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Characterization and parameters sensitivity analysis of worm gear rolling measurement

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Abstract

Gears rolling tests involve multiple error sources due to their consideration as a dynamic measurement. This paper presents an experimental characterization of a single and double-flank rolling tester for worm gears. A methodology is presented to determine the individual repeatability of the complete rolling tester and its components. The high repeatability of the individual and combined test series shows the mechanical reliability of the machine for both types of rolling tests, with error values similar to those of the measuring instruments used in the equipment. Additionally, a local sensitivity analysis quantifies the influence in the rolling parameters of the main error sources identified. In this way, contributions to the measurement uncertainty of the rolling equipment due to variables such as variation of the centre distance, height or perpendicularity between the axes are established.

KEYWORDS: Gear metrology; uncertainty; worm gear; sensitivity coefficients, rolling tests; repeatability

1 Introduction

Repeatability, according to the JCGM 100:2008 [1] guide to the expression of measurement uncertainty, is defined as the "closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement". The latter include the same measurement procedure, observer, measuring instrument used, location and finally that the repetition has been carried out in a short time period. In turn, the reproducibility concept is used to define the "closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement". It is necessary in this case to specify which conditions have varied: principle of measurement, method of measurement, the observer, measuring instrument, reference standard, location, conditions of use or the time [1]. The repeatability and reproducibility values are determined experimentally and allow us to quantify the different sources of error involved in a measurement result.

On the other hand, sensitivity analysis evaluates how the model output is affected by variations in input parameters, model parameters or boundary conditions. The objective of these analyses is to determine the sensitivity of the model, identifying the parameters that do or do not influence the model or that may interact with other factors. The identification of these factors and their quantification allows the reduction of the model complexity and facilitates the estimation of the uncertainty of the model [2,3]. Pianosi et al. [4] propose a workflow for the application of sensitivity analysis considering the following phases: select input factors and model output, choose SA method, define input variability space and sampling, check model behaviour and assess robustness and convergence computing sensitivity indices that indicate each parameter's sensitivity.

In gear metrology, individual influences due to temperature variations, shaft misalignments and eccentricities, calibration errors, resolution of measuring instruments, data filtering and even operator subjectivity can be incorporated into the estimation of measurement uncertainty [5,6]. The measurement uncertainty is a parameter that characterizes the dispersion of the values that could reasonably be attributed to the measurand [1]. The measurement uncertainty increases if the

repeatability of the equipment is low, but also if the knowledge of the most relevant parameters such as calibration, environmental conditions, alignment or mounting that may affect measurement is scarce [7].

In the case of rolling tests to verify gears functionally, it has been observed that different rolling results can be obtained among different machines and laboratories, due to small unintentional variations, even when the test conditions are maintained. This concept was already shown in [8] for cylindrical gears and is still maintained today [9,10]. These discrepancies in gear rolling tests results are possibly a consequence of the lack of a complete review of practical inspections, equipment characteristics and calibration procedures [11]. Nevertheless, there are recent studies in relation to gear metrology based on rolling principles for different gear geometries [12–14].

ANSI/AGMA 2116-A05 standard [15] focuses on the evaluation of double-flank testers for radial measurements and provides an overview of different methods for uncertainty estimation. The uncertainty concept is linked to the calibration process of the instrument but the standard recommends considering also other error sources in the measurement uncertainty expression such as the environmental conditions, the runout and positioning of mounting and the repeatability and reproducibility of the instrument, among others.

There is no standard or guideline applicable for assessing the uncertainty of single-flank rolling measurement. Nevertheless, evaluation and testing procedures in both methods are similar and therefore, ANSI/AGMA 2116-A05 standard could be also applied to single-flank rolling tests. In addition, ISO/TR 10064- 5: 2005 [5] identifies factors that can influence the uncertainty of a measurement process apart from the instrument itself so that its recommendations need to be also considered for the uncertainty estimation analysis.

Uncertainty budget for worm gear single or double-flank gear rolling tests could be structured according to equation (1). It gives an expression for the expanded uncertainty U_{95} considering a coverage factor, k=2 for the 95% confidence interval [16,17]. The following terms are accounted in the uncertainty expression: (i) the tester initial calibration (u_0); (ii) the movements of the master worm and worm gear holder carriages from the calibration point to the test execution centre distance (u_{cd}) and height (u_h) respectively; (iii) the assembling of the gears (u_{gear}); (iv) the test performance (u_{test}). The advantage of this method is that the uncertainty is broken down into its most elementary components, allowing the influence of each source of error to be known. In this way, it is possible to improve the accuracy of a piece of equipment by minimizing the sources of error with the greatest impact. However, a priori it is not possible to know what is the real influence of each source of error on the uncertainty of the rolling parameters until all the terms are quantified and brought into the uncertainty expression (1). In fact, measurement sensor uncertainties are only part of the total uncertainty of the measurement process.

$$U_{95}(k=2) = 2\left[\left(u_0^2 + u_{cd}^2 + u_h^2 + u_{gear}^2 + u_{test}^2\right)^{0.5}\right] \qquad (1)$$

Therefore, a complete study of the characterisation of the rolling tests, as well as of the verification equipment itself, enables to identify the main terms of the measurement uncertainty and to calculate the sensitivity coefficients that quantify the influence of the main sources of error on the budget. The data obtained together with the data from the calibration and evaluation process of the equipment [18] can contribute to the proper estimation of the measurement uncertainty of these rolling testers.

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This work presents an experimental characterization procedure of the behaviour of a gear rolling test machine and single and double- flank rolling tests for worm gears. By means of the execution of multiple structured tests, maintaining the operating conditions, it has been possible to determine the individual repeatability of each component and the complete tester. Finally, sensitivity coefficients that quantify the influence of the main error sources in the rolling parameters have been determined by means of a one-at-a-time (OAT) local sensitivity analysis method. In this way, the contributions to the measurement uncertainty of the rolling equipment due to the variation of the gears centre distance, the meshing height or the perpendicularity between the axes are established.

2 Tester experimental characterization and repeatability evaluation

The gear rolling tester presented in this study is capable of performing both single and double-flank rolling tests for worm gears [19]. Due to its structure, the movement of the worm holder column regulates the height position between the gears, while the distance between centres depends on the movement of the worm gear holder carriage. A second carriage, located on the previous one, incorporates the elastic system that performs the back and forth movement of the double-flank test (Figure 1).

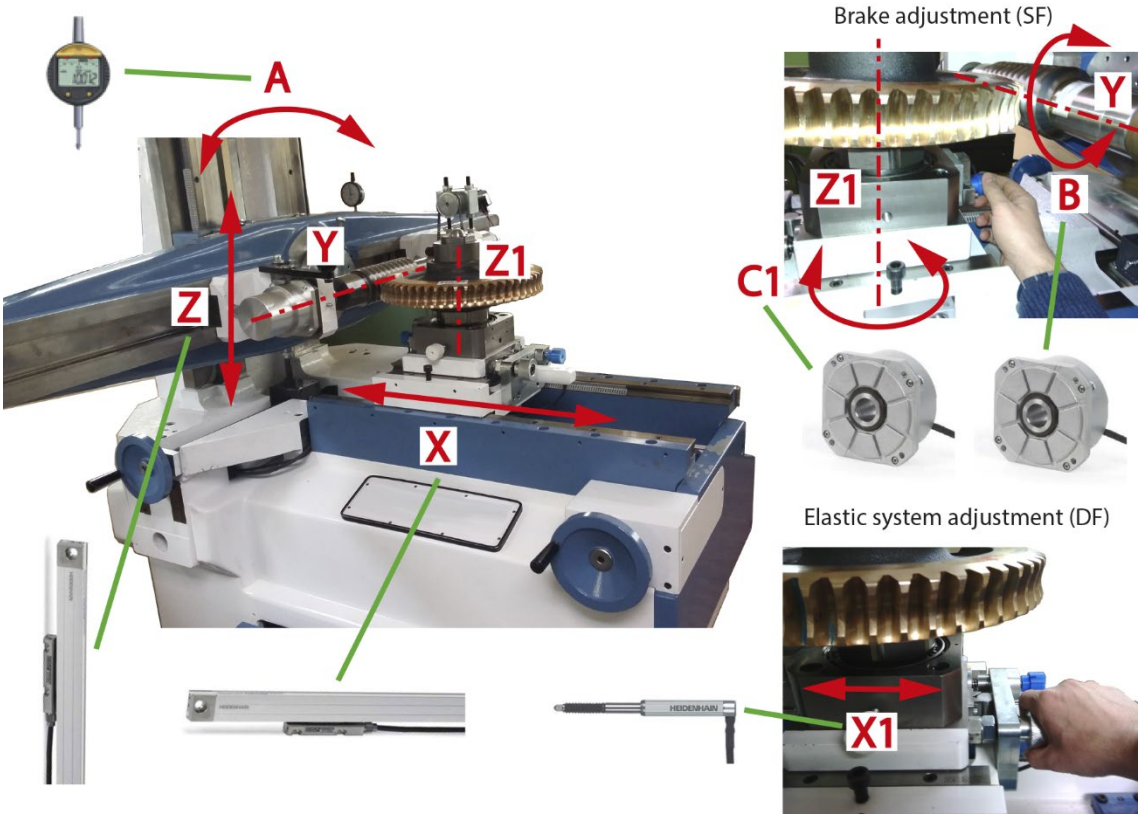

Figure 1. Worm gear single and double flank rolling tester

The master worm is arranged between points to be dragged, thus acting as a driving wheel. The worm gear to be tested is then placed on its corresponding shaft, ensuring that the back-and-forth movement is in the locked position. Afterwards, both gears are placed in their corresponding nominal height (z-axis) and centre distance position (x-axis), being able to execute the single-flank test. For the double-flank test, the worm gear holder carrier must be released after adjusting the

elastic system, which forces both gears to be in contact at the tight mesh centre distance during the whole rotation (x1-axis).

Two linear encoders Heidenhain LF481C with ± 3 µm accuracy were used to control the position of the main carriages (x-axis, y–axis). A length gauge model Heidenhain ST1288 with ± 1 µm accuracy measured the position of the secondary horizontal carriage (x1-axis) and a dial gauge TESA, with ± 1 µm accuracy, was used to calibrate the perpendicularity between the gears' axes. In addition, two angle encoders Heidenhain RON 285C of ± 2.5 arcsec accuracy measured the rotation angles of the master worm and the worm gear in the tests (B-axis, C1-axis) (Figure 1).

2.1 Testing conditions

In the characterization of the equipment, a one-start master worm and a 55-tooth worm gear have been used. The reference diameters of both gears are 59 and 281 mm respectively, with 170 mm being the nominal distance between centres and 45 mm the height between the reference surface of the worm gear and the hobbing plane. In addition, the teeth of the evaluated transmission have a pressure angle of 20°, a normal module of 5.09 mm and a helix angle of 4.949°. This transmission represents an intermediate size of the equipment measuring range, as it is capable of measures worm gear diameters from 100 to 600 mm and modules between 2 and 12 mm. For a 281 mm reference diameter and 5 mm normal module worm gear, Table 1 shows the reference values of single and double-flank rolling parameters according to the accuracy grade and size of the test gear following ISO 1328-1:2013 and ISO 1328-2:1997 [20,21].

Table 1. Radial and tangential composite parameters tolerances [20,21]

Accuracy Grade	F_i' (µm)	F_p' (µm)	f_i' (µm)	F_i'' (µm)	F_r'' (µm)	f_i'' (µm)
5	32	25	7	42	23	15
6	45	36	10	60	32	22
7	64	51	14	84	45	31
8	91	72	19	119	64	44
9	129	101	28	154	91	63

The rotation speed was 25 rpm on the worm gear, always turning it clockwise. For the single-flank tests, a small braking force of 3 Nm was applied on the worm gear holder carrier to avoid the teeth meshing separation due to the rotation inertia. In the double-flank tests, the force exerted by the elastic system was 85 N. In both types of tests, the same initial relative position was maintained between the gears during the execution of the repeatability series, in order to compare measurements made on the identical rolling.

The validity of the tests to characterize the behaviour of the rolling tester goes through guaranteeing the repeatability conditions. In this study, a single operator was responsible for carrying out the tests on the same tester with identical measurement and calibration procedure, in a metrological laboratory with a clean environment and a controlled temperature of 20 ± 0.5 °C. Under these conditions, we considered as negligible the influence of temperature variation in the testing performed.

2.2 Measurement parameters

2.2.1 Single-flank rolling test

In the single-flank rolling test, the test results are represented by a sinusoidal graph following ISO 1328-1:2013 [20]. Its main parameters are the tangential composite deviation (*Fi'*); the tooth-to-tooth tangential composite deviation (*fi'*) and the long-wave component of the tangential composite deviation (*Fp'*)[22]. The other component resulting from the Fourier series, the short-wave component of the tangential composite deviation (*fk'*), has a value very close to *fi'* and is therefore usually considered equal. Although its interpretation is complex, the short-wave component includes accumulated pitch and eccentricity errors, while the high frequency component includes tooth-hobbing errors.

2.2.2 Double-flank rolling test

For the double-flank test, the parameters used are *Fi", fi"* and *Fr"* . In the sinusoidal graph showing the results, *Fi"* represents the total radial composite deviation that is the difference between the maximum and minimum values of the centre distance when the test gear is rotated in tight mesh contact with the master gear through one complete revolution. The tooth-to-tooth radial composite deviation (*fi"*) is the greatest change in the centre distance within an angle of rotation corresponding to the pitch [22]. The difference between the maximum and minimum value of the long-wave component of *Fi"* gives as a result the *Fr"* parameter known as runout in the composite test. Although the results may appear very similar in both tests, their interpretation is not, as they operate under completely different measurement principles. In fact, the double-flank test has a much simplified interpretation and can quickly detect unsteady clamping as well as tooth defects [23].

2.2.3 New parameter (Aa").

In the case of the double-flank rolling test, a new test measurement parameter associated with the mounting tolerance is proposed. It is called the rolling distance discrepancy between flanks *(Aa").* This parameter is defined as the difference between the real (a''_{real}) and the theoretical *(a")* tight mesh centre distance and depends on the backlash between the transmission teeth. Although its analysis is similar to that of tooth thickness measurement by radial composite tests for cylindrical gears in ISO 10064-2:1996 [24], it is not covered or regulated by any standard. Nevertheless, it is being used in industrial environments because of its convenience, since it allows the prediction of gear operating incompatibilities, even when its grinding is of good quality and is validated by double-flank rolling tests [25].

By definition, the double-flank rolling parameters are the result of the difference between a maximum and a minimum value. The smaller the difference, the better the quality of the tested gear unit. However, this is only a partial indication of the future behaviour of the transmission in its final assembly. It is therefore necessary, based on experience [24], to establish the most suitable limits for the tooth backlash of the worm gear, given that the mesh is carried out with a production worm and coupled to a mechanism with pre-determined mounting tolerances.

For a transmission to work free of interference, there must be a certain amount of backlash between the gear teeth. For this purpose, the teeth thickness must be less than the size of the gap between the teeth. The difference measured on the reference diameter is called the circular or tangential backlash (j_t). On the other hand, the normal backlash (j_n) is the shortest distance between the non-active flanks of two gears, when the active flanks are in contact. It is related to the previous one according to equation (2) being j_{nt} the tangential backlash projected on a plane parallel to the normal one, β the helix angle and α_n the normal pressure angle (Figure 2). It is the one considered as manufacturing process reference backlash.

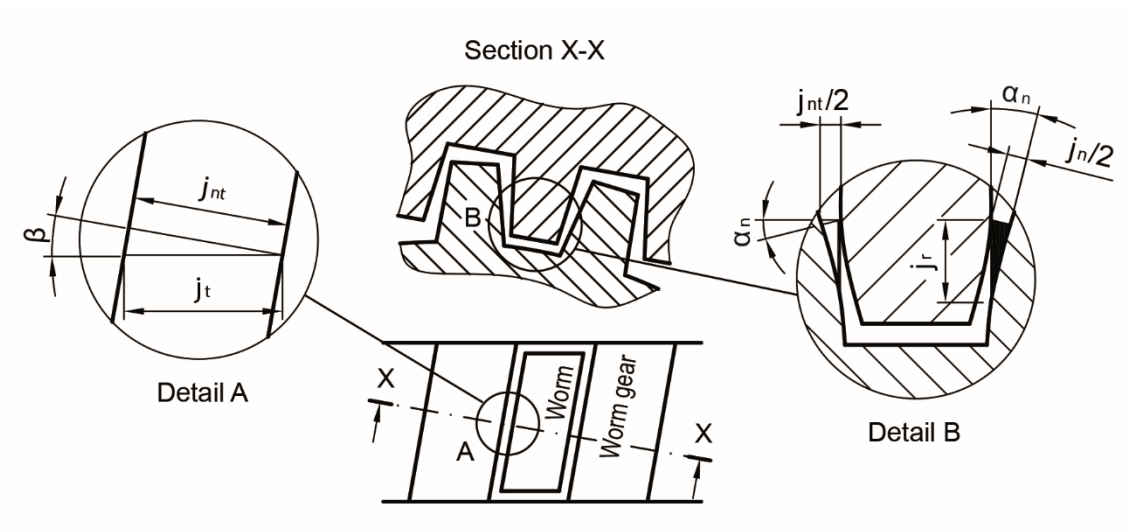

Figure 2. Tangential, normal and radial backlash relation for worm gears

Also, the radial backlash (j_r) is the amount that the nominal centre distance would have to decrease to keep the right and left flanks of a transmission in contact (Figure 2). It can be calculated from the normal backlash (j_n) and the normal pressure angle (α_n) (3).

$$j_n = j_{nt} \cdot cos(\alpha_n) = [j_t \cdot cos(\beta)] \cdot cos(\alpha_n) \tag{2}$$

$$j_r = \frac{j_n}{2 \cdot sin(\alpha_n)} \tag{3}$$

Normal and radial backlash are closely linked to the double-flank rolling tests. It is possible to establish the limits of the rolling values to ensure the correct operation. Greater backlash in the teeth implies that the centres are closer together, whereas less backlash holds them further away from each other.

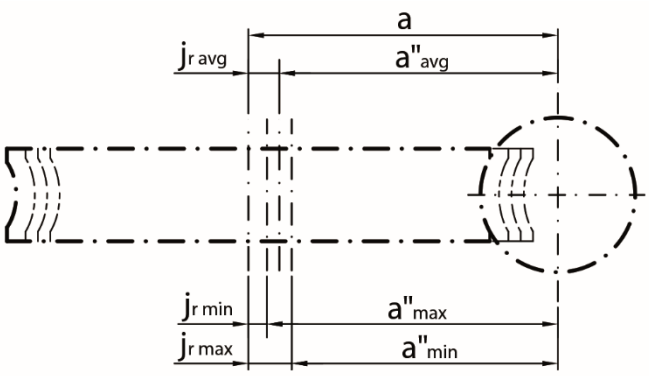

Figure 3. Maximum and minimum radial backlash

From the normal, minimum and maximum backlashes specified in the worm gear design, the values of the radial backlash (equations (4) and (5)) can be constrained and thus limit the maximum and minimum positions of the theoretical tight mesh centre distance. These values are obtained as the difference of the nominal centre distance and the corresponding calculated radial internal backlashes (equations (6) , (7)) and Figure 3. Though there is nothing specified in this regard, it is

advisable to use the average value of the mounting tolerances (a) as the nominal centre distance value.

$$j_{r\,min} = \frac{j_{t\,min} \cdot cos\,(\beta)}{2 \cdot tan(\alpha_n)} \tag{4}$$

$$j_{r\,max} = \frac{j_{t\,max} \cdot cos\,(\beta)}{2 \cdot tan(\alpha_n)} \tag{5}$$

$$a''_{max} = a - j_{r\,min} \tag{6}$$

$$a''_{min} = a - j_{r\,max} \tag{7}$$

where:

$j_{r\,min}$, minimum radial backlash;

$j_{r\,max}$, maximum radial backlash ;

a''_{max}, maximum tight mesh centre distance ;

a''_{min}, minimum tight mesh centre distance;

a, nominal centre distance .

The rolling distance discrepancy between flanks *(Aa")* (8), is calculated as the difference of the real tight mesh centre distance *(a''_{real})* and the average value of the limits of theoretical tight mesh centre distance *(a''_{avg})* (9).

$$Aa'' = a''_{real} - a''_{avg} \tag{8}$$

$$a''_{avg} = \frac{a''_{max} + a''_{min}}{2} \tag{9}$$

In this way, a''_{avg} is considered to be the reference value of a symmetrical interval whose upper limit *(Aa''_{upper})* and lower limit *(Aa''_{lower})* are equal but changed in sign (equations (10) , (11)).

$$Aa''_{upper} = a''_{max} - a''_{avg} \tag{10}$$

$$Aa''_{lower} = a''_{min} - a''_{avg} \tag{11}$$

To assure the validation of the gear quality grade, its rolling parameter *Aa"* must be within these limits (Figure 4). Values below the lower limit imply tighter backlashes, which can lead to interference and even blocking of the transmission. Conversely, exceeding the upper limit reduces the tooth contact, resulting in loss of performance and vibration generation due to excessive tooth backlash.

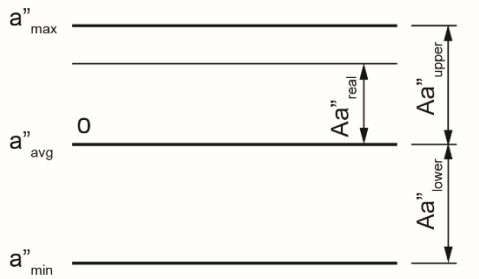

Figure 4. Aa" parameter upper and lower limits

2.3 Methodology and testing

2.3.1 Tester basic repeatability testing

In the evaluation process of the equipment, we carried out series of 10 repetitions for single (SF) and double-flank (DF) rolling tests, placing the gears in the nominal position between centres and maintaining the operating conditions. Although it is clear that small random variations are introduced, it is impossible to apply the two measuring principles simultaneously in a single test, unless several master gears are used at the same time [13,26,27]. The latter would imply other different sources of error. The results of the tests are shown in row 1 of Table 2 (basic repeatability). The data obtained from the testers' basic series show the reliability of the system, with standard deviation values within the reading error of the instruments. In the case of the single-flank test parameters (SF), the angular errors are indicated in micrometers over the reference diameter of the worm gear.

2.3.2 Components repeatability testing

Following the methodology of the basic tests, consecutive series of 10 repetitions have been carried out modifying one or more parameters to study their influence on the behaviour of the equipment. The operating conditions and the initial test point are maintained.

Firstly, the position of the worm gear holder carriage has been modified before the start of each test to evaluate its influence. For this purpose, the gears were separated by at least 100 mm from the centre distance and returned to the nominal position between tests. In the second set of series, the master worm holder vertical carriage has been moved (see Table 2, rows 2 and 3). In between measurements, the worm has been moved 10 mm upwards or downwards, alternatively, returning it to the nominal test height before starting the next measurement.

The following set of tests includes the combined movement of the main movements of the machine, i.e. before starting the corresponding test, the three carriages have been moved: the worm holder carrier, the main worm gear carrier and the secondary carrier. This collects information about what happens in the real test set-up (Table 2, row 4). Subsequently, the repeatability of the system restart has also been checked. These series add to the previous ones the resetting of the measuring instruments that forces to move the elements until the reference marks are recognized. This process is necessary every time the machine is switched on (Table 2, row 5).

The analysis of the results demonstrates the stability and repeatability of the system both individually and combining several effects simultaneously. The calculated error in the basic series within the error of the measuring instruments is kept, and even tends to decrease when several influences are added together, possibly because of the errors compensation.

2.3.3 Gears assembling repeatability testing

In the final phase of the equipment evaluation, the influence of the gears mounting process has been assessed. This requires the manipulation by the operator of both the master worm and the test worm gear, which can lead to uncontrolled sources of error.

Two sets of ten repetitions following the methodology of the basic tests have been carried out before changing the starting conditions. The first set of tests consisted of disassembling and reassembling the master worm, bringing it back to the same starting position between each testing sequence (Table 2, row 6). In the second set, both the assembly of the worm gear and the starting point between test sequences, have been done in a random way since the gear eccentricity can be given in any orientation (Table 2, row 7). Therefore, multiple positions have been established

between the tests to evaluate 360°. Thus, it is necessary to move the worm gear horizontal holder carrier 100 mm between each test sequence to be able to manipulate the gears without risk of damaging them. As the above sections have shown the movement repeatability of the tester components, it can be assumed that these last tests also include the repeatability of the entire system.

Table 2. Repeatability tests results (SF/DF)

Test type	SF/DF	F_i' (µm)	F_p' (µm)	f_i' (µm)	F_i'' (µm)	F_r'' (µm)	f_i'' (µm)	Aa" (µm)
Basic repeatability	Average	86.5	75.9	26.0	33.3	21.7	17.3	-16.1
	Std. dev	1.8	2.2	0.5	0.9	0.8	0.4	0.7
Worm gear carriage displacement	Average	82.7	71.6	27.5	32.9	23.2	14.4	-15.6
	Std. dev	0.1	0.3	0.6	0.2	0.3	0.8	0.7
Master worm carriage displacement	Average	87.0	76.4	26.0	33.1	23.6	13.9	-17.8
	Std. dev	0.5	0.3	0.8	0.3	0.2	0.5	0.6
Three carriages displacement	Average	86.6	76.5	25.6	33.0	23.4	14.4	-17.4
	Std. dev	0.1	0.2	0.3	0.2	0.1	0.6	0.7
System restart	Average	87.0	76.5	25.6	32.3	22.3	14.3	-18.0
	Std. dev	0.2	0.7	0.6	1.1	1.3	0.4	0.8
Master worm assembling	Average	87.4	76.5	25.9	32.1	23.2	13.5	-16.8
	Std. dev	4.6	4.0	1.0	0.3	0.2	0.3	0.8
Worm gear assembling	Average	86.3	74.9	23.6	28.0	18.6	17.3	-16.1
	Std. dev	7.8	8.0	1.3	3.8	3.3	0.5	1.1

Finally, Figure 5 and Figure 6 show graphically the results of Table 2. In particular, Figure 5 corresponds to the results of the repeatability tests of the single-flank rolling parameters (Fi', Fp' and fi'). The x-axis shows the seven test series (basic, component repeatability and gear assembly) while the y-axis shows the measured parameter value in micrometers. For each series, we represent the median, average values and interquartile range box representing the middle 50% of the results data of the ten repetitions. Likewise, the graphs in Figure 6 represent the repeatability tests of the double-flank rolling parameters (Fi", Fr", fi" and Aa"). In this way, we compare graphically the results obtained in each test per rolling parameter, showing the shape, central tendency, and variability of sample distributions.

The results show that when the gears are manipulated, there are variations in the results of the rolling parameters, as well as an increase in the range of uncertainty. The backlash between axis, bush and worm gear required for assembly, as well as the associated eccentricity of these, is the main cause. In the double-flank test, the worm gear could rotate free, meanwhile in the single-flank test it must rotate together with its shaft that moves in accordance with the angle encoder rotor. The standard deviation of the master worm assembling shows a similar error range to the rest of the machine elements in the double-flank rolling parameters, and a slight increase in the *Fi'* and *Fp'* parameters that shows changes in the contact points between the teeth. On the other hand, if the positioning of the worm gear is random, which usually happens in the measuring process, the uncertainty of the parameters *Fp'* and *Fr"*, and therefore also that of the total radial and tangential composite deviations, *Fi'* and *Fi"*, increase considerably due to the variation of the orientation of the clamping eccentricity. This could be seen in the spread increase for these parameters in the columns corresponding to worm gear assembling in Figure 5 and Figure 6. Furthermore, the standard deviation of the tooth to tooth parameters *fi'* and *fi"*, remains within the error of the measuring instruments. However, it can be observed that some of the values of these parameters can be found so close to the filter cut frequency that can move from one component to another, due to the effect of

the change of the rotation axis, modifying the average value. Finally, the standard deviation of the parameter *Aa"* remains within the same order of magnitude in all series, some variations being observed mainly due to the change of the eccentricity orientation.

As regards the standard deviation of the remaining rolling parameters, both *fi'* and *fi"*, which define the hobbing of the teeth, and *Aa"*, which determines their average backlash, remain constant and are only marginally affected. Only the assembling of the master and worm gear affect *fi'* and *Aa"* parameters showing bigger dispersion in their results due to the cut frequency used to separate the short and long wave components.

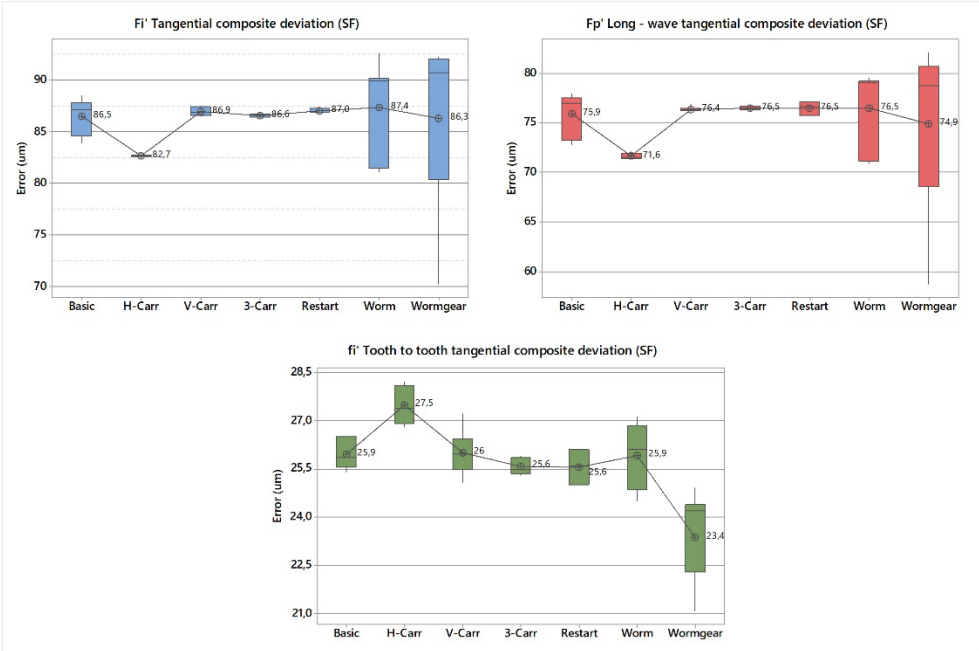

Figure 5. Single-flank repeatability tests results boxplot

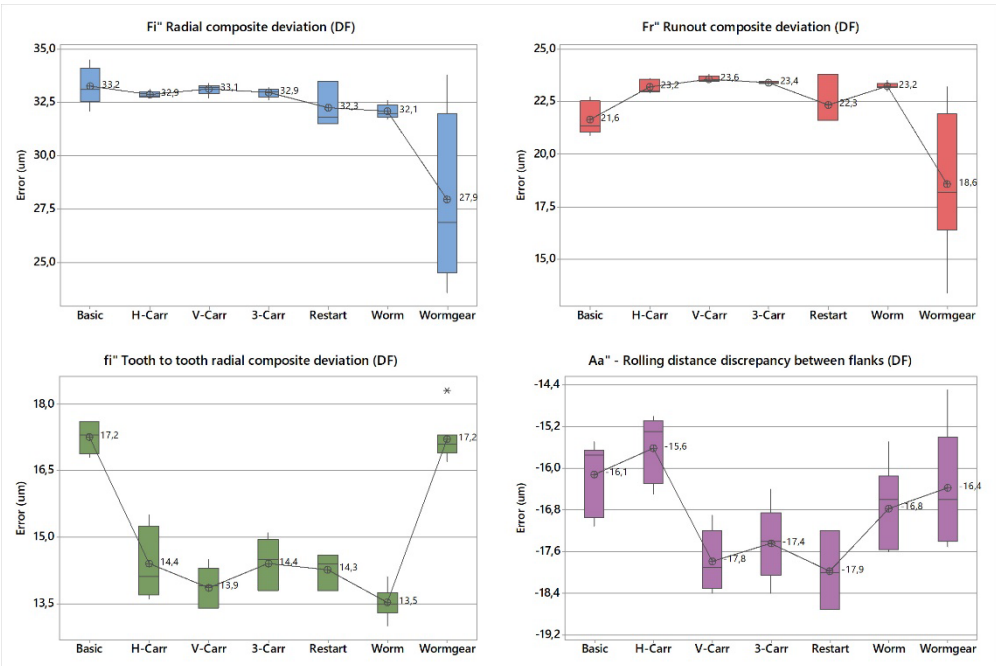

Figure 6. Double-flank repeatability tests results boxplot

3 Sensitivity coefficients estimation in the measurement uncertainty budget.

According to GUM [1], in the calculation of the combined standard uncertainty $u_c(y)$ for uncorrelated input quantities, the partial derivatives describe how the output estimate (y) varies, depending on the variations in the values of the input estimates ($x_1, x_2, \ldots, x_N$). These derivatives are often referred to as sensitivity coefficients (c_i). The combined variance $u_c^2(y)$ is considered as a sum of terms, each representing the estimated variance associated with (y), due to the estimated variance associated with each input estimate (x_i) (equation (12)).

$$u_c^2(y) = \sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) = \sum_{i=1}^{N} [c_i u(x_i)]^2 \equiv \sum_{i=1}^{N} u_i^2(y) \qquad (12)$$

"Global sensitivity analysis (GSA) study how the uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input" as stated in [28]. Local sensitivity analysis (LSA) usually proceed by changing one variable or assumption at a time comparing against a known 'baseline' output [29], [30]. This common method of local sensitivity analysis is usually known as one-at-a-time (OAT) method. In this type of LSA, the value of a single parameter is changed individually maintaining all other parameters at their baseline nominal values. Thus, the changes in the output could be attributed to the perturbing factor.

The sensitivity coefficients are sometimes determined experimentally: one measures the change in (y) produced by a change in a particular (x_i) while maintaining the remaining input quantities constant. In this case, the function (f) is accordingly reduced to an empirical first-order Taylor series expansion based on the measured sensitivity coefficients [1].

From the experimental characterization of the rolling tester, it has been possible to establish the sensitivity coefficients between the different sources of error and the rolling parameters, both in single and double-flank tests. These relations serve to transform the possible variations found in the calibration of the equipment in parameters such as the centre distance, height or perpendicularity between axes, into valid terms of rolling measurement uncertainty. In addition, the experimental characterization also includes the study of the influence of other variables types such as the gear rotation speed, the braking system force of the worm gear carrier shaft and the elastic system force in the secondary carriage.

3.1 Experimental sensitivity coefficients

It seems reasonable to consider that to achieve accurate and precise rolling measurements it is essential to carry out tests in the theoretical nominal position. However, in the results of machine calibration, non-numerically compensated errors appear involving unexpected variations in the nominal centre distance, in the height and in the perpendicularity between the axes. These do not necessarily affect the rolling parameters equally, having a different influence on the uncertainty of measurement [16,17]. For example, a lack of perpendicularity is equivalent to errors in the helix hobbing angle that change the functional tooth thickness. The centre distance and height errors change the contact between the master gear and the test worm gear and mainly affect the tooth to tooth rolling parameters. In this study, the proportion that each source of error contributes to the uncertainty of the rolling parameters was determined experimentally by means of repeatability tests in different positions, heights and orientations.

For this purpose, a methodology to experimentally calculate the sensitivity coefficients was defined. A single-flank test (SF) is performed followed by a double-flank test (DF), without making any changes and always starting from the same position. Five repetitions of this series of tests are performed. The parameters whose influence is not compensated for in the tester calibration (perturbing factors), vary gradually from series to series so that their impact on the uncertainty (model output) is evaluated.

3.1.1 Centre distance influence analysis

First, after an initial series at the nominal centre distance (170 mm), measurements were made by incrementally changing the centre distance. These were carried out in 7 different positions, keeping the position of the master worm fixed and moving the main worm gear holder carriage in 0.050 mm intervals.

Table 3 shows the measurement results corresponding to the variations in the various rolling parameters when changing the centre distance. An asterisk on the value indicates the values that generates greater influence per rolling parameter with comparison to the corresponding reference of the nominal position at 170 mm. These values were used for the calculation of the sensitivity coefficients shown in the last row of the table according to equation (13), which shows the example of the calculation of the sensitivity coefficient of *Fi'* for the variation of the nominal centre distance, named c_{cd} *(Fi')*. We calculate the steepest gradient considering the range of variation of the Fi' parameter due to the centre distance variation range in the testing from the nominal testing position.

$$c_{cd}(F_i') = \frac{\Delta F_i'}{\Delta cd} = \left|\frac{(Fi'_{170.2} - Fi'_{170})}{(170.2 - 170.000)\cdot 1000}\right| = \left|\frac{(98.4 - 85.9)}{(170.200 - 170.000)\cdot 1000}\right| = \frac{12{,}5\ \mu m}{200\ \mu m} = 0.063 \qquad (13)$$

Table 3. Sensitivity coefficients of rolling parameters due to centre distance variation

Centre distance (mm)	Fi' (µm)	Fp' (µm)	fi' (µm)	Fi" (µm)	Fr" (µm)	fi" (µm)	Aa" (µm)
169.900	88.2	77.3	25.1	31.0*	21.5	14.4	-18.3
169.950	87.1	76.9*	25.5	31.8	22.0*	14.8*	-18.6*
170.000	**85.9**	**75.7**	**26.1**	**32.3**	**23.1**	**13.8**	**-18.0**
170.050	85.2	74.9	33.4*	32.5	23.4	13.6	-18.1
170.100	85.4	74.8	39.9	32.6	23.4	13.5	-17.1
170.150	90.9	74.4	47.2	32.6	23.4	13.7	-17.4
170.200	98.4*	74.0	54.3	32.4	23.4	13.5	-17.0
$C_{cd\ max}$ (µm/ µm)	0.063	0.024	0.147	0.013	0.022	0.019	0.013

This can best be seen graphically in Figure 7 where regardless of the apparent trend lines generated, whether linear or polynomial, the sensitivity coefficients have been calculated from the maximum absolute slopes from the nominal value to the average values of each test series. In this way, the most unfavourable options are considered which include all possible values and therefore integrate the highest admissible uncertainty.

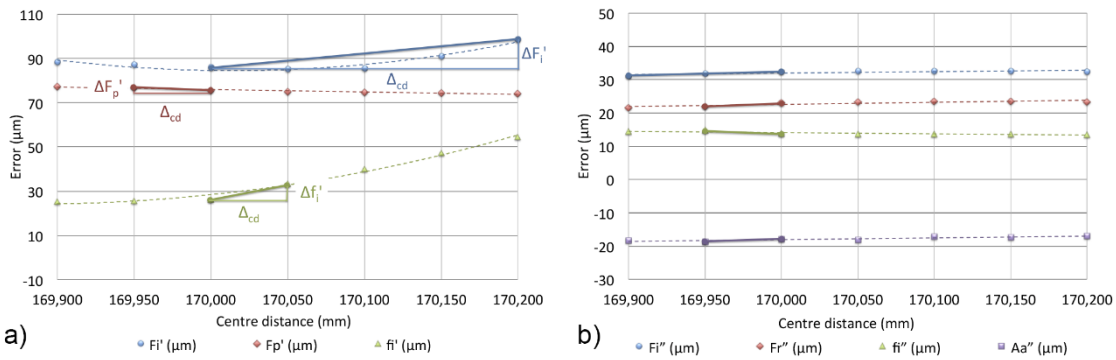

Figure 7. Single-flank (7.a) and double-flank (7.b) rolling parameters change due to gears centre distance variation

In general, it is observed that the influence of the variation of the centre distance is relatively small. It could be noted that the tooth-to-tooth transmission error *(fi')* tends to grow as the distance between centres increases and so does the total error *(Fi')*. On the other hand, there are hardly any appreciable variations in the double-flank rolling parameters. However, the final effect on the estimation of uncertainty will depend on the amount of residual error in the calibration process.

3.1.2 Height influence analysis

Sensitivity coefficients due to variations in the relative position in height between the worm gear hobbing plane and the axis of the master worm have been established experimentally. This time, seven measurement series have been carried out leaving the position of the worm gear fixed and moving the master worm carrier vertically in 0.050 mm intervals from the nominal height value of 45 mm.

Table 4 shows the average values obtained for each rolling parameter in each series carried out at different heights. The values with an asterisk represent the points with the greatest slope from the nominal position of 45 mm and allow the corresponding calculation of the sensitivity coefficients following the procedure in the equation (13).

Table 4. Sensitivity coefficients of rolling parameters due to height variation

Height (mm)	Fi' (µm)	Fp' (µm)	fi' (µm)	Fi" (µm)	Fr" (µm)	fi" (µm)	Aa" (µm)
44.850	92.0	78.0	25.5	37.9*	25.0*	12.3	-7.2*
44.900	90.0	75.7	25.0	35.1	24.0	13.1	-12.2
44.950	84.7	73.5	25.3	33.2	23.3	13.7	-15.9
45.000	**82.3**	**72.4**	**24.9**	**31.9**	**23.1**	**14.0**	**-17.2**
45.050	87.1*	75.3*	37.8*	31.0	22.4	13.9	-16.8
45.100	91.2	67.2	45.2	30.0	22.3	17.2*	-15.7
45.150	95.0	65.0	48.2	30.9	21.9	15.7	-11.9
$C_{h\ max}$ (µm/ µm)	0.096	0.056	0.258	0.040	0.013	0.032	0.067

The modules of the maximum possible slopes of each of the rolling parameters are considered as the most unfavourable situation to include the maximum theoretical uncertainty as it is graphically shown in Figure 8.

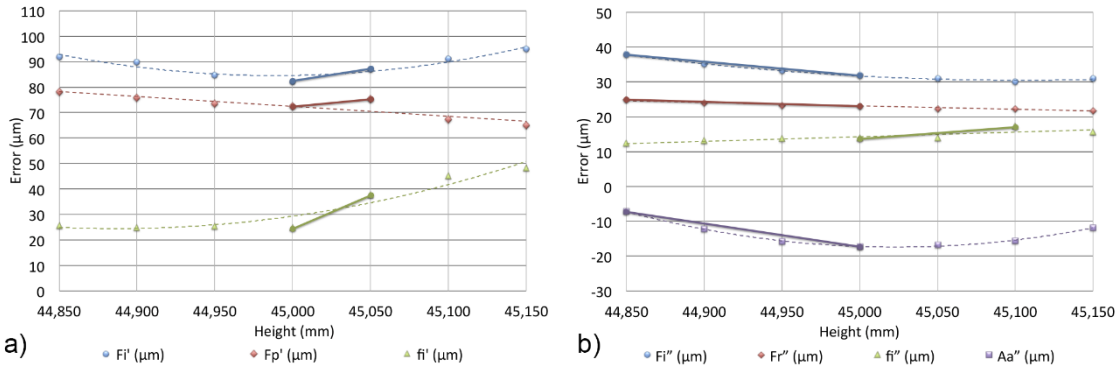

Figure 8. Single-flank (8.a) and double-flank (8.b) rolling parameters change due to gears centre distance variation

When varying the height between the gears, there is an increasing trend in the tangential composite deviation *(Fi')*, in either direction, because of a progressive decrease in the low frequency error *(Fp')* and a gradual increase in the high frequency error *(fi')* with rising height. On the other hand, a general decrease in radial composite deviation *(Fi")* is observed, derived from the decrease of the low frequency error and the growth of the tooth-to-tooth error *(fi").* The minimum value of parameter *(Aa")* is in the nominal position, increased when leaving the position. Nevertheless, although the sensitivity coefficients due to the effect of height variation are larger than the previous ones, around double, it is not possible to determine beforehand their impact on the uncertainty estimation because they affect several terms.

3.1.3 Lack of perpendicularity between axes influence analysis

To calculate the sensitivity coefficients due to the variation of the angle between the axes, measurements have been made every 50 µm with a dial indicator, which means variations of approximately 37" in the inclination of the axis. Considering the horizontal as the reference position (0° 0' 0"), values have been obtained from 15 series within a range of -258" to +258", taking as positive the counter-clockwise rotation, looking at the crosspiece from the front. Between series, the worm gear has remained in its nominal position maintaining also the other test conditions.

As in the other influences studied above, Table 5 gives the average values of the measurements for each series, and indicates with an asterisk the values used to calculate the sensitivity coefficients for the different rolling parameters according to equation (13).

Table 5. Sensitivity coefficients of rolling parameters due to angle between axes variation

Angle (arcsec)	Fi' (µm)	Fp' (µm)	fi' (µm)	Fi" (µm)	Fr" (µm)	fi" (µm)	Aa" (µm)
-258	89.5	77.0	37.4	35.6	25.0	16.2	-14.9
-221	89.0	77.5	36.7	35.1	24.8	15.3	-16.5
-184	82.9	71.6	35.3	35.7	25.5	16.3	-16.6
-147	82.6	72.1	33.8	34.3	24.8	15.3	-18.0
-111	84.0	73.3	34.5	33.9	24.5	15.8	-16.6
-74	85.2	75.0	30.8	33.2	24.1	16.3	-15.7
-37	85.0	75.1	27.6	32.8	23.9	15.8	-16.0
0	**86.0**	**75.8**	**28.2**	**32.0**	**23.5**	**15.2**	**-17.3**
37	86.0	75.8	24.6*	30.5*	22.1*	14.4*	-14.6*
74	86.8	76.3	24.0	29.6	21.8	14.1	-13.0
111	91.2*	79.8*	23.6	30.0	22.2	14.1	-10.5
147	92.3	80.5	24.4	29.3	21.5	13.7	-9.8

184	93.9	81.0	25.7	29.1	20.6	14.1	-10.4
221	92.6	78.5	26.0	28.6	19.4	13.4	-8.9
258	95.6	80.7	25.4	28.4	19.6	13.2	-7.3
$C_{p\ max}$ (µm/ arcsec)	0.046	0.036	0.097	0.039	0.037	0.021	0.074

The general trend of each rolling parameter can be seen in Figure 9, where the steepest possible lines, in absolute terms, are also represented with regard to the reference position.

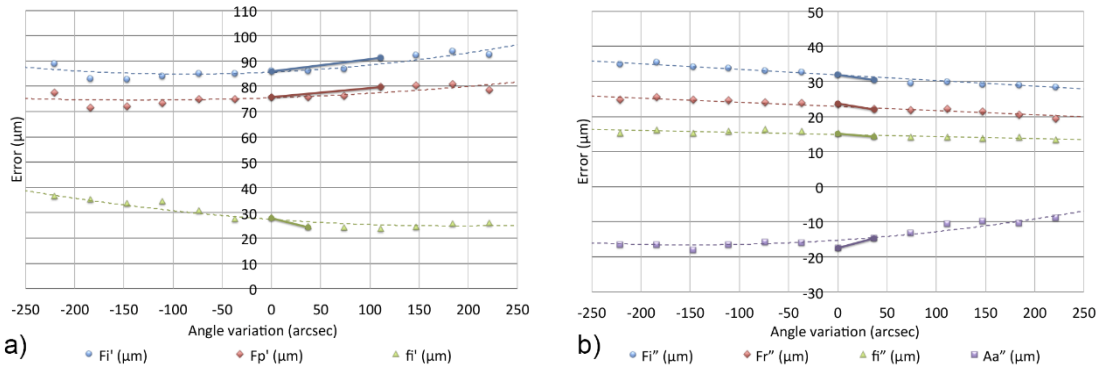

Figure 9. Single-flank (9.a) and double-flank (9.b) rolling parameters change due to perpendicularity variation

The sensitivity coefficients of the variation of the angle between the axes are in the same order of magnitude as those of the other influences. An increasing trend is observed, both in the tangential composite deviation *(Fi')* and in the short-wave component *(Fp'),* as the axes move away from perpendicularity. On the contrary, the long-wave component of the tangential composite error *(fi'),* despite his parabolic behaviour, tends to decrease with the positive angles. With a linear tendency, *(Fi"), (Fr")* and *(fi")* also tend to decrease with positive angles. In contrast, the values of *(Aa")* increase with positive angles between the axes. It should be noted that, in this case, positive angles are considered those that make the worm gear rotate counter clockwise seen from the worm gear, which implies an increase in the angle of its helix as it is a right-hand gear.

3.1.4 Other variables influence

In the rolling tests, some variables, although they do not modify the geometry of the gears, act on the dynamics and the test conditions. In this study, the influence of the rotation speed, the braking system force and the behaviour of the elastic system of the double-flank test are analysed. The rotation speed of the master gear and the braking force of the worm gear carrier shaft are generally specified in VDI/VDE 2608 [22]. In practice, test speeds below 30 rpm and braking torques of 1 to 5 Nm are recommended (for single-flank tests), but nothing is specified about gear types and sizes. Therefore, we carried out tests to analyse any kind of influence of these variables even within the established values.

Following the procedure of the previous sections, five series of measurements (SF/DF) have been made considering nominal position for the gears at different speeds. The average values, calculated from five repetitions per series, as well as the standard deviation of the resulting rolling parameters are shown in Table 6.

Table 6. Rotation speed influence on rolling parameters

Rotation speed (rpm)	Fi' (µm)	Fp' (µm)	fi' (µm)	Fi" (µm)	Fr" (µm)	fi" (µm)	Aa" (µm)

5	87.7	77.2	26.0	32.2	23.6	12.3	-18.5
10	88.8	78.7	25.0	32.3	23.3	13.8	-18.2
15	89.0	78.5	25.3	32.2	23.8	13.6	-17.8
20	88.9	78.2	24.9	32.6	23.8	14.7	-17.5
25	88.5	77.9	25.4	31.4	22.4	15.5	-18.1
Std. deviation	0.5	0.6	0.4	0.4	0.6	1.2	0.4

Figure 10 shows that, except for the tooth to tooth radial composite deviation *(fi")* which seems to grow slightly as the test speed increases, there are no appreciable variations in the results below the speed recommended by the standards.

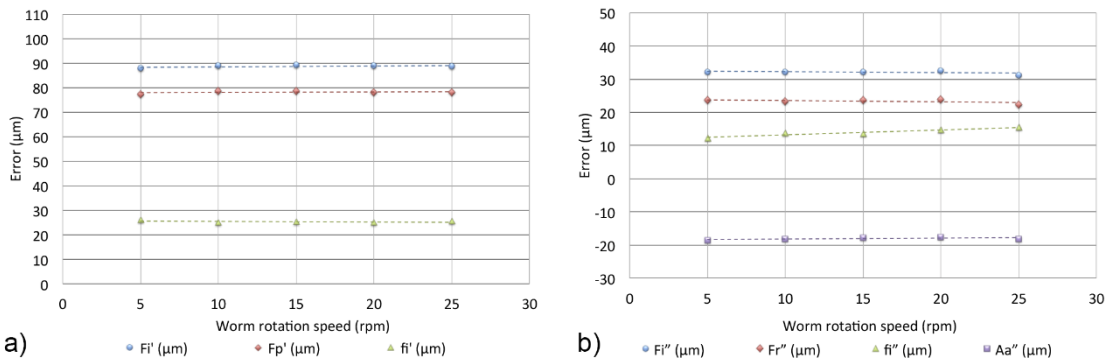

Figure 10. Rotation speed influence on rolling parameters single-flank (10.a) and double-flank (10.b)

Likewise, tests have been carried out modifying the braking torque of the worm gear shaft carrier, which is the one that prevents the loss of contact between the teeth due to the inertia effect in the single-flank test. Table 7 and Figure 11a show the results. No significant differences are observed when testing at different forces, within the range established by the standards.

Table 7. Braking force influence on single-flank rolling parameters

Braking torque (Nm)	Fi' (µm)	Fp' (µm)	fi' (µm)
1	87.5	73.7	24.7
2	91.0	78.4	24.3
3	87.2	74.8	25.7
4	90.3	78.7	25.9
Std. deviation	1.9	2.5	0.8

Regarding the behaviour of the elastic system in the double-flank test, neither VDI/VDE 2608 [22] nor any ISO standard specifies the force to be applied in the tests. They only indicate that full tooth contact must be ensured throughout the measurement without deforming the contact. AGMA 915-2-A05 [31] is the only one that includes recommended values, in a very general way, depending on the gear size. To check whether there is any relationship between the applied force and the rolling parameters, we made 12 series of tests varying the tension of the secondary carriage elastic system.

The results shown in Table 8 and Figure 11b, reveal a significant variation of parameter *Aa"*. The higher the applied force, the lower the parameter, which means that the centre distance tends to decrease. On the other hand, the rest of the parameters *(Fi", Fr"* and *fi")* do not suffer changes due to the effect of the force of the elastic system. This suggests that there might be a small local deformation in all the contacts equally or that there is a sort of mechanical hysteresis in some of the elements.

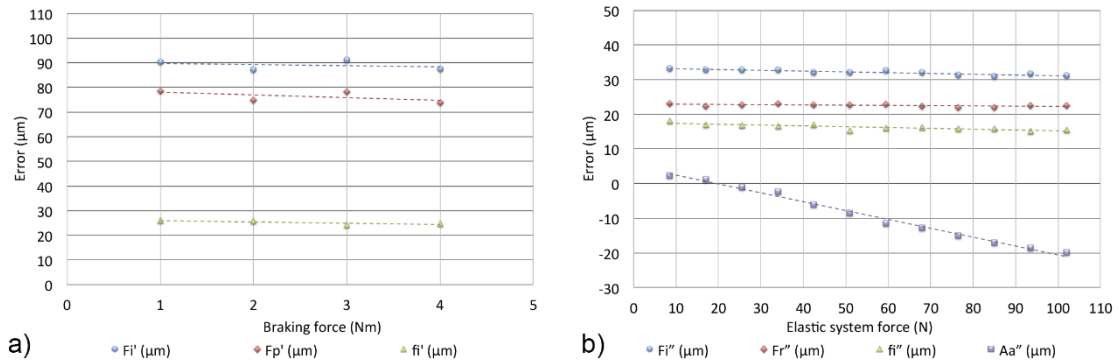

Figure 11. Braking force and elastic system force influence on rolling parameters for single-flank (11a) and double-flank tests (11b).

Table 8. Elastic system force influence on double-flank rolling parameters

Force (N)	Fi" (µm)	Fr" (µm)	fi" (µm)	Aa" (µm)
9	33.3	23.2	18.1	2.4
17	32.8	22.3	17.0	1.2
26	32.8	22.8	16.8	-1.1
34	32.8	23.1	16.6	-2.4
43	32.1	22.7	17.1	-6.2
51	32.0	22.8	15.3	-8.5
60	32.5	22.9	16.1	-11.5
68	32.1	22.4	16.2	-12.8
77	31.3	21.9	15.9	-15.0
85	30.9	22.1	15.8	-17.1
94	31.7	22.6	15.2	-18.6
102	31.0	22.5	15.4	-19.9
Std. deviation	0.8	0.4	0.9	7.9

4 Conclusions

In this work, as a previous step to perform an uncertainty budget estimation of single and double-flank rolling tests for worm gears, the behaviour of the tester and its components has been experimentally characterized. Since it is not possible to make a partial estimate of how the sources of error affect independently, the values obtained in the tester characterization need to be evaluated together with the errors of the measuring instruments and the calibration data to be able to estimate the influence of the variables analysed on an individual basis. In addition, the fact of carrying out single and double-flank tests on the same kinematic structure (same equipment) allows to obtain equivalent data for the uncertainty budget because of maintaining the same test conditions and influencing error sources.

The methodology is based on multiple consecutive repetitions, first making variations component by component and then combining effects among them. The high repeatability of the individual and combined test series reflects the mechanical reliability of the tester used in both types of rolling test. The error range obtained coincides with the error range of the measuring instruments, and even tends to be lower when several influences are added together due to errors compensation.

However, the single-flank rolling parameters error is slightly higher than the double-flank ones because it is more sensitive to error sources. Likewise, the backlash between the shaft and the worm gear is what generates the greatest variation in the rolling results, due to the effect of the assembling eccentricity.

Besides the usual rolling parameters, this study also includes in its results a new proposed parameter obtained in the double-flank test that is closely related to mounting tolerances. This is the discrepancy of the rolling distance between flanks *(Aa")* which limits the maximum and minimum values of the rolling for a given transmission backlash. It enables to predict operating incompatibilities in gear pairs, even if the cuttings are of good quality. Despite being used in some sectors of the gear industry, it does not appear as rolling parameter in the standards.

In addition, the sensitivity coefficients of the rolling parameters to different error sources have been established on an experimental basis. Involuntary variation in the centre distance, height or angle between the gear axes does not affect the rolling parameters equally, but each of them has a different influence on the measurement uncertainty. To this end, repeated tests were carried out to modify these variables in a controlled way, determining their influence by means of sensitivity analysis. In general, the effect of any error on most of the rolling parameters is below 10%, i.e. for each micrometer of error it would mean a maximum of 0.1 μm variation. However, the contribution on the tooth to tooth tangential deviation *(fi'),* which represents the pitch and pressure angle error, rises to 15% for the variation in centre distance and up to 25% for the variation in height. However, the weight of each uncertainty term on the rolling parameters will be evaluated in the measurement uncertainty budget. Finally, a similar methodology has been used to check that the dynamic effect does not affect the results as long as the tests are executed within the values of speed, braking force and elastic system force recommended by the technical standards.

Further studies could be addressed to other gear geometries, dimensions and transmission types, broadening the knowledge about rolling test measurement and benefiting gear industry, main user of this type of equipment.

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