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The optimal container selection problem for parts transportation in the automotive sector

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ABSTRACT

Today's automotive factories are essentially assembly plants that receive parts from a vast network of suppliers around the world. Transporting thousands of part types over very long distances is a major logistic problem whose solution is a critical factor in the factory management. In this study we have developed a statistical and optimization methodology implemented in a software tool to help the decision makers select the most appropriate container for each part. A key element is to determine the number of parts that fit in a given container. Two optimization procedures have been developed, depending on the type of part, and used to calculate costs of each container. These costs include not only transporting parts from supplier to factory, but also the costs of handling parts within the factory and returning the empty containers to the supplier.

1. Introduction

The automotive industry comprises a wide range of companies that are involved in the design, manufacture and marketing of motor vehicles. A car is made up of several thousand parts and car factories do not make most of them, if any, but receive individual parts from a complex network of suppliers and sub-assemblies from other factories in the same group. This creates major logistics problems. In the case of European car plants, it involves trucks and trains for transport across Europe and containers and ships for intercontinental transport. In 2015, Renault, together with ESICUP, the EURO Working Group in Cutting and Packing, launched the challenge of minimizing the number of trucks and containers required for moving parts between factories in the group. There was a point-to-point transportation, so no routing was involved, and the problem was a container loading problem with very hard constraints. Many research teams all over the world developed algorithms and some of them have been published (Correcher, Alonso, Parreño, & Alvarez-Valdes, 2017; Toffolo, Esprit, Wauters, & Berghe, 2017). In 2020, Renault launched a new challenge, this time with ROADEF (the French Operational Research Society), proposing large-scale logistics problems, including also the temporal planning of shipments from suppliers to factories.

The problem studied in this paper concerns the relationship between an automobile factory and each of its large set of parts suppliers, scattered over many different countries. It is a problem with important economic and environmental implications, integrated in the general trend of looking for sustainable supply chains (Nieuwenhuis, Beresford, & Choi, 2012; Siems, Land, & Seuring, 2021). From among a given set of available containers, the right container for each part must be selected. For each container, its dimension, maximum load weight, and cost are known. The cost includes all logistic costs involved for transporting the container from the supplier to the assembly line of the manufacturer. For each part, its dimension, shape, and weight are given, and to calculate the cost per part the number of parts fitting into each container must be estimated. In other words, for each pair part-container a three-dimensional packing problem has to be solved. When only one out of a few containers has to be selected to transport a single part, the problem can be solved by physically testing the capacity of all containers if enough units of the part are available, or by using computer simulation packages. In both cases, estimating the number of parts in each container is time consuming and expensive. However, when the problem must be solved for hundreds or even thousands of different parts, and there are dozens of possible containers, then the problem cannot be solved using the above ways.

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A similar problem has been studied by Knoll, Neumeier, Prüglmeier, and Reinhart (2019), using data from BMW AG, who propose a machine learning approach based on the company's previous knowledge. The problem of selecting the right packaging or container for a part has been studied in the automotive industry, but from a different perspective. Pålsson, Finnsgård, and Wänström (2013) developed a model to compare single-use and returnable containers according to economic and sustainability criteria, and apply it to a part sent from Turkey to a Volvo factory in Sweden. Na, Sim, and Lee (2019) studied whether returnable containers are preferable to expendable ones depending on factors such as the uncertainty of the demand and the container return time. More recently, Škerlič and Muha (2020) have extended previous studies comparing several types of containers and developed a practical decision-making tool for manufacturing companies. All these studies help to choose the right type of containers, depending of the specific circumstances of each case, but they assume that the number of parts per container is known and fixed, so they do not answer the problem considered here. The container selecting problem also arises in other areas. Bortolini, Galizia, Mora, Botti, and Rosano (2018) address the selection of suppliers and packages for fresh food, considering disposable and reusable containers, and develop a bi-objective mixed integer linear programming model. Cochran and Ramanujam (2006) study a problem in electronics manufacturing in which the third-part logistics

For the first time, to the best of our knowledge, the Container Selection Problem (CSP) – entailing the selection of the suitable container for shipping each part from its supplier to the manufacturer – is approached with a comprehensive perspective. The problem takes into account the transportation cost of components in containers from the supplier to the factory, the handling cost to transport the components to the assembly line, and the expense of returning the empty containers to the supplier location. A key element in selecting the optimal container is the estimation of the maximum number of parts that can be transported in a container, whether in bulk or through strategic individual placement.

We propose a method for calculating the number of parts within a container, considering the two cases, when parts are manually placed and when they are loaded in bulk. The problem of estimating the number of parts through individual placement involves optimizing their arrangement and is consequently related to the container loading problem. This issue has been extensively studied across various contexts and with diverse constraints, remaining a current research topic (Li, Chen, & Huo, 2022). Besides the basic geometric constraints, preventing boxes from overlapping and exceeding the container dimensions, there are many other practical requirements for a safe transportation, related to weight distribution and stability. The surveys by Bortfeldt and Wäscher (2013) and Zhao, Bennell, Bektaş, and Dowsland (2016) review the most important constraints and how they have been taken into account, but many others are continuously arising in practical applications, such as the use of multi-compartment trucks (Ranck Júnior, Yanasse, Morabito, & Junqueira, 2019) or the use of robots in the loading and unloading processes (Jiao, Huang, Song, Li, & Wang, 2024). Our problem differs somewhat from those previously explored, because to simplify packing at the supplier and handling in the factory, parts must be arranged in layers and these layers should have simple patterns easy to understand and manipulate, but even with this simplification there are two NP-hard problems to be solved. The article proposes a methodology that requires geometric processing of the part and then combines two procedures – one heuristic and one exact – to rapidly provide a solution that also determines the manner in which the pieces should be packed.

The study of bulk container loading has not received much attention in the operational research literature, which, due to the nature of the problems it deals with, is more focused on solving optimization problems to find the best way to position pieces through their placement. In the field of theoretical and applied geometry, packing studies are found that deal with the packing of simple and regular geometric shapes such as spheres, tetrahedra, ellipsoids, cylinders, etc.

Initially, the problem focused on spheres and attempted to solve the arrangement that achieved the maximum density. The origin of these studies dates back to the 17th century and originates from Kepler's conjecture, who believed that the face-centered cubic lattice is the densest packing structure of identical spheres, and the packing density is $\pi/\sqrt{18} \approx 0.7405$ (the proof of the conjecture had to wait until 1998 when Hales (2005) proved it with the help of a computer program).

Random placement, related to the old problem of how much grain can be stored in a barrel, has been also experimentally studied and simulated; Weitz (2004) summarized that the random packing of spheres achieves densities ranging from 0.56 to 0.64; Donev et al. (2004) show that ellipsoids can randomly pack more densely, ranging from 0.68 to 0.71 for spheroids with an aspect ratio close to that of M&M's Candies, and even approach 0.74 for ellipsoids with other shapes.

In fact, the shape of the element being packed is a decisive factor determining the packing density. In the case of cones, the random packing density depends on the ratio H/D, where H is the height and D is the diameter of the circumference. Numerical simulation results by Li, Zhao, Lu, and Xie (2010) show that the peak value of the packing density of cones is 0.6664, for a ratio H/D of 0.8, but ranges from 0.61 for other cone shapes. The same authors, in Li, Zhao, and Zhou (2008), present results for regular tetrahedra reporting densities ranging from 0.62 to 0.68, although Dong and Ye (1993) reported densities less than 0.5 in their experiments. In the case of cylinders, the packing density depends on the ratio between height and diameter: Zhang, Thompson, Reed, and Beenken (2006) obtained a maximum density of 0.66, while in Li et al. (2010), a range of values from 0.66 to 0.718 was obtained. From these results, we conclude that space utilization depends on the geometric shape of the packed component, and randomness can impact the variations, producing more compact or more dispersed packings.

Because of the absence of a formula or method that consistently gives the number of units of a part that can be packed into a container, our paper introduces an innovative bulk loading estimation method that includes, in addition to historical data, the use of a database of the number of archetypal pieces that fit in containers of different sizes. These archetypal pieces include regular geometric figures and others obtained by graphic design. The proposed method yields satisfactory results, robust enough for practical use. The accuracy of the results is expected to improve as the database used in the estimation process, continuously updated by the company's logistics engineers, is expanded.

We embed the proposed algorithms into a stand-along application that can be used for a carmaker company to decide the best container for each new or existing part designed for each car model.

The main contributions of the paper are:

- A methodology for calculating the cost of using each container that includes the cost of transporting the parts from the supplier to the factory, the cost of handling the parts within the factory, and the cost of returning the empty containers to the supplier.
- Two methods for calculating the number of parts that fit in a container, distinguishing between small parts that are loaded in bulk from larger parts that are placed manually.
- The development of a software program that automatizes the whole process, connecting to the database, calling the optimization procedures, calculating the costs, and providing the decision maker with a list of containers ordered by cost.

The remainder of the paper is structured as follows. The problem is formally described in Section 2. The optimization procedures for calculating the number of parts fitting into a container are developed in Section 3 for parts to be placed manually and in Section 4 for parts to be loaded in bulk in both cases and illustrative example is also shown. A global container selection example as well as a detailed description of the elements of the software program developed are detailed in Section 5. In Section 6, some conclusions are drawn and future work is outlined.

2. Description of the problem

Given a set of containers $\mathbb{C} = \{C^i; i = 1, \ldots, K\}$ and a part P^* that needs to be transported, the core challenge in the optimal Container Selection Problem lies in identifying the most suitable container that minimizes the cumulative expenses associated with transporting and managing parts P^* along their journey from the supplier's location to the assembly line at the manufacturing plant. These expenses encompass various components, namely: the transportation in trucks from the supplier location until the assembling plant, the posterior handling cost of the containers to get the parts to the assembly line and the transportation of the empty containers back to the suppliers.

Truck Transportation: This covers the cost of ferrying from the supplier's location to the assembly plant via trucks.

Handling Costs: Subsequent to transportation, there are handling costs incurred during the unloading and transfer of parts from containers to the assembly line. This becomes especially pertinent when the containers used are of the *GLT* type (*GLT* denoting large load carriers). In such cases, an additional step is required: unloading the *GLT* containers into smaller containers before integration into the assembly line. This extra logistic operation escalates the overall transportation cost of P^* from the supplier to the assembly line. However, when the part arrives to the assembly plant in containers of *KLT* type (*KLT* stands for small load carrier), they can be directly placed in the assembly line (Müllerklein, Fontaine, & Ostermeier, 2022).

Container Return: Following use, the containers employed for transporting units of P^* need to be transported back to the supplier's location, incurring additional expenses (Glock, 2017).

Furthermore, it is essential to take into consideration the constraints of the transportation process, specifically the maximum weight W_T and volume V_T that can be loaded in a single truck.

The objective function is designed to encapsulate the comprehensive cost per unit of P^*, and comprises the three aforementioned terms: the cost of transporting the part from the supplier to the assembly plant, the internal handling costs associated with the containers used for P^*, and the cost entailed in returning the empty containers to the supplier's base:

$$\min_{C_i \in \mathbb{C}} \frac{Z_T}{N_{C_i,T} \times N_{P^*,C_i}} + \frac{Z_{C_i,N_{P^*,C_i}}}{N_{P^*,C_i}} + \frac{Z_T}{N_{C_i^e,T} \times N_{P^*,C_i}} \tag{1}$$

Where,

Z_T is the cost of sending a truck from supplier location to assembly plant location and vice versa.

N_{P^*,C_i} is the number of units of part P^* that can be transported in container C_i fulfilling the constraints of weight and volume of container C_i.

$N_{C_i,T}$ is the number of containers of type C_i loaded with N_{P^*,C_i} units of part P^* that can be transported in a truck fulfilling the constraints of weight and volume.

$N_{C_i^e,T}$ is the number of empty containers of type C_i that can be transported in a truck fulfilling the constraints of weight and volume.

$Z_{C_i,N_{P^*,C_i}}$ is the cost of handling a container of type C_i with N_{P^*,C_i} units in the assembly factory to get the units to the assembly line.

The number $N_{C_i,T}$ of containers C_i transported in the truck is calculated as the minimum of the number of containers that fit considering both constraints, volume and weight:

$$N_{C_i,T} = \min\left\{\left\lfloor \frac{V_T}{V^0_{C_i}} \right\rfloor, \left\lfloor \frac{W_T}{W^0_{C_i} + (N_{P^*,C_i} \times W_{P^*})} \right\rfloor\right\} \tag{2}$$

where W^0_C is the weight of the empty container C, and V^0_C its outer volume.

The number $N_{C_i^e,T}$ of empty containers C_i transported in the truck is also calculated as the minimum of the number of containers that fit considering both constraints, volume and weight:

$$N_{C_i^e,T} = \min\left\{\left\lfloor \frac{V_T}{V^{0,e}_{C_i}} \right\rfloor, \left\lfloor \frac{W_T}{W^0_{C_i}} \right\rfloor\right\} \tag{3}$$

where $V^{0,e}_C$ is the volume of a container C empty and folded.

The main challenge to solve this problem is to calculate the number of units of the part P^* that fit in each container C_i, that is the values N_{P^*,C_i}. The methodology developed to carry out these calculations is presented in the following two sections. Section 3 considers the optimization process that determines the number of units that can be transported in a container when they are placed manually following a strategy that seeks to maximize the use of space. However, when parts are transported in bulk, i.e. the container is filled by a pour process, without any unit placement strategy, the maximum number of parts that can be transported may differ from one fill to another. The methodology for the estimation of the number of units that may be transported in bulk is presented in Section 4.

The Optimal Container Selection Problem plays a pivotal role in addressing both economic and environmental concerns, offering the potential for significant cost savings and minimizing the ecological footprint of the transportation process. The main notation used throughout the paper is shown in Table 1.

3. Transportation of parts in containers by placement

This section describes how the number of units that can be manually placed in a container can be approximated using a fast procedure. Section 3.1 presents the geometric analysis leading to the calculation of the minimum cuboids when the part is placed individually, by interlocking multiple parts together, or by stacking. Then, these cuboids are packed into the container using a two-phase optimization procedure (Section 3.2), and finally, an example of packing is presented (Section 3.3).

3.1. Geometric processing of the parts

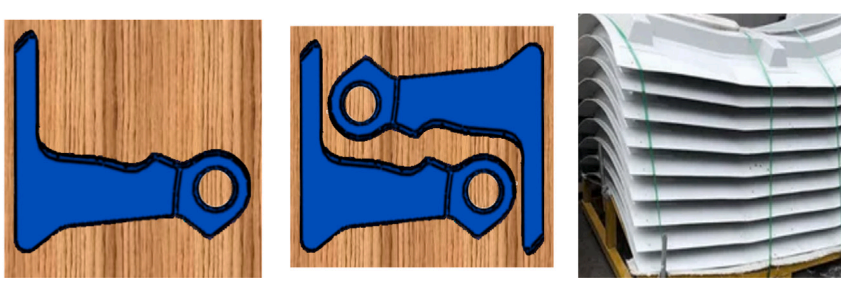
When placing a part into a container, the following arrangements are possible: a single unit can be placed in a specific orientation (Fig. 1(a)), a group of a certain number of parts can be nested (as seen in the example in Fig. 1(b)), or several units can be stacked (Fig. 1(c)). Quality control plays a crucial role in determining when a part can be stacked, the maximum number of units stacked, and the possibility of nesting multiple units to make better use of the space. Once the possibilities for grouping and stacking parts are known, the smallest bounding box for each case is determined using the algorithm proposed by Chan and Tan (2001).

The bounding boxes defined can rest on some or all of their faces: the xy plane, the xz plane or the yz plane, due to stability or quality criteria. This fact is considered in the optimization procedure explained in the next section.

Fig. 2 illustrates an example with a non-stackable part and their associated bounding boxes. The part has three potential oriented bounding boxes: BB_1, the minimum bounding box for an individual part, with

Table 1
Mathematical notation.

Parts	
P	Denotes a part.
W_P	Weight of the part P.
$V_{CH}(P)$	Volume of the convex hull of part P.
$V_{BB}(P)$	Volume of the minimum bounding box of part P.
BB_1^*	Minimum bounding box of part P^* with dimensions $b^*_{x1}, b^*_{y1}, b^*_{z1}$.
BB_i^*	Additional bounding box of part P^* with dimensions $b^*_{xi}, b^*_{yi}, b^*_{zi}$.
B^*	List of bounding boxes associated to part P^*.
I_{xi}, I_{yi}, I_{zi}	Increments in axis X, Y and Z, respectively, of BB_i^* for each unit added to the stack.
Containers	
C	Container, with dimensions C_x, C_y, C_z $(C_x \geq C_y \geq C_z)$.
W_C^0, V_C^0	Weight and outer volume of an empty container C, respectively
$V_C^{0,e}$	Volume of empty and folded container (if C cannot be folded, $V_C^{0,e} = V_0$).
W_C, V_C	Maximum weight and volume that can be loaded in C, respectively.
$\mathbb{C}$	Set of all containers used by the company.
$\rho(P, C)$	No. of units of part P that fit into the container C per unit of volume.
$\mathbb{S}(P^*)$	Set of parts similar in shape to part P^*.
Trucks	
T	Truck for transporting parts in containers.
W_T, V_T	Maximum weight and volume that can be loaded in truck T.
Packing	
N_{P,C_i}	No. of units of part P that can be transported in the container C_i fulfilling its constraints of weight (W_{C_i}) and volume (V_{C_i}).
$N_{C_i,T}$	No. of C_i containers loaded with N_{P,C_i} units of part P that can be transported in the truck fulfilling the weight and volume constraints.
Costs	
Z_T	Cost of sending a truck between supplier and assembly plant locations.
$Z_{C_i,N_{P^*,C_i}}$	Cost of handling a container of type C_i with N_{P^*,C_i} units in the assembly factory to get the units to the assembly line.



(a) (b) (c)

Fig. 1. Possible arrangements of a part.

dimensions $(b_{x_1}, b_{y_1}, b_{z_1})$. Bounding box BB_2 comprises two parts that fit together, resulting in dimensions $(b_{x_2}, b_{y_2}, b_{z_2})$. Bounding box BB_3 also contains a composition of two parts, with dimensions $(b_{x_3}, b_{y_3}, b_{z_3})$. In this case, the parts are packed with a diagonal separator, resulting in larger dimensions than the second bounding box, BB_2.

Fig. 3 shows the case of a part that is stackable. BB_1 is the minimum bounding box for the part. In BB_2, each additional part stacked increments the oriented bounding box only along the y-axis ($Ix_i = 0, Iy_i > 0, Iz_i = 0$), and there is no maximum limit on the number of parts that can be stacked, restricted only by the dimensions of the container.

3.2. Optimizing unit placement in containers

Once the bounding boxes of a part are determined, for each container in which the part can be packed, the problem consists in finding the way in which these bounding boxes are packed into the container so that the number of parts packed is maximized.

As the boxes must be packed forming layers, the problem can be decomposed into two phases.

- Phase 1: Generate all valid layers considering all possible orientations of the bounding boxes.
- Phase 2: Select the combination of layers that maximizes the number of parts packed, satisfying the container height and weight limits.

The pseudocode of the procedure is shown in Algorithm 1. Its two main algorithms are described in detail in the next subsections.

3.2.1. Generating layers

Building a layer is a two-dimensional problem with two types of boxes, corresponding to a bounding box in a given orientation and its rotation, keeping fixed the height and exchanging length and width. To solve this problem, we use the 5-block placement algorithm described

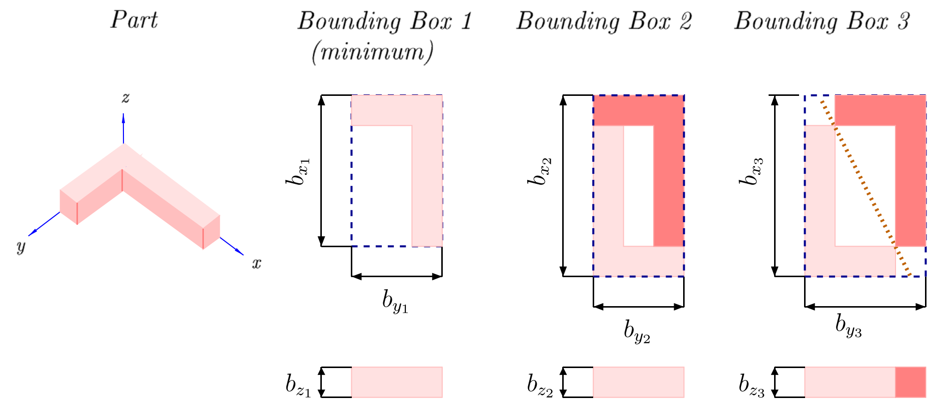


Fig. 2. Example of bounding box process. Non-stackable part.

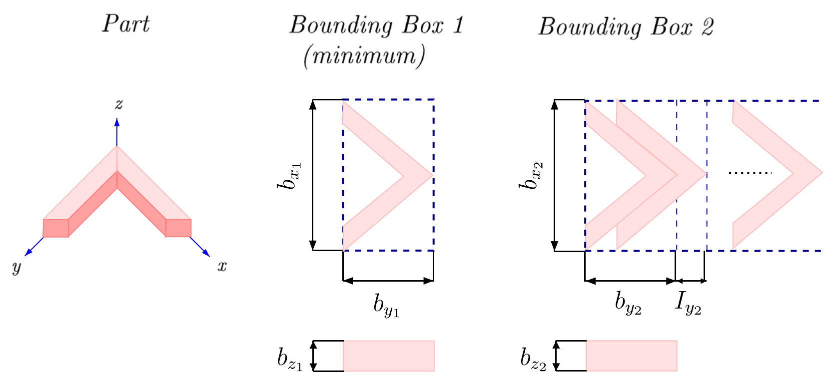


Fig. 3. Example of bounding box process. Stackable part.

Algorithm 1: Optimized Box Packing.

Data: List of bounding boxes $B = \{BB_1, BB_2, \dots, BB_n\}$
Data: Set of containers $\mathbb{C} = \{C_1, C_2, \dots, C_m\}$

foreach *container* $C_i \in \mathbb{C}$ **do**
  $R \leftarrow \emptyset$ `// List of layers`
  foreach *bounding box* $BB_j \in B$ **do**
    if BB_j *fits in* C_i *and for each allowed orientation* **then**
      $layers \leftarrow$ **GenerateLayers**(BB_j, C_i)
      Add $layers$ to R
  $chosenLayers \leftarrow$ **Knapsack**(R, C_i) `// Choose optimal layers`
  Pack $chosenLayers$ into C_i

in Bischoff and Dowsland (1982). This approach constructs homogeneous blocks of boxes in a clockwise arrangement starting from the bottom left corner, as described in the Algorithm 2.

Specifically, we first place block 1 (B_1). Block 2 (B_2) is then built to the right of block 1 using the same box rotated 90 degrees to fill the width. Block 3 (B_3) is placed above block 2 with the box again rotated 90 degrees. Block 4 (B_4) goes to the left of block 3 and above block 1, with the same orientation as block 2. The center area is reserved for block 5 (B_5), which is placed in the orientation that occupies the largest area. The algorithm explores all possible combinations of rows and columns for each block and returns the one with the largest number of boxes. Each block has a pre-fixed orientation, except for block 5 whose orientation is postfixed. Fig. 4(a) shows the block structure created by the algorithm for bounding box 1.

Additionally, if there is any empty space between blocks 1–2, 2–3, 3–4, or 1–4, other bounding boxes with the same height can be inserted in this space to increase the number of parts packed. The solution in Fig. 4(b) is built using only bounding box 1. In contrast, the one in Fig. 4(c) is initially built using bounding box 3, but an empty space would remain to the right and this space can be used to pack several copies of bounding box 1.

By generating layers for all box orientations and 5-block patterns, we construct a list of candidate layers R to pack into the container. Each layer $l_j \in R$ has height H_j and weight Q_j.

3.2.2. Knapsack layer selection

Once all feasible layers R for a part in a container have been built, the problem is how to fill the container with these layers to maximize the number of parts packed while satisfying the container limits.

We model this as a knapsack problem. Let the variables x_j indicate the number of layers of type j to pack, $j = 1, \dots, |R|$. Let m_j be the maximum number of layers of type j fitting vertically ($\lfloor H/H_j \rfloor$, where H is the height of the container), n_j the number of parts per layer, Q_j its weight, and hl_j the height of protective interlayers.

The layer selection problem is formulated as:

$$\max \sum_{j=1}^{|R|} n_j x_j \tag{4}$$

$$\text{s.t.} \sum_{j=1}^{|R|} (H_j + hl_j) x_j \leq H \tag{5}$$

$$\sum_{j=1}^{|R|} Q_j x_j \leq W_C \tag{6}$$

$$x_j \in 0, 1, \dots, m_j \quad \forall j \in 1, \dots, |R| \tag{7}$$

Algorithm 2: Layer generator (GenerateLayers).

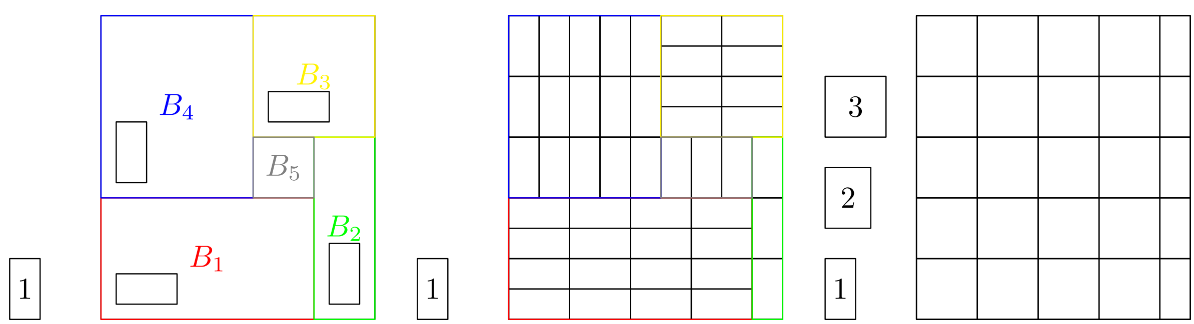
Data: A bounding box $BB = \{b_x, b_y, b_z\}$
Data: A container $C = \{C_x, C_y, C_z\}$

$lb = max((\lfloor C_x/b_x \rfloor \lfloor C_y/b_y \rfloor), (\lfloor C_x/b_y \rfloor \lfloor C_y/b_x \rfloor))$

$ub = \lfloor (C_x C_y)/(b_x b_y) \rfloor$

for $i \leftarrow 1, \lfloor C_x/b_x) \rfloor$ **do**
  for $j \leftarrow 1, \lfloor C_y/b_y) \rfloor$ **do**
    $B_1 = (L_1, W_1) = (ib_x, jb_y)$
    for $k \leftarrow 1, \lfloor C_y/b_y) \rfloor$ **do**
      $B_2 = (L_2, W_2) = (\lfloor (C_x - L_1)/b_y \rfloor b_y, kb_x)$
      for $l \leftarrow 1, \lfloor C_x/b_x) \rfloor$ **do**
        $B_3 = (L_3, W_3) = (lb_x, \lfloor (C_y - W_2)/b_y \rfloor b_y)$
        $B_4 = (L_4, W_4) = (\lfloor (C_x - L_3)/b_y \rfloor b_y, \lfloor (C_y - W_1)/b_x \rfloor b_x)$
        if *There is no overlap, that is if* $(L_1 + L_3 > C_x, W_1 + W_3 > C_y)$ *and* $(W_2 + W_4 > C_y, L_2 + L_4 > C_x)$ *are false* **then**
          $B_5 = (L_5, W_5) = (C_x - L_2 - L_4, C_y - W_1 - W_3)$.
          Determine $z = \sum_{i=1}^{5} z_i$, where $z_i = \lfloor L_i/b_x \rfloor \lfloor W_i/b_y \rfloor$, $i = 1, 3$
          $z_i = \lfloor L_i/b_y \rfloor \lfloor W_i/b_x \rfloor$, $i = 2, 4$
          $z_5 = max(\lfloor L_5/b_x \rfloor \lfloor W_5/b_y \rfloor, \lfloor L_5/b_y \rfloor \lfloor W_5/b_x \rfloor)$.
          if $z > lb$ **then**
            Update lb to the maximum number of boxes obtained so far.
            if $lb = ub$ **then**
              Add lb, b_z to $layers$. Exit

Add lb, b_z to $layers$.



(a) 5-block structure. (b) Solution with one bounding box. (c) Solution with two bounding boxes.

Fig. 4. 5-blocks algorithm.

The objective (4) maximizes the number of packed parts. Constraints (5) and (6) enforce limits on container height and weight. This yields the optimal set of layers to pack into the container. Therefore, the number of units than can be packed in container C is the optimum of the objective function $N_{P,C} = \sum_{j=1}^{|R|} n_j x_j$.

Regarding the efficiency of the two-phase packing procedure, as the second phase uses an exact integer lineal model, the only possible source of inefficiency could be in the first phase, in which a heuristic algorithm, the 5-block algorithm is used. However, the 5-block algorithm has been used many times in packing applications and as a part of more complex procedures and all the authors report its high efficiency. Nevertheless, to check whether it works well for the type of containers and parts used in this application, we have carried out experiments with the europallet and 30 different bounding boxes and the results obtained are on average less than 1% away from an upper bound.

3.3. Packing example

Fig. 5 shows an example part with dimensions (150, 150, 50) mm and weight 200 grams that can be grouped together to be packed by joining up to 5 units, yielding bounding boxes BB_1 to BB_5 with dimensions listed in Table 2. The bounding boxes are shown in Fig. 6. They can be rotated in any direction, so there are 15 configurations to be used to build layers. The container size is (1200, 1200, 1200) mm with a 2500 kg weight limit.

Table 2
Bounding box dimensions.

BoxLLL	Dimensions (mm)	Parts
BB_1	142, 231, 50	1
BB_2	213, 231, 50	2
BB_3	284, 231, 50	3
BB_4	355, 231, 50	4
BB_5	426, 231, 50	5

With each bounding box and each possible orientation, we generate a layer using the 5-block algorithm. As can be seen in Fig. 7, 15 layers have been generated. Each row shows the layers of each bounding box, from 1 to 5. The figure captions show the dimensions of the part (length, width, height) and the number of parts contained in the layer.

It can be observed that there are layers that dominate other layers, that is, layers with the same height but in which more parts fit. For

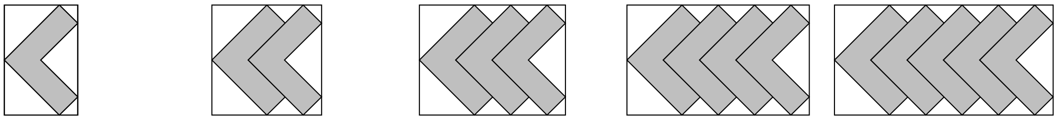


Fig. 5. One part and the ways in which it can be stacked.

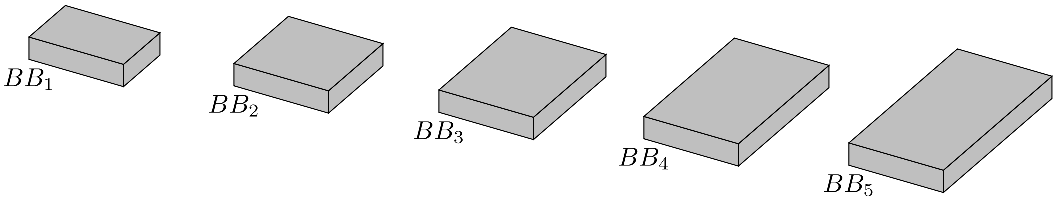


Fig. 6. Bounding boxes.

example all the layers in the column on the right have height 50 but the one with the most parts is the layer with configuration l, in which 66 parts fit. Nevertheless, the other layers of the same height also go to the layer selection problem because it may happen that the layer with the maximum number of parts does not fit in the container due to the maximum weight constraint and other layer with fewer parts can occupy that space.

The 2D knapsack model selects the optimal combination of layers that maximizes the number of parts without exceeding the height and weight limits. The optimal solution consists of:

- 2 layers of type (l), with dimensions $(231, 355, 50)$ and 66 parts each.
- 1 layer of type (n), with dimensions $(426, 50, 213)$ and 323 parts.
- 2 layers of type (m), with dimensions $(50, 213, 426)$ and 610 parts each.

This arrangement achieves:

- Total parts packed: $1675 = 2 \times 66 + 1 \times 323 + 2 \times 610$.
- Total height: $1165 = 2 \times 50 + 1 \times 213 + 2 \times 426$.
- Total weight: $335 = 1675 \times 0.2$.

4. Transportation of parts in bulk

This section presents a statistical method for fast estimating the number of parts that can be transported in a container when dumped in bulk, which is used, as in the case of Section 3.2, as an input of the optimization problem of selecting the container that minimizes the transportation and the "in house" handling costs. The statistical method is based on performing non-parametric regressions where the regressor variables are the number of parts, similar but not necessarily identical to the considered one, that can be transported in bulk in containers with known dimensions (which do not have to match the dimensions of the container considered). In Section 4.1 we present the sources and characteristics of the data needed to perform the estimation and in Section 4.2 we expose the statistical methodology. In Section 4.3 an example is presented to illustrate the proposed methodology.

4.1. Acquisition of data. Reference set of parts

The estimation of the number of units of a particular part that can be accommodated in a specific container is based on data collected from other parts for which the number of units that can fit in one or more containers, not necessarily identical to our target container, is known. It is important to note that the capacity for accommodating these parts within the container is primarily constrained by the container volume, regardless of any weight restrictions. This number of units of a part P that can be bulk-packed into a container C is denoted as $N(P, C)$.

These data come from two distinct sources. Firstly, from empirical data pertaining to parts that are currently (or have been in the past) transported in bulk in specific containers, for which the capacity is already established. This empirical data can also be generated through manual experimentation within the manufacturing facility, involving the determination of the number of part units that can be accommodated within a container. This method of data collection is time consuming and requires an adequate quantity of units for satisfactory testing.

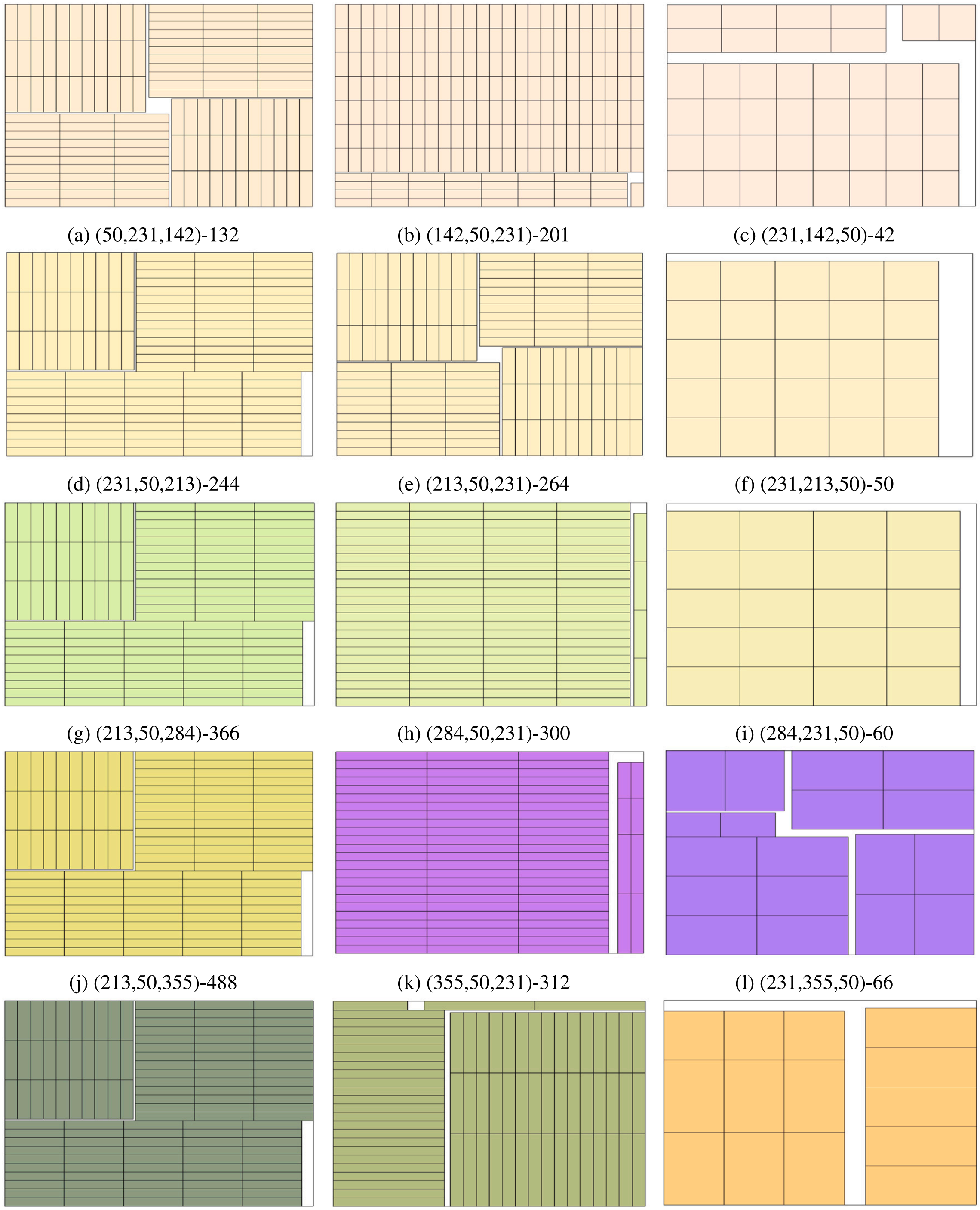
Secondly, information about the number of units that can be contained in a container can be obtained through computer simulation. Various simulation programs replicate the placement of each unit inside the container, calculating their positions until the container reaches its capacity. In such cases, when the simulation experiment is repeated, there may be variability in the number of units accommodated in the container. Similar variations can be observed when the experiment is replicated in a real context. This method of data gathering is also time-consuming, especially in scenarios where hundreds or even thousands of units can be accommodated inside the container. Moreover, due to the variability in the results, it is advisable to repeat the experiment several times (approximately 5–10 times) and select the maximum observed number of units in the container as the result to populate the database. This approach assumes that in a real-world filling scenario, the container may be agitated to accommodate the maximum possible number of units. An example of a program that simulates the filling in bulk of containers is Pack Assistant, (Fraunhofer SCAI, 2023).

To expand the range of available data for part shapes, computer-aided design (CAD) programs have been employed to design simple-shaped parts with a diverse range of forms (see Fig. 8). These serve as approximations for new parts for which data are not available. Using simulation programs, as described earlier, experiments have been conducted to fill containers with these archetypal parts, enriching the database that will be used in the estimation procedure outlined in the next section.

The primary aim of this data collection is to establish a comprehensive database encompassing a diverse array of parts, particularly in terms of their shapes, for which estimates regarding the number of units that can be accommodated in various containers are available. As explained in the next section, the completeness of this database significantly enhances the precision of the statistical estimation. Consequently, this database assumes a dynamic role within the decision support system, continuously evolving as new data on additional parts and/or containers become accessible.

4.2. Estimation of bulk placement in containers

As we have previously stated, given a container C^* with dimension vector (C^*_x, C^*_y, C^*_z) and a part P^*, the problem consists of estimating



(a) (50,231,142)-132 (b) (142,50,231)-201 (c) (231,142,50)-42
(d) (231,50,213)-244 (e) (213,50,231)-264 (f) (231,213,50)-50
(g) (213,50,284)-366 (h) (284,50,231)-300 (i) (284,231,50)-60
(j) (213,50,355)-488 (k) (355,50,231)-312 (l) (231,355,50)-66
(m) (213,50,426)-610 (n) (426,50,213)-323 (o) (426,213,50)-64

Fig. 7. Layers for all bounding box configurations.

the number $N(P^*, C^*)$ of units of part P^* that fit in container C^*. The algorithm requires an initial set of data $\mathbb{I} = \{N(P_j, C_i)\}$ composed by known values $N(P_j, C_i)$ for the reference set of parts $P_j, j = 1, \ldots, J$ and a certain set of containers $C_i, i = 1, \ldots, I$. It is not necessary to have the values $N(P_j, C_i)$ for all pairs (j, i). We denote by $\mathbb{C}(P_j) = \{C_i; i = 1, \ldots, I_{P_j} | N(P_j, C_i)$ is known$\}$ the set of containers for which $N(P_j, C_i)$ is known, being I_{P_j} the size of this set.

The algorithm consists of four main steps. In the first step, we select a subset of parts that exhibit some degree of similarity to the part P^* from the reference set of parts P_j, $j = 1, \ldots, J$. Next, for each selected part, we estimate the number of units of these parts that can fit in C_j^*, a scaled version of C^*. In the third step, using the estimates obtained in the previous step, we derive different estimates for the number of parts P^* that can fit in C^*. Finally, in the last step, we calculate a single estimate of $N(P^*, C^*)$ through a weighted average of the estimates obtained in step 3, which takes into consideration the similarity between the part P^* and the parts selected in step 1. Below are commented details of the steps of the Algorithm 3.

Step 1. Selection of a set of parts similar in shape to part P^*.

In this step the user selects a set of parts $\mathbb{S}(P^*)$, called the reference set for P^*, containing parts with similar shape to part

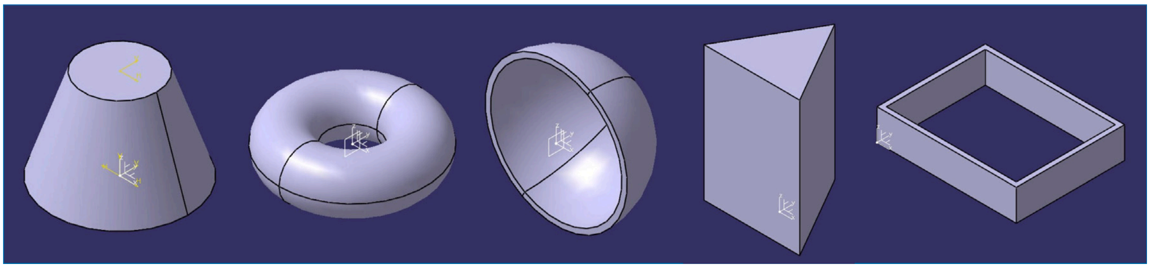


Fig. 8. Computer-aided designed simple-shaped part examples.

Algorithm 3: Estimation of capacity bulk process

Data: P^* part, C^* container.
$\mathbb{I} = \{N(P_j, C_i)\}$ known values for a certain $P_j,\ j = 1, \dots, J$ and $C_i, i = 1 \dots, I$.
$\mathbb{C}(P_j) = \{C_i, i = 1, \dots, I_{P_j} \mid \text{for which } N(P_j, C_i) \text{ is known})\}$.

Step 1 *Build reference set $\mathbb{S}(P^*)$ and assign similarity factors*
$\mathbb{S}(P^*) \subseteq \{P_j,\ j = 1, \dots, J \mid \mathbb{C}(P_j) \neq \emptyset\}$
Assign weighting factor $w_i\ i = 1, 2, 3$, for all $P_j \in \mathbb{S}(P^*)$
foreach $P_j \in \mathbb{S}(P^*)$ **do**

Step 2 *Estimation of the number of parts per unit volume in C_j^**
$C_{xj}^* = C_x^* \frac{b_{xj}}{b_x^*};\ C_{yj}^* = C_y^* \frac{b_{yj}}{b_y^*};\ \ C_{zj}^* = C_z^* \frac{b_{zj}}{b_z^*}$
$\rho(P_j, C_j^*) = \left(\sum_{C_i \in \mathbb{C}(P_j)} K\left(\frac{d(C_j^*, C_i)}{h}\right) \rho(P_j, C_i)\right) / \left(\sum_{C_i \in \mathbb{C}(P_j)} K\left(\frac{d(C_j^*, C_i)}{h}\right)\right)$

Step 3 *Estimation of the number of parts P^* in C^* based on P_j information*
$\rho_{P_j}(P^*, C^*) = \rho(P_j, C_j^*) \frac{V_{CH}(P_j)}{V_{CH}(P^*)}$
$N_{P_j}(P^*, C^*) = \rho_{P_j}(P^*, C^*) \times C_x^* \times C_y^* \times C_z^*$

Step 4 *Estimation of the number of parts P^* in C^* using all the information of P_j*
$N(P^*, C^*) = \left(\sum_{j=1}^{J} w_j N_{P_j}(P^*, C^*)\right) / \left(\sum_{j=1}^{J} w_j\right)$

P^* and having at least one pair $N(P_j, C_i) \in \mathbb{I}$. $\mathbb{S}(P^*) = \{P_j; j = 1, \dots, J\}$.

For each part $P_j \in \mathbb{S}(P^*)$ its similarity with part P^* is assessed on an ordinal scale, for example, {identical, very similar, similar}. Each category of the scale is assigned a weight that will be used in the regression step: "identical" → w_1, "very similar" → w_2 and "similar" → w_3.

Step 2. Estimation of the number of units in container C_j^* for each part in $\mathbb{S}(P^*)$.

Let b_{xj}, b_{yj}, b_{zj} be the dimensions of the minimal bounding box of the part $P_j \in \mathbb{S}(P^*)$.

We define a container C_j^*, with dimensions $C_{xj}^*, C_{yj}^*, C_{zj}^*$, that keeps a ratio with the dimensions of the container C^* equal to the ratio of the dimensions of P^* and P_j:

$$C_{xj}^* = C_x^* \frac{b_{xj}}{b_x^*};\quad C_{yj}^* = C_y^* \frac{b_{yj}}{b_y^*};\quad C_{zj}^* = C_z^* \frac{b_{zj}}{b_z^*} \tag{8}$$

Therefore, when the part P_j is a resizing of the part P^*, the number of units of P^* that fit in C^* would be the same as the number of units of P_j that fit in C_j^*. When the shapes of P_j and P^* are quite similar, the number of units of P_j that fit in C_j^* is an estimation of the number of units of P^* that fit in C^*.

The number of units of part P_j that fit in C_j^* is estimated from the number of units per unit of volume, $\rho(P_j, C_j^*)$, which is obtained by using the non-parametric Nadaraya–Watson estimator (Hastie, Tibshirani, & Friedman, 2009):

$$\rho(P_j, C_j^*) = \frac{\sum_{C_i \in \mathbb{C}(P_j)} K\left(\frac{d(C_j^*, C_i)}{h}\right) \rho(P_j, C_i)}{\sum_{C_i \in \mathbb{C}(P_j)} K\left(\frac{d(C_j^*, C_i)}{h}\right)} \tag{9}$$

Where,

- $K(\cdot)$ is the Epanechnikov Kernel (Tsybakov, 2009), $K(u) = \frac{3}{4}(1 - u^2)\mathbb{1}_{|u| \leq 1}$.
- $d(C_j^*, C_i)$ is the Euclidean distance between dimension vectors of C_j^* and C_i.
- $h = \frac{d_{max}}{5}$ and $d_{max} = \max_{i,j \in \mathbb{C}}\{d(C_i, C_j)\}$.

Step 3. Estimation of the number of units of P^* in container C^*, $N_{P_j}(P^*, C^*)$, using data from the part $P_j \in \mathbb{S}(P^*)$.

The number of units of part P^* per unit of volume, $\rho_{P_j}(P^*, C^*)$, is estimated by scaling the result $\rho(P_j, C_j^*)$ for P_j using the ratio between the volume of their convex hull:

$$\rho_{P_j}(P^*, C^*) = \rho(P_j, C_j^*) \frac{V_{CH}(P_j)}{V_{CH}(P^*)} \tag{10}$$

Therefore, the number of units of P^* in container C^*, based on data from part P_j, is estimated as:

$$N_{P_j}(P^*, C^*) = \rho_{P_j}(P^*, C^*) \times C_x^* \times C_y^* \times C_z^* \tag{11}$$

Step 4. Estimation of the number of units of P^* in container C^* using data from all parts $P_j \in \mathbb{S}(P^*)$.

The number of units of P^* in container C^*, $N(P^*, C^*)$, is estimated by the composition of the estimations obtained from each part $P_j \in \mathbb{S}(P^*)$:

$$N(P^*, C^*) = \frac{\sum_{j=1}^{J} w_j N_{P_j}(P^*, C^*)}{\sum_{j=1}^{J} w_j} \tag{12}$$

Fig. 9. Part P^*, Internal star head cylinder screw.

Table 3
Dimensions and volume of the convex hull for part P^* and three analogous parts to P^*.

Part	b^*_{x1} (mm)	b^*_{y1} (mm)	b^*_{z1} (mm)	$V_{CH}(P)$ (cm^3)
P^*	33.00	15.94	15.96	5.010
P_1	60.00	39.66	39.66	74.54
P_2	187.8	26.25	26.53	54.53
P_3	19.25	14.92	14.16	1.520

The weights are chosen to give more importance to data coming from parts that are more similar. In the proposed implementation, we assume $w_1 = 1$, $w_2 = 0.3$, and $w_3 = 0.1$. It is at the discretion of the logistics engineers to classify the selected parts in the database according to their level of similarity. The proposed categorization is straightforward and only uses three categories for the sake of simplicity, but it could easily be replaced by a continuous scale in the interval $(0, 1]$, where the logistics engineer assigns the weight value, **w**, reflecting the degree of similarity between the parts and uses these weights in the Eq. (12).

Once the number of parts of type P^* that fit in C^* has been estimated, we adjust this number taking into consideration the limitation imposed by the available load in the container in order to obtain the value N_{P^*,C^*} by means of Eq. (13).

$$N_{P^*,C_i} = \min\left\{ N(P^*, C_i), \left\lfloor \frac{W_{C_i}}{W_{P^*}} \right\rfloor \right\} \tag{13}$$

4.3. Example and experimental study of bulk container capacity estimation

Let us consider a transport scenario in which the component depicted in Fig. 9, denoted as P^* (internal star head cylinder screw), needs to be transported, and container A, with inner dimensions of 246, 163, and 130 millimeters, has to be assessed for its bulk transportation. We apply the method exposed in Section 4.2, step by step, to estimate the number of P^* units that can be accommodated within container A.

Initially, the reference parts database (those for which $N(P_j, C_i)$ data is available) is examined, from which a subset, $\mathbb{S}(P^*)$, *consisting of parts similar to* P^*, is selected. Let us assume three parts, $P_j, j = 1, 2, 3$, shown in Fig. 10: (a) P_1 is a cylinder (from the set of the archetypal shapes), (b) P_2 is a double stud with a hexagon drive (from the screw set), and (c) P_3 is a round head self-tapping screw (also from the screw set). It is decided that P_1 is very similar, P_2 exhibits an intermediate degree of similarity, and P_3 has a lower level of resemblance. We assign weights $w_1 = 1, w_2 = 0.3, w_3 = 0.1$, to parts P_1, P_2, and P_3, respectively, to weigh the estimations in Eq. (12).

The dimensions of parts P^* and $P_j, j = 1, 2, 3$, are provided in Table 3. For each of the selected similar parts, the number of units $N(P_j, C_i)$ that can be transported in a set of containers (third column of Table 4) is known, which is displayed in the fourth column in Table 4. For each part $P_j, j = 1, 2, 3$, the dimensions of container C^*_j is calculated in accordance with Eq. (8). Using the data in columns 3 and 4, the Eq. (9) in Step 2 provides $\rho(P_j, C^*_j)$ the density of parts of P_j in the container C^*_j, which is shown in column 5 of Table 4. Each value $\rho(P_j, C^*_j), j = 1, 2, 3$ is the input to Eq. (10) in Step 3 to calculate $\rho_{P_j}(P^*, C^*), j = 1, 2, 3$ which, by using Eq. (11), provides an estimation of the number of parts P^* in C^*, $N_{P_j}(P^*, C^*), j = 1, 2, 3$, (see column 6th in Table 4). Finally the three estimations $N_{P_j}(P^*, C^*), j = 1, 2, 3$ are combined according to Eq. (12) to get the estimation of the number of parts P^* in container C^*, $N(P^*, C^*) = 613.93$ (last column in Table 4), that means an associated density of 119.23 units of P^* per dm^3, which is 4.86% lower than the observed experimental result of 125.32 units per dm^3.

Experimental Study

To assess the accuracy of the estimation method and the influence of the database quality, we present the results of a computational study conducted with archetypal shapes. These shapes allow the reader to evaluate similarities between figures. The study involves eight parts (with geometric shapes) and 8 containers. The geometric shapes and dimensions are illustrated in Fig. 11 and Table 5: truncated cone, cone 1 and cone 2, sphere, quarter toroid, cylinder, triangular prism, and quadrangular prism; the container dimensions are given in Table 6.

The experimental study aims to estimate the number of parts shaped like a truncated cone (P^*) that can be transported in each of the 8 containers using estimates from three of the remaining seven parts. Thus, there are 35 different combinations of predictor parts (different ways to select 3 elements from a set of 7). Among the seven figures, we label the two cones as very similar, the three parts with curved surfaces as similar (cylinder, sphere, and quarter toroid), while the two with flat surfaces (both prisms) are considered less similar. To observe the influence of having at least one part labeled as very similar, we categorize the 35 possible combinations into two groups: those with at least one cone among the three (a total of 25 combinations) and those that do not have a cone (10 possible combinations).

The summary of the results from the 280 estimates conducted show that when the predictor parts include at least one very similar part, the estimations are good, with a mean difference of 3.35% and a standard deviation of 2.93%. However, results deteriorate when there is no part with a high level of similarity among the predictor parts. In this case, the mean error increases to 11.0% (with standard deviation of 3.93%). The disaggregated data exhibited a consistent behavior across all containers, with better results observed in larger containers, as shown in Table 7, where the smallest container has a mean estimation error of 7% while the largest container has only a 1.68%

5. Selection of the optimal container for transporting parts from supplier to assembly line

As stated in Section 2, in order to select the most appropriate container for transporting a certain part, an optimization problem (Eq. (1)) has to be included in the process and solved.

5.1. Description of the optimal container selection process

As primarily stated in Section 1, the *CSP* involves selecting the most suitable container for shipping each part from its supplier to the manufacturer, based on the estimated transportation and handling costs. We are now in a position to demonstrate how this optimization process works. A global description of the processes involved in the selection of the optimal container is shown in Fig. 12.

All the required information for this process is already available. Given a part P^* to be transported, the initial elements considered by the algorithm are the set of containers $C_i, i = 1, \dots, n$ that could be used to transport our target part P^*. The algorithm then estimates the transportation and handling cost for this type of part using each type of container in the set in order to select the most economical one.

The procedure to be followed is as follows:

- The dimensions and cargo availability of each candidate container are known.

(a) P_1 part (b) P_2 part (c) P_3 part

Fig. 10. Shapes of three similar parts to P^* used for estimating $N(P^*, C^*)$. P_1 exhibits the highest similarity, P_2 demonstrates an intermediate level of resemblance, and P_3 presents a lower degree of similarity.

Table 4
Data (4 columns on the left) and calculations (3 columns on the right) used to determine the number of units of part P^* within container A.

Input data to the Algorithm 3				Output of the Algorithm 3		
Part	C_i	C_i dimensions	$N(P_j, C_i)$	$\rho(P_j, C_j^*)$	$N_{P_j}(P^*, C^*)$ $(\rho_{P_j}(P^*, C^*))$	$N(P^*, C^*)(\rho(P^*, C^*))$
P_1	C_2	[243, 163, 130]	34			
	C_3	[344, 261, 258]	191			
	C_4	[345, 260, 130]	80			
	C_5	[730, 550, 315]	1084	7.52	35.87 (111.92)	
	C_6	[545, 359, 130]	181			
	C_7	[541, 360, 262]	411			613.93 (119.23)
	C_8	[532, 346, 125]	167			
P_2	C_1	[1160, 275, 132]	674	11.45	927 (124.63)	
P_3	C_2	[243, 163, 130]	3008			
	C_3	[344, 261, 258]	13 119	580.56	1448.13 (176.14)	
	C_4	[345, 260, 130]	6840			

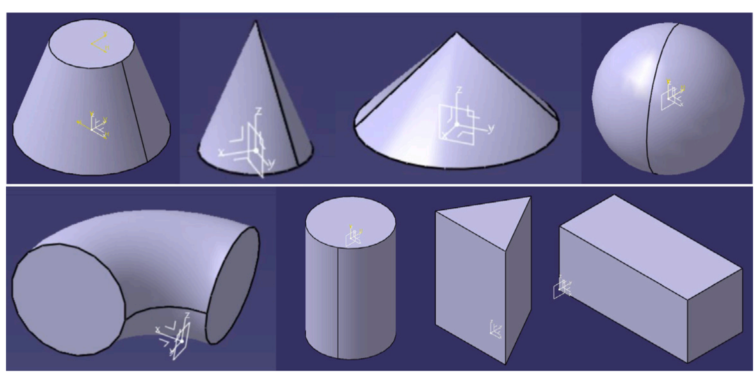


Fig. 11. Archetypal parts used in the experimental study of bulk loading.

Table 5
Dimensions of the parts used in the experimental study.

Part	b_{x1}^* (mm)	b_{y1}^* (mm)	b_{z1}^* (mm)	$V_{CH}(P)$ (cm^3)
P^* Truncated cone	59.62	59.62	59.62	0.068544
P_1: Cone 1	53.06	37.13	36.86	0.020706
P_2: Cone 2	79.61	79.61	40.00	0.066591
P_3: Cylinder	90.00	59.62	59.62	0.252339
P_4: Sphere	39.87	39.41	39.32	0.032564
P_5: Quarter Toroid	45.00	45.00	30.00	0.035562
P_6: Rectangular Prism	70.00	30.00	30.00	0.063000
P_7: Triangular Prism	70.00	50.00	43.30	0.075777

Table 6
Type and dimensions of the containers used in the experimental study.

Dimensions	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
Length	243	344	541	730	920	987	1210	1720
Width	163	258	360	550	520	577	810	1120
Height	130	261	262	315	325	310	789	797

Table 7
Disaggregated results for the difference between predicted and observed number of parts for the smallest and largest containers. Results are expressed in percentages.

Container	C_1	C_8
Minimum	0.33	0.07
Mean	7.00	1.68
Standard deviation	4.90	0.91
Range	14.56	3.38

Table 8
Container information. *99 loaded and 186 empty containers because E containers are collapsible.

C^*	Type	$C_x^*/C_y^*/C_z^*$	$W_{C^*}^0$	W_{C^*}	No. in truck
A	KLT	243/163/130	0.57	14.43	9000
B	GLT	730/550/315	56	250	384
C	GLT	920/520/325	61	517	260
D	GLT	987/577/310	52	1000	312
E	GLT	1210/810/789	90	1000	99/186*

Table 9
Remaining parameters of the system for the cost optimization example.

Weight of the part of Fig. 9 (W_P)	0.027 Kg.
Capacity of the truck trip (W_T)	24 000 Kg.
Cost of a truck trip (Z_T)	€2000
Handling cost per container ($Z_{C,N_{P,C}}$)	GLT €3, KLT €1

Table 10
Capacity analysis results for part P^* in containers A to E.

C^*	$\rho(P^*, C^*)$		Difference	$N(P^*, C^*)$	$N_{P^*,C*}$
	Estimated	Observed	(%)		
A	119.23	125.32	4.86	613.93	534
B	133.28	130.10	2.45	16 856.63	9259
C	135.72	134.14	1.18	21 101.86	19 148
D	136.02	132.31	2.80	24 012.99	24 012
E	136.60	130.44	4.72	105 630.46	37 037

- For each container, starting with the first one ($i = 1$ in Fig. 12), and while not all containers have been considered ($i < n$), the following processes are carried out based on how the parts are stored in it:
  - If placement is used, all desired bounding boxes of the part are generated through geometric processing (Section 3.1). A design program such as CATIA (Dassault Systemes, 2023) can be utilized for this purpose. Subsequently, layer generation using these preprocessed bounding boxes (Section 3.2.1) and the selection of layers that maximize the number of parts in the container (Section 3.2.2) are applied. As a result, the number of transportable parts is obtained for each candidate container.
  - If bulk is used, the first step involves selecting parts from the Reference Set that maintain a certain similarity to the part P^* (Section 4.1). Once these parts have been chosen, the estimation process developed in Section 4.2 is applied to obtain the estimated number of loaded parts.
- After finishing the calculation of the number of parts that can be loaded in each candidate container, the transport and handling costs are evaluated. According to Eq. (1), all the required elements for its evaluation are available: Z_T and $N_{C_i^e,T}$ are static information about the trucks; $N_{C_i,T}$ is obtained from the truck information and the number N_{P*,C_i} just calculated; and $Z_{C_i,N_{P*,C_i}}$ is obtained using the value N_{P^*,C_i} and the information provided by the company.
- The output of the algorithm provides the cost for each container, and the one with the lowest cost is selected and presented to the decision maker.

5.2. Example of the optimal container selection problem: Small case study

This section presents a brief case study to illustrate the container selection problem. Let us assume that, similar to the example in Section 4.3, we need to transport the screw represented in Fig. 9 using bulk loading. Information is available regarding the number of units that can be transported for the three similar parts shown in Fig. 10, with dimensions provided in Table 3. In Section 4.3, we addressed the problem of estimating the number of screws that could be transported in a single container (Table 4). Now, we consider four additional containers and pose the problem of selecting the optimal container among them. The dimensions of the five containers are shown in Table 8. Additional data needed to solve the problem (weight of the transported piece, maximum truck load capacity, and costs of container transportation and handling in the plant) are presented in Table 9. Applying the procedure outlined in Section 4.2 and illustrated in 4.3, we obtain estimates of the number of units that can be transported in each of the five containers Table 10. With information on the effective capacity of each container,

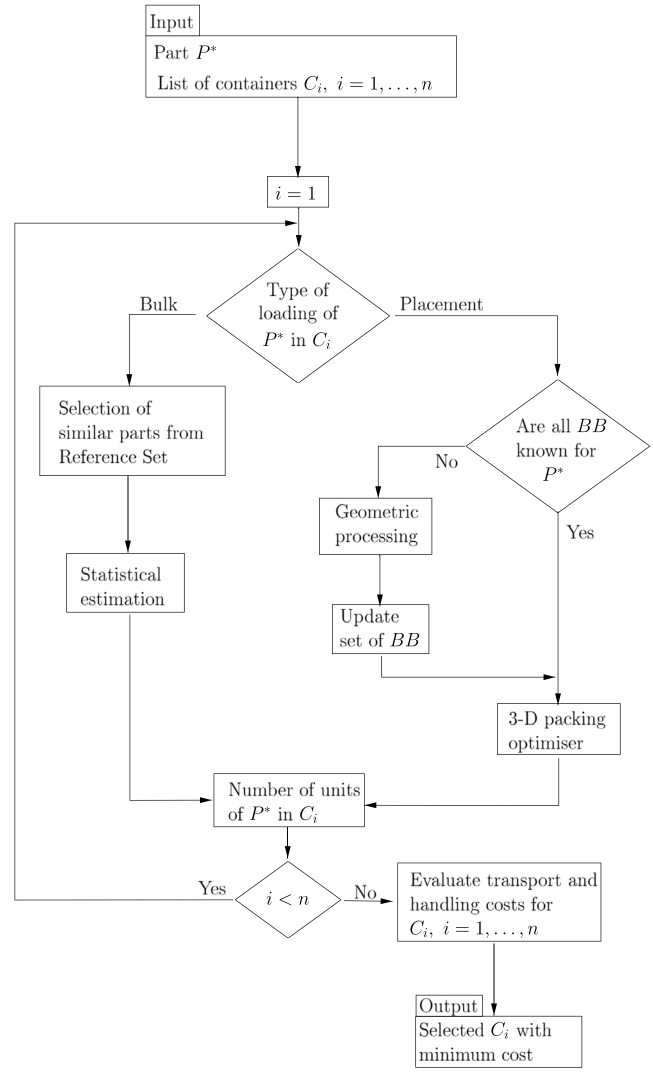


Fig. 12. Flow chart of the optimization process.

which considers the weight constraint (last column of Table 10) and various costs, calculations leading to the evaluation of the objective function, Eq. (1), are organized in Table 11.

The first column of Table 11 shows the type of container for which we estimate the transportation and handling costs. The second column shows the number of parts that can be loaded into the container, accounting form volume and weight limitations. The third column displays the total number of full containers that can be transported in the truck, considering both weight and volume limitations (Eq. (2)). The fourth through sixth columns display the three cost terms defined in Eq. (1) (per 1000 parts transported): transportation from the supplier location to the plant, cost of handling the parts inside the plant, and cost of returning the empty container. The seventh column contains the total cost. It is important to note that for the cost associated with the third term, we use the number of empty containers the truck can carry, $N_{C_i^e,T}$ (this information is in the last column of Table 8).

We detail the calculations of the cost for every 1000 units transported using container A:

- The number of parts transported in container A is $N_{P^*,A} = 534$ (obtained in the example of Section 4.3, Table 10).
- The number of containers loaded with 534 units of part A that can be transported in a truck, fulfilling the constraints of weight

Table 11
Information for the evaluation/selection of the containers.

C^*	N_{P^*,C_i}	$N_{C_i,T}$	C. Transport	C. Handling	C. Return	Total
A	534	1601	2.339	1.873	0.416	4.628
B	9259	78	2.769	0.324	0.563	3.656
C	19 148	41	2.548	0.157	0.402	3.106
D	24 012	34	2.450	0.125	0.267	2.842
E	37 037	22	2.455	0.081	0.290	2.826

and volume, is given by (Eq. (2))

$$N_{A,T} = \min\left\{9000, \frac{24000}{534 \times 0.027 + 0.57}\right\} \approx 1601$$

- The transportation cost is

$$1000 \times \frac{Z_T}{N_{A,T} \times N_{P^*,A}} = \frac{1000 \times 2000}{1601 \times 534} \approx 2.339$$

- The handling cost, considering that the type of container A is KLT, is

$$1000 \times \frac{Z_{A,N_{P^*,A}}}{N_{P^*,A}} = \frac{1000 \times 1}{534} \approx 1.873$$

- The cost of returning empty containers to the supplier's is

$$1000 \times \frac{Z_T}{N_{C_i^e,T} \times N_{P^*,C_i}} = \frac{1000 \times 2000}{9000 \times 534} \approx 0.416$$

Based on the results obtained and shown in Table 11, the recommended container for transporting the selected type of parts would be the fifth one, which is the E container. The total estimated cost for 1000 units using this type of container is €2.826, and it would be the one proposed to the decision maker as the preferred choice.

5.3. Implementation of the methodology in a decision support system

The methodology presented in the previous sections has been implemented in a decision support system that manages the entire logistics of transporting parts in containers from the supplier to the assembly line. It takes into account transportation costs, handling costs at the assembly factory, and the subsequent costs of returning the empty containers to the suppliers.

The primary objective of this software is to provide a tool capable of generating an ordered list of containers with their corresponding total transportation costs for a specific type of part, assuming that the containers are filled to their maximum capacity based on volume and weight constraints. Additionally, when bulk transportation is not feasible, the software also offers the optimal arrangement for placing the parts inside the container. This information equips decision-makers with the data necessary to make informed decisions.

Together with all the algorithms described above, to achieve the proposed goal, the software incorporates a comprehensive database containing all the information required for the calculations: parts, containers, supplier locations, truck costs, handling costs, reference parts for bulk loads, and also a database of bounding boxes associated with each processed part when bulk loading is not possible.

Next, each specific database table and its associated processes are briefly commented:

- **Containers:** This table contains a list of containers, each described by its type (GLT or KLT), its outer and inner dimensions, weight, maximum load capacity, volume when empty and folded (if applicable), rental cost per day, and the number of containers that can be transported in a truck (both folded and unfolded). Additionally, for certain containers which have to be grouped into packs, the table also provides information on the number of containers per pack. The program includes options for managing this information, such as adding new containers, modifying or removing existing ones, creating lists of containers, and batch importing lists. This information is crucial for the estimation process of determining the number of parts that can fit inside the container, whether in bulk or not, as well as for the evaluation of the total cost associated to the transport of the different parts.
- **Parts:** This table stores all the parts to be transported to the assembly line. Each part is described by its code, part type (bulk/non-bulk), supplier and physical dimensions. It also includes the results of its initial geometric processing: volume of the convex hull and minimum bounding box. The geometric processing is done by using a design program such as CATIA (Dassault Systemes, 2023), which is automatically linked from this software. Additionally, it may contain information about other bounding boxes designed for the part, either manually or by using CATIA software, as in the case of the minimum bounding box. This information will be stored in the bounding box table of the database. As in the previous item, the software offers options for adding, modifying, removing, listing, etc.
- **Supplier locations:** This table contains a list of suppliers, along with their mailing addresses, the costs associated with truck transportation to and from their locations, and the container rental duration.
- **Reference parts**. The reference parts table stores all those parts transported in bulk for which there is information about the number that fits in a certain subset of the containers (one register per part and container). This table is linked to the parts and containers tables, showing all their physical characteristics. These parts are grouped by families and are the ones that provide the needed information to carry out the statistical estimation for the transportation of parts in bulk.
- **Logistics data**. This table contains information about the labor costs associated handling containers within the factory necessary to take the parts to the assembly line. These costs are broken down by container type. It also records data on the transport capacities of the trucks.

All the database tables can be updated by introducing new elements, deleting elements, or modifying their information. It is also possible to import batch information and generate different lists.

The distinguishing features of the software concern the various process presented in the previous sections: building of the minimum bounding boxes; building of more complex bounding boxes that incorporate more than one part (placeable or stackable); calculation of the maximum number of parts in each container in bulk or by placement; transport optimization.

All these processes have been presented before except the geometric processing designed to obtain the different bounding boxes from the minimum one to most complex ones. This important feature of the software is a module that has as input the design of the part in format $.CATPArt$ or $.CATProduct$ of the 3D design in CATIA, and it extracts and calculates the physical properties of the part such as weight, actual volume, volume of the convex hull, and dimensions of the minimum bounding box. The geometric process also allows the combination of several part units, with the aim of making better use of space, for which it calculates the minimum bounding box for the grouping of parts. Many combinations of units are possible, different in number of units and relative position, and therefore many different bounding boxes can be associated to a single part, which serve as input to the optimization problem of parts transported by placement. The geometric processing of a part is carried out only once. The results of the geometric analysis are stored in the bounding box table of the database.

The reference parts table is initialized with the available data on the current ways of transporting the parts and with the results obtained from some simulation packages. It is a table designed to be expanded with new information, because the more data on more different parts and more containers are available, the more accurate the results of

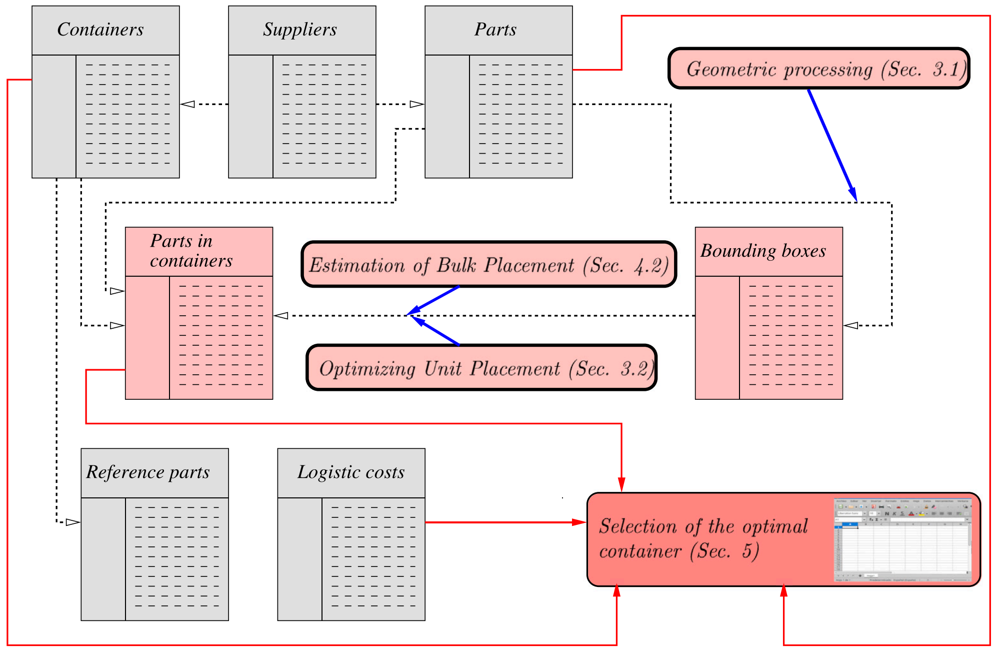


Fig. 13. Global schema of the relations in the computer tool.

the statistical estimation of the units of a part that fit in a specific container will be. The analysis of the transportation of a part can be done in two different ways, by positioning the part in the container or by dumping the units in bulk into the container, though, generally, the characteristics of the part (size, material, cost,…) and quality criteria determine the way it should be transported, by bulk or by placement.

The results of the analysis are exported in files with complete information about the positioning of the units and the costs. Specifically, the software provides the summary of the results for all the containers sorted by ascending price. The total cost is broken down into the costs in the different processes: transportation from supplier to factory, handling of the units in the factory, return of empty containers. Also, the weight of the units in each container and the volume of the container they occupy, the units transported in a full truck, and the number of containers returned in a full truck are included.

The global relationships between the various elements of the entire process are depicted in Fig. 13. Gray tables correspond to raw data, which do not require any processing (static information about containers, suppliers, part descriptions, etc.), and dotted lines indicate the existence of a relationship between the linked tables. Pink tables are constructed using information from other tables (black dotted lines) and through one of the process developed. For instance, to create the bounding boxes, it is necessary to perform the geometric processing of Section 3.1, and the use of these processes is indicated by blue lines. Finally, the selection of the optimal container for a given part is carried out using the information from the tables connected via red lines and the cost calculation Eq. (1).

6. Conclusions

This paper addresses the logistics challenge of improving the transportation of parts from suppliers to a major car assembly factory. Despite considerable experience, the factory's logistics department identified room for enhancement in selecting optimal containers for each part. Manual methods or simulation-based approaches, suitable for evaluating individual alternatives, become impractical with the increasing number of parts and container types, especially with the rise of new car models, such as the upcoming production of electric cars. The container optimization problem considers the cost of transporting the container, the costs of handling the parts, and the cost of returning the empty container.

Our methodological approach distinguishes between two modes of part transportation: individual placement inside a container and bulk loading. The key step in selecting the optimal container is to estimate the number of units each container can hold. For individual placement, we propose decomposing the problem into two optimization sub-problems, solved using heuristic and exact algorithms, respectively. For bulk transportation, a learning algorithm estimates the number of units that each container can carry, using information about the company's current methods of transporting parts and simulation data.

To efficiently address the problem, all optimization and estimation methods were implemented in software that interacts with logistics engineers who can transfer their knowledge to the program. This allows them to specify how the part should be transported, to define permissible groupings of units for individual placement and to suggest similar parts for the learning procedure.

All procedures have been integrated into a software program that extracts the necessary data from the database, estimates the number of parts per container, and calculates the costs. The program functions satisfactorily, assisting the decision-makers at the car factory. Future work should involve integrating this tool into the company's entire logistics system, connecting it with the route planning and truck loading modules.

CRediT authorship contribution statement

Marta Cildoz: Software, Validation, Formal analysis, Investigation, Visualization, Methodology, Writing – review & editing. **Pedro M. Mateo:** Software, Validation, Data curation, Visualization, Methodology, Writing – review & editing. **Maria Teresa Alonso:** Software, Formal analysis, Investigation, Visualization, Methodology, Writing – review & editing. **Francisco Parreño:** Validation, Formal analysis, Visualization, Writing – original draft, Methodology, Writing – review & editing. **Ramon Alvarez-Valdes:** Conceptualization, Formal analysis, Writing – original draft, Project administration, Methodology, Writing – review & editing. **Fermin Mallor:** Conceptualization, Formal analysis, Writing – original draft, Supervision, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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