RELATIVE AND ABSOLUTE STATION-KEEPING FOR 2D LATTICE FLOWER CONSTELLATIONS

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I. Introduction

Flower Constellations present beautiful and interesting dynamical features that allow exploration of a wide range of potential applications, such as telecommunications, Earth and deep space observation, global positioning systems, and distributed space systems. The Flower Constellations Theory [1–3] was developed in 2004 by Prof. Mortari. A Flower Constellation (FC) is a set of satellites following the same trajectories with respect to a rotating frame of reference. It includes the classic Walker Constellations [4] but without the necessity of having circular orbits. This FC theory was substantially improved by the 2D-Lattice Flower Constellations [5] (2D-LFCs), making the theory independent of any reference frame (inertial or rotating), and achieving a minimal parametrization.

In the Keplerian model, 2D-LFCs remain 2D-LFCs over time, that is, the initial lattice of the constellation and its symmetries are maintained. However, when a perturbation is considered, such as the J_2 effect due to the non-spherical shape of the Earth, the initial structure slowly changes. Thus, instead of trying to compensate the J_2 perturbation by orbital maneuvers, we intend to include this perturbation in the design process [6].

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Although the relative distance between two satellites is time changing, the Lattice-preserving Flower Constellations method [7] allows to control the lattice and to preserve its initial configuration and symmetries through a two-step procedure. In the first place, a slight modification of the semi-major axis to control the orbital period of each satellite is applied; next, the values of the eccentricity and inclination are adjusted to minimize the non secular rates. This way, it is possible to preserve for several months the initial properties of the constellation. Nonetheless, the lattice-preserving property is not valid indefinitely and some orbit-maintenance maneuvers must be planned in order to compensate the ground track shift [8].

In this paper we deal with the concepts of relative and absolute station keeping for a 2D-LFC. Relative station-keeping is attained using the lattice-preserving methodology. Absolute station-keeping is achieved by an impulsive-maneuvering strategy to compensate the shifting in the relative-track. This methodology is then applied to a real constellation of satellites: the Galileo Constellation, showing the Δv that would be required in order to perform an absolute station keeping in an example of application. This method can be extended to any constellation of satellites using optimization tools and the application of Lambert problem solvers, making the methodology very attractive from an economical point of view (low fuel consumption) and from a practical point of view (fixed initial configuration).

The paper is organized as follows; first, the 2D Lattice Flower Constellation Theory is summarized. Then, the effect of the J_2 perturbation on the (Ω, M)-space is presented, and the method to obtain the lattice-preserving Flower Constellation (relative station-keeping) is shown. After that, an example of design is presented, in particular, the Galileo Constellation is built based on the lattice-preserving Flower Constellation technique. Moreover, a study of the effects of the J_3 and the Sun perturbations are considered to show the differences from the initial design. Finally, the Δv and absolute station-keeping concepts are introduced for the 2D Lattice Flower Constellations and then, an example of application is computed taking Galileo as the constellation of study. This way, a complete station keeping strategy is defined, taking the lattice-preserving Flower Constellations as a base and including the absolute station keeping methodology.

II. Preliminaries

This section describes the main tools used throughout the paper: the theory of Flower Constellations and the concepts of relative station-keeping and absolute station-keeping.

A. 2D-Lattice Flower Constellation Theory

A 2D Lattice Flower Constellation (2D-LFC) is described by nine parameters: three integers and six continuous parameters. The first three parameters are the number of inertial orbits (N_o), the number of satellites per orbit (N_{so}) and the configuration number (N_c), which is a parameter that satisfies $N_c \in [0, N_o - 1]$ and governs the phasing of the constellation. In particular, the location of the satellites of a 2D-LFC corresponds to a lattice in the (Ω, M)-space [9]. This space can be regarded as a 2D torus (both axes, Ω and M, are modulo 2π), and coincides with the solutions of the following system of equations:

$$\begin{pmatrix} N_o & 0 \\ N_c & N_{so} \end{pmatrix} \begin{pmatrix} \Omega_{ij} - \Omega_{00} \\ M_{ij} - M_{00} \end{pmatrix} = 2\pi \begin{pmatrix} i \\ j \end{pmatrix}, \tag{1}$$

where $i = 0, \cdots, N_o - 1$, $j = 0, \cdots, N_{so} - 1$, and $N_c \in [0, N_o - 1]$. Indices (i, j) represent the j-th satellite on the i-th orbital plane.

Finally, the other six parameters are the semi-major axis (a), the eccentricity (e), the inclination (i) and the argument of perigee (ω) (which are the same for all the satellites of the constellation), and the longitude of the ascending node and the mean anomaly of the first satellite of the constellation i. e. Ω_{00} and M_{00}.

Other important concepts to introduce in this section are the relative and absolute station-keeping. The application of these concepts to a constellation of satellites are the aim of the paper, so, it is required to present them first.

B. Relative Station-Keeping

The relative station-keeping is based on the upkeep of the configuration as a constellation. The lattice-preserving property [7], applied to FCs, means that the initial configuration and initial symmetries are preserved. This kind of station-keeping is not related to any frame of reference, it is a property of the constellation as a whole. In fact, achieving the relative station-keeping generates a rigid configuration of satellites that will be able to move or rotate with respect to the Earth and the inertial frame of reference.

C. Absolute Station-Keeping

The absolute station-keeping considered in this paper consist of the upkeep of the relative-tracks of the constellation over the Earth surface in a certain available range. This range is part of the orbit design requirement of each particular mission and it is usually chosen due to payload constraints of the satellite, the telecommunications subsystem or the Earth target area to study. As such, it is necessary to relate the inertial positions of the constellation satellites with the rotating frame of reference fixed in the Earth.

III. Relative station-keeping in Flower Constellations

This section describes how to achieve the relative station-keeping in a Flower Constellation by means of a proper initial design, no requiring further orbital corrections to maintain this property. Normally, when perturbations are considered in the 2D-LFC, the lattice of the constellation departs from the initial configuration. However, the theory of lattice-preserving Flower Constellations [7] describes a methodology to design a 2D-LFC in such a way that the initial lattice and symmetries are maintained over a long period of time when the J_2 perturbation is taken into account. Lattice-preserving FCs are achieved in two steps. First, the semi-major axis of all satellites are slightly modified to make them have the same slope of the secular part of their osculating elements (i.e. the secular perturbation is the same for all the satellites). Next, the values of their orbital eccentricity and inclination are computed so that they minimize the non-secular perturbation of the osculating elements as much as possible. Hence, it is possible to obtain a constellation where all the satellites are perturbed in a similar way under the J_2 effect, preserving the initial configuration and the initial symmetries.

In this work we apply the Lattice-preserving FCs methodology for the circular case in order to show a simple and clear example of station keeping that is also widely used in constellations. The lattice-preserving property degenerates over time and, in consequence, some orbit-maintenance maneuvers must be planned in order to compensate the ground track error as we will see later.

A. Lattice-preserving methodology

Lattice-preserving Flower Constellations [7] only consider the J_2 effect, which represents the second harmonic of the Earth's gravitational potential, since it is almost 1000 times larger than the higher harmonic terms of the Earth's gravitational potential. We briefly summarize below the methodology followed in the paper mentioned above because we intend to apply it in the following section.

From the formulation of 2D Lattice Flower Constellations (Eq. (1)), all the orbital elements of all the satellites are equal except for the right ascension of the ascending node and the mean anomaly. Thus, taking into account the J_2 perturbation, we conclude that the slopes of the secular component of the osculating elements $(\dot{a}, \dot{e}, \dot{i}, \dot{\Omega}, \dot{\omega}, \dot{M})$ do not depend on the initial right ascension of the ascending node. However, they depend on the initial mean anomaly of each satellite.

To overcome this difference we take into account that the secular motion of the mean anomaly and the semi-major axis are related by the following equation:

$$\dot{M}_{sec}(t) = n = \frac{2\pi}{T}, \tag{2}$$

where, n is the mean motion and T is the orbital period, directly related to the semi-major axis. Then, the correction method states that, if we take the semi-major axis and the rate of change of the mean anomaly corresponding to the first satellite of the constellation ($\dot{M}_{00}^{sec}$) as reference values, it is possible to obtain the same rate of change for the mean anomaly of all the satellites of the constellation by slightly modifying the semi-major axis of all the satellites as follow:

$$a_{ij} = a \left(\frac{\dot{M}_{sec}^{ij}}{\dot{M}_{sec}^{00}} \right)^{\frac{2}{3}}, \tag{3}$$

where ij represents the j-th satellite on the i-th orbital plane. This slightly modification on the semi-major axis is useful to control the secular variation of the satellites in order to make them experience the same secular variation. Finally, the initial values of the eccentricity and inclination are slightly modified in such a way that the non-secular variations of the osculating elements are minimized as much as possible. For a fully detailed analysis see Casanova et al. [7].

B. Application to Galileo constellation

Galileo constellation [10] is Europe's own global navigation satellite system. It consists of 27 satellites positioned in three circular Medium Earth Orbit planes at an orbit inclination of 56 degrees respect to the equator. This constellation can be described using the Flower Constellation theory with the design parameters following Eq. (1): $N_o = 3$, $N_{so} = 9$, $N_c = 2$, $a = 29600.137$ [km], $e = 0$, $i = 56°$, $\omega = 0°$, $\Omega_{00} = 0°$ and $M_{00} = 0°$.

Firstly, the evolution of the lattice of the constellation in the (Ω, M)-space is shown. Figure 1 (left) illustrates the initial lattice at time $t = 0$ [s]. Remark that each point represents one satellite of the constellation. Figure 1 (right) presents the position of the satellites after one year of propagation under the J_2 effect.

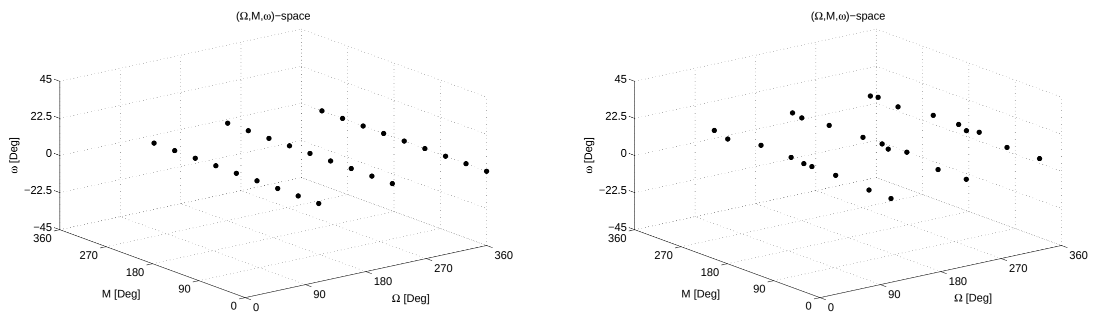

Fig. 1 Satellite distribution for Galileo Flower Constellation over 1 year of propagation.

It is observed that, at time $t = 0$ [s] the lattice is perfectly distributed. Nevertheless, after 1 year, the position of the satellites depart from the initial configuration. We use the lattice-preserving correction method with the aim of maintaining the lattice of the constellation over time.

Subsequently, we apply the lattice-preserving FCs method to all the satellites of the constellation. Through the semi-major axis correction we are able to control the long-term dynamics. Thus, all the satellites have the same slope for the osculating elements and they are perturbed in the same way. In addition, we have computed the values of the eccentricity and the inclination that minimize the non-secular component of the osculating elements of the satellites in the constellation. Therefore, the new lattice-preserving Galileo constellation has the orbital parameters: $e = 0.01$, $i = 56.0009°$, $\omega = 0°$ for all the satellites. The values for the right ascension of the ascending node and the mean anomaly are computed following Eq. (1). Meanwhile, the values of the corrected semi-major axis are given in Table 1. Additionally, the table shows the secular variation of the osculating elements for each satellite after one year of propagation. Note that satellite (i, j) represents the j-th satellite on the i-th orbital plane.

Table 1 Corrected semi-major axis and slopes of the osculating elements of lattice-preserving Galileo FC.

$\mathrm{Sat}_{(i,j)}$	a [km]	$\dot{a}_{sec}$ [km/s]	$\dot{e}_{sec}$ [s^{-1}]	$\dot{i}_{sec}$ [rad/s]	$\dot{\omega}_{sec}$ [rad/s]	$\dot{\Omega}_{sec}$ [rad/s]	$\dot{M}_{sec}$ [rad/s]
$(0,0)$	29600.137	$-2.833\cdot10^{-11}$	$-8.944\cdot10^{-17}$	$-3.227\cdot10^{-16}$	$2.661\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(0,1)$	29598.872	$8.298\cdot10^{-12}$	$-1.198\cdot10^{-17}$	$9.466\cdot10^{-17}$	$2.638\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(0,2)$	29597.165	$3.114\cdot10^{-11}$	$3.891\cdot10^{-16}$	$3.549\cdot10^{-16}$	$2.629\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(0,3)$	29597.843	$2.525\cdot10^{-12}$	$9.406\cdot10^{-16}$	$2.883\cdot10^{-17}$	$2.620\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398265\cdot10^{-4}$
$(0,4)$	29599.783	$-3.029\cdot10^{-11}$	$8.598\cdot10^{-16}$	$-3.450\cdot10^{-16}$	$2.622\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(0,5)$	29599.784	$-1.305\cdot10^{-11}$	$-3.276\cdot10^{-16}$	$-1.486\cdot10^{-16}$	$2.617\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(0,6)$	29597.845	$2.569\cdot10^{-11}$	$-1.847\cdot10^{-16}$	$2.928\cdot10^{-16}$	$2.612\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398265\cdot10^{-4}$
$(0,7)$	29597.166	$2.198\cdot10^{-11}$	$-4.364\cdot10^{-16}$	$2.506\cdot10^{-16}$	$2.620\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(0,8)$	29598.874	$-1.811\cdot10^{-11}$	$-1.044\cdot10^{-15}$	$-2.062\cdot10^{-16}$	$2.629\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,0)$	29599.524	$-2.764\cdot10^{-11}$	$-7.800\cdot10^{-16}$	$-3.148\cdot10^{-16}$	$2.628\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,1)$	29599.976	$-1.926\cdot10^{-11}$	$-3.502\cdot10^{-16}$	$-2.194\cdot10^{-16}$	$2.643\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,2)$	29598.165	$2.096\cdot10^{-11}$	$1.680\cdot10^{-16}$	$2.389\cdot10^{-16}$	$2.636\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,3)$	29597.085	$2.650\cdot10^{-11}$	$4.320\cdot10^{-16}$	$3.020\cdot10^{-16}$	$2.623\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,4)$	29598.519	$-1.175\cdot10^{-11}$	$9.635\cdot10^{-16}$	$-1.338\cdot10^{-16}$	$2.623\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,5)$	29600.101	$-3.061\cdot10^{-11}$	$5.169\cdot10^{-16}$	$-3.488\cdot10^{-16}$	$2.622\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,6)$	29599.218	$1.081\cdot10^{-12}$	$-5.280\cdot10^{-16}$	$1.245\cdot10^{-17}$	$2.612\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,7)$	29597.329	$3.094\cdot10^{-11}$	$1.278\cdot10^{-17}$	$3.526\cdot10^{-16}$	$2.607\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(1,8)$	29597.554	$9.650\cdot10^{-12}$	$-8.179\cdot10^{-16}$	$1.101\cdot10^{-16}$	$2.624\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,0)$	29598.167	$-4.737\cdot10^{-12}$	$-1.048\cdot10^{-15}$	$-5.379\cdot10^{-17}$	$2.627\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,1)$	29599.978	$-3.129\cdot10^{-11}$	$-2.438\cdot10^{-16}$	$-3.565\cdot10^{-16}$	$2.630\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,2)$	29599.522	$-6.135\cdot10^{-12}$	$-1.903\cdot10^{-16}$	$-6.979\cdot10^{-17}$	$2.637\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,3)$	29597.553	$2.915\cdot10^{-11}$	$3.121\cdot10^{-16}$	$3.322\cdot10^{-16}$	$2.634\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,4)$	29597.329	$1.624\cdot10^{-11}$	$6.098\cdot10^{-16}$	$1.851\cdot10^{-16}$	$2.614\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,5)$	29599.216	$-2.352\cdot10^{-11}$	$1.012\cdot10^{-15}$	$-2.680\cdot10^{-16}$	$2.622\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,6)$	29600.101	$-2.443\cdot10^{-11}$	$7.844\cdot10^{-17}$	$-2.783\cdot10^{-16}$	$2.620\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$
$(2,7)$	29598.522	$1.497\cdot10^{-11}$	$-3.629\cdot10^{-16}$	$1.707\cdot10^{-16}$	$2.609\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398265\cdot10^{-4}$
$(2,8)$	29597.085	$2.962\cdot10^{-11}$	$-6.970\cdot10^{-17}$	$3.376\cdot10^{-16}$	$2.614\cdot10^{-9}$	$-5.228\cdot10^{-9}$	$1.2398266\cdot10^{-4}$

The illustration of the satellite distribution of the corrected Galileo Flower Constellation is not depicted here because its representation coincides with Figure 1 (left). Yet, there are remarkable differences highlighted on Table 2. In particular, the table presents the relative distance (in mean anomaly) between satellites. It is depicted the values of the mean anomaly of each of the nine satellites of orbit $i = 1$ along with their relative variations. Note that we have only shown the relative mean anomalies between satellites in the first orbit due to the similarity of the satellites distribution in the other two orbital planes ($i = 2, 3$). Table 2 shows the mean anomaly variations after one year of propagation of the lattice in case the lattice-preserving technique is applied or not. Thus, we validate the proposed method and quantify the accuracy. Accordingly, it is concluded that the initial lattice and the initial symmetries are maintained for a certain period of time. Thus, the relative station-keeping of the satellites in the Galileo constellation states.

Table 2 Relative variations in Mean anomaly [deg] of the satellites $(i = 0, j)$ after one year.

Orbit $i = 0$	M_{00}	M_{01}	M_{02}	M_{03}	M_{04}	M_{05}	M_{06}	M_{07}	M_{08}
$t = 0$	0	40	80	120	160	200	240	280	320
$\lvert M_{0j} - M_{0(j-1)}\rvert$	-	40	40	40	40	40	40	40	40
$t = 1$y.	145.42	170.34	185.43	234.69	303.21	341.88	356.94	28.15	91.41
$\lvert M_{0j} - M_{0(j-1)}\rvert$	-	24.91	15.09	49.25	68.52	38.66	15.07	31.20	63.26
Corrected constellation									
$t = 1$y.	147.75	186.94	226.64	267.62	307.11	346.90	27.87	67.28	106.80
$\lvert M_{0j} - M_{0(j-1)}\rvert$	-	39.19	39.69	40.98	39.48	39.79	40.96	39.41	39.52

It is worth making a couple considerations about the methodology presented. As it is stated before, lattice-preserving FCs procedure is valid only for J_2 perturbation, the most important perturbation at the altitude of study (23222 [km]) as seen in Figure 2. Figure 3 shows the evolution of the right ascension of the ascending node under other perturbations such as zonal harmonic J_3 and the Sun and the Moon as a third bodies. We observe an error of $1°$ in the right ascension of the ascending node. Therefore, we can expect a similar behavior of the constellation if other perturbations such as the zonal harmonics J_3 and the Sun and the Moon as third bodies, are included in the dynamical model.

Finally, it is worth noting that lattice-preserving property degenerates over time. Table 3 derives the mean anomaly variations after five years of propagation. If we compare this table with Table 2 we observe that the lattice of the constellation in a very long propagation is not preserved and so, some orbit-maintenance maneuvers must be planned in order to compensate this in the long term.

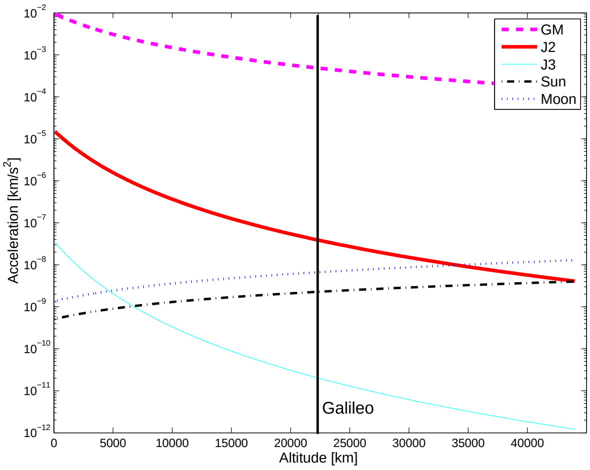

Fig. 2 Comparison of the disturbing accelerations acting on an Earth satellite.

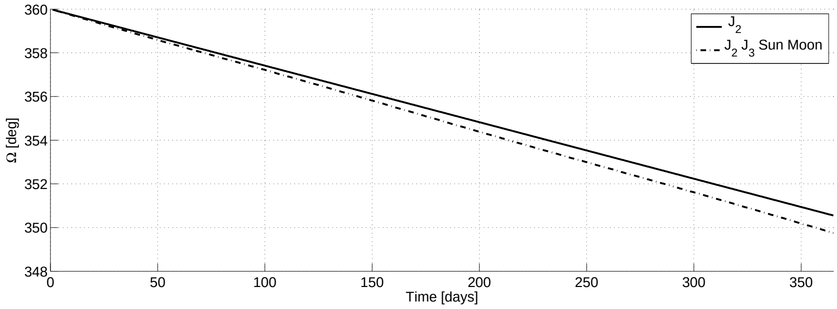

Fig. 3 Evolution of RAAN over 1 year of propagation for different perturbations.

Table 3 Relative variations in Mean anomaly [deg] of the satellites $(i = 0, j)$ after five years.

Orbit $i = 0$	M_{00}	M_{01}	M_{02}	M_{03}	M_{04}	M_{05}	M_{06}	M_{07}	M_{08}
$t = 0$	0	40	80	120	160	200	240	280	320
$\lvert M_{0j} - M_{0(j-1)}\rvert$	-	40	40	40	40	40	40	40	40
Corrected constellation									
$t = 5$y.	90.21	124.45	164.74	210.32	245.36	285.33	330.50	5.09	44.50
$\lvert M_{0j} - M_{0(j-1)}\rvert$	-	34.24	40.28	45.57	35.04	39.97	45.16	34.59	39.40

IV. Absolute station-keeping in Flower Constellations

The previous section has presented a correction method to maintain the symmetries and the structure of the 2D-Lattice Flower Constellations i.e. the satellites of the constellation display a relative station-keeping. The main idea of lattice-preserving Flower Constellations is that all the satellites in the constellation are perturbed in a similar way, and consequently, the initial lattice is maintained over time.

However, some missions require absolute station-keeping, which means that each satellite of the constellation remains in a predefined mathematical box relative to the Earth. This section analyses the feasibility of an absolute station-keeping in a lattice-preserving Flower Constellation. In particular, a study has been made to estimate the required velocity change (Δv) to maintain an absolute station-keeping.

A. Orbital Maneuvers Required

Thus, the objective now is to correct the satellite orbits of the whole constellation to maintain the absolute station-keeping between the boundaries of a particular effective range. Let $\Delta\psi_{max}$ be the maximum permitted deviation of the relative-track in longitude with respect to the initial instant, where $\Delta\psi_{max}$ is chosen due to constellation design in order to fulfill the mission requirements.

First, it is important to know how much time each satellite requires to reach this boundary. Let T_c be the repetition cycle time, that is a time equal to N_p satellite revolutions (T) or N_d sidereal days (T_d):

$$T_c = N_p T = N_d T_d. \tag{4}$$

The deviation suffered by the relative-track of each satellite in the time T_c can be obtained by propagating each constellation satellite for a time equal to $2T_c$ and calculating the angle between the two passings of the satellite over the Earth equator, one at the beginning of the cycle and another one at the end, in the rotating frame of reference. Let $\Delta\psi_c$ be that angle (see Figure 4).

Now, a relation between $\Delta\psi_c$ and the orbital parameters must be established. That relation appears between the right ascension of the ascending node and the deviation of the relative-track:

$$\Delta\Omega = -\Delta\psi_c. \tag{5}$$

Thus, the orbital maneuvers must be based on a plane shift applied when the deviation of the relative-track reaches its maximum allowed ($\Delta\psi_{max}$) i.e. the orbital maneuvers over the right ascension of the ascending node is:

$$\Delta\Omega = -\Delta\psi_{max}. \tag{6}$$

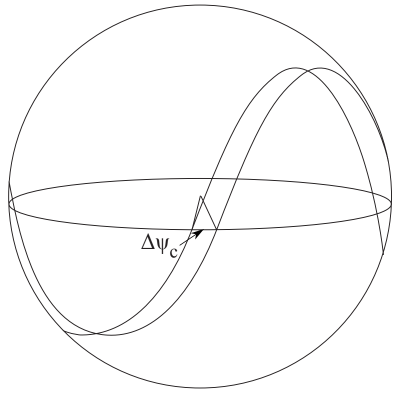

Fig. 4 Representation of $\Delta\psi_c$.

The frequency of this correction depends on the value of $\Delta\psi_c$ obtained, which gives an idea of the time when the next orbital maneuver must be performed. Let $\dot{\psi}$ be the angular velocity of the relative-track drift, then:

$$\dot{\psi}_c = \frac{\Delta\psi_c}{T_c}. \tag{7}$$

Thus, the time to make the orbital maneuvers, which happens when $\Delta\psi_c$ is near to get equal to $\Delta\psi_{max}$, is equal to:

$$t_{correction} = \frac{\Delta\psi_{max}}{\dot{\psi}_c} = \frac{\Delta\psi_{max}}{\Delta\psi_c} T_c. \tag{8}$$

B. Maneuvering Strategy

After the previous analysis it is possible to confirm that the variation of the parameter Ω is the only one required to be changed. Now, a maneuvering strategy is set in order to achieve the absolute station keeping maintaining the lattice of the Flower Constellation. The terms of the final configuration will be denoted with a superscript $*$.

As a requirement of the correction, the lattice of the constellation has to be maintained. Thus, the new configuration (Ω^*_{ij}, M^*_{ij}) has to follow the 2-D Lattice Flower Constellation theory equation, so, using Eq. (1), the following expression can be obtained:

$$\begin{aligned} \Omega_{ij} &= \Omega_{00} + \frac{2\pi}{N_o} i; \\ M_{ij} &= M_{00} + \frac{2\pi}{N_{so}} j - \frac{N_c\left(\Omega_{ij} - \Omega_{00}\right)}{N_{so}}; \end{aligned} \tag{9}$$

where Ω_{00} and M_{00} represent the origin of the constellation distribution. Let $\Delta\Omega_{ij}$ and ΔM_{ij} be the distribution of each particular satellite given by:

$$\begin{aligned} \Delta\Omega_{ij} &= \frac{2\pi}{N_o} i, \\ \Delta M_{ij} &= \frac{2\pi}{N_{so}} j - \frac{2\pi N_c}{N_{so} N_o} i, \end{aligned} \tag{10}$$

then, if they are introduced in Eq. (9), Eq. (11) is obtained:

$$\begin{aligned} \Omega_{ij} &= \Omega_{00} + \Delta\Omega_{ij}; \\ M_{ij} &= M_{00} + \frac{2\pi}{N_{so}} j - \frac{2\pi N_c}{N_{so} N_o} i = \\ &= M_{00} + \Delta M_{ij}. \end{aligned} \tag{11}$$

As such, the constellation is positioned fixing a reference satellite and then distributing the satellites respect to that satellite. Note that this definition is established for a given instant due to the fact that the mean anomaly M is a function of time.

Following Eq. (11), the parameters of the Flower Constellation $\Delta\Omega_{ij}$, ΔM_{ij}, N_o, N_c and N_{so} must be the same in the new constellation in order to maintain its configuration, so the only free variables are Ω_{00} and M_{00} of the new configuration, i.e. Ω_{00}^* and M_{00}^*. Thus, they are the parameters that have to be modified in order to position the constellation over the initial relative-track.

In that respect, the new value of Ω_{00}^* is:

$$\Omega_{00}^* = \Omega_{00} + \Delta\Omega, \tag{12}$$

and using Eq. (6) to relate Ω_{00}^* with the deviation of the relative-track:

$$\Omega_{00}^* = \Omega_{00} - \Delta\psi_{max}. \tag{13}$$

Since Ω_{00} and $\Delta\psi_{max}$ are common for all the satellites of the constellation, the value of Ω_{00}^* is the same for all the satellites in the constellation.

Now, we have to prove that the value of M_{00}^* is the same for all the satellites of the constellation for the orbital maneuvers chosen. This allows maintaining the lattice and configuration of the new constellation with respect to the original one. In that sense, the value of M_{00}^* has only the constraints due to Eq. (1). However, M_{00}^* of the constellation is a function of time, so, if a transfer orbit is required from the initial constellation to the final one, the time that it takes to travel through it affects the new value of M^*_{00} of each satellite (that has to be the same for all of them) and as such, the constellation distribution. That means that having the same reference mean anomaly, M_{00}, as a design parameter of the Flower Constellation, it is necessary to generate transfer orbits that present the same transfer time.

This situation can be solved by the application of a Lambert's problem numerical solver that includes in its formulation the perturbations considered. That way, it is required to calculate the initial and final positions, and the transfer time of each satellite of the constellation. These conditions are given by the maintenance of the lattice of the constellation, and the lattice-preserving semi-major axis correction for each satellite. As it can be seen, this methodology requires a numerical method able to generate several iterations until a feasible solution is found. However, in order to show an analytical and compact solution to this problem, we consider a simplified case based on constellations whose initial and final orbits have at least an intersection. If that constraint is fulfilled, it is possible to reconfigure the constellation in just one impulse per satellite. That means that the time that each satellite spends in the transfer orbit is zero, so the problem does not require any iteration. As an example of this case are constellations whose satellites have circular orbits.

In order to prove that M^*_{00} is the same for all the satellites in the constellation, it is first required to calculate the positions in which the impulse is applied. Let M' be the mean anomaly of the point of each initial orbit in which the impulse is done, and let M^* be the mean anomaly of the same point in the final orbit. Taking into account the constraint proposed, we will demonstrate that M' and M^* are independent of each satellite due to the constellation symmetries in the problem. Let $\mathbf{x}'$ be the inertial positions of the initial orbits in the instant of the impulse and let $\mathbf{x}^*$ be the inertial positions of the final orbits in the same instant, then, in the moment where the impulse is made:

$$\mathbf{x}^* = \mathbf{x}', \tag{14}$$

then, Eq. (15) is obtained, where $\mathcal{R}_3$ and $\mathcal{R}_1$ are the rotational matrices, f' and f^* are the true anomalies of the initial and the final orbits, $r^* = \|\mathbf{x}^*\|$ and $r' = \|\mathbf{x}'\|$:

$$\mathcal{R}_3\left(\Omega_{ij}^*\right)\mathcal{R}_1(i)\,\mathcal{R}_3(\omega)\begin{bmatrix} r^*\cos f^* \\ r^*\sin f^* \\ 0 \end{bmatrix} = \mathcal{R}_3(\Omega_{ij})\,\mathcal{R}_1(i)\,\mathcal{R}_3(\omega)\begin{bmatrix} r'\cos f' \\ r'\sin f' \\ 0 \end{bmatrix}; \tag{15}$$

Eq. (15) can be simplified using the property that both points are the same in the inertial frame of reference and thus $r^* = r'$. Thereby, using Eq. (13) that states:

$$\Omega_{ij}^* = \Omega_{ij} - \Delta\psi_{max}, \tag{16}$$

Eq. (15) can be written as a function of $\Delta\psi_{max}$:

$$\mathcal{R}_1(i)\,\mathcal{R}_3(\omega)\begin{bmatrix} \cos f^* \\ \sin f^* \\ 0 \end{bmatrix} = \mathcal{R}_3(\Delta\psi_{max})\,\mathcal{R}_1(i)\,\mathcal{R}_3(\omega)\begin{bmatrix} \cos f' \\ \sin f' \\ 0 \end{bmatrix}. \tag{17}$$

Thus, from Eq. (17), it is possible to conclude that if the inclination and argument of perigee are the same in all the satellites of the constellation in that moment (which is the case), the values of f^* and f' are independent on the satellite in study, because $\Delta\psi_{max}$ is the same for the whole constellation. Therefore, M^* and M' are also independent on the satellite due to the fact that they are only related to the true anomalies through the eccentricity, which is the same for all the constellation.

That way, a relation can be established between the time (t'_{ij}) that each satellite requires to reach M' and the mean anomaly itself (M_{ij}) that is the initial distribution of the constellation:

$$M' = M_{ij} + n_{ij}t'_{ij}; \tag{18}$$

it is possible to calculate t'_{ij} as a function of the rest of parameters:

$$t'_{ij} = \frac{M' - M_{ij}}{n_{ij}}. \tag{19}$$

On the other hand, with respect of the final orbits, the satellites will appear with this distribution over time (note that the order of the satellites must be the same in the final constellation):

$$t^*_{ij} = T_{ij} - t'_{ij}, \tag{20}$$

where T_{ij} is the orbital period of each satellite. That way, the final distribution of M^*_{ij} (the positions of each satellite in the new configuration), can be obtained by:

$$M^*_{ij} = M^* + n_{ij} t^*_{ij}; \tag{21}$$

and using Eq. (19) and Eq. (20):

$$M^*_{ij} = M^* - M' + M_{ij} + n_{ij} T_{ij}. \tag{22}$$

Since $n_{ij} T_{ij} = 2\pi$, that term can be eliminated due to the modular nature of the mean anomaly, simplifying the equation:

$$M^*_{ij} = M^* - M' + M_{ij}. \tag{23}$$

If the expression is expanded using Eq. (9), this final relation is obtained:

$$M^*_{ij} = M^* - M' + M_{00} + \Delta M_{ij}. \tag{24}$$

Eq. (24) relates the final distribution (M^*_{ij}) with the original spacing of the satellites (ΔM_{ij}). As it can be seen, $(M^* - M' + M_{00})$ is common for the whole constellation (it does not depend on each particular satellite), i.e., the position of the reference satellite coincides for the whole constellation, and corresponds with the new value of M^*_{00}:

$$M^*_{00} = M^* - M' + M_{00}. \tag{25}$$

Hence, it is proved that all the satellites in the constellation share the same value of M^*_{00}, which was the goal sought in the orbital maneuvers in order to maintain the lattice of the constellation.

Therefore, it has been proved that the initial and the final distribution are equivalent and Eq. (9) is fulfilled by the new configuration, so the lattice has been maintained:

$$M^*_{ij} = M^*_{00} + \Delta M_{ij}. \tag{26}$$

Thus, we have demonstrated that the lattice of the constellation has been maintained during the orbital transfer which means that the initial and final constellation distribution are equivalent.

C. Example of Absolute Station-Keeping maneuvering

Once the maneuvering strategy for absolute station-keeping has been shown in the previous section, it is time to apply this strategy to the constellation studied in this work. Using Eq. (17), it is possible to obtain f' and f^*, the true anomalies of the initial and the final orbits. From those anomalies, the value of Δv of each satellite is obtained by calculating the velocities in the initial and final orbits. Let $(v^*_{x_{ij}}, v^*_{y_{ij}}, v^*_{z_{ij}})$ be the components of the velocity in the final orbit and let $(v'_{x_{ij}}, v'_{y_{ij}}, v'_{z_{ij}})$ be the components of the velocity in the initial orbit. Then, the Δv required for each satellite is:

$$\Delta v_{ij} = \sqrt{\left(v^*_{x_{ij}} - v'_{x_{ij}}\right)^2 + \left(v^*_{y_{ij}} - v'_{y_{ij}}\right)^2 + \left(v^*_{z_{ij}} - v'_{z_{ij}}\right)^2}. \tag{27}$$

Since each satellite has a slightly different semi-major axis (see Table 1) due to the lattice-preserving constellation property obtained, each satellite requires a particular impulse to make the orbital maneuver as Table 4 shows.

Table 4 Impulses required for each satellite.

Sat. (i,j)	a [km]	Δv [km/s]	Sat. (i,j)	a [km]	Δv [km/s]	Sat. (i,j)	a [km]	Δv [km/s]
(0, 0)	29600.137	0.1592758	(1, 0)	29599.524	0.1592775	(2, 0)	29598.167	0.1592811
(0, 1)	29598.872	0.1592792	(1, 1)	29599.976	0.1592763	(2, 1)	29599.978	0.1592762
(0, 2)	29597.165	0.1592792	(1, 2)	29598.165	0.1592811	(2, 2)	29599.522	0.1592775
(0, 3)	29597.843	0.1592820	(1, 3)	29597.085	0.1592840	(2, 3)	29597.553	0.1592828
(0, 4)	29599.783	0.1592768	(1, 4)	29598.519	0.1592802	(2, 4)	29597.329	0.1592834
(0, 5)	29599.784	0.1592768	(1, 5)	29600.101	0.1592759	(2, 5)	29599.216	0.1592783
(0, 6)	29597.845	0.1592820	(1, 6)	29599.218	0.1592783	(2, 6)	29600.101	0.1592759
(0, 7)	29597.166	0.1592838	(1, 7)	29597.329	0.1592834	(2, 7)	29598.522	0.1592802
(0, 8)	29598.874	0.1592792	(1, 8)	29597.554	0.1592828	(2, 8)	29597.085	0.1592840

It is also interesting to know which is the time in which these orbital maneuvers have to be done. In order to do that, it is necessary to compute the deviation that the relative-track experiences each day as it has been pointed out before. If the deviation of the relative-track of each satellite over the Earth equator is calculated, a maximum deviation of $5.31 \cdot 10^{-4}$ rad a day is obtained for the constellation. Knowing that the allowed range of the Galileo constellation is about 3 degrees [11], which is the value of $\Delta\psi_{max}$, and using Eq. (8), we obtain that an orbit correction is required to be made each 98.5 days.

Figure 5 shows the initial and final distributions of the lattice of the constellation in the maneuvering process. Those distributions are shown at two times: one before the impulses are done, i.e. the initial configuration (circles), and the other after, i.e. the final configuration (asterisks). As it can be seen, the lattice of the constellation has been maintained during the process, obtaining a new lattice-preserving Flower Constellation, equivalent to the original, that present the same relative-track that the initial configuration had at $t = 0\,[s]$.

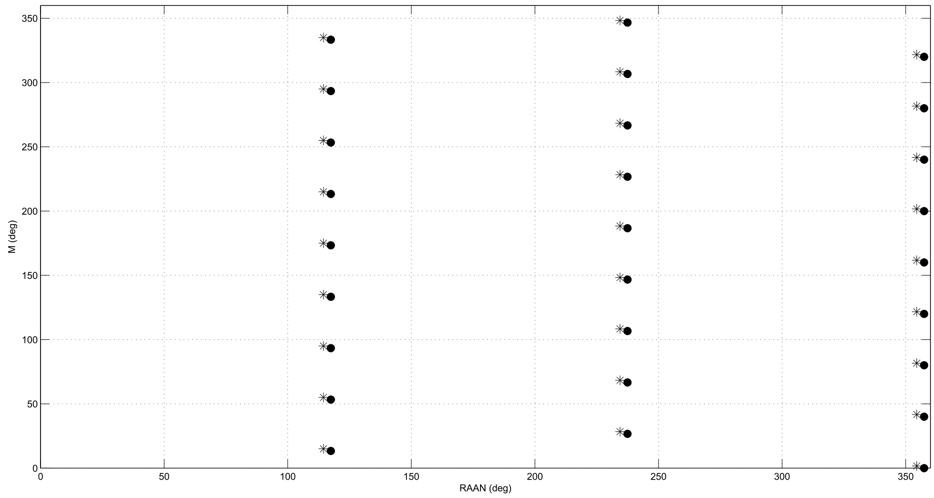

Fig. 5 Initial and final lattice for the *corrected* Galileo Flower Constellation before and after the orbital maneuvers to achieve the absolute station-keeping at time $t = 98.5\,days$.

V. Conclusion

The 2D lattice-preserving Flower Constellations is a novel way to design Flower Constellations that maintain the initial distribution of satellites and the initial symmetries over time i.e. relative station-keeping. The main characteristic is that all the satellites in the constellation are perturbed in a similar way, and consequently, the initial distribution of the satellites (initial lattice), and specially its symmetries are time-preserving. This constellation design maintains the relative station keeping without orbital maneuvers for longer periods of time than one year.

This design is expanded with the addition of the absolute station keeping. That way, with a two-step maneuver design process, the absolute station keeping of a lattice-preserving Flower

Constellation can be achieved against the effects of the J_2 geopotential perturbation and without losing the lattice and the relative station-keeping of the constellation in the process. This is an important property because it allows the maintenance of the constellation in the relative to Earth position that a mission could require, despite of being the satellites subjected to certain orbital perturbations. Moreover, the amount of fuel required to achieve the absolute station-keeping with this procedure is very low. In the case of the Galileo Constellation, this strategy results in a Δv consumption of less than 0.16 km/s per satellite each 3 months, which proves that the absolute station keeping is feasible in this design.

With this two properties, relative and absolute station-keeping, a complete maintenance of the constellation is established for the J_2 perturbation. Furthermore, if the J_3 effect and the Sun perturbation are considered, these techniques are still valid, although the times to perform the absolute and relative station-keeping maneuvers may change slightly.

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