Research in International Business and Finance 69 (2024) 102263

Contents lists available at ScienceDirect

Research in International Business and Finance

journal homepage: www.elsevier.com/locate/ribaf

Sequential management of energy and low-carbon portfolios

Pilar Gargallo [a], Luis Lample [b], Jesús A. Miguel [a], Manuel Salvador [a,*]

[a] Department of Applied Economics, Faculty of Economics and Business Studies, Universidad de Zaragoza, Gran Vía 2, Zaragoza 50005, Spain
[b] Department of Accounting and Finance, Faculty of Economics and Business, University of Zaragoza, Gran Vía 2, Zaragoza 50005, Spain

A R T I C L E I N F O

Keywords:
Sequential analysis
Risk management
Portfolio selection
ADCC-GARCH
EU ETS
Clean energy

A B S T R A C T

This study explores the ability of clean energy and European Union Allowance (EUA) assets to diminish portfolio risk when mixed with unclean energy assets. We use a family of Asymmetric Dynamic Conditional Correlation-Generalized AutoRegressive Conditional Heteroskedastic (ADCC-GARCH) models and provide a flexible and adaptive estimation and model selection framework based on a sequential strategy with differently sized estimation and validation windows, as well as different model update frequencies. Through this procedure, we obtain accurate estimations of the conditional covariance matrices of day-to-day asset returns and build adequate optimal minimum variance portfolios. The analyzed period (Jan. 2010–May. 2022) includes the latest crisis episodes (Sovereign debt crisis, Brexit, COVID-19, and the Russian–Ukrainian war). Our findings show that since the 2015 Paris Agreement (the only exception being the pandemic period), investing in clean energy companies and EUAs is an attractive investment in terms of return-risk. These results should provide investors with more incentives to decarbonize their portfolios.

1. Introduction

1.1. *Climate change and the urgency for sustainable solutions*

The greenhouse effect is the primary driver of climate change, leading to well-documented and detrimental consequences associated with global warming (Intergovernmental Panel on Climate Change., 2023). Taking measures is costly, but acting now will be much less expensive than in years to come. By reducing Greenhouse Gas (GHG) emissions rapidly, and on a large scale, it will be possible to limit climate change and its effects. Therefore, all countries worldwide must activate plans to cut emissions, to eradicate them, and thus stop the average temperature of the planet from increasing. The Kyoto protocol, endorsed in 1997 by the United Nations Framework Convention on Climate Change, represented the first commitment by signatory countries to reduce their average emissions by 5% in the 2008–2012 period, compared to 1990 levels. After several summits, in 2015 (COP 21),[1] the Paris Agreement was adopted, which established that growth in the worldwide mean temperature should be kept below 2°C, and it invited countries to

Abbreviations: ABEKK, Asymmetric Baba, Engle, Kraft and Kroner; ADCC, Asymmetric Dynamic Conditional Correlation; CCC, Constant Conditional Correlation; DCC, Dynamic Conditional Correlation; EU, European Union; EUA, European Union Allowance; EU ETS, European Union Emissions Trading Scheme; GARCH, Generalized AutoRegressive Conditional Heteroskedasticity; GHG, Greenhouse Gas; MSR, Market Stability Reserve; VAR, Vector Autoregressive.

* Corresponding author.
E-mail address: salvador@unizar.es (M. Salvador).

[1] The twenty-first session of the Conference of the Parties (COP)

https://doi.org/10.1016/j.ribaf.2024.102263
Received 17 July 2022; Received in revised form 29 January 2024; Accepted 31 January 2024
Available online 10 February 2024

carry out additional efforts to keep the increase below 1.5°C. In line with these global commitments, the European Union (EU) also aims to achieve climate neutrality by 2050. In December 2020, the European Council endorsed the target of reducing EU net emissions by 2030, cutting them by at least 55% compared to 1990—a necessary intermediate step to reach the 2050 goal. To undertake this, the EU has strategically focused on both the energy sector and the carbon market. This comprehensive approach seeks to reduce GHG emissions while promoting sustainable energy practices and leveraging the carbon market as a valuable tool in achieving these environmental goals.

With this awareness, it was determined that existing carbon markets needed urgent reform to favor the achievement of more ambitious mitigation measures. Among these markets is the European Union Emissions Trading Scheme (EU ETS); since its inception in 2005, it has undergone important reforms, both in scope, due to the incorporation of new sectors, and in operation, governance, and supply containment mechanisms. The EU ETS encourages EU constituents to lower emissions, and it has grown into a crucial instrument for reducing CO_2, penalizing the technologies and industries that pollute the most (Bing et al., 2015). It covers over 12,000 fixed sources, representing about 45% of the EU's total CO_2 allowances. The EU ETS sets a cap on all the carbon emission allowances assigned per year to entities covered by the system. Enterprises that do not adapt to their emission permits face severe penalties; however, a firm can emit less carbon than its target, giving it excess allowances. The fourth phase of the EU ETS, which covers 2021–2030, centers on decreasing the overall number of emission permits by 2.2% yearly. Moreover, phase 4 extends the MSR (Market Stability Reserve), in which excess permits are transferred to steady the prices. The MSR aims to eliminate in the short term the excess of emission allowances generated in the economic crisis of 2008 and subsequent years and to adjust their supply in the event of great scarcity in the long term.

With respect to the energy sector, the EU produces 75% of GHGs due to burning fossil fuels (European Commission, 2021). To successfully limit global warming, the EU needs to urgently switch to clean energy sources for transport, heating, and cooling. Renewable energies are unbeatable and inimitable resources supplied by nature, and, in contrast to fossil fuels, they do not produce GHG emissions; therefore, they do not alter the climate. Furthermore, renewables are a local resource, so they eliminate the need to import fossil fuels from oil and gas-rich countries, thus avoiding territorial and geopolitical conflicts. Following the Russian invasion of Ukraine, the need for a rapid transition toward renewables has never been more pressing and evident.

1.2. The role of finance in accelerating the sustainable energy transition

The financial sector plays a decisive role in the decarbonization of economic sectors by channeling investment flows from emitting activities to clean activities. Urgent investment is required to achieve a successful energy transition in line with the climate targets of the Paris Agreement. Therefore, the financial system and asset owners must be essential drivers in accelerating the transition to a sustainable and low-carbon economy. The international regulatory environment and EU are making progress, recognizing the need for greater transparency of investment portfolios concerning the alignment of investment strategies with the goals of the Paris Agreement. Therefore, shifting financial markets toward less risky and more sustainable assets can benefit the planet and investors.

The requirement to cut emissions does not shut out the usage of fossil fuels yet, given that we are still not ready to completely replace them with renewables; however, it demands a meaningful change of course, as the current situation is inconsistent with the required emissions cut in world energy systems. Therefore, although portfolios only composed of clean energy are still thought to be unprofitable, investment portfolios that at least combine fossil fuels and renewables are needed if we want to help fight climate change and support a sustainable energy transition. Expanding the portfolio investment strategy to include clean energy can provide appealing returns and help to address the climate emergency.

1.3. The carbon market: a financial investment opportunity

In addition to renewables, carbon rights have become indispensable assets to fight climate change for investment purposes. These carbon allowances were initially planned as an economic stimulation to effectively stem GHG emissions. However, now they imitate some features of investment products and are becoming a promising asset in the construction of portfolios (Zhang et al., 2017). A company may emit less carbon than its target, originating a surplus that can be traded to firms whose emissions are higher than their allowances. These surpluses are known as European Union Allowances (EUAs), a tradable commodity in the stock market (Benz and Trück, 2006; Uddin and Holtedahl, 2013). EUAs were classified as financial assets under the Markets in Financial Instruments Directive (MIFID II), attracting many investors seeking assets related to Environmental, Social, and Governance policies; hence, the importance of this asset, which can be considered in any market as an alternative investment for any stock or commodity portfolio (Zhang and Wei, 2010; Venmans, 2016). When considering a new class of assets, the first incentive for any investor is the assumption that the price of an emissions right will rise. Regarding carbon, there is increased trust in this taking place due to the need to put pressure on the industry to cut emissions and invest in less contaminating technologies and manufacturing processes (Zhang and Zhang, 2020).

1.4. The need to adapt investment strategies

Diversification is critical to balancing and constructing a profitable investment portfolio. If political or economic affairs damage one enterprise or industry, clever diversification can protect all other portfolio assets from risk. Investing in renewable energy and emission allowances would finance a stable and profitable future. However, investor allocations to both assets remain limited. Investors' hesitancy has been based on a lack of information and clarity regarding environmental and climate policy, a limited understanding of specific risks, and a lack of sufficient data to assess this asset class. For this reason, these clean investment initiatives must be encouraged, carefully monitored, and expanded where successful. It is critical to help market participants make informed decisions about a low-carbon transition. This research encourages investors to gradually abandon fossil fuels and commit to renewables and carbon markets. A transformation in the financial markets is urgently needed to change the current trend and reduce investments with high carbon content. Investors must realize that what is profitable today might not be profitable in the near future due to the physical effects of global warming and regulatory impacts that a low-carbon economy represents for investments in fossil fuels.

Sadorsky (2012) already justified the inclusion of carbon assets in investment portfolios concerning the energy market, identifying optimal hedges in fossil fuel portfolios. Dutta et al. (2018) and Lin and Chen (2019) highlighted the benefits of diversifying with carbon permits in portfolios of clean energy companies. Moreover, studies like Wang and Guo (2018) have shown the dynamic relationship over time and transmission of volatility between dirty and clean energy markets. Therefore, due to the time-varying interdependence among these assets, portfolio management must be dynamic, allowing frequent weight rebalancing to achieve an acceptable risk over time. That is to say, rather than using a buy-and-hold strategy, i.e., passively investing, we propose that investors should actively decide which asset classes are over- or underweight to be able to adapt their investment strategies and holdings based on their market outlook. Consequently, this study's weight allocation is enclosed in a volatility-timing context, which responds to changing market environments by taking different portfolios at different times. This approach is particularly relevant because financial markets, especially those related to clean energy and carbon assets, can experience considerable fluctuations and shifts in risk profiles over time (Arslan-Ayaydin and Thewissen, 2016). By employing a sequential methodology for portfolio allocation, we can adapt to evolving market conditions.

1.5. Development of a sequential portfolio selection methodology

Concerning the methodology adopted in this paper, we must point out that since the pioneering paper of Markowitz (1952), mean-variance optimization has been the classic way to choose stocks, and this method is regularly applied in academic research. The covariance matrix of asset returns is the crucial entry to obtain the best portfolio weightings, thus making its precise estimate decisive to take appropriate decisions in risk management and the pursuit of optimal returns. Since the economic scenario and investment markets are continually switching, the adaptability of covariance modeling becomes a highly engaging feature to be incorporated into today's portfolio management. In mean-variance analysis, it is fundamental to have a balance between risk and profit; hence, any decrease in risk results in an expected rise in returns. Ledoit and Wolf (2003) prove why focusing only on the covariance matrix is entirely justified without being concerned about the expected returns. For this reason, we propose a sequential scheme to select the model at any given time, making it possible to obtain a better estimate of covariance matrices of day-to-day asset returns. With that aim, and to calculate the composition of the optimal minimum risk portfolio, we provide a highly flexible and adaptive model estimation and selection framework based on a sequential strategy that considers the possibility that the model used for estimating the covariance matrix could change. This sequential procedure helps investors to perform intelligent management, enabling them to adapt and learn more each day by conducting a sample evaluation.

Furthermore, in the estimation process, our procedure consider expanding or rolling observation windows and analyses their impact on the construction of minimum risk investment portfolios. The rolling window slides over the values, using the same number of observations each time to estimate the model parameters. In contrast, the expanding window accumulates data on the values and increases the number of observations each time. Thus, the sample size used to estimate the unknown parameters changes at each point in time. We instinctively know that if the model has not changed, it will be preferable to use an expanding window, since the more data available, the better the estimate, the less noise, and the better the uncertainty quantification. Nonetheless, if the model has changed either in its parameters or shape, it will be preferable to use a rolling window, since more recent observations will provide more reliable information than observations further back in time, i.e., the past, in this case, will be irrelevant.

Another issue analyzed by our procedure is the influence of the updating frequency of model estimation when building the optimal portfolio. Covariance matrices are estimated from data available on a specific date, the weights of the best portfolio are calculated from this estimate, and then the portfolio is built at that time and maintained until the forthcoming reweighting occurr. Frequent updates (for instance, daily) can be time-consuming in computation terms, and the risk of capturing local noise patterns may lead to an increase in both the risk and cost of the selected portfolio due to an excessive change in portfolio weightings. However, if the model is not updated, we risk not capturing some relevant patterns in the risk evolution of the assets, which may result in an incorrect portfolio selection with high-risk levels.

The variance of the optimal portfolio appraises the efficiency of the estimator of the asset returns' covariance matrix after it has been created. Therefore, to select the best model to estimate said covariance matrix, we use the moving averages of the observed one-step-ahead portfolio volatilities as a comparison criterion obtained with each model. This criterion is based on Engle and Colacito (2006), who showed that the true model is the one that obtains the best values of the average volatilities for various scenarios. Therefore, another parameter that our sequential procedure considers is the size of the out-of-sample validation period used to build this moving average and its influence on building the optimal portfolio.

Finally, and as the proposed model estimation and selection strategies were diverse, to select the most appropriate strategy, we used both overall and local evaluation processes to calculate the risk of the selected portfolios based on the works of Engle and Colacito (2006) and Giacomini and Rossi (2010).

1.6. Data analysis and study period

We considered the daily closing prices of five assets from January 19 2010, to May 5 2022. Specifically, we took two fossil fuel series (oil and gas), two indexes of clean and dirty energies, and the EUA series. Our analysis period was long enough to consider several crisis episodes (sovereign debt crisis, Brexit, COVID-19, and the Russo–Ukrainian War).

To achieve our objective, we used the Asymmetric Dynamic Conditional Correlation-Generalized AutoRegressive Conditional Heteroskedastic (ADCC-GARCH) family of models (Cappiello et al., 2006), which enabled us to obtain reasonable estimates of the conditional covariance matrices of the day-to-day asset returns. This family of models collects models as particular cases with constant correlations between assets (CCC), models that incorporate changes in the correlation of assets (DCC), and models that capture evidence of asymmetric responses to negative returns (ADCC). Furthermore, we considered the possibility of a change in the conditional distribution of the error terms, alternating between Gaussian and Student's *t*-distributions; the first is more appropriate for calm periods, and the second is better for agitated periods thanks to its heavier tails. We proposed a dynamic sequential model selection strategy to capture all possible options, which could provide a more accurate valuation of the portfolio risk. This strategy enables more straightforward and parsimonious models to be chosen in moments of calm in the market, with more precise and reliable estimations, and more complicated models to be selected at times of turbulence or crisis.

1.7. Contributions

This paper's main contributions are the following:

1) We introduce a novel sequential methodology for building portfolios and monitoring their out-of-sample performance in the context of carbon and energy financial markets. This methodology offers the distinct advantage of adaptability to dynamically changing market conditions. We achieve this adaptability using a family of ADCC-GARCH models, which provide a flexible, robust, and adaptive process of model selection and estimation. Furthermore, our framework incorporates a model selection strategy based on a sequential approach with varying estimation and validation window sizes, as well as different model update frequencies. This innovative approach not only enhances our ability to capture evolving market dynamics but also empowers us to make informed and timely adjustments to portfolio composition. As such, it provides a more resilient and responsive framework for portfolio management in these volatile markets, enhancing our ability to navigate and optimize investment strategies in the face of uncertainty.
2) Our sequential methodology incorporates estimation, prediction, and validation windows that change over time. Therefore, from our point of view, it may be a valid strategy to analyze the robustness of our results. These changing windows help us evaluate how conclusions hold up across different conditions and periods. As our study addresses a wide range of scenarios and evaluates how conclusions change in response to different conditions, this strengthens the validity of our results, thus ensuring the robustness of our findings.
3) Our study encompasses a relevant temporal context, as it spans significant events such as the sovereign debt crisis, Brexit, the COVID-19 pandemic, and the Russo-Ukrainian war. This underscores the study's relevance against a critical economic and financial backdrop.
4) We provide actionable strategies for investors and portfolio managers seeking to align their portfolios with energy transition trends, contributing to a more sustainable and prosperous future.

1.8. Document structure

This paper is structured as follows: Section 2 provides a literature review. A description of the data is included in Section 3. Section 4 sets up the problem and presents the methodology. It explains the procedures used to obtain the weights of a minimum risk portfolio and compare the risk of two portfolios, and it details the sequential algorithm used to dynamically determine the optimum minimum risk portfolio. Section 5 applies the proposed methodology to study the European carbon and energy markets and discusses the results. Finally, Section 6 concludes with main remarks, policy implications, and future research directions.

2. Literature review

Numerous academic investigations have examined the interdependent relationship between the prices of CO_2 emissions allowances and the fossil energy market. In the initial phases of the EU ETS 2005–2010, Chevallier (2011) found, via Markov-switching VAR models, a relationship between EUA and fossil fuels along with economic activity as measured by the aggregated industrial production in the EU 27. Subsequently, Reboredo (2013) and Marimoutou and Soury (2015) investigated the relationship between the carbon and fossil fuel markets using copula models. Their research sheds light on the significance of portfolio diversification and volatility transmission among these markets. Additionally, studies by Liu and Chen (2013), Zhang and Sun (2016) and Gargallo et al. (2021), contributed to the understanding of dynamic correlations between EUAs and fossil fuels by means of MGARCH models, providing a more comprehensive view of the relationship between these assets. Other authors, such as Castagneto-Gissey (2014), Hammoudeh et al. (2014), (2015), and Balcilar et al. (2016), added the electricity market to the analysis. In general, these works confirm the dynamic nature of the interconnection between the EUA market and different fossil fuels. However, the influence of the carbon market remained limited for a long time after the 2008 crisis, determining the need to increase the carbon price for the transition to clean energy.

The pursuit of optimal portfolio diversification and risk mitigation in the energy market has driven research into the intricate relationship between energy commodities and stock markets. Understanding correlation patterns and volatility transmission is crucial for effective hedging and portfolio optimization, with notable findings emerging from studies of portfolio management in the dirty energy market. Choi and Hammoudeh (2010), using VAR DCC-GARCH models, analyzed the evolving connections between commodities and the S&P 500 index, showing declining correlations since the Iraq War in 2003, which implied changing dynamics between those markets. Arouri et al. (2012), using a bivariate VAR CCC GARCH model, explored how oil price volatility propagates to European financial markets, providing insights into portfolio diversification and risk management. Antonakakis and Filis (2013), by means of DCC-GARCH models, found that oil prices have a negative effect or do not affect the stock markets of oil importing/exporting countries. Khalfaoui et al. (2015), using BEKK-GARCH and a wavelet-based MGARCH approach, found significant effects of volatility transmission from the price of oil to the stock markets. More recently, Xiao and Wang (2020), by means of multiple methodologies originated from information theory and physics, found significant time-varying, nonlinear, bidirectional, causal relationships between crude oil and stock returns, with the most common pattern featuring stronger information flows during the period of financial crisis. Jebabli et al. (2022) used VAR models to measure total and directional volatility spillovers and employed DCC-GARCH for portfolio design. Their findings indicated that during the 2008 global financial crisis, stock markets primarily transmitted volatility to energy markets. However, the COVID-19 crisis exhibited different patterns. The study also highlighted asymmetric volatility spillovers between these markets and suggested that, on average, natural gas was more effective in hedging stock market risks compared to crude oil.

Several studies have explored the potential of utilizing carbon assets for effective risk management, focusing on EUAs. They have investigated this in the context of a challenging global economic environment and their interactions with various capital markets. These investigations have been conducted by researchers such as Subramaniam et al. (2015), Uddin et al. (2018), Wen et al. (2017), Balcilar et al. (2016), and Zhang and Sun (2016). In this line, Oberndorfer (2009), using regression panel data models with GARCH error terms, highlighted the existence of significant relationships between the profitability of the main electricity companies and EUA prices. Bushnell et al. (2013) delved into the interplay between electricity prices and carbon markets, particularly examining the impact of EUAs on electricity prices. In a similar vein, Jong et al. (2014), Tian et al. (2016), and Ji et al. (2019) extended this exploration by examining the effects of EUAs on stock returns in the electricity sector, revealing its dynamic nature and offering essential insights for investors and policymakers. Finally, Tan et al. (2020) used advanced tools to explore the interconnectedness between carbon and energy markets, revealing shifting dynamics and structural changes. This connection was more pronounced with equity and non-energy commodity markets than with bond markets, providing valuable guidance for diversification strategies.

Research in energy market portfolio management, encompassing clean and traditional energy sources, features noteworthy contributions by various scholars. Henriques and Sadorsky (2008) highlighted the tech and clean energy sectors' strong linkage, eclipsing oil's influence, best captured by the DCC model. Sardosky (2012) revealed intensified connections between oil prices, tech stock values, and clean energy stock values post-2008. Managi and Okimoto (2013) identified a substantial 30% systemic risk contribution from oil prices to renewable energy markets. Bondia et al. (2016) explored separate impacts of oil prices and tech stock values on clean energy stocks, echoed by Reboredo (2015). Kumar et al. (2012) employed a copulas-based approach to underscore the asynchronous effects of oil prices and tech stocks on clean energy firms, contributing to a holistic understanding of energy portfolio management. Broadstock et al. (2012), using BEKK-GARCH models, emphasized the impact of oil prices on energy-related stock returns in China, particularly for new energy firms. More recently, Dutta et al. (2020) used DCC-GARCH models to analyze clean and traditional energy asset relationships, discovering a strong, time-varying connection between technology and clean energy markets, particularly notable in the long term and for returns of the same sign, especially negative ones. Chen et al. (2020) studied the impact of oil prices on clean energy stocks using VAR and GARCH models, revealing oil's significant influence during geopolitical crises. Asl et al. (2021) explored energy sector interactions with VAR and BEKK-MGARCH(1,1) models, suggesting the oil sector's potential to diversify risk in renewable energy portfolios. Niu (2021) identified a strong and lasting impact of technology and clean energy markets on clean energy stocks using Time-Dependent Intrinsic Correlation (TDIC). Kuang (2021) demonstrated improved performance by diversifying between clean energy and oil and gas in clean energy portfolios. Wan et al. (2021) highlighted clean energy's resilience during the COVID-19 pandemic. Gargallo et al. (2022a) showed that from 2020 onwards investing in clean energy does not significantly increase risk levels even after the first period of pandemic crisis, most likely due to the influence on investors of governments' green recovery plans. Gargallo et al. (2022b), and Cheikh and Zaied (2023) advocated clean energy diversification in response to geopolitical risk in complex asset relationships.

Researchers such as Gronwald et al. (2011) have delved into the connection between portfolios incorporating EUAs and the clean energy market. Employing a copula approach, they identified a time-varying, substantial relationship between EUAs and various factors, including oil, gas, coal, and equity and energy indices. These connections were notably stronger during crisis periods. da Silva et al. (2016) used cointegration and panel data methods to determine EUA's significant long-term impact on the energy sector of the stock market, which also includes some renewable energy companies. Ji et al. (2018) conducted an analysis using a VAR model, with a specific focus on forecasting error variance decomposition. The results highlighted a noteworthy transmission of volatility among EUA prices, fossil fuels, electricity, and the clean energy market. Dutta et al. (2018) employed Bivariate VAR-GARCH and VAR-AGARCH models, finding a strong link of between volatilities in EUA and European clean energy price indices, suggesting potential benefits of portfolio diversification. Lin and Chen (2019) examined the dynamics between carbon emissions trading, clean energy markets, and coal prices in China. They used various models (VAR-DCC GARCH, VAR-BEKK GARCH) to explore correlations, asymmetric effects, and optimal portfolio strategies. They found significant time-varying correlations and long-term persistence of shocks, together with dynamic linkages and spillover effects between the three markets.

This literature review demonstrates a growing concern regarding the interaction between financial and energy markets, especially in the context of the transition to cleaner energy sources and carbon emissions regulations. The studies reviewed underline the relevance of these relationships in investment decision-making and portfolio management, given the increasing volatility of energy prices and associated risks. One primary conclusion of this review is the significance of portfolio diversification. Oil prices and assets related to both dirty and green energy have a substantial impact on stock returns. Investors and portfolio managers must carefully consider how these relationships can affect their risk exposure and investment returns. The literature suggests that proper diversification can enhance portfolio performance and reduce market volatility exposure. Furthermore, the literature has identified dynamic relationships and asymmetries in volatility transmission between financial and energy markets. Economic crises and geopolitical events (Russia-Ukraine or Israel-Palestine wars) can have a significant impact on these relationships, highlighting the need for robust risk management. Additionally, considering the evolution of environmental regulations and policies, such as the Paris Agreement, will be crucial for anticipating changes in these relationships. Ongoing research in this area will remain critical in guiding investors and portfolio managers in an ever-changing financial environment.

Finally, regarding methodology, the studies reviewed have employed a wide range of approaches (VAR and MGARCH models, variance decomposition of forecasting error techniques, panel data regression and cointegration models, optimal portfolio selection and hedge ratios estimation, analysis of the time evolution of correlation matrices, etc.). Some studies (Caporin and McAleer, 2014) suggest that there is no single, universally optimal approach, and methodology selection should be careful and aligned with specific research objectives (Lucheroni et al., 2019). For these reasons, it is essential to continue researching and developing methods and models that provide a more comprehensive understanding of the interaction between financial and/or energy markets. Therefore, in this paper, we introduce a novel sequential methodology for building portfolios and monitoring their out-of-sample performance in the context of carbon and energy financial markets. This methodology offers the distinct advantage of adaptability to dynamically changing market conditions. Section 4 describes it, and Section 5 applies it to the European carbon and energy markets.

3. Data

We consider the daily closing prices of five assets from January 19, 2010, to May 5 2022 (totaling 2978 observations). The last part of the selected period collects the two current major crises: the COVID-19 pandemic and the Russian–Ukrainian war. We considered the Brent oil futures (OIL series) and the Natural Gas Futures (GAS series), listed on the United Kingdom stock exchange market, as the leading European references. Furthermore, we selected the S&P Global Clean Energy Index (CLEAN series) and the EURO STOXX® Oil & Gas Index (OIL_GAS series). The first index measures the performance of 31 firms worldwide dedicated to the clean energy business. The second index provides information on the 12 biggest European firms dedicated to mining, perforation, production, refining, distribution, and retail sale of oil and gas. Finally, we consider the European Unit Allowances (EUA series) prices obtained from SENDECO2 (European CO_2 Trading System).

Fig. 1 displays the day-to-day excess returns series corresponding to the five assets, and Tables 1A and 1B shows the descriptive statistical analysis, and the study of unit roots, stationarity, and ARCH effects, respectively. The stationarity hypothesis is accepted in all the series. The mean return of the series is not significantly different from zero; all series are strongly leptokurtic and have significant but not very strong asymmetries. Furthermore, all the series are heteroscedastic and show the typical volatility clustering of financial series. The less volatile series tend to be CLEAN and OIL_GAS, given that they are stock indices. Additionally, the kurtosis of OIL_GAS tends to be significantly higher than that of CLEAN due to the greater trend of OIL_GAS to have a heavier left tail, reflecting a tendency to take more extreme negative excess returns. The most volatile series tends to be GAS and EUA. In the case of GAS, the higher volatility is mainly due to the last 2019–2022 period (see Fig. 1) and more intense in the period corresponding to the Russo–Ukrainian war; while in the case of EUA, its higher volatility reflects the different changes that occurred in the CO_2 emission allowances granting

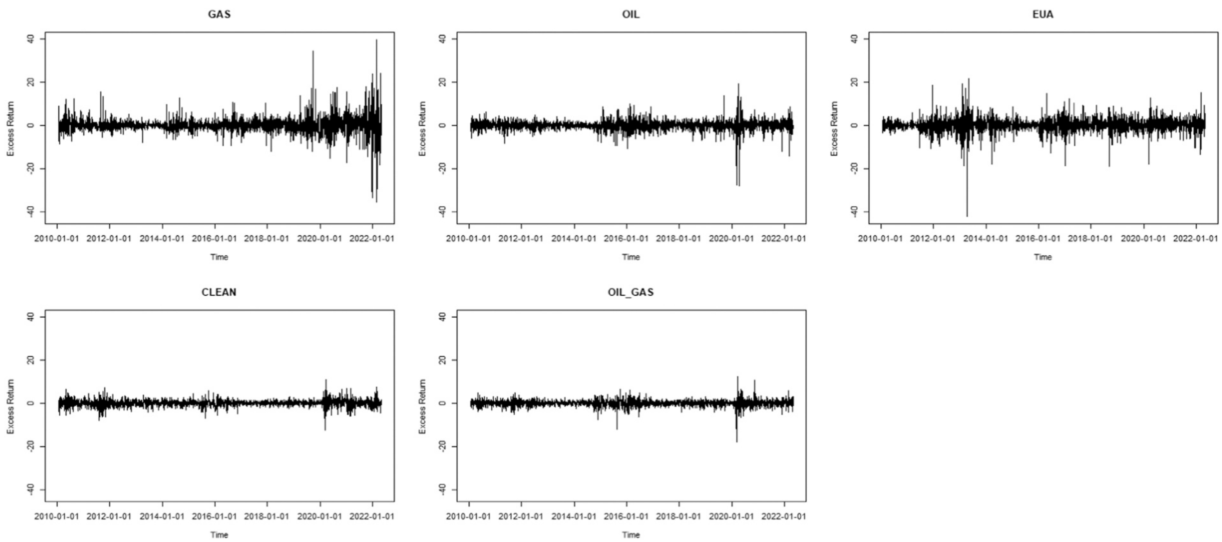

Fig. 1. Evolution of the day-to-day returns of the five series.

Table 1A
A descriptive study of the day-to-day returns of the five series.

	Minimum	Maximum	Mean	Std. Dev.	Skewness	Kurtosis
GAS	−35.467	39.533	0.051	3.780	0.364*	18.446*
OIL	−27.975	19.079	0.009	2.328	−1.019*	18.898*
EUA	−42.256	21.582	0.060	3.222	−0.939*	14.722*
CLEAN	−12.495	11.034	−0.007	1.574	−0.448*	6.315*
OIL_GAS	−17.951	12.388	−0.005	1.511	−1.003*	16.084*

*** Significant at 1%.

Table 1B
Unit root, stationarity, and ARCH tests.

Variables	ADF	PP	KPSS	LB (1)	LB (2)	ARCH-LM (1)	ARCH-M (10)
GAS	-14.355**	-50.460***	0.0943	23.648***	113.78***	433.38***	744.58***
OIL	-13.824**	-51.592***	0.0904	9.8573***	38.913***	136.51***	321.37***
EUA	-15.787**	-55.953***	0.0303	2.6312***	53.22***	50.155***	137.09***
CLEAN	-12.317**	-48.705***	0.0677	55.567***	116.42***	125.53***	670.04***
OIL_GAS	-15.425**	-51.397***	0.0203	10.484***	70.844***	27.707***	504.3***

The notations ***, **, and * are used to indicate statistical significance at the 1, 5, and 10 percent levels, respectively. The abbreviations ADF, PP, and KPSS represent the statistical metrics derived from the Augmented Dickey and Fuller (1979), the Phillips and Perron. (1988), and the Kwiatkowski et al. (1992). The selection of lag order is determined using the Schwartz Information Criterion. LB (1) and LB (2) refer to the Ljung-Box test for autocorrelation, respectively. Additionally, the ARCH-LM (1) and ARCH-LM (10) tests, introduced by Engle (1982), check for the existence of autoregressive conditional heteroscedasticity (ARCH) effects.

systems.

4. Methodology

This section sets up the problem and presents the methodology. Here, we explain the procedures used to obtain the weights of a minimum risk portfolio and compare the risk of two portfolios (Section 3.1). We also detail the sequential algorithm used to determine dynamically the optimum minimum risk portfolio (Section 3.2).

4.1. Minimum risk portfolio

Online decisions for portfolio selection are important as related market information arrives sequentially, and the allocation decision must be made instantly. Buy and hold is a strategy where an investor invests in an initial portfolio and never rebalances it; however, the portfolio weights should change as time goes by because the underlying assets change in price due to the changing conditions of the markets. Therefore, a sequential portfolio selection strategy with minimum risk is needed. To that aim, we had to establish a way to measure this risk.

Markowitz (1952) developed a model based on rational investor behavior. That is, the investor seeks profitability and rejects risk. A well-diversified portfolio enables investors to take advantage of the long-term increase of all markets while decreasing the effect of short-term risk in individual markets, recovering from severe vicissitudes due to speculative movements. Furthermore, it is evident that the risk of the portfolio, measured as its variance, depends on the individual variances of the returns of its different assets and their covariances. An efficient portfolio can be calculated by solving an optimization problem consisting of minimizing the variance of the portfolio subject to a given expected return. Within this context, our first problem was selecting the weights of the single assets of an investment portfolio, which depend on the hypotheses underlying the model adopted to generate the daily assets return.

Let $\left\{\boldsymbol{r}_t = \left(r_{1,t},\ldots,r_{n,t}\right)'; t=1,\ldots,T\right\}$ be the series of day-to-day financial return vectors with $r_{i,t} = 100 \bullet \log\left(\frac{p_{i,t}}{p_{i,t-1}}\right)$ and pi,t is the closing price of the i-th asset in period t for i = 1,…,n.

Let $\mathscr{F}_t = \{\boldsymbol{r}_1,\ldots,\boldsymbol{r}_t\}$ be the information set in period t.

Let $\boldsymbol{\Omega}_t = \mathrm{var}(\boldsymbol{r}_t|\mathscr{F}_{t-1})$ be the conditional covariance matrix in period t.

Therefore, we solve the following optimization problem:

$$\min_{\mathbf{w}_t}\mathrm{Var}\left(\mathbf{w}_t'\mathbf{r}_t \quad |\mathscr{F}_{t-1}\right) = \min_{\mathbf{w}_t}\mathbf{w}_t'\boldsymbol{\Omega}_t\mathbf{w}_t$$

$$\mathrm{s.t.}\mathbf{w}_t'\boldsymbol{\mu} = \mu_0$$

where $\boldsymbol{\mu}$ is a given expected return, $\mu_0 > 0$ is the required return. $\mathbf{w}_t = \left(w_{1,t},\ldots,w_{n,t}\right)'$ is the vector of portfolio weights for time t chosen at time t–1 with $w_{i,t}$ being the share on asset i for time t for i = 1,…,n. The solution to this problem is

$$\mathbf{w}_t = \frac{\boldsymbol{\Omega}_t^{-1}\boldsymbol{\mu}}{\boldsymbol{\mu}'\boldsymbol{\Omega}_t^{-1}\boldsymbol{\mu}}\mu_0$$

Note that $\sum_{i=1}^{n} w_{i,t}$ does not generally need to add 1. Indeed, 1-$\sum_{i=1}^{n} w_{i,t}$ is the weight corresponding to the risk-free asset. In the following, we take $\mu_0 = 1$.

The parameters of this problem are the expectation and the dispersion of asset returns and the correlation between them; however, it is not easy to obtain the expected return vector $\boldsymbol{\mu}$ (Engle and Colacito, 2006). For this reason, and to take into account the uncertainty associated with this problem, we perform the optimization procedure for a collection of imaginary time-changeless vectors of expected returns $\mu \in E$. Set **E** tries to capture several scenarios where yields could be elevated, and others could be small or even zero. Once we have determined **E**, we calculate the minimum variance portfolio weights for each $\mu \in E$. Our proposed final portfolio is built as an equally weighted portfolio of minimum variance obtained with each scenario, so it collects information about all the solutions. In this way, we consider the uncertainty associated with the value of the vector of expected returns, providing a smoothing of the asset weights.

Note that the optimal allocation for each $\mu \in E$ requires the risk evaluation of the portfolio; the portfolio variance represents the risk, thus the correct specification of the covariance matrix of the day-to-day asset returns, $\boldsymbol{\Omega}_t$, is of paramount importance. Engle and Colacito (2006) demonstrated that if σ_t is the standard deviation of the minimum variance portfolio calculated in period t using an estimation $\mathbf{H}_t$ of $\boldsymbol{\Omega}_t$, then $\sigma_t \geq \sigma_t^*$ where σ_t^* is the standard deviation of the minimum risk portfolio computed with $\boldsymbol{\Omega}_t$ and, hence, $\frac{1}{T}\sum_{t=1}^{T}(\sigma_t^*)^2 \leq \frac{1}{T}\sum_{t=1}^{T}(\sigma_t)^2 \ \forall \mu$**.** Hence, bad estimating $\boldsymbol{\Omega}_t$ implies an augmentation of the portfolio's risk or, equivalently, a decrease of the required return μ0 for a fixed risk level. Therefore, $\boldsymbol{\Omega}_t$ had to be estimated as accurately as possible, so we used sequential estimation procedures that approximate $\boldsymbol{\Omega}_t$ the best as possible. The following section describes this paper's procedures, which we name strategies.

4.2. Estimation and model selection strategies

This paper used statistical models that describe the evolution of the analyzed series. Given that market conditions are changing, models can change their parameters and their form. In an investment context, the estimation of the models is generally sequential, and the unknown parameters are updated over time. The frequency with which parameters are updated depends on data processing costs and the expected profit of the updates. When processing costs are insignificant, the parameters are updated with the arrival of new data; however, day to day updates can take a long time, and it is not clear what the expected benefit is. Therefore, we wanted to analyze whether updating the unknown parameters on a daily, monthly, quarterly, or yearly basis, which could imply a computational saving, also has a positive or negative effect on portfolio volatility. Furthermore, we also wanted to determine if estimating with expanding or rolling observation windows affects the construction of the investment portfolios. The rolling window is a window that slides over the values, which uses the same amount of observations each time to estimate the model parameters. In contrast, the expanding window accumulates the values, which increase with the number of observations each time, so the sample size used to estimate the unknown parameters changes at each point in time. Contrary to the rolling window technique, in which the oldest observations are removed, in expanding windows, all observations are retained, and the new one is added. Finally, we analyze the effect of the size of the out-of-sample validation period used to select the best-performing model.

We have used different model estimation and selection strategies. Each strategy determines the estimation window, update frequency, and the size of the out-of-sample validation period used to select the best-performing model according to the volatility of its minimum variance portfolios. In this way, we provide a very flexible and adaptive framework of model estimation and selection procedures to capture the most significant changes in the evolution of the risk of the considered assets.

Let $\mathbf{M} = \{M_1, \ldots, M_K\}$ be the set of models considered in the paper and $\mathbf{S} = \{S_1, \ldots, S_L\}$ be the set of the strategies used to estimate and select the best model. Let $\mathbf{E} = \{\mathbf{m}_i; i = 1, \ldots, R\}$ be the set of plausible expected return vectors.

For each period t, each $S \in \mathbf{S}$ specifies a vector $(t_{end,t}, h_{estim,t}, h_{val,t})$, where tend,t < t-hval,t+1 is the end of the estimation period of each model and determines its update frequency, $0 < h_0 \leq h_{estim,t} \leq t_{end,t}$ is the size of the estimation window, so that $\{t_{end,t}\text{-}h_{estim,t}+1, \ldots, t_{end,t}\}$ is the estimation period for each model in **M**, and h_0 is the minimum sample size necessary to estimate each $M \in \mathbf{M}$ reliably. Furthermore, $h_{val,t} \geq 0$ is the size of the validation window used to evaluate the out-of-sample performance of each model $M \in \mathbf{M}$, which is carried out using the period $\{t\text{-}h_{val,t}+1, \ldots, t\}$. If $h_{val,t} = 0$, we understand that S only considers one model type, so no model selection process is performed.

So, for instance, if the strategy vector is (t-21, t-21, 21), we are considering a strategy that updates each model daily and uses all the previous data up to the current period t to estimate it (expanding window) and a monthly validation period[2] {t-20,…, t}. In contrast, if the strategy vector is ($t_{q(t)}$-1, 63, 63), where $t_{q(t)}$ is the initial period of the quarter where the current period t is located, the strategy updates the estimation of the models quarterly, with a quarterly sliding estimation window (rolling window) and a quarterly validation period {t-62,…., t}.

We set $h_{val,t} = 0 \ \forall t$ if we considered that model M had not changed over time; hence, we did not perform any model selection process, and we only updated the estimation period of the considered model M.

[2] We assume that a month has 21 days, two months have 42 days, and so on.

For each strategy $S \in \mathbf{S}$, we propose the following sequential procedure to obtain the composition of the optimum portfolio's evolution.

4.2.1. Sequential procedure to calculate the weights and volatilities of optimal portfolios

Step 0: Inputs.

Fix $S = (t_{end,t}, h_{estim,t}, h_{val,t})$ the estimation and model selection strategy

For $t = h_0+1,\ldots,T$ perform steps 1–3.

Step 1: Determination of the minimum risk portfolio for each model.

For each $M \in \mathbf{M}$ and for $u = t\text{-}h_{val,t},\ldots,t$ execute steps 1 A and 1B.

Step 1 A: Estimation of the variance and covariance matrix

$$\mathbf{H}_{u,M,S} = \mathrm{Cov}\left(\mathbf{r}_u | M, \widehat{\boldsymbol{\theta}}_{M,t_{end,u},h_{estim,u}}, \mathscr{F}_{u-1}\right)$$

where $\widehat{\boldsymbol{\theta}}_{M,t_{end,u},h_{estim,u}}$ is the maximum likelihood estimator of $\boldsymbol{\theta}_M$ corresponding to the estimation period $\left\{t_{end,u} - h_{estim,u} + 1, \ldots, t_{end,u}\right\}$

Step 1B: Determination of the weights and the variance of minimum risk portfolios.

For $i = 1,\ldots, R$ calculate.

- The vector of optimal weights $\mathbf{w}^{i}_{u,S,M} = \frac{\mathbf{H}^{-1}_{u,S,M}\mathbf{m}_i}{\mathbf{m}_i^{'}\mathbf{H}^{-1}_{u,S,M}\mathbf{m}_i}$

- The variance of the minimum risk portfolio $\sigma^{2,i}_{u,S,M} = \mathbf{w}^{i'}_{u,S,M}\mathbf{H}_{u,S,M}\mathbf{w}^{i}_{u,S,M}$

If $h_{val,t}>0$ (i.e.,., we perform a model selection process), go to step 2; otherwise, put $M_{t,S,opt} = M$, and go to step 3.

Step 2: Selection of the best model

$$\mathrm{Calculate} M_{t,S,opt} = \min_{M \in \mathbf{M}} \frac{1}{R}\frac{1}{h_{val,t}}\left(\sum_{i=1}^{R}\sum_{u=t-h_{val,t}+1}^{t}\sigma^{2,i}_{u,S,M}\right)$$

Step 3: Determination of weights and expected variances of the final portfolio strategy

$$\mathrm{Calculate}\mathbf{w}_{t,S} = \frac{1}{R}\sum_{i=1}^{R}\mathbf{w}^{i}_{t,S,M_{t,S,opt}} \mathrm{and} \sigma^2_{t,S} = \mathbf{w}^{'}_{t,S}\mathbf{H}^{-1}_{t,S,M_{t,S,opt}}\mathbf{w}_{t,S}$$

Note that the output of the algorithm is

$$\left\{\left\{\mathbf{w}^{i}_{t,S,M_{t,S,opt}}\right\}_{j=1}^{R}; \mathbf{H}^{-1}_{t,S,M_{t,S,opt}}, \mathbf{w}_{t,S}; \sigma^2_{t,S}\right\}_{t=h_0+1}^{T} \tag{1}$$

4.2.2. Comparison of strategies

This section explains the procedures used to compare the performance of two strategies, S1 and S2 $\in \mathbf{S}$. To that aim, we compare the risk levels (volatilities) of the portfolios built for each one. This comparison is made in two ways: an overall comparison, using an evaluation of the risk differences throughout the entire period (Engle and Colacito, 2006), and a local comparison, where a study of the time evolution of these differences is conducted (Giacomini and Rossi, 2010).

4.2.2.1. Global performance. To select the best strategy, we have applied a procedure based on Engle and Colacito (2006), which conducts a pairwise comparison of the risk of minimum variance portfolios. We use the outputs (1) of each strategy, and we perform the regression:

$$\mathbf{V}_{t,S_1,S_2} = \beta_{S_1,S_2}\mathbf{1}_{Rx1} + \boldsymbol{\varepsilon}_{t,S_1,S_2} \text{for } t = h_0 + 1, \ldots, T \tag{2}$$

where $\mathbf{V}_{t,S_1,S_2} = \left(v^1_{t,S_1,S_2}, \ldots, v^R_{t,S_1,S_2}\right)^{'}$ is the vector of standardized risk differences:

$$v^{j}_{t,S_1,S_2} = \frac{\left(\pi^{(j)}_{t,S_1}\right)^2 - \left(\pi^{(j)}_{t,S_2}\right)^2}{D\left[\left(\pi^{(j)}_{t,S_1}\right)^2 - \left(\pi^{(j)}_{t,S_2}\right)^2\right]} \text{for } j = 1, \ldots, R$$

with $\pi^{(j)}_{t,S_i} = \mathbf{w}^{i'}_{t,S,M_{t,S,opt}}(\mathbf{r}_t - \bar{\mathbf{r}}_T)$ where $\bar{\mathbf{r}}_T = \frac{1}{T}\sum_{v=1}^{T}\mathbf{r}_v$ for i=1, 2, are the observed return of the minimum variance portfolio built by strategy Si assuming an expected return $\mu = \mathbf{m}_j \in \mathbf{E}$. We used the approximation

$$D\left[\left(\pi^{(j)}_{t,S_1}\right)^2 - \left(\pi^{(j)}_{t,S_2}\right)^2\right] \approx \left[0.5\left(\mathbf{m}_i^{'}\mathbf{H}^{-1}_{t,S_1,M_{t,S_1,opt}}\mathbf{m}_i\right)\left(\mathbf{m}_i^{'}\mathbf{H}^{-1}_{t,S_2,M_{t,S_2,opt}}\mathbf{m}_i\right)\right]^{-1/2}$$

We tested hypothesis H_0: $\beta_{S_1,S_2} = 0$ against H_1: $\beta_{S_1,S_2} \neq 0$ using the t-test of Diebold and Mariano (1995) with a robust Newey–West estimator of the standard error of $\widehat{\beta}_{S_1,S_2}$. If we accepted H_0, we concluded that there were no significant differences between the risks of the compared portfolios; hence, no significant differences exist between the estimations of $\boldsymbol{\Omega}_t$ provided by both strategies (S_1 and S_2). In contrast, if we accept that $\beta_{S_1,S_2} > 0$, we concluded that $\left\{\mathbf{H}_{t,S_2,M_{t,S_2,opt}}; t = h_0, \ldots, T\right\}$ described the evolution of $\boldsymbol{\Omega}_t$ over time better than $\left\{\mathbf{H}_{t,S_1,M_{t,S_1,opt}}; t = h_0, \ldots, T\right\}$. The opposite happens if we accept that $\beta_{S_1,S_2} < 0$.

4.2.2.2. Local performance. Regression model (2) compares the overall variance efficiency of the two considered portfolios but not

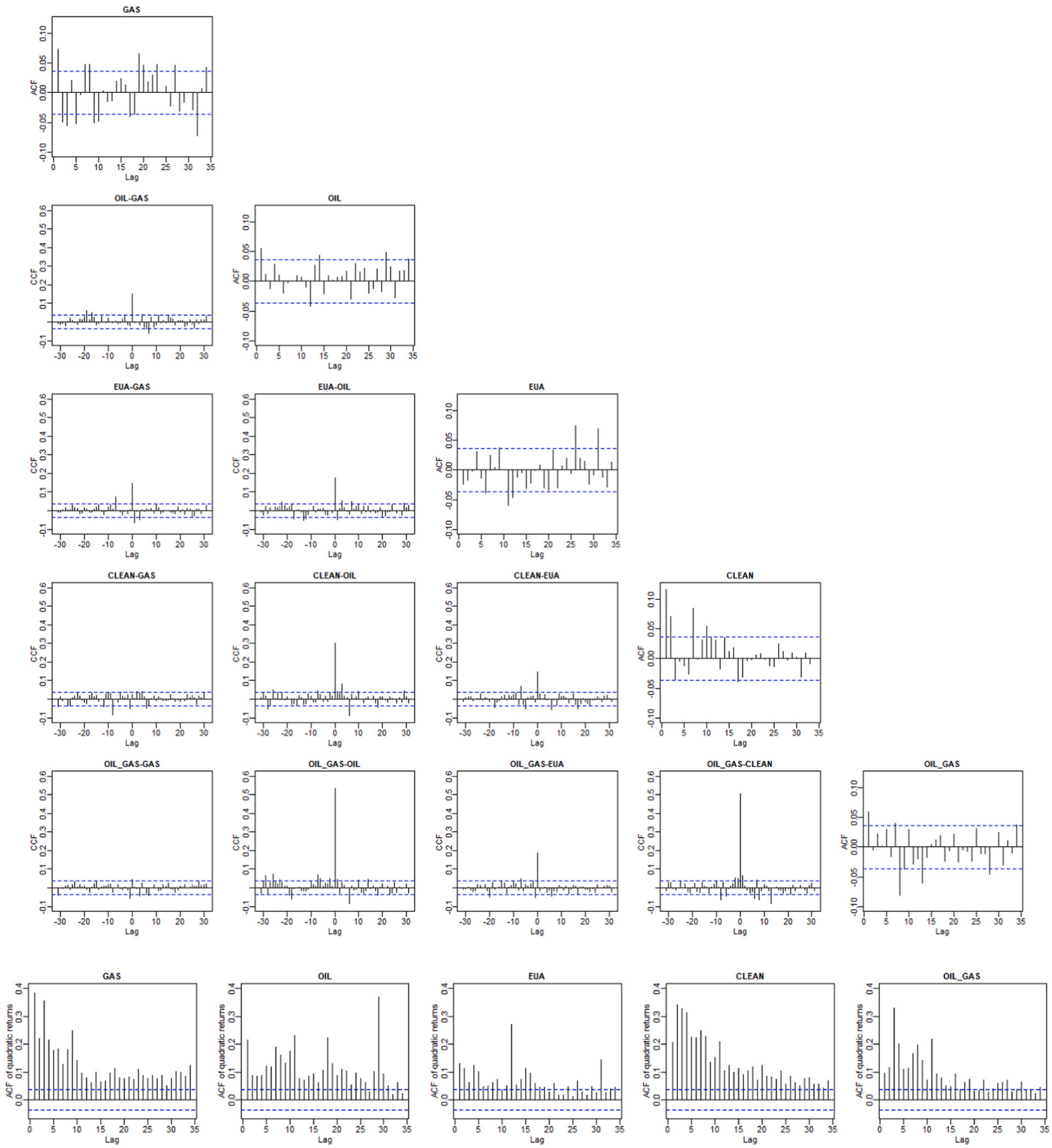

Fig. 2. Two-panel graphic—Top panel: cross-correlation function of the returns of the five series in the off-diagonal cells and autocorrelation function of the returns in the diagonal; Bottom panel: autocorrelation function of the quadratic returns.

their local performance at each point. Therefore, we also conducted the fluctuation test presented by Giacomini and Rossi (2010) to identify temporary changes in the risk differences of the compared portfolios and to check the hypothesis that these differences are null each time. To simplify the analysis results, we applied this test to the final portfolio strategy instead of the minimum variance portfolios obtained for each plausible expected return vector $\mathbf{mi} \in \mathbf{E}$. Therefore, we used the following loss functions:

$$\mathrm{loss}_{i,t} = \pi^2_{t,S_i} \quad \text{for } i = 1, 2; \; t = h_0 + 1, \ldots, T$$

where $\pi_{t,S_i} = \mathbf{w}'_{t,S_i}(\mathbf{r}_t - \overline{\mathbf{r}}_t)$ where $\overline{\mathbf{r}}_t = \frac{1}{t}\sum_{v=1}^{t}\mathbf{r}_v$.

5. Empirical analysis

This section applies the proposed methodology to the study of European carbon and energy asset markets and discusses the results. The selection of a model for the vector of expected returns and its covariance matrices, as well as the choice of the investment strategy to be assumed, are two critical aspects when building an investment portfolio. Covariance matrix modeling of stock returns is critical to the portfolio selection problem. To determine the most appropriate family of models, Fig. 2 represents a two-panel chart in which the top panel shows the cross-correlation function of all the asset returns on off-diagonal elements and their autocorrelation functions on the diagonal cells. These cells distinguish that most autocorrelations are small and non-significant, representing this kind of financial time series.

Thus, we propose a Vector Autoregressive (VAR) model to explain the relations between the returns of the five series, concretely a VAR(1). Furthermore, the bottom panel shows the autocorrelation function of the quadratic returns, indicating significant positive values for all the lags, which reflects the presence of volatility clustering and the tendency of these changes to persist in all the series. Therefore, we use a GARCH to model the volatility of each analyzed series. Finally, the off-diagonal cells in the top panel of Fig. 2 contain the correlations between returns, where significant contemporaneous positive correlations can be observed between most of the series, suggesting that multivariate GARCH modeling can enable correlation between all series. We decided to use ADCC-GARCH models to capture the possible asymmetric impact of recent market information. These models were chosen for their parsimoniousness, adaptability, flexibility, and capacity to handle time-varying parameters and model selection, and they have shown their empirical superiority to other multivariate GARCH models (Sadorsky, 2012, Dutta et al., 2020). These characteristics are instrumental in capturing the evolving relationships within our selected carbon and energy assets and align with the objectives of our research, which focuses on portfolio performance and risk management in dynamic financial markets.[3]

5.1. Family models

We suggest using a VAR(1)-ADCC(1,1)-GARCH(1,1) family to estimate $\mathbf{\Omega}_t$; Cappiello et al. (2006) introduced this family, which is a key reference due to its flexibility and feasibility of implementation, as conditional volatilities and conditional correlations are specified separately. Concretely, we assume that:

$$\mathbf{r}_t | \mathscr{F}_{t-1} = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\mu}_t = E(\boldsymbol{r}_t | \mathscr{F}_{t-1})$ is given by the VAR(1) expression:

$$\boldsymbol{\mu}_t = \mathbf{\Phi}_1 \mathbf{r}_{t-1}$$

and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \ldots, \varepsilon_{nt})'$ is a conditional heteroscedastic error term with $\boldsymbol{H}_t = \mathrm{var}(\boldsymbol{\varepsilon}_t | \mathscr{F}_{t-1})$ given by

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t$$

where $\mathbf{D}_t = \mathrm{diag}\left(\sqrt{h_{11,t}}, \cdots, \sqrt{h_{nn,t}}\right)$ with $h_{ii,t} = \mathrm{var}(\varepsilon i,t | \mathscr{F}_{t-1})$ for i=1,…,n. $\mathbf{R}t$ is the conditional correlation matrix of $\mathbf{r}_t$. The conditional variances, $h_{ii,t}$, are given by n independent GARCH(1,1) models, written in vector form as

$$\mathrm{diag}(\mathbf{H}_t) = \mathbf{\Omega} + \mathbf{A}_1 \boldsymbol{\varepsilon}_{t-1} \odot \boldsymbol{\varepsilon}_{t-1} + \mathbf{B}_1 \mathrm{diag}(\mathbf{H}_{t-1})$$

where $\mathbf{\Omega} = \mathrm{diag}(\omega_i)$, $\mathbf{A}_1 = \mathrm{diag}(\alpha_i)$ and $\mathbf{B}_1 = \mathrm{diag}(\beta_i)$ are non-negative diagonal nxn matrices. $\odot$ denotes the Hadamard operator. Regarding the time-varying correlation matrix, $\mathbf{R}_t$, we assume that

$$\mathbf{R}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1} \quad \text{with} \mathbf{Q}_t^{*} = \mathrm{diag}(\mathbf{Q}_t)$$

and $\mathbf{Q}_t$ is given by

$$\mathbf{Q}_t = \overline{\mathbf{Q}} + a\left(\mathbf{z}_{t-1}\mathbf{z}'_{t-1} - \overline{\mathbf{Q}}\right) + b(\mathbf{Q}_{t-1} - \overline{\mathbf{Q}}) + g\mathbf{z}_t^{-}\mathbf{z}_t^{-'}$$

[3] We decided not to use other multivariate GARCH models, such as the ABEKK-GARCH, because they are subject to the so-called "curse of dimensionality" resulting from the increase in the number of covariance terms. This makes the estimation of the covariance matrix very difficult, especially if the number of analysed series is greater than 2, as is our case.

where a and b are non-negative scalars verifying the condition that $a + b < 1$, which is imposed to ensure stationary and positive definiteness of $\mathbf{Q}_t$. Furthermore, $\mathbf{z}_t = \mathbf{D}_t^{-1}\boldsymbol{\varepsilon}_t$ are the standardized residuals, $\mathbf{z}_t^- = \mathbf{z}_t I(\mathbf{z}_t < 0)$ to capture asymmetric effects, $\overline{\mathbf{Q}}$ is a definitive positive symmetrical matrix and $\mathbf{Q}_0$ is the starting value of $\mathbf{Q}_t$, which must be positive-definite to guarantee that $\mathbf{H}_t$ is positive-definite. This model's parameters are estimated using a three-step procedure based on Engle and Sheppard (2001). In the first step, a multivariate VAR(1) model for $\mathbf{r}_t$ is estimated, and we obtain an estimated $\widehat{\boldsymbol{\varepsilon}}_t$ of the residual $\boldsymbol{\varepsilon}_t$. In the second step, univariate GARCH(1,1) models are estimated separately for each residual univariate time series. In the third step using the estimated standardized residuals $\widehat{\mathbf{z}}_t = \widehat{\mathbf{D}}_t^{-1}\widehat{\boldsymbol{\varepsilon}}_t$, computed from the estimated volatilities from the second stage, we take $\overline{\mathbf{Q}} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\mathbf{z}}_t\widehat{\mathbf{z}}_t'$ and we estimate a, b and $\mathbf{Q}_0$ using maximum likelihood. The two last steps are used to estimate the elements in $\mathbf{H}_t$ separately. First the diagonal elements and then using these to estimate the off-diagonal elements.

To select the most appropriate models, we consider the following three subfamilies of VAR(1)-ADCC(1,1)-GARCH(1,1) models:

- The VAR(1)-CCC(1,1)-GARCH(1,1) model uses the Constant Conditional Correlation (CCC) model proposed by Bollerslev (1990). This model supposes that a constant correlation matrix links the univariate models for conditional variances GARCH(1,1) to one another. It assumes that $a = b = g = 0$ and, hence, $\mathbf{R}_t = \mathbf{R}\ \forall t$.
- The VAR(1)-DCC(1,1)-GARCH(1,1) model uses the Dynamic Conditional Correlation (DCC) model proposed by Engle (2002) and Tse and Tsui (2002). This model generalizes the CCC model because the assumption of constant conditional correlations may not seem realistic for many practical financial applications. The DCC allows for the dynamic evolution of correlations and assumes that the asymmetric effects are insignificant ($g = 0$).
- The VAR(1)-ADCC(1,1)-GARCH(1,1) model uses the ADCC model, allowing us to examine the degree to which changes in asset correlation show evidence of asymmetric responses to negative returns.

Therefore, we considered $k = 6$ possible models resulting from the combination of the CCC, DCC, and ADCC models with the multivariate normal distribution (N) and Student's t (T) for the conditional error distributions $\varepsilon_t|\mathscr{F}_{t-1}$.These models are denoted CCC_N, DCC_N, and ADCC_N when the errors are jointly normally distributed, and CCC_T, DCC_T, and ADCC_T when the multivariate Student's t is used for the distribution.

We also considered multivariate Student's t because the Gaussian assumption is rejected in many finance-related applications because of conditional leptokurtosis. This fact is especially interesting for risk analysis, where the tail properties of return distributions are of primary concern. In each period, we were interested in choosing the model out of the six that improved portfolio performance in terms of risk.

5.2. *Strategies*

We considered strategies with a constant size of the validation window ($h_{val,t} = h_{val}$) with ($hval > 0$) and without ($h_{val} = 0$) model selection process. We chose typical update frequencies for investors (daily, monthly, quarterly and yearly) for both strategies. Regarding the type of estimation window, we considered two possibilities: a recursive expanding window that includes all the previous periods and a recursive rolling window that includes the last h0 observations. In our case, we chose $h_0 = 504$, which corresponds to a period of approximately two years, due to the complexity of the models to be compared. The initial estimation period corresponds to January 3, 2012. Section 4.2.1 describes the strategies used in further detail without carrying out a model selection process, while Section 4.2.2 is devoted to the strategies with a model selection process.

5.2.1. *Strategies without a model selection process*

For each model M, we considered eight estimation strategies that result from the combination of the parameter update frequency and the selected type of estimation window. These combinations are the following:

- Daily update and recursive expanding window, denoted as *Daily Expanding*, where $t_{end,t} = h_{estim,}$ $t= t$
- Daily update and recursive rolling window, denoted as *Daily Rolling*, where $tend,t = t$, $hestim,t = h_0$
- Monthly update and recursive expanding window, denoted as *Monthly Expanding*, where $t_{end,t} = h_{estim,t} = t_{m(t)}$, where $t_{m(1)}$ denotes the initial period of the month which contains t
- Monthly update and recursive rolling window, denoted as *Monthly Rolling*, where $t_{end,t} = t_{m(t)}$, $h_{estim,t} = h_0$
- Quarterly update and recursive expanding window, denoted as *Quarterly Expanding*, where $t_{end,t} = h_{estim,t} = t_{q(t)}$, where $t_{q(t)}$ denotes the initial period of the quarter which contains t
- Quarterly update and recursive rolling window, denoted as *Quarterly Rolling*, where $t_{end,t} = t_{q(t)}$, $h_{estim,t} = h_0$
- Yearly update and recursive expanding window, denoted as *Yearly Expanding*, where $_{tend,t} = h_{estim,t} = t_{y(t)}$, where $t_{y(t)}$ denotes the initial period of the year which contains t
- Yearly update and recursive rolling window, denoted as *Yearly Rolling*, where $_{tend,t} = t_{y(t)}$, $h_{estim,t} = h_0$

5.2.2. *Strategies with a model selection process*

In this case, we considered strategies with $h_{val} \in \{1,21,63,252\}$ larger than the updating estimation frequencies for each updating estimation frequency. For instance, for quarterly selection strategies, we have considered the following six possibilities: *Daily rolling* ($h_{end,t}=t$, $h_{estim,t} = h_0$, $h_{val,t} = 63$), *Daily Expanding* ($h_{end,t}= h_{estim,t} = t$, $h_{val,t} = 63$), *Monthly rolling* ($h_{end,t}=t_{m(t)}$, $h_{estim,t} = h_0$, $h_{val,t} = 63$), *Monthly Expanding* ($h_{end,t}= h_{estim,t} = t_{m(t)}$, $h_{val,t} = 63$), *Quarterly rolling* ($h_{end,t}=t_{q(t)}$, $h_{estim,t} = h_0$, $h_{val,t} = 63$), and *Quarterly Expanding* ($h_{end,t}= h_{estim,t} = t_{q(t)}$, $h_{val,t} = 63$).

We considered the 6 models described in Section 4.1 in all cases; therefore, the number of strategies with a model selection process was equal to 20 (2 with $h_{val} = 1$; 4 with $h_{val} = 21$; 6 with $h_{val} = 63$; 8 with $h_{val} = 252$).

5.3. Empirical results

This section provides the empirical results. Sub-Section 4.3.1 presents the results of the strategies that do not perform any model selection process. Sub-Section 4.3.2 presents the results of the strategies that perform a sequential model selection process.

5.3.1. Strategies without a model selection process

For the DCC_T model, Table 2 shows the results of the comparison procedure described in Section 3.2.2.1, applied to the eight strategies, which differ in the updating frequency and the size of the model estimation window. The results obtained with the other five models are similar, which do not show for brevity. Table 2 contains the values of the t statistics of the regression coefficient model (1) for each pairwise comparison of strategies. The bottom row of the table displays the index numbers corresponding to the average of the estimated standard deviation of the observed one-step-ahead returns $\left\{\mathbf{w}_{t,S}^{j\prime}\mathbf{r}_t; t = t_0, \ldots, T\right\}$ of the selected portfolios by each strategy S for each return $\mathbf{m}_j \in \mathbf{R}$, taking the minimum average as a reference.

The best strategy has a quarterly model update frequency and is estimated with an expanding window. These results are according to the frequency with which many businesses, analysts, and government agencies release critical new data about various markets or economic indicators at the end of each quarter. In contrast, the selection of an expanding window reveals the estimated models' stability to describe the joint evolution of the series, which can be explained by the implicit loss of relevance of the most remote past implied by GARCH models.

Table 3 shows the corresponding results when we compare the best strategies obtained for each model, indicating that the best strategy corresponds to using a DCC_T model. This result highlights the existence of changing correlations among the series over time, the conditional leptokurtosis typical of financial series, and the practical non-relevance of incorporating asymmetric effects in the selection of the best portfolios. However, the improvements in terms of portfolio volatility are modest. With the only exception of CCC models, where improvements of around 6% are appreciated in volatility, these improvements are all under 0.2% for the rest of the

Table 2
Engle and Colacito test applied to the DCC_T model.

	Daily Expanding	Daily Rolling	Monthly Expanding	Monthly Rolling	Quarterly Expanding	Quarterly Rolling	Yearly Expanding	Yearly Rolling
Daily Expanding		-3.940	19.354	16.861	**19.709**	13.835	19.063	15.940
Daily Rolling			18.283	17.888	**18.593**	15.384	17.870	16.317
Monthly Expanding				-0.992	**3.434**	-0.442	2.121	-1.276
Monthly Rolling					**2.440**	0.123	1.729	-0.556
Quarterly Expanding						-1.460	**-2.162**	**-3.062**
Quarterly Rolling							1.052	-0.459
Yearly Expanding								-2.394
Yearly Rolling								
Volatility Ratio	113.302	114.789	100.938	100.196	**100.000**	100.338	100.759	100.917

Blue (red) bold signals the best strategy t statistics, which rejects the null hypothesis because $\beta_{S_1,S_2} > 0$ ($\beta_{S_1,S_2} < 0$).

models.

To graphically show the effect of the estimation update window, Figs. 3 to 6 contain the evolution of the parameter estimation of the DCC_T model. Specifically, Figs. 3 to 5 contain the estimation of parameters ω_i, α_i, and β_i, corresponding to the individual conditional variances of each series modeled by independent GARCH(1,1). Fig. 6 contains the evolution of the estimation of parameters *a,* and *b*, which capture the time-varying correlation matrix, indicating that the green series, corresponding to a quarterly update, adaptively reflects the evolution of the series parameters, smoother than daily and monthly updates. However, a yearly update (cyan series) is somewhat too adaptive and moves away in some periods from the real evolution of the model parameters; this movement does not occur with the annual update, which is too little adaptive.

5.3.2. Strategies with model selection

Table 4 shows the results from the procedure described in Section 3.2 applied to the comparison of strategies for a model selection process, classified according to the size of the validation period (daily, monthly, quarterly, and yearly). Again, the best strategies correspond to those that use an expanding estimation window and the largest updating frequency, whose best behavior corresponds to a quarterly model update frequency. The sole exception is the yearly strategy, which uses a yearly validation period.

Fig. 7 shows the results of the sequential model selection processes for these best strategies, classified according to the size of the validation period. As expected, the strategies that carry out model selection processes with smaller validation periods tend to better adapt to the series' evolution and change the model more frequently. The most selected models tend to be the DCC_T, CCC_T, and CCC_N with considerable validation periods (quarterly and yearly) and the ADCC_T, DCC_T, and CCC_T with smaller validation periods (daily and monthly). This result is also logical because the larger (smaller) the validation period is, the weaker (stronger) the influence of atypical periods, manifested in the selection of more (less) parsimonious models; however, the improvements in volatility are modest and usually less than 2%, mainly if the size of the validation period is large. This result reveals that the determination of the best model is not very sharp.

Fig. 8 compares the final portfolio strategy selected by the best strategies with and without model selection, using the procedures described in Section 3.2.2 to apply to each pair of strategies. The figure shows that the best performance corresponds to the strategy that does not carry out any model selection process; it assumes that the model is equal to DCC_T for the entire analyzed period, followed by the strategy whose validation period is equal to a quarter. No significant differences were observed between both strategies throughout the period analyzed.

The best performance, regarding the volatility ratio, corresponded to the strategy that did not perform any model selection and used the DCC_T model to calculate the variance and covariance matrices **H**t. Its final portfolio was at least 1% lower than the final portfolios selected by the remaining strategies. This result is due to the excellent behavior of this model over time, which is usually among the two or three best models (see Fig. 7), with slight differences in the average value of volatilities concerning the best model.

5.3.3. Evolution of the selected portfolio weights and their volatilities

This sub-section compares the final portfolio weights and their volatilities selected by the two best strategies: a DCC_T model without model selection, quarterly updating, and expansive estimation window (Strategy S_1), and quarterly model selection and updating with expansive estimation window (Strategy S_2). Fig. 9 shows their weights, predicted volatilities $\{\sigma^2_{t,S}; t = t_0, \ldots,$

Table 3
Comparison of the best strategies which do not perform any model selection process (hval,t=0).

	CCC_N	DCC_N	ACCC_N	CCC_T	DCC_T	ADCC_T
CCC_N		14.957	14.960	-0.432	**15.264**	15.231
DCC_N			-3.441	-14.957	**7.589**	7.195
ACCC_N				-14.960	**7.771**	7.396
CCC_T					**15.264**	15.231
DCC_T						**-4.343**
ADCC_T						
Volatility Ratio	106.085	100.179	100.187	106.085	**100.000**	100.019

Blue (red) bold signaled the best strategy t statistics, which rejects the null hypothesis because $\beta_{S_1,S_2} > 0$ ($\beta_{S_1,S_2} < 0$).

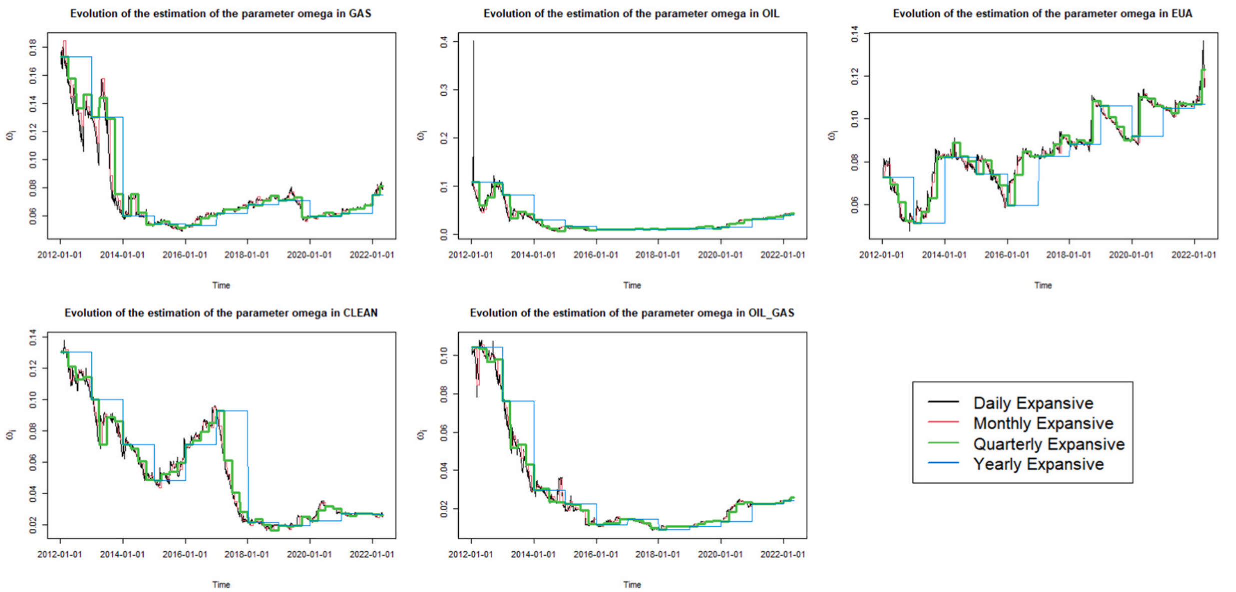

Fig. 3. Matrix graph with the daily evolution of the estimations of the parameter ω_i corresponding to the GARCH(1,1) model for the volatility of the five-asset series.

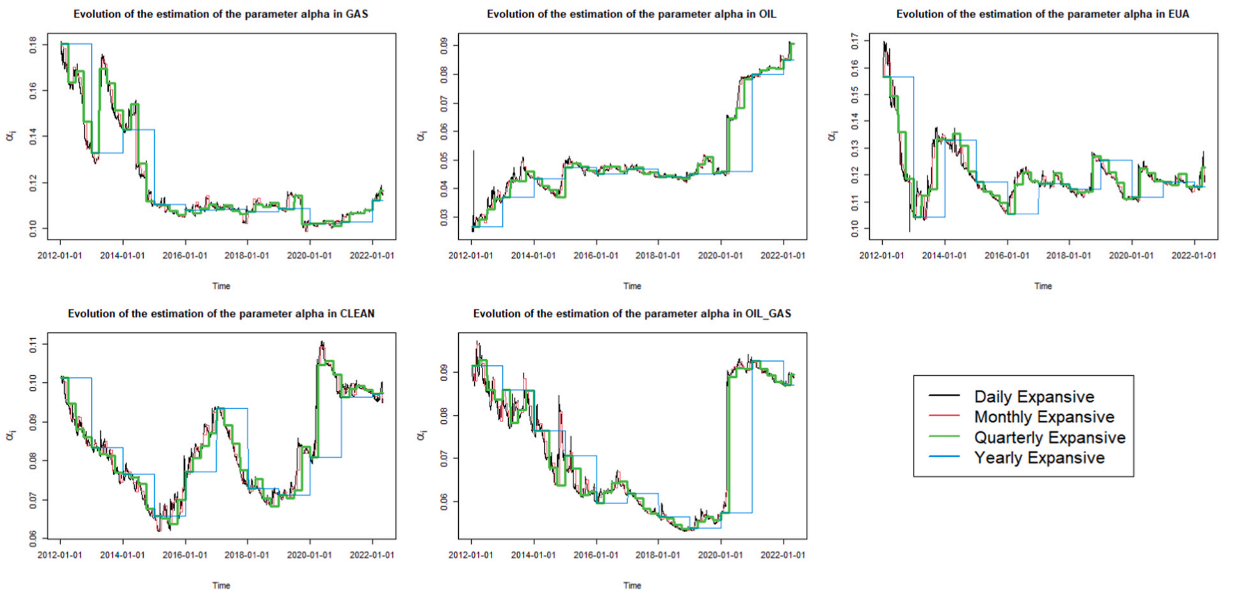

Fig. 4. Matrix graph with the daily evolution of the estimations of parameter α_i corresponding to the GARCH(1,1) model for the volatility of the five-asset series.

$T\Big\}$ with $\sigma^2_{t,S} = \mathbf{w}'_{t,S}\mathbf{H}_{t,S,M_{opt,t}}\mathbf{w}_{t,S}$, and their observed volatilities $\left\{\left(\mathbf{w}'_{t,S}\mathbf{r}_t\right)^2; t = t_0, \dots, T\right\}$. The figure shows that the weights of the selected portfolios and the expected volatilities are very similar for both strategies; therefore, the results are robust to both the model specification and its updating and estimation procedure, thus providing greater reliability and realism to this study.

In general, for most of the analyzed period (until 2021, with the sole exception of the COVID-19 pandemic period), the final portfolios took long positions in GAS, OIL, and CLEAN and short positions in OIL_GAS and the free risk asset. This result reflects the greater importance of GAS and OIL as fossil energies throughout the analyzed period and, in the case of CLEAN and OIL_GAS, the significantly positive correlation between them (around 0.5, see Fig. 2) and the lower skewness and kurtosis of the CLEAN series. The only exception to this rule occurred in the last sub-period (from January 2022 onwards), where the final portfolios selected long positions in CLEAN and OIL_GAS and short in the free risk asset due to the Russo–Ukrainian war. On the one hand, the decline in

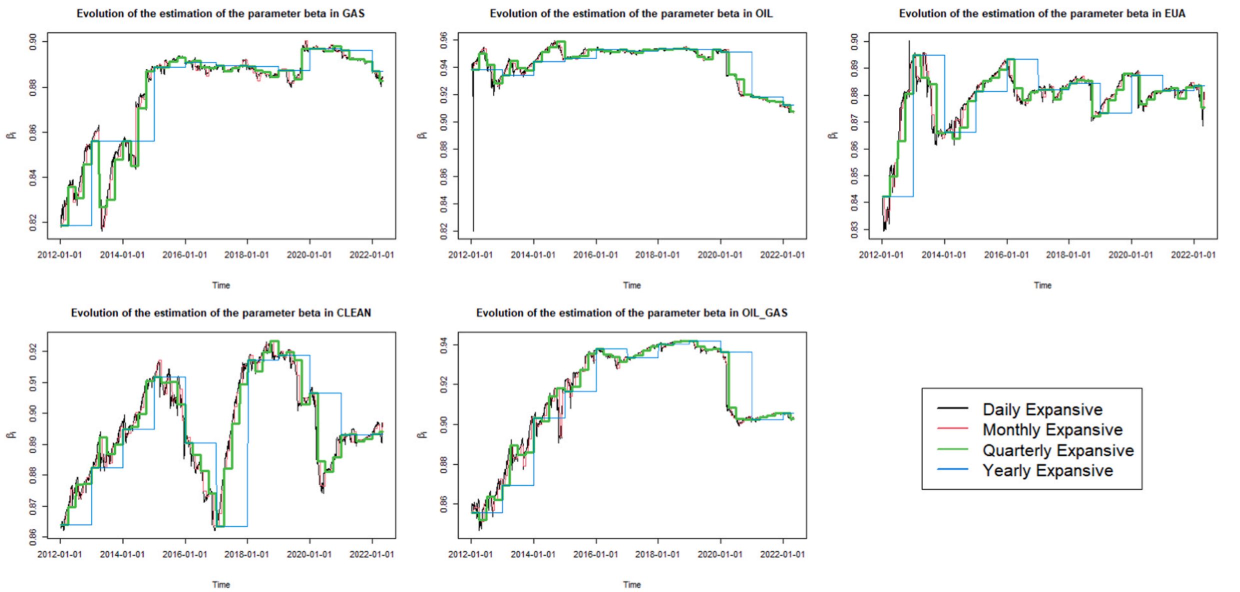

Fig. 5. Matrix graph with the daily evolution of the estimations of parameter β_i corresponding to the GARCH(1,1) model for the volatility of the five-asset series.

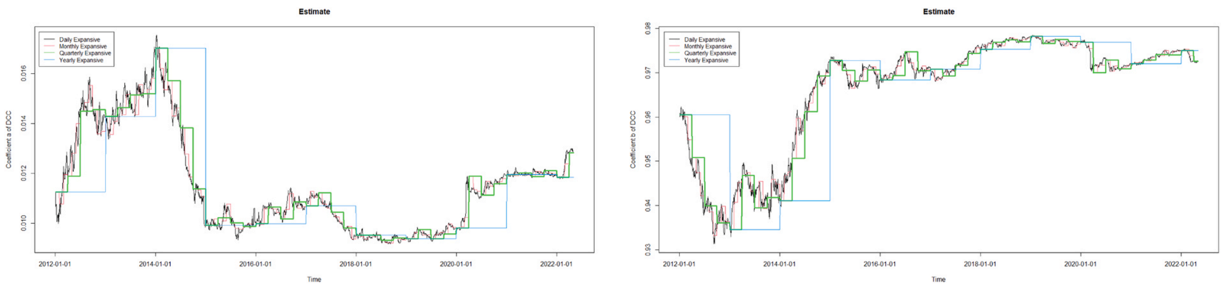

Fig. 6. Matrix graph with the daily evolution of parameters a and b estimations, which capture the time-varying correlation matrix.

Russian oil and gas exports to NATO countries has led to an increase in the market share of the large European energy companies included in the OIL_GAS index. On the other hand, the increase in inflation and, consequently, a probable scenario of rising interest rates have reduced the attractiveness of investing in fixed-income assets or government bonds. Therefore, the leverage effect that reflects the short position in risk-free assets shows that investment is made in clean energy and that long positions are occasionally opened in European energy companies listed on the equity market. Finally, EUA weights tend to have a trajectory similar to CLEAN weights, reflecting the relationship between EU ETS and renewable energy companies, but with lower weights due to their larger volatility (see Fig. 1).

Several periods can be differentiated. In the first period (2012–2016), GAS and, occasionally, OIL tended to have the highest weights in the portfolio. In this period, when the Sovereign debt crises (2012–2014) and the Brexit announcement (2015) took place, GAS became very important. GAS was considered a substitute refuge value for oil because it is a less dirty energy source (EIA, 2021) and, therefore, could be a better alternative than oil for the energy transition; however, since the Paris Agreement (December 2015), the optimal portfolio in each period tends to allocate the highest weight to CLEAN. The Paris Agreement was a milestone in the multifaceted fight process against global warming because it was the first binding agreement that brought all countries together in a mutual cause to engage in grandiose efforts to combat climate change and accommodate its effects. Therefore, the Paris Agreement underpinned the change toward an economy that can gradually be managed without oil and gas as support. Since 2016, the preponderance of CLEAN was only lost at certain times during the 2020–2021 period, coinciding with the most atypical years globally in recent generations due to the COVID-19 pandemic. During this period, all assets (GAS, OIL, CLEAN, and EUA) tended to have similar weights, and OIL appeared as a safe-haven asset in the portfolio. Despite all the difficulties and uncertainty in 2020 and the first half of 2021, with a high percentage of the European population vaccinated, since the second half of 2021, renewable energies have once again positioned themselves firmly with the most significant weights in the optimal portfolio with minimum risk. The reason was the search for a better, more equitable, resilient, clean, and fair future. In this final sub-period, OIL has moderate positive weights. The preponderance of CLEAN is associated with a fall in the importance of the GAS weights, accentuated in the last period by the increase in

Table 4
Comparison of the best strategies, which perform model selection, classified according to the validation period. Blue (red) bold signaled the best strategy t statistics, which rejects the null hypothesis because $\beta_{S_1,S_2} > 0$ ($\beta_{S_1,S_2} < 0$).

Monthly selection

	Daily Expanding	Daily Rolling	Monthly Expanding	Monthly Rolling
Daily Expanding		-4.152	**16.788**	14.558
Daily Rolling			**14.785**	15.745
Monthly Expanding				-1.078
Monthly Rolling				
Volatility Ratio	111.399	113.156	**100.000**	100.472

Quarterly selection

	Daily Expanding	Daily Rolling	Monthly Expanding	Monthly Rolling	Quarterly Expanding	Quarterly Rolling
Daily Expanding		-4.168	16.801	13.768	**17.228**	9.965
Daily Rolling			14.471	14.845	**14.922**	11.482
Monthly Expanding				-1.347	**2.981**	-1.204
Monthly Rolling					**2.345**	-0.490
Quarterly Expanding						**-1.969**
Quarterly Rolling						
Volatility Ratio	112.039	114.223	100.508	101.781	**100.000**	102.061

Daily selection

	Daily Expanding	Daily Rolling
Daily Expanding		**-3.704**
Daily Rolling		
Volatility Ratio	**100.000**	101.371

Yearly selection

	Daily Expanding	Daily Rolling	Monthly Expanding	Monthly Rolling	Quarterly Expanding	Quarterly Rolling	Yearly Expanding	Yearly Rolling
Daily Expanding		-4.160	17.029	13.564	**16.734**	11.170	15.900	12.734
Daily Rolling			15.192	15.846	**15.354**	12.004	14.570	13.079
Monthly Expanding				-1.011	**3.204**	-1.199	1.002	-2.218
Monthly Rolling					**1.955**	-0.835	1.189	-1.923
Quarterly Expanding						-1.883	**-3.846**	**-3.124**
Quarterly Rolling							1.370	-0.642
Yearly Expanding								-2.468
Volatility Ratio	112.019	113.748	100.523	100.905	**100.000**	101.827	100.490	102.797

its volatility due to the pandemic and the Russo–Ukrainian war.

The evolution of the expected and observed volatilities for the final portfolios of both strategies tends to be similar, confirming the validity of the volatility expectations of the selected model. A more significant increase in volatility is observed in specific periods that coincide with the start of unexpected events (Sovereign debt crisis, Brexit, COVID-19, and the Russo–Ukrainian war) that significantly raise risk levels. However, they are corrected by the selected portfolio, which quickly reduces these increases, showing that our procedure performs adequate risk management.

Fig. 10 shows the differences between the final portfolio weights of each asset, and Fig. 11 displays the differences between the expected and observed portfolio volatilities. These figures allow us to analyze the differences between both strategies.

The main differences occurred during the 2012–2014 and 2018–2019 crises, including when the market stability reserve was established within the EU ETS. Strategy S_2 selects CCC_N as the best model instead of DCC_T in both periods; however, as regards the impact of this selection on the sign of differences, the volatilities (right panel of Fig. 11) were not clear. No significant differences between the values of both volatilities were observed.

5.3.4. Discussion

Our results highlight the time-varying character of the covariance matrices between the clean energy, EUA, and dirty energy assets, with the best-performing strategy being that which assumed a constant model, DCC_T, for the entire analyzed period, with a quarterly model update frequency and that uses an expanding window to estimate the model. The selection of a DCC model coincides with other studies, such as Lee et al. (2014), Dutta et al. (2020) or, more recently, Zhong and Liu (2021), who compare different models to analyze dynamic correlations and volatility spillovers between stock and energy markets. Regarding the model update frequency, this finding aligns with how businesses, analysts, and government agencies often release important new data about different markets or economic indicators at the end of each quarter. These periodic updates serve several vital purposes, such as facilitating informed decision-making, allowing for necessary adjustments in strategies, and upholding transparency and accuracy in reports. Ensuring that stakeholders have access to the most current and relevant information is paramount for making well-informed financial and investment decisions. While a constant DCC_T model has emerged as the winner, quarterly updating with an expansive estimation window and model selection produced comparable results. This highlights the robustness of our portfolio optimization approach. This stability provides investors with a reliable tool for navigating uncertain market conditions, enhancing the credibility of our methodology.

In addition, our analysis indicates that the portfolios effectively managed risk, with an ability to mitigate increased risk levels

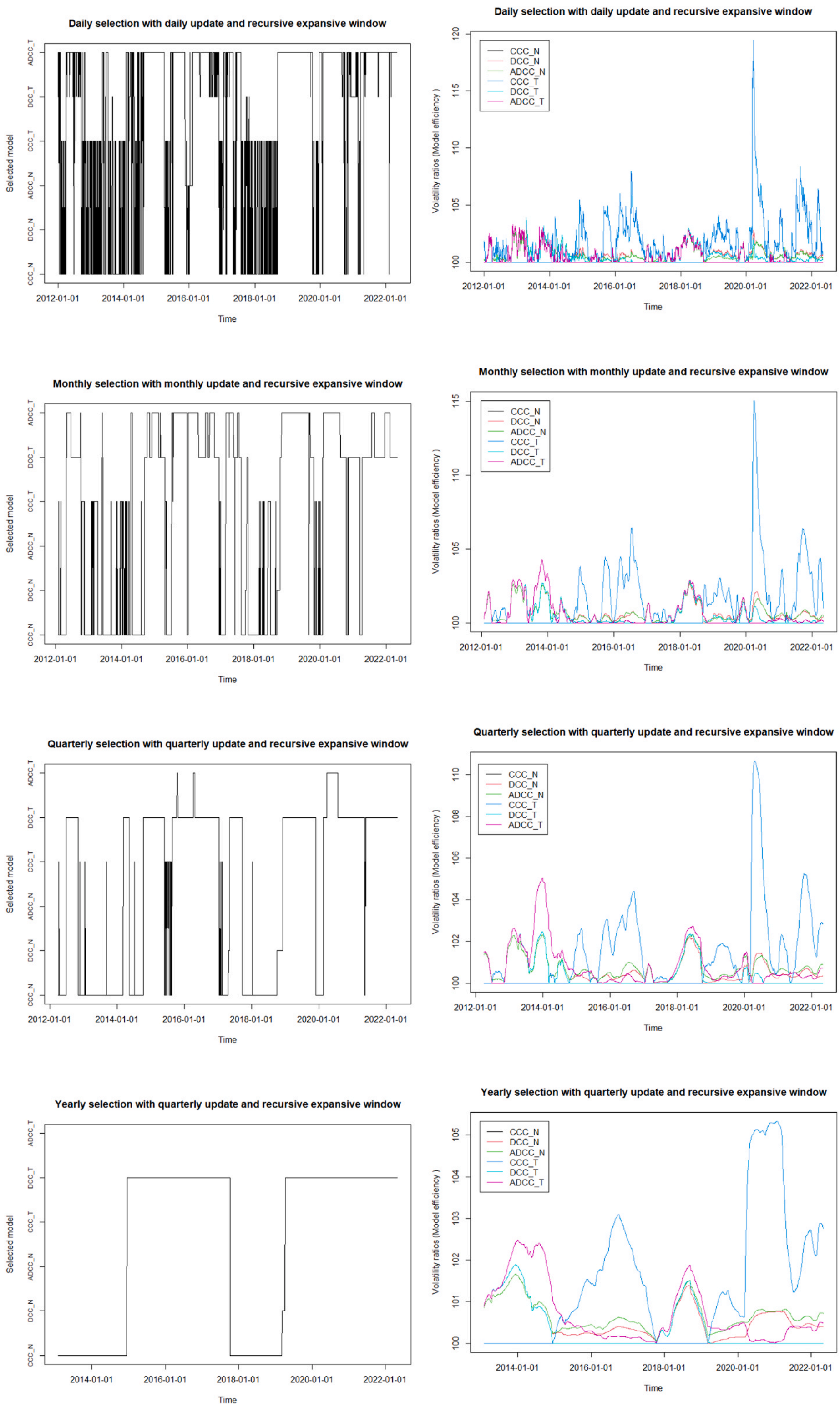

Fig. 7. Matrix graph with the model selected (left column) and their index numbers of the volatility averages, taking the minimum average as reference (x100) (right column) corresponding to the best strategies that perform a model selection process. The bottom row indicates the frequency of the model selection process.

Fig. 8. Matrix with the pairwise model comparisons of the best strategies through the t statistic of the Diebold and Mariano (lower triangular matrix) and the dynamic evolution of Giacomini and Rossi's fluctuation test (upper triangular matrix). The bottom row displays the volatility ratios to compare the final portfolio strategy. Blue (red) bold signaled the 95% significant statistics, which rejects the null hypothesis because $\beta_{S_1,S_2} > 0$ ($\beta_{S_1,S_2} < 0$).

during specific events, such as the sovereign debt crisis, Brexit, COVID-19, and the Russo-Ukrainian war (see Fig. 9, second and third rows). These results align with those of Kuang (2021), who demonstrated the advantages of risk diversification between clean energy subsectors and dirty assets. Furthermore, they are consistent with the recommendations by Asl et al. (2021) that suggested oil and gas portfolio investors should adjust their hedging strategies frequently and incorporate clean energy assets.

The composition of our optimal portfolio exhibited different patterns over the analyzed period, reflecting its adaptability to changing market dynamics. In this regard, Tan et al. (2020) demonstrated that information from other markets affects the European carbon market and highlighted that the type of information spillover varies over time horizons, which is in line with our findings. Until 2016, our optimal portfolios prominently featured assets associated with fossil fuels, such as GAS and OIL, reflecting their historical significance in energy markets. This result is consistent with Jebabli et al. (2022), however, this composition also included EUAs and CLEAN assets, underscoring the presence and contribution of sustainable investments to diversification. In essence, renewables and EUAs played pivotal roles within the portfolio, coexisting alongside fossil fuel assets, thus reflecting the prevailing market dynamics.

Balcılar et al. (2016) showed significant changes in the hedge effectiveness over the different phases of the European carbon market and suggested that transmission of risks to carbon markets leads to the need to implement stabilization policies for this market. In this sense, the Paris Agreement, enacted in December 2015, marked a pivotal moment in the transition toward a more sustainable economy. In line with this, our portfolio allocation began to favor CLEAN assets shortly after this agreement (see first row in Fig. 9).

This shift reflects the global commitment to combat climate change and reduce reliance on fossil fuels. While this trend was briefly interrupted during the atypical years of the COVID-19 pandemic, where switching from oil to gas for portfolio hedging was generally more effective (coinciding with Jebabli et al., 2022), it regained momentum in the second half of 2021, emphasizing the commitment to clean and renewable energy sources. Our findings align with prior research by Czech, Wielechowski (2021) and Wan et al. (2021), who highlight a strong commitment to clean energy companies. Notably, these studies also emphasize the resilience of the alternative energy sector compared to conventional energy during the COVID-19 pandemic.

The final sub-period (from January 2022 onwards) witnessed a noteworthy shift in portfolio composition. This shift can be attributed to the Russo-Ukrainian war and other factors, resulting in long positions in CLEAN. These findings underline the resilience of

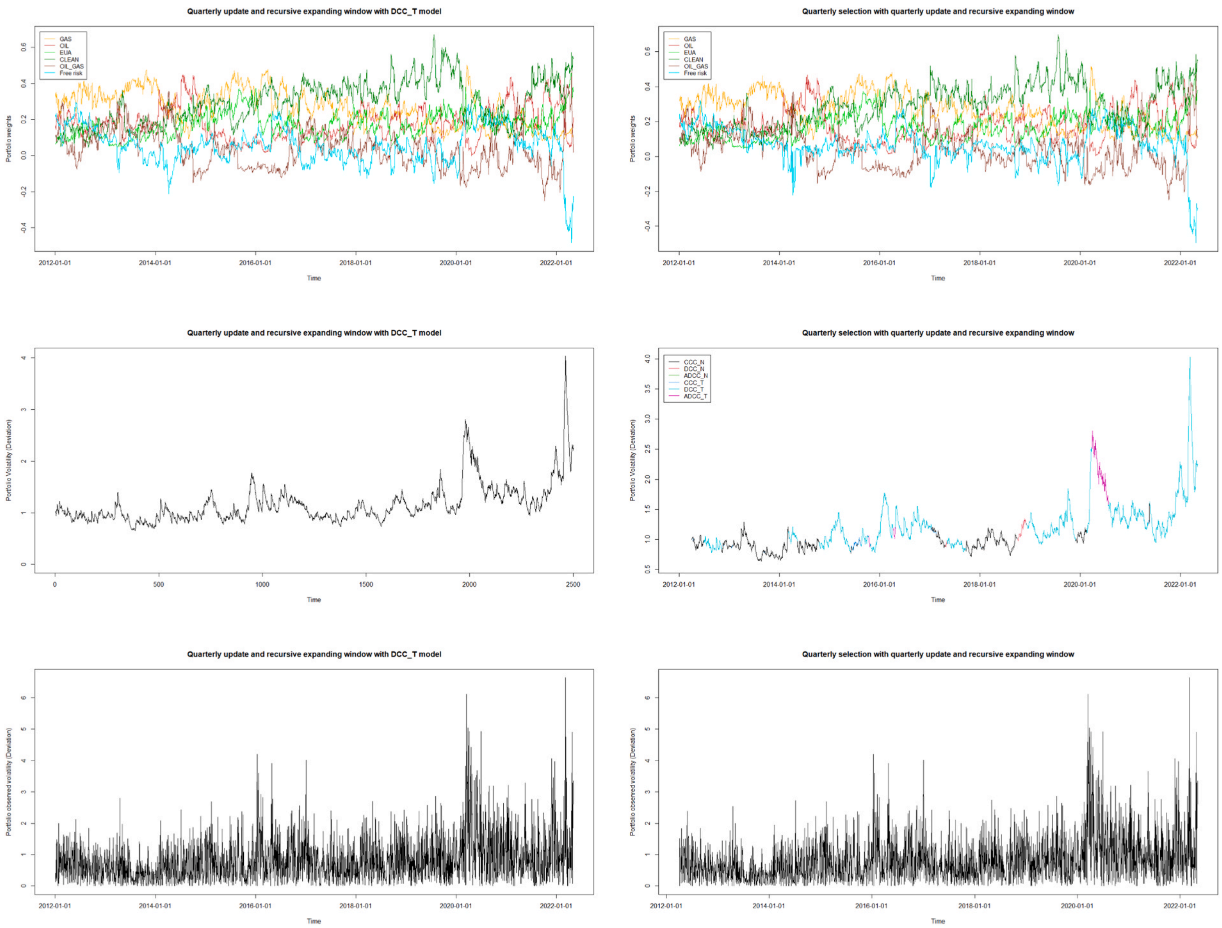

Fig. 9. Left: Strategy S1 and right: Strategy S_2); first row: evolution of the weights of the final portfolio strategy; second row: expected portfolio volatilities; third row: observed portfolio volatilities.

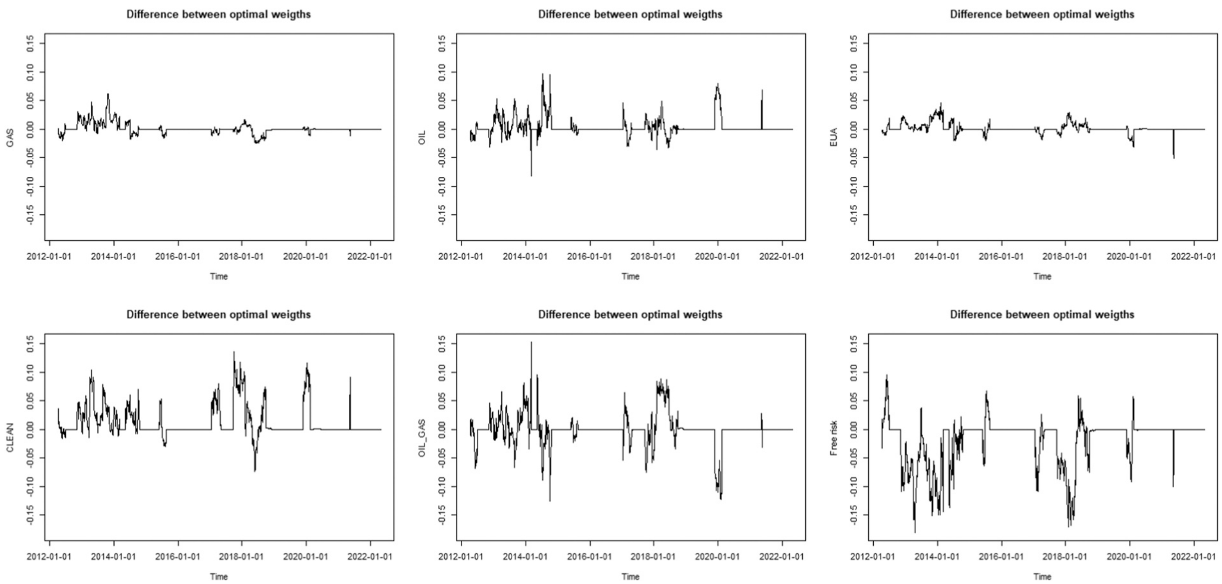

Fig. 10. Matrix graphs with the differences $\{\mathbf{w}_{k,t,S_1} - \mathbf{w}_{k,t,S_2}; k = 1, \ldots, 6\}$ between the final portfolio weights of the strategies S_1 and S_2 for each of the assets.

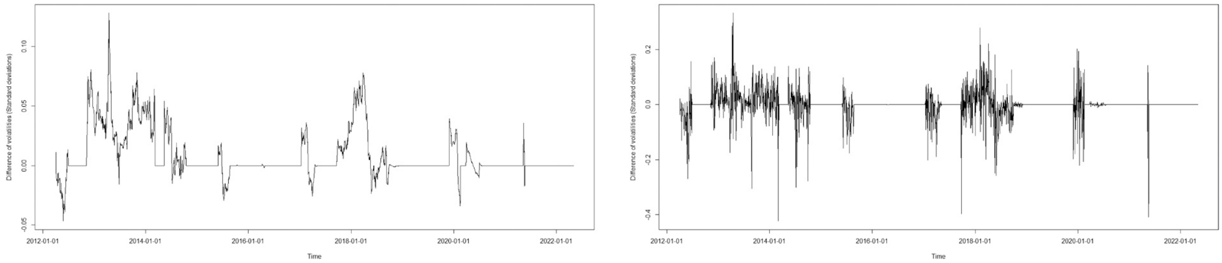

Fig. 11. Differences in the expected (left panel) and observed (right panel) volatilities of the final portfolios of the strategies S_1 and S_2.

clean energy investments during turbulent times, in line with the recent papers by Gargallo et al. (2022a) and (2022b), and Cheikh and Zaied (2023).

In conclusion, our results underscore the adaptability of our portfolio optimization approach, with a consistent DCC_T model proving effective across the entire analyzed period. These portfolios not only effectively manage risk, reducing exposure during significant events, but also align with the global trend towards sustainable energy. Notably, the recent shift towards long positions in CLEAN and EUA assets highlights the resilience of clean investments, even in challenging times. This research enhances our understanding of energy portfolio management and risk mitigation strategies. In summary, both renewable energies and EUAs played pivotal roles within the portfolio, with their individual contributions evolving in response to the ever-changing economic and political landscape.

6. Conclusions

This paper examined the appeal of investing in clean energy stocks and EUAs in the carbon and energy stock markets. Specifically, we conducted a statistical analysis of the daily evolution of minimum risk portfolio weights, paying particular attention to the importance of the above assets in the portfolio composition. To that aim, we have proposed a methodology to build dynamically minimum variance portfolios for a set of plausible mean return vectors. This methodology is based on a sequential algorithm to estimate as best as possible the evolution of the covariance matrices of the returns of a set of assets to obtain the lowest expected one-step ahead volatility of the minimum variance portfolios. We carried out a sequential model selection process where we determined the updating frequency, the estimation window of the model parameters, and the size of the out-of-sample model validation period. The methodology is very flexible and realistic, enabling different inversion strategies to be designed and compared to build portfolios with good risk properties.

Our findings showed that optimum minimum risk portfolios are obtained using multivariate DCC-GARCH models with fat-tailed error distributions, which capture changing correlations over time but update their unknown parameters every quarter. This result is in line with the regulation that requires investment funds to publish their portfolios at least every three months. Furthermore, quarters are relevant for investors because many enterprises, analysts, public authorities, governmental agencies, etc., emit quarterly information on diverse markets or economic measures. We also show that excessive updating of the model estimation and selection process can increase risk levels artificially or spuriously.

The analysis of the evolution of the weights of the constructed energy portfolios highlights that currently investing in clean energies and EUAs is gainful not only because it is a way of collaborating toward the transition of sustainable energy to renewable sources but also for its appeal from a financial viewpoint. Although the Russo–Ukrainian war has indeed changed the global scenario, clean energies are still the main bet to be able to fight climate change, and this war has created an even greater need to accelerate the energy transition. Specifically, Europe has realized that it is too dependent on energy, and this transition, initially intended to be carried out gradually, must be implemented much more quickly.

Our research yields practical implications that are closely tied to the results of our study. For investors and portfolio managers, our findings underscore the importance of considering clean assets, such as renewable energies and EUAs, in investment portfolios in order to achieve both financial gains and sustainability objectives. Furthermore, the resilience of clean energy assets during times of economic turmoil, such as the sovereign debt crisis, Brexit, and the COVID-19 pandemic, highlights their ability to mitigate risks. This insight empowers investors to make well-informed decisions about risk management and portfolio diversification while capitalizing on the sustainable investment landscape. The need for dynamic portfolio management, including quarterly model updates, is in line with common industry practices where businesses, analysts, and government agencies frequently release critical market data and economic indicators at the end of each quarter. For policymakers, our study provides valuable insights for striving to align financial systems with environmental sustainability goals. The prominence of clean energy assets in our optimized portfolios indicates the importance of policies that incentivize clean energy investments. Policymakers can draw inspiration from the Paris Agreement's impact on portfolio composition, as it prompted a notable shift toward clean energy assets. To support sustainable finance practices, governments can enact policies that facilitate clean energy adoption, aligning with international climate objectives. Furthermore, our research emphasizes the role of EUAs in portfolios, suggesting that policymakers can promote and expand emissions trading systems to encourage cleaner energy practices. In addition, and given that the transition to clean energy is evident in our portfolio compositions, reflecting a changing energy landscape, energy companies and professionals can use this insight to strategically pivot toward renewable energy sources and reduce carbon emissions. The positive performance of clean energy assets, even during times of global crisis, underscores their resilience and attractiveness to investors, policymakers, regulators, companies, and civil society. By aligning business strategies with sustainability goals and incorporating clean energy practices, industry professionals can position their organizations to thrive in the evolving energy market (Gutiérrez-López et al., 2022).

All these implications demonstrate the practical relevance of our results, offering actionable guidance for investors, policymakers, and industry professionals. Our study's insights into energy and low-carbon portfolios provide a valuable bridge between academic research and real-world applications, facilitating informed decision-making in a dynamic and sustainable investment landscape.

In this paper, only daily returns are considered. It would be interesting to extend the methodology to other investment horizons to analyze whether working with extended returns, such as weekly, monthly, or quarterly, is more worthwhile. Furthermore, the paper uses the Markowitz mean-variance set-up to build minimum variance portfolios. It would be interesting to work with other ways of managing risk, such as the conditional value-at-risk, to build minimum risk portfolios or generalizations of the Markowitz model that incorporates skewness and kurtosis into the classical mean-variance allocation framework (Uberti, 2023). Finally, to fight against climate change, there are other areas (electricity sector, water, natural resources, etc.) where it would also be necessary to encourage investment and the proposed methodology in this paper could be applied.

CRediT authorship contribution statement

Pilar Gargallo: Methodology; Software; Formal Analysis, Writing-Original Draft; Writing-Review & Editing; Visualization; Validation. **Luis Lample**: Data Curation; Writing-Review & Editing; Investigation; Validation. **Jesús Miguel**: Software; Data Curation; Visualization; Formal Analysis; Investigation. **Manuel Salvador**: Conceptualization; Methodology; Formal Analysis; Validation; Writing-Review & Editing; Supervision; Funding Acquisition.

Declaration of Competing Interest

The authors declare that there is not any conflict of interest

Data Availability

Data will be made available on request.

References

Antonakakis, N., Filis, G., 2013. Oil prices and stock market correlation: a time-varying approach. Int. J. Energy Stat. 01, 17–29. https://doi.org/10.1142/S2335680413500026.

Arouri, M.E.H., Jouini, J., Nguyen, D.K., 2012. On the impacts of oil price fluctuations on European equity markets: volatility spillover and hedging effectiveness. Energy Econ. 34, 611–617. https://doi.org/10.1016/j.eneco.2011.08.009.

Arslan-Ayaydin, Ö., Thewissen, J., 2016. The financial reward for environmental performance in the energy sector. Energy Environ. 27, 389–413. https://doi.org/10.1177/0958305×15627547.

Asl, M.G., Canarella, G., Miller, S.M., 2021. Dynamic asymmetric optimal portfolio allocation between energy stocks and energy commodities: evidence from clean energy and oil and gas companies. Resour. Policy 71, 101982. https://doi.org/10.1016/j.resourpol.2020.101982.

Balcılar, M., Demirer, R., Hammoudeh, S., Nguyen, D.K., 2016. Risk spillovers across the energy and carbon markets and hedging strategies for carbon risk. Energy Econ. 54, 159–172. https://doi.org/10.1016/j.eneco.2015.11.003.

Benz, E., Trück, S., 2006. CO2 emission allowances trading in. Eur. –Specif. a N. Cl. assets. Probl. Perspect. Manag 4, 30–40.

Bing, X., Bloemhof-Ruwaard, J., Chaabane, A., Van Der Vorst, J., 2015. Global reverse supply chain redesign for household plastic waste under the emission trading scheme. J. Clean. Prod. 103, 28–39. https://doi.org/10.1016/j.jclepro.2015.02.019.

Bollerslev, T., 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model. Rev. Econ. Stat. 72, 498. https://doi.org/10.2307/2109358.

Bondia, R., Ghosh, S., Kanjilal, K., 2016. International crude oil prices and the stock prices of clean energy and technology companies: evidence from non-linear cointegration tests with unknown structural breaks. Energy 101, 558–565. https://doi.org/10.1016/j.energy.2016.02.031.

Broadstock, D.C., Cao, H., Zhang, D., 2012. Oil shocks and their impact on energy related stocks in China. Energy Econ. 34, 1888–1895. https://doi.org/10.1016/j.eneco.2012.08.008.

Bushnell, J.B., Chong, H., Mansur, E.T., 2013. Profiting from regulation: evidence from the european carbon market. Am. Econ. J.: Econ. Policy 5, 78–106. https://doi.org/10.1257/pol.5.4.78.

Caporin, M., McAleer, M., 2014. Robust ranking of multivariate GARCH models by problem dimension. Comput. Stat. Data Anal. 76, 172–185. https://doi.org/10.1016/j.csda.2012.05.012.

Cappiello, L., Engle, R.F., Sheppard, K., 2006. Asymmetric dynamics in the correlations of global equity and bond returns. J. Financ. Econ. 4, 537–572. https://doi.org/10.1093/jjfinec/nbl005.

Castagneto-Gissey, G., 2014. How competitive are EU electricity markets? An assessment of ETS Phase II. Energy Policy 73, 278–297. https://doi.org/10.1016/j.enpol.2014.06.015.

Cheikh, N.B., Zaied, Y.B., 2023. Investigating the dynamics of crude oil and clean energy markets in times of geopolitical tensions. Energy Econ. 124, 106861 https://doi.org/10.1016/j.eneco.2023.106861.

Chen, Y., Zheng, B., Qu, F., 2020. Modeling the nexus of crude oil, new energy and rare earth in China: an asymmetric VAR-BEKK (DCC)-GARCH approach. Resour. Policy 65, 101545. https://doi.org/10.1016/j.resourpol.2019.101545.

Chevallier, J., 2011. A model of carbon price interactions with macroeconomic and energy dynamics. Energy Econ. 33, 1295–1312. https://doi.org/10.1016/j.eneco.2011.07.012.

Choi, K., Hammoudeh, S., 2010. Volatility behavior of oil, industrial commodity and stock markets in a regime-switching environment. Energy Policy 38, 4388–4399. https://doi.org/10.1016/j.enpol.2010.03.067.

Czech, K., Wielechowski, M., 2021. Is the alternative energy sector COVID-19 resistant? comparison with the conventional energy sector: markov-switching model analysis of stock market indices of energy companies. Energies 14, 988. https://doi.org/10.3390/en14040988.

da Silva, P.P., Moreno, B., Figueiredo, N.C., 2016. Firm-specific impacts of CO2 prices on the stock market value of the Spanish power industry. Energy Policy 94, 492–501. https://doi.org/10.1016/j.enpol.2016.01.005.

Dickey, D.A., Fuller, W.A., 1979. Distribution of the estimators for autoregressive time series with a unit root. J. Am. Stat. Assoc. 74, 427–431. https://doi.org/10.1080/01621459.1979.10482531.

Diebold, F.X., Mariano, R.S., 1995. Comparing predictive accuracy. J. Bus. Econ. Stat. 13, 253–263. https://doi.org/10.1080/07350015.1995.10524599.

Dutta, A., Bouri, E., Das, D., Roubaud, D., 2020. Assessment and optimization of clean energy equity risks and commodity price volatility indexes: Implications for sustainability. J. Clean. Prod. 243, 118669 https://doi.org/10.1016/j.jclepro.2019.118669.

Dutta, A., Bouri, E., Noor, M.H., 2018. Return and volatility linkages between CO2 emission and clean energy stock prices. Energy 164, 803–810. https://doi.org/10.1016/j.energy.2018.09.055.

Engle, R., 2002. Dynamic conditional correlation. J. Bus. Econ. Stat. 20, 339–350. https://doi.org/10.1198/073500102288618487.

Engle, R., Colacito, R., 2006. Testing and valuing dynamic correlations for asset allocation. J. Bus. Econ. Stat. 24, 238–253. https://doi.org/10.1198/073500106000000017.

Engle, R., Sheppard, K., 2001. Theoretical and Empirical properties of Dynamic Conditional Correlation Multivariate GARCH (No. w8554). National Bureau of Economic Research, Cambridge, MA. http://www.nber.org/papers/w8554.

Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. Econometrica 50, 987–1007. https://doi.org/10.2307/1912773.

Gargallo, P., Lample, L., Miguel, J.A., Salvador, M., 2021. Co-movements between Eu Ets and the energy markets: a var-dcc-garch approach. Mathematics 9, 1787. https://doi.org/10.3390/math9151787.

Gargallo, P., Lample, L., Miguel, J., Salvador, M., 2022a. Dynamic comparison of portfolio risk: Clean vs dirty energy. Financ. Res. Lett. 47, 102957 https://doi.org/10.1016/j.frl.2022.102957.

Gargallo, P., Lample, L., Miguel, J.A., Salvador, M., 2022b. Dynamic risk management in European energy portfolios: evolution of the role of clean and carbon markets. Energy Rep. 8, 15654–15668. https://doi.org/10.1016/j.egyr.2022.11.146.

Giacomini, R., Rossi, B., 2010. Forecast comparisons in unstable environments. J. Appl. Econ. 25, 595–620. https://doi.org/10.1002/jae.1177.

Gronwald, M., Ketterer, J., Trück, S., 2011. The relationship between carbon, commodity and financial markets: a copula analysis. Econ. Rec. 87, 105–124. https://doi.org/10.1111/j.1475-4932.2011.00748.x.

Gutiérrez-López, C., Castro, P., Tascón, M.T., 2022. How can firms' transition to a low-carbon economy affect the distance to default? Res. Int. Bus. Financ. 62, 101722 https://doi.org/10.1016/j.ribaf.2022.101722.

Hammoudeh, S., Lahiani, A., Nguyen, D.K., Sousa, R.M., 2015. An empirical analysis of energy cost pass-through to CO2 emission prices. Energy Econ. 49, 149–156. https://doi.org/10.1016/j.eneco.2015.02.013.

Hammoudeh, S., Nguyen, D.K., Sousa, R.M., 2014. Energy prices and CO2 emission allowance prices: A quantile regression approach. Energy Policy 70, 201–206. https://doi.org/10.1016/j.enpol.2014.03.026.

Henriques, I., Sadorsky, P., 2008. Oil prices and the stock prices of alternative energy companies. Energy Econ. 30, 998–1010. https://doi.org/10.1016/j.eneco.2007.11.001.

Intergovernmental panel on climate change (I.P.C.C.), 2023. Climate Change 2021 – The Physical Science Basis: Working Group I Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge. https://doi.org/10.1017/9781009157896.

Jebabli, I., Kouaissah, N., Arouri, M., 2022. Volatility spillovers between stock and energy markets during crises: a comparative assessment between the 2008 global financial crisis and the covid-19 pandemic crisis. Financ. Res. Lett. 46, 102363 https://doi.org/10.1016/j.frl.2021.102363.

Ji, Q., Xia, T., Liu, F., Xu, J.-H., 2019. The information spillover between carbon price and power sector returns: evidence from the major European electricity companies. J. Clean. Prod. 208, 1178–1187. https://doi.org/10.1016/j.jclepro.2018.10.167.

Ji, Q., Zhang, D., Geng, J., 2018. Information linkage, dynamic spillovers in prices and volatility between the carbon and energy markets. J. Clean. Prod. 198, 972–978. https://doi.org/10.1016/j.jclepro.2018.07.126.

Jong, T., Couwenberg, O., Woerdman, E., 2014. Does EU emissions trading bite? An event study. Energy Policy 69, 510–519. https://doi.org/10.1016/j.enpol.2014.03.007.

Khalfaoui, R., Boutahar, M., Boubaker, H., 2015. Analyzing volatility spillovers and hedging between oil and stock markets: evidence from wavelet analysis. Energy Econ. 49, 540–549. https://doi.org/10.1016/j.eneco.2015.03.023.

Kuang, W., 2021. Which clean energy sectors are attractive? a portfolio diversification perspective. Energy Econ. 104, 105644 https://doi.org/10.1016/j.eneco.2021.105644.

Kumar, S., Managi, S., Matsuda, A., 2012. Stock prices of clean energy firms, oil and carbon markets: a vector autoregressive analysis. Energy Econ. 34, 215–226. https://doi.org/10.1016/j.eneco.2011.03.002.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., Shin, Y., 1992. Testing the null hypothesis of stationarity against the alternative of a unit root: how sure are we that economic time series have a unit root? J. Econ. 54, 159–178. https://doi.org/10.1016/0304-4076(92)90104-Y.

Ledoit, O., Wolf, M., 2003. Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. J. Empir. Financ. 10, 603–621. https://doi.org/10.1016/S0927-5398(03)00007-0.

Lee, Y.-H., Huang, Y.-L., Wu, Ch-Y., 2014. Dyn. Correl. Volatility Spillovers Crude Oil Stock Index Return.: Implic. Optim. Portf. Constr. Int. J. Enegy Econ. Policy 3 (3), 327–336.

Lin, B., Chen, Y., 2019. Dynamic linkages and spillover effects between CET market, coal market and stock market of new energy companies: a case of Beijing CET market in China. Energy 172, 1198–1210. https://doi.org/10.1016/j.energy.2019.02.029.

Liu, H.-H., Chen, Y.-C., 2013. A study on the volatility spillovers, long memory effects and interactions between carbon and energy markets: the impacts of extreme weather. Econ. Model. 35, 840–855. https://doi.org/10.1016/j.econmod.2013.08.007.

Lucheroni, C., Boland, J., Ragno, C., 2019. Scenario generation and probabilistic forecasting analysis of spatio-temporal wind speed series with multivariate autoregressive volatility models. Appl. Energy 239, 1226–1241. https://doi.org/10.1016/j.apenergy.2019.02.015.

Managi, S., Okimoto, T., 2013. Does the price of oil interact with clean energy prices in the stock market? Jpn. World Econ. 27, 1–9. https://doi.org/10.1016/j.japwor.2013.03.003.

Marimoutou, V., Soury, M., 2015. Energy markets and CO2 emissions: analysis by stochastic copula autoregressive model. Energy 88, 417–429. https://doi.org/10.1016/j.energy.2015.05.060.

Markowitz, H., 1952. Portfolio selection. J. Financ. 7, 77–91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x.

Niu, H., 2021. Correlations between crude oil and stocks prices of renewable energy and technology companies: a multiscale time-dependent analysis. Energy 221, 119800. https://doi.org/10.1016/j.energy.2021.119800.

Oberndorfer, U., 2009. EU Emission Allowances and the stock market: evidence from the electricity industry. Ecol. Econ., Particip. Eval. Sustain. River Basin Gov. 68, 1116–1126. https://doi.org/10.1016/j.ecolecon.2008.07.026.

Phillips, P.C.B., Perron, P., 1988. Testing for a unit root in time series regression. Biometrika 75, 335–346. https://doi.org/10.1093/biomet/75.2.335.

Reboredo, J.C., 2015. Is there dependence and systemic risk between oil and renewable energy stock prices? Energy Econ. 48, 32–45. https://doi.org/10.1016/j.eneco.2014.12.009.

Reboredo, J.C., 2013. Modeling EU allowances and oil market interdependence. implications for portfolio management. Energy Econ. 36, 471–480. https://doi.org/10.1016/j.eneco.2012.10.004.

Sadorsky, P., 2012. Correlations and volatility spillovers between oil prices and the stock prices of clean energy and technology companies. Energy Econ. 34, 248–255. https://doi.org/10.1016/j.eneco.2011.03.006.

Subramaniam, N., Wahyuni, D., Cooper, B.J., Leung, P., Wines, G., 2015. Integration of carbon risks and opportunities in enterprise risk management systems: evidence from Australian firms. J. Clean. Prod. 96, 407–417. https://doi.org/10.1016/j.jclepro.2014.02.013.

Tan, X., Sirichand, K., Vivian, A., Wang, X., 2020. How connected is the carbon market to energy and financial markets? a systematic analysis of spillovers and dynamics. Energy Econ. 90, 104870 https://doi.org/10.1016/j.eneco.2020.104870.

Tian, Y., Akimov, A., Roca, E., Wong, V., 2016. Does the carbon market help or hurt the stock price of electricity companies? further evidence from the European context. J. Clean. Prod., Prev. Smog Crises 112, 1619–1626. https://doi.org/10.1016/j.jclepro.2015.07.028.

Tse, Y.K., Tsui, A.K.C., 2002. A Multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. J. Bus. Econ. Stat. 20, 351–362. https://doi.org/10.1198/073500102288618496.

Uberti, P., 2023. A theoretical generalization of the Markowitz model incorporating skewness and kurtosis. Quant. Financ. 23, 877–886. https://doi.org/10.1080/14697688.2023.2176250.

Uddin, G.S., Hernandez, J.A., Shahzad, S.J.H., Hedström, A., 2018. Multivariate dependence and spillover effects across energy commodities and diversification potentials of carbon assets. Energy Econ. 71, 35–46. https://doi.org/10.1016/j.eneco.2018.01.035.

Uddin, N., Holtedahl, P., 2013. Emission trading schemes – avenues for unified accounting practices. J. Clean. Prod. 52, 46–52. https://doi.org/10.1016/j.jclepro.2013.03.017.

Venmans, F.M.J., 2016. The effect of allocation above emissions and price uncertainty on abatement investments under the EU ETS. J. Clean. Prod. 126, 595–606. https://doi.org/10.1016/j.jclepro.2016.02.108.

Wan, D., Xue, R., Linnenluecke, M., Tian, J., Shan, Y., 2021. The impact of investor attention during COVID-19 on investment in clean energy versus fossil fuel firms. Financ. Res. Lett. 43, 101955 https://doi.org/10.1016/j.frl.2021.101955.

Wang, Y., Guo, Z., 2018. The dynamic spillover between carbon and energy markets: New evidence. Energy 149, 24–33. https://doi.org/10.1016/j.energy.2018.01.145.

Wen, X., Bouri, E., Roubaud, D., 2017. Can energy commodity futures add to the value of carbon assets? Econ. Model. 62, 194–206. https://doi.org/10.1016/j.econmod.2016.12.022.

Xiao, D., Wang, J., 2020. Dynamic complexity and causality of crude oil and major stock markets. Energy 193, 116791. https://doi.org/10.1016/j.energy.2019.116791.

Zhang, F., Zhang, Z., 2020. The tail dependence of the carbon markets: the implication of portfolio management. PLoS ONE 15, e0238033. https://doi.org/10.1371/journal.pone.0238033.

Zhang, Y., Liu, Z., Yu, X., 2017. The diversification benefits of including carbon assets in financial portfolios. Sustainability 9, 437. https://doi.org/10.3390/su9030437.

Zhang, Y.-J., Sun, Y.-F., 2016. The dynamic volatility spillover between European carbon trading market and fossil energy market. J. Clean. Prod. 112, 2654–2663. https://doi.org/10.1016/j.jclepro.2015.09.118.

Zhang, Y.-J., Wei, Y.-M., 2010. An overview of current research on EU ETS: Evidence from its operating mechanism and economic effect. Appl. Energy 87, 1804–1814. https://doi.org/10.1016/j.apenergy.2009.12.019.

Zhong, Y., Liu, J., 2021. Correlations and volatility spillovers between China and Southeast Asian stock markets. Q. Rev. Econ. Financ. 81, 57–69. https://doi.org/10.1016/j.qref.2021.04.001.