A two-phase inspection policy with imperfect testing

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Abstract

This paper presents an inspection policy to detect failures of a single component system that remain hidden otherwise. Inspection reveals whether the unit is in good or failed state. The possibility of non perfect testing is assumed, thus, successive inspections may fail detecting a failure or result in a false alarm. The occurrence of false alarms is reported in optical fire detectors and inspection of printing circuit boards which are on the basis of electronic systems. A two-phase inspection schedule takes into account the changes in component's aging. The system may undergo different inspection frequencies to detect both early failures or those due to the natural deterioration in the system as time goes by. The examples reveal the advantages of a two-phase inspection when comparing with the unique interval inspection.

Keywords: Inspection, False alarm, Optimum policy, Reliability

1. Introduction

Systems that are not continuously monitored may undergo failures that are dormant and only detected by periodic tests or inspections. Safety systems and those that alternate working and idle periods are typical examples. For safety systems the consequences of an undetected failure can be assessed in terms of the expected cost of an accident that is likely to occur whereas in production lines the penalty is due to defective production. The cost incurred during the downtime period should be weighted against the maintenance costs in order to make the maintenance procedure to be profitable.

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There is a wide research on inspection policies that deal with maintenance optimization, focusing on availability, cost or both. Vaurio [1] analyzes the cost rate as well as the availability for a system that experiences periodic inspections at times $kT, k = 1, 2, \ldots$, solving optimum test and maintenance intervals. Biswas *et al* [2] studies the availability of a periodically inspected system which is maintained through a fixed number of imperfect repairs before being replaced. Nakagawa [3] consider periodic check intervals where the unit after check has the same age with probability p or is as-good-as-new with probability 1-p. Nakagawa and Yasui [4] proposes an inspection model under which the cost of replacement is higher when the number of failures exceeds a given threshold.

An imperfect testing may result in a false alarm, i.e., a test indicating a failure when no such a failure has occurred. They are responsible for shutdowns that may lead to unusually high costs.

Regarding protection systems, optical fire detectors can respond to many sources of ultraviolet (UV) and infrared (IR) radiations, including non fire sources. Goedeke [5] reports the occurrence of false alarms of this type of systems in an study carried out in aircraft hangars. Goedeke and Gross [6] identified possible sources of UV, IR and visible radiations that may cause false alarms in optical fire detectors, affecting its fire detection performance.

Moreover for highly complex systems, false alarms become a serious incovenient. This is an inherent problem with electronic systems, specially those aiming at prognostics. Electronic prognostics is commonly used in high-reliability and high-availability systems to detect failures. This technique can be found in cars where the state of main parts (airbags, stability control, oil) is checked by means of circuitry. Sometimes car users report on alarm signals which turn out to be due to a testing system failure rather than to a main system failure as a usual checking reveals. In turn, those false alarms often carry relevant costs. Chen *et al* [7] present an automatic optical system to inspect defects in flexible printing circuits (FPC). FPC is a sort of printing circuit board widely used in notebooks, cell phones and digital cameras. The automatic optical inspection satisfy the demand of quality and speed in electronic devices being progressively smaller and present a false alarm rate less than 10%.

Badía *et al* [8] present an inspection policy with periodic tests that have no effect on systems' reliability and consider the possibility of non-perfect inspections. A less than perfect testing is also considered in Badía *et al* [9] for a system that alternates operating and idle periods its failures being revealed and unrevealed, respectively. Thus, the inspection may not detect a failure that remain undiscovered after successive inspections or erroneously states that a failure has occurred, that is a false alarm. Periodic inspections do not provide a suitable correspondence to the change in unit's aging whereas a two-phase inspection policy does. Cavalcante *et al* [10] point out the efficacy of a two-phase inspection policy more frequent in early life and less frequent in later stages when component arises from a heterogeneous population, being a mixture of distributions the time to failure. Inspections corresponding to the first phase aim at preventing early failures. The second phase comprises a more relaxed inspection provided that the unit has passed the initial tests giving evidence of being strong enough. Scarf *et al* [11] consider a policy which combines inspections and age based replacements. Inspections aim at preventing early failures as in burn-in processes. The goal of preventive maintenance is to reduce failures due to wear-out in later life. Badía *et al* [8] consider a single inspection interval every T units of time. The model described in the present work extends the previously cited work as it presents a procedure with two different inspection intervals, T_1 and T_2. This policy adapts itself to the changes in the system reliability. When we want to detect 'infant' failures then $T_1 < T_2$. However the inspection frequency is the opposite, $T_1 > T_2$, provided that failures due to wear-out are of a major concern. Mandatory inspection of cars in Spain serves as an instance of $T_1 > T_2$. When a car reaches 4 years old it has to be inspected every two years until it is 10 years old. From that moment on the inspection takes place every year. This paper presents a pure two-phase inspection policy under which a perfect testing is no longer assumed.

The paper is organized as follows. Section 2 describes both the inspection policy and the model assumptions along with the calculations to obtain the cost function. It lays the foundations for further works. Section 3 analyzes the implications of different reliability classes for the optimum policy. The examples therein illustrate the proposed inspection procedure. The conclusions of this paper are presented in Section 4.

2. The inspection policy

Consider a single-unit system whose failures are detected by some type of testing. From new, M inspections are carried out at times $jT_1, j = 1, 2 \ldots M$. The unit is renewed whenever an inspection points out the occurrence of a failure and with no effect on its reliability otherwise. In the latter case the unit is considered to be 'as-good-as-old'. If the unit reaches MT_1 and no failure has been detected, inspections from MT_1 on are carried out at times $jT_2, j = 1, 2, \ldots$.

A less than perfect testing is considered. Thus an inspection may give a false alarm or on the contrary it can fail to detect a failed unit. False alarms and undetected failures correspond, respectively, to type I and type II statistical errors.

We take into account the costs derived from inspections, renewal of the unit, type I errors as well as those due to the downtime incurred while a failed unit remains undetected.

The following notation will be used:

X time to failure of the unit

$R(x)$ reliability function $R(x) = P(X > x)$

μ mean time to failure

T_1 time between scheduled inspections in phase 1

T_2 time between scheduled inspections in phase 2

T_{0j} random duration of the jth inspection

t_0 mean duration of the jth inspection

T_r random time for renewal of a failed unit

t_r mean time of the renewal

K_1 number of inspections in phase 1 previous to failure or to the beginning of phase 2 whatever comes first

K_2 number of inspections in phase 2 (from MT_1 on) previous to failure

K_3 number of inspections in phase 1 after failure until its detection or to the beginning of phase 2 whatever comes first

K_4 number of inspections in phase 2 after failure until its detection

α probability of a false alarm

β probability of not detecting a failure in an inspection item n_I number of false alarms in a cycle

c_0 unitary cost of inspection

c_I cost of a false alarm

c_r cost of the renewal of the unit

c_d cost-rate due to the downtime

The following calculations aim at obtaining the probability distribution of K_1. The range of K_1 is $\{0, 1, 2, \ldots, M\}$

For $i = 0, \ldots, M-1$

$$P(K_1 = i) = P(iT_1 \leq X < (i+1)T_1) = R(iT_1) - R((i+1)T_1)$$

For $i = M$

$$P(K_1 = M) = P(X \geq MT_1) = R(MT_1)$$

Next, the expected value of K_1 is obtained

$$E[K_1] = \sum_{i=1}^{M-1} i(R(iT_1) - R((i+1)T_1)) + MR(MT_1) =$$

$$\sum_{i=1}^{M} R(iT_1) \tag{1}$$

The number of inspections in phase 2 previous to failure, K_2, takes on the values $\{0, 1, 2, \ldots\}$ with the corresponding probabilities given below

$$P(K_2 = 0) = P(X < MT_1 + T_2) = 1 - R(MT_1 + T_2)$$

For $i = 1, 2, \ldots$

$$P(K_2 = i) = P(MT_1 + iT_2 \leq X < MT_1 + (i+1)T_2) = R(MT_1 + iT_2) - R(MT_1 + (i+1)T_2)$$

The mean value of K_2 is given as follows

$$E[K_2] = \sum_{i=1}^{\infty} i(R(MT_1 + iT_2) - R(MT_1 + (i+1)T_2)) =$$

$$\sum_{i=1}^{\infty} R(MT_1 + iT_2) \tag{2}$$

The range of K_3 (number of inspections in phase 1 after failure until it is detected or inspections in phase 2 begin whichever occurs first) is $\{0, 1, 2 \ldots, M\}$. Its probability function is obtained next with β denoting the probability of not detecting a failure in an inspection.

$$P(K_3 = 0) = P(X > MT1) = R(MT_1)$$

For $i = 1, 2, \ldots, M$

$$P(K_3 = i) =$$
$$(R(0) - R((M-i)T_1))\beta^{i-1}(1-\beta) + (R((M-i)T_1) - R((M-i+1)T_1))\beta^{i-1}$$

Therefore

$$P(K_3 = i) = \beta^{i-1}(1-\beta) + R((M-i)T_1)\beta^i - R((M-i+1)T_1)\beta^{i-1}$$

Now, the expectation of K_3 is obtained

$$E[K_3] = (1-\beta)\sum_{i=1}^{M} i\beta^{i-1} + \sum_{i=1}^{M} i\beta^i R((M-i)T_1) - \sum_{i=1}^{M} i\beta^{i-1} R((M-i+1)T_1)$$

In addition

$$\sum_{i=1}^{M} i\beta^{i-1} = \frac{1 - M\beta^M + M\beta^{M+1} - \beta^M}{(1-\beta)^2}$$

$E[K_3]$ can also be expressed as follows

$$E[K_3] = \frac{1 - M\beta^M + M\beta^{M+1} - \beta^M}{(1-\beta)} + M\beta^M - \sum_{i=1}^{M} \beta^{M-i} R(iT_1) =$$
$$\frac{1-\beta^M}{1-\beta} - \sum_{i=1}^{M} \beta^{M-i} R(iT_1) \qquad (3)$$

We denote by A the following event: a failure occurs in $(0, MT_1)$ but it is not detected in this interval.

$$P(A) = \sum_{i=1}^{M} (R((i-1)T_1) - R(iT_1))\beta^{M-(i-1)} = S(T_1, M, \beta)$$

In addition B represents a failure occurring in $(0, MT_1)$ and being detected in that interval.

$$P(B) = \sum_{i=1}^{M} (R((i-1)T_1) - R(iT_1))(1 - \beta^{M-(i-1)})$$

The foregoing probabilities are used in further calculations involving the number of inspections in phase 2 after failure until it is detected, K_4. The random variable K_4 takes on the values $\{0, 1, 2, \ldots\}$ with the probabilities given by

$$P(K_4 = 0) = P(B) = \sum_{i=1}^{M} (R((i-1)T_1) - R(iT_1))(1 - \beta^{M-(i-1)})$$

For $j = 1, 2, \ldots$

$$P(K_4 = j) = [S(T_1, M, \beta) + R(MT_1)]\, \beta^{j-1}(1 - \beta)$$

And its corresponding expected value

$$E[K_4] = (S(T_1, M, \beta) + R(MT_1)) \sum_{j=1}^{\infty} j\beta^{j-1}(1 - \beta) =$$
$$(S(T_1, M, \beta) + R(MT_1)) \frac{1}{1 - \beta} \tag{4}$$

The length of a cycle, τ is given by

$$\tau = (K_1 + K_3)T_1 + (K_2 + K_4)T_2 + \sum_{j=1}^{K_1+K_2+K_3+K_4} T_{0j} + T_r$$

The expressions in (1), (2), (3), (4) lead to

$$E[\tau] = (T_1 + t_0) \left(\sum_{i=1}^{M} (1 - \beta^{M-i}) R(iT_1) + \frac{1 - \beta^M}{1 - \beta} \right) + t_r$$
$$(T_2 + t_0) \left(\sum_{i=1}^{\infty} R(MT_1 + iT_2) + (S(T_1, M, \beta) + R(MT_1)) \frac{1}{1 - \beta} \right) \tag{5}$$

The period of downtime, D, is

$$D = \tau - X$$

hence

$$E[D] = E[\tau] - \mu \tag{6}$$

The limiting expected proportion of time that the system is up, known as limiting average availability constitutes a measure of interest. It is defined as follows

$$AV = \frac{E[U]}{E[U] + E[D]}$$

with $E[U]$ representing the expected uptime during a cycle. Moreover

$$E[U] + E[D] = E[\tau]$$

In this case the foregoing expression turns out to be

$$AV = \frac{\mu}{E[\tau]}$$

The following calculations aim at obtaining the cost of a cycle:

The number of inspections in a cycle is $K_1 + K_2 + K_3 + K_4$ with c_0 being the unitary cost. Therefore, the mean cost of inspections in a cycle, C_{in}, turns out to be

$$C_{in} = c_0 \left(\sum_{i=1}^{M} (1 - \beta^{M-i}) R(iT_1) + \frac{1 - \beta^M}{1 - \beta} \right) +$$
$$c_0 \left(\sum_{i=1}^{\infty} R(MT_1 + iT_2) + (S(T_1, M, \beta) + R(MT_1)) \frac{1}{1 - \beta} \right) \tag{7}$$

Next we consider the number of false alarms in a cycle, n_I. Its conditional distribution given $K_1 + K_2$ is binomial with parameters $n = K_1 + K_2$ and α. Then

$$E[n_I] = E[E[n_I | K_1 + K_2]] = \alpha \left(\sum_{i=1}^{M} R(iT_1) + \sum_{i=1}^{\infty} R(MT_1 + iT_2) \right) \tag{8}$$

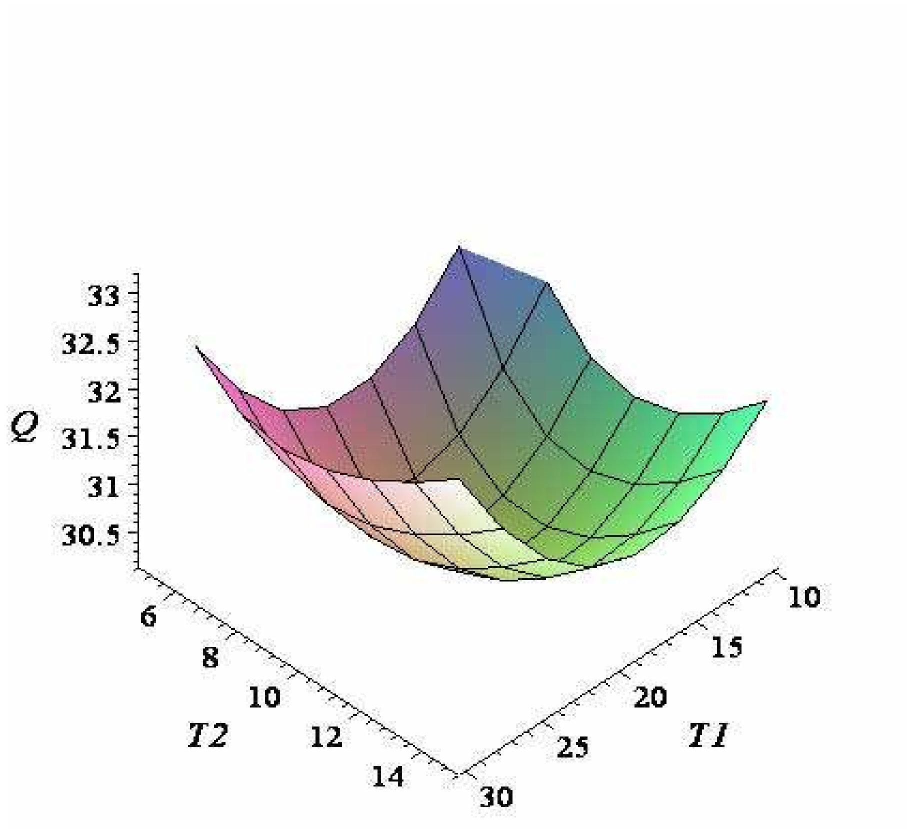

Figure 1: Cost Function: $c_0 = 1$, $c_I = 50$, $c_r = 25$, $c_d = 100$, $M^\star = 1$

The cost of a cycle is

$$C(\tau) = c_0(K_1 + K_2 + K_3 + K_4) + c_I n_I + c_r + c_d D$$

From (5),(6), (7) and (8), straightforward calculations lead to $E[C(\tau)]$ which is expressed as follows

$$E[C(\tau)] = C_{in} + c_I \alpha \left(\sum_{i=1}^{M} R(iT_1) + \sum_{i=1}^{\infty} R(MT_1 + iT_2) \right) + c_r + c_d E[D] \qquad (9)$$

The objective function considered will be the cost per unit of time in the long run, that is

$$Q(T_1, T_2, M) = \frac{E[C(\tau)]}{E[\tau]}$$

a	b	μ	$M^\star$	$T_1^\star$	$T_2^\star$	$Q^\star$	$T_0^\star$	$Q_0^\star$
40	1.5	36.11	1	13.9	7.3	5522.65	8.4	5608.28
40	2.5	35.49	1	22.1	5.9	5269.44	8.4	5649.75
30	1.5	27.08	1	11.6	6.2	6371.38	7.3	6462.91
30	2.5	26.62	1	17.8	4.9	6102.43	7.3	6507.94
20	1.5	18.05	1	9.0	5.0	7717.58	5.9	7813.52
20	2.5	17.75	1	13.1	3.9	7436.74	5.9	7861.16
10	1.5	9.03	1	5.8	3.3	10328.74	4.1	10418.35
10	2.5	8.87	1	7.7	2.5	10065.40	4.2	10464.15
40	1	40	0	8.58	8.58	5367.65	8.58	5367.65
30	1	30	0	7.39	7.39	6197.68	7.39	6197.68
20	1	20	0	5.98	5.98	7517.77	5.98	7517.77
10	1	10	0	4.15	4.15	10094.58	4.15	10094.58
6	0.9	6.31	1	2.9	3.4	11925.03	3.18	11929.86
5	0.9	5.26	1	2.7	3.0	12629.14	2.88	12633.42
2	0.8	2.27	1	1.5	2	15560.59	1.77	15571.86
2	0.9	2.10	1	1.6	1.9	15762.99	1.73	15764.94
1	0.8	1.13	1	1.1	1.4	17321.99	1.19	17327.28

Table 1: Optimum decision values, $M^\star$, $T_1^\star$, $T_2^\star$ and optimum cost, $Q^\star$, for a two phase inspection policy. Optimum $T_0^\star$ and $Q_0^\star$ under a single inspection interval

which is expressed as follows

$$Q(T_1, T_2, M) = c_d + \frac{C_{in} + c_I \alpha \left(\sum_{i=1}^{M} R(iT_1) + \sum_{i=1}^{\infty} R(MT_1 + iT_2) \right) + c_r - c_d \mu}{E[\tau]}$$

and $E[\tau]$ given in (5).

Note that $M = 0$ or $M = \infty$ lead to a unique inspection interval. This case corresponds to the single phase model given in Badía *et al* [8].

The plot in Figure 1 corresponds to the cost function when the time to failure follows a Weibull distribution with scale parameter $a = 40$ and shape parameter $b = 1.5$. The rest of the parameters are $M^\star = 1$, $t_0 = 2$, $t_r = 2.5$

3. Numerical examples

The following examples aim at providing some insight about the optimal inspection policy $(M^\star, T_1^\star, T_2^\star)$. Several conditions leading to $T_1^\star < T_2^\star$ or

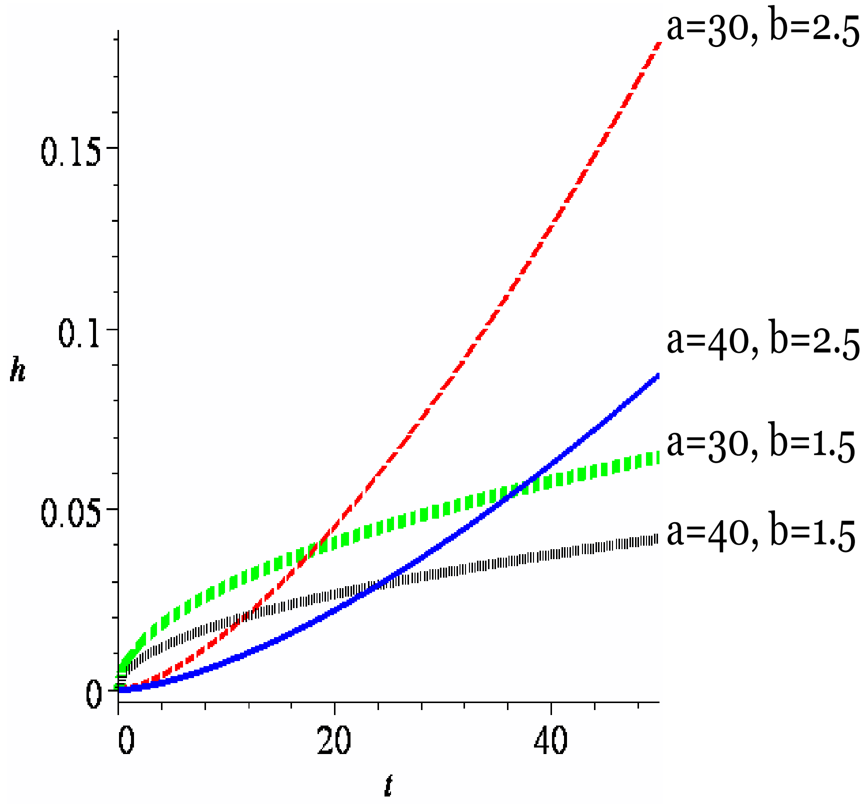

Figure 2: Failure Rate for IFR Weibull Distributions

$T_1^\star > T_2^\star$ are studied. The time to failure is a Weibull distribution function whose reliability function is

$$R(t) = e^{-(t/a)^b}$$

In addition the times of inspection and repair are, respectively, $t_0 = 1$, $t_r = 5$ along with the following the costs: $c_0 = 1$, $c_I = 50$, $c_r = 25$, $c_d = 20000$

Table 1 displays the optimum policy $(M^\star, T_1^\star, T_2^\star)$ and the optimum cost, $Q^\star$, for different IFR and DFR times to failure. In addition the optimum $T_0^\star$ for the model with a unique inspection interval (Badía *et al* [8]) and the corresponding optimum cost $Q_0^\star$ is also included.

The optimum policy $(M^\star, T_1^\star, T_2^\star)$ is obtained by the analytical procedure proposed by Nakagawa [12] and Zequeira and Bérenguer [13] and it verifies:

$$Q(T_1^\star, T_2^\star, M^\star) = \min_{T_1, T_2, M} Q(T_1, T_2, M) = \min_M Q(T_{1M}^\star, T_{2M}^\star, M)$$

where $(T^\star_{1M}, T^\star_{2M})$ represents the optimum (T_1, T_2) for a given M. The search for the optimum policy has been restricted to low values of M as they correspond to the the economically relevant region.

An inspection of Table 1 reveals the following interesting patterns:

For increasing failure rate (IFR) distributions ($b > 1$), the optimum inspection intervals verify $T^\star_1 > T^\star_0 > T^\star_2$. This makes sense provided that the proneness of failure increases as time goes by due to the natural deterioration of the system. Thus, it pays to increase the inspection frequency. The comparison of the optimum policy in the cases $b = 1.5$ and $b = 2.5$ leads to report that the two-phase policy constitutes an inspection procedure better adapted to the current reliability of the system than that based on a unique inspection interval. The corresponding failure rates, $h(t)$, for $b = 2.5$ and $b = 1.5$ and different values of the scale parameter a are depicted in Figure 2. It can be observed that $h(t)$ is smaller in $b = 2.5$ than $b = 1.5$ for low values of t whereas the situation is just the opposite when t is large. In the latter case $h(t)$ grows faster when $b = 2.5$ than $b = 1.5$. This shape of the failure rate determines a more relaxed inspection in the first stage for $b = 2.5$ than for $b = 1.5$ and the reversed policy in the second stage.

The cases corresponding to exponential distributions ($b = 1$) lead to $T^\star_1 = T^\star_2$, that is a single inspection interval. This result is consistent with a two-phase inspection policy, trying to adapt the inspection to the actual state of the system. Exponential distributions mean no deterioration neither improvement so a unique interval fits for those systems experiencing this invariant mode.

Regarding decreasing failure rate (DFR) distributions, the result is just the opposite to the IFR case with $T^\star_1 < T^\star_0 < T^\star_2$. Under this assumption the system experiences an improvement that determines a more relaxed inspection in the second stage. This result agrees with that obtained by Cavalcante *et al* [10] in the inspection and preventive maintenance of heterogeneous populations. These authors consider the time to failure to be a mixture of IFR distributions as an explanation for the system improvement. Such a mixture tends to decrease in the long run (Gurland and Sethuraman [14]). The comparison of the shape of $h(t)$ for $b = 0.9$ and $b = 0.8$ and common scale parameter provides a reasonable explanation of the differences between the inspection frequency in both cases. This study is similar to that given in the IFR case and reveals that the two-phase policy is also adapted to the reliability of the system when dealing with DFR distributions.

The optimum cost is not strictly monotonic respect to the mean time to

a	b	μ	$AV1$	$AV2$
40	1.5	36.11	0.7196	0.7239
40	2.5	35.49	0.7175	0.7366
30	1.5	27.08	0.6769	0.6815
30	2.5	26.62	0.6747	0.6949
20	1.5	18.05	0.6094	0.6142
20	2.5	17.75	0.6070	0.6282
10	1.5	9.03	0.4792	0.4836
10	2.5	8.87	0.4769	0.4968
6	0.9	6.31	0.4036	0.4038
5	0.9	5.26	0.3684	0.3686
2	0.8	2.27	0.2215	0.2221
2	0.9	2.10	0.2119	0.2120
1	0.8	1.13	0.1338	0.1341

Table 2: Limiting average availability under unique inspection interval (AV1) and the two-phase inspection model (AV2)

failure, μ, but in general it tends to be larger for those systems with lower mean time to failure, making more profitable the inspection of a system the longer his expected life. Cost savings can be observed when comparing the two-phase inspection policy with the model with a unique inspection interval (Badía *et al* [8]). The reduction in the optimum cost in the examples reaches 6%.

Table 2 presents the results concerning the limiting mean availability for the model with a unique inspection interval (AV1) and the two-phase inspection policy (AV2). The latter provides larger values up to 4%.

4. Conclusions

This paper presents a new inspection policy in two stages where also the possibility of less than perfect tests is assumed. The present work extends that of Badía *et al* [8]. The policy based on two inspection intervals turns out to be better adapted to the current reliability of the system than the unique inspection model provided that the inspection frequency varies in the former case. Thus, a more frequent testing can be carried out in the first phase if the tester is concerned with early failures or, on the contrary, during the second phase provided that use and wear out make failures more likely to occur. The examples dealing with IFR and DFR distribution give some insight about the way that a two-phase inspection policy capture the changes in the failure rate.

Apart from tests that fail detecting failures, false alarms are reported as a major problem in some types of security systems such as optical fire detectors as well as in several electronic systems. The unavailability and costs derived from erroneous tests can not be neglected. Therefore an inspection policy that takes into account the possibility of imperfect testing is suitable for those systems.

When comparing with the unique interval model, the examples reveal that the two-phase inspection policy produces both cost savings and a more lasting availability of the system making the two-phase model a preferable choice. This improvement occurs because this new inspection procedure suits better to the behavior of the failure rate. The comparison of a two-phase inspection policy that includes preventive maintenance with the corresponding to a unique inspection interval plus preventive maintenance (Badía *et al* [9]) constitutes a problem to be dealt with next.

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