Can one kilogram of iron float on water?

All general physics text books include a chapter regarding hydrostatics. Archimedes and Pascal principles are commonly covered but surface tension is merely named. Few people knows that surface tension contributes to the buoyancy of any object, with more or less relevance [1]. I think that including surface tension in general physics courses is important to understand all phenomena regarding hydrostatics and this is why every year, before starting the lesson regarding hydrostatics, I pose a question to the students: Can one kilogram of iron float on water? Usually, some of them answer instinctively saying “no, that is not possible”. Others, knowing that I am used to show them physics “tricks” in the classroom, remain thoughtful. Anyway, it is a question that would be answered negatively by around 90% of questioned people. Although, as we will show along the present manuscript, there are at least two ways to make one kilogram of iron floating. When we think about one kilogram of iron or other metal, we use to think about it as a compact cubic-shaped block. If that is the thought, the answer to the mentioned question is negative, since the weight overcomes the other forces, such as the buoyance force, among others [2-5].

However, what would happen if the kilogram of iron was an extremely thin sheet? Could we place it on the water and floating? To answer it, we need to look at the forces that would act on the iron sheet. Supposing that it does not penetrate into the water, the only forces acting to it are its weight and the surface tension exerted by the water surface, Figure 1. The force exerted by the surface tension [6,7], is given by

$$F = 2\pi r\gamma \,, \tag{1}$$

where r is the radius of the piece in contact with the water surface, supposed circular, and γ is the surface tension (γ=72.5·10^{-3} N/m for the water). This force should be enough to counteract the weight, given by $w = mg$, being g=9.8 m/s^2 the gravitational acceleration. By applying the static condition and considering the iron sheet as a circle, we can obtain the necessary surface to counteract the weight action, whose radius is

$$r = \frac{mg}{2\pi\gamma} \,, \tag{2}$$

resulting r=21.51 m. From the radius, the surface results S=1454 m^2. Considering that the iron density is 7874 kg/m^3, the thickness of the sheet should be t=87 nm. It results in an extremely thin film but, in theory, it would be possible.

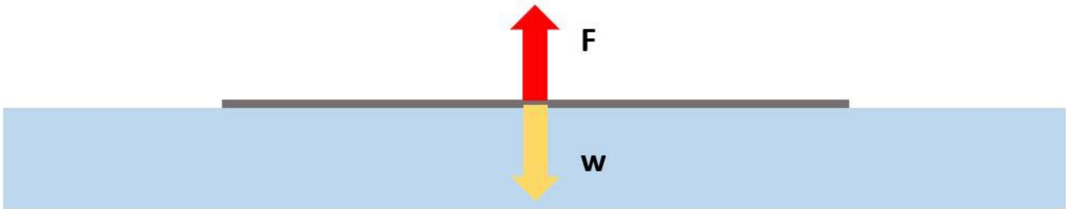

Figure 1.- Forces acting to the iron sheet on the water.

The other option is making the buoyance force equals the weight [8-10], Figure 2. This force is given by

$$B = \rho_l g V \, , \tag{3}$$

where ρ_l=1000 kg/m^3 is the density of water and V is the volume of water displaced by the iron. To compensate the weight, the displaced volume needs to be $V = m / \rho_l$, resulting V=10^{-3} m^3. In this case, the force due to the surface tension will be neglectable.

There are many ways to achieve the necessary displaced volume but we will assume a semi-spherical shape. The volume of a semi-sphere is given by $2\pi r_e^3 / 3$. Thus, the radius of the sphere can be obtained as

$$r_e = \sqrt[3]{\frac{3m}{2\pi\rho_l}} \, , \tag{4}$$

resulting r_e=7.8 cm. To achieve this exterior radius with one kilogram of iron, the sphere must be hollow. Then, we may obtain the interior radius as

$$r_i = \sqrt[3]{r_e^{\,3} - \frac{3m}{2\pi\rho_m}} \, , \tag{5}$$

being ρ_m=7874 kg/m^3 the iron density, and resulting r_i=7.4 cm. Supposing a shell with hemispherical shape, another valid expression for the inner radius results

$$r_i = r_e \sqrt[3]{1 - \frac{\rho_l}{\rho_m}} \, , \tag{6}$$

The thickness of the iron boat results t=4 mm, approximately and, in this way, the kilogram of iron will float.

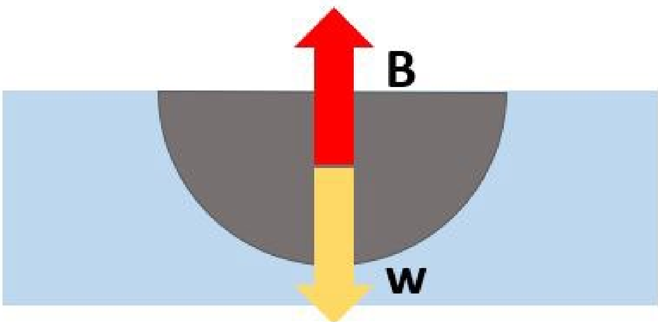

Figure 2.- Forces acting to the iron boat on the water.

Finally, since it is difficult to manage one kilogram of iron, we may perform a didactic experiment by using aluminum foil. Firstly, we cut a circular piece of foil and fill a basin with water. As we may observe in Figure 3a, the piece of aluminum remains on the surface of the water. We have measured the radius and mass of the piece of aluminum, resulting $r = 6.5\,cm$ and $m = 0.45\,g$, respectively. Then the surface tension, obtained with Eq. (1), and the weight result $F = 2.96 \cdot 10^{-2}\,N$ and $w = 4.4 \cdot 10^{-3}\,N$, respectively. As the surface tension is higher than the weight, it floats. Having a closer look at Figure 3a, we observe the effect of the surface tension in the light shadows and reflections around the aluminum piece. Following, we may decrease the surface tension of water by adding liquid soap. Thus, the aluminum piece does not float but falls to the bottom of the basin, Figure 3b. To finalize, making a shell with it, we may achieve it to float again, Figure 3c. This time is due to buoyance force, since surface tension is neglectable.

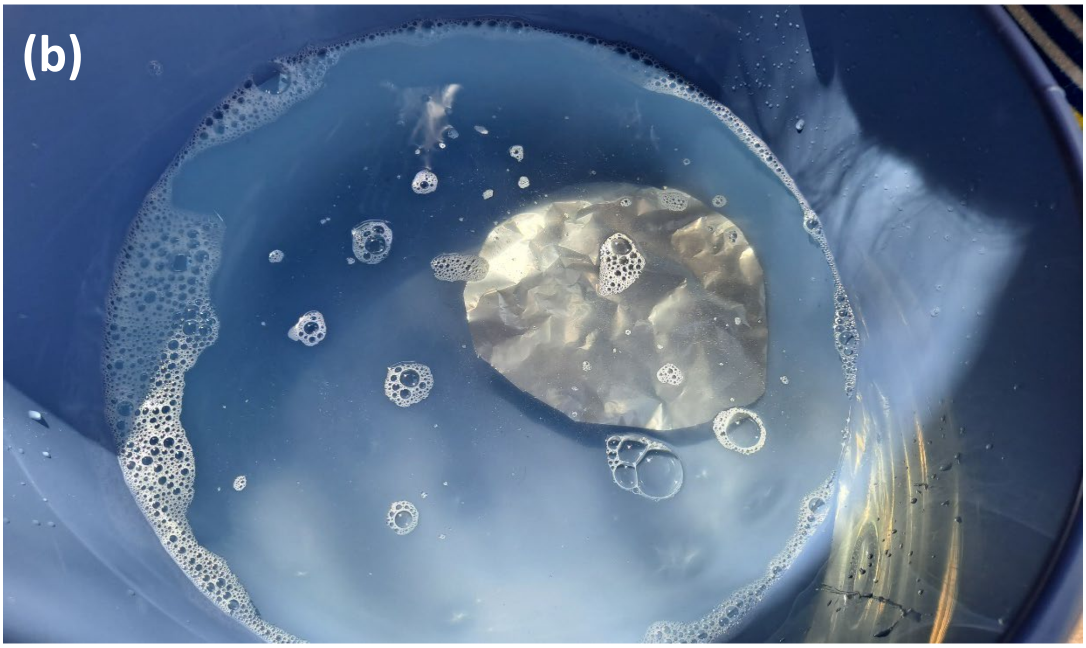

Figure 3.- (a) Smooth aluminum circular foil floating on the water due to the surface tension, (b) smooth aluminum circular foil dropped to the bottom of the water thanks to the soap, and (c) aluminum shell floating on the water due to the buoyance force.

In this manuscript, a question difficult to be answered, a priori, such as: Can one kilogram of iron float on water? is answered with no doubt. It is a question that cannot be answered without some knowledge about hydrostatics. We show that, there are at least two ways to achieve it. One of them by using the surface tension of water and the other by using the buoyance force or Archimedes theorem. In both cases, the kilogram of iron surprisingly floats. Finally, we prove it experimentally by using aluminum foil. This exercise and experimental demonstration could be interesting as example of both phenomena in undergraduate courses or even first course of university degrees.

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References:

[1] D. Naylor and S. S. H. Tsai, “Archimedes’ principle with surface tension effects in undergraduate fluid mechanics,” Int. J. Mechanical Eng. Educ. 50, 749 (2022).

[2] F. Thompson, “Archimedes and the golden crown,” Phys. Educ. 43, 396 (2008).

[3] J. A. Moreira, A. Almeida and P. S. Carvalho, “Two Experimental Approaches of Looking at Buoyancy,” Phys. Teach. 51, 96 (2013).

[4] J. Nelson and J. B. Nelson, “Buoyancy Can-Can,” Phys. Teach. 53, 279 (2015).

[5] Y. Feigel and N. Fuzailov, “Floating of a long square bar: experiment vs theory,” Eur. J. Phys. 42, 035011 (2021).

[6] R. D. Edge, “Surface tension,” Phys. Teach. 26, 586 (1988).

[7] G. S. Siahmazgi, “Some instructive experiments in surface tension,” Phys. Educ. 48, 142 (2013).

[8] R. E. Weston, “A Boatload of Pennies,” Phys. Teach. 40, 392 (2002).

[9] M. B. Larosi, “Floating Together on the Top,” Phys. Teach. 53, 93 (2015).

[10] A. Raymond Penner, “Suspension of a disk on a surface of water,” Am. J. Phys. Educa. 68, 549 (2000).