

# Analytical Formulation of Copper Loss of Litz Wire with Multiple Levels of Twisting Using Measurable Parameters

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**Abstract**—Litz wire has been widely utilized in power transformers and inductors as a wire with low copper loss at high-frequency operation. The Litz wire is commonly made of many thin isolated strands twisted in multiple levels. Due to its complicated structure, the copper loss prediction of the Litz wire has been difficult, hindering the design optimization of the Litz wire structure. To overcome this difficulty, preceding studies have investigated the analytical copper loss models of the constituting elements of the Litz wire, i.e. the strands and the bundles of strands. The purpose of this paper is to propose an analytical copper loss model of the Litz wire by utilizing these preceding knowledge. The proposed model is formulated only with parameters that can be measured by basic testing instruments. Besides, the proposed model considers the bundle structure of the Litz wire, which affects the local ac current distribution, and the twisting pitch, which causes the inclination of the Litz wire strands. The proposed model was tested by comparing the analytical prediction and experimental measurements of the ac resistance of commercially available Litz wires. As a result, the predicted ac resistance showed good agreement with the measured ac resistance, suggesting the appropriateness of the proposed model.

**Index Terms**—Ac resistance, analytical model, copper loss, Litz wire, proximity effect

## I. INTRODUCTION

Litz wire is widely utilized in power magnetic devices that carry high-frequency ac current, such as the heating coils of induction heating systems [1]–[3] and the transformers of high-frequency resonant converters [4]–[8]. The Litz wire is made by twisting many thin electrically-isolated strands. The strands are commonly twisted hierarchically in multiple levels in the Litz wire with a large cross-section area, particularly for large ac current applications. For example, a bunch of strands is firstly twisted to form the bundles, and then these bundles are again twisted to form the Litz wire. As pointed out in [9][10], a single twisting process can prevent axial inhomogeneity of the ac current distribution among the twisted strands. Therefore, by combining multiple twisting processes in the hierarchy, the ac current of the Litz wire is expected to flow equally in all the constituent strands. As a result, the Litz wire can mitigate the the copper loss in thick Litz wires even under high-frequency

operation.

However, excessive levels of twisting may increase manufacturing cost and decrease the packing factor, which expands the wire cross-section area, consequently occupying additional volume. Therefore, Litz wire designs commonly require searching for the simplest twisting structure for achieving effective suppression of the copper loss at a given operating frequency.

In this sense, the optimization of the Litz wire designs needs an accurate prediction of the copper loss based on the practically measurable parameters such as the strand diameter, the Litz wire diameter, the number of strands, the number of each level of the bundles, etc. However, the multiple levels of twisting of many thin strands forms a complicated structure, which hinders the numerical [11]–[15] and analytical [16]–[28] approaches to predict the copper loss of the Litz wire.

Certainly, recent progress in computing has enabled the copper loss prediction of the Litz wire with multiple levels of twisting using finite element analysis (FEA) [15]. However, these approaches still require an enormous effort to construct the physical model for the electromagnetic analysis and enormous calculation resources, both of which may still cause obstacles in the practical design optimization of the Litz wire. Meanwhile, the analytical approach tends to result in a simpler physical approximation model and require less calculation resource than the numerical approach, which may be promising for application to practical design optimization of the Litz wire.

In the analytical approach, preceding studies [16]–[21], [25]–[28] have investigated analytical copper loss models of the constituting elements of the Litz wire as the strands and the bundles of strands. These methods provide analytical models of the skin and proximity effect of the stands and the bundle. Among these methods, [26] developed the analytical copper loss model of the Litz wire with multiple levels of twisting based on the analytical models of the strands and the bundles of strands. However, this analytical model may still have difficulty in practical application to Litz wires with multiple levels of twisting because these methods contain parameters such as the packing factor of each level of the bundles, which are difficult to be measured directly.

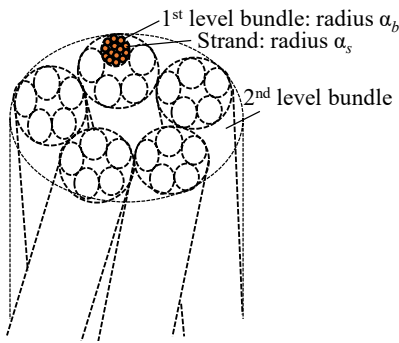


Fig. 1. Litz wire structure under consideration with 3 levels of twisting.

By utilizing these preceding knowledge, this paper derives an analytical copper loss model of the Litz wire based on the parameters that can be measured with basic testing instruments. The proposed model applies certain approximations to simplify the equation so that only physical constants and geometrical parameters are required for the copper loss prediction. In addition to these preceding knowledge, the proposed model further improves the consideration of the bundle structure of the Litz wire, which affects the local ac current distribution, and includes the consideration of the twisting pitch, which causes the inclination of the Litz wire strands.

The approximations of the proposed model are introduced by assuming typical Litz wire structure, as well as typical usage condition of the Litz wire, used for the power magnetic devices. Specifically, the proposed model is derived for the Litz wire under the following conditions:

1. The surface of the strands is coated with non-conductive material for electrical isolation each from the others;
2. The strands are twisted in two or more levels;
3. The lowest level of twisting, i.e. twisting the strands, twists much more than 5 strands to form a bundle, whereas the higher levels of twisting twists no more than 5 bundles to form a bundle of the bundles.
4. The Litz wire does not contain magnetic material, e.g. in the isolation material for coating the strand surface;
5. The Litz wire is used with a sufficiently longer length than the twisting pitch of each level of twisting so that each level of twisting ensures a rotationally symmetrical magnetic condition among the strands or the bundles to be twisted. (Please refer to the appendix for the estimation of the necessary length.)

It is worth noting that these conditions can be applied for many commercial Litz wires in many applications, although these conditions do not cover all the Litz wires and all the applications.

This paper is the updated version of the conference paper [29]. Compared with the conference paper, this paper further incorporates the detailed description of the derivation process of the proposed copper loss model. Besides, this paper added more experimental data for evaluating the proposed model.

The following discussion comprises 4 sections. Section II analytically derives the copper loss model of the Litz wire. In this section, we first briefly review the analytical copper loss model of the strand. Then, we derive the analytical copper loss

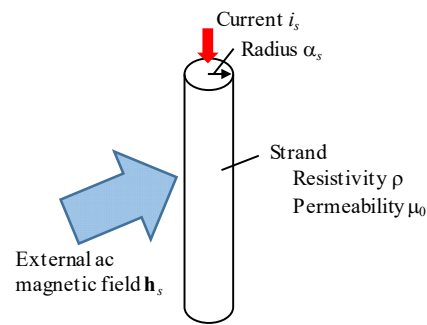


Fig. 2. Strand under consideration.

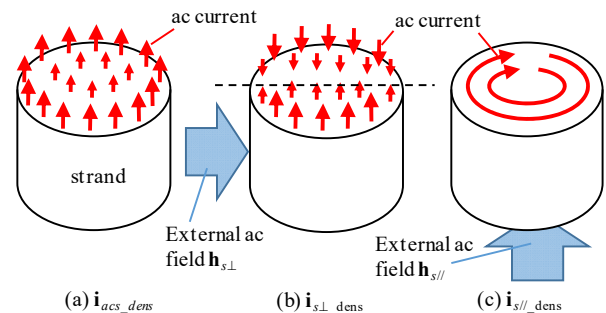


Fig. 3. AC current distributions of the ac current density vector  $\mathbf{i}_{acs\_dens}$ , which constitutes the strand current  $i_s$ , and the eddy current density vectors  $\mathbf{i}_{s\perp\_dens}$  and  $\mathbf{i}_{s\parallel\_dens}$ , which are generated by  $\mathbf{h}_{s\perp}$  and  $\mathbf{h}_{s\parallel}$ , respectively.

model of the bundle of the twisted strands. As a result of the detailed analytical discussions, this paper propose a slight modification of the previous copper loss model of the bundle proposed in [26]. Based on the model of the bundle, we construct the copper loss model of the Litz wire. Section III presents experiments performed to verify the proposed copper loss model of the Litz wire. Finally, section IV gives conclusions.

## II. COPPER LOSS MODEL OF LITZ WIRE

This section formulates the proposed copper loss model of the Litz wire with multiple levels of twisting. For this purpose, a typical Litz wire structure illustrated in Fig. 1 is considered as an example. This Litz wire is assumed to satisfy the requirement conditions listed in the introduction. Because the Litz wire is assumed not to contain magnetic material, linear media is assumed for the electromagnetic analysis.

The Litz wire of Fig. 1 has 3 levels of twisting. This Litz wire (3rd level bundle) is assumed to be formed by twisting no more than 5 bundles (2nd level bundle), each of which is formed by twisting no more than 5 bundles (1st level bundle). Therefore, in these twisting process, the twisted bundles are placed in the rotational symmetry in the cross-section, and they exchange their position rotationally along the Litz wire. However, the 1st level bundles are assumed to be formed by twisting many strands (much more than 5).

The commercial Litz wires commonly have, but are not limited to 2 or 3 twisting levels. This section derives the proposed model by analyzing the Litz wire with 3 twisting levels. However, as it is shown later, the proposed model can

be applied to the Litz wire with 2, 4, or more twisting levels. Nonetheless, the proposed model requires that the lowest twisting level involves much more than 5 strands and the higher twisting levels involve no more than 5 bundles.

The copper loss of a wire is generated by two causes: The ac current flowing through the wire, and the external ac magnetic field applied to the wire. Therefore, the copper loss model should incorporate both of these two factors. The Litz wire under consideration is assumed to carry a sinusoidal ac current  $i_L$  with the frequency  $f$ . Additionally, uniform external sinusoidal ac magnetic field  $\mathbf{h}_L$  with the frequency  $f$  is assumed to be applied in perpendicular direction to the Litz wire. The reason for assuming the same frequency for the ac current and the external magnetic field is that the external magnetic field is also generated by the ac current flowing in the Litz wire in common magnetic devices used in power electronics applications. ( $i_L$  and  $\mathbf{h}_L$  are the instantaneous current and the instantaneous magnetic field vector, respectively.)

Certainly, the actual current in the Litz wire, as well as the actual magnetic field applied to the Litz wire, is not necessarily sinusoidal. However, the copper loss of an arbitrary current waveform and an arbitrary external magnetic field waveform can be determined as the sum of the copper loss generated by their fundamental waves and each of their harmonics waves because this copper loss model is constructed on the electromagnetics in linear media.

The copper loss model was constructed based on the hierarchical structure of the Litz wire. Starting from the copper loss of the strand, the copper loss model of the bundle of each twisting level was formulated based on the copper loss model of its subcomponents. Specifically, the analytical copper loss model of a strand is firstly derived. Then, based on the model of the strand, the analytical copper loss model of the 1<sup>st</sup> level bundle is derived. Finally, the analytical copper loss model of the Litz wire is derived based on the copper loss model of the 1<sup>st</sup> level bundle.

#### A. Copper Loss Model of a Strand

The copper loss model of the strand is well known in the literature as the copper loss model of the round solid copper wire [16]–[21], [25], [26]. This strand is assumed to carry an ac current  $i_s$ . Additionally, an external magnetic field  $\mathbf{h}_s$  is assumed to be applied to the strand. ( $i_s$  and  $\mathbf{h}_s$  are the instantaneous current and the instantaneous magnetic field vector, respectively. The direction of  $\mathbf{h}_s$  is not limited to be perpendicular to the strand.) Magnetic field  $\mathbf{h}_s$  should include not only the magnetic field applied from outside the Litz wire but also the magnetic field generated by the ac current flowing through all the strands of the Litz wire under consideration as well as the eddy current generated in these strands. However, as discussed later in subsection II.C, this paper neglects the contribution of the eddy current to  $\mathbf{h}_s$  for simplifying the model construction.

The copper loss of the strand is the sum of the Joule loss generated by the local ac current inside the strand. Therefore, the instantaneous copper loss  $p_s$  per the unit of length of the strand can be expressed as

$$P_s = \int_{S_s} \rho \|\mathbf{i}_{s\_dens}\|^2 dS_s. \quad (1)$$

where  $\rho$  is the resistivity of the copper,  $S_s$  is the horizontal cross-section of the strand, and  $\mathbf{i}_{s\_dens}$  is the instantaneous ac current density vector inside the strand.

The instantaneous ac current density vector  $\mathbf{i}_{s\_dens}$  can be expressed as the sum of the eddy current density vector generated by  $\mathbf{h}_s$  under the absence of the strand current  $i_s$  and the ac current density vector  $\mathbf{i}_{acs\_dens}$  constituting  $i_s$  under the absence of  $\mathbf{h}_s$ . Therefore, by further dividing  $\mathbf{h}_s$  into  $\mathbf{h}_{s\perp}$  and  $\mathbf{h}_{s//}$ , which are the components of  $\mathbf{h}_s$  in perpendicular and parallel directions to the strand, respectively, (1) can be rewritten as

$$P_s = \rho \int_{S_s} \|\mathbf{i}_{acs\_dens} + \mathbf{i}_{s\perp\_dens} + \mathbf{i}_{s//\_dens}\|^2 dS_s, \quad (2)$$

where  $\mathbf{i}_{s\perp\_dens}$  and  $\mathbf{i}_{s//\_dens}$  is the instantaneous eddy current density vector generated by  $\mathbf{h}_{s\perp}$  and  $\mathbf{h}_{s//}$ , respectively.

Figure 3 depicts the distribution of the current density vectors  $\mathbf{i}_{acs\_dens}$ ,  $\mathbf{i}_{s\perp\_dens}$ , and  $\mathbf{i}_{s//\_dens}$  in the cross-section of the strand. Because of the axial symmetry of the electromagnetic condition when the ac current flows under the absence of the external magnetic field, the current density vector  $\mathbf{i}_{acs\_dens}$  must also have axial symmetry. Meanwhile, the eddy currents induced by the uniform external magnetic field must be perpendicular to the field. Furthermore, the surface integral of  $\mathbf{i}_{s\perp\_dens}$  and  $\mathbf{i}_{s//\_dens}$  over the strand cross-section must vanish because  $\mathbf{i}_{s\perp\_dens}$  and  $\mathbf{i}_{s//\_dens}$  are the eddy current density under the absence of the strand current  $i_s$ . Therefore, considering the two-fold symmetry of the electromagnetic condition caused by  $\mathbf{h}_{s\perp}$  with respect to the dashed line in Fig. 3(b), and the axial symmetry of the electromagnetic condition caused by  $\mathbf{h}_{s//}$ ,  $\mathbf{i}_{s\perp\_dens}$  must be in parallel to the strand and distributed in the two-fold antisymmetry and  $\mathbf{i}_{s//\_dens}$  must be in perpendicular to the strand and distributed in the axial symmetry. Consequently, the following relations can be obtained among  $\mathbf{i}_{acs\_dens}$ ,  $\mathbf{i}_{s\perp\_dens}$ , and  $\mathbf{i}_{s//\_dens}$ :

$$\begin{aligned} \int_{S_s} \mathbf{i}_{acs\_dens} \cdot \mathbf{i}_{s\perp\_dens} dS_s &= 0, & \int_{S_s} \mathbf{i}_{s//\_dens} \cdot \mathbf{i}_{s\perp\_dens} dS_s &= 0, \\ \int_{S_s} \mathbf{i}_{s//\_dens} \cdot \mathbf{i}_{acs\_dens} dS_s &= 0. \end{aligned} \quad (3)$$

By utilizing (3), (2) can be further rewritten as

$$\begin{aligned} P_s &= \rho \int_{S_s} \|\mathbf{i}_{acs\_dens}\|^2 dS_s + \rho \int_{S_s} \|\mathbf{i}_{s\perp\_dens}\|^2 dS_s \\ &\quad + \rho \int_{S_s} \|\mathbf{i}_{s//\_dens}\|^2 dS_s. \end{aligned} \quad (4)$$

According to (4), the time-averaged copper loss  $P_s$  per unit of length of the strand can be expressed as

$$P_s = R_s I_s^2 + G_{s\perp} H_{s\perp}^2 + G_{s//} H_{s//}^2, \quad (5)$$

where  $R_s$  is the strand-level loss coefficient caused by the strand current  $i_s$ , which includes both dc resistance and skin effect loss,  $G_{s\perp}$  and  $G_{s//}$  are the strand-level proximity effect loss coefficients, which expresses the eddy current losses caused by the external magnetic field  $\mathbf{h}_{s\perp}$  and  $\mathbf{h}_{s//}$ , respectively, and  $I_s$ ,  $H_{s\perp}$  and  $H_{s//}$  are the root-mean-square values of  $i_s$ ,  $\mathbf{h}_{s\perp}$ , and  $\mathbf{h}_{s//}$ , respectively.

These coefficients  $R_s$ ,  $G_{s\perp}$ , and  $G_{s//}$  can be obtained using analytical expressions. As reported in [16]-[26], the expressions for calculating these coefficients are as follows:

$$R_s = \frac{\rho}{\pi\alpha_s^2} F(\gamma_s), \quad (6)$$

$$G_{s\perp} = 4\pi\rho K(\gamma_s), \quad (7)$$

$$G_{s//} = 2\pi\rho K(\gamma_s), \quad (8)$$

where  $\alpha_s$  is the radius of the strand,  $\gamma_s$  is the parameter defined as (9), and  $F$  and  $K$  are functions of real number  $x$  defined as (10) and (11), respectively.

$$\gamma_s \equiv \alpha_s \sqrt{\frac{\omega\mu_0}{\rho}}, \quad (9)$$

$$F(x) \equiv \frac{x \operatorname{ber} x \operatorname{bei}' x - \operatorname{ber}' x \operatorname{bei} x}{2 \left( (\operatorname{ber}' x)^2 + (\operatorname{bei}' x)^2 \right)}, \quad (10)$$

$$K(x) \equiv -x \frac{\operatorname{ber}_2 x \operatorname{ber}' x + \operatorname{bei}_2 x \operatorname{bei}' x}{(\operatorname{ber} x)^2 + (\operatorname{bei} x)^2}, \quad (11)$$

where  $\omega$  is the angular frequency,  $\mu_0$  is the permeability of the air,  $\operatorname{ber}$  and  $\operatorname{bei}$  are Kelvin functions, which are the special functions related to the Bessel function [30].

### B. Copper Loss Model of First Level Bundle

Based on the analytical solutions of the copper loss of the strand, this subsection formulates the analytical copper loss model of the 1<sup>st</sup> level bundle. The 1<sup>st</sup> level bundles are made by twisting many strands. As a result of the twisting process, the strands located at the same distance from the center of the bundle rotationally exchange their position along the bundle. Consequently, by assuming that the length of the bundle is far longer than the twisting pitch, these strands can be approximated to have the same electromagnetic condition and therefore to carry the same strand ac current. Hence, the distribution of the strand ac current among the strands constituting the 1<sup>st</sup> level bundle can be approximated to have the axial symmetry in the cross-section of the bundle.

As discussed in the preceding study [26], not only the strand but also the bundle is susceptible to the proximity effect. In other words, the distribution of the strands ac current in the bundle is affected by the external ac magnetic field. This bundle-level proximity effect generates a strand current

distribution of the two-fold antisymmetry by the external ac magnetic field in perpendicular to the bundle. However, this strand current distribution is not acceptable in the bundle far longer than the twisting pitch because the long bundle requires the axial-symmetrical strand ac current distribution. Similarly, the bundle-level proximity effect generates the circulating current flow in the bundle cross-section by the external ac magnetic field in parallel to the bundle. However, this current flow is not acceptable because the surface of the strands is insulated and therefore the current cannot flow between the strands. Consequently, the bundle-level proximity effect is simply ignored in this paper by assuming that the bundle is far longer than the twisting pitch, although the strand-level proximity effect still causes the local eddy current  $\mathbf{i}_{s\perp, dens}$  and  $\mathbf{i}_{s//, dens}$  confined inside the strands. The necessary Litz length for this approximation is estimated in the appendix.

Because the proximity effect is considered only in the strand level, the time-averaged copper loss  $P_b$  of the 1<sup>st</sup> level bundle per unit of length can be expressed in the following form:

$$\begin{aligned} P_b &= \sum_j R_s I_{s-j}^2 + \sum_j G_{s\perp} H_{s\perp,j}^2 + \sum_j G_{s//} H_{s//,j}^2 \\ &= R_b I_b^2 + \sum_j G_{s\perp} H_{s\perp,j}^2 + \sum_j G_{s//} H_{s//,j}^2, \end{aligned} \quad (12)$$

where  $j$  is the index of the strands constituting the 1<sup>st</sup> level bundle,  $I_{s-j}$  is the root-mean-square value of the ac current flowing through strand  $j$ ,  $R_b$  is the bundle-level loss coefficient caused by the ac current of the bundle,  $I_b$  is the root-mean-square value of the ac current flowing through the bundle, and  $H_{s\perp,j}$  and  $H_{s//,j}$  are the root-mean-square values of the local magnetic field  $\mathbf{h}_{s\perp}$  and  $\mathbf{h}_{s//}$  at strand  $j$ .

As a result of twisting the strands, the strand must be slightly longer than the 1<sup>st</sup> level bundle. However, this paper simply neglects this effect in (12), although this paper only considers this effect for twisting the 2<sup>nd</sup> level bundles to form the Litz wire in the next subsection. The reason for this simplification is that the largest scale of twisting tends to mainly contribute to the length increase of the strand to the Litz wire.

The first term of the right-most side of (12), i.e.  $R_b I_b^2$ , corresponds to the total copper loss of the bundle under the absence of the external magnetic field. Hence,  $R_b$  can be determined by calculating the total copper loss of the bundle when no external magnetic field is applied to the bundle. For this purpose, this paper approximates that the ac current distribution among the strands in the bundle is the same as the ac current distribution in the solid wire with the same diameter as the bundle. This imaginary solid wire is assumed to be made of the uniform material with the resistivity  $\rho_{eff}$  defined as

$$\rho_{eff} = \frac{A_s}{\eta} R_s = \frac{\pi\alpha_s^2}{\eta} R_s, \quad (13)$$

where  $\eta$  is the packing factor of the bundle, which is the area rate of the copper in the cross-section of the bundle, and  $A_s$  is the cross-sectional area of the strand.

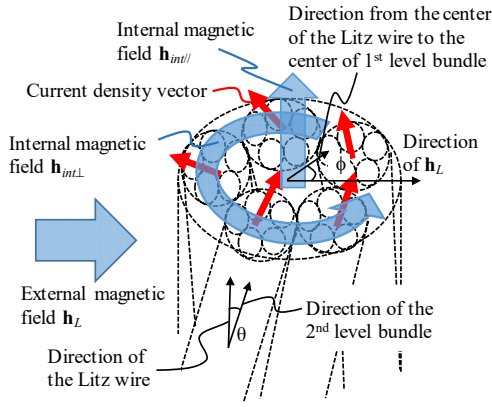


Fig. 4. Internal magnetic field in Litz wire generated as a result of twisting.

This approximation is consistent with the ac current distribution of the strands in the bundle according to the following three reasons. Firstly, the solid wire model results in the ac current distribution in which the direction of the ac current is in parallel to the bundle. Therefore, if the same current distribution is assumed for the ac current of the strands in the bundle, the ac current of a strand can be automatically confined to the strand. (Twisting the strands to form the 1<sup>st</sup> level bundle causes the slight inclination of the strands to the bundle. However, this inclination angle of the strand to the 1<sup>st</sup> level bundle is ignored because the diameter of the 1<sup>st</sup> level bundle is far smaller than that of the Litz wire.)

Secondly, the solid wire model results in an ac current distribution of the strands that ensures the ac electric potential to be the same at each strand in the cross-section of the solid wire. Therefore, the solution automatically meets the implicit boundary condition of the Litz wire that all the strands are connected in parallel at both ends of the Litz wire.

Thirdly, the solid wire model results in the same total copper loss as the bundle, when the same ac current distribution among the strands is assumed in the bundle. In fact, by determining the instantaneous strand current  $i_{s\_j}$  of strand  $j$  according to

$$i_{s\_j} = \frac{A_s}{\eta} i_{b\_dens\_j}, \quad (14)$$

where  $i_{b\_dens\_j}$  is the instantaneous ac current density of the solid wire model at the point that corresponds to the center of strand  $j$ . Therefore, the following relations can be obtained:

$$i_b = \sum_j i_{s\_j} = \frac{A_s}{\eta} \sum_j i_{b\_dens\_j} \approx \int_{S_b} i_{b\_dens} dS_b, \quad (15)$$

$$\begin{aligned} R_b I_b^2 &= \sum_j R_s I_{s\_j}^2 = R_s \left( \frac{A_s}{\eta} \right)^2 \sum_j I_{b\_dens\_j}^2 \\ &= \frac{A_s}{\eta} \sum_j \rho_{eff} I_{b\_dens\_j}^2 \approx \int_{S_b} \rho_{eff} I_{b\_dens}^2 dS_b, \end{aligned} \quad (16)$$

where  $i_{b\_dens}$  is the instantaneous ac current density of the solid wire model,  $S_b$  is the cross-section of the bundle, and  $I_b$ ,  $I_{s\_j}$ ,  $i_{b\_dens\_j}$ , and  $I_{b\_dens}$  are the root-mean-square values of  $i_b$ ,  $i_{s\_j}$ ,

$i_{b\_dens\_j}$ , and  $i_{b\_dens}$ , respectively.

Equation (15) indicates that the total ac current flowing through the bundle is the same as that in the solid wire model. Furthermore, (16) indicates that the total copper loss generated in the bundle is also the same as that in the solid copper model.

The copper loss of the solid wire model can be analytically calculated similarly as in the previous subsection. Hence,  $R_b$  can be calculated as

$$R_b = \frac{1}{I_b^2} \int_{S_b} \rho_{eff} I_{b\_dens}^2 dS_b = \frac{\rho_{eff}}{\pi \alpha_b^2} F(\gamma_b) = \frac{A_s F(\gamma_b)}{\eta \pi \alpha_b^2} R_s, \quad (17)$$

where  $\alpha_b$  is the radius of the 1<sup>st</sup> level bundle,  $\gamma_b$  is the parameter defined as

$$\gamma_b = \alpha_b \sqrt{\frac{\omega \mu_0}{\rho_{eff}}} = \alpha_b \sqrt{\frac{\omega \mu_0 \eta}{A_s R_s}} = \frac{\alpha_b}{\alpha_s} \sqrt{\frac{\omega \mu_0 \eta}{\pi R_s}}. \quad (18)$$

Finally, substituting (6) into (17) yields

$$\begin{aligned} R_b &= \frac{A_s F(\gamma_b)}{\eta \pi \alpha_b^2} \frac{\rho}{\pi \alpha_s^2} F(\gamma_s) = \frac{\rho}{\eta \pi \alpha_b^2} F(\gamma_s) F(\gamma_b) \\ &= \frac{\rho}{\pi \alpha_s^2 n_s} F(\gamma_s) F(\gamma_b), \end{aligned} \quad (19)$$

where  $n_s$  is the number of the strands contained in a 1<sup>st</sup> level bundle.

The basic idea of formulating  $R_b$  by the solid wire model has been proposed in the preceding study [26]. The aforementioned discussion is fundamentally the same as this preceding study, although there is a slight difference in the definition of the effective resistivity  $\rho_{eff}$ . The preceding study [26] defined  $\rho_{eff}$  as  $\rho_{eff} = \rho/\eta$ . However, this paper rather defines  $\rho_{eff}$  as (13) for ensuring the third feature of the analytical solution of the solid wire model. This difference, however, may have a subtle effect on the calculation result of  $R_b$  in many Litz wire because the strand is commonly designed to be sufficiently thinner than the skin depth and therefore the definitions of [26] and this paper tend to take similar values.

### C. Copper Loss Model of Litz Wire

Because each twisting process forming the 2<sup>nd</sup> bundle and the Litz wire involve no more than five 1<sup>st</sup> level bundles and 2<sup>nd</sup> level bundles, respectively, these twisted bundles rotationally exchange their position along the Litz wire. Therefore, these twisted bundles have the axial symmetry, thus carrying the same ac current. Consequently, all the 1<sup>st</sup> level bundles in the Litz wire can be assumed to carry the same ac current. In other words, the bundle-level proximity effect is also neglected in the 2<sup>nd</sup> level and 3<sup>rd</sup> level bundles, i.e. the Litz wire. The necessary Litz wire length for neglecting the bundle-level proximity effect in the 2<sup>nd</sup> and 3<sup>rd</sup> level bundles is estimated in the appendix.

The twisting process generally needs a longer length of the subcomponents to be twisted than the product of the twisting process. As a result, the strands must have a greater length than

the Litz wire. This effect occurs at any level of twisting. However, the largest scale of twisting tends to mainly contribute to the length increase of the strands. Therefore, this paper simply considers this effect only for the highest level of twisting, i.e. twisting the 2<sup>nd</sup> level bundles to form the Litz wire, and neglects the effect for the lower levels of twisting, i.e. twisting the strands to form the 1<sup>st</sup> level bundle and twisting the 1<sup>st</sup> level bundles to form the 2<sup>nd</sup> level bundle.

Because of the rotational symmetry among the 2<sup>nd</sup> level bundles, the 2<sup>nd</sup> level bundles are assumed to have a constant length ratio  $m$  to the Litz wire. Meanwhile, the strands and the 1<sup>st</sup> level bundles are approximated to have the same length as the 2<sup>nd</sup> level bundles. Hence, all the 1<sup>st</sup> level bundles (and therefore all the strands) are approximated to have the same length ratio  $m$  to the Litz wire. Consequently, the total copper loss  $P_L$  of the Litz wire can be expressed as

$$\begin{aligned} P_L &= mn_2n_1R_bI_b^2 + mG_{s\perp} \sum_{k,j} H_{s\perp kj}^2 \\ &\quad + mG_{s\parallel} \sum_{k,j} H_{s\parallel kj}^2 \\ &= mn_2n_1R_b \left( \frac{I_L}{n_2n_1} \right)^2 + mG_{s\perp} \sum_{k,j} H_{s\perp kj}^2 \\ &\quad + mG_{s\parallel} \sum_{k,j} H_{s\parallel kj}^2, \end{aligned} \quad (20)$$

where  $k$  is the index of the 1<sup>st</sup> level bundles contained in the Litz wire,  $I_L$  is the root-mean-square value of  $i_L$ , i.e. the ac current of the Litz wire,  $n_1$  is the number of the 1<sup>st</sup> level bundles in a 2<sup>nd</sup> level bundle,  $n_2$  is the number of the 2<sup>nd</sup> level bundles in a Litz wire, and  $H_{s\perp kj}$  and  $H_{s\parallel kj}$  are the root-mean-square value of the local ac magnetic field  $\mathbf{h}_{s\perp}$  and  $\mathbf{h}_{s\parallel}$ , respectively, at the strand  $j$  in the 1<sup>st</sup> level bundle  $k$ . (Because of the symmetry among the 1<sup>st</sup> level bundles in a 2<sup>nd</sup> level bundle and the symmetry among the 2<sup>nd</sup> level bundles in the Litz wire, the same value of  $m$  is assumed to all the 1<sup>st</sup> level bundles.)

Equation (20) still contains local parameters  $H_{s\perp kj}$  and  $H_{s\parallel kj}$ , which are dependent on the position of the 1<sup>st</sup> level bundle and the strand. Therefore, from this subsection hereafter  $\sum_{k,j} H_{s\perp kj}$  and  $\sum_{k,j} H_{s\parallel kj}$  are formulated by the global parameters that specify the Litz wire.

As a result of twisting, the strands generally have a slight inclination in the Litz wire. This inclination causes the derivation of the local magnetic field  $H_{s\perp kj}$  and  $H_{s\parallel kj}$  complicated. For example, the ac current flowing in a bundle forms the circulating current around the center of the bundle as a result of the twisting. This circulating current generates the magnetic field parallel to the bundle, which does not appear without twisting.

This effect occurs at any level of twisting. However, considering this effect at all levels may lead to extremely complicated calculations. Therefore, this paper again simply considers the effect only for the highest level of twisting and neglects the effect for the lower levels of twisting. The reason for this approximation is that the largest scale of the twisting mainly causes the inclination angle of the strands. Hence, all the

strands are assumed to be twisted to form the 1<sup>st</sup> level bundle with a negligible inclination angle to this bundle; and similarly, all the 1<sup>st</sup> level bundles are assumed to be twisted to form the 2<sup>nd</sup> level bundle with a negligible inclination angle to this 2<sup>nd</sup> level bundle. Nonetheless, all the 2<sup>nd</sup> level bundles are twisted with the constant inclination angles to the Litz wire. As a result, all the strands contained in the Litz wire are assumed to be inclined to the Litz wire at the same angle  $\theta$  as the 2<sup>nd</sup> level bundles.

The magnetic field inside the Litz wire can be expressed as the sum of the instantaneous external ac magnetic field  $\mathbf{h}_L$  and the instantaneous internal ac magnetic field  $\mathbf{h}_{int}$  generated by the ac current flow through the strands of the Litz wire as well as the eddy current generated inside these strands. However, this paper simply approximates  $\mathbf{h}_{int}$  to be the magnetic field generated by the uniform ac current distribution in the Litz wire cross-section because all the 1<sup>st</sup> level bundles carry the same ac current. This uniform ac current distribution assumes the ac current density vector inclined with respect to the Litz wire at the constant inclination angle  $\theta$  because of the assumption of the constant inclination of the 1<sup>st</sup> level bundles with respect to the Litz wire.

Certainly, this approximation of  $\mathbf{h}_{int}$  neglects the inhomogeneity of the ac current distribution inside a 1<sup>st</sup> level bundle at high frequency, which is modeled as the ac current distribution of the solid copper model, introduced in the previous subsection. Furthermore, this approximation also neglects the inhomogeneity of the ac current distribution inside a strand due to the strand-level skin effect as well as the eddy current generated inside the strand. However, these two types of inhomogeneity have a far smaller scale than the Litz wire cross-section. Therefore, they can scarcely affect  $\mathbf{h}_{int}$  at the point far more distant from the 1<sup>st</sup> level bundle or the strand than the scale of the inhomogeneity. Consequently, these two types of inhomogeneity cause the smaller scale of fluctuations in  $\mathbf{h}_{int}$  compared with the scale of the Litz wire cross-section. Therefore, this paper regards these small-scaled fluctuations to be ignorable when compared with the large-scaled magnetic field generated by the uniform current distribution among all the 1<sup>st</sup> level bundles. These assumptions are later validated by the good agreement with experimental results.

Under this approximation, the internal magnetic field  $\mathbf{h}_{int}$  can be expressed as the sum of the axially symmetric magnetic field  $\mathbf{h}_{int\perp}$  in perpendicular to the Litz wire, and the axially symmetric magnetic field  $\mathbf{h}_{int\parallel}$  in parallel to the Litz wire, as illustrated in Fig. 4. The absolute values of  $\mathbf{h}_{int\perp}$  and  $\mathbf{h}_{int\parallel}$  are

$$\|\mathbf{h}_{int\perp}\| = \frac{r}{2\pi\alpha_L^2} i_L, \quad (21)$$

$$\|\mathbf{h}_{int\parallel}\| = \frac{(\alpha_L - r)\tan\theta}{\pi\alpha_L^2} i_L, \quad (22)$$

where  $\alpha_L$  is the radius of the Litz wire,  $r$  is the distance from the center of the Litz wire, and  $\theta$  is the inclination angle of the 1<sup>st</sup> level bundles to the Litz wire.

All the strands are assumed to be inclined to the Litz wire at

the constant angle  $\theta$ . Therefore, if  $\mathbf{h}_{s\perp kj}$  and  $\mathbf{h}_{s\parallel kj}$  denote the instantaneous local ac magnetic field  $\mathbf{h}_{s\perp}$  and  $\mathbf{h}_{s\parallel}$  at the strand  $j$  in the 1<sup>st</sup> level bundle  $k$ , the square of the absolute values of  $\mathbf{h}_{s\perp kj}$  and  $\mathbf{h}_{s\parallel kj}$  can be obtained as

$$\begin{aligned} \|\mathbf{h}_{s\perp kj}\|^2 &= \left( -\|\mathbf{h}_L\| \sin \phi_{kj} \cos \theta + \|\mathbf{h}_{int\perp}\| \cos \theta - \|\mathbf{h}_{int\parallel}\| \sin \theta \right)^2 \\ &\quad + \|\mathbf{h}_L\|^2 \cos^2 \phi_{kj}, \end{aligned} \quad (23)$$

$$\|\mathbf{h}_{s\parallel kj}\|^2 = \left( -\|\mathbf{h}_L\| \sin \phi_{kj} \sin \theta + \|\mathbf{h}_{int\perp}\| \sin \theta + \|\mathbf{h}_{int\parallel}\| \cos \theta \right)^2, \quad (24)$$

where  $\phi_{kj}$  is the angle between  $\mathbf{h}_L$  and the vector in the Litz wire cross-section drawn from the center of the Litz wire to the center of strand  $j$  of the 1<sup>st</sup> level bundle  $k$ . Therefore, the summation of (23) and (24) over all the strands in the Litz wire can be obtained by approximating the summation by the area integration.

$$\begin{aligned} \sum_{k,j} \|\mathbf{h}_{s\perp kj}\|^2 &= \frac{n_{total}}{A_L} \int_{S_L} \|\mathbf{h}_{s\perp kj}\|^2 dS_L \\ &= \frac{n_{total}}{A_L} \int_0^{2\pi} d\phi_{kj} \int_0^{\alpha_L} r \|\mathbf{h}_{s\perp kj}\|^2 dr \end{aligned} \quad (25)$$

$$\begin{aligned} &= \frac{n_{total}}{A_L} \int_0^{2\pi} d\phi_{kj} \int_0^{\alpha_L} r \left\{ \left( -\|\mathbf{h}_L\| \sin \phi_{kj} \cos \theta \right. \right. \\ &\quad \left. \left. + \|\mathbf{h}_{int\perp}\| \cos \theta - \|\mathbf{h}_{int\parallel}\| \sin \theta \right)^2 + \|\mathbf{h}_L\|^2 \cos^2 \phi_{kj} \right\} dr, \end{aligned}$$

$$\begin{aligned} \sum_{k,j} \|\mathbf{h}_{s\parallel kj}\|^2 &= \frac{n_{total}}{A_L} \int_{S_L} \|\mathbf{h}_{s\parallel kj}\|^2 dS_L \\ &= \frac{n_{total}}{A_L} \int_0^{2\pi} d\phi_{kj} \int_0^{\alpha_L} r \|\mathbf{h}_{s\parallel kj}\|^2 dr \end{aligned} \quad (26)$$

$$\begin{aligned} &= \frac{n_{total}}{A_L} \int_0^{2\pi} d\phi_{kj} \int_0^{\alpha_L} r \left( -\|\mathbf{h}_L\| \sin \phi_{kj} \sin \theta \right. \\ &\quad \left. + \|\mathbf{h}_{int\perp}\| \sin \theta + \|\mathbf{h}_{int\parallel}\| \cos \theta \right)^2 dr, \end{aligned}$$

where  $n_{total}$  is the total number of the strands in the cross-section area of the Litz wire (Hence,  $n_{total} = n_s n_1 n_2$ ),  $A_L$  is the cross-section area of the Litz wire,  $S_L$  is the cross-section of the Litz wire.

Substituting (21) and (22) into (25) and (26) yields

$$\begin{aligned} \sum_{k,j} \|\mathbf{h}_{s\perp kj}\|^2 &= n_{total} \left\{ \frac{\|\mathbf{h}_L\|^2}{2} (1 + \cos^2 \theta) \right. \\ &\quad \left. + \left( \frac{1}{3 \cos^2 \theta} + \frac{11 \cos^2 \theta}{12} - 1 \right) \frac{i_L^2}{2 \pi^2 \alpha_L^2} \right\}, \end{aligned} \quad (27)$$

$$\sum_{k,j} \|\mathbf{h}_{s\parallel kj}\|^2 = n_{total} \left( \frac{\|\mathbf{h}_L\|^2}{2} + \frac{11}{24} \frac{i_L^2}{\pi^2 \alpha_L^2} \right) (1 - \cos^2 \theta) \quad (28)$$

Consequently, the 2<sup>nd</sup> and 3<sup>rd</sup> terms of the right-most side of

(20) can be obtained as

$$\begin{aligned} m G_{s\perp} \sum_{k,j} H_{s\perp kj}^2 &= m G_{s\perp} n_{total} \left\{ \frac{H_L^2}{2} (1 + \cos^2 \theta) \right. \\ &\quad \left. + \left( \frac{1}{3 \cos^2 \theta} + \frac{11 \cos^2 \theta}{12} - 1 \right) \frac{I_L^2}{2 \pi^2 \alpha_L^2} \right\}, \end{aligned} \quad (29)$$

$$m G_{s\parallel} \sum_{k,j} H_{s\parallel kj}^2 = m G_{s\parallel} n_{total} \left( \frac{H_L^2}{2} + \frac{11}{24} \frac{I_L^2}{\pi^2 \alpha_L^2} \right) (1 - \cos^2 \theta) \quad (30)$$

Because all the 1<sup>st</sup> level bundles are regarded to have a constant inclination angle  $\theta$  to the Litz wire, the following relation can be obtained between  $\theta$  and  $m$ :

$$m = \frac{1}{\cos \theta}. \quad (31)$$

Finally, the analytical copper loss model of the Litz wire is obtained by substituting (19), (29), (30), (31) into (20):

$$\begin{aligned} P_L &= m n_2 n_1 \frac{\rho}{\pi \alpha_s^2 n_s} F(\gamma_s) F(\gamma_b) \left( \frac{I_L}{n_2 n_1} \right)^2 \\ &\quad + 4 \pi \rho K(\gamma_s) n_{total} \left\{ \frac{H_L^2}{2} \left( m + \frac{1}{m} \right) \right. \\ &\quad \left. + \left( \frac{m^3}{3} + \frac{11}{12m} - m \right) \frac{I_L^2}{2 \pi^2 \alpha_L^2} \right\} \\ &\quad + 2 \pi \rho K(\gamma_s) n_{total} \left( \frac{H_L^2}{2} + \frac{11}{24} \frac{I_L^2}{\pi^2 \alpha_L^2} \right) \left( m - \frac{1}{m} \right). \\ \therefore P_L &= \frac{m \rho}{\pi \alpha_s^2 n_{total}} F(\gamma_s) F(\gamma_b) I_L^2 \\ &\quad + \frac{4 \pi \rho n_{total} K(\gamma_s)}{8 \pi^2 \alpha_L^2} \left( \frac{4m^3}{3} - \frac{13m}{6} + \frac{11}{6m} \right) I_L^2 \\ &\quad + 4 \pi \rho n_{total} K(\gamma_s) \left( \frac{3m}{4} + \frac{1}{4m} \right) H_L^2. \end{aligned} \quad (32)$$

As it can be seen in (32), the copper loss of the Litz wire has the form of  $P_L = R_L I_L^2 + G_L H_L^2$ , where  $R_L$  and  $G_L$  are the copper loss coefficients. These coefficients are therefore obtained as

$$\begin{aligned} R_L &= \frac{m \rho}{\pi \alpha_s^2 n_{total}} F(\gamma_s) F(\gamma_b) \\ &\quad + \frac{4 \pi \rho n_{total} K(\gamma_s)}{8 \pi^2 \alpha_L^2} \left( \frac{4m^3}{3} - \frac{13m}{6} + \frac{11}{6m} \right), \end{aligned} \quad (33)$$

$$G_L = 4 \pi \rho n_{total} K(\gamma_s) \left( \frac{3m}{4} + \frac{1}{4m} \right). \quad (34)$$



As we have seen, (33) and (34) were derived directly from the copper loss of the strand and the 1<sup>st</sup> level bundle, i.e. (7), (8), and (17), based on the two assumptions:

1. All the 1<sup>st</sup> level bundles carry the same ac current;
2. Only the highest level of twisting contributes to the inclination angle  $\theta$  as well as the length increase of the 1<sup>st</sup> level bundles.

Therefore, (33) and (34) can be applied to the Litz wire with any levels of twisting greater than 2 because these two assumptions are not restricted by the number of twisting levels.

#### D. Estimation of Copper Loss Coefficients Using Directly Measurable Parameters

Table I lists the parameters contained in the coefficients  $R_L$  and  $G_L$  obtained in expressions (33) and (34). Parameters  $\eta$ ,  $\alpha_b$ ,  $\omega$ , and  $\mu_0$  do not explicitly appear in (33) and (34). However, these parameters are indirectly included in  $\gamma_s$  and  $\gamma_b$ , defined in (9) and (18), for characterizing the ac current distribution inside the strand and the 1<sup>st</sup> level bundle. Among the Litz wire parameters listed in Table I, the strand radius  $\alpha_s$ , the Litz wire radius  $\alpha_L$ , and the number of strands  $n_{total}$  are commonly given by the Litz wire manufacturer or can be otherwise straightforwardly measured with basic instruments. However, the other parameters, the packing factor  $\eta$ , the radius of the 1<sup>st</sup> level bundle  $\alpha_b$ , and the length ratio of the 1<sup>st</sup> level bundle to the Litz wire  $m$  are difficult to be directly measured and therefore need estimation based on the directly measurable parameters for practical application of the copper loss model of the Litz wire. This subsection discusses the estimation method of these three parameters.

The packing factor  $\eta$  is introduced in subsection II.B as the ratio of the copper in the cross-section area of the 1<sup>st</sup> level bundle. However, this paper simply approximates  $\eta$  as the ratio of the copper in the cross-section area of the Litz wire because the strands are commonly distributed with the uniform density in the Litz wire cross-section in actual Litz wires. Under this approximation,  $\eta$  can be estimated as

$$\eta = \frac{n_{total} A_s}{A_L} = \frac{n_{total} \alpha_s^2}{\alpha_L^2}. \quad (35)$$

Next, the radius of the 1<sup>st</sup> level bundle  $\alpha_b$  is difficult to be physically determined in actual Litz wires because the 1<sup>st</sup> level bundles are commonly deformed in the twisting process. However, because the strands are distributed uniformly in the Litz wire cross-section, the cross-section area of the 1<sup>st</sup> level bundle can be calculated by dividing the cross-section area of the Litz wire by the number of the 1<sup>st</sup> level bundles contained in the Litz wire. Therefore, this paper simply approximates  $\alpha_b$  as the radius of the circle area that occupies the same area as the cross-section of the 1<sup>st</sup> level bundle. Under this approximation,  $\alpha_b$  can be estimated as

TABLE I

PARAMETERS CONTAINED IN INITIAL VERSION OF PROPOSED COPPER LOSS MODEL DERIVED IN SUBSECTION II.B

	Symbol	Meaning
	$\eta$	Packing factor of 1 <sup>st</sup> level bundle
	$\alpha_s$	Radius of strand
Litz wire	$\alpha_b$	Radius of 1 <sup>st</sup> level bundle
Parameters	$\alpha_L$	Radius of Litz wire
	$n_{total}$	Total number of strands in Litz wire
	$m$	Length ratio of 1 <sup>st</sup> level bundle to Litz wire
Physical	$\omega$	Angular frequency
Parameters	$\mu_0$	Permeability of air
	$\rho$	Resistivity of copper

TABLE II

PARAMETERS IN FINAL VERSION OF PROPOSED COPPER LOSS MODEL AFTER USING ESTIMATION

	Symbol	Meaning
	$\alpha_s$	Radius of strand
	$\alpha_L$	Radius of Litz wire
Litz wire	$n_s$	Number of strands in 1 <sup>st</sup> level bundle
Parameters	$n_{total}$	Total number of strands in Litz wire
	$l_L$	Length of Litz wire
	$R_{dc}$	DC resistance of Litz wire
Physical	$\omega$	Angular frequency
Parameters	$\mu_0$	Permeability of air
	$\rho$	Resistivity of copper

$$\frac{n_{total}}{n_s} \pi \alpha_b^2 = \pi \alpha_L^2. \quad (36)$$

$$\therefore \alpha_b = \sqrt{\frac{n_s}{n_{total}}} \alpha_L.$$

Finally, parameter  $m$  is the length ratio of the 1<sup>st</sup> level bundle to the Litz wire. However, direct measurement of this parameter is difficult because the measurement of the length of each 1<sup>st</sup> level bundle needs disassembling the Litz wire. For estimating  $m$  without disassembling the Litz wire, this paper utilizes the measurement result of the dc resistance  $R_{dc}$  of the Litz wire under consideration.

Because the dc current is uniformly distributed in all the strands, the length of the strand  $l_s$  can be estimated as

$$\frac{\rho}{n_{total} A_s} l_s = R_{dc}. \quad \therefore l_s = \frac{n_{total} A_s}{\rho} R_{dc} = \frac{n_{total} \pi \alpha_s^2}{\rho} R_{dc}. \quad (37)$$

Noting that parameter  $m$  is also assumed to be the length ratio of the strand to the Litz wire,  $m$  can be estimated by measuring the length of Litz wire  $l_L$  as

$$m = \frac{l_s}{l_L}. \quad (38)$$



TABLE III  
SPECIFICATIONS OF EXPERIMENTAL LITZ WIRES

Parameter	Wire A	Wire B	Wire C	Wire D	Wire E	Wire F	Wire G	Wire H	Wire I
$\alpha_s$ [mm]	0.025	0.025	0.025	0.355	0.355	0.0355	0.040	0.060	0.060
$\alpha_L$ [mm]	0.82	1.12	1.13	0.90	1.35	1.40	1.78	0.87	1.15
$n_s$	24	40	42	18	32	40	40	21	36
$n_{total}$	600	1000	1050	270	800	800	1000	105	180
$l_L$ [m]	3.01	3.01	2.95	3.01	3.00	2.95	3.01	3.01	3.01
$R_{dc}$ [m $\Omega$ ]	47.0	28.7	26.7	52.3	17.5	17.7	11.1	46.2	26.7
$\eta$	0.56	0.50	0.52	0.42	0.55	0.51	0.51	0.51	0.49
$m$	1.049	1.068	1.059	1.062	1.056	1.074	1.059	1.041	1.032

$\eta$  and  $m$  are estimated parameters based on (35) and (38), respectively.

To summarize, using the estimation presented in this subsection, the loss coefficients  $R_L$  and  $G_L$  can be calculated by the parameters listed in Table II. Consequently, the copper loss model of the Litz wire derived in this paper can be expressed by directly measurable parameters of the Litz wire and the physical constants.

#### E. Utilization of the Proposed Model for Copper Loss Estimation of Litz Wire Coil in Magnetic Devices

The copper loss of model, i.e. (33) and (34), directly includes the proximity effect caused by the strands in the adjacent part of the Litz wire under consideration. Meanwhile, the proximity effect caused by the other wires and the distant part of the same Litz wire, as well as and the magnetic field generated by other elements, is modeled using the external magnetic field  $H_L$ . The reason for this indirect modeling is that the magnetic field caused by the other wires and the distant part of the same Litz wire, as well as and other magnetic elements, is dependent on the magnetic structure surrounding the Litz wire coil. However, if the external magnetic field is given, the copper loss model of (33) and (34) can be utilized for copper loss estimation of the Litz wire coil in various magnetic devices.

Equations (33) and (34) give the Litz wire copper loss per length unit at a local point in the magnetic device. Therefore, the total copper loss  $P_{L\_total}$  of the Litz wire coil in a magnetic device can be formulated as

$$\begin{aligned} P_{L\_total} &= \int_C P_L ds = R_L l_L I_L^2 + G_L \int_C H_L^2 ds \\ &= R_L l_L I_L^2 + G_L l_L \overline{H_L}^2, \end{aligned} \quad (39)$$

where  $\int_C$  indicates the line integral along the Litz wire of the coil,  $ds$  is the line element along the Litz wire,  $\overline{H_L}$  is the root-mean-square of the external magnetic field  $H_L$ , defined as

$$\overline{H_L} = \sqrt{\frac{1}{l_L} \int_C H_L^2 ds}. \quad (40)$$

The root-mean-square external magnetic field  $\overline{H_L}$  is dependent on the magnetic structure, in which the Litz wire is incorporated. In some simple coil structures, as exemplified in

the next section,  $\overline{H_L}$  can be analytically determined. However, in more complicated structures as in many practical magnetic devices, numerical magnetic analysis may be needed to calculate  $\overline{H_L}$ . Nonetheless, this does not deny the benefit of the proposed analytical copper loss model of the Litz wire. The numerical calculation of  $\overline{H_L}$  needs the magnetic analysis covering the scale greater than the Litz wire, whereas the direct numerical calculation of the Litz wire copper loss will need the magnetic analysis covering the scale smaller than the strand diameter to calculate the eddy current flow inside the strand. Therefore, even with the numerical calculation of  $\overline{H_L}$ , the proposed model can simplify the copper loss prediction and reduce the necessary calculation resource.

### III. EXPERIMENT

Two experiments were carried out to verify the proposed copper loss model of the Litz wire. In these experiments, the ac resistance of various Litz wires was evaluated and compared with the theoretical values, which are analytically calculated using the loss coefficients  $R_L$  and  $G_L$  from the parameters listed in Table II. Specifications of these experimental Litz wires are presented in Table III. These experimental Litz wires were formed with two or three levels of twisting. The bundle-level proximity effect in these Litz wires was estimated to have a neglectable effect under the experimental conditions according to the discussion in the appendix.

The first experiment was targeted at evaluating the coefficient  $R_L$ . In this experiment, the Litz wire to be tested was wound on the core-less bobbin with the diameter of 100 mm to form a non-inductive solenoid coil, which was made of 4 clockwise turns and 4 counter-clockwise turns, as shown in Fig. 5(a). Then, the ac resistance of the non-inductive solenoid coil was measured using the LCR meter (NF Corp. ZM2376 with attachment ZM2363) to obtain the ac resistance per unit of length  $R_{ac1}$  of the wire of this solenoid coil. The frequency region for the measurement of  $R_{ac1}$  was 1 kHz–700 kHz for wire G and 1 kHz–2 MHz for the other experimental Litz wires, because wire G was predicted to violate the condition for neglecting the bundle-level proximity effect in the 1<sup>st</sup> level bundle at higher frequencies than 700kHz.

Thanks to this configuration, the total external field is cancelled. The wire of the non-inductive solenoid coil can be

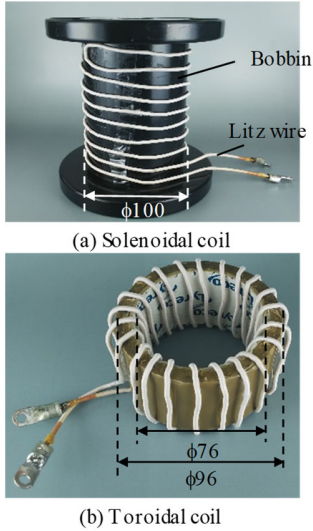


Fig. 5. Photographs of the experimental setup.

approximated to be free from external magnetic field  $H_L$  because the non-inductive solenoid coil does not have magnetomotive force. Therefore,  $R_{ac1}$  can be analytically predicted as

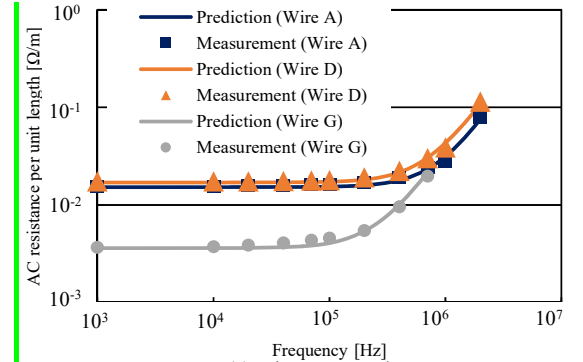
$$P_{L\_total} = R_{ac1} I_L I_L^2 = R_L I_L I_L^2. \therefore R_{ac1} = R_L. \quad (41)$$

The second experiment was targeted at evaluating the coefficient  $G_L$ . According to (39), the copper loss of the Litz wire coil is the sum of two terms: One is  $R_L I_L I_L^2$ , which is the loss characterized by  $R_L$ , and the other is  $G_L I_L \overline{H_L}^2$ , which is the loss characterized by  $G_L$ . The former loss can be estimated using  $R_L$ , which is evaluated in the first experiment. Therefore,  $G_L$  can be evaluated from the ac resistance of an arbitrary coil of the Litz wire as far as  $\overline{H_L}$  can be estimated.

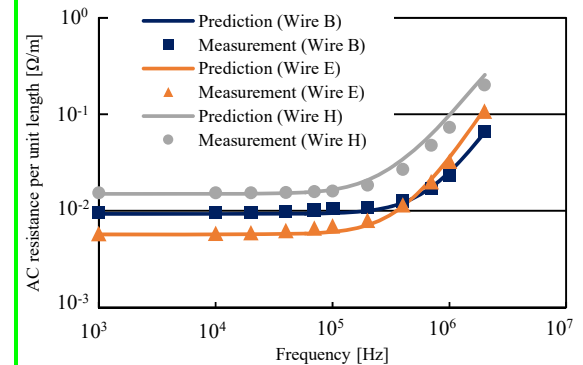
Estimation of  $\overline{H_L}$  is complicated in many magnetic device structures. Therefore, for simplifying this experiment, the coreless toroidal coil was adopted because  $\overline{H_L}$  for this coil structure can be analytically derived in a simple formula.

In this experiment, the Litz wire to be tested was uniformly wound on the coreless toroidal coil with 22 turns, as shown in Fig. 5(b). The inner and the outer diameter of the toroidal coil was 76 mm and 86 mm, respectively; the height was 50 mm. Then, the ac resistance of the toroidal coil was measured using the LCR meter to obtain the ac resistance per unit of length  $R_{ac2}$  of the wire of this toroidal coil. The frequency region for measurement of  $R_{ac2}$  was 1 kHz–700 kHz for wire G and 1 kHz–2 MHz for the other experimental Litz wires. Finally, the measured difference between  $R_{ac1}$  and  $R_{ac2}$  was compared with the theoretical prediction to evaluate the appropriateness of the coefficient  $G_L$ . The difference between  $R_{ac1}$  and  $R_{ac2}$  is the copper loss caused by the magnetic field generated in the toroidal coil. Because the difference  $R_{ac2} - R_{ac1}$  is small in thin wires, this experiment evaluated  $R_{ac2} - R_{ac1}$  for wires G-I.

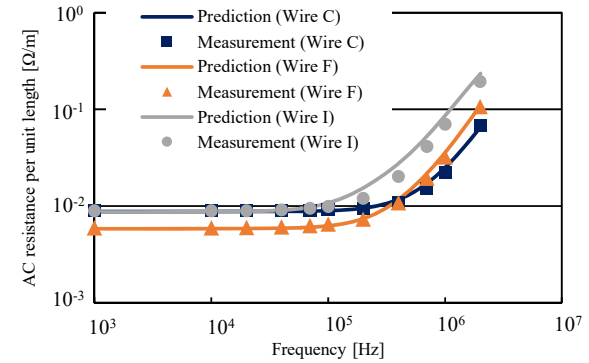
The toroidal coil generates the uniform ac magnetic field in



(a) Wires A, D, and G



(b) Wires B, E, and H



(c) Wires C, F, and I

Fig. 6. Comparison results between the theoretically predicted and experimentally measured ac resistance per unit of length of wire of the non-inductive solenoid ( $R_{ac1}$ ).

perpendicular to the wire in the inner side of the coil. If  $H_{toroid}$  denotes the root-mean-square of the ac magnetic field inside the toroidal coil,  $H_{toroid}$  can be calculated according to Ampere's law as

$$H_{toroid} = \frac{N I_L}{2\pi r_{eq}}, \quad (42)$$

where  $N$  is the number of turns wound on the toroid,  $r_{eq}$  is the representative distance from the center of the toroid to the coil. This experiment adopted  $r_{eq} = 40.5$  mm, which equals to the average of the inner and outer radius of the coreless toroidal coil.

The ac magnetic field outside the toroidal coil is zero. Therefore, the part of the Litz wire that faces the outside of the

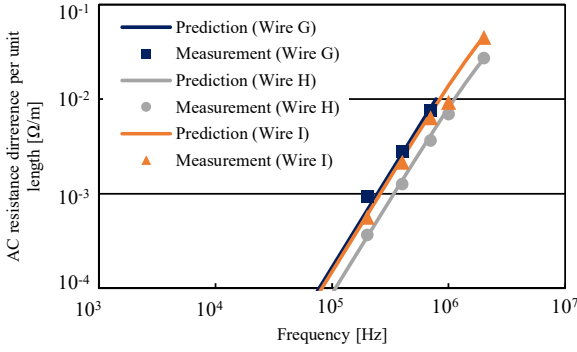


Fig. 7. Comparison results between the theoretically predicted and experimentally measured  $R_{ac1}-R_{ac2}$ , which is the ac resistance difference per unit of length of wire between the toroidal coil and the non-inductive solenoid coil.

toroidal coil has the zero ac magnetic field, whereas the part that faces the inside of the toroidal coil has the ac magnetic field  $H_{toroid}$ . However, this experiment approximates that the Litz wire is applied with the uniform representative external ac magnetic field, whose root-mean-square value equals to the root-mean-square average over the Litz wire. Consequently,  $\overline{H_L}$  can be expressed as follows by simply approximating that the ac magnetic field increases linearly from the outer side to the inner side of the toroidal coil:

$$\overline{H_L} = \frac{NI_L}{2\sqrt{3}\pi r_{eq}}, \quad (43)$$

By utilizing (43), the theoretical value of  $R_{ac2}$  can be calculated as

$$\begin{aligned} P_{L\_total} &= R_{ac2}I_L I_L^2 = R_L I_L I_L^2 + G_L I_L \overline{H_L}^2 \\ &= \left( R_L + G_L \frac{N^2}{12\pi^2 r_{eq}^2} \right) I_L I_L^2, \end{aligned} \quad (44)$$

$$\therefore R_{ac2} = R_L + G_L \frac{N^2}{12\pi^2 r_{eq}^2}.$$

Consequently, the difference between  $R_{ac1}$  and  $R_{ac2}$ , i.e.  $R_{ac2}-R_{ac1}$ , can be theoretically predicted as

$$R_{ac2} - R_{ac1} = G_L \frac{N^2}{12\pi^2 r_{eq}^2}. \quad (45)$$

Figure 6 shows the evaluation results of  $R_{ac1}$ . The maximum estimation error rate was 28% observed in wire I at 200 kHz. However, the proposed model well predicted the frequency dependence of different Litz wire constructions. Consequently, the result supported the model of  $R_L$ , derived as (33).

Figure 7 shows the evaluation result of  $R_{ac2}-R_{ac1}$ . Because the measured  $R_{ac2}-R_{ac1}$  was extremely small at low frequencies below 200 kHz, as is also expected from the proposed Litz wire copper loss model, Fig. 7 presents the results for the frequencies

higher than 200 kHz. The maximum estimation error rate for  $R_{ac2}-R_{ac1}$  was 50% observed in wire I at 1 MHz. However, as it can be seen in Fig. 7, the frequency dependence predicted by the proposed model also agreed well with the experimental result in both of the two experimental Litz wires, although drastic approximation was applied for calculating  $R_{ac2}-R_{ac1}$ . As a result, Fig. 7 supported the model of  $G_L$ , derived as (34).

Consequently, these experiments supported the proposed copper loss model of the Litz wire.

#### IV. CONCLUSIONS

The copper loss prediction of the Litz wire is important for the design optimization of the Litz wire. However, the Litz wire has a complicated structure with multi-level twisting, which hinders the analytical and numerical analysis of the copper loss. The preceding studies have long accumulated analytical insights for the copper loss modeling of the strands and the bundle of twisted strands. However, there is currently no analytical model available that allows the designer to calculate copper loss by using easily obtained constructive parameters. Therefore, this paper constructed an analytical copper loss model of the Litz wire, which is formulated only with parameters that can be measured by basic testing instruments.

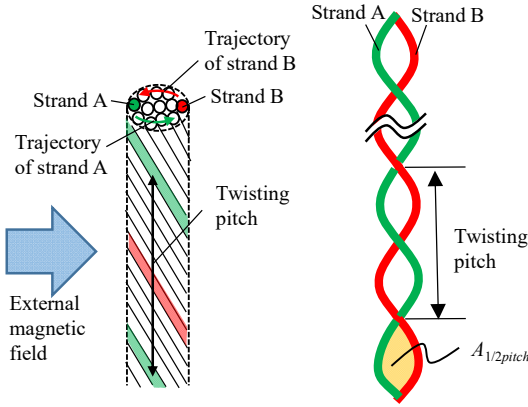
Based on these preceding insights, the proposed model introduced the frequency-dependent effective resistivity of the Litz wire for estimation of the local ac current distribution inside the Litz wire, which is affected by the bundle structure. Furthermore, the proposed model includes analysis of the proximity losses of strands caused by the parallel field due to twisting the strands.

Appropriateness of the proposed copper loss model was experimentally evaluated by comparing the ac resistance between the measurement and the analytical prediction. The result revealed the successful prediction of the frequency dependence of the ac resistance of the commercial Litz wires, supporting the proposed copper loss model.

#### APPENDIX

##### A. Estimation of Litz Wire Length for Neglecting Bundle-Level Proximity Effect in 1<sup>st</sup> Level Bundle

The bundle-level proximity effect deteriorates the rotationally symmetrical ac current distribution among the strands. The deterioration of this current distribution is caused by the difference of the voltage induction, which is caused by the external ac magnetic field applied to the 1<sup>st</sup> level bundle, among the strands in the 1<sup>st</sup> level bundle. For analyzing this phenomenon, a pair of strands is considered as shown in Fig. 8(a). Owing to the twisted structure of the bundle, the pair of the strands has the double helix structure as shown in Fig. 8(b). According to Faraday's law, the difference of the voltage induction between these two strands equals to the time derivative of the magnetic flux linkage with the closed-loop path formed of these strands. Because the two ends of the strands are connected at the Litz wire ends, the voltage difference generates circulating ac current, which deteriorates the rotationally symmetrical ac current distribution.



(a) Pair of strands located at the periphery of 1<sup>st</sup> level bundle (b) Double helix structure formed of 2 strands with the length of an integral multiple and a half of twisting pitch

Fig. 8. Condition when the maximum voltage difference occurs between a pair of strands located in a 1<sup>st</sup> level bundle.

Considering the external magnetic field in parallel to the bundle, this closed-loop path does not have total flux linkage with this field. This indicates that the bundle-level proximity effect does not occur for the external field applied in parallel to the bundle. Therefore, the external field applied perpendicular to the bundle is analyzed hereafter.

Because of the double helix structure of the pair of strands, the flux linkage equals to zero and therefore the bundle-level proximity effect does not occur if the bundle length equals to the integral multiplication of the twisting pitch. However, if the bundle length deviates from the accurate integral multiplication of the twisting pitch, the difference of the voltage induction between the two strands causes the circulating ac current, which appears as the bundle-level proximity effect.

The worst case, i.e. the condition when the maximum voltage difference occurs, takes place when two strands are located at the two opposite sides of the outer periphery of the bundle, as strand A and B in Fig. 8(a), and the bundle length equals to the integral multiple and half of the twisting pitch, as in Fig. 8(b). Therefore, if  $v_{diff\_max}$  denotes the maximum difference of the voltage induction in a pair of strands,  $v_{diff\_max}$  can be estimated as follows, considering that the trajectory of these strands along the Litz wire forms the circle in the bundle cross-section as shown in Fig. 8(a):

$$\begin{aligned} v_{diff\_max} &= A_{1/2pitch\_b} \frac{d\mu_0 H_{ext\_b}}{dt} \\ &= \frac{4}{\pi} \alpha_b \cdot \frac{1}{2} l_{pitch\_b} \cdot 2\pi f \mu_0 H_{ext\_b} = 4\alpha_b l_{pitch\_b} f \mu_0 H_{ext\_b}, \end{aligned} \quad (46)$$

where  $H_{ext\_b}$  is the root-mean-square value of the external ac magnetic field applied perpendicular to the bundle;  $A_{1/2pitch\_b}$  is the projected area of the region surrounded by two strands A and B of the 1/2 twisting pitch length with respect to a plane perpendicular to the external magnetic field, as defined in Fig. 8(b);  $\alpha_b$  is the radius of the bundle cross-section;  $l_{pitch\_b}$  is the twisting pitch length of the strands to form the 1<sup>st</sup> level bundle;  $f$  is the frequency of the external magnetic field;  $\mu_0$  is the

permeability of the air.

Therefore, if  $I_{cir\_b}$  denotes the ac current circulating between these two strands in the worst case,  $I_{cir\_b}$  can be estimated as

$$I_{cir\_b} = \frac{4\alpha_b l_{pitch\_b} f \mu_0 H_{ext\_b}}{2R_s l_s}, \quad (47)$$

where  $R_s$  is the ac resistance of the strand given in (6), and  $l_s$  is the strand length in the bundle. For neglecting the bundle-level proximity effect,  $I_{cir\_b}$  must be sufficiently smaller than the non-circulating ac current flow in the strand. The non-circulating ac current in the strands is affected by the bundle-level skin effect. However, if the average strand current in the Litz wire is taken as the representative non-circulating strand current for simplifying the discussion, this requirement can be expressed as

$$I_{cir\_b} = \frac{2\alpha_b l_{pitch\_b} f \mu_0 H_{ext\_b}}{R_s l_s} \ll \frac{I_b}{n_s} = \frac{I_L}{n_{total}}, \quad (48)$$

where  $I_b$  and  $I_L$  are the ac current flowing through the 1<sup>st</sup> level bundle and the Litz wire, respectively;  $n_s$  and  $n_{total}$  is the number of strands contained in the 1<sup>st</sup> level bundle and the Litz wire, respectively. (Because the bundle-level skin effect results in the larger strand current than the average strand current at the periphery of the bundle, the requirement in (48) is stricter than the actual requirement.)

The copper loss of the strands is proportional to the square of the ac current flowing through the strands. Therefore, this circulating ac current  $I_{cir\_b}$  should be smaller than 1/3 of the average strand current  $I_L/n_{total}$ , if the copper loss increase caused by  $I_{cir\_b}$  is less than approximately 10% of the total copper loss in the pair of the strands. Therefore, (48) would be expressed more specifically as follows in the practical point of view:

$$\frac{2\alpha_b l_{pitch\_b} f \mu_0 H_{ext\_b}}{R_s l_s} \leq \frac{I_L}{3n_{total}}. \quad (49)$$

By approximating that the largest scale of twisting tends to mainly contribute to the length increase of the strands, the strand length  $l_s$  can be regarded to be the same for all the strands, as discussed in subsection II.C. Therefore, the Litz wire length  $l_L$  can be expressed as  $l_L = l_s/m$  where  $m$  is the length ratio of the 1<sup>st</sup> level bundle to the Litz wire. Consequently, the necessary Litz wire length for neglecting the bundle-level proximity effect can be obtained from (49) as

$$l_L \geq \frac{6n_{total}\alpha_b l_{pitch\_b} f \mu_0 H_{ext\_b}}{mR_s} \frac{H_{ext\_b}}{I_L} \quad (50)$$

Finally, by utilizing (36), (50) can be rewritten as

$$l_L \geq 6\sqrt{n_{total}n_s} \frac{\alpha_L l_{pitch\_b} f \mu_0 H_{ext\_b}}{mR_s} \frac{H_{ext\_b}}{I_L} \quad (51)$$

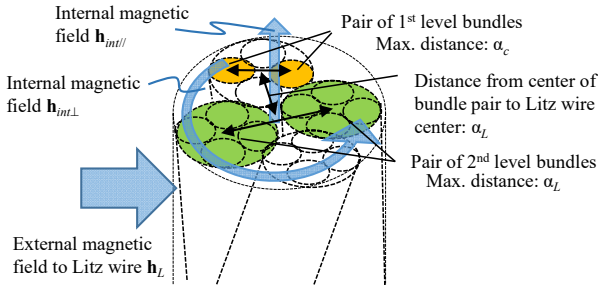


Fig. 9. Pairs of 1<sup>st</sup> level bundles and 2<sup>nd</sup> level bundles with the maximum distance in the 2<sup>nd</sup> and 3<sup>rd</sup> level bundles, i.e. the Litz wire, respectively.

In the Litz wire structure,  $H_{ext\_b}$  is the sum of the magnetic field  $H_L$  applied from outside the Litz wire and the internal magnetic field generated inside the Litz wire. Therefore, the former is dependent on the magnetic structure surrounding the Litz wire, whereas the maximum value of the latter is approximately given as  $I_L/2\pi\alpha_L$  at the outer periphery of the Litz wire, according to Ampere's law. Therefore, in the worst case where the two fields reinforce each other,  $H_{ext\_b}$  is given as

$$H_{ext\_b} = H_L + \frac{I_L}{2\pi\alpha_L}. \quad (52)$$

In section III, the Litz wires with 3 m length, used in the experiment, were all confirmed to satisfy the requirement of (51) in the experimental frequency region. For calculation of  $H_{ext\_b}$ ,  $H_L=0$  was adopted in the first experiment, whereas  $H_L$  given in (43) was adopted in the second experiment.

#### B. Estimation of Litz Wire Length for Neglecting Bundle-Level Proximity Effect in 2<sup>nd</sup> and 3<sup>rd</sup> Level Bundles

The necessary Litz wire length for neglecting the bundle-level proximity effect in the 2<sup>nd</sup> and 3<sup>rd</sup> level bundles can be calculated according to a similar discussion as the previous subsection. The magnetic field parallel to the bundle is again not considered according to the same reason as the previous subsection. The slight difference is that the distance between a pair of the bundles to be twisted in the worst case, i.e. the pair with the maximum distance, equals to the radius of the produced bundle because of the small number of bundles to be twisted, as shown in Fig. 9. (The distance of the strands in a 1<sup>st</sup> level bundle is  $2\alpha_b$  in the worst case.) Therefore, if  $I_{cir\_c}$  and  $I_{cir\_L}$  denote the ac current circulating between the pair of the two bundles to be twisted in the 2<sup>nd</sup> level bundle and 3<sup>rd</sup> level bundle, i.e. the Litz wire, respectively,  $I_{cir\_c}$  and  $I_{cir\_L}$  can be expressed as

$$I_{cir\_c} = \frac{\alpha_c l_{pitch\_c} f \mu_0 H_{ext\_c}}{R_b l_b}, \quad (53)$$

$$I_{cir\_L} = \frac{\alpha_L l_{pitch\_L} f \mu_0 H_{ext\_L}}{R_c l_c},$$

where  $\alpha_c$  is the radius of the 2<sup>nd</sup> level bundle;  $l_{pitch\_c}$  and  $l_{pitch\_L}$

are the twisting pitch length of the 1<sup>st</sup> level bundles to form the 2<sup>nd</sup> level bundle and that of the 2<sup>nd</sup> level bundles to form the 3<sup>rd</sup> level bundle, respectively;  $H_{ext\_c}$  and  $H_{ext\_L}$  are the root-mean-square value of the perpendicular external ac magnetic field applied to the 2<sup>nd</sup> and 3<sup>rd</sup> level bundles, respectively;  $R_c$  is the ac resistance of the 2<sup>nd</sup> level bundle;  $l_b$  and  $l_c$  is the length of the 1<sup>st</sup> level and 2<sup>nd</sup> level bundles, respectively.

By approximating that  $l_b=l_l/m$  and  $l_c=l_l/m$  according to the same reason as in the previous subsection, the necessary Litz wire length for neglecting the bundle-level proximity effect can be obtained as (54) for the 2<sup>nd</sup> level bundle and (55) for the 3<sup>rd</sup> level bundle.

$$\frac{\alpha_c l_{pitch\_c} f \mu_0 H_{ext\_c}}{m R_b l_L} \leq \frac{I_L}{3n_1 n_2}, \quad (54)$$

$$\therefore l_L \geq \frac{3n_1 n_2 \alpha_c l_{pitch\_c} f \mu_0 H_{ext\_c}}{m R_b I_L},$$

where  $n_1$  is the number of the 1<sup>st</sup> level bundles in a 2<sup>nd</sup> level bundle and  $n_2$  is the number of the 2<sup>nd</sup> level bundles in a Litz wire.

$$\frac{\alpha_L l_{pitch\_L} f \mu_0 H_{ext\_L}}{m R_c l_L} \leq \frac{I_L}{3n_2}. \quad (55)$$

$$\therefore l_L \geq \frac{3n_2 \alpha_L l_{pitch\_L} f \mu_0 H_{ext\_L}}{m R_c I_L}.$$

Parameter  $\alpha_c$  can be estimated as the radius of the circle area that occupies the same area as the cross-section of the 2<sup>nd</sup> level bundle. Furthermore,  $R_c$  is approximated as  $R_b/n_1$  to simplify the analysis. Then, the necessary Litz wire length for neglecting the bundle-level proximity effect is finally formulated as (56) for the 2<sup>nd</sup> level bundle and (57) for the 3<sup>rd</sup> level bundle.

$$l_L \geq \frac{3n_1 \sqrt{n_2} \alpha_c l_{pitch\_c} f \mu_0 H_{ext\_c}}{m R_b I_L}, \quad (56)$$

$$l_L \geq \frac{3n_1 n_2 \alpha_L l_{pitch\_L} f \mu_0 H_{ext\_L}}{m R_b I_L}. \quad (57)$$

As for the pair of the 1<sup>st</sup> level bundles in a 2<sup>nd</sup> level bundle, the center of the pair in the worst case is always located at the distance of  $1/2\alpha_L$  from the center of the Litz wire, as shown in Fig. 9. Meanwhile, a pair of the 2<sup>nd</sup> level bundles in a 3<sup>rd</sup> level bundle, i.e. the bundle of the highest level, does not interlink with the internal magnetic field because  $\mathbf{h}_{int\perp}$  has the rotational symmetry in the Litz wire cross-section and  $\mathbf{h}_{int\parallel}$  is in parallel to the pair. Therefore,  $H_{ext\_c}$  and  $H_{ext\_L}$  are approximately calculated as

$$H_{ext\_c} = H_L + \frac{I_L}{4\pi\alpha_L}, \quad H_{ext\_L} = H_L. \quad (58)$$



The necessary Litz wire length for neglecting the bundle-level proximity effect in Litz wires with twisting levels other than 3 can be calculated according to the similar discussion. For example, for Litz wires with 2 twisting levels, the necessary condition should be considered only for the 2<sup>nd</sup> level bundle. As a result, the necessary length can be calculated by substituting  $n_2=1$  and  $H_{ext,c}=H_L$  to (56). The reason for substituting  $H_{ext,c}=H_L$  is that the 2<sup>nd</sup> twisting is the highest level in these wires.

In section III, the Litz wires with 3-m length, used in the experiment, were all confirmed to satisfy the requirement of (56) and (57) in the experimental frequency region. For calculation of  $H_{ext,c}$  and  $H_{ext,L}$ ,  $H_L=0$  was adopted in the first experiment, whereas  $H_L$  given in (43) was adopted in the second experiment.

#### REFERENCES

- [1] I. Lope, J. Acero, and C. Carretero, "Analysis and optimization of the efficiency of induction heating applications with litz-wire planar and solenoidal coils," *IEEE Trans. Power Electron.*, vol. 31, no. 7, pp. 5089-5101, Jul. 2016.
- [2] M. Hataya, Y. Oka, K. Umetani, E. Hiraki, T. Hirokawa, and M. Imai, "Novel thin heating coil structure with reduced copper loss for high frequency induction cookers," in *Proc. IEEE Intl. Conf. Elect. Mach. Syst. (ICEMS2016)*, Chiba, Japan, Nov. 2016, pp. 1-6.
- [3] M. Hataya, K. Kamaeguchi, E. Hiraki, K. Umetani, T. Hirokawa, M. Imai, and S. Sadakata, "Verification of the reduction of the copper loss by the thin coil structure for induction cookers," in *Proc. IEEE Intl. Power Electron. Conf. (IPEC2018)*, Niigata, Japan, May 2018, pp. 410-415.
- [4] S. Inoue and H. Akagi, "A bidirectional isolated DC-DC converter as a core circuit of the next-generation medium-voltage power conversion system," *IEEE Trans. Power Electron.*, vol. 22, no. 2, pp. 535-542, Mar. 2007.
- [5] W. Shen, F. Wang, and D. Boroyevich, C. W. Tipton IV, "High-density nanocrystalline core transformer for high-power high-frequency resonant converter," *IEEE Trans. Ind. Appl.*, vol. 44, no. 1, pp. 213-222, Jan./Feb. 2008.
- [6] W. Water and J. Lu, "Improved high-frequency planar transformer for line level control (LLC) resonant converters," *IEEE Magn. Lett.*, vol. 4, 6500204, Nov. 2013.
- [7] E. L. Barrios, A. Ursua, L. Marroyo, and P. Sanchis, "Analytical design methodology for litz-wired high-frequency power transformers," *IEEE Trans. Ind. Electron.*, vol. 62, no. 4, pp. 2103-2113, Apr. 2015.
- [8] M. Leibl, G. Ortiz, and J. W. Kolar, "Design and experimental analysis of a medium-frequency transformer for solid-state transformer applications," *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 5, no. 1, pp. 110-123, Mar. 2017.
- [9] C. R. Sullivan and R. Y. Zhang, "Simplified design method for litz wire," in *Proc. IEEE Appl. Power Electron. Conf. Expo. (APEC2014)*, pp. 2667-2674, Mar. 2014.
- [10] B. A. Reese and C. R. Sullivan, "Litz wire in the MHz range: modeling and improved designs," in *Proc. IEEE Workshop Control Modeling Power Electron. (COMPEL2018)*, pp. 1-8, Jul. 2017.
- [11] A. Roßkopf, E. Bär, and C. Joffe, "Influence of inner skin- and proximity effects on conduction in litz wires," *IEEE Trans. Power Electron.*, vol. 29, no. 10, pp. 5454-5461, Oct. 2014.
- [12] R. Y. Zhang, J. K. White, and J. G. Kassakian, "Fast simulation of complicated 3-D structures above lossy magnetic media," *IEEE Trans. Magn.*, vol. 50, no. 10, pp. 7027416, Oct. 2014.
- [13] A. Roßkopf, E. Bär, C. Joffe, and C. Bonse, "Calculation of power losses in litz wire systems by coupling FEM and PEEC method," *IEEE Trans. Power Electron.*, vol. 31, no. 9, pp. 6442-6449, Sept. 2016.
- [14] S. Hiruma and H. Igarashi, "Fast 3-D analysis of eddy current in litz wire using integral equation," *IEEE Trans. Magn.*, vol. 53, no. 6, pp. 7000704, Jun. 2017.
- [15] E. Plumed, J. Acero, I. Lope and C. Carretero, "3D finite element simulation of litz wires with multilevel bundle structure," in *Proc. Annu. Conf. IEEE Ind. Electron. Soc. (IECON2018)*, Washington, DC, 2018, pp. 3479-3484.
- [16] J. A. Ferreira and J. D. van Wyk, "A new method for the more accurate determination of conductor losses in power electronic converter magnetic components," in *Proc. 3rd Intl. Conf. Power Electron. Variable-Speed Drives*, 1988, pp. 184-187.
- [17] J. A. Ferreira, "Analytical computation of AC resistance of round and rectangular litz wire windings," *IEE Proc. B – Elect. Power Appl.*, 1992, vol. 139, no. 1, pp. 21-25.
- [18] M. Bartoli, N. Noferi, A. Reatti, and M. K. Kazimierczuk, "Modeling Litz-wire winding losses in high-frequency power inductors," in *Proc. IEEE Power Electron. Specialists Conf.*, 1996, vol. 2, pp. 1690-1696.
- [19] X. Nan and C. R. Sullivan, "An improved calculation of proximity-effect loss in high-frequency windings of round conductors," in *Proc. IEEE Power Electron. Specialists Conf.*, 2003, vol. 2, pp. 853-860.
- [20] J. Acero, R. Alonso, J. M. Burdio, L. A. Barragan, D. Puyal, "Frequency-dependent resistance in litz-wire planar windings for domestic induction heating appliances," *IEEE Trans Power Electron.*, vol. 21, no. 4, pp. 856-866, Jul. 2006.
- [21] J. Acero, R. Alonso, J. M. Burdio, L. A. Barragan, C. Carretero, "A model of losses in twisted-multistranded wires for planar windings used in domestic induction heating appliances," in *Proc. IEEE Appl. Power Electron. Conf. and Expo.*, 2007, pp. 1247-1253.
- [22] X. Nan and C. R. Sullivan, "An equivalent complex permeability model for litz-wire windings," *IEEE Trans. Ind. Appl.*, vol. 45, no. 2, pp. 854-860, Mar./Apr. 2009.
- [23] D. Sinha, P. K. Sadhu, N. Pal, and A. Bandyopadhyay, "Computation of inductance and AC resistance of a twisted litz-wire for high frequency induction cooker," in *Proc. Intl. Conf. Ind. Electron. Control Robotics*, 2010, pp. 85-90.
- [24] R. P. Wojda and M. K. Kazimierczuk, "Winding resistance of litz-wire and multi-strand inductors," *IET Power Electronics*, vol. 5, no. 2, pp. 257-268, Jan. 2012.
- [25] V. Väisänen, J. Hiltunen, J. Nerg, and P. Silventoinen, "AC resistance calculation methods and practical design considerations when using litz wire," in *Proc. Annu. Conf. IEEE Ind. Electron. Soc. (IECON2013)*, 2013, pp. 368-375.
- [26] C. R. Sullivan and R. Y. Zhang, "Analytical model for effects of twisting on litz-wire losses" in *Proc. IEEE Workshop Control Modeling Power Electron. (COMPEL)*, 2014, pp. 1-10.
- [27] R. P. Wojda and M. K. Kazimierczuk, "Winding resistance and power loss of inductors with litz and solid-round wires," *IEEE Trans. Ind. Appl.*, vol. 54, no. 4, pp. 3548-3557, July-Aug. 2018.
- [28] Z. Liu, J. Zhu and L. Zhu, "Accurate calculation of eddy current loss in litz-wired high-frequency transformer windings," *IEEE Trans. Magn.*, vol. 54, no. 11, pp. 1-5, Nov. 2018.
- [29] K. Umetani, J. Acero, H. Sarnago, O. Lucia, and E. Hiraki, "Simple fully analytical copper loss model of Litz wire made of strands twisted in multiple levels," in *Proc. IEEE Appl. Power Electron. Conf. (APEC2019)*, Anaheim, CA, USA, Mar. 2019, pp. 1257-1264.
- [30] M. Abramowitz and I. A. Stegun, "Bessel functions of integer order" in *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Washington, DC: NBS, 1964, ch. 9, pp. 379-385.