t) = k(g;\mu,\sigma,s)$ then $f(g,t) = j_{\mathrm{N}}(g,t)$ solves the corresponding Fokker–Plank equation (10). Note that if μ,σ are constants, the drift term just becomes

$$-\frac{s^2}{2\sigma^2}(g-\mu)$$

so, up to a redefinition of s,σ, we recover as a special case the well-known *mean reverting process* of [49, 26, 50].

For the 2-mixture of normal distributions (2N), let us denote likewise

$$j_{2\mathrm{N}}(g,t) = f_{2\mathrm{N}}(g;\mu_1(t),\sigma_1(t),\mu_2(t),\sigma_2(t),p_1(t))\,,$$

the quantity

$$\pi_1(g,t) = \frac{\Phi(g;\mu_1(t),\sigma_1(t)) - \Phi(g;\mu_2(t),\sigma_2(t))}{j_{2\mathrm{N}}(g,t)}$$

and the time-dependent *posterior probabilities* (see, e.g., [35])

$$\tau_1(g,t) = p_1(t) f_{\mathrm{N}}(g;\mu_1(t),\sigma_1(t))/j_{2\mathrm{N}}(g,t)$$
$$\tau_2(g,t) = (1-p_1(t)) f_{\mathrm{N}}(g;\mu_2(t),\sigma_2(t))/j_{2\mathrm{N}}(g,t)$$

Then, let the diffusion and drift terms be defined by $a(g,t) = s^2$ for $s > 0$ constant, and

$$b(g,t) = k(g;\mu_1,\sigma_1,s)\tau_1(g,t) + k(g;\mu_2,\sigma_2,s)\tau_2(g,t) - \dot{p}_1\pi_1(g,t) \qquad (12)$$

so that we obtain that $f(g,t) = j_{2\mathrm{N}}(g,t)$ in this case is a solution of the corresponding Fokker–Planck equation (10). Note that the sign of the drift term depends on different contributions: The sign of the individual k's, that in particular depend on how $\mu_i(t),\sigma_i(t)$, $i = 1,2$, do behave, the $\tau_i(g,t)$, $i = 1,2$, are always positive (and take values between 0 and 1) and also the sign of the difference of the CDF's corresponding to the first component and the last one in the mixture, multiplied by the sign of the derivative of the mixing parameter $\dot{p}_1(t)$. So in general the drift term has no definite sign and a complex behaviour is allowed. This is also a time-dependent generalization of a stationary model presented for the first time, as far as we know, in [7].

For the 3-mixture of normal distributions (3N) it is rather similar to the case of the 2N. Let us denote then

$$j_{3\mathrm{N}}(g,t) = f_{3\mathrm{N}}(g;\mu_1(t),\sigma_1(t),\mu_2(t),\sigma_2(t),\mu_3(t),\sigma_3(t),p_1(t),p_2(t))\,,$$

[4] But such developments are available upon request from the authors.

the quantities

$$\pi_1(g,t) = \frac{\Phi(g;\mu_1(t),\sigma_1(t)) - \Phi(g;\mu_3(t),\sigma_3(t))}{j_{3\mathrm{N}}(g,t)}$$
$$\pi_2(g,t) = \frac{\Phi(g;\mu_2(t),\sigma_2(t)) - \Phi(g;\mu_3(t),\sigma_3(t))}{j_{3\mathrm{N}}(g,t)}$$

and the time-dependent posterior probabilities

$$\tau_1(g,t) = p_1(t) f_{\mathrm{N}}(g;\mu_1(t),\sigma_1(t))/j_{3\mathrm{N}}(g,t)$$
$$\tau_2(g,t) = p_2(t) f_{\mathrm{N}}(g;\mu_2(t),\sigma_2(t))/j_{3\mathrm{N}}(g,t)$$
$$\tau_3(g,t) = (1 - p_1(t) - p_2(t)) f_{\mathrm{N}}(g;\mu_3(t),\sigma_3(t))/j_{3\mathrm{N}}(g,t)$$

Then, let us take the diffusion and drift terms be defined respectively by $a(g,t) = s^2$ for $s > 0$ constant, and

$$b(g,t) = k(g;\mu_1,\sigma_1,s)\tau_1(g,t) + k(g;\mu_2,\sigma_2,s)\tau_2(g,t) + k(g;\mu_3,\sigma_3,s)\tau_3(g,t) - \dot{p}_1\pi_1(g,t) - \dot{p}_2\pi_2(g,t) \qquad (13)$$

so that we obtain that $f(g,t) = j_{3\mathrm{N}}(g,t)$ again in this case is a solution of the corresponding Fokker–Planck equation (10). The sign of the drift term is also indefinite on this occasion, and the interpretation of the contributing terms is similar to the one of the case of the 2N. This is once more a time-dependent generalization of the stationary 3N model of [7].

3. The datasets

We have taken the data from the source mentioned in [2] and [1], that is, the Oak Ridge National Laboratory (ORNL) for 207 countries and territories (in the cited reference these countries and territories are listed explicitly; we refer the reader therein for more information), although we have enlarged the previously considered years (2000, 2005, 2010) [2] to 1994, 1995, 1999, 2004 and 2009. We have considered both the CO_2 emissions and the CO_2 emissions per capita (CO_2pc). The descriptive statistics are shown, respectively, in Tables 1 and 2. On these tables, the skewness of the different samples is remarkable and above all, the kurtosis departs in a great measure from that of the normal distribution, and this fact may explain the previous difficulties in modelling statistically the log-growth rates of CO_2/CO_2pc emissions.

CO_2 lgr	Obs	Mean	SD	Skewness	Kurtosis	Min	Max
1994-1995	207	0.038	0.191	2.34	23.977	-0.971	1.358
1999-2000	207	0.03	0.124	2.161	22.643	-0.53	0.976
2004-2005	207	0.028	0.08	0.641	6.157	-0.251	0.348
2009-2010	207	0.052	0.113	0.323	7.361	-0.386	0.524
1994-1999	206	0.066	0.355	1.695	15.026	-1.145	2.526
1995-2000	207	0.13	0.291	0.323	7.886	-1.159	1.392
1999-2004	207	0.17	0.282	2.524	22.177	-0.693	2.416
2000-2005	207	0.168	0.259	2.77	26.285	-0.693	2.338
2004-2009	207	0.116	0.266	1.659	13.292	-0.536	1.964
2005-2010	207	0.14	0.247	1.72	13.201	-0.441	1.852
1994-2004	207	0.308	0.525	3.8	34.706	-0.976	5.063
1995-2005	207	0.298	0.422	2.521	20.517	-0.827	3.603
1999-2009	207	0.285	0.393	1.066	7.933	-1.039	2.295
2000-2010	207	0.307	0.382	1.521	9.975	-0.693	2.392
1995-2010	207	0.438	0.509	1.314	9.763	-0.882	3.596

Table 1: Descriptive statistics of the CO_2 emissions' log-growth rates samples.

4. Results

We have estimated the parameters of the five densities considered by maximum log-likelihood (ML). The results are available from the authors upon request. Among several possibilities we have defined four log-growth rates one year apart (short run), six five years apart (medium run), four ten years apart and one fifteen years apart (long run).

In Tables 3 and 4 we show standard Kolmogorov-Smirnov (KS), Cramér-von Mises (CM) and Anderson-Darling (AD) tests in order to assess the goodness-of-fit for CO_2 and CO_2pc log-growth rates, respectively. A first conclusion from the Tables stems from the fact that the estimation algorithm does not always converge and for some years,

CO_2pc lgr	Obs	Mean	SD	Skewness	Kurtosis	Min	Max
1994-1995	206	0.02	0.198	2.337	22.955	-0.981	1.386
1999-2000	206	0.021	0.12	3.168	25.526	-0.288	0.982
2004-2005	207	0.013	0.08	-0.28	9.282	-0.405	0.325
2009-2010	207	0.038	0.123	1.006	9.188	-0.405	0.693
1994-1999	206	0.066	0.355	1.695	15.026	-1.145	2.526
1995-2000	205	0.067	0.275	0.71	7.695	-1.113	1.293
1999-2004	207	0.105	0.268	2.631	23.093	-0.641	2.266
2000-2005	206	0.097	0.24	3.199	29.448	-0.565	2.174
2004-2009	207	0.033	0.262	1.711	15.821	-0.693	1.946
2005-2010	207	0.058	0.248	2.615	23.23	-0.553	2.079
1994-2004	206	0.171	0.498	4.105	38.165	-1.012	4.792
1995-2005	206	0.164	0.393	2.729	21.469	-0.693	3.272
1999-2009	207	0.137	0.361	1.091	8.812	-1.157	1.963
2000-2010	206	0.155	0.344	1.555	10.621	-0.693	2.079
1995-2010	206	0.222	0.457	1.539	10.388	-0.981	3.085

Table 2: Descriptive statistics of the CO_2pc emissions per capita log-growth rates samples.

	N			2N		
CO_2 lgr	KS	CM	AD	KS	CM	AD
1994-1995	0 (0.210)	0 (3.422)	0 (17.863)	**0.470 (0.059)**	**0.635 (0.090)**	**0.641 (0.607)**
1999-2000	0 (0.148)	0 (1.730)	0 (9.754)	**0.855 (0.042)**	**0.906 (0.045)**	**0.952 (0.281)**
2004-2005	0.003 (0.127)	0.004 (0.916)	0.003 (4.992)	**0.711 (0.049)**	**0.926 (0.041)**	**0.970 (0.250)**
2009-2010	0.017 (0.108)	0.008 (0.778)	0.004 (4.786)	**0.982 (0.032)**	**0.964 (0.033)**	**0.938 (0.300)**
1994-1999	0.001 (0.136)	0.001 (1.138)	0.001 (6.258)	**0.271 (0.070)**	**0.510 (0.117)**	**0.625 (0.624)**
1995-2000	0.007 (0.117)	0.019 (0.631)	0.010 (3.850)	**0.784 (0.046)**	**0.885 (0.049)**	**0.946 (0.289)**
1999-2004	0.003 (0.125)	0.004 (0.912)	0.002 (5.286)	**0.498 (0.058)**	**0.382 (0.152)**	**0.453 (0.839)**
2000-2005	0 (0.143)	0 (1.270)	0 (7.127)	**0.282 (0.069)**	**0.207 (0.236)**	**0.228 (1.315)**
2004-2009	**0.306 (0.067)**	**0.209 (1.535)**	**0.168 (1.535)**	–	–	–
2005-2010	0.043 (0.096)	**0.099 (0.349)**	**0.084 (2.067)**	–	–	–
1994-2004	0.001 (0.136)	0 (1.311)	0 (7.507)	**0.931 (0.038)**	**0.969 (0.032)**	**0.987 (0.212)**
1995-2005	0.013 (0.110)	0.013 (0.702)	0.007 (4.194)	**0.997 (0.028)**	**0.999 (0.018)**	**0.998 (0.155)**
1999-2009	**0.126 (0.082)**	**0.096 (0.355)**	**0.051 (2.482)**	**0.230 (0.072)**	**0.447 (0.133)**	**0.412 (0.904)**
2000-2010	**0.063 (0.091)**	**0.067 (0.413)**	0.043 (2.610)	–	–	–
1995-2010	**0.219 (0.073)**	**0.158 (0.276)**	**0.105 (1.892)**	**0.983 (0.032)**	**0.983 (0.028)**	**0.985 (0.216)**

	3N			adLN		
CO_2 lgr	KS	CM	AD	KS	CM	AD
1994-1995	**0.628 (0.052)**	**0.908 (0.045)**	**0.974 (0.244)**	**0.065 (0.091)**	0.047 (0.473)	0.031 (2.907)
1999-2000	**0.963 (0.035)**	**0.988 (0.026)**	**0.999 (0.148)**	**0.615 (0.053)**	**0.580 (0.101)**	**0.454 (0.838)**
2004-2005	**0.982 (0.032)**	**0.994 (0.022)**	**0.998 (0.155)**	**0.796 (0.045)**	**0.785 (0.065)**	**0.826 (0.422)**
2009-2010	**0.983 (0.032)**	**0.994 (0.022)**	**0.999 (0.143)**	**0.799 (0.045)**	**0.898 (0.046)**	**0.780 (0.467)**
1994-1999	–	–	–	–	–	–
1995-2000	**0.977 (0.033)**	**0.978 (0.030)**	**0.997 (0.170)**	**0.730 (0.048)**	**0.877 (0.050)**	**0.868 (0.380)**
1999-2004	**0.907 (0.039)**	**0.960 (0.034)**	**0.970 (0.251)**	–	–	–
2000-2005	**0.902 (0.040)**	**0.949 (0.037)**	**0.948 (0.286)**	–	–	–
2004-2009	–	–	–	**0.915 (0.039)**	**0.933 (0.040)**	**0.959 (0.270)**
2005-2010	–	–	–	**0.975 (0.033)**	**0.989 (0.025)**	**0.986 (0.214)**
1994-2004	**0.999 (0.023)**	**0.999 (0.013)**	**0.999 (0.128)**	**0.845 (0.043)**	**0.754 (0.070)**	**0.751 (0.495)**
1995-2005	–	–	–	**0.976 (0.033)**	**0.972 (0.031)**	**0.974 (0.243)**
1999-2009	**0.649 (0.051)**	**0.895 (0.047)**	**0.955 (0.275)**	**0.357 (0.064)**	**0.708 (0.077)**	**0.651 (0.597)**
2000-2010	–	–	–	**0.993 (0.030)**	**0.987 (0.026)**	**0.967 (0.256)**
1995-2010	**0.999 (0.023)**	**0.999 (0.013)**	**0.999 (0.116)**	**0.979 (0.033)**	**0.964 (0.033)**	**0.962 (0.265)**

	ETN		
CO_2 lgr	KS	CM	AD
1994-1995	**0.063 (0.092)**	0.045 (0.481)	0.028 (2.975)
1999-2000	**0.596 (0.053)**	**0.579 (0.101)**	**0.445 (0.852)**
2004-2005	–	–	–
2009-2010	–	–	–
1994-1999	**0.266 (0.070)**	**0.525 (0.113)**	**0.600 (0.653)**
1995-2000	0.005 (0.121)	0.008 (0.784)	0.006 (4.269)
1999-2004	**0.819 (0.044)**	**0.842 (0.055)**	**0.802 (0.446)**
2000-2005	**0.940 (0.037)**	**0.997 (0.020)**	**0.986 (0.215)**
2004-2009	**0.995 (0.029)**	**0.999 (0.018)**	**0.999 (0.150)**
2005-2010	**0.962 (0.035)**	**0.999 (0.017)**	**0.997 (0.162)**
1994-2004	**0.894 (0.040)**	**0.816 (0.060)**	**0.809 (0.439)**
1995-2005	**0.994 (0.029)**	**0.996 (0.021)**	**0.994 (0.186)**
1999-2009	0.015 (0.109)	**0.055 (0.446)**	**0.067 (2.254)**
2000-2010	**0.960 (0.035)**	**0.955 (0.035)**	**0.940 (0.298)**
1995-2010	–	–	–

Table 3: Outcomes of the KS, CM and AD tests in the format *p*-value (statistic) for the CO_2 log-growth rates' samples. Non-rejections at the 5% level are marked in bold.

	N			2N		
CO_2pc lgr	KS	CM	AD	KS	CM	AD
1994-1995	0 (0.207)	0 (3.687)	0 (18.796)	0.011 (0.112)	0.063 (0.423)	0.094 (1.978)
1999-2000	0 (0.187)	0 (2.550)	0 (13.151)	0.016 (0.109)	**0.157 (0.277)**	**0.224 (1.326)**
2004-2005	0 (0.152)	0 (1.749)	0 (9.032)	0.005 (0.121)	**0.118 (0.320)**	**0.186 (1.462)**
2009-2010	0 (0.136)	0 (1.421)	0 (7.885)	0.045 (0.096)	**0.303 (0.183)**	**0.425 (0.882)**
1994-1999	0.001 (0.137)	0.001 (1.132)	0.001 (6.241)	**0.372 (0.064)**	**0.531 (0.112)**	**0.642 (0.607)**
1995-2000	0.001 (0.138)	0.003 (0.964)	0.002 (5.518)	**0.279 (0.069)**	**0.666 (0.085)**	**0.794 (0.453)**
1999-2004	0.001 (0.138)	0.001 (1.254)	0 (6.855)	**0.323 (0.066)**	**0.201 (0.240)**	**0.234 (1.294)**
2000-2005	0.001 (0.141)	0 (1.403)	0 (7.727)	**0.206 (0.074)**	**0.305 (0.182)**	**0.339 (1.036)**
2004-2009	**0.148 (0.079)**	**0.075 (0.394)**	**0.069 (2.234)**	–	–	–
2005-2010	0.003 (0.126)	0.010 (0.740)	0.009 (4.011)	–	–	–
1994-2004	0 (0.151)	0 (1.594)	0 (8.827)	**0.751 (0.047)**	**0.581 (0.101)**	**0.676 (0.571)**
1995-2005	0.001 (0.133)	0.002 (1.090)	0.001 (6.014)	**0.334 (0.066)**	**0.594 (0.098)**	**0.652 (0.596)**
1999-2009	**0.103 (0.085)**	0.043 (0.487)	0.025 (3.081)	**0.082 (0.088)**	**0.320 (0.176)**	**0.376 (0.965)**
2000-2010	0.016 (0.109)	0.045 (0.478)	0.027 (3.026)	**0.888 (0.041)**	**0.951 (0.036)**	**0.967 (0.255)**
1995-2010	0.023 (0.104)	0.023 (0.593)	0.013 (3.637)	**0.401 (0.062)**	**0.466 (0.127)**	**0.603 (0.649)**
	3N			adLN		
CO_2pc lgr	KS	CM	AD	KS	CM	AD
1994-1995	0.019 (0.106)	**0.102 (0.344)**	**0.172 (1.518)**	–	–	–
1999-2000	0.007 (0.118)	**0.230 (0.221)**	**0.364 (0.987)**	–	–	–
2004-2005	–	–	–	–	–	–
2009-2010	0.035 (0.099)	**0.333 (0.171)**	**0.492 (0.785)**	–	–	–
1994-1999	–	–	–	–	–	–
1995-2000	**0.314 (0.067)**	**0.668 (0.084)**	**0.824 (0.424)**	0 (0.149)	0.002 (1.058)	0.001 (5.830)
1999-2004	–	–	–	–	–	–
2000-2005	**0.199 (0.075)**	**0.298 (0.185)**	**0.329 (1.056)**	–	–	–
2004-2009	–	–	–	–	–	–
2005-2010	–	–	–	–	–	–
1994-2004	–	–	–	**0.729 (0.048)**	**0.654 (0.087)**	**0.685 (0.562)**
1995-2005	**0.437 (0.061)**	**0.441 (0.134)**	**0.476 (0.807)**	–	–	–
1999-2009	**0.770 (0.046)**	**0.962 (0.034)**	**0.991 (0.198)**	**0.514 (0.057)**	**0.673 (0.083)**	**0.660 (0.587)**
2000-2010	**0.778 (0.046)**	**0.580 (0.101)**	**0.654 (0.593)**	**0.673 (0.050)**	**0.874 (0.050)**	**0.869 (0.379)**
1995-2010	**0.970 (0.034)**	**0.976 (0.030)**	**0.973 (0.246)**	**0.566 (0.055)**	**0.572 (0.103)**	**0.668 (0.579)**
	ETN					
CO_2pc lgr	KS	CM	AD			
1994-1995	0 (0.154)	0.013 (0.697)	0.017 (3.398)			
1999-2000	–	–	–			
2004-2005	–	–	–			
2009-2010	–	–	–			
1994-1999	–	–	–			
1995-2000	–	–	–			
1999-2004	**0.179 (0.076)**	**0.359 (0.161)**	**0.299 (1.122)**			
2000-2005	**0.305 (0.068)**	**0.383 (0.152)**	**0.369 (0.978)**			
2004-2009	–	–	–			
2005-2010	**0.336 (0.066)**	**0.687 (0.081)**	**0.733 (0.513)**			
1994-2004	**0.745 (0.047)**	**0.666 (0.085)**	**0.694 (0.552)**			
1995-2005	**0.517 (0.057)**	**0.640 (0.089)**	**0.688 (0.559)**			
1999-2009	**0.477 (0.059)**	**0.806 (0.061)**	**0.814 (0.434)**			
2000-2010	**–**	**–**	**–**			
1995-2010	**0.749 (0.047)**	**0.733 (0.073)**	**0.825 (0.424)**			

Table 4: Outcomes of the KS, CM and AD tests in the format p-value (statistic) for the CO_2pc log-growth rates' samples. Non-rejections at the 5% level are marked in bold.

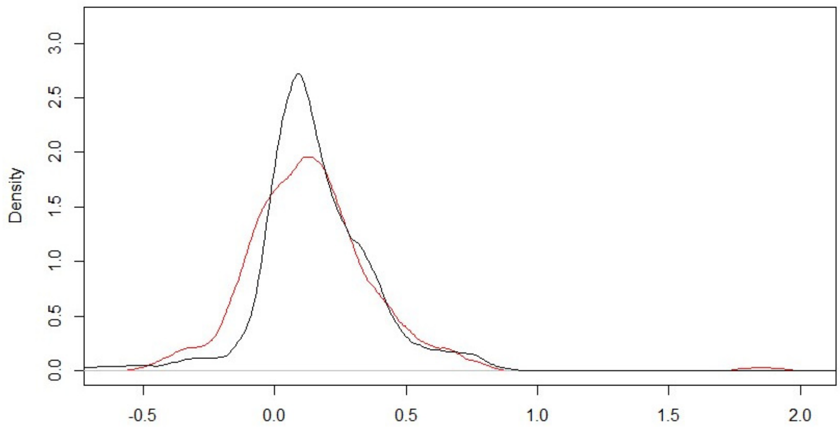

Figure 1: Kernels of CO_2 log-growth rates. 2000-2005 in black, 2005-2010 in red.

especially in Table 4, we simply cannot find the ML estimators. As stated above, the peculiar skewness and kurtosis of the data may explain this difficulty for achieving convergence. Another possible explanation, of a very different nature, may be related to the Kyoto Protocol, which entered into force since February 2005, and the Clean Development Mechanism (CDM), which was settled in 2006 and allowed purchases and sales of greenhouse gases between countries.

Moreover, other trading emissions mechanisms, as the European Union Emissions Trading Scheme, founded in 2005 as the first large emissions trading scheme in the world, were thought to reduce the CO_2 emissions for each industry.

The results of this paper suggest that there is a reallocation of CO_2 emissions in the studied countries every year, probably higher since the implementation of these policies. Many of the non-convergence periods coincide with those years and, therefore, both the Kyoto Protocol and the CDM might have had an impact on the CO_2/CO_2pc emissions, at least from a statistical point of view. Other works have already considered the period 2005-2009 as different from 1995-2005 regarding the CO_2 emissions. For instance, [3] show that in the 1995-2005 period there was an increase in emissions, in contrast to the decrease from 2005-2009. Nonetheless, they think that this shift is due to the impact of the Global Financial Crisis (GFC) on the sample. However, in contrast to our approach, neither the Kyoto protocol (2005) nor the CDM effects are considered.

In this context, we have explored whether the implementation of these trading mechanisms and the Kyoto's protocol has produced some effects in the intensity of the emissions. To do so, we have estimated two Epanechnikov kernels with adaptive bandwidth, one for the CO_2 log-growth rates (Figure 1) and one for the CO_2pc log-growth rates (Figure 2), distinguishing between two periods: pre-protocol (2000-2005) and post-protocol (2005-2010). In both cases, the kernel in red (2005-2010) is more platykurtic and is slightly but significantly shifted left, which reveals that the emissions in the post-protocol period show lower growth rates than those in the pre-protocol period.

Returning to the analysis of the estimated distributions, we can see that the N is nearly always rejected (non rejections in bold) and, therefore, it does not offer a good description of the data. The 2N performs well for CO_2 but not so for CO_2pc, with four rejections. The 3N behaves similarly to the 2N. The adLN, whenever it can be estimated, is in general non rejected. The ETN, when estimated, is rejected in three periods for CO_2 emissions and in one for CO_2pc. As we see, the evidence regarding the performance of the different densities is mixed and, in any case, not conclusive. Once the goodness-of-fit is examined, we turn our attention to the selection of the most appropriate model, out of those studied, by means of information criteria. We use the corrected Akaike Information Criterion like in [2]:

$$AIC_c = 2k - 2\hat{L} + \frac{2(k+1)(k+2)}{n-k-2}$$

where k is the number of estimated parameters, n is the sample size and $\hat{L}$ is the maximum log-likelihood of the

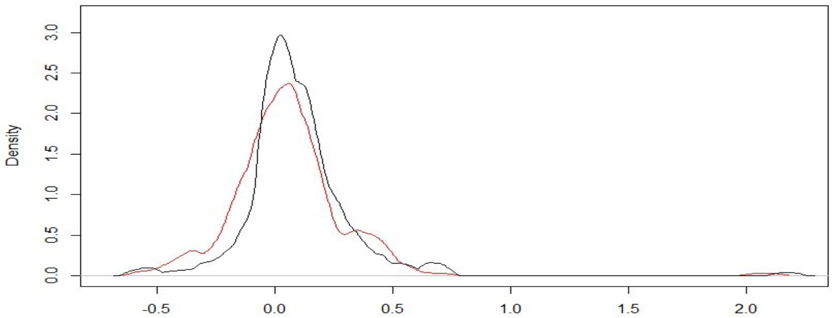

Figure 2: Kernels of CO_2pc log-growth rates. 2000-2005 in black, 2005-2010 in red.

corresponding model [24]. Likewise, we adopt as well the adjusted Bayesian or Schwartz Information Criterion:

$$BIC_a = k \ln\left(\frac{n+2}{24}\right) - 2\hat{L}$$

see [47] for details. The minimum values of each of these criteria point out to the selected model according to it.

For log-growth rates of CO_2 emissions we see in Table 5 that, according to both criteria, the 3N and the adLN are selected five times each, while the ETN is the best in three periods and the 2N in two. In Table 6 the variability is even larger. In fact, the 2N, the 3N and the ETN outperform the other densities four times each, the adLN two times and the N, due to the fact that it is the only distribution that can be estimated in that period, once.

As a complementary approach, we show in Figures 3 and 4 the plots of the log-ranks and log-coranks, respectively, for the last samples of each type of period (one, five, ten and fifteen years), both for CO_2 and CO_2pc emissions' log-growth rates. The variability in the selected models is covered with these choices, and there is at least one example of the non-rejected models (except the N) with these selections. When the 2N or the 3N is selected, we can observe remarkable curvatures at the tails, when the adLN is selected the curvature is lighter, and when the ETN is selected, we observe practically straight lines at the tails as expected. The presence of these curvatures or not at the tails might explain the variability in the most appropriate model in each case, and even from one year to another the curvature of the tails might change remarkably, making difficult to model these data sets with a single specification. However, in the shown graphs we observe very good fits for each selected model, so the case-by-case modelling yields good results. We recall that by taking the logs of the ranks or coranks the discrepancies at the tails are amplified (see, e.g., [19]). The fact that the tails of the log-growth rates are not always exponential (not straight lines in log-rank/corank plots) is not new; for example in [34] it has been observed for log-growth rates of city sizes, and in other fields, including economics, by [12, 13, 14, 8, 11].

5. Discussion

Now, the relevant question to be answered is as follows: What is really behind the growth rate of CO_2? Undoubtedly, speaking in general terms, there is some kind of underlying physical process. Maybe the process with higher explicative power is directly related to human activities. And human activities take place, with an increasing intensity, in urban areas. The effects of urbanization on CO_2 emissions have been analyzed in two ways: first, the urban scaling hypothesis, according to which city emissions are well described by a power law function of population [4]; second, the study of how population density influences CO_2 emissions per capita [23]. Both approaches ignore that population and area can be correlated and might affect emissions in an interconnected way, something that is explicitly considered in the framework defined by [45]. These authors conclude, on the one hand, that the key urban variable which has a greater effect on emissions is population changes and, on the other hand, for US areas, the larger the urban nuclei, the higher is the impact of changing its population. In this context, the effects of urbanization on CO_2 emissions, it is also important to take into account the concept of "active population" of a city, used by [30]. The

	N			2N		
CO_2 lgr	log-likelihood	AICc	BICa	log-likelihood	AICc	BICa
1994-1995	49.2459	-94.3737	-94.1633	158.3	-306.18	-305.778
1999-2000	138.323	-272.528	-272.318	204.924	-399.427	-399.026
2004-2005	228.655	-453.191	-452.981	257.467	**-504.515**	**-504.114**
2009-2010	157.532	-310.947	-310.736	188.281	-366.143	**-365.742**
1994-1999	-78.696	161.512	161.712	-38.753	87.928	88.303
1995-2000	-37.923	79.964	80.174	-11.126	**32.672**	**33.074**
1999-2004	-30.975	66.069	66.279	7.736	-5.052	-4.651
2000-2005	-13.888	31.894	32.104	32.41	-54.399	-53.998
2004-2009	-18.853	41.823	42.034	–	–	–
2005-2010	-3.907	11.932	12.142	–	–	–
1994-2004	-159.859	323.836	324.046	-105.387	221.194	221.596
1995-2005	-114.744	233.606	233.816	-80.768	171.956	172.358
1999-2009	-99.867	203.851	204.062	-82.198	174.816	175.217
2000-2010	-93.879	191.877	192.087	–	–	–
1995-2010	-153.389	310.897	311.107	-137.006	284.433	284.834
	3N			adLN		
CO_2 lgr	log-likelihood	AICc	BICa	log-likelihood	AICc	BICa
1994-1995	169.097	**-321.28**	**-320.879**	141.184	-274.07	-273.711
1999-2000	208.395	**-399.877**	**-399.476**	197.716	-387.134	-386.775
2004-2005	260.522	-504.13	-503.729	255.509	-502.72	-502.361
2009-2010	192.913	**-368.912**	**-368.512**	185.405	-362.512	-362.154
1994-1999	**–**	**–**	**–**	**–**	**–**	**–**
1995-2000	-9.439	35.792	36.193	-13.622	35.542	35.901
1999-2004	11.391	-5.867	-5.467	**–**	**–**	**–**
2000-2005	43.598	-70.281	-69.881	**–**	**–**	**–**
2004-2009	–	–	–	-0.966	**10.231**	**10.589**
2005-2010	–	–	–	15.625	**-22.952**	**-22.593**
1994-2004	-102.108	**221.129**	**221.53**	-106.599	221.497	221.856
1995-2005	–	–	–	-80.791	**169.88**	**170.238**
1999-2009	-74.127	**165.167**	**165.568**	-81.822	171.942	172.301
2000-2010	–	–	–	-71.939	**152.176**	**152.534**
1995-2010	-134.947	286.808	287.208	-136.614	**281.526**	**281.884**
	ETN					
CO_2 lgr	log-likelihood	AICc	BICa			
1994-1995	141.254	-269.946	-269.523			
1999-2000	197.782	-383	-382.578			
2004-2005	–	–	–			
2009-2010	–	–	–			
1994-1999	-34.596	**81.758**	**82.149**			
1995-2000	-13.426	39.414	39.837			
1999-2004	10.854	**-9.145**	**-8.722**			
2000-2005	44.869	**-77.175**	**-76.752**			
2004-2009	1.092	10.379	10.802			
2005-2010	17.038	-21.513	-21.09			
1994-2004	-105.68	223.923	224.346			
1995-2005	-80.096	172.754	173.177			
1999-2009	-84.675	181.912	182.335			
2000-2010	-71.821	156.205	156.627			
1995-2010	–	–	–			

Table 5: Maximum log-likelihoods, AIC_c and BIC_a for the CO_2 log-growth rates' samples. The preferred values for each criterion are marked in bold.

	N			2N		
CO_2pc lgr	log-likelihood	AICc	BICc	log-likelihood	AICc	BICc
1994-1995	42.156	-80.1927	-79.9926	154.155	-297.887	-297.512
1999-2000	144.596	-285.072	-284.872	221.843	-433.264	-432.889
2004-2005	230.881	-457.643	-457.432	283.317	**-556.215**	**-555.814**
2009-2010	141.365	-278.612	-278.401	187.956	**-365.492**	**-365.091**
1994-1999	-78.696	161.512	161.712	-38.753	**87.928**	**88.303**
1995-2000	-25.68	55.479	55.669	7.729	**-5.033**	**-4.684**
1999-2004	-20.917	45.953	46.163	21.9	-33.379	-32.98
2000-2005	1.914	0.292	0.492	52.217	-94.012	-93.636
2004-2009	-15.92	35.958	36.168	–	–	–
2005-2010	-4.499	13.117	13.327	–	–	–
1994-2004	-148.332	300.783	300.983	-88.462	187.347	187.722
1995-2005	-99.576	203.271	203.471	-61.937	134.295	134.67
1999-2009	-82.082	168.281	168.491	-61.095	132.61	133.011
2000-2010	-71.986	148.09	148.29	-48.371	107.164	107.539
1995-2010	-130.603	265.325	265.525	-106.845	224.112	224.487
	3N			adLN		
CO_2pc lgr	log-likelihood	AICc	BICc	log-likelihood	AICc	BICc
1994-1995	163.89	**-310.862**	**-310.505**	**–**	**–**	**–**
1999-2000	230.879	-444.839	-444.481	**–**	**–**	**–**
2004-2005	**–**	–	–	**–**	**–**	**–**
2009-2010	189.916	-362.918	-362.518	**–**	**–**	**–**
1994-1999	–	–	–	**–**	**–**	**–**
1995-2000	9.591	-2.259	-1.945	-26.234	60.77	61.087
1999-2004	–	–	–	**–**	**–**	**–**
2000-2005	53.038	-89.157	-88.8	**–**	**–**	**–**
2004-2009	–	–	–	**–**	**–**	**–**
2005-2010	–	–	–	**–**	**–**	**–**
1994-2004	–	–	–	-87.425	**183.15**	**183.488**
1995-2005	-60.874	138.665	139.023	**–**	**–**	**–**
1999-2009	-54.088	**125.089**	**125.49**	-60.238	128.774	129.133
2000-2010	-47.411	111.741	112.099	-49.084	**106.467**	**106.805**
1995-2010	-100.792	**218.502**	**218.86**	-106.634	221.567	221.905
	ETN					
CO_2pc lgr	log-likelihood	AICc	BICc			
1994-1995	157.934	-303.302	-302.911			
1999-2000	**–**	**–**	**–**			
2004-2005	**–**	**–**	**–**			
2009-2010	**–**	**–**	**–**			
1994-1999	**–**	**–**	**–**			
1995-2000	**–**	**–**	**–**			
1999-2004	27.748	**-42.934**	**-42.511**			
2000-2005	58.143	**-103.721**	**-103.33**			
2004-2009	**–**	**–**	**–**			
2005-2010	30.281	**-47.999**	**-47.576**			
1994-2004	-86.023	184.611	185.002			
1995-2005	-54.26	**121.086**	**121.477**			
1999-2009	-59.399	131.36	131.783			
2000-2010	–	–	–			
1995-2010	-103.568	219.702	220.093			

Table 6: Maximum log-likelihoods, AIC_c and BIC_a for the CO_2pc log-growth rates' samples. The preferred values for each criterion are marked in bold.

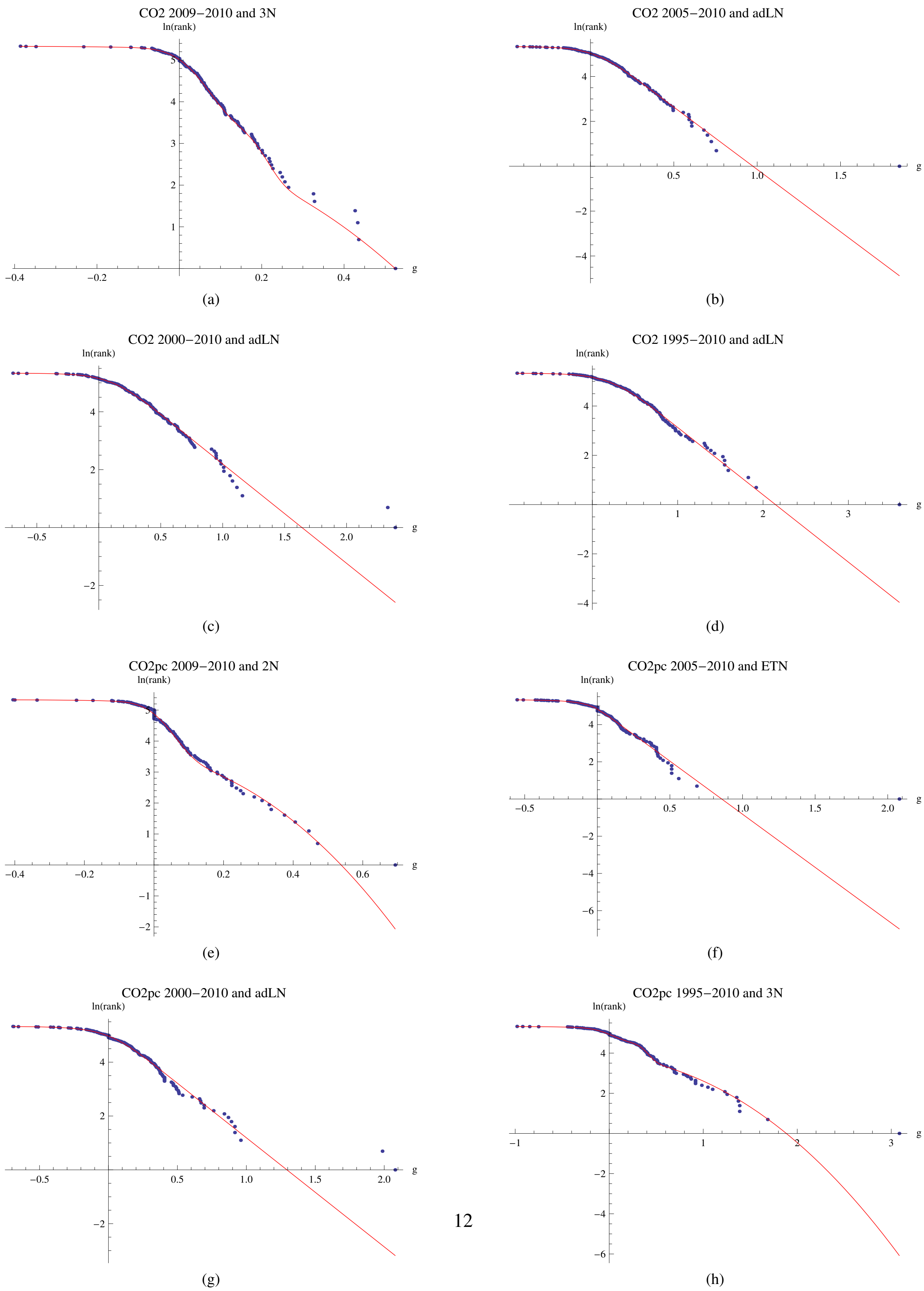

Figure 3: Log-rank plots for the whole samples of log-growth rates, using the best model in each case (red) and the empirical data (blue).

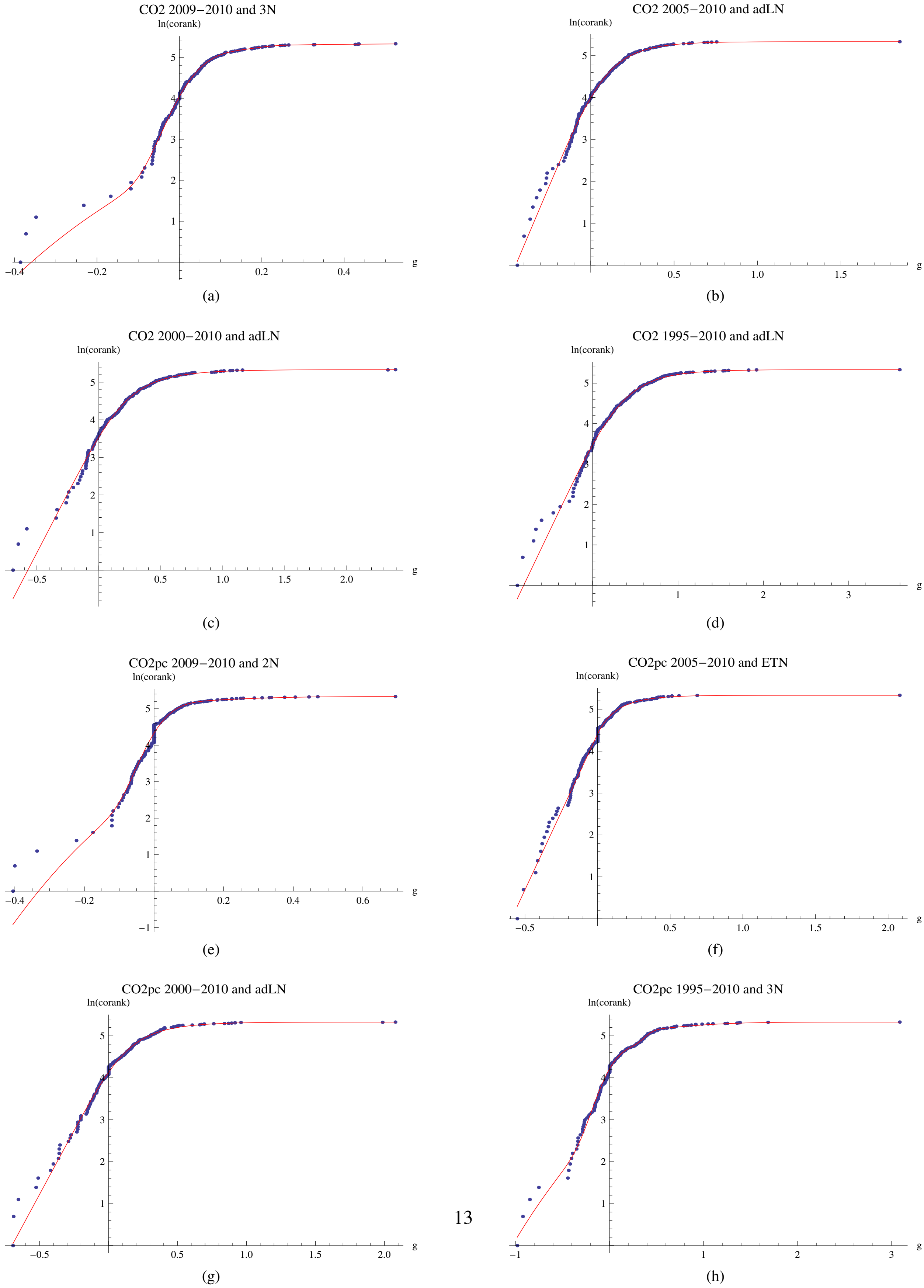

Figure 4: Log-corank plots for the whole samples of log-growth rates, using the best model in each case (red) and the empirical data (blue).

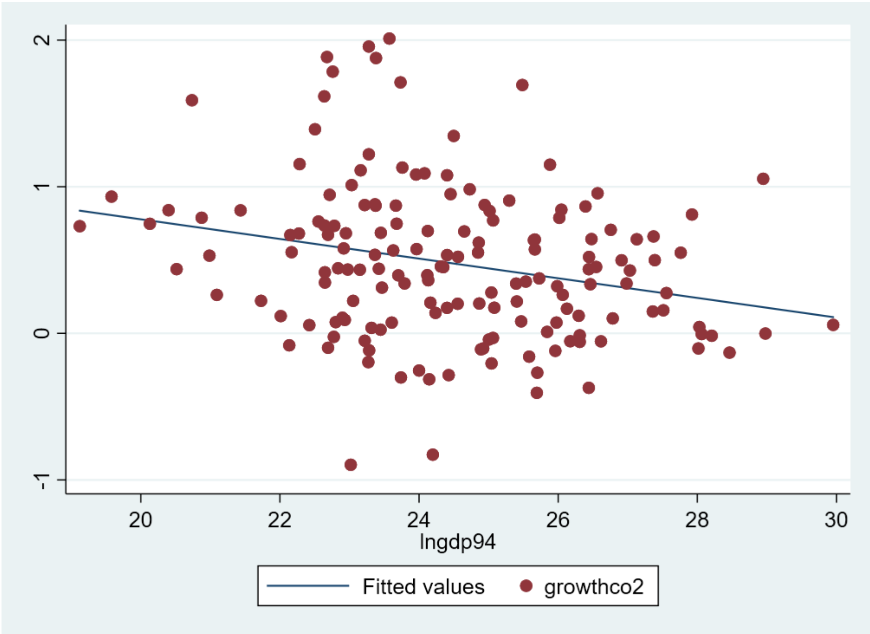

Figure 5: CO_2 log-growth rates between 1994 and 2010 as a function of log of GDP in 1994. Coefficient equal to -0.09, significative at the 5% level.

point is that it is not residential population the main factor that explains emissions, but “a mixture of residential and working population, according to the duration of their activities in the region” [30, page 2]. This is a better proxy for describing socioeconomic interactions, which are the principal origin of emissions. For example[5], in Washington D. C. the residential population is quite small, but the active population in the city is quite large (most of them come to D. C. for work, shopping, or tourism).

Another possible explanation for CO_2 emissions can be found in the heterogeneity in the economic size (Gross Domestic Product, GDP) of the different countries: We have done the analysis looking for a unique distribution of the log-growth rates of CO_2 for all countries, while it is true that there could be differences in how the level of GDP affects this growth. In order to explore this possibility, Figure 5 plots the relationship between the CO_2 log-growth rates from the first to the last period (1994 to 2010) against the log of GDP in 1994. As can be seen, both variables are negatively related.

Following with this Section, devoted to the discussion of the explicative mechanisms underlying CO_2 emissions and CO_2 log-growth rates, let us turn now to the statistical point of view. If we take for simplicity the (almost) never rejected models time-dependent 2N, 3N, we can model the time evolution of the log-growth rates g_t as a Itô (or Stratonovich, since the diffusion term has been chosen to be constant) differential equation [17]

$$dg_t = b(g_t, t)dt + s\, dB_t \tag{14}$$

where $s > 0$ is a real constant and B_t is a standard Brownian motion or Wiener process. Then, the drift term $b(g, t)$ can take the different forms (12) or (13), whose sign is indefinite but depends on clear-cut contributions that come from the different components in the (time-dependent) 2-mixture or 3-mixture of normal distributions. These models, albeit being relatively simple, exhibit an ample richness in the behaviour of the evolution of the log-growth rates, and many scenarios could take place. This diversity might explain as well why different behaviours do occur in practice, and that several models may fit the data at the same time in a reasonable manner. One could even consider a non-parametric point of view for the observed $f(g, t)$, by means of time-dependent stochastic kernels (see, e.g., [40]), and use (11) numerically to deal with the stochastic differential equation (14).

[5]We thank an anonymous referee for suggesting this illustrative example.

And moreover, the facts explained in the preceding three paragraphs are complementary. Indeed, the variation of human activities, urbanization, population, population densities, correlation of population and inhabited area, active population or other important factors not considered so far in the literature might lead to variations of the parameters of the probability density function at hand, or the dependencies of the non-parametric observed $f(g,t)$ if one takes that approach. In either case, a time-varying $f(g,t)$ is observed *a posteriori* but caused by the previous factors, and by means of the stochastic differential equation (14) and the formula for the drift part (11) one can explain the time evolution of the log-growth rates from a statistical mechanics point of view, and the door is open for the simulation of g_t and perhaps forecasting, being this task let aside for future work.

6. Conclusions

We have examined the statistical distribution of the log-growth rates of CO_2 and CO_2pc emissions from a parametric point of view. Three main outcomes emerge. First, this is not a simple task since, in a non-negligible number of cases, the distributions cannot be estimated, especially for CO_2pc emissions. Second, and more importantly, the best density is different depending on the period considered, in such a way that no distribution outperforms the others in a systematic way. This suggests an effective possible reallocation of CO_2 emissions since the establishment of new global institutional and policy measures such as the Kyoto protocol or the Clean Development Mechanism (CDM). Third, the tails of the distributions of the studied log-growth rates are not always exponential, and this happens independently of the length of the period considered, namely one, five, ten or fifteen years. The behavior thus may change from one year to the next one, and possibly this is influenced in a not small amount by policy measures that are taken regarding CO_2 emissions. These regulations are shown to have a clear impact on the changes in the emissions' distribution (that is, the distribution of log-growth rates), so countries and territories in fact do react to the different implementations of policies and carbon markets from one year to another. Therefore, a policy measure that can be derived from this paper is that there are still some countries that can improve the emission of CO_2 compared with others, as seen by the non-uniformity of the outperforming distribution. Also, the analysis of the spatial component of the CO_2 distribution among countries and the possibility of defining a spatial scaling law are interesting topics for future research.[6]

Author contributions

Guillermo Peña: Conceptualization, data curation, formal analysis, investigation, methodology, software, supervision, validation, visualization, writing-original draft, writing-review & editing. Miguel Puente-Ajovín: Conceptualization, data curation, formal analysis, funding acquisition, investigation, methodology, resources, software, validation, visualization, writing-original draft, writing-review & editing. Arturo Ramos: Conceptualization, data curation, formal analysis, funding acquisition, investigation, methodology, resources, software, validation, visualization, writing-original draft, writing-review & editing. Fernando Sanz-Gracia: Conceptualization, data curation, formal analysis, funding acquisition, investigation, methodology, resources, software, validation, visualization, writing-original draft, writing-review & editing.

Competing interests statement

The authors declare to have no competing interests concerning the research carried out in this article.

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References

[1] Ahundjanov, B. B. and Akhundjanov, S. B. (2019). Gibrat's law for CO_2 emissions. *Physica A: Statistical Mechanics and its Applications*, 526:120944.

[2] Akhundjanov, S. B., Devadoss, S., and Luckstead, J. (2017). Size distribution of national CO_2 emissions. *Energy Economics*, 66:182–193.

[3] Ali, Y., Pretaroli, R., Sabir, M., Socci, C., and Severini, F. (2020). Structural changes in carbon dioxide (CO_2) emissions in the United Kingdom (UK): an emission multiplier product matrix (EMPM) approach. *Mitigation and Adaptation Strategies for Global Change*, 25:1545–1564.

[4] Bettencourt, L. M. A. and West, G. (2010). A unified theory of urban living. *Nature*, 467:912–913.

[5] Butnar, I. and Llop, M. (2011). Structural decomposition analysis and input-output subsystems: Changes in CO_2 emissions of Spanish service sectors (2000-2005). *Ecological Economics*, 70:2012–2019.

[6] Băncescu, I., Chivu, L., Preda, V., Puente-Ajovín, M., and Ramos, A. (2019). Comparisons of log-normal mixture and pareto tails, GB2 or log-normal body of Romania's all cities size distribution. *Physica A: Statistical Mechanics and its Applications*, 526:121017.

[7] Campolieti, M. and Ramos, A. (2021). The distribution of strike size: Empirical evidence from Europe and North America in the 19th and 20th centuries. *Physica A: Statistical Mechanics and its Applications*, 563:125424.

[8] Cirillo, P. (2010). An analysis of the size distribution of Italian firms by age. *Physica A: Statistical Mechanics and its Applications*, 389:459–466.

[9] Dupire, B. (1993). Pricing and hedging with smiles. In *Proceedings of AFFI Conference, La Baule, June*.

[10] Dupire, B. (1994). Pricing with a smile. *Risk*, 7(1):271–275.

[11] Eliazar, I. (2014). From entropy-maximization to equality-maximization: Gauss, Laplace, Pareto, and Subbotin. *Physica A: Statistical Mechanics and its Applications*, 415:479–492.

[12] Fagiolo, G. and Luzzi, A. (2006). Do liquidity constraints matter in explaining firm size and growth? Some evidence from the Italian manufacturing industry. *Industrial and Corporate Change*, 15(1):1–39.

[13] Fagiolo, G., Napoletano, M., and Roventini, A. (2007). How do output growth-rate distribuitons look like? Some cross-country, time-series evidence. *European Physical Journal B*, 57:205–211.

[14] Fagiolo, G., Napoletano, M., and Roventini, A. (2008). Are output growth-rate distributions fat-tailed? Some evidence from OECD countries. *Journal of Applied Econometrics*, 23:639–669.

[15] Gabaix, X. (1999). Zipf's law for cities: An explanation. *Quarterly Journal of Economics*, 114:739–767.

[16] Gabaix, X. (2009). Power laws in Economics and finance. *Annu. Rev. Econ.*, 2009:255–293.

[17] Gardiner, C. W. (2004). *Handbook of stochastic methods for Physics, Chemistry and the Natural Sciences (third edition)*. Springer.

[18] Gikhman, I. I. and Skorokhod, A. V. (2007). *The theory of stochastic processes III. Reprint of the 1974 edition.* Springer-Verlag.

[19] González-Val, R., Ramos, A., and Sanz-Gracia, F. (2013). The accuracy of graphs to describe size distributions. *Applied Economics Letters*, 20(17):1580–1585.

[20] Gualandi, S. and Toscani, G. (2018). Call center service times are lognormal: a Fokker–Planck description. *Mathematical Models and Methods in Applied Sciences*, 28(8):1513–1527.

[21] Gualandi, S. and Toscani, G. (2019a). Human behavior and lognormal distribution. A kinetic description. *Mathematical Models and Methods in Applied Sciences*, 29(4):717–753.

[22] Gualandi, S. and Toscani, G. (2019b). Size distribution of cities: A kinetic explanation. *Physica A: Statistical Mechanics and its Applications*, 524:221–234.

[23] Gudipudi, R., Fluschnik, T., Ros, A. G. C., Walther, C., and Kropp, J. P. (2016). City density and CO_2 efficiency. *Energy Policy*, 91:352–361.

[24] Hurvich, C. M. and Tsai, C.-L. (1989). Regression and time series model selection in small samples. *Biometrika*, 76(2):297–307.

[25] Itô, K. and McKean Jr., H. P. (1996). *Diffusion processes and their sample paths (reprint of the 1974 edition)*. Springer.

[26] Kalecki, M. (1945). On the Gibrat distribution. *Econometrica*, 13(2):161–170.

[27] Kim, K. and Kim, Y. (2012). International comparison of industrial CO_2 emission trends and the energy efficiency paradox utilizing production-based decomposition. *Energy Economics*, 34(5):1724–1741.

[28] Kwong, H. S. and Nadarajah, S. (2019). A note on "Pareto tails and lognormal body of US cities size distribution". *Physica A: Statistical Mechanics and its Applications*, 513:55–62.

[29] Kyprianou, A. E. (2006). *Introductory lectures on fluctuations of Lévy processes with applications*. Springer-Verlag.

[30] Li, R., Dong, L., Zhang, J., Wang, X., Wang, W.-X., Di, Z., and Stanley, H. E. (2017). Simple spatial scaling rules behind complex cities. *Nature Communications*, 8:1841.

[31] Luckstead, J. and Devadoss, S. (2017). Pareto tails and lognormal body of US cities size distribution. *Physica A: Statistical Mechanics and its Applications*, 465:573–578.

[32] Luckstead, J., Devadoss, S., and Danforth, D. (2017). The size distributions of all Indian cities. *Physica A: Statistical Mechanics and its Applications*, 474:237–249.

[33] Manas, A. (2009). French butchers don't do quantum physics. *Economics Letters*, 103:101–106.

[34] Massing, T., Puente-Ajovín, M., and Ramos, A. (2020). On the parametric description of log-growth rates of cities' sizes of four European countries and the USA. *Physica A: Statistical Mechanics and its Applications*, 551:124587.

[35] McLachlan, G. and Peel, D. (2003). *Finite mixture models*. Wiley-Interscience.

[36] Ord, J. K. (1974). *Statistical Distributions in Scientific Work. Volume 2–Model Building and Model Selection*, chapter Statistical models for personal income distributions. D. Reidel Publishing Company.

[37] Otunuga, O. M. (2021). Time-dependent probability density function for general stochastic logistic population model with harvesting effort. *Physica A: Statistical Mechanics and its Applications*, 573:125931.

[38] Puente-Ajovín, M., Ramos, A., and Sanz-Gracia, F. (2020a). Is there a universal parametric city size distribution? Empirical evidence for 70 countries. *The Annals of Regional Science*, 65:727–741.

[39] Puente-Ajovín, M., Ramos, A., Sanz-Gracia, F., and Arribas-Bel, D. (2020b). How sensitive is city size distribution to the definition of city? The case of Spain. *Economics Letters*, 197:109643.

[40] Quah, D. T. (1997). Empirics for growth and distribution: Stratification, polarization, and convergence clubs. *Journal of Economic Growth*, 2:27–59.

[41] Ramos, A. (2017). Are the log-growth rates of city sizes distributed normally? Empirical evidence for the USA. *Empirical Economics*, 53:1109–1123.

[42] Reed, W. J. (2002). On the rank-size distribution for human settlements. *Journal of Regional Science*, 42:1–17.

[43] Reed, W. J. (2003). The Pareto law of incomes–an explanation and an extension. *Physica A: Statistical Mechanics and its Applications*, 319:469–486.

[44] Reed, W. J. and Jorgensen, M. (2004). The double Pareto-lognormal distribution–a new parametric model for size distributions. *Communications in Statistics-Theory and Methods*, 33(8):1733–1753.

[45] Ribeiro, H. V., Rybski, D., and Kropp, J. P. (2019). Effects of changing population or density on urban carbon dioxide emissions. *Nature Communications*, 10:3204.

[46] Schmalensee, R., Stoker, T. M., and Judson, R. A. (1998). World carbon dioxide emissions: 1950-2050. *Review of Economics and Statistics*, 80(1):15–28.

[47] Sclove, S. L. (1987). Application of model-selection criteria to some problems in multivariate analysis. *Psychometrika*, 52(2):333–343.

[48] Su, H.-L. (2020). On the city size distribution: A finite mixture interpretation. *Journal of Urban Economics*, 116:103216.

[49] Uhlenbeck, G. E. and Ornstein, L. S. (1930). On the theory of the Brownian motion. *Physical Review*, 36:823–841.

[50] Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5:177–188.