Thermal modeling, analysis and control using an electrical analogy

Edgar Ramírez-Laboreo*, Carlos Sagüés* and Sergio Llorente†
*Departamento de Informática e Ingeniería de Sistemas (DIIS) and
Instituto de Investigación en Ingeniería de Aragón (I3A),
Universidad de Zaragoza
Zaragoza 50018, Spain
Email: ramirlab@gmail.com, csagues@unizar.es
†Research and Development Department, Induction Technology,
Product Division Cookers, BSH Home Appliances Group,
Zaragoza 50016, Spain
Email: sergio.llorente@bshg.com

***Abstract*—Modeling and identification of thermal systems is a problem frequently treated in theoretical and application domains. Most of these systems have been modeled using black-box structures whose parameters are identified using temperature measurements. Although black-box models have achieved good results in terms of temperature evolution, they cannot model variables which had not been measured in the identification test. In this article we present a new method to build grey-box thermal models based on electrical equivalent circuits which not only give information about temperatures evolution, but also about heat fluxes and thermal energy stored in the system. The partially unknown parameters of the models are identified using temperature measurements and applying nonlinear optimization techniques. The obtained state space representation can be used to develop a deterministic state space temperature controller that provides better accuracy than classical PID controllers. Our proposal is complemented with various examples of a real application in an electric oven.**

I. Introduction

Reducing world energy consumption has become one of the main problems of human beings due to economical, environmental and sustainability reasons. Thermal systems like ovens, refrigerators or heating systems, which are widely used in industry and domestic environments, represent a large proportion of this energy consumption.

System identification is one of the fields from which thermal systems can be analyzed. Developing a good model allows the designer to build controllers for higher performance in terms of response time, accuracy or energy consumption. Depending on its structure, the model can also be used to analyze the system internal behavior and even to propose mechanical improvements or novel designs.

Different approaches have been adopted to build models of thermal systems. Black-box structures based on linear regressions, like *ARX* or *ARMAX*, have been used obtaining simple and precise models [1], [2]. Other authors have used grey-box continuous-time models based on the heat equation [3], [4] with similar results. Nevertheless, none of the already presented structures are adequate to study the internal behavior of the system. Some of the models have a physical basis, but the fact is that they cannot model certain variables of interest such as heat fluxes or thermal energy stored in the system. These variables are very useful to find the critical components and to study possible mechanical changes that reduce the system energy consumption.

In this article we use the thermal-electrical analogy to propose a new sort of models to give deeper information about the intrinsic elements of the system. These models provide useful data that other structures are unable to obtain, such as heat fluxes, energy stored in the different zones of the system or losses to the ambient, and they allow the engineer to analyze in a qualitative way the changes that a mechanical modification would produce. In addition, they permit the design of state space controllers because they include information about their internal behavior.

The analogy and the proposed modeling method are presented in section II. The building process has been structured into three simple steps that permit the designer to develop a thermal model as complex as desired. Then, another method is presented to automatically obtain its state space description. In section III some identification aspects are treated. The identifiability of our electrical diagrams is a difficult problem if many temperatures are to be modeled. However, if the model is small enough, it is possible to use differential algebraic methods and some recommendations are given in order to simplify the analysis. Nonlinear optimization algorithms must be used to identify the model parameters. We propose to use the Interior Point Method and we include some advice for obtaining a coherent model. In section IV we present two types of analyses that may permit the proposal of new designs or structural changes to improve the system. These analyses are only possible with our modeling method since there is a direct relationship between the elements of the electrical diagram and the components of the actual system. Finally, in section V we point up another benefit of our proposal: the possibility of designing a state space controller.

II. MODELING

A. Electrical analogy

The novel idea presented in this article is to build grey-box models based on the analogy that exists between thermal systems and electrical circuits. This analogy has been already explained (but not very widely) in some heat transfer textbooks [5] and it is basically sustained in the similarity of two pairs of thermal and electrical equations. The first pair consists of the equation that describes the charge of a capacitor (1) and the heat equation applied to a body with homogeneous temperature distribution (2).

$$C\frac{\mathrm{d}V(t)}{\mathrm{d}t} = I(t) \tag{1}$$

$$mc_p\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \dot{Q}(t). \tag{2}$$

The left-hand side of (1) describes the charge variation of the capacitor, which is proportional to the capacitance C and the derivative of its voltage V, and the right-hand side is the electric current I needed to produce this variation. In (2), the left-hand side of the balance represents the energy variation of the body, which is proportional to the mass of the body m, the specific heat of the material c_p and the derivative of its temperature T, and the right-hand side is the heat flux $\dot{Q}$ needed to produce this variation.

The second pair of equations are Ohm's law (3) and the one-dimensional form of Fourier's law (4).

$$I(t) = \frac{1}{R}V(t) \tag{3}$$

$$\dot{Q}(t) = \frac{kA}{l}\Delta T(t). \tag{4}$$

Ohm's law (3) states that the current I that flows between two points of a conductor is proportional to the potential difference V between these two points. The proportional constant is the electrical resistance R. Similarly, Fourier's law (4) states that the heat flux $\dot{Q}$ between two connected bodies is proportional to their temperature difference. The proportional constant is made up of the thermal conductivity k of the medium between the bodies, the distance l between them and the heat transfer area A.

The previous equations show that it is possible to establish an equivalence between electrical networks and those thermal systems which are directed by conduction heat transfer. In other words, a thermal system could be modeled using an electrical diagram. The components of the system would be represented as capacitors and the mediums between them would be included as resistances. These elements would not have electrical but thermal sense, and the variables of the electrical diagram would not be currents and voltages, but heat fluxes and temperatures, respectively.

The relationship between heat flux and temperature is also proportional if the heat transfer phenomenon is convection instead of conduction:

$$\dot{Q}(t) = hA\Delta T(t). \tag{5}$$

In this case, the proportional constant is composed of the convection coefficient h and the heat transfer surface area A. Given that (5) is also equivalent to (3), the electrical analogy is still valid.

The relationship between heat flux and temperature difference is completely different if the heat transfer phenomenon is radiation. In this case, heat flux is given by the Stefan-Boltzmann law (6), which is not a proportional relationship.

$$\dot{Q}(t) \propto \left(T_1(t)^4 - T_2(t)^4\right). \tag{6}$$

Consequently, the electrical analogy is not strictly valid if radiation is the dominant phenomenon. However, it can always be used as a linear approximation (with its corresponding limitations). As a final remark, it must be said that there exist many thermal systems in which the electrical analogy should be used very carefully. This is the case of systems in which mass movement or phase transition are not negligible. Some of these problems may be solved by adding disturbances to the model.

B. Diagram building method

In this section, a method to easily build an electrical network equivalent to a thermal system is presented. The diagram building process has three steps:

1) For each temperature to be modeled, include a capacitor in the diagram. The selection of the number of temperatures needed to be modeled is critical. A low number of temperatures would give a simple model, but it would not be accurate. On the other hand, a model with an excessive number of temperatures would be more accurate, but it would be more complex to be identified.
2) For each capacitor of the model, select one of its terminals and connect it to ground (temperature zero). Let this terminal be named *ground* terminal, while the other terminal will be the *temperature* terminal. Then, for every feasible thermal connection, place a resistance between the *temperature* terminals of thermally connected capacitors.
3) Finally, system inputs and disturbances can be either heat fluxes or temperatures. On the electrical diagram, place a power source for every external temperature input or disturbance, and a current source for every boundary heat flux. Then, connect these inputs and the capacitors by means of the corresponding resistances.

Example 1. To illustrate the method, a model of an electric oven has been developed. A real oven is a complex thermal system with non-homogeneous temperature in its interior, which may lead to infinite discrete temperatures. However, three characteristic temperatures have been identified: the heating element temperature, the internal cavity temperature and the external components temperature. As a result, the proposed model includes just those three temperatures. The heat flux produced by the heating element acts as input of the system and the losses to the ambient are the only existing disturbance.

The model diagram which contains all the previously explained elements is shown in Fig. 1. The current source p models the heat power produced by the heating element. This power is used to firstly increase the temperature of the heating element itself, modeled as the capacitor C_1. Then, through the resistance R_1, the heating element increases the temperature of the internal cavity, which is modeled as C_2. This component loses some energy to the external components of the oven, modeled as C_3, through the thermal resistance R_2. Finally, a fraction of the power is lost to the ambient temperature T_{amb} through R_3. Although the diagram represents p and T_{amb} as direct sources, they may be time-dependent.

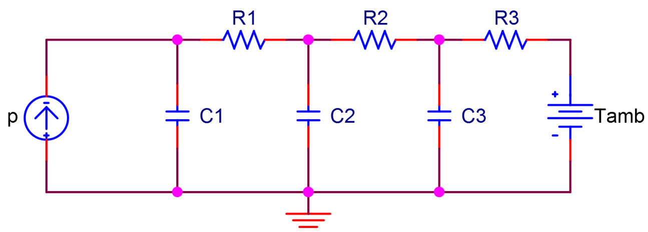

Fig. 1. Electrical equivalent circuit proposed for an electric oven. Capacitors, resistances and sources are directly related to the actual components of the system.

C. State space description

The proposed modeling method leads to models that are not linear regressions. However, an automatic method to build its continuous-time state space description (7) is presented in this section.

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + B_P\mathbf{p}(t). \tag{7}$$

Remember that this description of the model presents the derivative of the state vector $\dot{\mathbf{x}}$ as the sum of three terms: the product of state matrix A and state vector $\mathbf{x}$, the product of input matrix B and input vector $\mathbf{u}$ and the product of disturbance matrix B_P and disturbance vector $\mathbf{p}$.

Some previous definitions are needed to explain the state space description building process. Let C_1, C_2, ..., C_n be the thermal capacitors of the model. Let u_1, u_2, ..., u_m be the inputs and p_1, p_2, ..., p_q the disturbances. Let R_{ij} be the thermal resistance placed between the *temperature* terminals of capacitors C_i and C_j. Notice that $R_{ij} = R_{ji}$. If two capacitors C_i and C_j are not connected, consider that $R_{ij} = \infty$. Let R_{iuj} be the thermal resistance placed between the *temperature* terminal of C_i and input u_j. If a capacitor C_i is not connected to input u_j, keep in mind that $R_{iuj} = \infty$. Let also α_{iuj} be a coefficient which takes the value 1 if the capacitor C_i is connected to input u_j, either by means of a resistance or directly, and the value 0 in other case. In the same way, let R_{ipj} be the resistance between the *temperature* terminal of C_i and disturbance p_j. If a capacitor C_i is not connected to disturbance p_j, remember that $R_{ipj} = \infty$. Let also α_{ipj} be a coefficient with value 1 if C_i is connected to disturbance p_j and 0 in other case.

Our method requires that T_1, T_2, ..., T_n, which are the temperatures given by the *temperature* terminals of capacitors C_1, C_2, ..., C_n, are selected as state variables. It is also necessary that each external heat flux is thermally connected to only one capacitor. This condition is common in most thermal systems and provides simpler expressions. If these conditions are satisfied, the elements in row i and column j of matrices A, B, and B_P, which will be named a_{ij}, b_{ij} and b_{Pij}, respectively, can be obtained through the following expressions:

$$a_{ij} = \begin{cases} -\dfrac{1}{C_i}\left(\displaystyle\sum_{\substack{k=1\\k\neq i}}^{n}\frac{1}{R_{ik}} + \sum_{k=1}^{m}\frac{1}{R_{iuk}} + \sum_{k=1}^{q}\frac{1}{R_{ipk}}\right), & \text{if } i = j \\ \dfrac{1}{C_iR_{ij}}, & \text{if } i \neq j \end{cases} \tag{8}$$

$$b_{ij} = \begin{cases} \dfrac{1}{C_iR_{iuj}}, & \text{if } u_j \text{ is a temperature} \\ \dfrac{\alpha_{iuj}}{C_i}, & \text{if } u_j \text{ is a heat flux} \end{cases} \tag{9}$$

$$b_{Pij} = \begin{cases} \dfrac{1}{C_iR_{ipj}}, & \text{if } p_j \text{ is a temperature} \\ \dfrac{\alpha_{ipj}}{C_i}, & \text{if } p_j \text{ is a heat flux.} \end{cases} \tag{10}$$

Example 2. Back to the oven model of Fig. 1, the first step is to choose the temperatures given by thermal capacitors C_1, C_2 and C_3 as state variables. Let T_1, T_2 and T_3 be these temperatures:

$$\mathbf{x} = (T_1,\ T_2,\ T_3)^T.$$

Considering that p is the only input and T_{amb} the only disturbance, the state, input and disturbance matrices can be obtained using (8), (9) and (10), respectively:

$$A = \begin{pmatrix} \dfrac{-1}{C_1R_1} & \dfrac{1}{C_1R_1} & 0 \\ \dfrac{1}{C_2R_1} & \dfrac{-1}{C_2}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right) & \dfrac{1}{C_2R_2} \\ 0 & \dfrac{1}{C_3R_2} & \dfrac{-1}{C_3}\left(\dfrac{1}{R_2}+\dfrac{1}{R_3}\right) \end{pmatrix}$$

$$B = \begin{pmatrix} \dfrac{1}{C_1} \\ 0 \\ 0 \end{pmatrix} \qquad B_P = \begin{pmatrix} 0 \\ 0 \\ \dfrac{1}{C_3}\dfrac{1}{R_3} \end{pmatrix}.$$

III. IDENTIFICATION

A. Identifiability of the model

Model identifiability is a critical property that consists of two concepts. First, the possibility to reach a unique value of the model parameters in the identification process. And secondly, the validity of the identified model with respect to the real system.

Modeling a system directed by physical processes usually leads to continuous-time state space descriptions whose identifiability is not easy to analyze [6]. A method based on Ritt's algorithm [7] was proposed by Ljung and Glad [8]. However, the authors explain that the complexity of the problem increases rapidly with the size of the model and their implementation finds difficulties if the number of parameters and non-measurable variables exceeds 10.

For this reason, the identifiability of models based on the thermal-electrical analogy could still be a problem. In any case, if the system is small enough, it is possible to use the method of [8]. Then, two recommendations should be taken into account before executing Ritt's algorithm:

1) The method is simpler if conductances are used instead of resistances. The original parametrization in resistances and capacitances is nonlinear in parameters. However, this simple change of variables transforms the model equations obtaining a system which is linear in parameters. This property is beneficial to reduce the time complexity of Ritt's algorithm. Remember that conductance G is the inverse of resistance R.
2) It is desirable to use the following ranking:
$$u^{(\mu)} < y^{(\nu)} < G^{(o)} < C^{(\pi)} < x^{(\sigma)} \tag{11}$$
where u represents the group of measurable inputs and disturbances, y the group of measurable state variables and outputs, G the conductances, C the capacitors and x the group of non-measurable state variables, inputs and disturbances, for all derivative orders μ, ν, o, π and σ.

Example 3. In the electric oven model, this set of differential polynomials is obtained if resistances are substituted by conductances:

$$\begin{cases} C_1\dot{T}_1 &= -G_1T_1 + G_1T_2 + p \\ C_2\dot{T}_2 &= G_1T_1 + (G_1 + G_2)T_2 + G_2T_3 \\ C_3\dot{T}_3 &= G_2T_2 + (G_2 + G_3)T_3 + G_3T_{amb} \end{cases}$$

$$G_1 = \frac{1}{R_1}, \quad G_2 = \frac{1}{R_2}, \quad G_3 = \frac{1}{R_3}.$$

Suppose that it is possible to measure T_1, T_2, T_3, p and T_{amb}. According to (11), one of the possible rankings is selected:

$$p < T_{amb} < T_1 < T_2 < T_3$$
$$< G_1 < G_2 < G_3 < C_1 < C_2 < C_3.$$

Then, applying Ritt's algorithm, we obtain the characteristic set of differential polynomials of the model. The polynomials of the set which depend on the measurable variables and the unknown parameters (B_i) are presented below:

$$\begin{aligned} B_1 =&(T_1\ddot{T}_1 - \dot{T}_1^2 + \dot{T}_2\dot{T}_1 - T_2\ddot{T}_1)G_1 + \dot{T}_1\dot{p} - \ddot{T}_1 p \\ B_2 =&(\dot{T}_1\dot{T}_2 - T_1\ddot{T}_2 + T_2\ddot{T}_2 - \dot{T}_2^2)G_1 + \\ &+ (T_2\ddot{T}_2 - \dot{T}_2^2 + \dot{T}_3\dot{T}_2 - T_3\ddot{T}_2)G_2 \\ B_3 =&(\dot{T}_2\dot{T}_3 - T_2\ddot{T}_3 + \dot{T}_3^2 - T_3\ddot{T}_3)G_2 + \\ &+ (\dot{T}_3^2 - T_3\ddot{T}_3 + \dot{T}_{amb}\dot{T}_3 - T_{amb}\ddot{T}_3)G_3 \\ B_4 =&(T_2 - T_1)G_1 + p + (-\dot{T}_1)C_1 \\ B_5 =&(T_3 - T_2)G_2 + (T_1 - T_2)G_1 + (-\dot{T}_2)C_2 \\ B_6 =&(T_3 + T_{amb})G_3 + (T_2 + T_3)G_2 + (-\dot{T}_3)C_3. \end{aligned}$$

Note that all B_i are of order 0 and degree 1 in the leading parameter. This corresponds to the second situation described in [8]; therefore we can conclude that the model developed for the oven is globally identifiable.

B. Identification method

Except for some specific cases, there are not analytical methods to identify models which are based on physical processes because of their intrinsic structure. For this reason, nonlinear methods must be used to determine the optimum value of the parameters. In this article we recommend the use of the Interior Point Algorithm since it can be applied to minimize almost every error function. This method is implemented in many mathematical programs such as *MATLAB*.

The Interior Point Algorithm does not find global minimums, but local ones. As a consequence, the initial value of the parameters is critical. A preliminary study of the thermal properties of the system may provide useful information to properly set this value. For example, if one of the capacitors of the diagram is modeling a steel ($c_p \approx 0.47$ J/g·K) solid with an approximate mass of 1 kg, it is reasonable to use $C = 470$ J/K as initial value. The method permits the use of linear and nonlinear constraints. In order to obtain a physically coherent model, some of them must be introduced. For every thermal model, negative capacitors and resistances do not have physical sense. Additionally, depending on the specific properties of the model, more constraints may be introduced by the user.

Example 4. Experimental data from the real oven has been obtained in order to determine the value of the parameter set. Temperatures T_1, T_2 and T_3 as well as heating power p and temperature T_{amb} have been measured and registered during a 2 hour test.

The electric oven diagram includes three thermal capacitors that match with three specific components of the oven: the heating element, the internal cavity and the external components (mainly metal sheets). According to the estimated mass and specific heat of those elements, three initial values have been set for capacitors C_1, C_2 and C_3 (table I). The relationship between thermal resistances R_1, R_2 and R_3 and the components of the real oven is less clear. Resistance R_1, which is placed between the capacitors that model the heating element and the internal cavity, may be related with convection and radiation phenomena. R_2, which is between the internal cavity and the external parts, is probably related to the insulating material. Finally, R_3 is likely to be related with various heat transfer phenomena depending on the location of the oven. In any case, there is not too much information to estimate the values of these parameters. For this reason, their initial values have been set to 1 K/W.

The Interior Point Algorithm has been run in *MATLAB* to determine the optimum value of the parameters. The cost function used for this example was a weighted sum of the root mean squared errors of the three state variables (T_1, T_2 and T_3). Some numerical results are included in tables II and III. The adjustment of temperature T_2 is shown in Fig. 2.

Two conclusions are drawn from the results. Firstly, that the performance of the model is very good, basically because it is based on the same physical processes that control the real system. And, secondly, that the Interior Point Method works properly although some initial values are considerably far from the optimum.

TABLE I. INITIAL VALUE OF CAPACITORS C_1, C_2 AND C_3.

Capacitor	Modeled component	Estimated mass (kg)	Estimated specific heat (J/g·K)	Initial value (J/K)
C_1	Heating element	0.2	0.47	94
C_2	Internal cavity	4	0.47	1880
C_3	External parts	10	0.47	4700

TABLE II. IDENTIFICATION RESULTS. ERRORS.

Temperature	$RMSE$ (°C)	T_{max} (°C)	T_{min} (°C)	$\frac{RMSE}{T_{max}-T_{min}}$
T_1	19.4	699	20	2.9%
T_2	7.2	261	20	3.0%
T_3	1.9	75	20	3.5%

$RMSE$: Root Mean Squared Error

TABLE III. IDENTIFICATION RESULTS. PARAMETER SET.

Parameter	Initial value	Optimum value
C_1	94 J/K	112 J/K
C_2	1880 J/K	3690 J/K
C_3	4700 J/K	4590 J/K
R_1	1 K/W	0.228 K/W
R_2	1 K/W	0.181 K/W
R_3	1 K/W	0.252 K/W

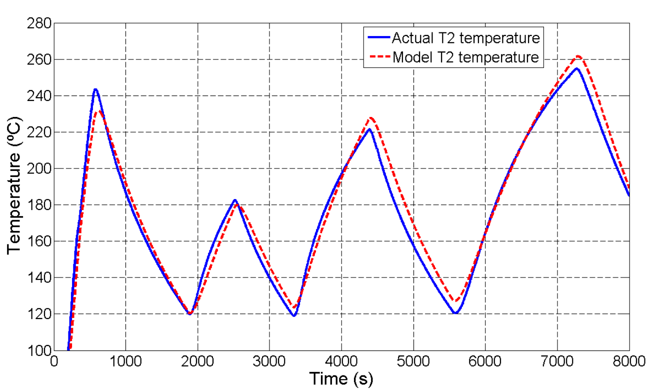

Fig. 2. Actual temperature T_2 and its corresponding model temperature.

IV. ANALYSIS

One of the main advantages of our proposal is that the identified models can be used to analyze the system. The analyses may be divided in two groups.

A. *Heat fluxes and stored energy*

Heat fluxes between the mechanical components of the actual system may be estimated with the electrical equivalent circuit. Remember that the heat flux $\dot{Q}$ between two connected points is proportional to their temperature difference ΔT and inversely proportional to the thermal resistance R_t of the medium between them (12).

$$\dot{Q}(t) = \frac{1}{R_t}\Delta T(t). \tag{12}$$

Then, if two components are modeled as thermal capacitors C_i and C_j, the heat flux through resistance R_{ij} of the model can be used as a good estimation of the real one. Losses to the ambient temperature may be calculated with the same procedure. The capacitors of the model can also be used to estimate the thermal energy stored in the system. Similarly to an electric capacitor, the thermal energy Q stored in a thermal capacitor is equal to the product of its thermal capacitance C_t and its temperature T (13).

$$Q(t) = C_t T(t). \tag{13}$$

Consequently, the thermal energy of a mechanical component which has been modeled as a thermal capacitor may be estimated by this method.

Heat fluxes and energies are essential in order to study whether the system is energetically good enough or not. The advantage of our proposal in this field is then evident since these variables are unable to be obtained with other model structures and even from experimental tests.

B. *Sensitivity analysis*

The identified model can also be used to study the influence of the parameter set. The designer may analyze the performance of the system when some specific thermal resistances or capacitors are increased or decreased. The analysis may be focused on the system energy consumption because of its importance, but it can also be centered on other facets such as the maximum temperatures or the response time.

Considering the direct relationship between the elements of the electrical diagram and the components of the actual system, the information given by this analysis may allow the engineer to identify the critical parts and, consequently, to suggest some mechanical changes that improve the system operation.

V. CONTROL

Finally, we want to highlight that the proposed model structure is also suitable for designing a state space temperature controller. Observability and controllability must be studied to determine the feasibility of this type of control technique. If possible, the performance of the system is likely to be improved respect to classical PID controlled systems.

Example 5. The electric oven model has been identified using data from an experimental test in which every state variable has been measured. However, in the oven real operation, only temperature T_2 is measured. Remember that this is the temperature of the internal cavity, that is the variable to be controlled in an oven.

The observability of the model has been analyzed and it has been proved that the system is observable. Then, a state observer which provides an estimation of the real value of every state variable has been designed. The observer has been improved by adding an estimator of the temperature disturbance T_{amb} which is not measured either. A feedback feedforward state space controller has been designed. While the state feedback places the closed-loop poles of the system, the reference and disturbance feedforward allows the system to reach the desired steady state behavior. The controller scheme is shown in Fig. 3.

Finally, the controller has been implemented in the real oven to validate its performance. The poles have been placed in order to obtain a response time of 400 seconds and a 5% overshooting, and the reference temperature has been set to 200ºC. The evolution of the cavity temperature is shown in Fig. 4. It can be seen that the response time has not been achieved, but the reason is that the actuator has saturated by reaching its upper limit. In any case, the overshooting and the steady state objectives have been accomplished properly.

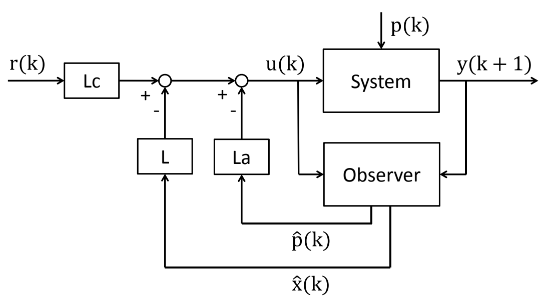

Fig. 3. State space controller with state feedback and reference and disturbances feedforward.

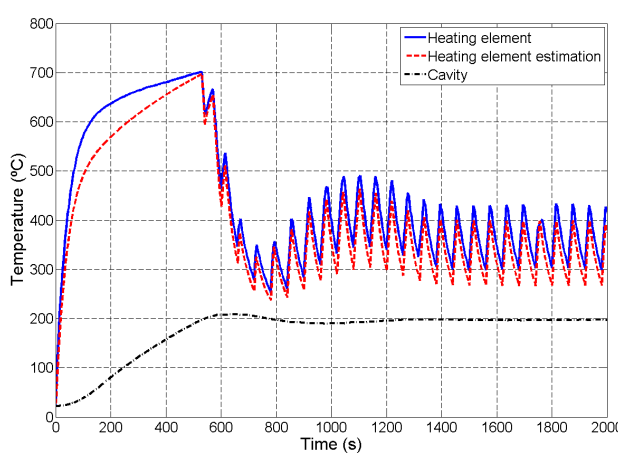

Fig. 4. Cavity and heating element temperatures during a real test with the proposed controller.

In Fig. 4 we also show the evolution of the heating element temperature and its corresponding estimation given by the observer. Although this temperature has been measured, the purpose is only to validate the observer and it has not been used to control the system. The observed ripple is caused by the use of *PWM* (*Pulse Width Modulation*) signals.

VI. Conclusions

The goal of this work has been to develop a new procedure to model thermal systems in order to analyze energy variables apart from temperatures. We have included a method to easily build the electrical equivalent diagram of a thermal system with as many temperatures as desired. Then, we have developed another method that can be used to automatically obtain its state space description. Although the identification of our models is not as easy as in linear regressions, some recommendations have been given in order to simplify the process.

Our proposal presents many benefits as it has been shown. Energy analysis, which takes an important role in the design of improved systems, is possible with our models. In addition, the state space description can be used to design advanced controllers with better performance. The applicability of the method has been demonstrated with some examples in a real electric oven.

Acknowledgment

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