Probabilistic siting and sizing of energy storage systems in distribution power systems based on the islanding feature

C. P. Delgado-Antillón*, J. A. Domínguez-Navarro

Department of Electrical Engineering, University of Zaragoza, Zaragoza, Spain

***Abstract*—Distributed storage systems embedded in distribution power systems could complement renewable generation and improve their operation, reducing peak power levels and providing supply support to island zones in the cases of outages. This paper proposes a multi-objective optimisation, which takes advantage of the possibility of operating in an island mode. The optimisation considers the siting and sizing of storage systems placed on power distribution systems with radial topology. The objectives to be minimised are: the amount of energy storage, power losses and expected energy not supplied (EENS). Loads and generators are uncertainty variables, and a probabilistic power flow based on the point estimate method helps with assessing the electrical parameters.**

The optimisation uses the IEEE-34 and IEEE-123 test feeders. The Monte Carlo simulation benchmarks some results. The final results show that storage systems could reduce the peaks of power required from the central network and improve other electrical parameters. In addition, those systems with higher interruption probabilities or those working near their operation limits could benefit from storage systems.

This point of view is different from other authors'. Here, storage energy systems and the island mode of operation are defined by a probabilistic point of view. The

siting and sizing of storage energy systems is decided from data obtained on the probabilistic power flow and the universal generating function.

***Keywords*—islanding, multi-objective optimisation, point estimate method, power losses, probabilistic power flow, reliability, uncertainty.**

ACRONYM

BSU	Balance Storage Use
CDF	Cumulative Distribution Function
EENS	Expected Energy Not Supplied
FOR	Forced Outage Rated
MPSI	Maximum Power at the System Input
MSS	Multi-State System
NSGA	Non-dominated Sorting Genetic Algorithm
PDF	Probability Density Function
PEM	Point Estimate Method
PPF	Probabilistic Power Flow
PVSI	Power Variation at System Input
SMC	Sequential Monte Carlo
TLL	Thermal Limits of the Lines
UGF	Universal Generating Function
VLB	Voltage Limit on Buses
VUB	Voltage Unbalanced on Buses

*Corresponding author, tel.: (34) 976 762401. Email addresses: carmend@hotmail.com (C. Delgado), jadona@unizar.es (J.A. Domínguez-Navarro).

1. INTRODUCTION

Distributed generation is growing more common with power distribution systems, especially those involving renewable sources, due to its benefits related to clean energy features and the possibility of energy independence for countries or customers who use it. Renewable generators allow distribution systems to reduce the power requirements placed on the central power system, thus decreasing power losses in transmission and distribution networks. Consequently, researchers developed methods for assessing the reliability of power distribution networks involving renewable energy sources [1, 3].

Reliability could be improved through the operation of these generators when some failure in the network occurs. One method that several authors [2, 4, 7] have suggested is creating zones with generators, also called microgrids, that provide the necessary energy for the operation of some or all loads in them. These microgrids can be temporarily isolated from the network while continuing to feed the loads. However, due to the uncertainty of power generated by renewable generators, the introduction of storage systems [5,6] in distribution networks is important for supporting renewable generators and island zones, making networks more reliable and reducing EENS. Storage systems could furthermore improve other electrical parameters placed on distribution networks; for example, they could help with peak shaving and voltage variability.

Some authors who manage the possibility of operating networks with an intentional isolated mode evaluate the reliability with different methods. The generation and load are modelled as a Markov process [2], and the authors calculate the reliability indices of the distribution network operated in an island mode. Other authors use Sequential Monte Carlo (SMC), in [3], to evaluate the reliability of a distribution generation system with the possibility of intentional isolation, or of a power distribution system with renewable distributed generation, energy storage and intentional islanding possibility [4-7]. Reference [8] tested the adequacy of a distribution system with solar and wind generation, considering two operation modes, isolated and connected to the network, and two methodologies, SMC and analytical.

Only a few authors treat the problem of the siting and sizing of the storage systems in the network with the island mode. The optimal mix and placement of storage systems for distribution networks focussing on optimising reliability cost, load priorities, and intentional isolation zones is described in [9], but authors use a deterministic power flow model, neglecting power losses, and consider an outage of a specific length.

Other authors optimise the siting and/or sizing of the storage systems in the classical distribution network to improve the reliability or the operation of the network, but they do not consider the isolated mode of operation. Some [10] apply the optimal allocation of batteries to store the surplus power that wind generators produce, using chronological series for wind velocities and load curve. Others [11] minimise the storage power and energy amount, the deviation of the voltage limit and the losses of rewards due to supplementary services and market exchange. In [13], the authors reused the optimisation model developed in [11] but added a voltage regulation parameter, power peak reduction and annual cost as objectives.

This paper develops a probabilistic model to optimise the siting and sizing of storage systems in distribution networks with renewable generators and islanding operation. The probabilistic model uses the point estimate method to evaluate the power flow, as well as the universal generating function to evaluate the EENS.

The rest of the paper is organised as follows. Section 2 describes the formulation of the problem. Section 3 introduces methodologies and tools for the estimation of objectives and indicators, the evaluation of the reliability index considering islanding, and the optimisation process. Section 4 presents the study cases and results. Finally, Section 5 presents the conclusions.

2. PROBLEM FORMULATION

2.1. Description

This model is based on two hypotheses: the elements of the network can be represented by their probabilistic distribution functions, or PDFs, and the network can operate in the island mode. In the following, the consequences of these hypotheses are explained.

Loads and generators are modelled with their respective PDFs, and therefore, a probabilistic power flow (PPF) is necessary. Due to the radial operation of the distribution networks, a Blackward-Forward Sweep method is used. In it, we have in each node *k* that the injected power to node *k*, $P_{in,k}$, is equal to the generated power,

$P_{gen,k}$, the demanded power, $P_{load,k}$, and the summation of the power that leaves node k to others m, $P_{k,m}$.

$$P_{in,k} = -P_{gen,k} + P_{load,k} + \sum_{m=1}^{nk} P_{k,m} \qquad (1),$$

Where $P_{gen,k}$, $P_{load,k}$, $P_{k,m}$ and $P_{in,k}$ are PDFs, and where nk are the nodes connected to node k.

If we introduce a storage system, $P_{sto,k}$ is the PDF of the power injected or consumed by the storage system, and eq. (1) is converted to eq. (2):

$$P_{in,k} = -P_{gen,k} + P_{load,k} + \sum_{m=1}^{nk} P_{k,m} + P_{sto,k} \qquad (2).$$

The strategy of the operation of a storage system determines its influence on the uncertainty of the node. If we want to reduce this uncertainty, then the storage connects in the node have to charge when the power is less than power LIMdown,k, and they have to discharge when the power is greater than power LIMup,k as shown in Figure 1. This working mode is called ‘peak saving’:

LIMdow,k = Pin,k(pdown) and LIMup,k = Pin,k(pup) (3),

Where pdow and pup are the percentiles defined by the operator of the network.

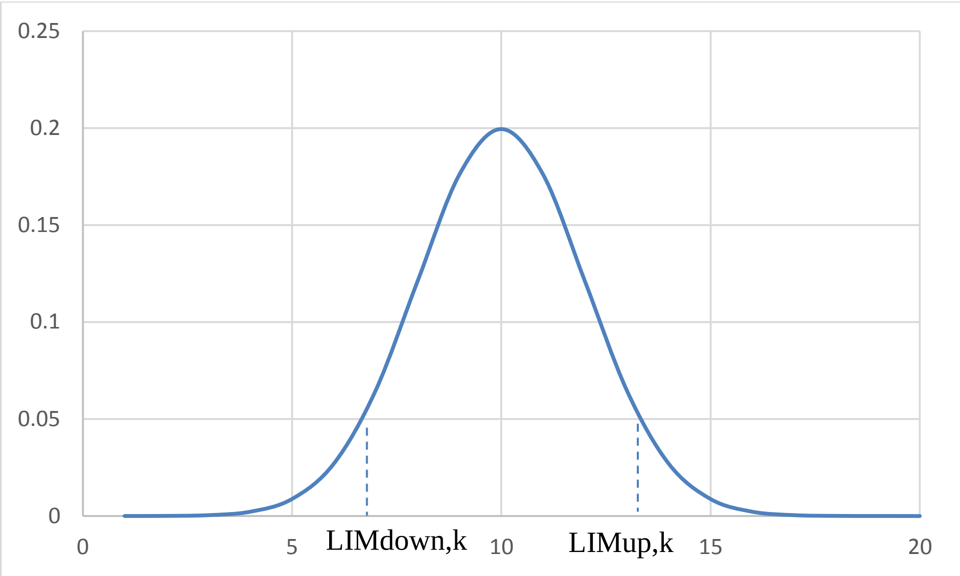
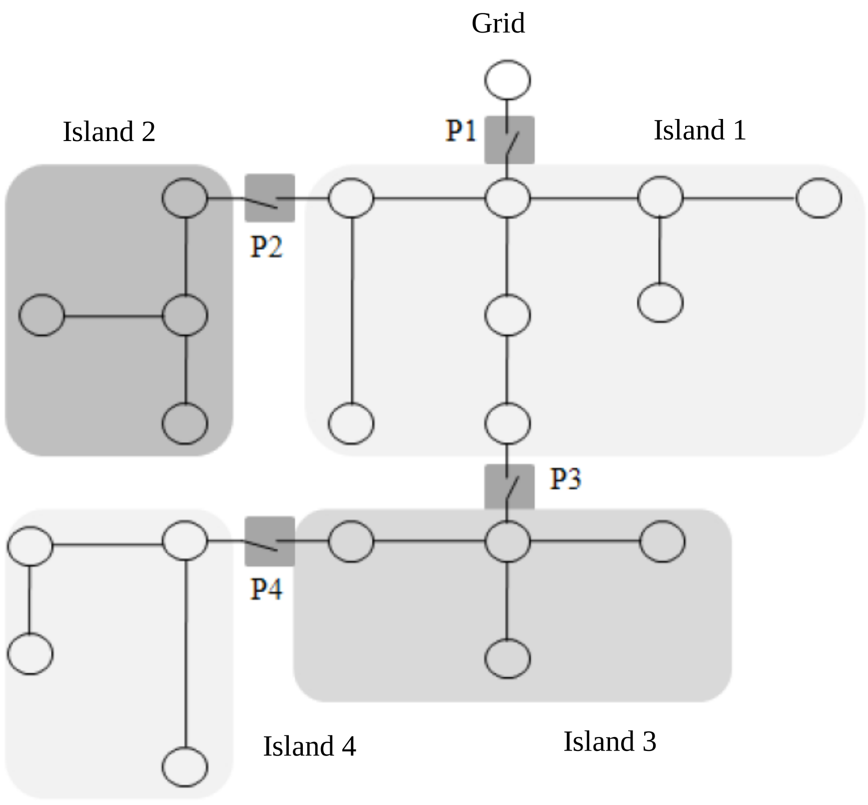


Figure 1. Operational strategy of storage system in ‘peak saving’ mode.

As a consequence of hypothesis 2 (the network can operate in the island mode), the distribution grid has several switches that divide the grid into zones. Depending on the positions of these switches (connect or disconnect), some zones are islanded or tighter with others, and this provides a number of possible island combinations created inside the distribution network that have different probabilities of being isolated from the main grid. The four switches integrated in the network of Figure 2 permit 16 different states of the network that have their own probabilities.



(a)

state	P1	P2	P3	P4	state	P1	P2	P3	P4
1	0	0	0	0	9	1	0	0	0
2	0	0	0	1	10	1	0	0	1
3	0	0	1	0	11	1	0	1	0
4	0	0	1	1	12	1	0	1	1
5	0	1	0	0	13	1	1	0	0
6	0	1	0	1	14	1	1	0	1
7	0	1	1	0	15	1	1	1	0
8	0	1	1	1	16	1	1	1	1

(b)

Figure 2. a) Distribution grid divided into island and b) grid states.

2.2. Optimisation model

This work involves developing a multi-objective probabilistic optimisation for the siting and sizing of storage energy systems in distribution electrical networks with distributed renewable generators, considering support for the island operation mode.

We prefer to conduct a multiobjective analysis and to compare technical variables, as a comparison of costs could hide the technical conclusions. The cost of the storage devices depends on the chosen technology, but the objective of the installation cost is proportional to the capacity of the storage devices. The cost of the EENS is not the same for all of the consumptions. If we introduce the cost of the storage, we also need to introduce the cost of the losses and the EENS to compare different solutions.

A probabilistic three-phase flow power [16] is used to calculate the power in the lines, and the reliability of the network with islands is calculated with the universal generating function method.

2.2.1. Objective Functions

Optimisation is based on three independent objectives to be minimised: installed storage energy size (*fstorage*), EENS (*feens*) and energy losses (*flosses*). These are evaluated through the following functions:

1) $Min \quad fstorage = \sum_{i=1}^{Nb} InstalledStorage_{bus(i)}$ (4),

2) $Min \quad feens = \sum_{s=1}^{Ns} EENS_s$ (5) and

3) $Min \quad flosses = \sum_{i=1}^{Nelem} LossesElem_i^{0.95}$ (6),

Where *Nb* is the number of buses at the network; *Ns* is the number of possible states created for the combinations of the island zones inside the distribution system when a failure occurs in the network; *Nelem* is the number of elements in the network; $InstalledStorage_{bus(i)}$ is the storage capacity installed at bus *i*; $EENS_s$ is the energy expected to be nonsupplied in state *s*; and $LossesElem_i^{0.95}$ are the losses of element *i* at percentile 0.95 because the energy loss calculated is a PDF.

2.2.2. Constraints

As power flows in the lines are modelled with PDFs, voltages and losses are also modelled with PDFs, and it is necessary to define and use several indexes that measure the grades of satisfaction of these constraints.

The constraints of the problem guarantee the operation of the system among the limits of normal operation (voltages on buses and power flows in lines at certain limits and the maximum power that supplies the substation is limited by the power of their transformers). As the system is a three-phase system, the voltage unbalance is also limited. In addition, we introduce two new constraints: the limitation of the power variation at the beginning of the network to determine the width of the peak shaving action, and the balanced use of storage devices to avoid having storage devices most of the time charged or discharged. We have as follows:

a) Power flow equations:

$$I_{in,k} = -I_{gen,k} + I_{load,k} + \sum_{m=1}^{nn} I_{k,m} + I_{sto,k} \qquad (7),$$

$$V_k = V_i + Z_{i,k} I_{i,k} \qquad (8),$$

b) Voltage limit on buses*:* $VLB_i \leq V_{lim}$ (9),

c) Thermal limits of the lines*:* $TLL_i \leq I_{lim}$ (10),

d) Voltage unbalance on buses: $VUB_i \leq V_{dlim}$ (11),

e) Maximum power at the system input: $MPSI_i \leq Ts_{lim}$ (12),

f) Power variation at the system input: $PVSI_i \leq Tf_{lim}$ (13) and

g) Balanced storage use: $BSU_i \leq Se_{lim}$ (14), Where:

$I_{in,k}$ is equal to the injected current in node *k*; $I_{gen,k}$ is the generated current in node *k*; $I_{load,k}$ is the load current in node *k*; $I_{sto,k}$ is the storage current in node *k*; and $I_{k,m}$ is the current that leaves node *k* to other node *m*.

V_k, and V_i are the voltages in nodes *k* and *I*; $Z_{i,k}$ is the impedance of the line that links node *i* with node *k*; and $I_{i,k}$ is the current that flows from node *i* to node *k*.

Voltage limit on buses (VLB): This indicator averages the times that voltages of buses are over or under the established limits. *Nf* is the number of phases, and *Nb* is the number of buses. The voltage is calculated for $V_{k,i} = \pm 3\sigma$, $V_{inf} = 0.93\, p.u.$ and $V_{sup} = 1.07\, p.u.$

$$VLB = \frac{\left(\begin{array}{c}\left[\sum_{k=1}^{Nf}\sum_{i=1}^{Nb}\begin{cases}1 & if\ V_{k,i} > V_{sup}\\ 0 & if\ V_{k,i} \leq V_{sup}\end{cases}\right] + \\ \left[\sum_{k=1}^{Nf}\sum_{i=1}^{Nb}\begin{cases}1 & if\ V_{k,i} < V_{inf}\\ 0 & if\ V_{k,i} \geq V_{inf}\end{cases}\right]\end{array}\right)}{2 * Nb * Nf} \quad (15)$$

Voltage unbalanced on buses (VUB): This indicator averages the difference in voltage between phases of buses. It is a simple indicator and is not based on direct or indirect sequences. *Nb* is the number of buses, and $\mu_{A,i}$, $\mu_{B,i}$ and $\mu_{C,i}$ are the averages of the voltage at phases *A*, *B* or *C*, respectively, for bus *i*:

$$VUB = \frac{\sum_{i=1}^{Nb}\sqrt{\left(\mu_{A,i} - \mu_{B,i}\right)^2 + \left(\mu_{B,i} - \mu_{C,i}\right)^2 + \left(\mu_{C,i} - \mu_{A,i}\right)^2}}{Nb} \quad (16).$$

Thermal limits of the lines (TLL): This indicator averages the times that the current on the lines is over the maximum limits for current intensity, in either the conventional sense or the reverse sense of flow. *Nf* is the number of phases, *Nl* is the number of lines and *Imax* is the maximum current. The current is calculated for $I_{k,i} = \pm 3\sigma$:

$$TLL = \frac{\left(\left[\sum_{k=1}^{Nf}\sum_{i=1}^{Nl}\begin{cases}1 & if\ I_{k,i} > I_{max_{k,i}}\\ 0 & if\ I_{k,i} \leq I_{max_{k,i}}\end{cases}\right] + \left[\sum_{k=1}^{Nf}\sum_{i=1}^{Nl}\begin{cases}1 & if\ I_{k,i} < -I_{max_{k,i}}\\ 0 & if\ I_{k,i} \geq -I_{max_{k,i}}\end{cases}\right]\right)}{2 * Nl * Nf} \quad (17).$$

Maximum power at the system input (MPSI): This indicator shows the maximum power entering the system through the substation transformer, relative to the maximum power of the substation transformer. P_{ST} or Q_{ST} are the active and reactive power at the substation transformer, and the number between parentheses is the maximum percentile (97.5) fixed by us. Max_{ST} is the nominal power of the substation transformer:

$$MPSI = \frac{\sqrt{P_{ST(97,5)}^2 + Q_{ST(97,5)}^2}}{Max_{ST}} \quad (18).$$

Power variation at system input (PVSI): This indicator shows the maximum variation of the power entering the system through the substation transformer defined by the maximum and minimum percentiles fixed by us (97.5 and 2.5), relative to its maximum power Max_{ST}:

$$PVSI = \frac{\sqrt{P_{ST(97,5)}^2 + Q_{ST(97,5)}^2} - \sqrt{P_{ST(2,5)}^2 + Q_{ST(2,5)}^2}}{Max_{ST}} \quad (19).$$

Balance storage use (BSU): This indicator is used to describe equality in the working mode of the storage (charging and discharging modes). Here, it is necessary to know the parameter *FCharging* (times that storage is charging) and *FDischarging* (times that storage is discharging) for each storage *i*. *PowSTGi* is the amount of the storage *i* = 1, ..., *n*, where *n* is the number of storage systems integrated:

$$BSU = \sum_{i=1}^{n}\left(\frac{\left|FCharging_i - FDischarging_i\right| * Pow_{STG_i}}{\sum_{j=1}^{n} Pow_{STG_j}}\right) \quad (20).$$

3. ADVANCED METHODS OF CALCULATION FOR PROBABILISTIC MODELS

The process involved in this work includes the following techniques:

- Probabilistic power flow with storage systems based on the Point Estimate Method PEM 2m+1,
- Reliability evaluation given the possibility of islanding based on the Universal Generating Function (UGF) method and
- Optimisation algorithm based on the Non-dominated Sorting Genetic Algorithm method (NSGA-II).

3.1. Probabilistic power flow evaluation with storage function

The power flow used in this work is based on [16], but it was necessary to improve it considering the storage systems. The basic power flow is a probabilistic power flow based on the PEM 2m+1 method for an unbalanced power distribution system with radial topology and correlated input variables as described in the Appendix. The input variables that could have uncertainty are the loads and generators (solar and wind types). The corresponding estimations of the Cumulative Distribution Functions (CDFs) or PDFs of the output random variables using approximations in normal expansions, such as Cornish-Fisher, Edgeworth and Gram-Charlier expansions, are detailed in [16].

This process is extremely difficult because sometimes the storage works in the charging mode (as a load), and the other works in the discharging mode (as a generator). The available data are: line parameters, loads, generators and storage capacities. In addition, it is necessary to define the operation strategy of the storage systems—when they will charge and when they will discharge. It is defined by percentiles (pdown = 0.15 p.u. and pup = 0.85 p.u. of power at bus) that we fix to control the operation of the storage, in charging and discharging modes respectively. The steps of this algorithm, Figure 3, are:

Step 1: The PPF is evaluated in the network without storage. One of the results is the PDF of Pin,k in each node *k*.

Step 2: LIMdown,k = Pin,k(pdown) and LIMup,k = Pin,k(pup) are defined. These powers define the operation of the storage devices.

Step 3: The PPF is calculated with storage. The methodology of the PEM 2m+1 used involves (2m+1) deterministic power flows. In each deterministic power flow, the storage sited in bus k will charge if Pin,k is less than LIMdow,k, and the storage will discharge if Pin,k is greater than LIMup,k.

In Step 4, the indices are calculated (equations from 15 to 20).

```
Input data:

Line parameters, loads, generators and storage capacities. Percentiles pdown and pup.

Step 1: Evaluate PPF without storage. Obtain PDF of Pin,k.

Step 2: For each bus k,

            LIMdown,k = Pin,k(pdown) and LIMup,k = Pin,k(pup)

        End_For

Step 3: Evaluate PPF with storage.

  For j <=  (2m+1)

    Do deterministic power flow

       Define the mode of each storage device k.

           If Pin,k_j > LIMup,k

              then STOmode,k = mode discharge and Increase FDischarging.

           If Pin,k_j < LIMdown,k

              then STOmode,k = mode charge and Increase FCharging.

  End_For

Step 4: Evaluate probabilistic indices.
```

Fig. 3. Assignment of storage devices.

3.2. Reliability evaluation in view of the possibility of islanding

In the case of an outage, the meshed power distribution systems could have different paths for supplying customers. Meanwhile, in radial distribution networks, when an interruption event occurs, the customers downstream of the element out of operation lose their supply. This affects the system's reliability. The possibility of isolating zones that contain distributed resources allows to work in the island mode. However, if distributed generation is renewable, the random behaviour of the power produced could result in reliability loss. Storage systems could therefore reinforce distributed renewable generation. Hence, here, we have integrated a model of an ideal storage system, combined with all stochastic elements as loads and generators to calculate the reliability index (EENS). For this, we use the UGF (as described in the Appendix). The UGF is a methodology successfully used in reliability evaluation at the generation level as shown in [17-18]. Here, it helps to compute the EENS for each islanding zone in the system, whether powered by the centralised system or not.

Next, the multi-state models of the stochastic elements required by the UGF are presented, and finally, the entire integration of different elements for assessing the EENS is described.

3.2.1. External source model (centralised power system)

The power that a centralised power system provides is modelled with two states: out of operation *(0* power) and normal operation (P_R). The probabilities of these states are *FOR* (Forced Outage Rated) and 1-*FOR*, respectively. The u-function is expressed as:

$$u_R(z) = FOR_R z^0 + (1 - FOR_R) z^{P_R} \quad (21).$$

3.2.2. Generators and load models

The Multi-State System MSS model for every element (generator or load) of the network is described by his u-function (8), where *i* is the subindex between 0 and the number of states *RP* associated with the element:

$$u_{GL}(z) = \sum_{i=0}^{RP} p_i z^{x_i} \quad (22)$$

Here *x* is the state (*0*,..., *i*,...,*RP*), and *p* is the probability of each state.

3.2.3. Storage system model

The multi-state modelling of different elements has the reliability assessment as the main objective—in particular, the evaluation of the EENS in this work. With the same purpose, the storage systems are modelled with three possible performance levels: 1) out of operation, 'zero power, *0*', with a probability equal to the FOR, 2) reserve energy with probability $PSTG_{n,1}$ and 3) maximum capacity of storage with probability $PSTG_{n,2}$.

Because storage systems are being considered ideal, as it is not possible to model the time evolution in this probabilistic model. However, the PPF provides information about the behaviour of each storage system, if it is charging or discharging energy, and this information is condensed in the indicator balanced storage use (*BSU*). This means if the *BSU* is small enough, the charging and discharging modes are balanced, but if the *BSU* is greater than a specified value, storage device would always be charged or discharged and would not operate as a storage device.

Then, if the *BSU* is smaller than a predefined value, the u-function of storage systems is in (23), and the probabilities for $PSTG_{n,1}$ and $PSTG_{n,2}$ both equal $\frac{(1-FOR)}{2}$. If the *BSU* indicator is equal to or greater than a predefined value, the u-function is in (24).

$$u_{STG_n}(z) = FOR_{STG_n} z^0 + \frac{(1 - FOR_{STG_n}) z^{P_{STGn,1}}}{2} + \frac{(1 - FOR_{STG_n}) z^{P_{STGn,2}}}{2} \quad (23)$$

$$u_{STG_n}(z) = FOR_{STG_n} z^0 + (1 - FOR_{STG_n}) z^{P_{STGn,1}} \quad (24)$$

3.2.4. Reliability evaluation of the system with islands

The proposal in this work requires the specific interruption probability for each combination of islands. This means that one zone could have trouble, requiring its isolation for correction, and it is possible that more than one zone could have such trouble. There are 2^n combinations of zones in the distribution system, where *n* is the number of isolated zones that the switches of the system can create. The steps of this algorithm, Figure 4, are:

Step 1: An island is disconnected from the rest when a failure occurs in one of the elements of the island. The island is disconnected with a determined combination of the switches, and every combination of these switches gives a state of the network that can be composed of one or several islands. As seen in the section 'Model for the Islanding mode', the states and their probabilities are obtained.

Step 2: Every device has a u-function that is obtained from its PDF, equations from (22) to (24). The u-function of a storage device is more complicated to obtain, as it is necessary to run a PPF for each state, equations (23) and (24).

Step 3: The EENS of each state is calculated by adding the EENS of each island formed in this state. Every *island* has u-functions from the total island load (U_L), and the total island generation + storage (U_{G+S}). These u-functions are the result of applying the addition operator ($\underset{\text{par}}{\otimes}$) to the u-functions of the loads, generators and storage belonging to that island.

The EENS function is applied to the u-function of the loads, generators and storage of each island ki:

$$EEN S(ki) = \emptyset_{\mathrm{EENS}}(\mathrm{U_L}, \mathrm{U_{G+S}}) = \mathrm{U_L} \underset{\mathrm{EENS}}{\otimes} \mathrm{U_{G+S}} =$$

$$\sum_{\mathrm{j=1}}^{\mathrm{k_{G+S}}} \sum_{\mathrm{i=1}}^{\mathrm{k_L}} \mathrm{p_{L_i}} \mathrm{p_{G+S_j}} \mathrm{z}^{\max(\mathrm{l_i} > \mathrm{g+s_j}, 0)} \quad (25).$$

Step 4: The reliability assessment process of the whole network is calculated considering all possible combinations and its occurrence probabilities. It is also considered that loads could be supplied from the central power system or not; each one of these possibilities has the same 2^n possible combinations:

$$EENS_{total} = \sum_{j=1}^{ns} p(j) * EENS(j) \quad (26).$$

Data:

PDF of each device (loads and generators)

Step 1: Obtain the states of the network $ks \in (1..ns)$ and their probabilities p(ks).

Step 2: Obtain the u-function of each device.

Obtain the u-functions of loads and generators from their PDF, eq (22).

For ks <= ns

Recover the results of the PPF with storage systems.

Obtain the u-functions of storage systems for each state with information of the PPF, eq (23) and (24).

End_For

Step 3: Obtain the EENS(ks) of each state ks.

For ks <= ns

//Obtain the EENS(ks) for state ks with u-functions: u-loads, u-generators, u-storage.

For ki <= ni

//Obtain the EENS(ki) for each island.

$$EENS(ks) = \sum_{ki=1}^{ni} EENS(ki)$$

End_For

End_For

Step 4: Calculate the EENS of the whole system.

$$EENS_{total} = \sum_{ks=1}^{ns} p(ks) * EENS(ks)$$

Fig. 4. Reliability evaluation considering all combinations.

3.3. Optimisation algorithms

The NSGA-II is the algorithm used in this work. It uses the grouping operator, elitism and sorting on the values obtained for the objectives of each solution. The implementation details of this method have been described in [19], and uses the binary tournament and the crossing operator to determine one random point in the whole chromosome length.

Figure 5 shows the particle structure. It has as many columns as number of buses *n*. In addition, each column has as many rows as number of phases, three phases here. If 3 kWh of storage are installed in each phase of each bus, each particle position has (3-3-3). If there is a different quantity in each phase, one particle position could have, for example, (1-0-2).

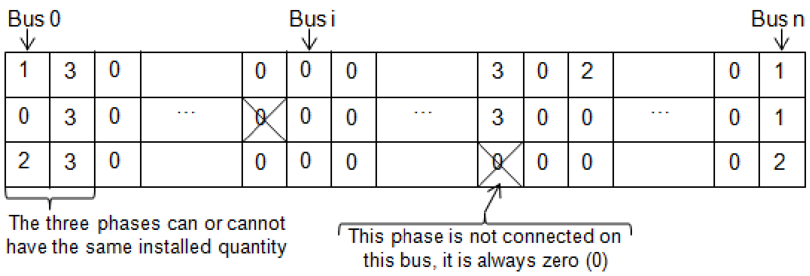


Fig. 5. Particle structure.

The optimisation problem has three objectives, but the optimisation algorithm uses a fitness function to guide the research to solutions with good values of these three objectives. This fitness function tries measure so that the value of the objectives satisfies the constraints with several indicators. The expression for computing these contributions is (27), and their parameters must be replaced with elements in Table I:

$$Cont_i = \begin{cases} \omega & if \quad (ind_i - ind_{lim}) \leq 0 \\ \omega * \dfrac{(base - ind_i)}{|base - ind_{lim}|} & if \quad (ind_i - ind_{lim}) > 0 \end{cases} \qquad (27).$$

Table I. Elements for the contribution to equation (27).

$Cont_i$	ω	$base$	ind_{lim}	ind_i
$contVLB_i$	$\omega\ V_s$	SV_{base}	V_{lim}	VLB_i
$contVUB_i$	$\omega\ V_{sd}$	SV_{bdase}	V_{dlim}	VUB_i
$contTLL_i$	$\omega\ I_s$	SI_{base}	I_{lim}	TLL_i
$contMPSI_i$	$\omega\ T_s$	LT_{base}	Ts_{lim}	$MPSI_i$
$contPVSI_i$	$\omega\ T_f$	Tf_{base}	Tf_{lim}	$PVSI_i$
$contBSU_i$	$\omega\ S_e$	Se_{base}	Se_{lim}	BSU_i
$contLosses_i$	1	$Losses_{base}$	$Losses_{lim}$	$Losses_i$
$contEENS_i$	1	$EENS_{base}$	$EENS_{lim}$	$EENS_i$

In (27), the particle is defined by the *i* sub-index; $Cont_i$ is the contribution of the *i* indicator to the constraint function; *w* is the weight of *i*; *base* is the value of *i* when storage systems are not integrated; ind_{lim} represents the limits established for *i* (defined on constraints); and ind_i is the value obtained for *i* with storage systems included.

Indicator contributions are joined in the constraint function (28):

$$Indsum_i = contVLB_i + contVUB_i + contTLL_i + contMPSI_i + contPVSI_i + contBSU_i \quad (28)$$

Finally, in (29) is the whole value of fitness function; here, *wLosses*, *wEENS*, *wCinst* and *wIndsum* are the global weights assigned to $contLosses_i$, $contEENS_i$, $contCInst_i$ and *Indsum*, respectively:

$$FA_i = wLosses * contLosses_i + wEENS * contEENS_i + wCinst * contCInst_i + wIndsum * Indsum_i \quad (29).$$

where: $$contCInst_i = \frac{(MaxInst - CantInst_i)}{MaxInst}$$

4. STUDY CASES AND RESULTS

4.1. Description of cases

To test this methodology, we use the IEEE test feeders with 34 buses. This test has been adapted to our problem: Regulators have been excluded to observe the effect of storage systems over a voltage profile, four switches have been integrated to define the isolated zones from them, and both solar and wind generators have been added as described in Table II and Fig. 6. Also, the correlation applied to wind velocities is 0.9 for turbines in the same site and 0.7 between different places; the solar radiation data have correlation values above 0.92 for all placements, and the correlation between loads is 0.7.

Table II. Additional generators for IEEE-34 bus system.

Node	Type	Number	Capacity	Parameters (m/s except *k*)				
				c	*k*	*Vci*	*Vr*	*Vco*
844	wind	3	100 kW	9	2.2	3	12	25
854	wind	1	100 kW	9	2.0	3	12	25
820	solar	1	50 kW*	Albuquerque meteorological station [20]				
860	solar	1	120 kW					

** All in phase A*

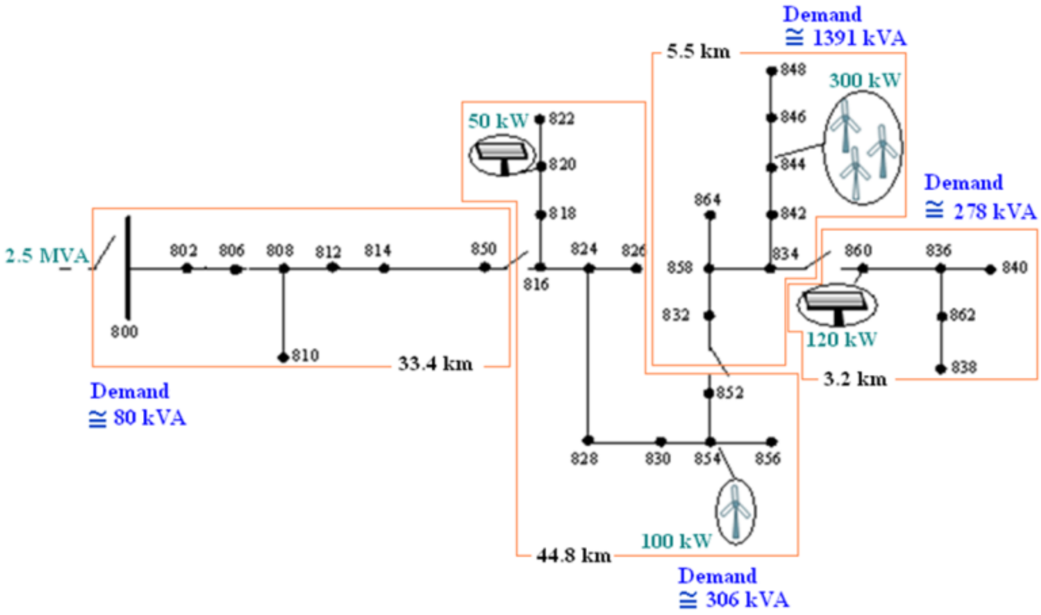


Fig. 6. IEEE-34 bus modified (isolated zones, wind and solar generation).

The four switches integrated in the network permit 16 different states of the network that have their own probabilities POF (probability of failure) as shown in Table III.

Table III. Probabilities of island combinations for IEEE 34 bus system.

Switch state to bus				POF	Switch state to bus				POF
800	816	832	860		800	816	832	860	
0	0	0	0	0.0	1	0	0	0	0.0
0	0	0	1	0.0	1	0	0	1	3x10-7
0	0	1	0	0.0	1	0	1	0	2x10-7
0	0	1	1	2.4x10-6	1	0	1	1	1.66x10-3
0	1	0	0	0.0	1	1	0	0	0.0
0	1	0	1	3x10-7	1	1	0	1	2.07x10-5
0	1	1	0	2x10-7	1	1	1	0	1.23x10-4
0	1	1	1	1.43x10-3	1	1	1	1	0.996573

4.2. Description of the multiobjective optimisation results

This methodology permits the obtaining all solutions of the Pareto front and the relations among the three objectives—storage capacity, losses and EENS as shown in

Figure 7. The EENS decreases from 27 to 22 MWh/year when the storage capacity increases from 0 to 700 kWh approximately as shown in Fig 7. The relation between losses and storage capacity is more complicated because losses decrease approximately from 170 to 140 kWh when the storage increases from 0 to 280 kWh, but for greater increments of the storage capacity, the losses return to increasing. This is due to the losses that are produced when the storage devices are charged and discharged.

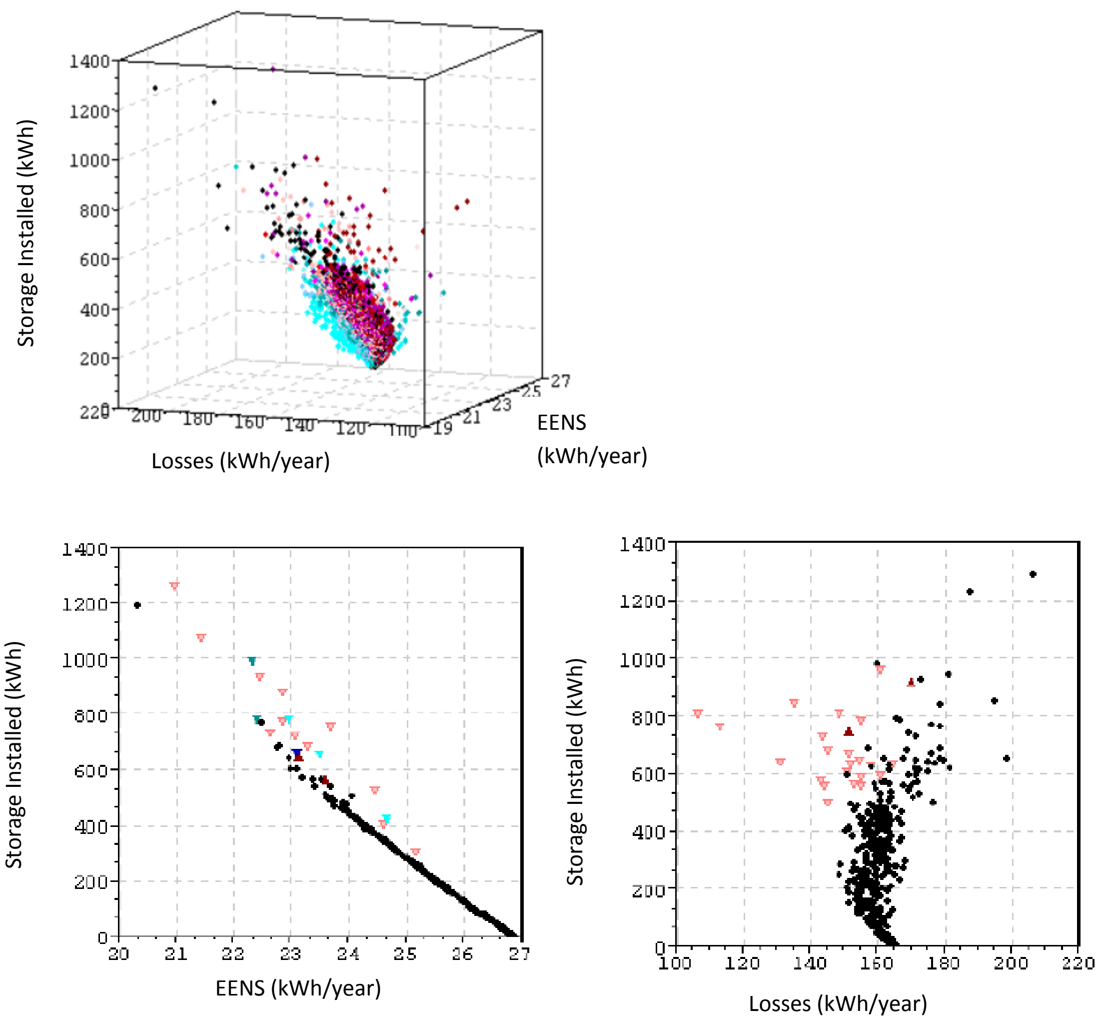


Fig. 7. Pareto front and two-dimensional (2D) projections of the three objectives.

Several of these solutions are presented in Table IV. In this table, the value of the objective functions and the value of the indices of the optimisation algorithm are shown.

In Table IV, there are values of probabilistic indices for several solutions. The VLB is about 0.4; say that voltages that are three times greater than the variance of the voltage at each node exceed the voltage limits only 40% of the time. The voltage that is unbalanced on buses, VUB, is about 0.25. The thermal limits of lines (TTL) is never exceeded. The maximum power at system input MPSI is about 70%. The power variation at system input PVSI is about 30%. The balance storage has very low values, which indicates that the storage is well used and balanced.

Table IV. Values of the objective functions and indices of several solutions non-dominated for IEEE 34 bus system.

N.º SOL	Storage (kWh)	Losses (kWh)	EENS (MWh/year)	VLB	VUB	TLL	MPSI	PVSI	BSU
1	0	167.01	18.25	0.3765	0.2392	0.00	0.6695	0.3360	0.0000
50	54	167.65	17.96	0.3796	0.2411	0.00	0.6712	0.3403	0.0007
100	110	167.13	17.68	0.3611	0.2434	0.00	0.6717	0.3450	0.0005
150	166	163.68	17.41	0.3611	0.2532	0.00	0.6674	0.3454	0.0024
200	220	150.68	17.22	0.3858	0.3041	0.00	0.6482	0.3246	0.0026
250	272	151.66	16.95	0.3858	0.3044	0.00	0.6507	0.3298	0.0021
300	316	161.27	16.66	0.3858	0.3046	0.00	0.6682	0.3538	0.0020
350	378	166.07	16.39	0.3642	0.2618	0.00	0.6767	0.3648	0.0008
400	1134	185.59	14.31	0.4197	0.2155	0.00	0.7518	0.5714	0.1135

4.3. Location of the storage systems

A list with the location and capacity of the storage devices in every solution is impossible due to the lack of space, but the times that the algorithm selects a node to put a storage device in it is represented in Figure 8. It is evident that some nodes are chosen more times than others are. A list of nine solutions distributed along the Pareto front with their distribution of storage devices is written for node in Table V and for island in

Table VI. The reader can see that the storage device is placed in areas most likely to be isolated.

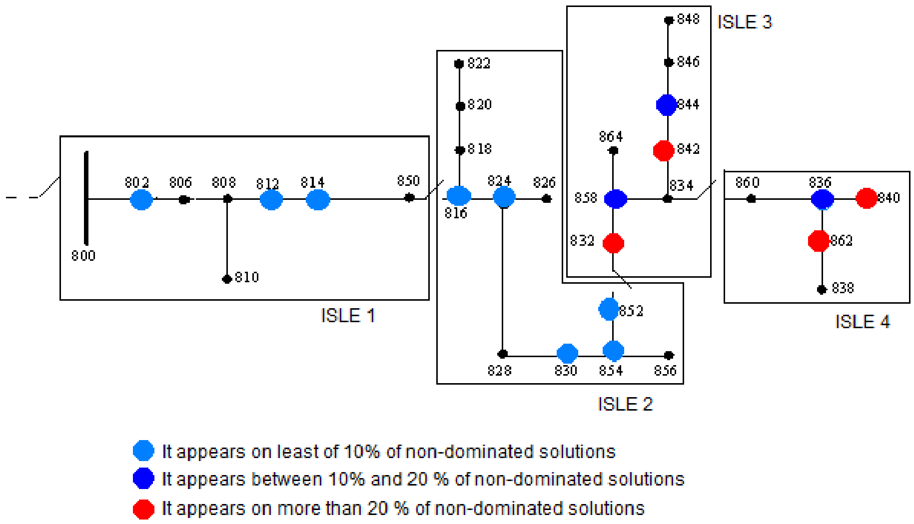


Fig. 8. Percentage of times that solutions have storage systems.

Table V. Probabilities of island combinations for IEEE 34 bus system.

N.º SOL	Total storage (kWh)	STORAGE IN EACH NODE									
		828	854	832	842	844	846	858	836	840	862
1	0	0	0	0	0	0	0	0	0	0	0
50	54	0	0	0	0	0	0	0	6	48	0
100	110	0	0	0	0	0	0	14	0	96	0
150	166	0	0	0	36	0	0	0	116	6	8
200	220	0	0	12	122	0	0	0	0	86	0
250	272	0	0	12	122	0	0	0	0	128	10
300	316	0	0	0	80	0	0	0	142	84	10
350	378	0	0	12	0	64	0	0	0	302	0
400	1134	296	120	72	14	190	142	30	142	128	0

Table VI. Storage installed in every island for IEEE 34 bus system.

Island	1	2	3	4
Prob. to leave isolated	0.0014329	0.0030934	0.0031141	0.0032371

N.º SOL				
1	0	0	0	0
50	0	0	0	27
100	0	0	7	48
150	0	0	18	65
200	0	0	67	43
250	0	0	67	69
300	0	0	40	118
350	0	0	38	151
400	0	208	224	135

4.4. Evaluation of one of the solutions

Now, to analyse the effect of the storage in a solution, it is necessary to choose one among all solutions that become the Pareto front. Several methods exist for doing this. We have chosen the compromise programming method [21], which consists of choosing one solution that is closer to the ideal point.

If the ideal and anti-ideal point are defined by $x_{ideal} = \{sto_{min}, eens_{min}, losses_{min}\}$ and $x_{anti\text{-}ideal} = \{sto_{max}, eens_{max}, losses_{max}\}$ respectively, then the distance metric L for a solution x is:

$$L(x) = \left\{\frac{fstorage(x) - sto_{min}}{sto_{max} - sto_{max}} + \frac{feens(x) - eens_{min}}{eens_{max} - eens_{min}} + \frac{flosses(x) - losses_{min}}{losses_{max} - losses_{min}}\right\} \qquad (30)$$

And the solution chosen by the compromise programming method minimizes,

$$Min \quad L(x) \qquad (31)$$

In this case, the ideal and anti-ideal points are $x_{ideal} = \{0, 125.52, 14.31\}$ and $x_{anti\text{-}ideal} = \{1134, 185.61, 18.24\}$, and the best solution is $x_{best} = \{290, 150.58, 16.88\}$ with a distance of 1.326.

This solution includes, in addition to renewable generators, 209 kWh of storage systems distributed on the buses: 850 (51 kWh), 842 (12 kWh), 844 (29 kWh), 860 (96 kWh) and 838 (63 kWh). With this storage system, the EENS is 16.88 kWh/year (18.25

kWh without storage), and the losses are 150.58 kWh/year (167.01 kWh/year without storage).

Figures 9 and 10 show the effect of the storage systems for voltage and active power magnitudes at every bus and line without and with storage systems, respectively. For each execution, we indicate the percentiles of 0.02, 0.50 and 0.98. The light lines show the response when storage systems are introduced and there are improvements.

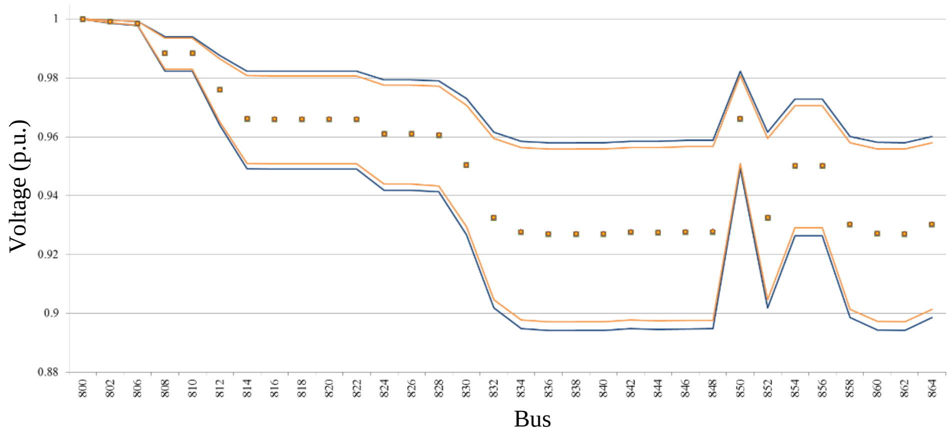


Fig. 9. Voltage profile without and with storage systems.

Fig. 10. Active power profile without and with storage systems.

5. CONCLUSIONS

This paper proposes a probabilistic methodology for optimising the location and size of the storage devices needed for an improvement operation of electrical network with the island mode of operation.

Several advanced techniques have been developed to attain this target: a probabilistic power flow based on the PEM for a network with storage devices, a reliability assessment method based on the UGF for a network with the isolated mode and an optimisation algorithm based on NSGA II with goodness indicators to accelerate the search.

Results obtained by the optimisation algorithm show the Pareto front of the three objectives: storage capacity installed, power losses and EENS. This information permits noting that a storage increase reduces the EENS and the losses in the network, that great quantities of storage can increase losses and that storage is placed in the areas with a greater probability of isolation.

When we apply a decision multiobjective method to choose only one solution from the Pareto solutions set, the chosen solution does not have much storage installed. This means that solutions that feature little storage and are cheaper can attain the expectations that decision-makers have in mind.

APPENDIX

A.1. Basic concepts of PEM 2m+1

Voltages, currents and powers are some of the random variables that are calculated by the probabilistic power flow from other independent random variables, such as the powers injected and consumed by generators and loads.

Let Z (voltages, currents,…) be a random variable function of m, independent random variables x_k (powers in each node k):

$$Z = f(x_1, x_2, \dots, x_m) \qquad (30).$$

For all $k = 1, 2, \dots, m$. Each x_k is composed by $h = 1, \dots, 6$, where 1, 3 and 5 are for real power, and 2, 4 and 6 are for the reactive power of phases A, B and C, respectively. For each $x_{k,h}$, there are $\mu_{k,h}$, $\sigma_{k,h}$, $\lambda_{k,h,3}$ y $\lambda_{k,h,4}$, which are, respectively, the mean, standard deviation, skewness and kurtosis coefficients of the $x_{k,h}$ variable. M_3' and M_4' are the third and fourth moments about the mean, and the skewness and kurtosis coefficients are defined by (20).

$$\lambda_{k,h,3} = \frac{M_3'(x_{k,h})}{\sigma_{k,h}^3} \text{ and } \lambda_{k,h,4} = \frac{M_4'(x_{k,h})}{\sigma_{k,h}^4} \qquad (31)$$

Each random variable in the system is represented with **three concentration points** with location $x_{k,h,i}$ and weight $p_{k,h,i}$:

$$x_{k,h,i} = \mu_{k,h} + \xi_{k,h,i}\sigma_{k,h} \qquad (32)$$

and

$$p_{k,h,i} = \frac{(-1)^{3-i}}{\xi_{k,h,i}(\xi_{k,h,1} - \xi_{k,h,2})}, \text{ for } i = 1, 2 \qquad (33).$$

$$p_{k,h,3} = \frac{1}{m} - p_{k,h,1} - p_{k,h,2} = \frac{1}{m} - \frac{1}{\lambda_{k,h,4} - \lambda_{k,h,3}^2}, \text{ for } i = 3$$

The $\xi_{k,h,i}$ that each location requires is computed with (23):

$$\xi_{k,h,i} = \frac{\lambda_{k,h,3}}{2} + (-1)^{3-i}\sqrt{\lambda_{k,h,4} - \frac{3\lambda_{k,h,3}^2}{4}}, \text{ for } i = 1, 2$$

$$\xi_{k,h,3} = 0, \text{ for } i = 3 \qquad (34).$$

These concentration points are used instead of the probabilistic functions of the random variables in the power flow algorithm. The process consists of evaluating the function for Z in each location $x_{k,i}$ (including all values h = 1, ..., 6 of each m input random variable). Then, it is necessary to do 2m+1 evaluations of the function Z with the deterministic power flow. For each evaluation l, all variables are fitted to the mean value μ_k except for one that has the value of one of her locations $x_{k,i}$

$$Z_l = f_l(\mu_1, \mu_2, \dots, x_{k,i}, \dots, \mu_{m-1,}\mu_m) \qquad (35)$$

The results for each location are then added to the results of other locations according to its corresponding weight, $p_{k,h,i}$. The approximate moments of each $Z_{l,h}$ are computed with (24), and its results are used for (26):

$$E(Z_{l,h}^j) \cong p_{0,h}\left(f_l(\mu_1, \mu_2, \dots, \mu_k, \dots, \mu_{n-1,}\mu_n)\right)^j \qquad (36),$$

$$+\sum_{k=1}^{m}\sum_{i=1}^{2} p_{k,h,i}\left(f_l(\mu_1, \mu_2, \dots, x_{k,i}, \dots, \mu_{m-1,}\mu_m)\right)^j$$

Where each location $x_{k,i}$ includes the six $x_{k,h,i}$ locations, and where each μ_k includes the mean of the real and reactive power values of all phases. The moments of output random variables go from j = 1, ..., 4. The weight $p_{0,h}$ is the result of (25) and includes all concentration points for i = 3:

$$p_{0,h} = \sum_{k=1}^{m} p_{k,h,3} = 1 - \sum_{k=1}^{m} \frac{1}{\lambda_{k,h,4} - \lambda_{k,h,3}^{2}} \quad (37).$$

Finally, the approximated mean (μ) and moments about the mean (μ_r') of each $Z_{l,h}$ are:

$$\mu(Z_{l,h}) = E(Z_{l,h}) \text{ and } \mu_r'(Z_{l,h}) \cong E[(Z_{l.h} - \mu_{l,h})^r] \quad (38).$$

The corresponding estimation of the CDFs or PDFs of the output random variables can be calculated using approximations in normal, Cornish-Fisher, Edgeworth and Gram-Charlier expansions.

Z_l could be whichever of the operation parameters is in the power flow. For example, if there are *s* buses, Z_l could be the voltage in *l* bus, for $l = 1,..., s$.

A.2. Multi-state system and universal generating function

A stochastic element, which can be a load, generator, or storage system, could be modelled as a discrete random variable, *X*, based on its different performance levels. This description has a number of possible values represented by a vector $x = (x0,\dots,xi)$, and each value in x is associated with some probability in the vector $p = (p0,\dots,pi)$. The resulting mapping from $xi \rightarrow pi$ is usually defined as the probability mass function (pmf) or performance distribution (PD) of *X*, and it represents the element as a MSS.

When the *z* transform is applied to the discrete random variable *X*, a polynomial is obtained (27), the u-function, $u(z)$, where k_X represents the number of possible states of performance of the variable *X* [22]:

$$u_X(z) = \sum_{i=1}^{k_X} p_i z^{x_i} \quad (39).$$

If two independent elements (*G* and *L*) modelled as MSS are considered—each one with its respective *u(z)* and related through the composition operator $\underset{\phi}{\otimes}$—the u-function of the entire system, denoted by *U(z)*, is obtained with:

$$U(z) = u_G \underset{\phi}{\otimes} u_L = \sum_{j=1}^{k_G} \sum_{i=1}^{k_L} p_{L_i} p_{G_j} z^{\phi(x_{L_i}, x_{G_j})} \quad (40).$$

If the number of independent multi-state elements in the system is *n*, such that:

$$u_1(z) = \sum_{i_1=1}^{k_1} p_{1,i_1} z^{x_{1,i_1}}, \ldots, u_n(z) = \sum_{i_n=1}^{k_n} p_{n,i_n} z^{x_{n,i_n}} \quad (41),$$

Then the u-function of the entire system will be:

$$U(z) = \sum_{i_1=1}^{k_1} \sum_{i_2=1}^{k_2} \cdots \sum_{i_n=1}^{k_n} \left(\prod_{j=1}^{n} p_{j,i_j} z^{\phi(x_{1,i_1}, \ldots, x_{n,i_n})} \right) \quad (42).$$

This technique based on the *z* transform and the composition operator $\underset{\phi}{\otimes}$, applied to a function with an arbitrary structure, is called UGF [23]. Because this technique is based on simple recursive procedures, it is a systematic method for the enumeration of system states that can replace complicated algorithms on the performance evaluation of an MSS, which can, in turn, be composed of several multi-state elements.

In particular, three composition operators are used. These are $\underset{par}{\otimes}$ (addition or parallel operator), $\underset{*}{\otimes}$ (multiple operator) and $\underset{EENS}{\otimes}$ (EENS assessment operator).

Operators are applied over two multi-estate elements and are repeated as many times as needed. This operation is described in (30) to (32):

$$\emptyset_{par}(u_L, u_G) = u_{L}\underset{par}{\otimes}u_G = \sum_{j=1}^{k_G}\sum_{i=1}^{k_L} p_{W_i} p_{G_j} z^{(l_i+g_j)}$$ (43),

$$\emptyset_{*}(u_L, u_G) = u_{L}\underset{*}{\otimes}u_G = \sum_{j=1}^{k_G}\sum_{i=1}^{k_L} p_{W_i} p_{G_j} z^{(l_i*g_j)}$$ (44)

and

$$\emptyset_{EENS}(u_L, u_G) = u_{L}\underset{EENS}{\otimes}u_G = \sum_{j=1}^{k_G}\sum_{i=1}^{k_L} p_{W_i} p_{G_j} z^{max(l_i>g_j,0)}$$ (45).

The EENS could be expressed as a single value through the expectation (33):

$$E(X) = \sum_{j=1}^{k_X} p_{x_i} * x_i$$ (46).

The estimation of reliability for the distribution system at the generation level is one of the main objectives pursued in this work and achieved through the UGF, based on the EENS. First, it is presented to obtain the performance distribution of each element in the system, and later, the combination of these u-functions is explained to determine the reliability of the whole system.

ACKNOWLEDGMENTS

The first author would like to thank CONACyT (Consejo Nacional de Ciencia y Tecnología) for its financial support.

This work was supported by the “Ministerio de Ciencia e Innovación” of the Spanish Government under Project ENE2013-48517-C2-1-R.

REFERENCES

[1] Li, Yan Fu and Zio, Enrico, “A multi-state model for the reliability assessment of a distributed generation system via universal generating function, ”*Reliability Engineering & System Safety*, volume 106, no. 0, pp. 28 – 36, 2012.

[2] S. Kennedy. Reliability evaluation of Islanded microgrids with stochastic distributed generation. IEEE PES *General Meeting, 2009,*pp. 1-8.

[3] Zhang, Xiaochen, Bie, Zhaohong, and Li, Gengfeng, “Reliability Assessment of Distribution Networks with Distributed Generations using Monte Carlo Method, ”*Energy Procedia*, volume 12, no. 0, pp. 278–286, ISSN 1876-6102, 2011.

[4] H., Liang, J., Su, and S., Liu, “Reliability evaluation of distribution system containing microgrid,” Electricity Distribution (CICED), 2010 China International Conference, 2010.

[5] Li, G., Bie, Z., Hua, B., and Wang, X., “Reliability evaluation of distribution systems including micro-grids considering demand response and energy storage,” Universities Power Engineering Conference (UPEC), 2012 47th International, Sept. 2012.

[6] Xu, Yixing and Singh, C., “Adequacy and Economy Analysis of Distribution Systems Integrated with Electric Energy Storage and Renewable Energy Resources, ”*IEEE Transactions on Power Systems,* volume 27, no. 4, pp. 2332–2341, 2012.

[7] Bie, Z., Zhang, P., Li, G., Hua, B., Meehan, M., and Wang, X., “Reliability Evaluation of Active Distribution Systems Including Microgrids, ”*IEEE Transactions on Power Systems,* volume 27, no. 4, pp. 2342–2350, Nov. 2012.

[8] Atwa, Y.M., El-Saadany, E.F., Salama, M. M A, Seethapathy, R., Assam, M., and Conti, S., “Adequacy Evaluation of Distribution System Including Wind/Solar DG

During Different Modes of Operation, ”*IEEE Transactions on Power Systems,*volume26, no. 4, pp. 1945–1952, 2011.

[9] Gantz, J.M., Amin, S.M., and Giacomoni, A.M., “Optimal mix and placement of energy storage systems in power distribution networks for reduced outage costs,” Energy Conversion Congress and Exposition (ECCE), *2012 IEEE,* 2012.

[10] Atwa, Y.M. and El-Saadany, E.F., “Optimal Allocation of ESS in Distribution Systems With a High Penetration of Wind Energy, ”*IEEE Transactions on Power Systems,* volume 25, no. 4, pp. 1815–1822, nov. 2010.

[11] Geth, F., Tant, J., Haesen, E., Driesen, J., and Belmans, R., “Integration of energy storage in distribution grids,” Power and Energy Society General Meeting, *2010 IEEE,* 2010.

[12] Aguero, J.R., Chongfuangprinya, P., Shao, Shengnan, Xu, Le, Jahanbakhsh, F., and Willis, H.L., “Integration of Plug-in Electric Vehicles and distributed energy resources on power distribution systems,” Electric Vehicle Conference (IEVC), *2012 IEEE,* 2012.

[13] Tant, J., Geth, F., Six, D., Tant, P., and Driesen, J., “Multiobjective Battery Storage to Improve PV Integration in Residential Distribution Grids, ”*IEEE Transactions on Sustainable Energy,* volume 4, no. 1, pp. 182–191, 2013.

[14] Hong, H., "An efficient point estimate method for probabilistic analysis", Reliability Enginnering & System Safety, volume 59, no. 3, pp. 261-267, 1198.

[15] Wallace, D. L., "Asymptotic Approximations to Distributions". The Annals of Mathematical Statistics, volume 29, no. 3, pp. 635-654, 1958.

[16] Delgado, C. and Domínguez-Navarro, J.A., “Point Estimate Method for Probabilistic Load Flow of an Unbalanced Power Distribution System with

Correlated Wind and Solar Sources, ”*International Journal of Electrical Power and Energy Systems*, volume 61, 267–278, 2014.

[17] Y. Ding, P. Wang, L. Goel., P. Chiang, et al. Long term reserve expansion of power systems with high wind power penetration using universal generating function methods. *IEEE Transactions on Power Systems*, volume 26, no. 2, pp. 766–774, 2010.

[18] Y. Li, E. Zio. *A Multi-State Model for the Reliability Assessment of a Distributed Generation System via Universal Generating Function.* eprint arXiv:1206.6808, 2012. .

[19] K. Deb, A. Pratap, A. Agarwal, T. Meyarivan. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, volume 6, no. 2, pp.182–197, 2002.

[20] National Solar Radiation Database, Las Cruces Intl Station (722695). Year 2010. http://rredc.nrel.gov/solar/old_data/nsrdb/1991-2010. Last accessed date: June 2013.

[21] J.L. Cochrane and M. Zeleny. Multiple Criteria Decision Making. University of South Carolina Press, Columbia, 1973.

[22] A. Lisnianski, G. Levitin. Multi-state system reliability assessment, optimisation, applications, *World Scientific*, 2003.

[23] G. Levitin, *The Universal Generating Function in Reliability Analysis and Optimization*, Springer, 2005.