

How does corporate altruism affect oligopolistic competition?

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ABSTRACT

This paper examines firms' corporate social responsibility (CSR) competition in a duopoly model with product differentiation. Firms exhibit different degrees of altruism, which is a characteristic positively valued by consumers; however, as each firm's degree of altruism (type) is private information, using CSR activities to signal the types becomes an additional tool for competition. We analyze a game where firms first signal about their types and then compete in standard price competition. Consumers observe the firms' CSR activities and infer their degree of altruism. Firms then simultaneously set prices, and consumers purchase goods from one firm or the other. In equilibrium, CSR is determined by the consumers' valuation of corporate altruism and the degree of product differentiation. The model examines the interaction between these two determinants, finding that more competition in the market (less product differentiation) might be detrimental to CSR when consumers' concern about firms' altruism is high enough.

1. Introduction

Corporate social responsibility (CSR) can be considered as companies' activities that are consciously directed to positively affect society. CSR activities must be voluntary and exceed the firms' legal obligations to qualify. Most CSR activities can be considered contributions to public goods valued by stakeholders (customers, employees, suppliers, investors, or shareholders). A wide range of CSR activities can be directed toward the environment, animal welfare, charities, improving labor conditions, etc. Such programs can be individual, like when Yoplait launched a pink-lid promotion in 2004, "Save lids to save lives," that committed up to 1.2 million United States dollars in contributions to breast cancer research. Alternatively, CSR activities can be collective, like joining the Red initiative, where participating companies donated 50% of the profits from product sales to purchase and distribute antiretroviral medicine to fight AIDS in Africa. The increasing importance of CSR activities in the 21st century might have been facilitated by the higher empowerment of civil society (i.e., NGOs) and the success of the responsible investment movement and fair-trade initiatives. However, a considerable variation exists in the observed firms' engagement with CSR activities. While some companies have a long-established reputation of being "altruistic," others devote little resources to CSR. For instance, Bosch is known for its pioneering policies in protecting the

environment, and Coca-Cola for its strong focus on sustainability. This paper proposes a theoretical model to account for the variation in CSR activities across companies that compete in prices in a duopolistic market. We seek a rationale for CSR investments by companies such as PepsiCo and The Coca-Cola Company, Google and Apple, or Starbucks and Dunkin Donuts.

We consider a model in which firms can voluntarily engage in CSR activities. Consumers buy goods or services from the firms and are also concerned about the perceived altruism of the firm with which they choose to interact. The two firms compete for consumers, who have different a priori preferences concerning the variety of each firm's products. Apart from their initial bias toward one firm, consumers also care about the prices offered to them by both firms in the relevant market. Furthermore, because consumers are sensitive to corporate altruism, firms that are perceived as socially concerned enjoy a competitive advantage; however, information about the firms' characteristics is not public. The only way for firms to credibly convey this information to consumers is by undertaking costly social actions.

Firms build their reputations by investing in visible CSR activities, which can be directed toward any prosocial goal that, we assume for simplicity, does not directly affect the quality or attributes of the good or service traded in the relevant market. For instance, CSR in our model might be related to improving the environment, donating to charities, or

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offering better labor conditions for firms' workers. Consumers observe the actions of all firms related to CSR and then make inferences about the firms' types based on these observations.

We consider a multistage game. In stage one, firms compete in CSR. The second stage is standard price competition; once consumers have assessed the firms' types, firms simultaneously set up prices in the relevant market. In the third stage, each consumer decides to buy the product from one firm or the other based on the observed vector of prices and on firms' assessed types. Following a backward induction rationale, firms internalize the equilibrium outcome of subsequent stages when competing for reputation in the first stage. The first stage of the game is a simultaneous signaling game where the two firms are the senders, and the consumers are the receivers of the signals. At this point, the signal consumers observe is a pair of equilibrium actions undertaken by the firms, which can anticipate the effect of each possible bi-dimensional signal on consumers' beliefs. Consumers understand that the two senders interact strategically and update information about their types from each pair of observed actions; however, because the firms' set of possible signals is dichotomic (to conduct a social activity or not), the types are not fully revealed, and consumers can only infer the interval in which the type of each firm lies.

We use the concept of perfect Bayesian equilibrium (PBE) and solve the game by backward induction. We then focus on how the equilibrium aggregate level of CSR offered in the market depends on the degree of (horizontal) product differentiation and consumers' preferences regarding firms' altruism. In the paper's final section, we carry out a comparative statics analysis that analyzes the interaction between these two variables and the influence of both on the equilibrium level of firms' engagement in CSR. We find that changes in the extent of price competition (product differentiation) can encourage or discourage firms' investment in CSR depending on the degree of consumers' concern for corporate altruism. Suppose this concern is low enough. Then, when the market becomes more competitive, the equilibrium levels of CSR increase. This theoretical prediction is confirmed by the empirical literature, showing that firms in more competitive markets exhibit a higher degree of CSR (Fernández-Kranz and Santaló, 2010; Flammer, 2015; Acabado et al., 2020; Forgione and Migliardo, 2020; Leong and Yang, 2020).

From a theoretical point of view, a similar result is obtained in Amin et al. (2021) in a model where consumers value the CSR activities and not the type of the firm. Unlike their approach, our model assumes that consumers directly value the degree of altruism of companies but not their prosocial activities. In this context, we find a counterintuitive result: the aggregate level of CSR may decrease due to increased market competition when consumers' weight on the type of firm is sufficiently high. The reason is that a higher valuation of the firms' types leads to most firms prioritizing social responsibility. Then, undertaking a social initiative does not significantly boost their reputation. Each firm must weigh the impact on their profits between reducing prices and investing in reputation-building activities; therefore, if the product becomes increasingly homogenous, the best competitive approach for firms will be to reduce prices and forgo CSR initiatives.

The paper is organized as follows. Section 2 frames our paper on oligopolistic competition in the literature. In Section 3, we present a model in which the two firms engage in competition in CSR and prices. Section 4 defines and characterizes our multistage game's (perfect Bayesian) equilibrium. In Section 5, we carry out comparative statics analysis and study how the equilibrium level of CSR depends on consumers' preferences and the degree of product differentiation. Finally, Section 6 concludes.

2. Related literature

This paper's signaling model considers vertical differentiation based on perceived corporate altruism by consumers in a duopolistic market. The interaction between the degree of product differentiation, price

competition, and strategic investment in CSR has also been studied recently by Amin et al. (2021) in a bi-dimensional Hotelling-type model where consumers value the investment made by firms in CSR but not the firms' degree of altruism. They conclude that firms make CSR investments simultaneously when the importance of CSR for consumers is high enough compared to the transportation cost parameter that measures the degree of product differentiation, and the higher the latter, the lower the CSR investment. Our approach finds the same result when consumers' concern for the type of company is low, but the opposite result when consumers place a sufficiently high weight on the company's reputation, generating differential predictions that may serve to test the theories with empirical evidence.

Other related literature analyzes duopolistic competition when consumers cannot observe the quality of the product. For example, in Fluet and Garella (2002), Hertzendorf and Overgaard (2001), or Yehezkel (2008), firms may signal quality through prices and dissipative advertising; however, their approach differs substantially from ours. First, prices can be used as signals when the unobserved variable is the quality of the product; however, prices cannot provide any meaningful information about the degree of social concern of the firm, except in cases where the costs of social activities are not type-contingent. Second, these papers assume that one firm produces high quality and the other low quality and that this fact is known to consumers; thus, if one firm is perceived as high quality, the other must be of low quality for sure, and vice versa. This situation constitutes a strong restriction on consumers' beliefs. In contrast, our approach is more flexible as we analyze equilibria for every possible combination of degrees of altruism between firms.

This paper introduces a critical element for our understanding of firms' investments in CSR as we consider a signaling process embedded into firms' price competition. Our approach is related to Fisman et al. (2006), who analyze different types of signaling equilibria in Hotelling duopoly. In their approach, one firm is socially indifferent, the other is socially responsible, and consumers cannot distinguish between them. In Fisman et al. (2006), the socially responsible firm uses social expenditures as a signal of safe products to consumers; however, no competition in CSR exists between firms, and one firm always has a competitive advantage over the other. In contrast, in our setup, firms exhibit different degrees of social responsibility and engage in strategic competition, carrying out their CSR decisions simultaneously. We characterize equilibrium outcomes in which both firms undertake CSR activities, neither of them does, or one firm does CSR, and the other does not. Each firm's decision regarding CSR is simultaneously informative (for consumers) and strategic (concerning the rival firm). Consumers understand how firms compete, and beliefs are consistently formed from the observed action vector.

Our paper also relates to other works where market competition interacts with firms' strategic decisions to invest in CSR. For example, Planer-Friedrich and Sahm (2021) use an asymmetric Cournot duopoly where firms may commit to incorporate the consumer surplus with profits as part of their objectives, paying a fixed cost interpreted as a credible commitment to CSR expenditure. They prove that this concept of CSR investment may strategically serve the most efficient firm to dominate the market and even deter competitors in the Stackelberg case (Planer-Friedrich and Sahm, 2020). Lambertini and Tampieri (2015) also consider a Cournot oligopoly with homogeneous goods and pollution. These authors analyze how a firm concerned with the environment (CSR firm) competes against pure profit-maximizing firms. As the CSR firm is concerned with consumer surplus, its profits can increase because choosing to be the CSR firm implies committing to higher output. Becchetti et al. (2014) consider CSR a new competitive dimension, where firms benefit from accumulating "ethical capital" and consumers are heterogeneous in their ethical distance to the firm from which they buy. In a dynamic optimization model where firms compete in price and ethical locations, these authors analyze the case where a profit-maximizing incumbent reacts to a not-for-profit ethical pioneer's

(potential) entry. In contrast with the mentioned papers, asymmetric information is central to our approach, and the signaling process that takes place before price competition determines the total supply of CSR in the market.

In a monopolistic competition setup, Hiller and Raffin (2020) develop a model where firms use CSR to attract socially responsible workers and thus increase firms' productivity. Our approach differs from the one in Hiller and Raffin (2020) in the role of CSR in firms' competition. In their model, some workers are intrinsically motivated to work for a firm that carries out CSR; hence, firms use CSR instrumentally to attract these (productive) workers. Conversely, in our model, firms have different degrees of altruism, and consumers prefer to buy from "ethical" (high reputation) firms. Hiller and Raffin (2020) consider CSR a screening device from (selfish) firms intended to attract responsible (productive) workers. In contrast, in our approach, CSR works as a signal sent by altruistic firms to attract self-interested consumers who value firms' social concern but not CSR activities per se.

3. Model

We consider two firms, labeled by $i = A, B$ that operate in the same market. Consumers are characterized by their a priori preferences concerning both firms. For instance, firms produce a different variety of the same good, or they produce the same good but are located at different places, so consumers prefer the firm closer to their location. For simplicity, we fix locations at the extremes and consider that firm A is located at 0 and firm B is located at 1. Abusing notation, $x \in [0, 1]$ denotes a consumer located at point x . We assume that consumers are uniformly distributed on the interval $[0, 1]$, and that this distribution is common knowledge in the game.

Consumers also care about the firms' degree of altruism (or social concern). Let $\theta_i \in \Theta_i \equiv [0, 1]$ represent firm i 's degree of altruism, which we call the type of firm $i = A, B$. Firm types are not known to consumers, but the distribution of the types is common knowledge. Types are independently distributed according to a continuous cumulative distribution function, $F(\theta_i)$, with associated density function $f(\theta_i)$. Let $\theta = (\theta_A, \theta_B) \in [0, 1] \times [0, 1]$ be a profile of types. We assume that the profile θ is known to both firms but not the public. The assumption that each firm observes the vector θ is justified by the fact that firms are competitors in the same market and have long interacted strategically; as a consequence of this relationship, the firms may have learned each other's type in the long run.

Our model takes a novel approach to how consumers are concerned with CSR. We depart from the idea that consumers value prosocial activities and assume that consumers respond positively to firms credibly perceived as more altruistic. This approach is similar to how people perceive individuals. This assumption is justified by several social psychology and behavioral economics experiments, which show contexts where agents systematically reciprocate when interacting with other agents (Akerlof and Yellen, 1990; Berg et al., 1995; Fehr et al., 1997; Fehr and Gächter, 1998; Brühlhart and Usunier, 2012). The halo effect might also arise (Sen and Bhattacharya, 2001); this is a cognitive bias whereby positive impressions of a firm in one aspect (e.g., the firm's investment in CSR) influence consumers' overall assessment of the firm. Moreover, as Bénabou and Tirole (2010) discuss extensively, consumers' concerns about social image or self-image could motivate the propensity to buy from socially responsible firms.

Consumers care about firms' types, but these types cannot be observed directly. Firms know the consumers' preferences and may signal their degree of altruism through investment in CSR-related activities. We consider that each firm ($i = A, B$) can either carry out a social activity ($a_i = 1$) or not ($a_i = 0$). This simple dichotomic decision captures firms' actions like asking (or not) for a voluntary external certification for social and environmental standards, developing (or not) a program to promote volunteering in the community, participating (or not) in charitable giving or engaging (or not) in a program to improve

employees' labor conditions. Let $a = (a_A, a_B) \in \{0, 1\} \times \{0, 1\}$ be a vector of social activities or CSR vector. Consumers observe this vector and update their beliefs about firms' types accordingly. The decision about which firm to buy from also depends on the prices, p_A and p_B , set by the firms and on the preference for each variety (distance).

We propose a multistage game of imperfect information. Nature selects the types of both firms at the beginning of the game. Each firm observes its type and that of its rival firm, so information is complete for the two firms. The timing of events is the following:

Stage 1: Firms learn θ , and simultaneously decide whether to undertake a CSR activity. Consumers observe the profile of actions $a = (a_A, a_B)$, which they use to make an inference on the type of each firm.

Stage 2: Firms choose prices, p_A and p_B , simultaneously.

Stage 3: Each consumer who holds beliefs regarding both firms' types observes the prices set up by the firms and demands one unit of the good or service from firm A or firm B.

After the initial move of Nature selecting a given θ , the proposed game has three stages of simultaneous moves in each; the two firms are the only players in the first and second stages, and consumers play in the final stage. The first stage of the game is a signaling game where each firm is the sender of a signal, $a_i \in \{0, 1\}$, and consumers are receivers of a bi-dimensional signal (a_A, a_B) . A system of beliefs represents the consumers' assessment regarding the type of each firm, given by the posterior density function, $f(\theta_i|a)$. The expected type of firm $i = A, B$ is

$$E[\theta_i|a] = \int_0^1 \theta_i f(\theta_i|a) d\theta_i. \quad (1)$$

According to expression (1), consumers form beliefs about firm types conditional on the vector of observed actions. We sometimes refer to this expected type as the firm's reputation. This model considers horizontal product differentiation; thus, consumer x buys one unit of the good either from firm A or firm B, based on the distance of this firm to their ideal point x (Hotelling, 1929) and the vectors $a = (a_A, a_B)$ and $p = (p_A, p_B)$. Let $d_i(x)$ be the distance from consumer x to firm $i = A, B$, and V the consumer's valuation of the good. The utility of a consumer x who buys from firm i is given by the following expression:

$$u(a, p, i, x) = V + \gamma E[\theta_i|a] - p_i - d_i(x). \quad (2)$$

For any given choice by consumer x , $u(\cdot)$ represents the degree of satisfaction received from buying the good, measured in terms of some numeraire. Since the type of the firm is a conceptual measure ranging from 0 to 1, reflecting the firm's degree of altruism, parameter $\gamma > 0$ adjusts the relative weight consumers place on a firm's altruism compared to other variables that influence their utility. This valuation can represent the consumers' social awareness level or the extent to which a firm's reputation is visible to consumers. As consumers value purchasing the good from a firm perceived as being altruistic, parameter γ is assumed to be positive.¹

Firm A is located in 0, and firm B is located in 1. Then, we have $d_A(x) = tx$ and $d_B(x) = t(1 - x)$, where parameter $t > 0$ (the transport cost in the Hotelling model) can be interpreted as the degree of the firm's (horizontal) differentiation. The value of t can be reasonably related to the degree of market competition in this oligopolistic market. If t is close to zero, we are dealing with a perfectly competitive market. In contrast, if t is high enough, each firm is a local monopoly in a neighborhood of its location.

The constant $V > 0$ represents a baseline trade value, which is common to all consumers and high enough to allow each consumer to

¹ The plausible range of values for γ is determined by technical requirements. In particular, the upper bound of this parameter is implicitly defined by the requirement that the equilibrium characterized is interior.

demand one unit of the good or service traded in the market. Conversely, if V were small enough, the market would not be fully covered,² and each firm would be a local monopoly. As we argue in Footnote 2, a sufficient condition for full coverage is to consider $V \geq t$. García-Gallego and Georgantzis (2009) studied changes in the market structure and its consequences on social welfare, analyzing the effect of an increase in the consumers' willingness to pay for goods produced by socially responsible firms. In a duopoly model of vertical product differentiation, Rodríguez-Ibeas (2007) assumed that some consumers are more environmentally concerned and are willing to pay more for the goods produced by the "prosocial firms" than others. In this case, partial coverage of the market plays a role in the price competition stage, and promoting environmental concerns on the part of consumers might lead to a lower market share of the firm that produces the environmental good.

We denote by $x_i(a, p_A, p_B)$ the market demand faced by firm $i = A, B$. There are no production costs, and function $c_i(\cdot)$ represents the cost to firm θ_i of taking action $a_i \in \{0, 1\}$. The profit function of firm $i = A, B$ is given by

$$\pi^i(a, p_A, p_B, \theta_i) = p_i x_i(a, p_A, p_B) - c_i(a_i, \theta_i). \quad (3)$$

We assume that cost function $c_i(\cdot)$ is continuous, differentiable, and strictly decreasing in θ_i . Moreover, $c_i(0, \theta_i) = 0$ for all $\theta_i \in \Theta_i$. The following is a simple linear form for this function³:

$$c_i(a_i, \theta_i) = \begin{cases} (1 - \theta_i)c & \text{if } a_i = 1 \\ 0 & \text{if } a_i = 0 \end{cases}. \quad (4)$$

Parameter $c > 0$ represents the maximum possible cost of a social activity. The cost of increasing the signal (from 0 to 1) varies across types, meaning that the more concerned with social issues a firm is, the lower the cost it faces from carrying out CSR. Therefore, genuinely altruistic firms enjoy a competitive advantage. In making this assumption, we combine two complementary visions of CSR, as discussed in Bénabou and Tirole (2010). On the one hand, prosocial behavior is a strategy that maximizes profits in the long run (vision 1 in B&T, 2010). On the other hand, the firm's decision to engage in CSR may emerge directly from its managers' ethical or altruistic values (vision 3 in B&T, 2010). It is reasonable to think that genuinely altruistic managers should face lower costs for prosocial activities.

4. Equilibrium of the reputation-price competition game

The equilibrium concept we shall use here is PBE. Firms' and consumers' strategies must be optimal given the consumers' system of beliefs. Furthermore, beliefs are updated from the equilibrium strategies using Bayes' rule whenever possible.

First, we define a strategy for a consumer and a firm. A strategy for consumer x is a function, $\tau: \{0, 1\}^2 \times \mathbb{R}_+^2 \times [0, 1] \rightarrow \{A, B\}$. Here, $\tau(a, p, x) = A$ denotes that consumer x buys from firm A , and $\tau(a, p, x) = B$ denotes that consumer x buys from firm B . Let T be the set of all possible consumers' strategies. A strategy s_i for firm $i = A, B$ is described by the following tuple: $s_i(\theta, a) = (a_i(\theta), p_i(a))$. Here, $a_i(\theta) \in \{0, 1\}$

² To guarantee full market coverage in equilibrium, we must impose that the lowest utility for a rational consumer that buys the unit from firm $i = A, B$ is nonnegative. Note that this is the consumer's utility that is indifferent between the two firms. From the next section of the paper, the reader can check that this utility is given by $u(a, p_A, p_B, i, x') = V - \frac{p_A + p_B}{2} + \frac{\gamma(E[\theta_A|a] + E[\theta_B|a])}{2}$. As we show later, the duopoly equilibrium holds $p_A^* + p_B^* = 2t$, so the lowest possible utility in equilibrium is $u^*(a) = V - t + \frac{\gamma(E[\theta_A|a] + E[\theta_B|a])}{2}$. The last term is always positive and depends on the distribution of firms' types. Therefore, a sufficient condition for guaranteeing full market coverage in the model, valid for every distribution, is $V \geq t$.

³ In Section 5, we use this particular cost function to perform comparative statics analysis, considering that $c > 0$ is high enough to guarantee a high enough differential cost of the social activity across types.

represents the choice of firm i in stage 1, and $p_i(a)$ is the price set by firm i in stage 2. Let S_i be the set of all possible strategies for firm $i = A, B$. A vector of strategies for all players in the game is denoted as $s = (s_A, s_B, \tau) \in S_A \times S_B \times T$. Subscript $-i$ denotes firm B when $i = A$ and vice versa. Next, we define a PBE for our game.

Definition 1. A PBE comprises a vector of strategies $s^* = (s_i^*, s_{-i}^*, \tau^*) \in S_i \times S_{-i} \times T$ and a system of beliefs $f^*(\theta|a)$ such that

- (i) For all $x \in [0, 1]$, and given beliefs $f^*(\theta|a)$, $\tau^* = \operatorname{argmax}_{\{i\}} u(a, p, i, x)$ for all $a \in \{0, 1\}^2$, $p \in \mathbb{R}_+^2$.
- (ii) Beliefs $f^*(\theta|a)$ are formed using the strategies s_i^* for $i = A, B$, applying the Bayes' rule whenever possible.
- (iii) Given the consumers' strategies $\tau^* \in T$ and the system of beliefs $f^*(\theta|a)$, for all $\theta \in \Theta_A \times \Theta_B$, $a \in \{0, 1\}^2$, strategy $s_i^* = (a_i^*(\theta), p_i^*(a)) \in S_i$ for $i = A, B$ is such that

$$p_i^*(a) = \operatorname{argmax}_{\{p_i\}} \pi^i(a_i, a_{-i}, p_i, p_{-i}^*, \theta_i),$$

and

$$a_i^*(\theta) = \operatorname{argmax}_{\{a_i\}} \pi^i(a_i, a_{-i}^*, p_i^*(a_i, a_{-i}^*), p_{-i}^*(a_i, a_{-i}^*), \theta_i).$$

In the last stage of the game, each consumer buys from the firm that yields a higher utility. The consumer's decision is based on the distance to each firm, the estimation of each firm's reputation, and the prices set by both firms. In the game's second stage, after the types of both firms have been assessed, firms set up prices simultaneously. Firms anticipate the equilibrium response of consumers to every possible pair of prices, given the reputation each firm acquires. In the first stage of the game, firms take into account the beliefs held by consumers regarding their types, and they also discount the equilibrium strategies (prices and consumer decisions) of the last two stages of the game. Given the equilibrium strategies of consumers in stage 3, the equilibrium price strategies of stage 2, and the consumers' system of beliefs, each firm makes an optimal decision regarding a CSR activity for any given decision made by the other firm. Beliefs are derived from the equilibrium strategies using the Bayes rule whenever possible.

To solve the game, we apply a backward induction logic.

Stage 3: We call x' the type of consumer indifferent between buying from firm A or firm B . Formally, given a system of beliefs $f(\theta|a)$, x' is such that $u(a, p, A, x') = u(a, p, B, x')$ for all $a \in \{0, 1\}^2$, $p \in \mathbb{R}_+^2$. Specifically, we have

$$x' = \frac{1}{2} - \frac{p_A - p_B}{2t} + \frac{\gamma(E[\theta_A|a] - E[\theta_B|a])}{2t}. \quad (5)$$

Therefore, $\tau^*(a, p, x) = A$ for all consumers x such that $x \leq x'$. The demand from the firm i , $x_i(a, p_A, p_B)$, is equal to x' if $i = A$ and equal to $1 - x'$ if $i = B$.

In this case, the linearity of transportation costs does not preclude equilibrium existence; see d'Aspremont et al. (1979) for a discussion of equilibrium existence problems in the Hotelling duopoly. It is easy to check that the expression in (5) would remain the same with quadratic costs. In any case, in this model, firms' location is fixed by assumption (representing two different varieties of the same product, like Coca-Cola and Pepsi-Cola), and we focus our attention on firms' decisions regarding CSR and prices.

Stage 2: In the second stage, given $a = (a_A, a_B)$ and $f(\theta|a)$, firms consider the consumers' equilibrium strategies $\tau^* \in T$, and choose prices simultaneously. Equilibrium price $p_i^*(a)$ for $i = A, B$, is such that $p_i^*(a) = \operatorname{argmax}_{\{p_i\}} \pi^i(a_i, a_{-i}, p_i, p_{-i}^*, \theta_i)$, and its value is given by

$$p_i^*(a) = t + \frac{\gamma}{3}(E[\theta_i|a] - E[\theta_{-i}|a]). \quad (6)$$

The equilibrium price strategy for each firm depends on the degree of product differentiation, measured by t , and the difference between both

firms' reputations, weighted by parameter γ . The firm with the highest expected type maximizes profits by charging a price premium. The size of the gap between the price set by a firm with a high reputation and that set by a firm with a low reputation depends on consumers' weight on firms' altruism. This result resembles the case where customers are willing to pay a higher price for goods that include credence attributes related to CSR (Calveras and Ganuza, 2016). Although firm i knows its type and also the type of the rival, its price strategy does not depend on the difference of the realized types. Instead, it depends on consumers' expectations about these types; thus, the firms need to signal the types through CSR activities.

The profit function of firm $i = A, B$ evaluated at the equilibrium price strategies is given by

$$\pi^i(a_i, a_{-i}, p_i^*, p_{-i}^*, \theta_i) = \frac{1}{2t} \left[t + \frac{\gamma}{3} (E[\theta_i|a] - E[\theta_{-i}|a]) \right]^2 - c_i(a_i, \theta_i). \quad (7)$$

Profits for firm i depend positively on the difference in reputations. Then, firms would like consumers to perceive them as being altruistic. To make an optimal decision regarding CSR, firms must trade off the beneficial effects of a higher reputation (they can sell more of the good at higher prices) against the cost of engaging in CSR activities.

Stage 1: At this stage, each firm $i = A, B$ with type $\theta_i \in \Theta_i$ sends a signal $a_i \in \{0, 1\}$ at a cost $c_i(a_i, \theta_i)$. Provided that the decision to be made by each firm is dichotomous (to carry out a social action or not), the set of possible signals is given by $A = \{(1, 1), (1, 0), (0, 1), (0, 0)\}$. The signal observed is correctly interpreted by consumers as the equilibrium result from the interaction between firms taking their decisions strategically, aware of the beliefs each pair of actions will induce in consumers. Only four possible signals exist, and the types belong to a continuum; therefore, the signal does not allow consumers to infer the exact type of each firm. Instead, signals convey information about the interval where the type of each firm belongs.

We are interested in characterizing PBE in which the firms' equilibrium choices reveal some information to consumers and call it a semiseparating PBE. The subsequent lemma states that if consumers' out-of-equilibrium beliefs satisfy the intuitive criterion (Cho and Kreps, 1987), no pooling equilibria exist in the game.

Lemma 1. No pooling PBE survives Cho and Kreps' (1987) intuitive criterion.

Proof. See Appendix.

We next characterize a semiseparating equilibrium for our game. Let us consider beliefs defined over a partition of the type space $[0, 1]$ into three subintervals: $[0, \alpha_0]$, $[\alpha_0, \alpha_1]$, and $[\alpha_1, 1]$, with $0 \leq \alpha_0 < \alpha_1 \leq 1$. The equilibrium beliefs in this game are given by the conditional density function, $f^*(\theta_i|a)$, such that

- (i) If $a(\theta) = (1, 1)$, then, $f^*(\theta_i|a) > 0 \Leftrightarrow \theta_i \geq \alpha_1$ for $i = A, B$
- (ii) If $a(\theta) = (1, 0)$, then $f^*(\theta_A|a) > 0 \Leftrightarrow \theta_A \geq \alpha_0$, and $f^*(\theta_B|a) > 0 \Leftrightarrow \theta_B < \alpha_1$
- (iii) If $a(\theta) = (0, 1)$, then $f^*(\theta_A|a) > 0 \Leftrightarrow \theta_A < \alpha_1$, and $f^*(\theta_B|a) > 0 \Leftrightarrow \theta_B \geq \alpha_0$
- (iv) If $a(\theta) = (0, 0)$, then, $f^*(\theta_i|a) > 0 \Leftrightarrow \theta_i < \alpha_0$ for $i = A, B$.

According to the system of beliefs represented by function $f^*(\theta_i|a)$, threshold α_1 represents the type of a firm $i = A, B$ that is indifferent between choosing $a_i = 1$ or $a_i = 0$, given that the other firm $-i$ has chosen $a_{-i} = 1$. Similarly, threshold α_0 represents the type that leaves firm i indifferent between $a_i = 1$ or $a_i = 0$, provided that firm $-i$ has chosen $a_{-i} = 0$. Consumers' equilibrium beliefs are such that if both firms take a CSR activity, both firms must have types higher than α_1 (case [i]). If neither firm carries out a CSR activity, then both firms must have types lower than α_0 (case [iv]); hence, if the equilibrium actions of

both firms coincide, the intervals where the types of the two firms must lie are the same. In contrast, if one firm carries out a social action but not the other (cases [ii] and [iii]), the firm that chooses the social action is believed to have a type above threshold α_0 (the lowest of the two), and the other firm is believed to have a type below threshold α_1 (the highest of the two). That is, consumers (correctly) believe that the CSR firm has a type relatively high (above α_0) and the other has a type relatively low (below α_1).

We seek to characterize pairs of actions following the logic of a Nash equilibrium. That is, the action of each firm must be optimal given the other firm's action. To determine the best reply for each firm, we must first find the value of firms' reputations where only one firm conducts CSR activities, not the other. For $i = A, B$, we have

$$E[\theta_i|a_i = 1, a_{-i} = 0] = r_H(\alpha_0) = \frac{\int_{\alpha_0}^1 \theta f(\theta_i) d\theta_i}{\int_{\alpha_0}^1 f(\theta_i) d\theta_i} \quad (8)$$

$$E[\theta_i|a_i = 0, a_{-i} = 1] = r_L(\alpha_1) = \frac{\int_0^{\alpha_1} \theta f(\theta_i) d\theta_i}{\int_0^{\alpha_1} f(\theta_i) d\theta_i}. \quad (9)$$

Let us define $\Delta r(\alpha_0, \alpha_1) = r_H(\alpha_0) - r_L(\alpha_1)$. Function $\Delta r(\cdot)$ represents the increase in reputation earned from being the only firm in the market that does not carry out CSR activities to being the only one that does. The following lemma establishes some valuable properties of this function.

Lemma 2. For all $\alpha_0, \alpha_1 \in [0, 1]$ with $\alpha_1 > \alpha_0$, we have (i) $r_H'(\alpha_0) > 0$ and $r_L'(\alpha_1) > 0$; and (ii) $\Delta r(\alpha_0, \alpha_1) > 0$. If types are distributed uniformly, then $r_H'(\alpha_0) = r_L'(\alpha_1) = \frac{1}{2}$.

Proof. See Appendix.

Now we evaluate the profit function in the equilibrium strategies of stages 2 and 3 and rename the second stage equilibrium profits, $\pi^i(p_i^*(a), p_{-i}^*(a), a, \theta_i)$, as $B^i(a_i, a_{-i}, \theta_i)$ for $i = A, B$. To simplify notation in the expression for the equilibrium profits, we define function $z(\alpha_0, \alpha_1) = \frac{\gamma \Delta r(\alpha_0, \alpha_1)}{3t}$. Note that, from Lemma 2, we have $z(\alpha_0, \alpha_1) > 0$. The firms' payoffs in the first stage of the game, given the second stage equilibrium price strategies and the consumers' beliefs described above, can be written, for $i = A, B$, as

$$B^i(a_i, a_{-i}, \theta_i) = \begin{cases} \frac{t}{2} - a_i c_i(a_i, \theta_i) & \text{if } a_i = a_{-i} \\ \frac{t}{2} [1 + (2a_i - 1)z(\alpha_0, \alpha_1)]^2 - a_i c_i(a_i, \theta_i) & \text{if } a_i \neq a_{-i} \end{cases} \quad (10)$$

Let us analyze the equilibrium actions for firms in the first stage of the game, given the equilibrium play of stages 2 and 3. We can represent the strategic interaction between both firms in stage 1 as a normal form game (see Fig. 1)

We now establish the incentive conditions. If firm $-i$ has chosen $a_{-i} = 1$, firm i (weakly) prefers $a_i = 1$ over $a_i = 0$ if

$$\frac{t}{2} - c_i(1, \theta_i) \geq \frac{t}{2} [1 - z(\alpha_0, \alpha_1)]^2. \quad (11)$$

Similarly, if firm $-i$ has chosen $a_{-i} = 0$, firm i (weakly) prefers $a_i = 1$ over $a_i = 0$ if

$$\frac{t}{2} [1 + z(\alpha_0, \alpha_1)]^2 - c_i(1, \theta_i) \geq \frac{t}{2}. \quad (12)$$

The two inequalities above are the incentive conditions of a semiseparating PBE equilibrium characterized by thresholds α_0 and α_1 . Notice that $a_i = 1$ is a dominant strategy for any firm with type $\theta_i \geq \alpha_1$, and $a_i = 0$ is a dominant strategy if the type of firm i is $\theta_i < \alpha_0$. In the case where the type θ_i is in between α_0 and α_1 , firm i has the best response to each possible action a_{-i} , as specified in the incentive conditions (11) and (12). To simplify notation, $c_i(\theta_i)$ denotes the cost of undertaking the social activity for a firm with type θ_i , that is, $c_i(\theta_i) =$

$c_i(1, \theta_i)$. Equilibrium thresholds, α_1^* and α_0^* , are implicitly defined as the solution for α_1 and α_0 of the following equations system:

$$\frac{t}{2} - c_i(\alpha_1) = \frac{t}{2}[1 - z(\alpha_0, \alpha_1)]^2 \quad (13)$$

$$\frac{t}{2} + c_i(\alpha_0) = \frac{t}{2}[1 + z(\alpha_0, \alpha_1)]^2 \quad (14)$$

The system of beliefs represented by $f^*(\theta_i|a)$ where thresholds α_1 and α_0 satisfy equations (13) and (14) is consistent with the equilibrium actions $a^*(\theta) = (a_A^*(\theta), a_B^*(\theta))$ whenever these actions fulfill the incentive conditions expressed in conditions (11) and (12). Therefore, we have:

$$a^*(\theta) = \begin{cases} (1, 1), & \text{if } \theta_A, \theta_B \geq \alpha_1^* \\ (1, 0), & \text{if } \theta_A \geq \alpha_0^* \text{ and } \theta_B < \alpha_1^* \\ (0, 1), & \text{if } \theta_A < \alpha_1^* \text{ and } \theta_B \geq \alpha_0^* \\ (0, 0), & \text{if } \theta_A, \theta_B < \alpha_0^* \end{cases} \quad (15)$$

We are now ready to establish the main result of this section.

Proposition 1. All semiseparating PBE equilibria of the CSR game are characterized by strategies $s^* = (s_i^*, s_{-i}^*, \tau^*) \in S_i \times S_{-i} \times T$ and beliefs described by function $f^*(\theta_i|a)$, with $\tau^* = A$ (B) for all consumers x such that $x \leq x'$ ($x \geq x'$). The value for x' appears in equation (5), and $s_i^* = (a_i^*(\theta), p_i^*(a)) \in S_i$ for $i = A, B$. Here, $p_i^*(a)$ is given in equation (6), $a^*(\theta)$ is given in (15) and thresholds α_0^* and α_1^* are implicitly defined by equations (13) and (14).

Proof. See Appendix.

5. Product differentiation, consumers' preferences, and aggregate CSR in the market

This section analyzes the interaction between product differentiation and consumers' concern with firms' altruism and how these factors affect the aggregate level of CSR supplied in the market by both firms. For this purpose, we perform comparative statics analysis on the equilibrium thresholds of the separating PBE concerning parameters γ and t . Parameter γ denotes the weight consumers place on the degree of altruism of the firm from which they purchase goods or services. This parameter can be interpreted as a reflection of the consumer's concern regarding the altruistic nature of the firm or as the degree of visibility of a firm's reputation. Disseminating yearly CSR reports, promotional campaigns, or the propagation of CSR-related information through media channels can augment this visibility. Parameter t represents the degree of horizontal product differentiation and measures the extent to which the goods produced by both firms are perceived as distinct based on attributes other than price. This parameter can be interpreted as the minimum monetary compensation for the most loyal consumer of firm i to buy the unit from the rival firm $-i$ when prices are equal. A higher value for t is related to a lower level of price competition. As t goes to zero, the market becomes perfectly competitive, and firms' equilibrium prices tend to marginal production costs. Conversely, a high enough value for t leads to a local monopoly.

Fig. 2 presents the relationship between equilibrium thresholds and the total amount of CSR supplied in the market. The PBE in Proposition 1 associates to each possible profile of types $\theta = (\theta_A, \theta_B)$ a vector of equilibrium actions $a^*(\theta) = (a_A^*(\theta), a_B^*(\theta))$. Therefore, the aggregate level of CSR at equilibrium is a function of α_0^* and α_1^* . We are interested in how the size of each region in the graph responds to changes in parameters γ and t .

The aggregate level of CSR, as a function of α_0^* and α_1^* , is equal to

$$CSR(\alpha_1^*, \alpha_0^*) = F^2(\alpha_1^*) - F^2(\alpha_0^*) + 2(1 - F(\alpha_1^*)). \quad (16)$$

Firm B

		Firm B	
		$a_B = 1$	$a_B = 0$
Firm A	$a_A = 1$	$B^A(1, 1, \theta_A), B^B(1, 1, \theta_B)$	$B^A(1, 0, \theta_A), B^B(0, 1, \theta_B)$
	$a_A = 0$	$B^A(0, 1, \theta_A), B^B(1, 0, \theta_B)$	$B^A(0, 0, \theta_A), B^B(0, 0, \theta_B)$

Fig. 1.

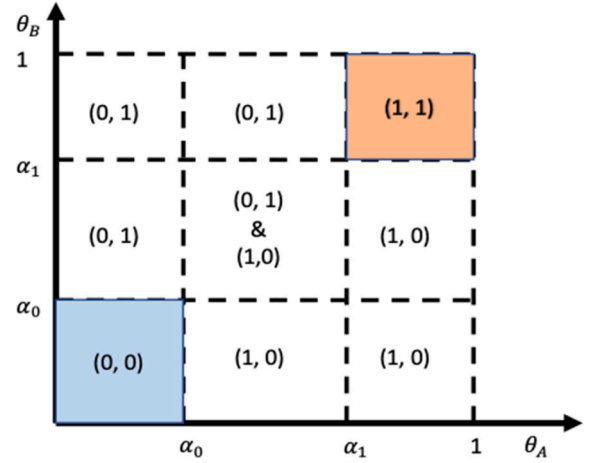


Fig. 2.

Lemma 3. The aggregate level of CSR is monotonically decreasing in both α_0^* and α_1^* .

Proof. The sign of the following derivatives proves the result

$$\frac{\partial CSR(\alpha_1^*, \alpha_0^*)}{\partial \alpha_0^*} = -2F(\alpha_0^*)f(\alpha_0^*) < 0, \quad (17)$$

$$\frac{\partial CSR(\alpha_1^*, \alpha_0^*)}{\partial \alpha_1^*} = -2[1 - F(\alpha_1^*)]f(\alpha_1^*) < 0. \quad (18)$$

For ease of exposition, we shall consider a scenario characterized by a linear cost function (as described in equation [41]) and uniform distribution of the firms' types.⁴ To simplify notation, we write $\Delta r(\alpha_0^*, \alpha_1^*)$ as Δr , and $r_H'(\alpha_0^*)$ and $r_L'(\alpha_1^*)$ as r_H' and r_L' , respectively. For the case of the uniform distribution of the types, it holds $F(\theta_i) = \theta_i$, $f(\theta_i) = 1$, and $r_H' = r_L' = \frac{1}{2}$. We also have $z = z(\alpha_0^*, \alpha_1^*) = \frac{\Delta r}{3r}$, with $\Delta r = \frac{1}{2}(1 + \alpha_0^* - \alpha_1^*)$. As equilibrium prices in equation (6) must be positive, then $z \leq 1$. Computing the total differential of the equations system consisting of equations (13) and (14) and considering $dt = 0$ yields:

$$\frac{d\alpha_0^*}{d\gamma} = -\frac{\Delta r}{D}(1 + z), \quad (19)$$

$$\frac{d\alpha_1^*}{d\gamma} = -\frac{\Delta r}{D}(1 - z), \quad (20)$$

where $D = \gamma z + 3c > 0$. From (19) and (20), we conclude that the equilibrium thresholds are decreasing in γ , and that α_0^* decreases faster than α_1^* , given that $\left|\frac{d\alpha_0^*}{d\gamma}\right| > \left|\frac{d\alpha_1^*}{d\gamma}\right|$, because $z \in (0, 1]$. The negative sign of expressions (19) and (20) implies that the aggregate CSR responds positively to a marginal increase in the consumers' concern on altruism, γ , for all values of parameters γ , t , and c compatible with the existence of

⁴ A general comparative statics analysis concerning parameters γ and t is carried out in the Appendix.

an interior equilibrium.

$$\frac{dCSR^*(\gamma)}{d\gamma} = \frac{2\Delta r}{D} [\alpha_0^*(1+z) + (1-\alpha_1^*)(1-z)] > 0. \quad (21)$$

In this particular scenario, the role played by the variable γ is clearly determined. If we start from a minimal value for γ and consider successive increases in γ , the optimal thresholds move gradually from values close to one (when $\gamma \rightarrow 0$) to end up with $\alpha_0^* = 0$ and some α_1^* , such that $1 > \alpha_1^* > 0$. Hence, a high γ is associated with low thresholds, and this implies high levels of CSR. The contrary occurs for low values of γ . As argued below, this relationship is crucial in explaining when the degree of product differentiation affects positively or negatively the aggregate supply of CSR.

We now undertake the comparative statics analysis of equilibrium thresholds and CSR concerning the degree of product differentiation, t . Following a similar routine as before, we obtain:

$$\frac{d\alpha_0^*}{dt} = \frac{3z^2}{2D} \left(1 - \frac{\gamma}{3c}\right), \quad (22)$$

$$\frac{d\alpha_1^*}{dt} = -\frac{3z^2}{2D} \left(1 + \frac{\gamma}{3c}\right). \quad (23)$$

The marginal change in the aggregate level of CSR when t changes marginally is given by

$$\frac{dCSR^*(t)}{dt} = -\frac{3z^2}{D} \left[\alpha_0^* \left(1 - \frac{\gamma}{3c}\right) - (1 - \alpha_1^*) \left(1 + \frac{\gamma}{3c}\right) \right]. \quad (24)$$

Expression (24) can be rewritten more conveniently for its interpretation. Manipulating conditions (13) and (14) and considering the uniform distribution for the types, we have $\frac{\gamma}{3c} = \frac{2-\alpha_0^*-\alpha_1^*}{1-\alpha_1^*+\alpha_0^*}$. The expression in (24) can be written simply as $\frac{dCSR^*(t)}{dt} = -\frac{3z^2}{D} [2(\alpha_0^* + \alpha_1^*) - 3]$. Since $D' > 0$, the sign of $\frac{dCSR^*(t)}{dt}$ depends on the sign of the term in brackets. Therefore, we have $\frac{dCSR^*(t)}{dt} < 0$ whenever $\alpha_0^* + \alpha_1^* > \frac{3}{2}$ and vice versa. This situation means that when $\alpha_0^* + \alpha_1^*$ is high enough, more horizontal differentiation leads to lower levels of CSR, and the contrary occurs when $\alpha_0^* + \alpha_1^*$ is low enough. When γ is small (and the level of CSR is low), a reduction of price competition (i.e., a higher t) implies less aggregate investment in CSR in the sector. This theoretical prediction is supported by empirical evidence about the relationship between investment in CSR and the extent of price competition. For instance, [Fernández-Kranz and Santaló \(2010\)](#) found that different market concentration proxies are negatively related to social impact ratings. In the banking sector, [Forgione and Migliardo \(2020\)](#) showed that the Lerner index is correlated negatively with the banks' engagement in social responsibility. [Flammer \(2015\)](#) used a natural experiment on tariff reduction to conclude that domestic companies respond to this reduction by increasing their engagement in CSR. [Leong and Yang \(2020\)](#) empirically showed that firms in more competitive markets exhibit better overall social performance, and [Acabado et al. \(2020\)](#) found that a more competitive setup favors investments in all CSR categories. The theoretical prediction of duopoly models that include product differentiation à la Hotelling and CSR (understood as valued by consumers in itself) is that more competition (lower t) always leads to higher investment in CSR by all firms ([Amin et al., 2021](#)). However, in their case, CSR is only undertaken when γ is above a certain threshold.

Our model yields an entirely different prediction when the value of γ is high enough. In this case, as the market increases in competitiveness, firms' engagement in CSR tends to decrease. The reason is that competition in CSR is substituted by price competition when the levels of CSR in the market are high enough. To grasp some intuition for this latter result, suppose that each firm must decide whether to produce an environmentally friendly good, and the cost of producing this good may differ across companies; however, this "technology" is not observable, and consumers must infer its value from the equilibrium actions of both

companies. Parameter γ measures the impact on consumers' utility from engaging in a trade relationship with an environmentally concerned firm. Suppose that γ is relatively large, so most firms have adopted green technology. The expected types of these firms are lower than the ones consumers had inferred in a context where only a few firms had adopted green technology. In other words, when most companies are green, becoming green does not suppose a substantial enhancement of reputation. Provided that each firm must trade off the effects on their profits from a strategy of price reduction versus investment in activities that yield reputation, if the product becomes less differentiated, the optimal competitive response of firms will be to lower prices and renounce CSR activities.

6. Conclusion

We introduce firms' competition in CSR in a duopoly Hotelling-type model with (horizontal) product differentiation and price competition. Consumers prefer to buy goods from altruistic firms because they can be expected to behave ethically regarding their relationship with their customers; however, consumers do not observe the degree of altruism of each firm (the type of firm). Then, competition in CSR occurs as a signaling process, where firms invest in prosocial activities (CSR) to persuade consumers about their true social motivations. In equilibrium, there is a partial sorting of the firms' types, and a certain aggregate amount of CSR is achieved in the market. The model provides a rationale for corporate decisions regarding CSR in the context of oligopolistic competition and shows the relationship between the aggregate amount of CSR, consumers' preferences, and the extent of product differentiation.

We describe the competitive interaction between the two firms as a multistage game in which firms decide whether to undertake social action and then compete in prices. Our equilibrium concept is that of PBE, and we solve the game by backward induction. The equilibrium levels of CSR depend on the degree of product differentiation and the consumers' valuation of firms' reputations. In a context where consumers are primarily concerned with the altruism of firms, we find that more market competition may reduce the firms' engagement in CSR.

Compliance with ethical standards

Disclosure of potential conflict of interest:

- The authors did not receive support from any organization for the submitted work.
- The authors have no conflicts of interest to declare relevant to this article's content.

Data sharing does not apply to this article as no datasets were generated or analyzed during the current study.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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APPENDIX

Proof of Lemma 1. We now develop a formal argument to rule out pooling equilibria when the consumers' beliefs fulfill Cho and Kreps' intuitive criterion (Cho and Kreps, 1987). Since the model only considers two actions for each firm, only two possible candidates exist for a pooling equilibrium. The first candidate is when all firms, regardless of their types, choose $a_i = 1$. As any firm makes the same decision, the choice of social action does not provide any information about the types, and the only possible beliefs in this situation are given by the prior distribution of the types. The expected type for each firm would then be the expected value of θ_i on interval $[0,1]$; however, since the social activity is costly, any firm i that individually deviates to $a_i = 0$ will get a higher payoff because it avoids the cost of the social activity, but its reputation keeps being $E(\theta_i)$. Therefore, a situation where all firms choose $a_i = 1$ cannot be a pooling equilibrium.

The other possible pooling equilibrium is when all firms select $a_i = 0$, regardless of their types; as before, the equilibrium beliefs are the prior distribution of the types. This situation would undoubtedly constitute a pooling equilibrium without restricting out-of-equilibrium beliefs. No firm wants to deviate from $a_i = 0$, given the other firms' choices and the beliefs supporting this behavior; however, we argue that this pooling equilibrium would not survive a refinement like Cho and Kreps' intuitive criterion. Suppose that one firm deviates from this equilibrium. As the social action is more costly for the lower types, there will be a set of types S for which this deviation is equilibrium-dominated (provided the cost of the social action is high enough). In other words, even for the most optimistic beliefs such a deviation may induce on consumers, no firm within a specific set of low types S would ever be interested in carrying out the social action. Hence, if a deviation from the pooling equilibrium occurs, it can only come from a firm with a type higher than those belonging to set S . However, if consumers hold these out-of-equilibrium beliefs, then some of the most altruistic firms will find it profitable to deviate to $a_i = 1$; thus, the initial pooling equilibrium is destroyed.

Proof of Lemma 2. (i) The positive sign of $r'_H(\alpha_0)$ and $r'_L(\alpha_1)$ follows immediately from the expressions:

$$r'_H = \frac{f(\alpha_0)}{1 - F(\alpha_0)} [r_H(\alpha_0) - \alpha_0] > 0$$

$$r'_L = \frac{f(\alpha_1)}{F(\alpha_1)} [\alpha_1 - r_L(\alpha_1)] > 0$$

(ii) We prove now that $\Delta r(\alpha_0, \alpha_1) = r_H(\alpha_0) - r_L(\alpha_1) \geq 0$, where $r_H(\alpha_0)$ and $r_L(\alpha_1)$ are given by the expressions (8) and (9) in the main text. Let us

consider the extreme values $\alpha_1 = 1$ and $\alpha_0 = 0$. Clearly, for these values, we have $r_H(\alpha_0) = r_L(\alpha_1)$ because $r_H(0) = \frac{\int_0^1 \theta f(\theta) d\theta}{\int_0^1 f(\theta) d\theta} = r_L(1)$. Let us call $r = r_H(0) = r_L(1)$. Notice that r is the unconditional expected type of θ_i on interval $[0,1]$. Then, for $\alpha_1 = 1$ and $\alpha_0 = 0$, it holds $\Delta r(\alpha_0, \alpha_1) = r - r = 0$. Now, consider any pair $(\alpha_0, \alpha_1) \in [0,1]$ with $\alpha_0 > 0$ and $\alpha_1 < 1$. Considering that $r'_H(\alpha_0) > 0$ and $r'_L(\alpha_1) > 0$, we must have now $r_H(\alpha_0) > r > r_L(\alpha_1)$. Therefore, for (α_0, α_1) such that $0 < \alpha_0 < \alpha_1 < 1$, it holds that $\Delta r(\alpha_0, \alpha_1) > 0$. Therefore, for (α_0, α_1) such that $0 \leq \alpha_0 < \alpha_1 \leq 1$, we always have $\Delta r(\alpha_0, \alpha_1) \geq 0$.

Finally, if the types follow a uniform distribution on the interval $[0,1]$, we have $f(\alpha_0) = f(\alpha_1) = 1$, $F(\alpha_0) = \alpha_0$ and $F(\alpha_1) = \alpha_1$. From the definition of $r_H(\alpha_0)$ and $r_L(\alpha_1)$ in (8) and (9), we obtain $r_H(\alpha_0) = \frac{1+\alpha_0}{2}$ and $r_L(\alpha_1) = \frac{\alpha_1}{2}$. Then, $r_H(\alpha_0) = r_L(\alpha_1) = \frac{1}{2}$.

Proof of Proposition 1. We first prove that the only possible semiseparating PBE in our model must be such that the space of types $[0,1]$ is partitioned into the three in subintervals $[0, \alpha_0^*]$, $[\alpha_0^*, \alpha_1^*]$, and $[\alpha_1^*, 1]$ with $0 \leq \alpha_0^* < \alpha_1^* \leq 1$. For this purpose, since there are only two possible actions, no separating equilibrium can exist where the interval $[0,1]$ is not partitioned. If this were the case, the belief induced by choosing $a_i = 0$ would be the same as the belief induced by the action $a_i = 1$. Then, provided that the social activity is costly to the firm, no firm would ever choose the social action; thus, we would face a pooling equilibrium (which has been ruled out in Lemma 1).

The first statement we prove is that no separating PBE is supported by a partition where consumers believe that choosing $a_i = 1$ ($a_i = 0$) may signal a type lower (higher) than choosing $a_i = 0$ ($a_i = 1$). This result occurs because the cost of social action strictly decreases in type. Note that if a firm of type $\tilde{\theta}_i$ chooses $a_i = 0$ (given any other choice by the other firm), then this firm would also choose $a_i = 0$ if their type were below $\tilde{\theta}_i$. If the firm of type $\tilde{\theta}_i$ chooses $a_i = 0$, the following inequality must hold: $\gamma E[\theta_i | a_i = 0, a_{-i}] \geq \gamma E[\theta_i | a_i = 1, a_{-i}] - c(a_1, \tilde{\theta}_i)$. If this firm had a type $\hat{\theta}_i < \tilde{\theta}_i$, and provided that $c(a_1, \hat{\theta}_i) > c(a_1, \tilde{\theta}_i)$, we have $\gamma E[\theta_i | a_i = 0, a_{-i}] \geq \gamma E[\theta_i | a_i = 1, a_{-i}] - c(a_1, \hat{\theta}_i)$ and the firm would also choose $a_i = 0$. Similarly, any firm with type $\hat{\theta}_i$ that chooses $a_i = 1$, given the choice of the other firm a_{-i} , must also choose $a_i = 1$ if its type is $\hat{\theta}_i > \tilde{\theta}_i$. As we proved before if the firm with type $\hat{\theta}_i$ had chosen $a_i = 0$, it should have also chosen $a_i = 0$ when its type was $\hat{\theta}_i < \tilde{\theta}_i$; however, since the firm chose $a_i = 1$ when its type was $\hat{\theta}_i$, there is no equilibrium choice other than a_1 when its type is $\hat{\theta}_i > \tilde{\theta}_i$. Therefore, any possible partition of the types in a semiseparating equilibrium must leave higher types above some threshold and lower types below some other threshold, and the monotonicity of the cost function drives this result. Furthermore, since the cost of the social action is decreasing in the firm's type, having more than two thresholds is impossible.

We next prove that $\alpha_0^* = \alpha_1^* = \alpha$ is not possible in equilibrium. Suppose $\alpha_0^* = \alpha_1^* = \alpha$. Then, the types above α choose $a_i = 1$, and the types below α choose $a_i = 0$. In this case, for any cdf $F(\theta)$ over the types $\theta \in [0,1]$, the expected type for a firm that chooses $a_i = 1$ when the other firm has chosen $a_{-i} = 0$ is given by equation (8) with $\alpha_1^* = \alpha$. The expected type for a firm that chooses $a_i = 0$, given that the other has chosen $a_{-i} = 1$ is given by equation (9), with $\alpha_0^* = \alpha$. Therefore, we have $\Delta r(\alpha) = r_H(\alpha) - r_L(\alpha) > 0$ and, consequently, $z(\alpha) = \frac{\gamma \Delta r(\alpha)}{3r} > 0$. We next consider the two conditions, (13) and (14) that define implicitly the thresholds α_0^* and α_1^* . Manipulating these equations, we obtain $\frac{c_i(\alpha)}{c_i(\alpha)} = \frac{2-z(\alpha)}{2+z(\alpha)} = 1$. For this equality to hold, we must have $z(\alpha) = 0$, which is a contradiction to the fact that $z(\alpha) > 0$. Therefore, no separating PBE exists where $\alpha_0^* = \alpha_1^* = \alpha$. We conclude that any separating PBE must be supported by beliefs over a partition on the type space, which can only be determined by a pair of thresholds α_0^* and α_1^* with $\alpha_0^* \neq \alpha_1^*$.

We prove now that $\alpha_1^* > \alpha_0^*$, proceeding again by contradiction. Suppose that $\alpha_0^* > \alpha_1^*$, and consider a firm with type θ_i such that $\alpha_1^* < \theta_i < \alpha_0^*$. As the type θ_i is below α_0^* , this firm chooses $a_i = 0$; however, as the type θ_i is also above α_1^* , the firm also chooses $a_i = 1$. It is not possible that the best choice for firm i is $a_i = 0$ and $a_i = 1$ at the same time; therefore, we must have $\alpha_1^* > \alpha_0^*$.

So far, we have shown that beliefs $f^*(\theta_i|a)$ are consistent with the equilibrium strategies s^* . The analysis in the main body of the paper shows that, given the system of beliefs $f^*(\theta_i|a)$, the strategies $s^* = (s_i^*, s_{-i}^*, \tau^*)$ are optimal. First, both the price strategies $p_i^*(a)$ in equation (6) and the choices $a^*(\theta)$ in (15) maximize the profits of firms given the beliefs $f^*(\theta_i|a)$. Second, from equation (5), each consumer maximizes utility by choosing the firm that yields the highest expected utility.

Comparative statics in the general case:

We now present a comparative statics analysis based on equations (13) and (14) that define equilibrium thresholds α_0^* and α_1^* . We consider a generic cdf. $F(\theta_i)$, and a cost function $c(\theta_i)$, such that $c'(\theta_i) < 0$. Let us first consider the two equations that implicitly define the equilibrium thresholds α_0^* and α_1^* :

$$\frac{t}{2} - c(\alpha_1) = \frac{t}{2} \left[1 - \frac{\gamma \Delta r}{3t} \right]^2. \quad (13')$$

$$\frac{t}{2} + c(\alpha_0) = \frac{t}{2} \left[1 + \frac{\gamma \Delta r}{3t} \right]^2. \quad (14')$$

For notational simplicity, let us call $z = \frac{\gamma \Delta r}{3t}$. Adding up (13') and (14') and subtracting (14') from (13') we obtain:

$$c(\alpha_0) - c(\alpha_1) = tz^2 \quad (13'')$$

$$c(\alpha_0) + c(\alpha_1) = 2tz. \quad (14'')$$

To determine how equilibrium thresholds respond to marginal variations in the parameters γ and t , we differentiate equation (13'') and (14''), considering that $z = \frac{\gamma \Delta r}{3t}$.

$$c'(\alpha_0)d\alpha_0 - c'(\alpha_1)d\alpha_1 = \frac{2\gamma}{9t}\Delta r^2(\alpha_0, \alpha_1)d\gamma - \frac{\gamma^2}{9t^2}\Delta r^2(\alpha_0, \alpha_1)dt + \frac{2\gamma^2}{9t}\Delta r(\alpha_0, \alpha_1)[r_H'(\alpha_0)d\alpha_0 - r_L'(\alpha_1)d\alpha_1] \quad (13''')$$

$$c'(\alpha_0)d\alpha_0 + c'(\alpha_1)d\alpha_1 = \frac{2}{3}\Delta r(\alpha_0, \alpha_1)d\gamma + \frac{2\gamma}{3}[r_H'(\alpha_0)d\alpha_0 - r_L'(\alpha_1)d\alpha_1]. \quad (14''')$$

Let us start with the comparative statics regarding γ . To analyze the impact of a marginal change in γ on the equilibrium thresholds, we consider $dt = 0$ and solve the equations system for $\frac{d\alpha_0^*}{d\gamma}$ and $\frac{d\alpha_1^*}{d\gamma}$. The comparative statics analysis yields the following results:

$$\frac{d\alpha_0^*}{d\gamma} = -\frac{\Delta r}{D}(1+z),$$

$$\frac{d\alpha_1^*}{d\gamma} = -\frac{\Delta r}{D}(1-z),$$

where $D' = \gamma[(1+z)r_H' - (1-z)r_L'] + 3c$. To discuss the sign of $\frac{d\alpha_0^*}{d\gamma}$ and $\frac{d\alpha_1^*}{d\gamma}$, note that z is such that $0 < z \leq 1$. We have $z > 0$ because $\alpha_1 > \alpha_0$ and then $\Delta r > 0$. Provided that the equilibrium prices (see equation [6]) are $p_i^*(a) = t + \frac{\gamma \Delta r}{3}$ and $p_{-i}^*(a) = t - \frac{\gamma \Delta r}{3}$, we have $\frac{p_i^*(a)}{t} = 1 + z$, and $\frac{p_{-i}^*(a)}{t} = 1 - z$. As prices cannot be negative, $z \leq 1$; thus, the sign of $\frac{d\alpha_0^*}{d\gamma}$ and $\frac{d\alpha_1^*}{d\gamma}$ depends only on the sign of D' .

The comparative statics over the aggregate amount of CSR yields the following result:

$$\frac{dCSR^*(\gamma)}{d\gamma} = \frac{2\Delta r}{D} [F(\alpha_0^*)f(\alpha_0^*)(1+z) + [1 - F(\alpha_1^*)]f(\alpha_1^*)(1-z)].$$

The sign of $\frac{dCSR^*(\gamma)}{d\gamma}$ depends on the sign of D' , and this sign depends on the distribution function of the types, which determines (jointly with the values α_0^* and α_1^*) the magnitude of r_H' and r_L' .

We proceed with a similar routine in the comparative statics for t , obtaining the following results:

$$\frac{d\alpha_0^*}{dt} = \frac{3z^2}{2D} \left(1 - \frac{2\gamma r_L'}{3c} \right),$$

$$\frac{d\alpha_1^*}{dt} = -\frac{3z^2}{2D} \left(1 + \frac{2\gamma r_H'}{3c} \right),$$

with $D' = \gamma[(1+z)r_H' - (1-z)r_L'] + 3c$.

$$\frac{dCSR^*(t)}{dt} = -\frac{3z^2}{D} \left[F(\alpha_0^*)f(\alpha_0^*) \left(1 - \frac{2\gamma r_L'}{3c} \right) - [1 - F(\alpha_1^*)]f(\alpha_1^*) \left(1 + \frac{2\gamma r_H'}{3c} \right) \right].$$

Again, the sign of $dCSR^*/dt$ depends on the distribution function for firms' types. [Not Available in CrossRef] [Not Available in Internal Pubmed] [Not Available in External Pubmed]

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