An evolutionary growth model with banking activity

by

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Abstract

In this paper we propose an evolutionary growth model in which an innovative production sector interacts with a simplified banking sector. We explore the relationships between long-term sources of growth (technological change), and short-term/mid-term factors (such as price dynamics and interest rates). The model suggests new explanations for the endogenous emergence of sharp crises with profound effects in the long run. An interesting aspect of the model is that these crises appear in a strictly private economy, in which everything produced is sold, and there are no government distortions nor exogenous shocks. The crises emerge from the interactions between uneven innovation rates and market reactivity. In fact, high reactivity in financial markets can amplify the (initially small) effects of innovative competition, leading to a destabilization of economic growth. Drawing on the results of the model we suggest some policy implications.

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1 Introduction

With the Great Recession, doubts have appeared regarding the capacity of mainstream models to explain real financial interactions, and the large impact of intense short-run fluctuations on economic growth (Dosi, 2011; Krugman, 2011; Stiglitz, 2011). Interesting non-mainstream models have been proposed to face this challenge. These models combine short-run/long-run sources of change, and incorporate new links between innovation, production and banking activity (Dosi et al., 2013; Assenza and Delli Gatti, 2013; Assenza, Delli Gatti and Grazzini, 2015).

Drawing upon these works, we present an evolutionary growth model with banking activity. Our model follows the Neo-Schumpeterian tradition (Nelson and Winter, 1982; Silverberg and Verspagen, 2005; Metcalfe et al., 2006; Dosi et al., 2010; Saviotti and Pyka, 2013) and allows us to explore *sharp* crises whose effects may alter long-run growth.

The model we propose looks at the relationships between a productive sector—made up of innovative firms—and a banking sector which funds the needs of the productive sector and carries out additional banking activities to obtain a profit. For simplicity, and because we want to highlight new sources of capitalist fragility, we model a private economy with no public sector, in which firms sell all they can produce, and the banking system can resort to external funding and operate in interbank external markets. As we will see, even in this "seemingly innocuous" economy, sharp crises arise generated by the interactions between the innovative real economy and banking.

Given the complicated structure of the model, we begin our analysis by obtaining some formal properties (Section 3) and, then, we complete the analysis (Section 4, see also Annex) via simulations. In turn, the simulations are discussed in the light of the analytical results. The deterministic character of the model facilitates the obtaining of results, and excludes random shocks as destabilizing factors.

Regarding the results, we will see that the simulations and formal analysis allow us to establish conditions for which sustained growth trajectories emerge. Taking these *virtuous scenarios* as a reference, we modify some parametric values and we find that the links between the productive sector and the financial system become fragile. In fact, *intense crises may appear* in the model. These episodes emerge from the interactions between uneven technological change, and market (mostly financial markets) reactivity. Under certain conditions, this reactivity amplifies the (initially small) effects of innovative competition and destabilizes growth.

Our model has clear links with recent contributions. On the one hand, we find similarities between our model and the *Schumpeter-meeting-Keynes* (S+K) models in Dosi et al. (2010, 2013). On the other hand, there are commonalities with the *capital-credit* model in Assenza et al. (2015) and, in general, with the ABMs in Delli Gatti et al. (2008).

More precisely, regarding the S+K models in Dosi et al. (2010, 2013), these authors present two-sector ABMs with a bank, where complementarities between aggregate demand and technological change are analyzed. Depending on the (mis-) matching between innovation and demand, and considering alternative (functional) income distributions, distinct growth regimes emerge. We follow these models in the way that we also consider firm innovation and a bank, but using a one-sector model (ours is a lower scale model). We also find balance-sheet/bank-lending effects underlying crises (Dosi et al., 2013). The main difference between Dosi et al. (2010, 2013) and our approach is that we propose different pricing and supply-demand coordination mechanisms in a more stylized framework. Ours is a one-sector model with a bank inspired by Nelson and Winter (1982) and Delli Gatti et al. (2008), but considering sticky prices (Blinder et al., 1998). We have reduced the scale of the model to gain transparency in our analysis.

Likewise, the similarities between our proposal and Assenza et al. (2015) are clear. In both cases, crises arise from the amplification—via externalities—of initially small events that eventually lead to large contractions. A difference between our model and Assenza et al. (2015) is that, whereas they find upstream-downstream vertical externalities underlying contractions, we show that purely horizontal externalities—with innovative firms linked through a bank—can also generate amplifying effects that transform small events into large crises. We believe that our stylized proposal adds a complementary perspective to Dosi et al. (2010, 2013) and Assenza et al. (2015)—along with the germane work by Delli Gatti et al. (2008).

The paper is organized as follows: we present the model in Section 2. In Section 3, we explore key theoretical mechanisms of the model and we obtain some formal results. In Section 4, we carry out the simulations for the case of three firms and a bank. Finally, in Section 5, we summarize our conclusions.

2 The model

We propose an evolutionary growth model to study the interactions between a productive sector and a schematic banking sector. The economy consists of *households*, boundedly-rational innovative *firms in an industrial sector* and a boundedly-rational *private bank*. Households include skilled and non-skilled workers who can be hired by firms or the bank, earning a salary for their work. Both productive and banking activities can generate profits. A part of the profits is shared with shareholders (who are also households). All the households' rents (salaries and shared profits) are fully devoted to consumption.

In the productive sector of our model, skilled workers are hired not to produce but to generate firm-specific knowledge (to improve technologies), and their salaries come from R&D spending. Non-skilled workers are hired to develop operational tasks of the firms. Their salaries, together with depreciation and other input costs, make up the operational costs of firms. We propose a labor-knowledge economy in which only one single good is produced and sold at one unique price at any time. By selling the good, firms may obtain positive profits. The part of the profits which is not distributed among the shareholders (i.e. a firm's savings) is maintained by firms as bank deposits.

The main role of the bank is to channel resources from profitable firms (which save a share of their profits as deposits in the bank) to firms with financial needs (which ask for loans and pay interests to the bank). The bank remunerates deposits of each firm with the same interest rate, and extends loans to firms at different interest rates depending on each firm's relative risk. When deposits (net of required reserves) are insufficient to fund the internal credits of the economy, the bank can ask for external (international) funding, paying an interest rate for these extra funds. Likewise, when the internal needs of funds are already satisfied, the bank can lend the surplus of deposits (net of required reserves) to the international financial markets, obtaining interest on these funds.

Therefore, in this economy, there are heterogeneous competing firms which differ in their production technologies, in their financial conditions, in their operational costs, and in their R&D to sales ratios. Firms devote their R&D spending to hiring skilled workers who lead knowledge creation and technological change. Apart from production firms, there is one single bank in the economy which channels savings from profitable firms to other firms presenting financial needs.

In this economy, the overall levels of production, technology and consumption tend to grow in the long-run but, in certain scenarios, the interactions between the productive and the banking sectors generate deep crises. To avoid the reiteration of crisis-generating factors in the literature (shocks, demand shortages, policy mistakes) and seeking to highlight new aspects of capitalist change, we propose a deterministic model for a private economy in which everything produced is sold, and crises emerge from unpredictable unsustainable paths involving innovation, market competition and the financial counterpart.

We present the model as follows: in the first sub-section, we explain the productive sector. Then, in the second sub-section, we connect productive activity with the banking system. Finally, in the third sub-section, we obtain relevant aggregate magnitudes which will be of help to focus the simulations and to understand the interactions between the productive and the financial sector.

2.1 The productive sector

2.1.1. Production, Costs and Profits

We assume there is a number n_t of firms in the productive sector at $t = 0, 1, 2, \dots$, and n_t decreases when a firm becomes bankrupt. We consider that the *physical capital* K_i of each firm i $(i = 1,\dots,n_t)$ is constant (after covering depreciation) and it is equal to 1, and we adopt physical capital as *numeraire*. Unlike with physical capital, we suppose that *effective capital,* $A_{i,t}$, differs among firms, depending on the capacity of firms for generating and applying technology. Each firm's *effective capital tells us what the corresponding unit of physical capital yields* (in terms of final output) *depending on the firm's specific technology*, that is, the productivity of firm physical capital.

In the model, firms belong to owners/households seeking to make profits using effective capital. They produce and sell a *homogeneous good* (whose price will be expressed in physical capital/numeraire units). The initial price of the good is $p_0 = 1$. The good can be used both as a consumption good, and for operational and depreciation uses (with firm-specific and constant unit-requirements). We assume that the price of the good is unique for all its uses.

The flow of income (after costs) generated by each firm is the firm *profit*. Part of this profit will be shared out among owners, who will spend it on consumption, whereas the rest of the profit will become savings for the firm. These savings will be maintained as deposits in the bank, who will pay a constant interest rate to firms for maintaining these deposits. If the flow of income (after costs) is negative and the accumulated savings of the firm are not enough to cover the negative profits (losses), the firm can ask for a loan from the bank, paying a variable and firm-specific interest rate.

Formally, assuming that the production of each firm is determined by its effective capital, and supposing that all that is produced is sold (Delli Gatti et al., 2008; Nelson and Winter, 1982; Winter, 1984), we propose the following *production function*:[1]

$$q_{i,t} = A_{i,t} \tag{1}$$

Equation (1) allows us to represent the production function of firms in a very stylized way.

The technological level (or effective capital of each firm) $A_{i,t}$ evolves via innovation (equation (7)).

We shall assume that the firms have three kinds of *costs*: costs of innovation (equation (6)); financial costs (equation (13)); and operational costs which includes the payments of non-skilled workers' salaries, depreciation costs and other input costs. The costs of each firm at *t* will be:

$$CT_{i,t} = R_{i,t} + F_{i,t} + \delta_i A_{i,t} p_t, \quad \text{with } 0 < \delta_i < 1 \tag{2}$$

$R_{i,t}$ is R&D spending. We assume that this will go to paying skilled workers who make innovation possible and spend their income on consumption. $F_{i,t}$ are the financial costs of the firm (equation (13)). Finally, $\delta_i A_{i,t} p_t$ are the specific costs of each firm associated to the operational working of production plants and to depreciation. We consider that the unit-requirements of the good for production and depreciation are a firm-specific and constant-through-time trait, which we represent by (δ_i) *per unit of output*. We denote the price of the unique homogeneous good for all uses (consumption and intermediate good) in the economy with p_t. The total unit cost of each firm at *t* will be:

$$c_{i,t} = \frac{CT_{i,t}}{q_{i,t}} \tag{3}$$

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[1]Notice that, since physical capital is equal to 1, effective capital can be also identified as the productivity of physical capital for each firm. If we had worked under different premises, we could also have reached this function. Thus, if we assumed a Leontief-type technology in which the only inputs are non-qualified labor and capital, we would obtain a function $q_i = \min(B_iN_i; A_iK_i)$. If we suppose that labor is always abundant, we see that $q_i = A_iK_i$. Assuming $K_i = 1$ for all firms, we end up with $q_i = A_i$.

Production activity generates a flow of net income equal to $\left(p_t - c_{i,t}\right)q_{i,t}$, which, together with the interests obtained, as we shall see later, from deposits $D_{i,t}$, allows us to define the total profits of each firm at every moment in time as:

$$\pi_{i,t} = \left(p_t - c_{i,t}\right)q_{i,t} + r_t^D D_{i,t} \tag{4}$$

where r_t^D is the interest rate received at t for the deposits.

With respect to the price dynamics we assume the existence of a price index in the economy (for simplicity, we will refer to it as the price of the good) with the following evolution:

$$p_{t+1} = Max\left(0; p_t + \sigma\left(\frac{\sum A_{i,t+1}}{\sum A_{i,t}} - (1+r_t)\right) = p_t + \sigma\left(g_t - r_t\right)\right), \quad \text{with} \quad \sigma > 0 \tag{5}$$

where, r_t and g_t are, respectively, the interest rate of the economy (which is determined in equation (16)) and its growth rate. This equation aims to capture the macroeconomic relationship between the prices of the goods (in this case, of just one good) and the growth rate of the economy. The price tends to increase when the growth rate of economic activity (g_t) is higher than the potential cost of taking on debt (r_t). The price changes with a degree of stickiness captured by σ (Blinder, 1998). As defined, we can affirm that the price or price index is never negative. It is important to notice that the price from (5) is not determined in the model by supply/demand interactions in the goods market (as this market is not explicitly defined on a microeconomic level), nor by mechanisms of direct competition between price-setting firms which later try to satisfy the market; in other words, it is not grounded in microeconomics. This price, p_t , must be seen as the result of global macroeconomic dynamics, competition in technical progress rates underlying production growth, financing needs which affect r_t , and the search for profits by the firms and the bank[2]. That is to say, variable p_t represents a price index rather than the explicit aggregation of prices obtained on a microeconomic level.

2.1.2. Innovation

[2] The competition described in our model for the product's market is implicit as there is no explicit-standard market for the good. Our model only sets out explicitly the supply mechanisms of the firms, and we assume that supply generates the required demand. The produced output is bought in our model by shareholders with the profits they receive, by workers with their salaries, by firms for their operational needs, and perhaps by an external market.

Firms devote resources to innovation with the aim of improving technology, and increasing effective capital and production $(A_{i,t})$. They hire skilled workers to innovate, and they do so by devoting a specific share $\gamma_i \in (0,1)$ of their sales to R&D (Dosi and Nelson, 2010). That is,

$$R_{i,t} = \gamma_i p_t q_{i,t} \tag{6}$$

Furthermore, we suppose that firms innovate following an innovation equation (Nelson, 1982; Almudi et al., 2013; Fatas-Villafranca et al., 2009, 2014) where $R_{i,t}$ is R&D spending (which differs among firms), and $z_{i,t}$ is R&D productivity:

$$\left(A_{i,t+1} - A_{i,t}\right) p_t = z_{i,t} R_{i,t} \Leftrightarrow A_{i,t+1} - A_{i,t} = z_{i,t} \gamma_i A_{i,t}, \quad \text{with } z_{i,t} = 1 + \varphi \frac{A_t^{\text{Max}} - A_{i,t}}{A_{i,t}}, \quad \varphi \in (0,1) \tag{7}$$

In equation (7), new knowledge comes from two sources: inner learning activities, and knowledge spill-overs (captured by φ and the gap-expression in (7), where $A_t^{Max} = Max\left\{A_{1,t}, \ldots, A_{n_t,t}\right\}$). Let us note that $\frac{A_t^{\text{Max}}}{A_{i,t}}$ will always be higher than or equal to 1. Here we want to anticipate that, although according to equation (7) the rate of effective capital growth seems to be always higher than or equal to "0", the overall model dynamics will generate situations in which firms may become bankrupt and exit the market (see below). Therefore, as we will see, positive output growth is not always the case.

2.1.3. Finance

In our model, the financial structure of firms matters (Greenwald and Stiglitz, 1993). Thus:

a) *If firm i at time t obtains positive profits or null profits,* $\pi_{i,t} \geq 0$:

In this case, the firms share out a proportion $\theta_i \in (0,1)$ of profits amongst their owners, who devote these incomes to consumption. We suppose that the remaining profits are put into savings of the firms. They keep these savings as bank deposits $D_{i,t}$ obtaining an interest rate r_t^D. Formally we have:

$$D_{i,t+1} = D_{i,t} + \left(1 - \theta_i\right) \pi_{i,t} \tag{8}$$

Furthermore, each firm can have bank loans within its liabilities. We call this stock $L_{i,t}$. In firms with positive or zero profits (see below for firms with negative profits), the net worth will be the difference between total assets and liabilities, aggregating the retained profits in *t* (remember that $K_i = 1$):

$$N_{i,t} = 1 + D_{i,t} - L_{i,t} \tag{9}$$

Now, we have to consider debt-repayment. If a profitable firm has previous debts, its debts will not increase and it evolves as:

$$L_{i,t+1} - L_{i,t} = -\chi_i L_{i,t} \tag{10}$$

with $\chi_i \in (0,1)$ being the proportion of the debt which must be returned. This debt devolution, together with interests (at rate $r_{i,t}$) which must be paid to the bank, constitutes the financial costs of the firm $F_{i,t}$, see (13).

Thus, the corporate balance-sheet for each single *i*-firm is

Firm *i*	
Assets	**Liabilities**
1	$N_{i,t+1}$
$D_{i,t}$	$L_{i,t+1} = L_{i,t} - \chi_i L_{i,t}$
$(1-\theta_i)\pi_{i,t}$	

b) <u>*If firm i at time t has losses*</u> $\pi_{i,t} < 0$:

In this case, the banking deposits of the firm will be:

$$D_{i,t+1} = Max\{0; D_{i,t} + \pi_{i,t}\} \tag{11}$$

That is, with losses $\pi_{i,t} < 0$, the firm turns to its pre-existing deposits to cover these losses. It will make use of its deposits until they are used up. When deposits are no longer sufficient, we assume that the banking system will lend to the firm. Therefore, debt evolution will be given as:

$$L_{i,t+1} - L_{i,t} = Max\{0; |\pi_{i,t}| - D_{i,t}\} - \chi_i L_{i,t} \tag{12}$$

Note that firms with recurrent losses can seriously erode their net worth given by $N_{i,t} = 1 + D_{i,t} - L_{i,t}$

Equation (13) of financial costs is valid for both profitable firms and loss-making ones:

$$F_{i,t} = \left(r_{i,t} + \chi_i\right) L_{i,t} \tag{13}$$

where $r_{i,t}$ is the interest rate paid by firm *i*, and, as we shall see later, it depends on both r_t, the referential interest rate of the economy, and the firm's level of debt or risk. Finally, we can compute the profits of the productive sector, $\pi_t = \sum_{i=1}^{n_t} \pi_{i,t}$, and the aggregate deposits and loans, $D_t = \sum_{i=1}^{n_t} D_{i,t}$, $L_t = \sum_{i=1}^{n_t} L_{i,t}$

2.1.4. Exit

Any firms whose net worth is not positive, $N_{i,t} \le 0$, leave the market (they go bankrupt). Furthermore, the bank (the only creditor) keeps the assets of the firm so as to recover as much debt as possible.

2.2 Banking activity

2.2.1. The commercial bank

We suppose that there is a private bank which starts out with an initial net worth N_0^b. We know that firms put their savings in deposits and receive interests at rate $r_t^D < r_t$, and also obtain credit at a rate $r_{i,t}$. Regarding the bank balance-sheet, the *Liability* part has three components: (i) net worth N_t^b; (ii) firm deposits D_t, (iii) the volume of debt with other (external) banks L_t^b. Regarding *Assets*, the bank maintains a proportion of deposits wD_t, with $w \in (0,1)$, as required reserves. The rest of the deposits, $(1-w)D_t$, are funds which can be loaned to firms or can remain as bank reserves if the firms do not need them. We suppose that the bank will cover the needs of the productive sector by resorting, firstly, to its loanable volume of deposits $(1-w)D_t$. If it is not sufficient, the bank turns to external interbank funds to be able to lend to firms. This interbank system lends at a rate r_t^b. If the bank has excessive resources, it can lend them to external banks at interest rate r_t^b. The bank will pay a lower interest rate for these deposits than the one it pays for external assets, so we assume $r_t^D = \alpha^D r_t^b > r_t^b$, $\alpha^D < 1$.

Finally, we also suppose that if the bank has a net value N_t^b during time period *t*, the bank has operating costs of $\delta^b N_t^b$. These costs can be associated with the salary costs of a firm's labor force and the devaluation of the assets they own.

2.2.2. External Interbank loans

We use L_t^b to label the needs for external interbank loans which may emerge as a consequence of the fact that the private bank does not have enough deposits to cover all the credits firms need. Thus,

$$L_t^b = Max\{0; L_t - (1-w)D_t\} \tag{14}$$

In the Bank *Assets* we have: (i) the bank reserves $\Gamma_t = wD_t$; (ii) credits loaned to firms L_t; (iii) credits loaned to external (foreign) banks $E_t^b = Max\{0;(1-w)D_t - L_t\}$; (iv) initial resources N_0^b.

2.2.3. Net worth of the Bank

Banking activity seeks to obtain profits which, if positive, we suppose will be shared out in proportion θ^b, and devoted to consumption by shareholders, who may be workers, or not, of the firms or the bank itself; the remaining proportion $1-\theta^b$ will go to the bank's net worth. To define profit π_t^b, we focus on the differential between: yields for credits awarded to firms and external foreign banks; payments for deposits and resources obtained from external interbank loans; operating costs; and losses associated with firms who had previously received credit going bankrupt. Notice that the interactive dynamics linking the productive sector and the bank may, at any time, generate a set of firms which go bankrupt. If this happens, we assume that the bank loses the credits it had given to this firm but, in compensation, it will keep the physical capital and any deposits the firm may have had. Thus, we see:

$$\pi_t^b = \sum_i r_{i,t} L_{i,t} + r_t^b E_t^b - \alpha^D r_t^b D_t - r_t^b L_t^b - \delta^b N_t^b - u_t \tag{15}$$

where $u_t = -\sum_{i \in \Xi_t} \left(1 + D_{i,t} - L_{i,t}\right)$, with Ξ_t denoting the set of firms bankrupted at *t*. The variable u_t represents the losses the bank incurs as a result of being unable to recover credits lent to bankrupt firms but taking into account that the bank keeps the firm's installations (physical capital) and their deposits.

This allows us to know the net value of the bank for both a profit-making bank and a bank which does not have profits. If it has positive profits:

$$N_{t+1}^b = N_t^b + \left(1-\theta^b\right)\pi_t^b$$

If its profits are negative—that is, it has losses—the net value will be given by:

$$N_{t+1}^b = N_t^b + \pi_t^b$$

In this case the new net value will be lower than the previous one due to the losses which also include operating costs.

In agreement with the above-mentioned, the balance sheet for the bank with non-negative profits is the following:

Bank	
Assets	**Liabilities**
N_t^b	N_{t+1}^b
wD_t	D_t
L_t	$\alpha^D r_t^b D_t$
E_t^b	$r_t^b L_t^b$
$\sum_i r_{i,t} L_{i,t}$	$\delta^b N_t^b$
$r_t^b E_t^b$	u_t
	$\theta^b \left(\sum_i r_{i,t} L_{i,t} + r_t^b E_t^b - \alpha^D r_t^b D_t - r_t^b L_t^b - \delta^b N_t^b - u_t \right)$

It is similar in the case of negative profits, or losses, but the expression

$$\theta^b \left(\sum_i r_{i,t} L_{i,t} + r_t^b E_t^b - \alpha^D r_t^b D_t - r_t^b L_t^b - \delta^b N_t^b - u_t \right)$$

does not appear, as benefits are not shared out amongst shareholders.

Clearly, N_t^b must always be positive for the activity to continue. If it is negative, we face a collapse of the banking system.

2.2.4. Interest rates

Firstly, we suppose that the interest rate $r_{i,t}$ the bank charges each firm depends on both the firm's risk conditions ($L_{i,t} - D_{i,t}$), and the general state of the economy captured by a referential interest rate r_t . This rate rises as the bank is overrun in its capacity for loaning $(1-w)D_t$, and has to resort to external interbank loans. The dynamics will be given by:

$$r_{t+1} = Max\left\{ r_{\min}; r_t \left[1 + \alpha \frac{L_t - (1-w)D_t}{m_t} \right] \right\}, \tag{16}$$

with $\alpha \in (0,1)$, $m_t = Max\{L_t;(1-w)D_t\}$ if it is positive, and $m_t = 1$ otherwise. Furthermore, we suppose that the specific rate for each firm is given by:

$$r_{i,t} = Max\left\{ r_{\min}, r_t \left(1 + \rho \frac{L_{i,t} - D_{i,t}}{M_t} \right) \right\} \tag{17}$$

with $\rho > 0$, $M_t = Max\{L_{i,t};D_{i,t}\}$ if it is positive, and $M_t = 1$ otherwise.

The parameter $r_{\min}$ is the minimum rate below which a bank would never give a loan. Of course $r_{\min}$ depends on institutional and socioeconomic circumstances underlying credit interest rates (i.e. $r_{\min}$ is assumed to be lower, the higher the level of judicial security, the higher the respect for the rule of law, and the more reliable the legislation and regulation supporting credit contracts in the economy. All these factors moderate the risk premium and allow for $r_{\min}$ to be low, Greenwald and Stiglitz, 1993). The aforementioned institutional aspects are crucial in determining, together with national financial factors, the range within which the interest rate will move. As we can see below, the cost of external interbank lending also determines the floor-level $r_{\min}$. Furthermore, $r_{\min}$ could also be related to monetary policy. Nevertheless, in this paper, we prefer to highlight the role of institutional determinants which are not related to monetary policy; namely, the rule of law, judicial security, or interbank lending costs. These factors have been studied less than monetary policy, but they seem to be crucial in banking-productive interactions and in the dynamics of economic growth (Stiglitz, 2011).The parameter α captures the intensity with which the bank reacts via interest rates to changes in the structure of its balance-sheet (i.e. the demand for credit *vs* lending resources).

The weight ρ indicates the greater or lesser influence that debts have on the rate firms will pay. With this definition, the interest rate of a firm will never be above $(1+\rho)$ times the reference rate, nor will it ever be zero as when there are no debts, it will be equal to the minimum value of the reference rate. This undoubtedly changes the evolution of firms, making things worse for debtors and even more so when they are more in debt. The mechanism operates as a firm-specific risk-premium.

Secondly, we suppose that the interest the bank pays for external funding increases when the needs of the bank increase. Thus,

$$\begin{aligned} r_{t+1}^{b} &= Max\left\{ r_{\min}^{b} ; r_{t}^{b}\left[1+\beta \frac{L_{t+1}^{b}-L_{t}^{b}}{h_{t}}\right]\right\} \\ &= Max\left\{ r_{\min}^{b} ; r_{t}^{b}\left[1+\beta \frac{Max\{0; L_{t+1}-(1-w)D_{t+1}\}-Max\{0; L_{t}-(1-w)D_{t}\}}{h_{t}}\right]\right\}, \end{aligned} \tag{18}$$

with $\beta \in (0,1)$, $h_{t}=Max\{Max\{0; L_{t+1}-(1-w)D_{t+1}\}; Max\{0; L_{t}-(1-w)D_{t}\}\}$ if it is positive, and $h_{t}=1$ otherwise. Parameter $r_{\min}^{b}$ is the minimum rate of the interbank system, with all the aforementioned institutional aspects being pertinent here too. We assume, as we stated above, that $r_{\min}^{b} \leq r_{\min}$.

2.3 Aggregate magnitudes

From the proposed model, we can now obtain the standard national account expressions with the specific form that they take in our modelled economy. These expressions, together with the balance-sheets and the interactive effects linking production and banking can be of help to locate all expenditure, income and added-value generating activities in our specific economy. Thus, we start out from the Net National Product (NNP) or the production of net added value in our economy, whose expression is:

$$NNP_{t}=\left(p_{t} \sum_{i} A_{i,t}-\sum_{i} \tilde{\delta}_{i} p_{t} A_{i,t}\right)-r_{t}^{b} L_{t}^{b}-\tilde{\delta}^{b} N_{t}^{b}-u_{t}$$

where $\sum_{i} \tilde{\delta}_{i} p_{t} A_{i,t}$ captures the non-salary components of the operating costs of firms which are given together as $\sum_{i} \delta_{i} p_{t} A_{i,t}$. In a similar way, $\tilde{\delta}^{b} N_{t}^{b}$ is the non-salary part of the costs of the bank's operations and u_{t} represents the losses incurred by banks due to firms going bankrupt. Notice that from this expression, the aggregate national income is generated as follows:

$$\left(p_t \sum_i A_{i,t} - \sum_i \tilde{\delta}_i p_t A_{i,t} \right) - r_t^b L_t^b - \tilde{\delta}^b N_t^b - u_t = \sum_i R_{i,t} + \sum_i (\delta_i - \tilde{\delta}_i) p_t A_{i,t} + \sum_i (\delta^b - \tilde{\delta}^b) N_t^b + \sum_i \pi_{i,t} + \pi_t^b$$

If we now move on to the expression of disposable income, we can see the links between income and consumption $(Cons_t)$ in our economy. Consumption by shareholders and workers, qualified or otherwise, is given by:

$$\sum_i R_{i,t} + \sum_i (\delta_i - \tilde{\delta}_i) p_t A_{i,t} + \sum_i (\delta^b - \tilde{\delta}^b) N_t^b + \sum_i \theta_i \pi_{i,t} + \theta^b \pi_t^b = Cons_t$$

We can obtain the Gross Domestic Product (GDP) in our specific economy as follows:

$$GDP_t = p_t \sum_i A_{i,t} = Cons_t + \sum_i \left(1 - \theta_i\right) \pi_{i,t} + \left(1 - \theta^b\right) \pi_t^b + \sum_i \tilde{\delta}_i p_t A_{i,t} + r_t^b L_t^b + \tilde{\delta}^b N_t^b + u_t$$

3 Key theoretical mechanisms

3.1 The model dynamics

Once we have presented the model, we start out our analysis by exploring specific elements of its theoretical structure. Thus, let us begin by looking at expression (19) which represents the main equations for firm dynamics, price evolution, and interest rates:

$$
\left.
\begin{aligned}
&A_{i,t+1} = A_{i,t} + \gamma_i \left[1 + \varphi \frac{A_t^{\mathrm{Max}} - A_{i,t}}{A_{i,t}} \right] A_{i,t} \\
&D_{i,t+1} = \begin{cases} D_{i,t} + (1-\theta_i)\pi_{i,t}, & \text{if} \quad \pi_{i,t} = p_t A_{i,t} - \gamma_i p_t A_{i,t} - r_{i,t} L_{i,t} - \chi_i L_{i,t} - \delta_i A_{i,t} p_t + r_t^D D_{it} \geq 0 \\ Max\{0; D_{it} + \pi_{it}\}, & \text{if} \quad \pi_{i,t} < 0 \end{cases} \\
&L_{i,t+1} = \begin{cases} L_{i,t} - \chi_i L_{i,t}, & \text{if} \quad \pi_{i,t} \geq 0 \\ L_{i,t} + Max\{0; |\pi_{i,t}| - D_{i,t}\} - \chi_i L_{i,t}, & \text{if} \quad \pi_{i,t} < 0 \end{cases} \\
&p_{t+1} = Max\left(0; p_t + \sigma \left(\frac{\sum A_{i,t+1}}{\sum A_{i,t}} - (1+r_t) \right) \right) \\
&r_{t+1} = Max\left\{ r_{\min}; r_t \left[1 + \alpha \frac{\sum L_{i,t} - (1-w)\sum D_{i,t}}{m_t} \right] \right\}, \\
&r_{i,t+1} = Max\left\{ r_{\min}, r_{t+1} \left(1 + \rho \frac{L_{i,t+1} - D_{i,t+1}}{M_{t+1}} \right) \right\}, \\
&r_{t+1}^b = Max\left\{ r_{\min}^b; r_t^b \left[1 + \beta \frac{Max\{0; \sum L_{i,t+1} - (1-w)\sum D_{i,t+1}\} - Max\{0; \sum L_{i,t} - (1-w)\sum D_{i,t}\}}{h_t} \right] \right\} \\
&r_{t+1}^D = \alpha^D r_{t+1}^b
\end{aligned}
\right\} \quad (19)
$$

As it can be seen in (19), together with (8) and (11), profits lead firms to accumulate deposits, thus strengthening the financial position of the firm. Note that according to (8) and (9), the net worth of a firm (without debt) is $N_{i,t+1} = 1 + D_{i,t+1} = 1 + D_{i,t} + (1-\theta_i)\pi_{i,t}$, so profits swell deposits and increase net worth. On the other hand, losses—especially if they are large and persistent—erode net worth and, eventually, may lead firms to exit the market. Moreover, notice that deposits build up bank reserves. This mechanism increases the banks' capacity to lend to the production sector without resorting to external funding; see (12), (14) and (19). Keeping all this in mind, we will now comment on other mechanisms which will allow us to understand the dynamics of our model better.

3.1.1. Effects on the evolution of profits

From (1)-(6) and (13), and also (19), we can obtain the profit equation in two different situations:

$\pi_{i,t} = p_t A_{i,t}(1 - \gamma_i - \delta_i) + r_t^D D_{it}$, if firms have no financial costs (20)

$$\pi_{i,t} = p_t A_{i,t}\left(1-\gamma_i-\delta_i\right)-\left(r_{i,t}+\chi_i\right)L_{i,t}+r_t^D D_{it}\text{, if firms bear financial costs} \tag{21}$$

Notice the following effects we can see in (20) and (21): i) although high levels of $\left(\gamma_i+\delta_i\right)$—R&D spending and functioning costs—may be essential for firms to continue within highly innovative and competitive economies, they can also lead firms into debt; ii) the component $\left(r_{i,t}+\chi_i\right)L_{i,t}$ incorporates a negative-feedback effect on firm profits; iii) the factor $\left(p_t A_{i,t}\right)$ spurs revenues but, additionally, it increases non-financial costs. Thus, from (20) and (21) we see that *increasing output* and *rising prices* affect firm *profits* and *net-worth* in a different way depending on whether the firm is profitable or not. In the case of non-profitable firms with a high $\left(\gamma_i+\delta_i\right)$, rising values of $\left(p_t A_{i,t}\right)$ may generate intense losses and increasing debts, thus unchaining a negative-feedback effect via $\left(r_{i,t}+\chi_i\right)L_{i,t}$. As we will show in the simulations, this process can spread across the productive sector via the banking system and can jeopardize economic growth.

3.1.2. Market shares and R&D investments

In (22) we present the process driving firms' market shares—defined as $s_{i,t}=\dfrac{q_{i,t}}{q_t}=\dfrac{q_{i,t}}{\sum_i q_{i,t}}$:

$$\frac{s_{i,t+1}-s_{i,t}}{s_{i,t}} \cong \gamma_i z_{i,t}-\left(\overline{\gamma z}\right)_t = g_{i,t}-\sum_j s_{j,t} g_{j,t}, \quad \text{with } \left(\overline{\gamma z}\right)_t = \sum_j s_{j,t}\gamma_j z_{j,t} = \sum_j s_{j,t} g_{j,t} = g_t \tag{22}$$

This expression is obtained from (1) and (7) and by bearing in mind that the rate of change of market shares is, approximately, the growth rate of each firm minus the average growth rate. There are two ideas that we want to remark on from (22). *Firstly*, (22) shows that an intense effort in R&D (high γ_i) is a source of leadership only if the productivity of R&D ($z_{i,t}$) is sufficiently high—compared with the average value. Since R&D spending is a cost for firms and, according to (7), the productivity of R&D depends on internal efforts but also on knowledge spill-overs, firms in the model face a dilemma: i) either they carry out a high level of R&D to foster innovation; or ii) they try to benefit from knowledge spill-overs without bearing such a cost. Both strategies are possible, but they may produce different aggregate outcomes.

Secondly, note from (22) that, since the evolution of the growth rate $\left(\overline{\gamma z}\right)_t$ depends on both the distribution of firm-level R&D results (via $z_{i,t}$), and the correlation between R&D investments (γ_i) and market shares ($s_{i,t}$), the aggregate growth effects of innovative competition could be different.

3.1.3. Growth and price evolution

Finally, we want to reflect on equation (5) —see also equation (19)— by expressing it as follows:

$$p_{t+1} - p_t = \sigma\left[\left(\overline{\gamma z}\right)_t - r_t\right] = \sigma(g_t - r_t) \tag{23}$$

If we combine (23) with (20) or (21), and with (22), an interesting intuition appears (which will be explored later on). That is, high growth rates at an aggregate level (high $g_t = \left(\overline{\gamma z}\right)_t$) tend to generate: *i)* very fast market transformations (see (22)) which quickly erode the share of less competitive firms; and *ii)* fast price increases through (23) which add to the deterioration of less competitive firms ((20) and (21)). What is worrying about this is that both effects (typical of innovative fast-growing economies) may lead a significant part of the firms into debt, and even into bankruptcy, thus converting (what would have been) a smooth transformation of the economy, into a broken growth path (a "*big rip*" in the productive sector). Furthermore, the reaction of the banking system—who raise interest rates as creative destruction spurs the need for credit—may intensify the process. Moreover, depending on the extent to which the bank resorts to external funding, and on the pattern of a firm's exit, the banking system itself may become seriously damaged. We will explore some of these elements in detail later on.

3.1.4. Stationary state conditions and stylized guiding facts

Finally, we would like to complete this heuristic subsection 3.1 by, firstly, obtaining (as a guide for further analysis) the characterization of possible stationary states of the dynamics. Secondly, we are going to put forward several well-known empirical facts which our model can reproduce, as we will show in the simulations (Section 4). Both the characteristics of stationary states and these empirical facts are useful to understand the behaviour of our model and to design empirically-relevant simulations in Section 4. Thus, firstly, let us begin by characterizing possible *conditions for stationary state* in the model. According to the market equation (22), the model can only remain in stationary conditions if all firms that remain in the market grow at the same rate. For instance, we may consider that there is a leader growing at a rate $\gamma_i^{\max}$, and other non-leading firms which, although they may have $\gamma_j < \gamma_i^{\max}$, grow at the maximum rate (that of the leader) because they assimilate knowledge through imitation (see equation (7), Proposition 1 in this Section, and simulations in Section 4). We can see that the absorption of knowledge through imitation by the non-leading firms is:

$$\Delta\gamma_j = \gamma_j \varphi \left(\frac{A_t^{\text{Max}}}{A_{j,t}} - 1 \right)$$

Thus, all firms grow in the stationary state at a common growth rate $\gamma_i^{\max}$ (which is also the overall growth rate in steady state). It is interesting to realize that, in the stationary state, the firms may in general co-exist with identical growth rates but they maintain very different market shares.

Now, let us look at the price in the stationary state. Although we already know that the global output growth rate is $g = \gamma_i^{\max}$ we still have to work out the stationary value for the interest rate (see the price equation (5) or also (23)). Thus, assuming that our stationary firms are profitable, they could (if this is the case) re-pay their debts at a rate $-\chi_i$. Therefore, at the limit, there would be no corporate debt. In this situation, according to our equation (16), we can see that $r = r_{\min}$ in the stationary state. Following equation (5), we will have increasing, resting, or decreasing prices in stationary state depending on whether $\gamma_i^{\max} >,=,< r_{\min}$. Since the variable $r_{\min}$ depends (as we have seen in Section 2) on institutional factors, we can obtain alternative price stationary dynamics depending on $\gamma_i^{\max} >,=,< r_{\min}$. Finally, let us note that according to equation (18), as long as the corporate debt can be fully re-paid in the long run, and the firms' deposits can grow, then $r^b = r_{\min}^b$. Let us remember that in the model we assume that $r_{\min}^b \leq r_{\min}$. These are stationary conditions for certain dynamic paths of the model. There are alternate more abrupt dynamics, as we will see in the simulations.

Secondly, we shall now anticipate certain *well-known stylized facts* from the literature (see Dosi et al., 2010; 2013) which our model can reproduce in a qualitative manner. We will take a deeper look at this aspect later on in Section 4 (simulations). For now, though, drawing on our heuristic exploration of the model, we can see that our system generates dynamics in which the following well-known facts arise: i) in our model dynamics, firms with different technologies compete and can co-exist; ii) from what we have already presented, it can be seen that our model can generate exponential growth paths, even displaying fluctuations and sudden breakdowns (crises; explained in more detail in the simulations below); iii) we can obtain dynamic trajectories, in which heterogeneous firms grow unevenly, underlying the aggregate growth path; iv) in our model dynamics and even in stationary states, both innovation and imitation processes underlie knowledge accumulation and output (and effective capital) growth; v) in the transient dynamics of the model, the productive system and the bank co-evolve, in the sense that the bank channels corporate savings (retained profits) to cover other firms' financial needs, and the resulting growth rates of output *vs* interest rate modulate price evolution. Besides this, corporate and bank savings (overall retained profits) are the key flows in the model driving the evolution of the stock of wealth (i.e. the dynamics of savings fuel the aggregate evolution of net-worths, according to the balance-sheets in Section 2; see also the flow-stock results in Dosi et al., 2013).

3.2 Some formal results

To end this section, we shall now obtain some formal results (proposition 1) regarding long-run growth trends in the model. These results formalize some of the heuristic intuitions discussed in Section 3.1, and allow us to focus the simulations in Section 4 better. For simplicity, we will consider the case of three firms and a bank.

Proposition 1. The economy described in previous sections verifies:

1) *If a firm sustains its growth in the long term, its growth rate tends towards the highest* γ_i *among those firms that survive.*

2) *If any firm i has no debts initially (t=0) and* $1 > \gamma_i + \delta_i$ *(i.e. the sum of its operative unit costs, plus unit-R&D, is smaller than 1), then its unit profit (profit rate) from productive activity* ε_i *(the firm can also get profits from its deposits) is always positive and constant. In these situations, in the long term, the growth rate of the economy will be* $\tilde{\gamma} = \underset{i}{Max}\left(1-\delta_i-\varepsilon_i\right)$*, the interest rate will*

be the minimum r_{min}, and the price will be increasing, constant, or decreasing depending on whether $\tilde{\gamma} >,=,< r_{min}$.[3]

Proof:

For 1)

a. Firstly, we shall see whether it is verified under the conditions: $A_{1,0} \geq A_{2,0}$, $A_{1,0} \geq A_{3,0}$ and $\gamma_1 \geq \gamma_2 \geq \gamma_3 > 0$. Under these conditions, for t=0 and for any t such that $A_{1,t} \geq A_{2,t}$ and $A_{1,t} \geq A_{3,t}$, (7) allows us to express the dynamic equations for firm growth in the following way:

$$T: \begin{cases} A_{1,t+1} = A_{1,t} + \gamma_1 A_{1,t} \\ A_{2,t+1} = A_{2,t} + \gamma_2 A_{2,t} + \gamma_2 \varphi \left(A_{1,t} - A_{2,t} \right) \\ A_{3,t+1} = A_{3,t} + \gamma_3 A_{3,t} + \gamma_3 \varphi \left(A_{1,t} - A_{3,t} \right) \end{cases}$$

This dynamic system is lineal and we can solve it through the Jacobian matrix:

$$JT = \begin{pmatrix} 1+\gamma_1 & 0 & 0 \\ \gamma_2 \varphi & 1+\gamma_2(1-\varphi) & 0 \\ \gamma_3 \varphi & 0 & 1+\gamma_3(1-\varphi) \end{pmatrix}$$

whose eigenvalues are $\lambda_1 = 1+\gamma_1$, $\lambda_2 = 1+\gamma_2(1-\varphi)$, $\lambda_3 = 1+\gamma_3(1-\varphi)$, being $\lambda_1 > \lambda_2 \geq \lambda_3$.

The corresponding eigenvectors are:

$$v_1 = \begin{pmatrix} 1 \\ \gamma_2 \varphi \big/ \left(\gamma_1 - \gamma_2 + \gamma_2 \varphi \right) \\ \gamma_3 \varphi \big/ \left(\gamma_1 - \gamma_3 + \gamma_3 \varphi \right) \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

which lead to the trajectory of the dynamic system T:

[3] The case $\tilde{\gamma} < r_{min}$ is not relevant from an economic point of view because in this case the equilibrium price tends to go to zero, but it is formally significant to point out the possibility of price deflation in our model.

$$\begin{cases} A_{1,t} = A_{1,0}\left(1+\gamma_1\right)^t \\ A_{2,t} = A_{1,0}\dfrac{\gamma_2\varphi}{\gamma_1-\gamma_2+\gamma_2\varphi}\left(1+\gamma_1\right)^t + A_{2,0}\left(1+\gamma_2(1-\varphi)\right)^t - A_{1,0}\dfrac{\gamma_2\varphi}{\gamma_1-\gamma_2+\gamma_2\varphi}\left(1+\gamma_2(1-\varphi)\right)^t \\ A_{3,t} = A_{1,0}\dfrac{\gamma_3\varphi}{\gamma_1-\gamma_3+\gamma_3\varphi}\left(1+\gamma_1\right)^t + A_{3,0}\left(1+\gamma_3(1-\varphi)\right)^t - A_{1,0}\dfrac{\gamma_3\varphi}{\gamma_1-\gamma_3+\gamma_3\varphi}\left(1+\gamma_3(1-\varphi)\right)^t \end{cases}$$

where $A_{1,0}$, $A_{2,0}$ and $A_{3,0}$ are the initial values.

Note that these functions verify the relations $A_{1,t} \geq A_{2,t}$ and $A_{1,t} \geq A_{3,t}$ for all t (we carried it out for index 2 but it is identical for 3):

$$\begin{aligned} A_{1,t} - A_{2,t} &= A_{1,0}\left(1+\gamma_1\right)^t - A_{1,0}\frac{\gamma_2\varphi}{\gamma_1-\gamma_2+\gamma_2\varphi}\left(1+\gamma_1\right)^t \\ &\quad + A_{1,0}\frac{\gamma_2\varphi}{\gamma_1-\gamma_2+\gamma_2\varphi}\left(1+\gamma_2(1-\varphi)\right)^t - A_{2,0}\left(1+\gamma_2(1-\varphi)\right)^t \\ &\geq A_{1,0}\left(1+\gamma_1\right)^t\left[1-\frac{\gamma_2\varphi}{\gamma_1-\gamma_2+\gamma_2\varphi}\right] - A_{2,0}\left(1+\gamma_2(1-\varphi)\right)^t\left[1-\frac{\gamma_2\varphi}{\gamma_1-\gamma_2+\gamma_2\varphi}\right] \\ &\geq \left(A_{1,0}-A_{2,0}\right)\left(1+\gamma_2(1-\varphi)\right)^t\left[1-\frac{\gamma_2\varphi}{\gamma_1-\gamma_2+\gamma_2\varphi}\right] \geq 0 \end{aligned}$$

Then, we can confirm that the system T under conditions $A_{1,0} \geq A_{2,0}$, $A_{1,0} \geq A_{3,0}$ and $\gamma_1 \geq \gamma_2 \geq \gamma_3$, is expansive and grows at a rate tending to the highest value of γ_i

b. What happens when the system starts out from other initial conditions?

For the sake of brevity, let us assume $A_{3,0} = Max\left(A_{i,0}\right) > A_{1,0}$. For the case $A_{2,0} = Max\left(A_{i,0}\right) > A_{1,0}$ we have a similar proof. If $\gamma_1 = \gamma_3$, we can carry out the previous demonstration again, this time using γ_3 instead of γ_1, thus proving the proposition for this case. Given that, all that remains to be proved next is the proposition for the case: $A_{3,0} = Max\left(A_{i,0}\right) > A_{1,0}$ and $\gamma_1 > \gamma_2 \geq \gamma_3 > 0$. From (7), we will obtain at $t = 0$ and at any t while the previous conditions are fulfilled, that:

$$A_{3,t+1} = A_{3,t}(1+\gamma_3),\ A_{1,t+1} = A_{1,t}(1+\gamma_1 z_{1t}),\ A_{2,t+1} = A_{2,t}(1+\gamma_2 z_{2t}),\ \text{being } z_{1t} > 1,\ z_{2t} \geq 1$$

Therefore, either $A_{1,t+1}$ becomes no lower than the other values, or at least it reduces the distance with the leader in percentage terms of $A_{1,t+1}$ more than $\frac{\gamma_1-\gamma_3}{1+\gamma_1}>0$.[4] Although there may be changes in the firm with the highest $A_{i,t}$, this process will end up in finite time in a situation in which $A_{1,t}=Max\left(A_{i,t}\right)$. Then, we are back in the case already seen in 1).

For 2)

We know that condition

$$p_t q_{it} = \delta_i p_t q_{i,t} + \gamma_i p_t q_{i,t} + \chi_i L_{i,t} + r_t L_{i,t} + \pi_{i,t}^{prod}, \quad \text{with} \quad \pi_{i,t}^{prod} = \pi_{i,t} - r_t^D D_{it},$$

must hold with $\pi_{i,t}^{prod}$ being the profit obtained exclusively from productive activity. If $L_{i,0}$ is null for all firms, it follows that $1=\delta_i+\gamma_i+\varepsilon_i, \forall i$ at $t = 0$, with ε_i being the productive profit per value unit of production. Moreover $\varepsilon_i = 1-\gamma_i-\delta_i>0$ and constant, and $L_{i,t}, \forall i$ and $\forall t$, is also null. Then the growth rate of the economy will be $\tilde{\gamma}=\underset{i}{Max}\left(1-\delta_i-\varepsilon_i\right)$. The result regarding interest rates and prices follows from (16) and (5). □

Note that in 2), if $1>\gamma_i+\delta_i, \forall i,$ the condition "no initial debts" is sufficient but not necessary, as we will see later in Figure 1. Then, a path which verified $1>\gamma_i+\delta_i, \forall i,$ could achieve the stationary path while the initial debts are not too big. Moreover, according to 1), the stationary state can be achieved by several firms (not only one) although other firms may become bankrupt in the process due to their debts (see Figure 3). Thus, we can see indirectly in the proposition that it focuses on the relevance of financial constraints, which are an essential component of the model. This proposition also states that all firms in the model (apart from those that exit the market) tend to assimilate "best-practice" technologies as time goes by. This assimilation mechanism is endogenous in the model, along the lines of Schumpeter (1939) and modern growth theory. Note that the proposition does not require that less innovative firms exit the

[4] If $A_{3,t+1}=(1+\tilde{\eta})A_{1,t+1}$ and $A_{3,t}=(1+\eta)A_{1,t}$, then:

$$\tilde{\eta}\, A_{1,t+1}=A_{3,t+1}-A_{1,t+1}=\left[\frac{1+\eta}{1+\gamma_1 z_{1t}}(1+\gamma_3)-1\right]A_{1,t+1}\Rightarrow\tilde{\eta}<\frac{1+\eta}{1+\gamma_1}(1+\gamma_3)-1\Rightarrow\tilde{\eta}+(1+\eta)\frac{\gamma_1-\gamma_3}{1+\gamma_1}<\eta\Rightarrow\tilde{\eta}+\frac{\gamma_1-\gamma_3}{1+\gamma_1}<\eta$$

market; in fact, as knowledge spills over, the productive sector changes and deploys technological transformations, even allowing for the co-existence of firms with different technologies and market shares (as long as new knowledge is spread and assimilated by the followers at a sufficiently fast rate).

To sum up, the approach in 3.1 and the formal results in 3.2 anticipate important aspects of the dynamics of our model. There seems to be a ceiling for the growth rate, with the most innovative firms applying pressure but, at the same time and under certain conditions, fuelling the less innovative ones; prices tend to rise with growth and generate different effects upon profitable *vs* non-profitable firms; and the existence of debts conditions the manner in which innovation, competition and market structure develop. Finally, these features take place in a context heavily shaped by market reactivity ((5), (16) and (18); see (19)). In Section 4, we use the insights obtained above to design different simulation experiments. More precisely, we shall look at the following properties of the model:

- The model can deploy virtuous stationary paths, characterized by sustained output growth, moderate interest rates, a very diverse behaviour in terms of the accumulation of debts and loans, and an intense process of technological absorption or imitation. We can identify these "virtuous growth paths" with Schumpeter's idea of referential steady state trajectories. See 4.1.
- We will see the key role of technological change in highly competitive economies. Nevertheless, the need to be highly innovative to compete may end up being detrimental (in certain settings) not only for the individual firm, but also for the economy as a whole. See 4.2.
- The key role of institutional conditions underlying the production-financial links. The complex feedbacks between the real and financial spheres force us to sharpen specific conditions for the viability of the economy and the possibility of sustained growth. See 4.3.
- The role of prices in growth and the multiple interactions with several parameters in the model. Clear policy implications follow. See 4.4.
- The processes of creative destruction as being highly dependent on financial conditions, as Schumpeter (1939) and (recently) some Neo-Schumpeterians (Dosi et al., 2013) argue. See 4.5.

4 Simulations

Drawing on Section 3, we will now run the model in different settings for a better understanding of its properties. For the sake of simplicity we consider three firms and a bank in the following baseline scenario (see Table 1: Baseline Scenario I), with these parametric values and initial conditions:

	γ_i	δ_i	θ_i	φ	χ_i	$A_{i,0}$	$D_{i,0}$	$L_{i,0}$	$K_{i,0}$
First firm	0.1	0.87	0.98	0.1	0.1	1	1	1.8	1
Second firm	0.08	0.90	0.90	0.1	0.1	1	1	1.5	1
Third firm	0.07	0.92	0.90	0.1	0.1	1	1	1.2	1
Other values of the economy and the bank: $\sigma=0.5; w=0.1; \theta^b=0.3; p_0=1; N_0^b=0.5; r_0=r_{\min}=0.02; r_0^b=r_{\min}^b=0.005; \alpha=0.1; \rho=1; \beta=0.2; \alpha^D=0.75; \delta^b=0.1$									

Table 1: *Baseline scenario I*

In *Baseline scenario I*, we have tried to delineate a setting that, at least in a qualitative way, may resemble real conditions. Note that R&D to sales ratios, operating cost rates, proportions of profits to be shared out, rates of debt devolution, minimum interest rates, and reserve ratios all have plausible values (see the simulations in Winter, 1984; Delli Gatti et al., 2008; Dosi et al., 2010, 2013; Almudi et al., 2013; Fatas-Villafranca et al., 2009, 2014). Likewise, we have set market reactivity coefficients that maintain the orders of magnitude. Notice also that capital, a bank's initial net-worth, and deposits are of compatible size, and can be assimilated to similar values in Delli Gatti et al., 2008; Fatas-Villafranca et al., 2009, 2014; Assenza et al., 2015. Moreover, *Baseline scenario I* seeks to be a useful departure point to run simulations in which we can study the following issues: virtuous growth paths, in 4.1; potential problems in highly innovative economies with uneven firm costs, 4.2; the role of institutional conditions underlying production-banking interactions in growth, 4.3; prices and growth, 4.4; and, in 4.5, highly stylized representations of what Schumpeter (1939) called *creative destruction* processes, exploring the links in the model between uneven innovation rates in the productive sector and banking aspects, as well as the possible roles of these links in large contractions with structural effects in long-run growth. Note in Table 1 that we shall consider uneven innovation efforts: firm 1 is the one with the highest innovating effort (high γ_1), followed by firm 2 and, finally, firm 3 is the one with the lowest effort of innovation. We also assume that the firms share out a significant part of their profits, especially firm 1 (the most innovative one) with $\theta_1 = 0.98$. We shall now explore the afore-mentioned issues in different subsections. As we will see, the results allow us to extract some policy implications, and they support the formal and heuristic results and the stylized facts that we anticipated in Section 3.

4.1 Virtuous growth paths

To start with, we can confirm the results from Proposition 1: the existence of virtuous growth paths in the model. Figure 1 shows the paths (for 60 periods) which emerge for the main variables when we run the model from *Baseline scenario I*. In the long term, the growth rate is the same for the three firms, $\tilde{\gamma} = \underset{i}{Max}\,\gamma_i = \gamma_1 = 0.1$ (in accordance with Proposition 1 and Table 1), the interest rate achieves the minimum value, $r_{\min} = 0.02$, and the price grows because the growth rate $\tilde{\gamma}$ is higher than $r_{\min}$. These results in Figure 1 support our results in Section 3.

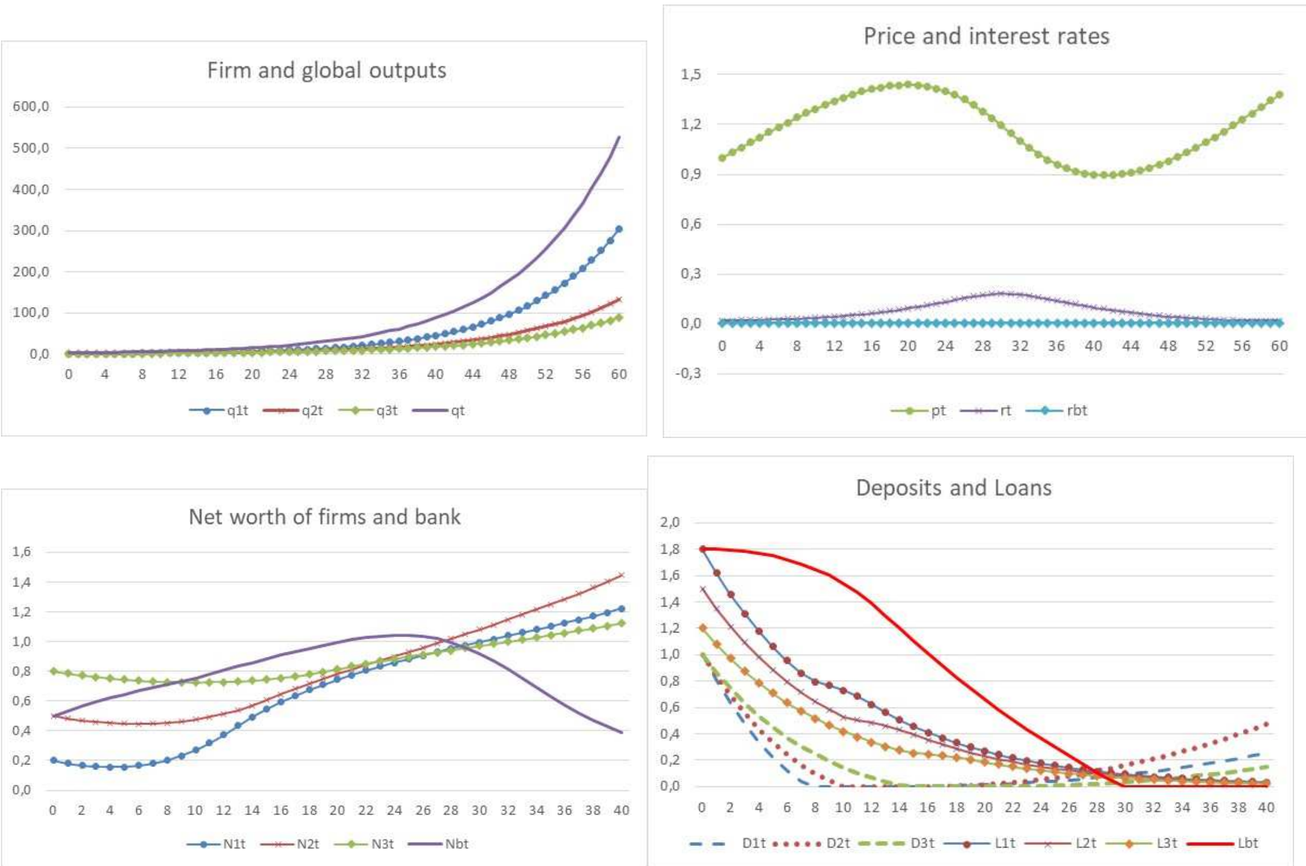


Figure 1. Baseline scenario I

Now we set out from this virtuous setting in which sustained growth emerges. We may wonder what happens when some firms increase their R&D efforts in this highly competitive economy; in other words, when we fix higher levels for γ_i in Table 1. The results we obtained from Proposition 1 are very clear: if the non-financial costs are viable ($1 > \gamma_i + \delta_i$), the long term growth rate should increase with the value of $\tilde{\gamma} = \underset{i}{Max}\, \gamma_i$. This result fits in well with the traditional support for high R&D in standard models. In Figure 2 we confirm this conclusion by a simulation of 60 periods for setting I, but when γ_1 goes from 0.1 up to 0.11, and γ_2 increases from 0.08 to 0.12, while maintaining the non-financial cost, $\gamma_1 + \delta_1$ and $\gamma_2 + \delta_2$ do not change. As we could expect, the long term growth rate is now higher, $\tilde{\gamma} = \gamma_2 = 0.12$, than in Figure 1, and firm 2's output surpasses that of firm 1, revealing the relevance of R&D carried out by firms in these conditions.

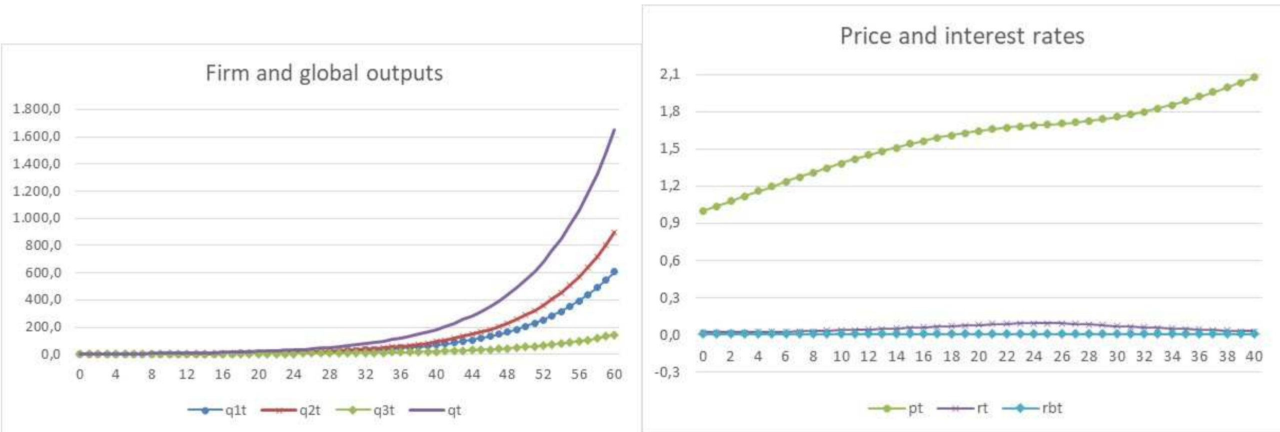


Figure 2. Baseline scenario I except $\gamma_1 = 0.11$, $\delta_1 = 0.86$, $\gamma_2 = 0.12$, and $\delta_2 = 0.86$

4.2 *High non-financial costs*

What happens if, instead of setting the preceding conditions regarding non-financial costs, we consider that a firm (in this case the leading firm) reaches levels of R&D spending together with the accompanying unit-operative costs so that $1 < \gamma_i + \delta_i$? We start out from Table 1 but we now set $\gamma_1 = 0.14$ instead of $\gamma_1 = 0.1$, while also reducing $L_{1,0}$ from 1.8 to 1.6, and we show the results in Figure 3. Notice that in highly innovative radically-uncertain environments in which firms are competing very hard, it is usually not possible to calculate the R&D spending or the total unit costs exactly, and this is especially so for the case of path-breaking innovative firms exploring new ways to do things. Here we often find deviations in the cost, or even some firms may push too hard to accelerate their innovativeness and growth. These are plausible situations in highly innovative and competitive frames. What the model shows in these conditions, in Figure 3, is really surprising.

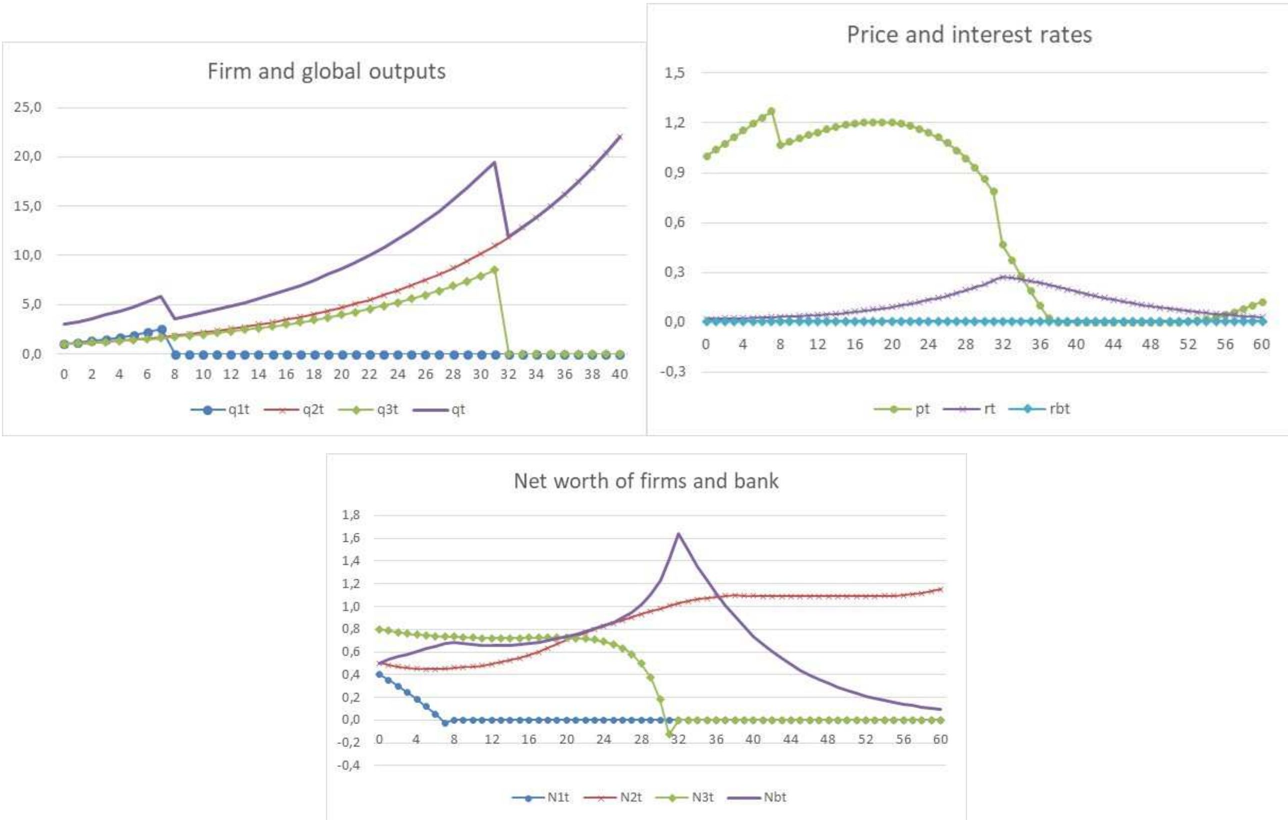


Figure 3. Baseline scenario I except $\gamma_1 = 0.14$ and $L_{1,0} = 1.6$

We can see in Figure 3 that firm 1's non-financial costs (mostly R&D increases) lead it to disaster; but what is striking is that the problems spread to firm 3 which also becomes bankrupt, just because of a negative externality coming from firm 1. Only the second firm survives, paying its debts and leading long-term growth with a growth rate equal to 0.08 and with a long-term interest rate equal to 0.02, the minimum level. This *negative externality* spreads from the disaster of the first firm to the third one through two channels: firstly, the collapse of the most innovative firm reduces the flow of knowledge spill-overs (from firm 1, the leader, to the others). This flow of knowledge had been fuelling the surviving performance of firm 3 and the loss of this knowledge (and the corresponding decrease in revenues) is crucial. Secondly, the rise in interest rates (that we see in Figure 3) which begins with the financial problems of firm 1 increases the burden of debt for firm 3, which then needs more credit, and this, in turn, feeds even more the increase in interest rates. Therefore, what is initially a failure in the leading firm (firm 1), unrelated to its financial costs but with an excess (deliberately or not) of R&D ambition, ends up spreading to other parts of the economy. And the diffusion of this negative externality occurs both through a reduction of revenues via the erosion of knowledge spill-overs from the leaders, and through the contagion of firm 1's problems via the financial system.

We may believe that firm 1's bankruptcy is due to its initial debts, but this is not the case as we can see in the following graph, Figure 4, where the setting is the same as in Figure 3, but this time assuming there are no debts for the three firms. Here, firm 1 also becomes bankrupt but we notice that the third firm survives due to its lack of debts, which reduces the impact it experiences due to firm 1's bankruptcy.

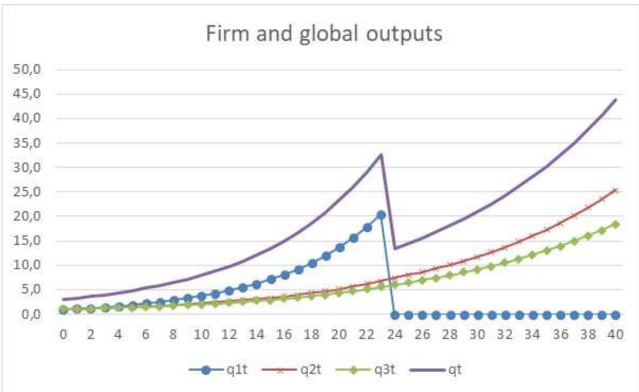


Figure 4. Baseline scenario I except $\gamma_1 = 0.14$ and $L_{i,0} = 0$, $i = 1, 2, 3$

4.3 Are the institutions underlying production-banking links relevant?

We have previously seen the relevance of the financial component on the results of the model, as happens in the real economy. The model also allows us to answer an additional question with serious policy implications. Do debts have the same impact on any economy? Or does the institutional frame influence the effects of debt on growth? In our model there are three parameters which clearly represent (as we have seen in Section 2) institutional and socio-economic framing conditions: firstly, the minimum interest rate,

$r_{\min}$, which, as we saw, delimits the dynamic range of variation for the market interest rate. This parameter may depend (as we argued in Section 2) on the perceived risks (lack of trust, lack of respect for the rule of law), a higher or lower level of institutional solidity, and of course, on monetary policy. Secondly, we have another (firm-specific) set of parameters capturing the re-payment or amortization rate of debts, as given by the parameters χ_i. And finally, parameter ρ reflects the perception of risk in the economy via the interests firms pay for their debts. The simulations will show us their relevance.

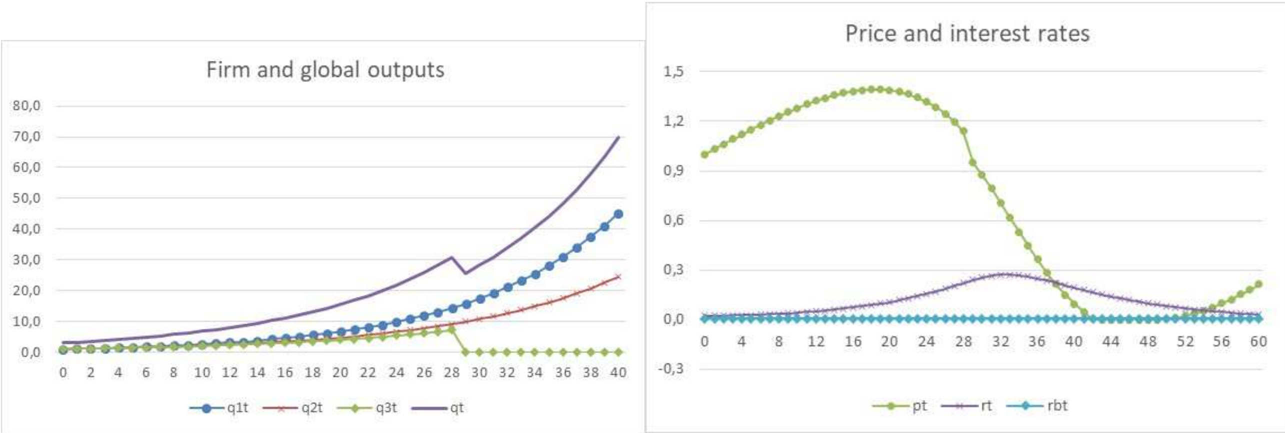
Thus, on the one hand, as we would expect, the lower the minimum interest rate, the greater the survival of firms and the higher the rhythm of overall growth, reducing the number of bankruptcies. The same is true for χ_i, the reductions of which could be identified with debt-restructurings and changes in the amortization periods. Furthermore, the lower the value of ρ, the lower the interests firms deeply in debt will have to pay and the easier it is for them to survive.

However, the simulations show once again that the effects of the changes of $r_{\min}$, χ_i and ρ overflow the firm affected, revealing the strong co-evolution of all firms which are linked by several financial and productive mechanisms.

Let us begin in Figure 5 with the analysis for $r_{\min}$. Starting out from the setting in Table 1, we run the model for three values of $r_{\min}$, 0.02, 0.022 and 0.025.

In Figure 1 we already saw the results for $r_{\min} = 0.02$; in Figure 5 we can see the results obtained for $r_{\min} = 0.022$ and 0.025. The differences in growth, price dynamics, and the evolution of net-worths show the overall influence of this institutional parameter, $r_{\min}$.

For $r_{\min} = 0.022$



For $r_{\min} = 0.025$

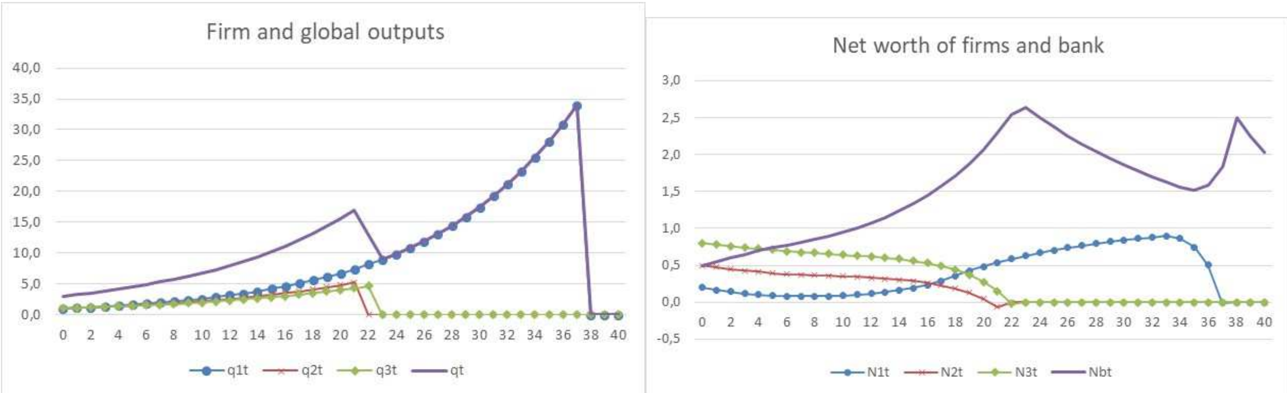


Figure 5. Baseline scenario I except $r_{\min} = 0.022$ and $r_{\min} = 0.025$

As we see in Figure 5, the viability of the firms decreases when the minimum value of the interest rate increases; thus, in the case of 0.02 (Figure 1) all firms survive and reach the stationary growth path, with a growth rate equal to $\tilde{\gamma} = \gamma_1 = 0.1$. On the contrary, for $r_{\min} = 0.022$ (in Figure 5), firm 3 becomes bankrupt in period 28. In the third simulation (see also Figure 5 for $r_{\min} = 0.025$), firms 2 and 3 both become bankrupt in periods 21 and 22 respectively while the first one seems to survive. Nevertheless, the initially surviving firm does not reach a situation of balanced growth, because the rate of interest is very high and financial costs choke it; eventually, it goes bankrupt in period 37. The firms and the economy as a whole have already collapsed. The net worth picture shows this process clearly; firms 2 and 3 reduce their net worth up until bankruptcy, while firm 1 increases it up to period 33 but it actually becomes bankrupt due to its negative profits (losses). The implications of these results show the danger of risk premium or the deterioration of institutional frames in which a cascade of bankruptcies can emerge.

Via simulations we can also see the institutional role χ_i plays. In other words, the role of the repayment rhythm of debts for each firm; see Figure 6. The reference evolution is given by Baseline scenario I, $\chi_i = 0.1,\ i = 1,2,3$, which we showed in Figure 1. In this case all firms survived and the long term growth rate g was 0.1.

If we now start out from the setting in Table 1 but increasing χ_1 and χ_2 from 0.1 to 0.15, we can see the new results in Figure 6. This change implies harder conditions for the re-payment of debt (a shorter time for amortization) for both the leading firm 1 and firm 2.

What is surprising—as we can see in Figure 6—is that firm 3, which does not change its rate of debt amortization, becomes bankrupt in period 13 when the conditions for firms 1 and 2 change. This effect reveals again the horizontal spread of effects and the strong coevolution of firms through the bank. A higher value of χ_1 and χ_2 reduces the capacity of the leading firms to generate profits and put resources into deposits during the growth process. This reduction in the bank deposits of firms 1 and 2 generates a higher ratio between financial needs and lending resources in the bank, and increases the interest rate. This leads to the bankruptcy of firm 3 and a reduction in the aggregate growth pattern, which falls from 527.07 in period 60 of Scenario I to 437.77 in this Scenario. The surviving firms 1 and 2 tend to the steady state with g equal to 0.1 and an interest rate equal to $r_{\min}$. As a policy action, we can indicate that a small reduction of the debt of firm 3, from 1.2 to 0.9 (a slight restructuring of the debt), means the crisis is avoided in the simulation and all the firms reach the long term with g equal to 0.1 and an interest rate equal to $r_{\min}$.

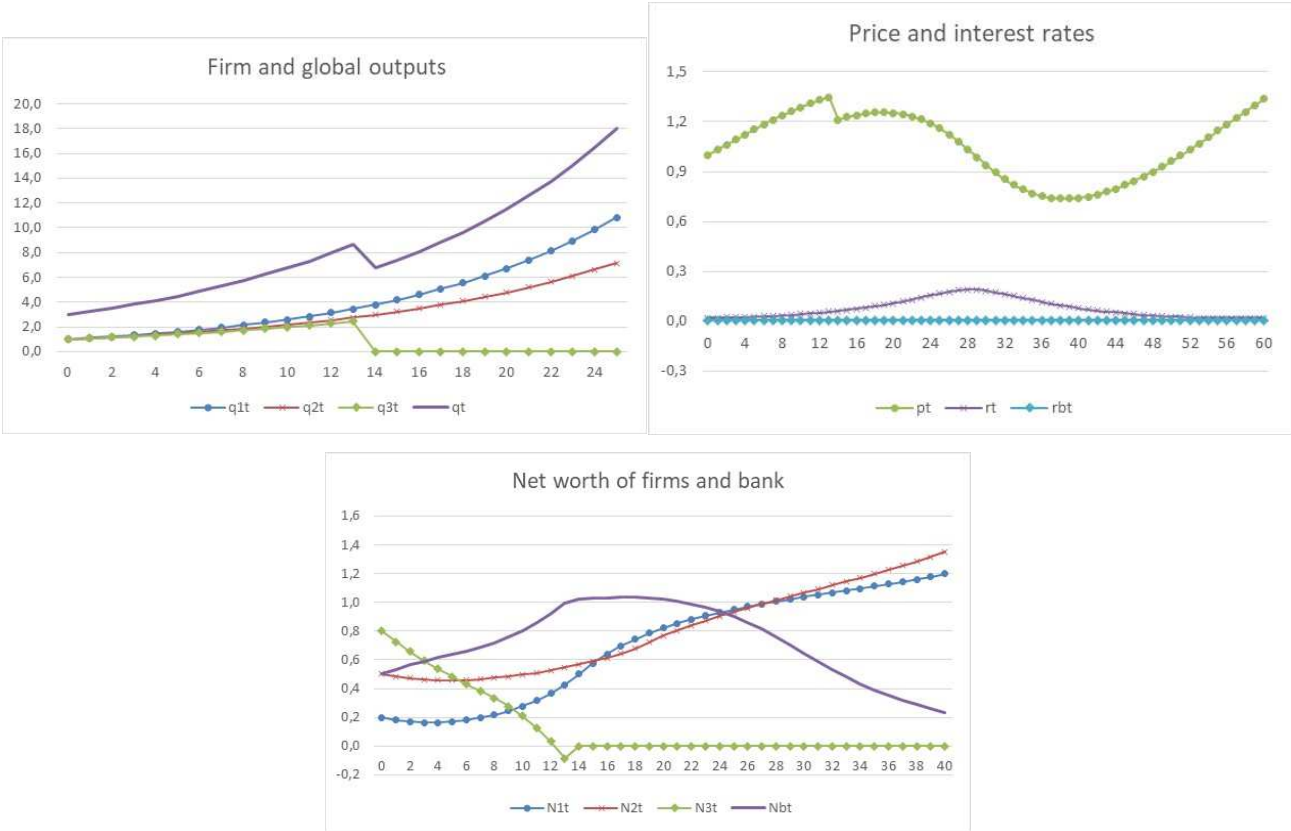
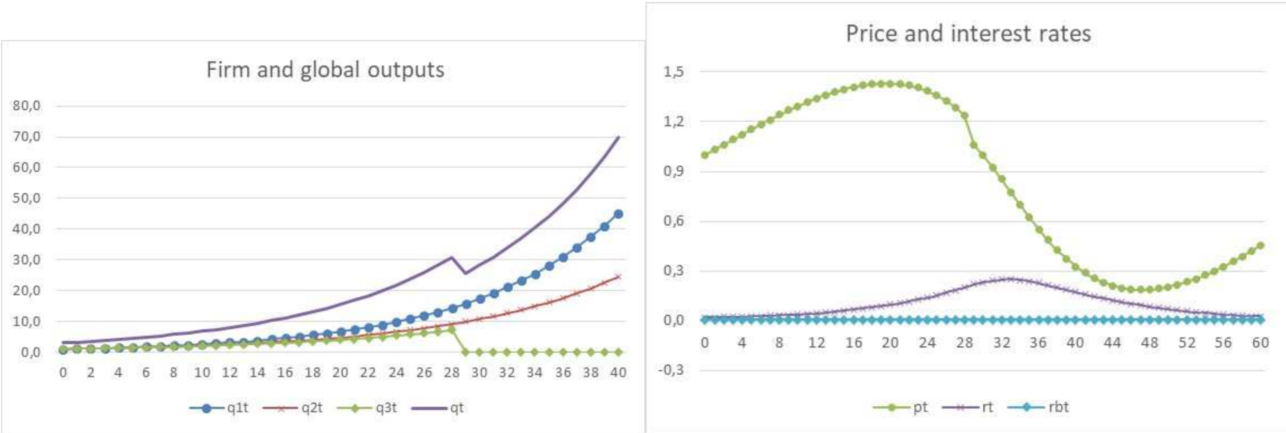


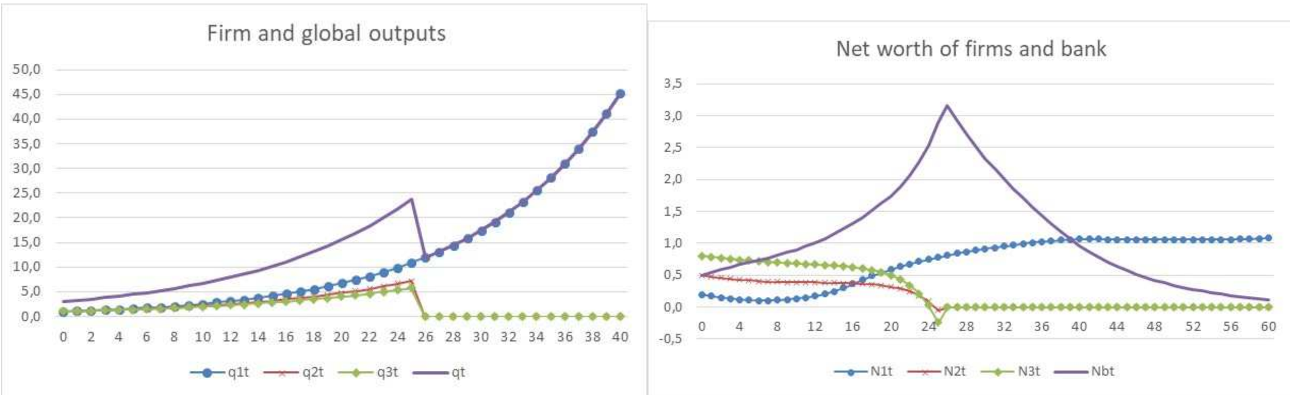
Figure 6. Baseline scenario I, except $\chi_1 = 0.15$ and $\chi_2 = 0.15$

If we now look at how the Banking System incorporates firms' risks, we can see how the changes in parameter ρ influence the dynamics of the firms. It is to be expected that higher values of ρ impede growth as it becomes more expensive to pay off debts. The smaller the value is, the less of a burden the debts become and the greater the chances of survival. Setting out from our Baseline scenario I, where ρ is equal to 1, we shall now see what happens when this parameter takes values of 1.3, 1.5 and 1.7. The results, seen in Figure 7, confirm the abovementioned; for a value of ρ equal to 1 (baseline scenario), the three firms will survive indefinitely; for the case of ρ equal to 1.3, firm 3 becomes bankrupt in period 28; for a value of 1.5, firms 2 and 3 become bankrupt in period 25. Finally, if ρ is 1.7, the three firms will become bankrupt during periods 41, 20, and 23, respectively, as they are unable to survive with these debts.

For $\rho = 1.3$



$\rho = 1.5$



$\rho = 1.7$

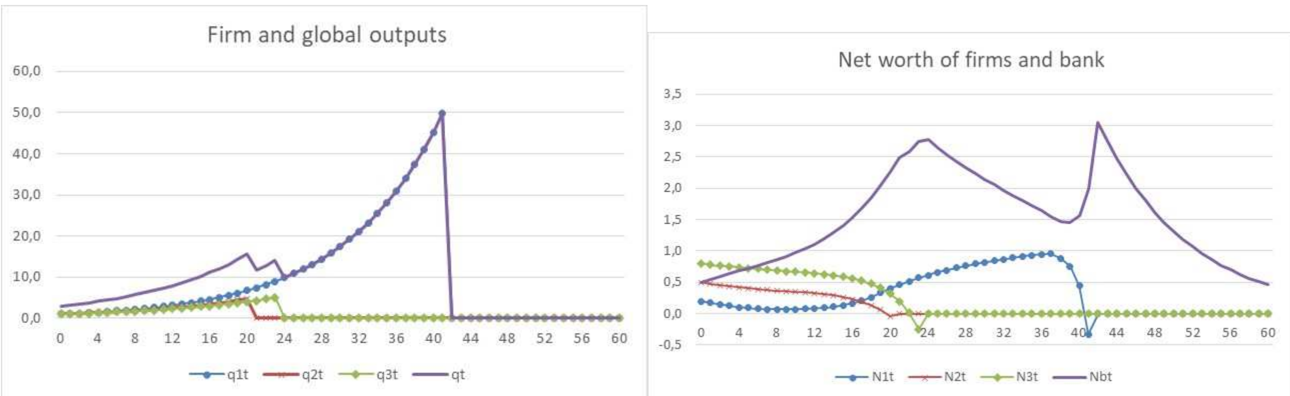


Figure 7. Baseline scenario I, except ρ = 1.3, 1.5 and 1.7

4.4 The relevant role of price dynamics

In Section 3, especially in sub-sections 3.1.3, 3.1.4 and in Proposition 1, we have discussed the relevant role played by prices in the global evolution of the economy, and how prices are linked to growth paths and the dynamics of the interest rate (see, for example, equation (5)). We have also seen how balanced growth leads to a minimum value of the interest rate, as well as the fact that there may be inflationary processes (rising prices) even if stationary growth is achieved. As can be seen in equation (5), prices depend on growth and interest rates, but they also depend on a structural-reactivity parameter σ. In the previous sub-sections 4.1, 4.2 and 4.3 we have already seen interactions between growth and the interest rate. Let's now look at how σ, which controls the speed of change in prices and, therefore, the intensity of possible inflationary or deflationary processes, affects the model dynamics. The higher σ, the higher the speed of adjustment in equation (5). We will now see that the effects of inflation or deflation are very much dependent on the specific institutional framework we consider regarding $r_{\min}$.

In Figures 8 and 9 the results of scenario I can be seen when σ changes from 0.5 to 0.2 and 0.8 respectively; these results can be compared to those in Figure 1 that correspond to $\sigma = 0.5$. In the three settings we have a relatively low level of $r_{\min} = 0.02$.

Figure 8. Baseline scenario I except $\sigma = 0.2$

In Figure 8 we see how the price increases more slowly (as compared with Figure 1) during the initial phase of the process, although it allows for a slow rise in production costs. Therefore, in Figure 8 we see a smooth price path but one that is associated to a much slower pattern of economic growth (q_t is 69.72 at period 40 in Figure 8 versus 87.62 in Figure 1). The consequence is that, even in less inflationary conditions, the weaker firm (firm 3) ends up failing and exiting the market (instead of surviving as we saw in the initial setting in Figure 1). Figure 8 shows how more stable price settings are not always the best scenario. Price stability does not seem to be always the advisable policy goal if we compare Figure 8 with Figure 1 where the three firms survive. This is an important policy lesson.

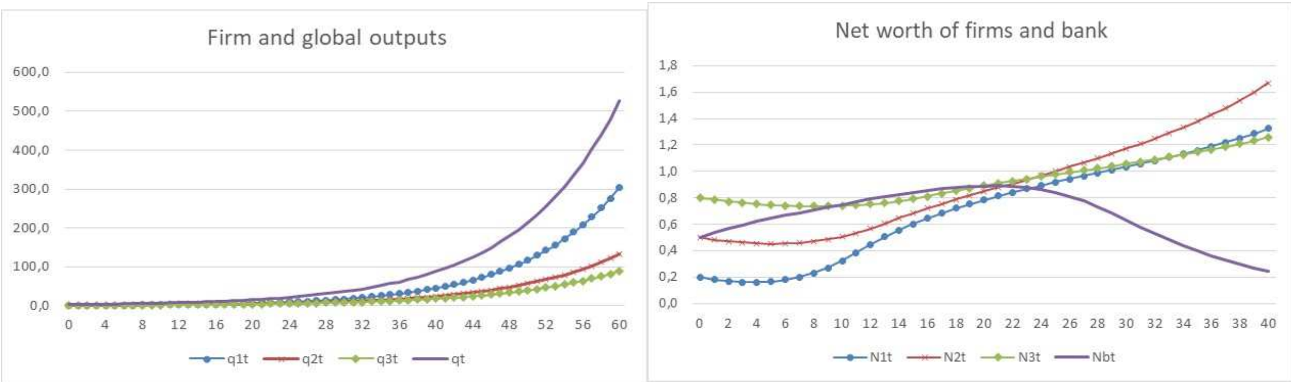


Figure 9. Baseline scenario I except σ= 0.8

Regarding Figure 9, we set σ= 0.8 so that we generate a more price-reactive setting. The results are very similar to those shown in Figure 1 and support our previous insights even further. We obtain a more inflationary path accompanying a nice growth pattern in which all firms survive. The policy lesson is that a moderate level of inflation allows for high revenues, high R&D and more deposits. The overall economy is more solvent, with a lower degree of financial needs. In this frame, there is not much high-opportunity business for the bank (compare the net-worth path for the bank in Figure 9 with the peak during the crisis of firm 3 in Figure 8) and we just see smooth solvent growth.

4.5 *Creative destruction and banking*

Once we have a good understanding of multiple interactions in the model dynamics, we want to explore (at least in a stylized manner) those episodes which Schumpeter (1939) called processes of creative destruction. We shall see that our model allows us a close look at four facts which are very characteristic of Schumpeterian thinking: growth through innovative firms; destruction of older firms which do not or cannot adapt to new production conditions created by innovation; imitation processes; and strong links between uneven innovation in the productive sector and banking aspects, as well as the possible roles of these links in economic contractions.

We have already seen previously that the model accelerates the growth of firms via innovation and how this can make a firm become leader, with the surviving firm being the one that sets the growth rate and invests most in R&D (see Proposition 1). We have also seen that if a firm survives, although it is not the R&D investment leader, as there is an imitation mechanism or, in other words, a Schumpeterian process of diffusion and assimilation, firm ends up growing at the same rate as the surviving firm which keeps its place as leader in terms of investment. This is also ensured by the above mentioned Proposition. Thus, our model captures or allows us to come close to two of the characteristics mentioned above. Now, we shall look at the other two.

We set out again from *Baseline scenario I* which led to Figure 1, with the only difference being that the initial provision of physical capital in firm 3, the one which invests the least in technology, is double that of the others. We shall identify increases in R&D investment in the first two firms as a technological boom, such that $\gamma_1 = \gamma_2 = 0.12$ rather than $\gamma_1 = 0.1$ and $\gamma_2 = 0.08$. As firm 2 increases its R&D investment, it will reduce its operating costs at the same time from $\delta_2 = 0.9$ to $\delta_2 = 0.86$, with its R&D unit costs and operating costs becoming $\gamma_2 + \delta_2 = 0.98$ compared to the costs of 0.99 of firm 1. Firm 3 makes no technological change despite having the advantage of a greater initial size. The results of the simulations can be seen in Figure 10.

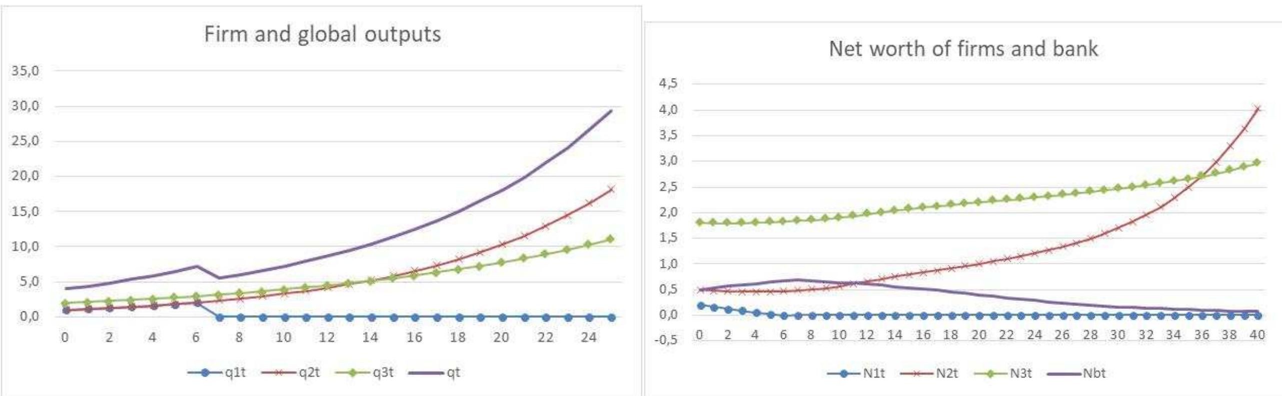


Figure 10. Baseline scenario I, except $\gamma_1 = 0.12$, $\gamma_2 = 0.12$, $\delta_2 = 0.86$ and $K_{3,0} = 2$

As can be seen in Figure 10, firm 2 clearly becomes the leader, setting a new growth rate, 0.12, and surpassing the other two firms in questions of market quota and net worths. On the other hand, firm 1 becomes bankrupt in period 7 although its R&D investment rate is as high as that of firm 2; it dies off as it is incapable of covering its debts, which were initially very high. As is well-known, being innovative is no guarantee of survival. The initial size of firm 3, even though it has initial values of net worth higher than those of the other firms, does not help it win the competitive process either. It survives but firm 2 surpasses its net worth in period 36. What happens globally is that firm 2, leader in R&D, is also the firm with least unit costs of investment and operations in the mid-term, which means it surpasses in questions of output and net worth firm 1, which is innovative but has higher costs. This opens the way for leadership changes, a situation which is empirically relevant—see Dosi and Nelson (2010). This is confirmed in Figure 11, where firm 2 once again easily outperforms firm 1 in output and net worth, and firm 1 does not enter bankruptcy thanks to its debts being much lower than before and lower than those of firm 2, $L_{1,0} = 1.2 < L_{2,0} = 1.5$.

It is also easy to see the key role of financial costs in the failure of firm 1 and the survival of firm 2. Thus, if we reduce its debt from $L_{1,0} = 1.8$ to $L_{1,0} = 1.2$, firm 1 does not exit the market, since it overcomes the burden of its debt (see Figure 11). However, firm 2 outperforms it in terms of net worth in period 20. As a normative prescription we suggest that firms should keep debt as low as possible because in this way they have more time and flexibility when adapting to fast-changing circumstances, in particular due to the fast technological change of rival firms.

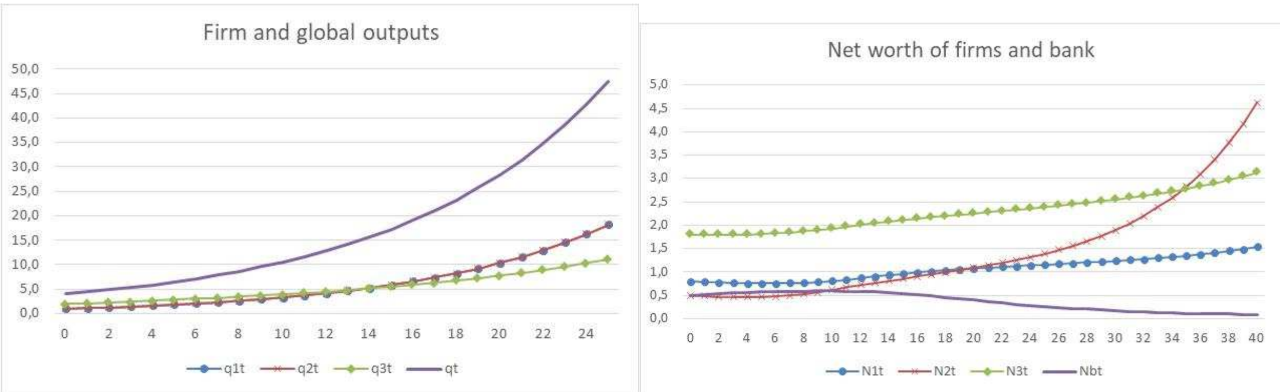


Figure 11. Baseline scenario I, except $\gamma_1 = 0.12$, $\gamma_2 = 0.12$, $\delta_2 = 0.86$, $K_{3,0} = 2$ and $L_{1,0} = 1.2$

We can obtain the same conclusion for firm 3; its survival depends on the financial conditions it faces the changes with. If its debts were higher, $L_{3,0} = 1.8$ instead of $L_{3,0} = 1.2$, this firm would also become bankrupt, as can be seen in Figure 12, in period 20, with firm 2 now being the only one remaining and tending to grow at 12% as the interest rate tends to $r_{\min} = 0.02$.

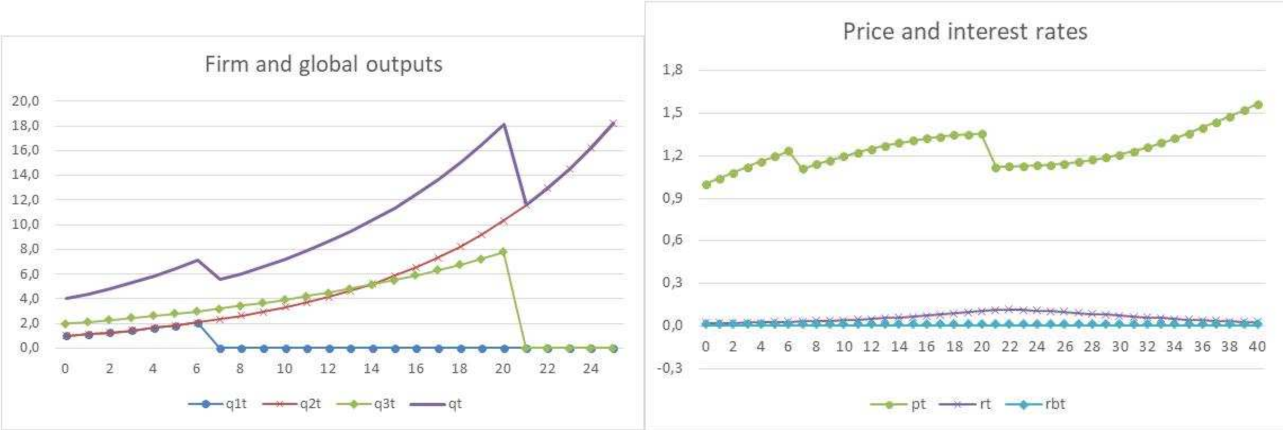


Figure 12. Baseline scenario I, except $\gamma_1 = 0.12$, $\gamma_2 = 0.12$, $\delta_2 = 0.86$, $K_{3,0} = 2$ and $L_{3,0} = 1.8$

5 Final conclusions

We have presented an evolutionary growth model in which an innovative productive sector interacts with a bank. By exploring the links between long-term sources of growth (technological change) and short-term/mid-term factors (debt accumulation, strategic parameters) we have found new drivers for the emergence of crises with long-run effects. Departing from the characterization of virtuous paths in which the interactions between banking and productive activity flow in a sustainable way, we have also explored settings in which (initially) small crises generate negative externalities and diffusion among firms. These processes end up generating large-scale output contractions. The role of the bank and the financial structure of firms is crucial in amplifying (or not) the unavoidable competitive effects of innovative firms which outperform inferior rivals, thus seeding the conditions either for progressive growth, or for the sudden collapse of a significant part of the productive sector.

We started out from the heuristics in 3.1 and the formal results in 3.2 in order to obtain a general overview of the model dynamics. The results indicate that there seems to be a ceiling for the economy growth rate—with the most innovative firms applying pressure, but also fuelling weaker rivals. It is worth noting that the price tends to rise with growth and generates very different effects upon profitable and non-profitable firms. On the other hand, the existence (or not) of debts significantly conditions the manner in which innovation, competition and market structure develop. Also, all of this seems to happen in a context heavily shaped by market reactivity parameters.

Despite the highly stylized manner in which we have linked innovative competition and banking activity in the model, we have seen that these connections are absolutely essential. Although we have found simulations in which banking provides firms with the time and resources to develop technology and overcome the burden of debts, we have also obtained trajectories in which fast innovation in strong competitive environments unchained pressures through rising prices and interest rates which ended up ripping the process of growth. Thus, we have shown in a simple evolutionary model that innovation, growth, prices, savings, banking reactivity and interest rates are interlinked variables which involve intricacies that must be understood; otherwise, we may pursue the wrong policy goals which may turn out to be counterproductive. The final message we want to put forth is that innovation and growth are not the *panacea* for all socio-economic problems. They are part of a highly complex mechanism which, in balanced conditions, can foster progress; but if out of control, may flow into collapse.

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Annex

Table A.1: Time evolution of the model with three firms for baseline scenario I, except $\gamma_1 = 0.12$, $\gamma_2 = 0.12$, $\delta_2 = 0.86$, $K_{3,0} = 2$ and $L_{3,0} = 1.8$ (Figure 12)

			γ_1 =	ρ =		δ_1 =	α^D =	φ =	θ_1 =	χ_1 =				γ_2 =			δ_2 =		φ =	θ_2 =	χ_2 =	
			0,12	1,00		0,87	0,75	0,10	0,98	0,10				0,12			0,86		0,10	0,90	0,10	
t	**$K_{1,t}$**	**$q_{1,t}$=$A_{1,t}$**	**$R_{1,t}$**	**$r_{1,t}$**	**$F_{1,t}$**	**$CT_{1,t}$**	**$\pi_{1,t}$**	**$z_{1,t}$**	**$D_{1,t}$**	**$L_{1,t}$**	**$N_{1,t}$**	**$K_{2,t}$**	**$q_{2,t}$=$A_{2,t}$**	**$R_{2,t}$**	**$r_{2,t}$**	**$F_{2,t}$**	**$CT_{2,t}$**	**$\pi_{2,t}$**	**$z_{2,t}$**	**$D_{2,t}$**	**$L_{2,t}$**	**$N_{2,t}$**
0	1,00	1,00	0,12	0,03	0,23	1,22	-0,22	1,10	1,00	1,80	0,20	1,00	1,00	0,12	0,03	0,19	1,17	-0,17	1,10	1,00	1,50	0,50
1	1,00	1,13	0,14	0,03	0,21	1,38	-0,20	1,09	0,78	1,62	0,16	1,00	1,13	0,14	0,03	0,17	1,33	-0,15	1,09	0,83	1,35	0,48
2	1,00	1,28	0,17	0,04	0,20	1,57	-0,18	1,08	0,58	1,46	0,12	1,00	1,28	0,17	0,03	0,16	1,52	-0,13	1,08	0,69	1,22	0,47
3	1,00	1,45	0,19	0,04	0,18	1,79	-0,17	1,07	0,40	1,31	0,09	1,00	1,45	0,19	0,03	0,15	1,73	-0,11	1,07	0,56	1,09	0,46
4	1,00	1,63	0,23	0,04	0,17	2,04	-0,15	1,06	0,24	1,18	0,06	1,00	1,63	0,23	0,04	0,14	1,99	-0,10	1,06	0,44	0,98	0,46
5	1,00	1,84	0,26	0,05	0,16	2,34	-0,14	1,05	0,09	1,06	0,02	1,00	1,84	0,26	0,04	0,13	2,29	-0,08	1,05	0,35	0,89	0,46
6	1,00	2,07	0,31	0,06	0,16	2,69	-0,13	1,04	0,00	1,01	-0,01	1,00	2,07	0,31	0,05	0,12	2,63	-0,07	1,04	0,27	0,80	0,47
7	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	2,33	0,31	0,05	0,11	2,64	-0,06	1,04	0,20	0,72	0,48
8	0,00	0,00	0,00	0,03	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	2,62	0,36	0,06	0,10	3,03	-0,04	1,03	0,14	0,65	0,50
9	0,00	0,00	0,00	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	2,95	0,41	0,07	0,10	3,47	-0,03	1,02	0,10	0,58	0,52
10	0,00	0,00	0,00	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	3,31	0,48	0,08	0,09	3,97	-0,01	1,02	0,07	0,52	0,55
11	0,00	0,00	0,00	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	3,71	0,54	0,08	0,09	4,54	0,00	1,01	0,06	0,47	0,58
12	0,00	0,00	0,00	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	4,16	0,62	0,09	0,08	5,17	0,02	1,01	0,06	0,42	0,63
13	0,00	0,00	0,00	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	4,67	0,71	0,10	0,08	5,89	0,04	1,00	0,06	0,38	0,68
14	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	5,23	0,81	0,11	0,07	6,69	0,06	1,00	0,06	0,34	0,72
15	0,00	0,00	0,00	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	5,86	0,92	0,12	0,07	7,58	0,09	1,00	0,07	0,31	0,76
16	0,00	0,00	0,00	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	6,56	1,04	0,12	0,06	8,57	0,11	1,00	0,08	0,28	0,80
17	0,00	0,00	0,00	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	7,35	1,18	0,13	0,06	9,68	0,14	1,00	0,09	0,25	0,84
18	0,00	0,00	0,00	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	8,23	1,33	0,13	0,05	10,91	0,17	1,00	0,10	0,23	0,88
19	0,00	0,00	0,00	0,09	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	9,22	1,50	0,13	0,05	12,26	0,20	1,00	0,12	0,20	0,92
20	0,00	0,00	0,00	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	10,32	1,68	0,13	0,04	13,74	0,24	1,00	0,14	0,18	0,96
21	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	11,56	1,56	0,11	0,03	12,75	0,23	1,00	0,16	0,16	1,00
22	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	12,95	1,75	0,09	0,03	14,31	0,26	1,00	0,19	0,15	1,04
23	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	14,50	1,96	0,07	0,02	16,07	0,31	1,00	0,21	0,13	1,08
24	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	16,24	2,21	0,05	0,02	18,05	0,35	1,00	0,24	0,12	1,12
25	0,00	0,00	0,00	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	18,19	2,48	0,04	0,02	20,30	0,40	1,00	0,28	0,11	1,17

			γ_3 =			δ_3 =		φ =	θ_3 =	χ_3 =		σ =				w =			δ^b=	α =	r_{min} =	$r_{min}{}^b$=	β =		θ_b =	
			0,07			0,92		0,10	0,90	0,10		0,50				0,10			0,10	0,10	0,02	0,005	0,20		0,30	
t	$K_{3,t}$	$q_{3,t}$=$A_{3,t}$	$R_{3,t}$	$r_{3,t}$	$F_{3,t}$	$CT_{3,t}$	$\pi_{3,t}$	$z_{3,t}$	$D_{3,t}$	$L_{3,t}$	$N_{3,t}$	p_t	q_t	R_t	π_t	D_t	L_t	Γ_t	N_t^b	L_t^b	r_t	r_t^b	E_t^b	π_t^b	C_t^b	u_t
0	2,00	2,00	0,14	0,03	0,23	2,21	-0,21	1,00	1,00	1,80	1,20	1,00	4,00	0,38	-0,59	3,00	5,10	0,30	0,50	2,40	0,02	0,01	0,00	0,07	0,02	0,00
1	2,00	2,14	0,16	0,03	0,21	2,42	-0,19	1,00	0,79	1,62	1,17	1,04	4,40	0,44	-0,53	2,41	4,59	0,24	0,55	2,42	0,02	0,01	0,00	0,07	0,02	0,00
2	2,00	2,29	0,17	0,03	0,20	2,65	-0,17	1,00	0,60	1,46	1,15	1,08	4,85	0,51	-0,48	1,87	4,13	0,19	0,60	2,45	0,02	0,01	0,00	0,06	0,02	0,00
3	2,00	2,45	0,19	0,04	0,18	2,90	-0,15	1,00	0,43	1,31	1,12	1,12	5,34	0,58	-0,43	1,39	3,72	0,14	0,64	2,46	0,02	0,01	0,00	0,06	0,02	0,00
4	2,00	2,62	0,21	0,04	0,17	3,18	-0,14	1,00	0,28	1,18	1,10	1,16	5,88	0,67	-0,39	0,96	3,35	0,10	0,68	2,48	0,02	0,01	0,00	0,06	0,02	0,00
5	2,00	2,81	0,24	0,05	0,16	3,49	-0,13	1,00	0,14	1,06	1,08	1,20	6,48	0,76	-0,35	0,57	3,01	0,06	0,72	2,49	0,03	0,01	0,00	0,06	0,02	0,00
6	2,00	3,00	0,26	0,06	0,15	3,82	-0,11	1,00	0,02	0,96	1,06	1,24	7,14	0,87	-0,31	0,28	2,76	0,03	0,76	2,51	0,03	0,01	0,00	0,05	0,02	0,01
7	2,00	3,21	0,25	0,06	0,16	3,68	-0,12	1,00	0,00	0,96	1,04	1,11	5,54	0,56	-0,18	0,20	1,67	0,02	0,80	1,50	0,03	0,01	0,00	0,01	0,00	0,00
8	2,00	3,44	0,27	0,07	0,17	4,04	-0,13	1,00	0,00	0,98	1,02	1,14	6,06	0,63	-0,17	0,14	1,63	0,01	0,81	1,50	0,03	0,01	0,00	0,02	0,01	0,00
9	2,00	3,68	0,30	0,08	0,18	4,43	-0,13	1,00	0,00	1,01	0,99	1,17	6,62	0,71	-0,16	0,10	1,59	0,01	0,82	1,50	0,04	0,01	0,00	0,03	0,01	0,00
10	2,00	3,93	0,33	0,08	0,19	4,85	-0,14	1,00	0,00	1,04	0,96	1,20	7,24	0,80	-0,16	0,07	1,57	0,01	0,84	1,51	0,04	0,01	0,00	0,03	0,01	0,00
11	2,00	4,21	0,36	0,09	0,21	5,30	-0,15	1,00	0,00	1,08	0,92	1,22	7,92	0,91	-0,15	0,06	1,55	0,01	0,86	1,50	0,05	0,01	0,00	0,04	0,01	0,00
12	2,00	4,50	0,39	0,10	0,22	5,79	-0,17	1,00	0,00	1,13	0,87	1,25	8,67	1,02	-0,15	0,06	1,55	0,01	0,89	1,50	0,05	0,01	0,00	0,05	0,02	0,00
13	2,00	4,82	0,43	0,11	0,25	6,31	-0,19	1,00	0,00	1,18	0,82	1,27	9,49	1,14	-0,14	0,06	1,57	0,01	0,93	1,51	0,05	0,01	0,00	0,07	0,02	0,00
14	2,00	5,16	0,47	0,12	0,27	6,86	-0,21	1,00	0,00	1,25	0,75	1,29	10,39	1,28	-0,14	0,06	1,59	0,01	0,98	1,54	0,06	0,01	0,00	0,08	0,02	0,00
15	2,00	5,52	0,51	0,13	0,31	7,46	-0,24	1,01	0,00	1,33	0,67	1,31	11,37	1,43	-0,15	0,07	1,64	0,01	1,03	1,58	0,07	0,01	0,00	0,10	0,03	0,00
16	2,00	5,91	0,55	0,14	0,35	8,09	-0,27	1,01	0,00	1,44	0,56	1,32	12,47	1,59	-0,16	0,08	1,71	0,01	1,10	1,64	0,07	0,01	0,00	0,12	0,04	0,00
17	2,00	6,33	0,59	0,16	0,40	8,77	-0,32	1,02	0,00	1,56	0,44	1,34	13,67	1,77	-0,18	0,09	1,81	0,01	1,19	1,73	0,08	0,01	0,00	0,15	0,04	0,00
18	2,00	6,78	0,64	0,17	0,47	9,50	-0,38	1,02	0,00	1,72	0,28	1,35	15,00	1,97	-0,21	0,10	1,95	0,01	1,29	1,86	0,09	0,01	0,00	0,19	0,06	0,00
19	2,00	7,26	0,69	0,19	0,56	10,27	-0,46	1,03	0,00	1,93	0,07	1,35	16,47	2,18	-0,25	0,12	2,13	0,01	1,42	2,02	0,09	0,01	0,00	0,24	0,07	0,00
20	2,00	7,78	0,74	0,21	0,67	11,11	-0,56	1,03	0,00	2,19	-0,19	1,35	18,10	2,42	-0,33	0,14	2,37	0,01	1,59	2,25	0,10	0,01	0,00	0,11	0,03	0,19
21	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,12	11,56	1,56	0,23	0,16	0,16	0,02	1,66	0,02	0,11	0,01	0,00	-0,15	0,00	0,00
22	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,13	12,95	1,75	0,26	0,19	0,15	0,04	1,52	0,00	0,11	0,01	0,02	-0,14	0,00	0,00
23	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,13	14,50	1,96	0,31	0,21	0,13	0,08	1,38	0,00	0,11	0,01	0,06	-0,13	0,00	0,00
24	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,13	16,24	2,21	0,35	0,24	0,12	0,12	1,25	0,00	0,11	0,01	0,10	-0,12	0,00	0,00
25	0,00	0,00	0,00	0,10	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,14	18,19	2,48	0,40	0,28	0,11	0,17	1,13	0,00	0,10	0,01	0,14	-0,11	0,00	0,00

Table A1 shows the evolution of three firms during 25 periods. In the first two rows (shaded in gray), we can see the parametric values used for the simulation, and in the third row we see the variables of each column (also shaded in gray)—variables defined in the main text using the same notation to avoid misunderstandings. In the fourth row, some cells are shaded in gray revealing the initial values used for the corresponding variables; whereas the non-shaded ones are deduced directly from the parametric values and from the other initial values.

The definition of each variable is the one that appears in the text. The model is completely explicit through the equations found in the main text and it has been solved with Excel. Where necessary the condition of non-negativity has been imposed.

For two firms, a part of its $N_{i,t}$ column appears shaded in gray, indicating that the firm has already gone bankrupt.