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DESIGN OF A SYNTHETIC POPULATION OF GEOSTATIONARY SPACE DEBRIS BY STATISTICAL MEANS

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We can observe and track objects located at the geostationary region with a size of approximately 1 meter by telescope means. However, a huge population of space debris still remains unknown for us. In this work, we propose to generate a synthetic (artificial) population of individual space debris, whose global characteristics are the same as the real one. We use two different tools; the first one, a combination of orbit propagators, fragmentation models, and historical data, and the second one, an Iterative Proportional Fitting (IPF) procedure, which allows to reconstruct the current population from partial data and statistical information.

INTRODUCTION

The number of space debris has increased significantly in the last years, and consequently the risk of collision between different orbiters (space debris - satellites, space debris - space debris) also. The means to observe and track in-situ measurements of space debris have evolved since the beginning of the nineties. All these observations have a common goal: to model and to calibrate our estimations of space debris population to prevent from a worsening situation.

The network of radar and optical telescopes of the U.S. Space Surveillance Network (USSSN) is available to observe, identify and track different types of objects. In particular, objects bigger than 10 cm in Low Earth Orbits (LEO), and objects bigger than 1 m in Geosynchronous Earth Orbits (GEO). The United States Strategic Command (USSTRATCOM) provides a catalog where all these objects are presented thought the well known Two Line Elements (TLE) catalog, which is available via the website *www.space-track.org*. The TLE catalog provides information about the officially cataloged objects such as satellites, upper-stages or space debris. The number of these cataloged objects (currently in orbit) ascend to approximately 16.000 objects. However, there are small fragments of debris, that are non-trackable by telescope means, which still remain unknown for us. Besides, this population of small fragments (1 cm) has been estimated to several hundreds of thousands of objects. For this reason, the estimation by numerical simulations or in-situ measurements of the real population of space debris orbiting the Earth is an important challenge nowadays.

In this paper, we focus on the study of the GEO region, which is defined as the region close to the 1:1 gravitational resonance, where the semi-major axis of an orbit is equal to 42,164 km, i.e, the orbital period of an object corresponds to one sidereal day (23h 54 min 4 s). This region is attractive for different space missions because satellites have a fixed position in the sky with

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respect to a ground observer. Thus, it is very useful for weather and communication missions since ground-track antennas do not have to rotate to establish communication with them.

The orbital surveys about space debris, that have been performed since the nineties by different space agencies, show that numerous non-cataloged space debris exist with a mean orbital motion of about 1 revolution per day.[1,2] These observations lead to the discovery of an unexpected population of objects with large eccentricity. This fact was studied and it was explained by the effect that the solar radiation pressure produces on this population.[3,4] In particular, the acceleration caused by the solar radiation pressure drastically affects the evolution of the eccentricity and inclination of these objects, and it is directly proportional to the area-to-mass ratio.

In this paper, we propose two complementary approaches to simulate and infer different populations of space debris. Both methodologies share the same characteristics: the creation of a synthetic (artificial) population of space debris whose characteristics and global properties are similar to the ones of the real population. The first approach is based on different tools: the use of orbital propagators,[5–7] the use of historical fragmentation data of past space activities[8] such as launches, collisions or explosions events, the use of the NASA Breakup Model,[9] and the use of other analytical tools developed at the University of Namur.[10,11] The second approach infers a synthetic population from simulated data and observational constraints at a given time by statistical means. In particular, we use an Iterative Proportional Fitting (IPF) procedure[12] to create a synthetic population of space debris.

This paper is organized as follows: first we present the current situation of space debris, making special emphasis in the GEO region. Second, we present the model to create the synthetic population by means of a deterministic approach, and we give some results. Third, we present the Iterative Proportional Fitting (IPF) process, and we apply the tool for creating the synthetic population from the simulated data and small survey of real data as input. Finally, we show the most interesting results, and we draw some conclusions.

CURRENT SITUATION OF SPACE DEBRIS IN THE GEO REGION.

Nowadays, according to NASA, the number of cataloged space debris is about 16.000. More precisely, in October 1, 2016 we count, in the GEO region, 1273 objects divided into three different types: 1031 satellites (active or inactive), 188 upper-stage and 54 debris.

The first geostationary satellite was Syncom 3, it was launched in 1964 to telecast the Summer Olympics in Tokyo. Since then, the number of satellites in this particular region has increased substantially. Unfortunately, two known breakups happened in the past years, and in 2016, two other ones took place.

The first one took place on June 25, 1978 caused by a malfunction of the battery of satellite Ekran 2 (weight 1750 kg). The location of this fragmentation correspond to an altitude of 35,790 km (0.0° North, 98.7° East). Immediately after the event, no object was detected by the USSSN. Several years later (about 1992) Russia confirmed and associated 4 objects to this explosion. The second fragmentation was caused by a failure of the upper stage Titan 3C Transtage (weight 2500 kg) on February 21, 1992. This explosion took place at an altitude of approximately 35,600 km ($\approx$ 197° East). The rocket body completed successfully its mission, but exploded 281 months after. Just after the explosion, 20 objects were detected.

More recently, on January 16, 2016 an upper-stage Breeze-M (dry mass 1220 kg) breakup took place in a region close to the geostationary ring. It happens at an altitude of 34,866 km (0.17° South,

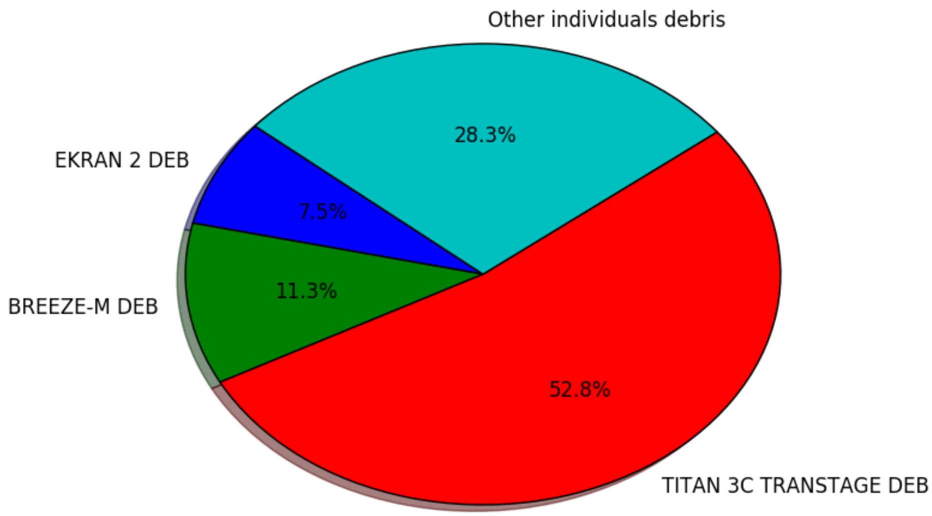

Figure 1. Proportion of space debris in the geostationary region corresponding to different breakups.

223° East). Just 10 fragments were observed but several hundreds are expected. A second breakup took place last year, on June 26, 2016 when the satellite BeiDou G2 (dry mass 1,100 kg) underwent a fragmentation in the GEO region.[13] The resulting orbit presents an apogee of 36,137 km, a perigee of 35,384 km, an inclination of 4.7°. In particular, at least five fragments were observed but no one was officially cataloged.

In the TLE catalog of USSTRATCOM, very few pieces of space debris corresponding to the previous mentioned fragmentation events are included. More precisely, 54 pieces of space debris are cataloged in this particular region: 28 are associated to Titan 3C Transtage, 6 corresponds to Breeze-M, another 4 are associated to Ekran2, and finally, other 16 fragments are considered individual objects. Figure 1 represents the proportion of each cloud of debris among the 54 cataloged pieces.

Another way to illustrate these populations of space debris consists in giving the distribution in space of the 54 cataloged pieces. Figure 2 shows the inclination versus the Right Ascension of the Ascending Node of each piece of debris, where it is possible to identify at least three different clusters.

Thanks to the optical surveys performed since the end of the nineties, numerous uncatalogued objects and unexpected clusters have been determined. Thus, an important issue is to determine the origins of these unexpected clouds and to estimate if the explanation could be the result of observational bias.[14] The comparisons between observations and artificial populations generated with a deterministic model such as MASTER-2005 show that the addition of at least 8 unexpected fragmentations to the known population of space debris give the best fit with the observations performed.[15]

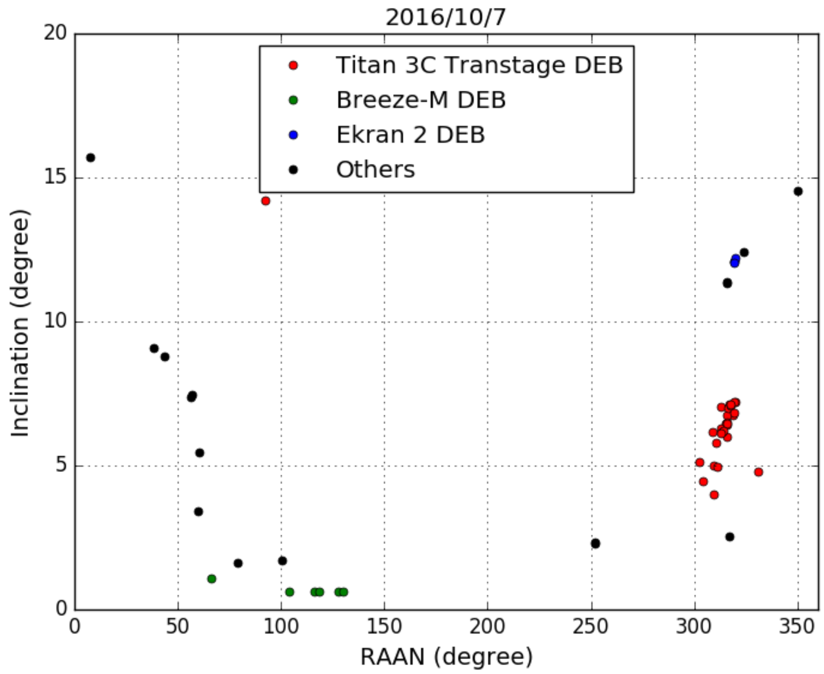

Figure 2. Inclination versus Right Ascension of the Ascending Node representation of 54 objects cataloged by USSTRATCOM, corresponding to the fragmentation of Ekran and Transtage, Breeze-M and some individual fragments.

MODEL OF THE GEO REGION BY A DETERMINISTIC APPROACH

Thanks to the historical observations provided by the USSTRATCOM, and the past activities (launches, fragmentations) that took place in a particular region, we are able to generate the current situation of the population of space debris until the actual date by means of an orbit propagation of each individual object created during the mentioned past activities.

This approach is followed by the main space debris models such as SMD, MASTER[16] or LEGEND.[17] An important challenge is the modelization of the different sources of debris such us explosions, ejection of Solid Rocket Motor particles (SRM) or the creation of paint flakes. Many efforts have been made to perform propagations over long time scales (several decades, or hundreds of years) of large populations of objects with limited computational resources. Two different ways are possible. One of them consist of using analytical or semi-analytical theories, which not only simplify the dynamics but also the calculation process. The other one consists of reducing the calculation process by means of different parallelization techniques allowed in the hardware. In this section, we describe the different tools that have been used to model the population of space debris with a size upper than 1 cm in the GEO region.

The NASA breakup model

A breakup model gives the size, the cross-section, the mass, and the increment of velocity of the fragments after an explosion or a collision. The methodology is based on the observations of past fragmentation events that took place in orbit and also is based on different hypervelocity impact tests made in laboratory. The observations and radar surveys performed during the eighties and nineties allowed the creation of this kind of model to simulate a breakup event. In particular, a powerful NASA breakup model has been developed and integrated in the software EVOLVE 4.0,[9] becoming nowadays the reference breakup model.

Following this model, after a breakup event it is possible to obtain the number of particles bigger than a particular size thanks to the following equation:

$$N(L_c) = S \cdot 6L_c^{-1.6} \tag{1}$$

where L_c represents a size, S is a dimensionless scale factor, and $N(L_c)$ represents the number of objects with a size bigger than L_c. This equation has been validated with numerous breakup events such as explosions or anti-satellite tests.

The second part of this model is to determine the area-to-mass ratio and the increment of velocity of all the pieces generated by a breakup event. The area-to-mass ratio of the new pieces of debris can be determined as follows:

$$D_{A/M}(\lambda_c, \chi) = \alpha(\lambda_c) N(\mu_1(\lambda_c), \sigma_1(\lambda_c), \chi) + (1 - \alpha(\lambda_c)) N(\mu_2(\lambda_c), \sigma_2(\lambda_c), \chi) \tag{2}$$

where $\lambda_c = \log_{10}(L_c)$, $\chi = \log_{10}(\frac{A}{M})$, N is a normal law, and the parameters α, μ_1, μ_2, σ_1, σ_2 depend on the parent body (upper-stage or spacecraft).

In a similar way, the module of the increment of velocity can be obtained through the following normal law distribution:

$$D_{A/M}(\lambda_c, \chi) = N(\mu(\lambda_c), \sigma(\lambda_c), \chi) \tag{3}$$

where $\lambda_c = \log_{10}(L_c)$, $\chi = \log_{10}(\frac{A}{M})$, N is a normal law, and the paremeters μ and σ depend on the parent body (upper-stage or spacecraft).

NIMASTEP : a numerical orbit propagator

The University of Namur has developed an orbit propagator named NIMASTEP. It uses cartesian coordinates and integrates numerically the equations of motion taking into account the geopotential until order and degree five, the Sun and the Moon, and the solar radiation pressure as disturbing functions.[5] An updated version includes the atmospheric drag with a model of space weather to propagate an object in LEO over several decades in the future.[7] The equations of motion are not averaged and no resonance is neglected. All computations are performed with a parallelized version of NIMASTEP that has been implemented with the library MPI (acronym for Message Passing Interface) and has been run on supercomputers. This way, it is possible to propagate the orbit of several hundreds of objects at the same time on different CPU.

Propagation of a population of space debris generated with historical data

We propose to create a population of space debris considering different historical breakup data. For that purpose we use the software NIMASTEP, which was initially designed to propagate a population of space debris. This software, was substantially improved with the help of a cluster of calculation, it was also improved with the possibility of managing the reentry or ejection of

Table 1. Scheduler of the breakup event in the GEO region.

Date	Type	Name	a	e	i	Ω	ω	M	Mass
1978/06/25	Explosion	EKRAN 2	42,163.500	$1{,}779 \cdot 10^{-4}$	0.100	78.390	325.277	78.390	1750
1992/02/21	Explosion	OV2-5 R/B	41,826.000	$8{,}488 \cdot 10^{-3}$	11.900	21.802	076.279	284.560	2500
2016/01/16	Explosion	PROTON-M/BRIZ-M	40,979.800	$2.868 \cdot 10^{-2}$	0.174	135.143	5.856	221.106	1600
2016/06/29	Explosion	BEIDOU G2	42,138.874	$8.934 \cdot 10^{-3}$	4.716	61.365	195.139	164.399	3060

space debris pieces and also by including the possibility of managing explosion or collision events planned. The last ones with the help of a file containing different information such as date, type of object, position of the parent(s) body(ies), and the scale factor necessary to use the breakup model. In our study case of GEO population we use a file with four reentries summarized in Table 1.

The simulation starts in 1978, and it is performed until the current date taking into account all different forces described previously except the atmospheric drag. At the end, we obtain a file containing all the objects created, their final state vector, and their area-to-mass ratio. In Figure 3, we plot the inclination versus the argument of the ascending node for the TLE debris population and the artificial population described previously.

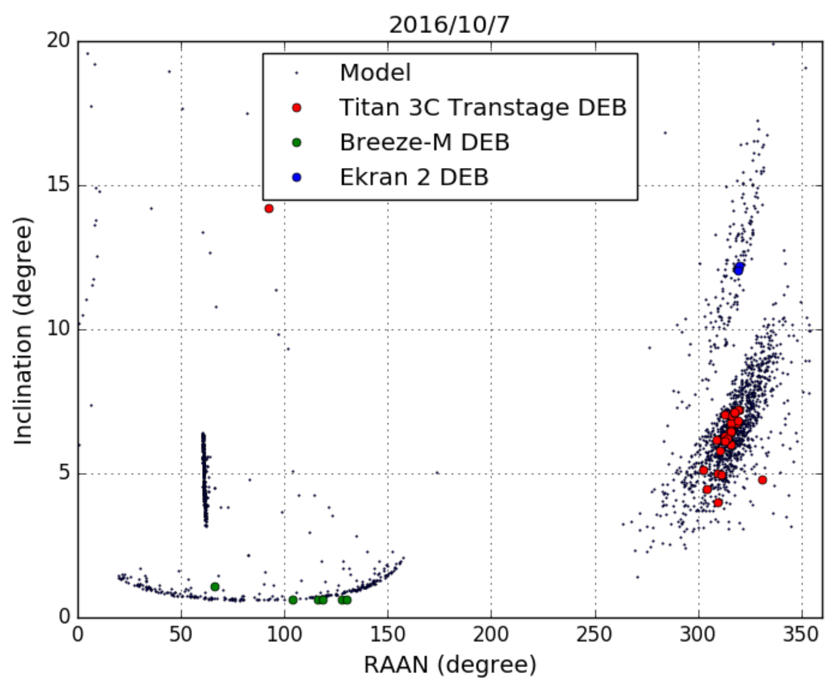

Figure 3. Inclination versus Right Ascension of the Ascending Node representation of 50 objects cataloged by USSTRATCOM, and the artificial population simulated

We find good agreement between the small TLE clusters and the simulated population. This method allows us to check several hypotheses such as an unknown fragmentation. However, with the new surveys performed last decade the number of catalogued objects grows. We propose to use new statistical means, to infer a lager synthetic population, closer to the real one, and constrained by the observations.

INFERENCE OF THE GEO POPULATION WITH THE IPF PROCESS

In this section we introduce an innovative way to create a synthetic population of space debris from a simulated catalog and using constrains from the real survey. The simulated catalog is created with the method previously explained. It is imperfect because some aspects like ejection velocities of the fragments, or the total number of objects created are unknow. However, real data can be used with an Iterative Proportional Fitting procedure to constrain the simulated catalog and at the end the new synthetic population will converge towards the statistical charateristics of the real population.

Iterative Proportional Fitting procedure

The most widely used and mature deterministic method to allocate individuals to zones is the Iterative Proportional Fitting (IPF) procedure. This method has a long history: it was demonstrated by Deming and Stephan in 1940 for estimating internal cells based on known marginals.[12] The theory behind IPF doesn't restrict the method to allocation into real geographical zones. Indeed, we could use the same process to classify objects to categories, such as planets, debris and satellites for example. In this study, the individuals are space debris.

The IPF method has two types of data as input:

- an initial individual population, containing all needed characteristics exept the "zone" (this can be a sample or a part of the population);
- the distributions of each characteristic per zone (these are called the constraints).

The goal of this method is to consider the correlations between the characteristics thanks to the non localised data, in such a way that the main characteristics of the artificial population mimics the characteristics of the real and complete one (thanks to the constraints). In the following section, we briefly describe how to design the new population (virtual pieces of space debris) by means of the IPF method with a small survey.

Creation of a synthetic population though an IPF method

In this section we present the different steps that are required to design the virtual population from the real survey.

Step 1. Transform the individual's characteristics into categorical variables. The initial population is taken from the fragment associated to fragmentation of the upper-stage Titan 3C. We count 104 pieces of debris whose characteristics are completely known. The 7 variables of the problem are: the 6 classical orbital elements (semi-major axis, eccentricity, inclination, right ascension of the ascending node, argument of perigee, mean anomaly) plus the area-to-mass ratio of each individual.

The first step of the problem is to allocate this information into different categories, to be able to create a contingency table. For example (similar with each variable of the problem) the semi-major axis can be divided in four different regions, meaning that if we take one piece of debris of the initial population it is possible to allocate the semi-major axis to one of the four regions. Consequently, we are able to "discretize" the problem since from now on the semi-major axis is not a real number

Space Debris ID	sma	inc	RAAN	A/m
1	1	2	1	1
2	4	1	1	1
3	1	3	3	2
⋮	⋮	⋮	⋮	⋮
104	1	2	4	3

Table 2. Allocate the individuals into different regions. Semi-major axis, inclination and RAAN are allocated in four different regions, while area-to-mass ratio is classified into three different regions

anymore, it is just an integer from 1 to 4, which means that it is located in region 1, region 2, region 3 or region 4.

This idea is applied to the different variables of the problem; the inclination and the right ascension of the ascending node are also allocated into 4 different regions, while the area-to-mass ratio is allocated in just 3 different regions. For the moment we do not consider the eccentricity, the argument of perigee, and mean anomaly as variables of the problem in order to simplify the methodology and to have a simple example of usage.

The result after this allocation is a matrix where all the data are stored, containing the categorical information of each individual. This matrix has 104 rows (number of pieces of debris) and 4 columns indicating the characteristics of the piece of space debris. Furthermore, the number of different regions can be enlarge as much as required (please note that the more you add classes, the more objects you will need to avoid the zero-cell problem[12]). The reason of these limitations is because we are dealing with a preliminary version of the methodology. The format of the information is stored in a similar way as presented in Table 2. The final idea is to calibrate the initial simulated population (add or reduce the number of debris with particular characteristics).

Step 2. Generating the contingency table. Once the individual characteristics are allocated into different zones it is time to generate the contingency table. This table is useful to count the number of individuals that presents a particular characteristic. The contingency table π is a 4-dimensional matrix (4 variables) and each cell counts the number of individuals that have a particular characteristic. For example, the number of objects whose semi-major axis is of type 1, the eccentricity is of type 1, the right ascension of the ascending node is of type 1, and the area-to-mass ratio is of type 1, are stored in the cell (1,1,1,1) of this matrix. As an example, if the contingency table matrix indicates that $\pi_{i,j,k,l}$ = 5 means that there are 5 objects whose semi-major axis is of type i, inclination of type j, right ascension of the ascending node of type k, and area-to-mass ratio of type l. Again, in this preliminary version of the IPF method, the contingency table is a 4-dimensional table since we are dealing with 4 variables. If the number of variables is enlarged to 7, the cross-table will be a 7-dimensional table.

Step 3. Statistical information of the simulated zone(s) (constraints). We need to relate each variable of the problem to a particular distribution*. This way, the new population characteristics will be according to the real population. For example, in this particular example we will have a constraint for the semi-major axis distribution. For this, we use a known distribution (from survey

*Note that in this study, we use the IPF for only one "zone" or simulations). For example, the semi-major axis distribution is fitted with a Normal distribution law, while the area-to-mass ratio is fitted with a Lognormal distribution law (see Figure 4).

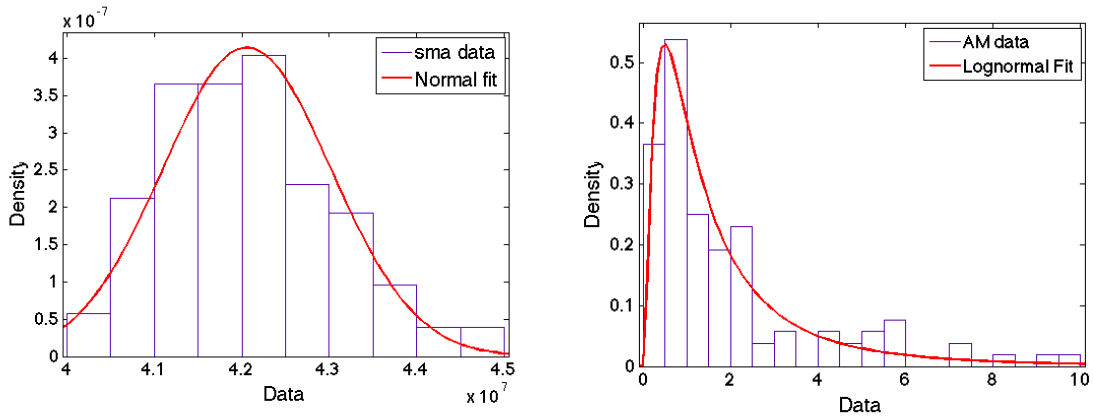

Figure 4. ***Left.*** **Semi-major axis fitted with a Normal distribution** $N(\mu, \sigma^2)$**, where** $\mu = 4.20569 \times 10^7$ **km, and** $\sigma = 963212$ **km.** ***Right.*** **Area-to-mass ratio fitted with a Lognormal distribution** $Log - N(\mu, \sigma^2)$**, where** $\mu = 0.216305\ m^2/s$**, and** $\sigma = 0.96853$ m^2/s

Step 4. Creation of the new population. Given a number of artificial objects (500 in our example), it is possible to create the characteristics of the new population according to the previous statistical information and using a Monte-Carlo method. Once the characteristics of the new population are created it is time to allocate the new population into different regions as in step 1. At this point, we know two things from the real population: the number of objects in each region and the contingency table, and from the virtual population we know the number of objects in each region.

Step 5. Applying the IPF method. We have all the required information to apply the IPF method and infer the contingency table of the synthetic population of space debris. Let $\pi^0_{i,j,k,l}$ the initial value of the cell of coordinate (i,j,k,l). The fitting process is performed on each dimension. To simplify, we explain the IPF method only for two dimensions i, j. We start with a i-fitting implemented as :

$$\pi^t_{i,j,k,l} = \pi^{t-1}_{i,j,k,l} \frac{m_{i+}}{\pi^{t-1}_{i+}} \tag{4}$$

where π^{t-1}_{i+} is the summation of cell of the contingency table at the $t-1$-th iteration, and with the index i, and m_{i+} is the constraint for the i-characteristic. The j-fitting is implemented as :

$$\pi^t_{i,j,k,l} = \pi^{t-1}_{i,j,k,l} \frac{m_{+j}}{\pi^{t-1}_{+j}} \tag{5}$$

where π^{t-1}_{+j} is the summation of cell of the contingency table at the $t-1$-th iteration, and with the index j, and m_{+j} is the constraint for the j-characteristic.

We iterate the IPF until convergence is reached, i.e. when the distance between the new contingency table and the contingency table at the last iteration is smaller than a parameter $\epsilon = 10^{-7}$. We compute the previous distance by using the following equation :

$$D(\pi^t_{i,j,k,l}, \pi^{t-1}_{i,j,k,l}) = \sum_{i,j,k,l} |\pi^t_{i,j,k,l} - \pi^{t-1}_{i,j,k,l}| \tag{6}$$

with $\pi^t_{i,j,k,l}$ the contingency table at the t-th iteration and i, j, k, l are the four variables of our problem, as explained previously. We plot the evolution of the distance in Figure 5, which reaches a convergence criterion at the iteration 61.

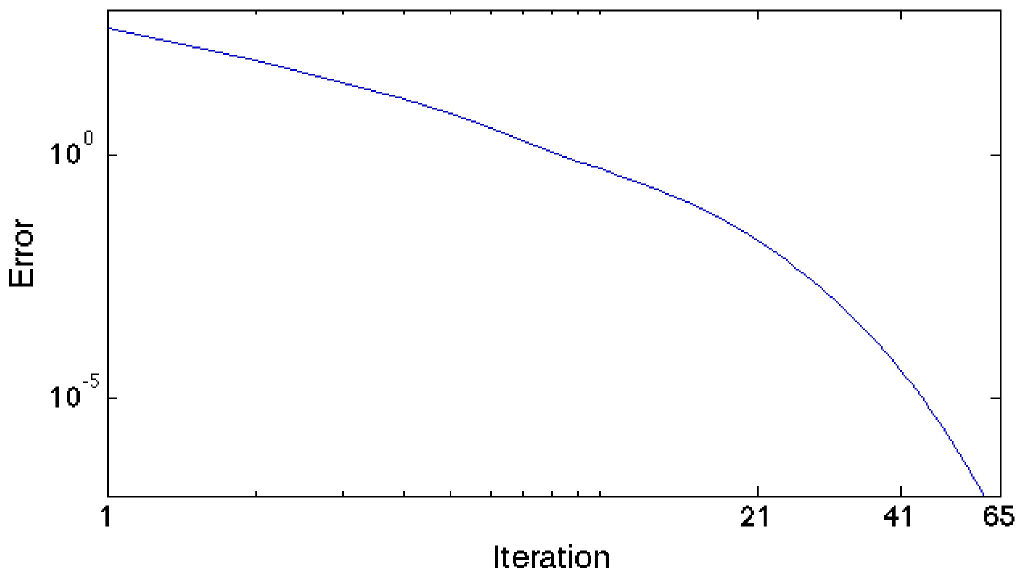

Figure 5. Convergence of the IPF method with a log scale.

Step 6. Truncate, Replicate and Sample. The new contingency table have real numbers but integers are required. For this reason we apply the Truncate, Replicate and Sample method[18] to transform each real vaue in each cell in an integer number and at the end, a total population of 500 items.

Step 7. Create the new population. Once the contingency table of the new population is designed with integer numbers it is time to create the new population with a Monte Carlo method. The final value of a variable in chosen randomly in the range of the type associated.

The flowchart shown in Figure 6 summarizes all the process.

Example of creation of a new population of Space Debris

After the explanation of how to build a synthetic population, it is time to apply the method to a particular population of space debris. Then, it is considered an initial set of 104 pieces of space debris obtained by the simulation show in Figure 3. To obtain the constrains we infer the new distribution from TLE data and after that we apply the IPF method. We can see the convergence of the process in Figure 5. At the end we have a new synthetic population adjusted on the observational data. In Figure 7 we plot the simulated population of space debris (104 pieces of debris), plus the synthetic population (500 pieces of debris) generated thanks to the IPF method.

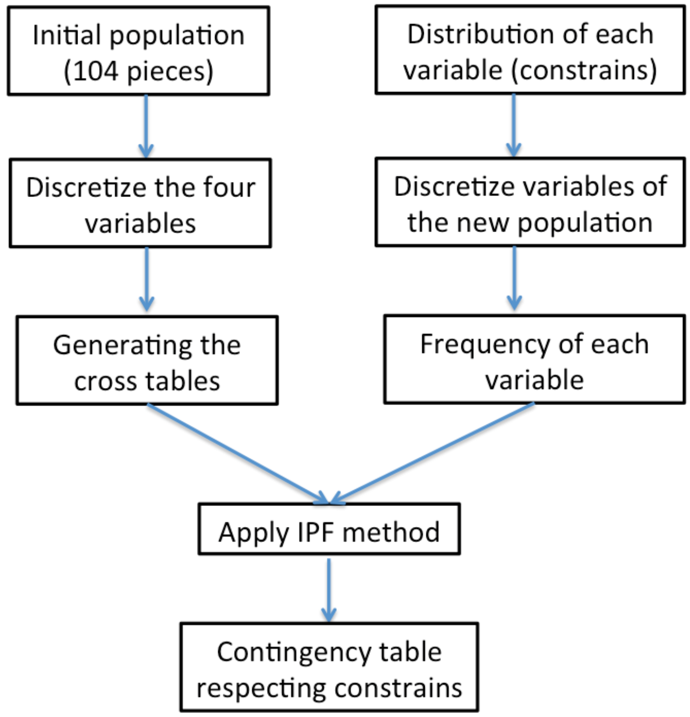

Figure 6. Flowchart indicating the applied method to create the new population of space debris from a particular survey.

CONCLUSION

Many efforts have been made in the last two decades to improve models and optical survey to better known the population of space debris at the GEO region. We present the main features of the space debris populations and our means to model them. In particular, we produce a deterministic simulation using different tools developed at the University of Namur like the orbit propagator NIMASTEP and our implementation of the NASA Breakup model. Using historical data of fragmentations in the GEO region, we find an artificial catalog of space debris objects in accordance with the TLE catalog.

In a second part we provide an innovative method to improve the simulation of space debris population by the creation of a synthetic population, which is constrained by observational data. Indeed, some aspects of a deterministic simulation by means of a breakup model given discrepancy with the reality. The IPF process correct the simulated catalog to give a synthetic population with statistical characteristics closer to the real one.

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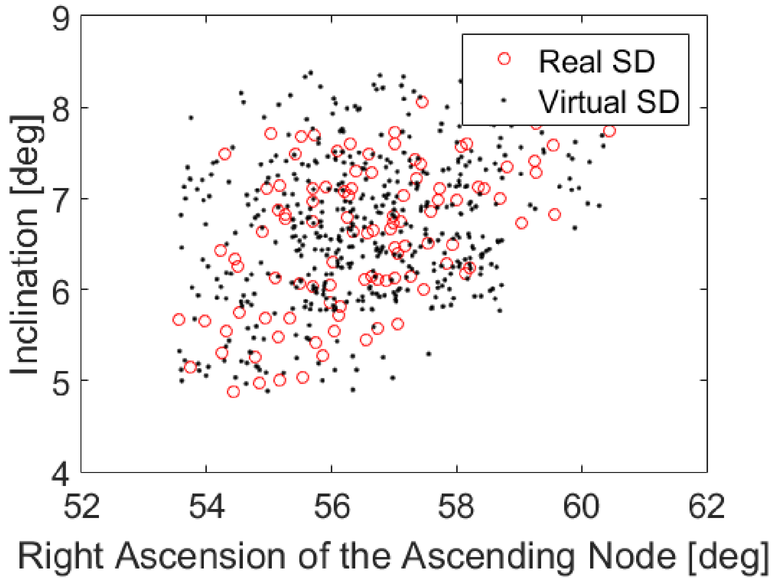

Figure 7. Inclination versus Right Ascension of the Ascending Node of the simulated population of debris (circles), plus the synthetic population of space debris generated by the IPF method (black dots).

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