The concrete-representational-abstract sequence for the acquisition of the cardinal principle in preschool children with autism

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Abstract

This study assessed the effectiveness of concrete-representational-abstract sequence (CRA) in teaching children with autism spectrum disorder (ASD) the acquisition of the cardinal principle. Three Spanish preschool children with ASD, aged 4, 4, and 5, participated in the study. The research followed a single-subject, multiple-probe across-participants design. The results showed a functional relationship between CRA sequence and the acquisition of the cardinal principle by the three participants. The acquired skills were maintained up to 16 weeks after the completion of the intervention. Moreover, children were able to generalize the skills to situations other than the instructional context. Social validity data collected from the children's families and teachers indicated that both groups had a positive perception of the teaching experience.

Keywords: Autism Spectrum Disorder; ASD; learning disabilities; mathematics; counting; cardinality; Concrete-Representational-Abstract sequence; CRA

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Introduction

Autism spectrum disorder (ASD) is a neurobiological disorder that appears during the first years of life and lasts throughout the life cycle. According to the Diagnostic and Statistical Manual of Mental Disorders (DSM-V; American Psychiatric Association, APA, 2013) the main symptoms are: persistent deficits in communication and restrictive and repetitive patterns of behavior, interests or activities. The symptoms of the disorder manifest themselves heterogeneously, often being the case that individuals interpret language literally (Happé, 1993), may have difficulties inferring mental states about others (i.e., theory of mind, Frith et *al*., 1994), or may show a low profile in executive functions such as planning, working memory or attention (Ozonoff and Schetter, 2007). On the other hand, people with ASD generally benefit from what is known as visual thinking (Grandin, 1995). Limitations specific to the disorder may imply for some students with ASD to present mathematical difficulties. Some studies (Ozonoff and Schetter, 2007) report an association between executive functions and mathematical skills in individuals with ASD. Verbal skills and theory of mind (Polo-Blanco et *al*., 2024) have also been shown to be predictors of mathematical problem-solving performance in students with ASD.

In recent decades there has been an increase in the prevalence of autism which is reflected in general education classrooms, where there is a growing presence of students with ASD. Children with ASD often face difficulties in their academic trajectory, and teachers are challenged with delivering adaptations to accommodate their diverse learning needs. In particular, a significant number of students with ASD present difficulties in mathematics (Polo-Blanco et *al*., 2024). Due to the crucial role of a strong mathematics foundation in facilitating later learning, numerous studies emphasize the significance of early mathematics intervention for children facing learning difficulties (Duncan et *al*., 2007), especially those with ASD (Root et *al*., 2020).

Early Numeracy

For typically developing children, various authors have shown that early numeracy skills are predictors for future mathematical learning and for other contents besides mathematics (Duncan et *al*., 2007). The notion of 'early numeracy' encompasses a wide range of acquired skills, including explicit understanding of numbers, such as identifying them, counting, recognizing patterns, making comparisons, use symbols, performing operations, measurement and making estimations (Jimenez et *al*., 2013). Among previous studies on early numeracy, the work of Gelman and Gallister (1978) stands out, establishing the following principles in the counting process: *Abstraction principle*, which refers to the fact that any collection of objects is countable regardless of its nature; the *stable-order principle*, which implies that in the counting process, the numerical series must be followed in the order established between its terms; the *one-one principle*, which requires that to each element of the collection a single term of the numerical series is associated; the *order-irrelevance principle* refers to the fact that the order in which objects are counted does not influence the final count result; and the *cardinal principle,* which consists in recognizing that the last number recited corresponds to the cardinal of the collection. For instance, when asked how many elements are in a collection of three, the child may count them (one, two, and three), recognizing that the last number counted, 'three', corresponds to the number of elements in the set. Many children acquire counting informally through daily activities but others require specific instruction to develop this ability. Specifically, the cardinal principle typically develops between the ages of 3 to 5 years, but interventions that provide guidance on its teaching are sometimes recommended. In order to improve early numeracy skills and specifically, aid in the acquisition of counting and cardinality, Paliwal and Baroody (2018) conducted three different interventions with 49 children aged 2 to 5 years. Each intervention aimed to address a set of actions involving a collection of elements: (1) label and then count, (2) count, emphasize, and repeat the last word, and (3) counting only. To evaluate the cardinal principle in these interventions, two distinct tasks were performed: 'how-many' tasks involved showing the children a collection of objects and asking them to determine the quantity, while 'give-me-n' tasks required the children to provide a specific number of objects. The authors concluded that the first intervention provided a more effective way of modeling the cardinal principle. In the same line, Johnson et *al*. (2019) recommend working with give-me-*n* tasks in assessments on cardinality acquisition. The authors point out that these tasks are more complex than simply counting sets, as it requires the child to remember the designated number, correctly label each object taken, maintain awareness of their counting progress, and cease counting once they have reached the correct number of objects.

Early Numeracy and ASD

Despite the significance of developing mathematical skills from an early age, there have been few studies focusing on early mathematical competence in children with ASD. Titeca et *al*. (2014) analyzed the early numeracy skills of children with ASD without intellectual disability. The authors determined that both the capacity for verbal subitization (known as the rapid and automatic identification of small quantities of objects) and counting abilities were indicative of future mathematical proficiency. In a similar vein, Fernández-Cobos and Polo-Blanco (in press), conducted a comparative study to analyze the early mathematical competence of 34 students (17 ASD and 17

TD) enrolled in first to fourth grade and matched by age, grade and classroom. The results showed that participants with ASD had more difficulties in mathematics than their TD peers, both in informal (e.g. early numerical skills without formal school instruction) and formal (e.g. numerical skills, arithmetic, and concepts learned in academic contexts) mathematical skills. These difficulties became more pronounced with age, and were particularly noticeable in fourth graders.

Morrison and Rosalez-Ruiz (1997) found that give-me-*n* tasks involving collections of up to 10 items resulted in a higher number of correct responses in a child with both ASD and moderate intellectual disability when taught with objects of medium to low preference, as opposed to high preference ones, which resulted in more stereotypic behavior and fewer accurate responses. Jowett et *al*. (2012) showed the efficacy of an iPad-based video modeling package in teaching a 5-year-old student with moderate ASD number identification, writing, and quantity comprehension for numbers 1 to 7, tailored to the student's specific interests. Jimenez and Kemmery (2013) and Root et al. (2020) demonstrated the effectiveness of the Early Numeracy curriculum (Jimenez et al., 2013) for children with and without intellectual disabilities, in grade two to four, and in kindergarten, respectively. This structured curriculum applies a research-based early numeracy skill trajectory and evidence-based practices, tailored specifically for students with severe disabilities, by including activities related to counting, use of symbols, and number identification. Similarly, Jimenez and Besaw (2020) demonstrated the efficacy of employing virtual manipulatives and a narrative context in 'give-me-n' tasks involving collections of up to 5 elements for two elementary students with ASD and mild intellectual disabilities, aged 8 and 9. In the same line, Ingelin et *al*. (2023) demonstrated the efficacy of an adapted version of *Number Talks* (Humphreys and Parker, 2015) in enhancing number recognition, understanding number parts, and subitizing for three preschool children with ASD. *Number Talks* promote a deep understanding of foundational math concepts through peer discussions facilitated by adults, thereby enhancing mathematical discourse and comprehension.

Concrete Representational Abstract Sequence

Given that previous studies demonstrate the effectiveness of early numeracy interventions for students experiencing difficulties in mathematics, especially those with ASD, it is essential to explore methodologies that can further support the learning of these students. The Concrete-Representational-Abstract (CRA) sequence stands out as an effective methodology for teaching mathematical concepts by fostering the connection between specific objects (concrete level), their visualizations (representational level), and symbols (abstract level) (Flores and Hinton, 2022).

The explicit nature and characteristic scaffolding of the CRA sequence, guiding learners from the concrete to the abstract, can benefit students facing challenges in mathematics. In the specific context of students with ASD, various successful practices have utilized the CRA sequence as a framework. For instance, focusing on early mathematical skills, Kahveci and Akkus (2023) demonstrated the effectiveness of a tablet-mediated intervention using the CRA sequence to teach pre-addition skills to three students with ASD aged 7, 8 and 8. Other studies have employed the CRA sequence for the acquisition of arithmetic operations by children with ASD, with and without intellectual disabilities (Kahveci and Akkus, 2023; Kaya and Yildiz, 2023; Polo-Blanco et al., 2019).

Objectives of the study

Different authors observe that there is a gap in research that specifically focuses on preschool-aged students with ASD, particularly in the acquisition of the cardinal principle (Gevarter et al., 2016).) Recognizing the significance of early mathematical learning for future academic performance (Duncan et al., 2007), we aimed to assess the efficacy of the Concrete-Representational-Abstract (CRA) sequence in teaching the cardinal principle to three preschool children with ASD.

We specifically addressed the following research questions: (1) Is there a functional relationship between the use of the CRA sequence and the acquisition of the cardinal principle in three preschool children with ASD? (2) Do children with ASD generalize the acquired skills to situations other than the instructional context? (3) Do they maintain what they have learned over time? and (4) How do families and teachers of the participating children perceive the study conducted?

Method

Participants

The participants were three children between 4 and 5 years of age, schooled in the same general education classroom of an inclusive public school in Spain, and who met the following inclusion criteria: (1) ASD diagnosis according to DSM-V (APA, 2013); (2) Sufficient visual discrimination for object and image selection; (3) Demonstrated ability to follow simple instructions, including the capacity to point to objects when prompted for counting tasks; (4) Motor skills for manipulating objects; (5) Lack of proficiency in understanding the cardinality principle. The first criterion was confirmed with the clinical history of the children. The second, third, and fourth criteria were verified based on children's records and the professional judgment of the teachers at the school. The fifth criterion was assessed with a task involving five sheets, each containing a set of 1 to 5 objects drawn. The children were required to provide the cardinal number for each set. The criterion was considered met if the child could not determine the cardinality of at least 4 out of the 5 collections.

A girl and two boys (Elena, Daniel and Andres, pseudonymous) met both criteria and, after their families provided informed consent, they were selected for the study. Elena was 5 years old at the beginning of the study, and was diagnosed with ASD at the age of 4 years. She showed global developmental delay according to the Battelle Developmental Inventory, with significant delay in the communicative and social areas. She had a CIT of 67 according to Wechsler Preschool and Primary Scale of Intelligence (WPPSI-IV). According to a psycho-pedagogical report from the public services, Elena's level of concentration was low in situations that were not of interest to her and had marked interests such as stacking, aligning, or wander around. Prior to the beginning of the study, Elena showed knowledge of recognition of symbols from 1 to 5. She showed errors related to the process of counting, in particular in the recitation of the number sequence and in the assignment of cardinality.

Daniel was a boy aged 4 years and 3 months at the beginning of the study, who at 2 years and 9 months was diagnosed with ASD. He showed global developmental delay according to the Battelle Developmental Inventory, with risk of delay in cognitive and motor areas and delay in social and communication. His level of attention was conditioned by the interest of the situation and he had a strong preference for lining up, stacking or sorting games. Prior to the study, Daniel showed recognition of the symbols from 1 to 5, subitized quantities of one, two and three elements. However, he made errors when reciting the numerical series and with the one-to-one correspondence principle when assigning the cardinality to sets of more elements.

Alex was a boy aged 4 years and 6 months at the beginning of the study who at 2 years and 4 months was diagnosed with ASD, along with difficulties in the language area. He showed global developmental delay according to the Battelle Developmental Inventory, with risk of delay in the cognitive area and delay in the remaining ones.

Alex's comprehension was closely linked to context and interest, presenting significant difficulties at the expressive level. His level of attention was very low. Prior to the study, he did not demonstrate recognition of any symbols from 1 to 5, consistently naming them as ‘one’ and ‘three’ randomly on almost all occasions. He made errors when reciting the numerical series and with the one-to-one correspondence principle when assigning the cardinality to sets of any number of elements.

The instructor who carried out the sessions with the participants was a school educational therapist with more than 20 years of experience in working with children with special needs. At the time of the study, she was employed at the school where the instruction took place and was already familiar with the three participants from the previous school year, as she had treated them as a specialist in hearing and language.

The study received ethical approval by the Clinical Research Ethics Committee of Cantabria, code 2020.252.

Setting and Sessions

The research took place in a dedicated classroom, free from distractions, at the participants' school during regular school hours. The classroom, typically reserved for specialists in therapeutic pedagogy, was familiar to the three participants who frequently used it for these purposes. Instructional sessions were conducted exclusively with the student and the instructor present. Each participant attended three weekly individual sessions lasting between 15 and 20 minutes. The instructor had prior meetings with families and teachers to gather additional details about the participants’ preferences and interests, and to provide them with information about the instruction.

The materials employed included: (1) various everyday objects for the concrete phase; (2) a template allowing placement of each object used in the concrete phase; (3) color-printed worksheets featuring 1 to 5 everyday objects for the representational phase; (4) a whiteboard with an erasable marker for the representational phase; and (5) laminated symbols from 1 through 5 for the abstract phase; (6) two color-printed and laminated pictograms representing the questions 'how-many' or the action 'put' (give me); Figure 1 illustrates some of the materials used.

Design and Data Collection

A single case, multiple probe design was conducted among participants to test the efficacy of the CRA sequence in the acquisition of the cardinal principle in three children with ASD. There were four experimental conditions: (1) baseline, (2) intervention following the CRA sequence, (3) generalization, and (4) maintenance.

In order to show a functional relationship between instruction and participant improvement, and following the multiple-probe design, participants were sequentially introduced to instruction. Specifically, the first child began instruction after showing a stable baseline (same score or decreasing trend on at least two consecutive probes). After the instruction began and once improvement was observed, the second child was introduced to instruction after having verified that his baseline was stable, and so on.

Dependent and Independent Variables

The dependent variable was the percentage of correct answers by each child when solving six cardinality tasks (i.e. ‘how many and label’ tasks, as detailed in the procedures) involving collections up to 5 elements. The decision about the quantities to work on was made taking into consideration the usual Spanish lesson plans of the teachers and the school manuals, as well as the stage of learning of the children. The independent variable was the CRA sequence taught with explicit instruction, which is detailed in the procedure section.

Interobserver Agreement and Procedural Fidelity

A member of the research team coded all evaluation probes across all conditions and participants by reviewing recorded videos during independent practice. A second member of the research team re-evaluated about 50% of the children's performance in all conditions. Interobserver agreement was calculated for each condition by dividing the number of agreements by the total number of data points and multiplying the result by 100.

Procedural fidelity data was collected by the second researcher from about 35% of the instructional sessions. This data measured the instructor's performance in executing planned behaviors, which included (1) providing the agreed-upon materials for each session, (2) offering the stipulated support in each phase (C, R and A), (3) allowing the children to independently solve the problems, (4) emphasizing the key actions for each type of task (how-many, and give-me-*n*), and (5) providing verbal praise at the end of each session. Procedural fidelity was calculated for each child by dividing the number of observed teacher behaviors by the number of planned behaviors and multiplying it by 100.

Social Validity

Analyzing social validity is an important aspect of research to address the concerns of the children involved, their families, and teachers. In this study, social validity interviews were conducted with the families of the children and their teachers, including both regular and support tutors, before and after the intervention. In both interviews, the questions were open ended and revolved around the aspects of mathematics that the children enjoyed the most and those in which they encountered the most difficulties. During the subsequent interview, teachers and family members were asked if they had observed any changes in the children's understanding of mathematical concepts and whether they believed that the CRA sequence had positively impacted them. They were encouraged to share as much information as they desired and provide examples whenever possible.

Procedure

Baseline

During baseline, the performance of the three children was assessed through probes consisting of six cardinality tasks (i.e. 'how many and label'), featuring pictures of sets containing 1 to 5 objects. The task was deemed correct when the child identified the cardinal symbol of the set, either through counting or subitizing. The task was considered incorrect if the child made an error in counting or failed to identify the cardinal symbol of the set, even if the count itself was correct.

Intervention

The intervention adhered to explicit instruction, characterized as a series of supports and scaffolds wherein students are guided through the learning process in incremental stages. They receive clear explanations and demonstrations of the targeted skill and are provided with practice along with feedback until mastery is achieved (Archer and Hughes, 2011). Following the CRA sequence, the initial intervention sessions involved the use of manipulative materials, followed by sessions incorporating graphic representations such as drawings or pictures, and finally, sessions employing numeric symbols to assign the cardinality of sets of either objects or drawings. The instruction transitioned phases (from concrete to representational and from representational to abstract) when the child had been assessed in at least two consecutive sessions, achieving a mean success rate of at least 75% in the last two sessions.

In addition, aspects of the instruction were adapted considering possible difficulties and strengths characteristic of ASD: (1) showing topics of interest to each child; (2) favoring concentration through short and clear instructions, rewards, frequent change of activities and materials and avoiding the variety of colors that could be distracting; (3) facilitating comprehension and verbal expression with known vocabulary; and (4) encouraging good visual processing through the use of pictograms to indicate questions or actions. In addition, and in line with previous studies (Paliway and Baroody, 2018), in each session, how-many and give-me-*n* tasks were worked on. In the former, the child was asked how many objects there were, and in the latter, the child was asked to provide a number of objects or draw them, in both cases with quantities from 1 to 5.

Based on previous studies in which the CRA methodology was successfully applied (Kaya and Yildiz, 2023, Flores and Hinton, 2022), a 'model, guided practice, and independent practice' sequence was followed in each session for all three phases. The number of tasks during the model and guided practice phases varied based on the child's concentration and understanding. During the modeling phase, the instructor solved one to three tasks of each type (‘how many’ and ‘give-me-*n*’ tasks). During the guided practice, the instructor directed and assisted the child in completing three to six tasks of each type. At the end of each instructional session during independent practice, the child performed a total of six tasks: three how-many tasks and three give-me-*n* tasks, without the instructor assistance (see Figure 2).

The following is a detailed description of the procedures distinguishing each phase.

Concrete Phase. The children had at their disposal a template with aligned squares on which the instructor placed the different everyday objects to facilitate counting. During each session, explicit instruction was provided in the following manner:

How-many tasks: During modeling, the instructor arranged the objects in a row using the template and asked herself aloud, ‘how many objects are here?’ while pointing to the ‘how many’ pictogram (see Figure 1, left). She then counted aloud and gestured while emphasizing the cardinality (Paliwal and Baroody, 2018). For example, with a set of 4 elements, she would say, ‘one, two, three, four; there are four elements.’ The instructor guided the child similarly, explicitly asking about the number of objects and providing support if needed. In independent practice, the child was asked directly about the number of objects without additional assistance. The task was considered correct if the child correctly stated the cardinality, either through subitizing or counting.

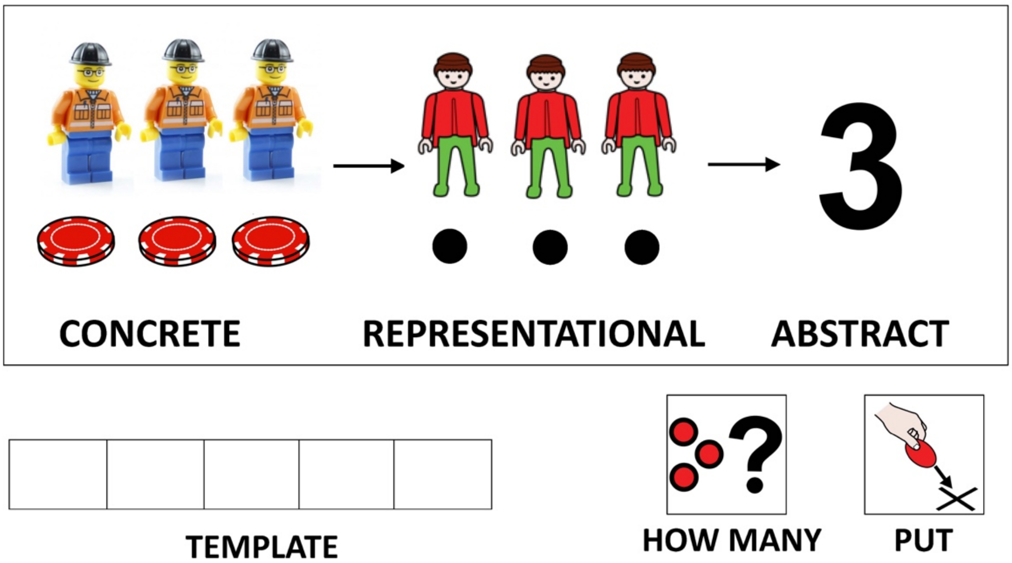
Give-me-n tasks: During the modeling phase, the instructor explicitly told the child she would provide a specific number of objects (e.g., ‘I'm going to give you three soldiers’) while pointing to the ‘give-me’ pictogram. She then recited the sequence, emphasizing the quantity of objects. When guiding the child, the instructor instructed them to choose a specific number of objects and either place them on the template or hand them over, using commands like 'put,' 'place,' and 'give me.' Gestural prompts were used (Flores and Hinton, 2022) to signal when the child should stop placing objects. In the independent practice, tasks were similar to the guided phase but without assistance. The tasks were considered correct if the child accurately provided the specified number of objects.

Representational Phase. In each session, the how-many tasks followed a similar format as in the concrete phase, but using pictures instead of objects. For give-me-*n* tasks, actions like ‘draw’ or ‘sketch’ were used on a whiteboard with an erasable marker. Each session ended with independent practice with tasks similar to the concrete phase with either drawings or pictures.

Abstract Phase. Because cardinality is associated with a collection of objects (represented manipulatively or pictorially, Paliwal and Baroody, 2018), in this study, we considered an adaptation of the abstract phase, where tasks incorporated symbolic representations with written numbers 1 to 5, but also maintained either objects or pictures representing the elements. In how-many tasks, participants were required to first express the cardinality of the collection verbally and then select the corresponding written number (see Figure 1). In give-me-*n* tasks, the instructor would present the child with a symbol and ask him/her to draw or place that number of objects (e.g. 'Can you draw three circles?').

Figure 1

Materials Used During Instruction: CRA Sequence (Top), Template (Bottom Left) and Pictograms (Bottom Right).



Posttest

Immediately after concluding the intervention, the dependent variable was measured in one session with a probe consisting of six cardinality tasks under the same conditions as during the baseline.

Generalization

The generalization of acquired learning was assessed in two contexts distinct from the instructional setting during three sessions: one in a play-based environment within the regular classroom during the school period, and two held outside the educational environment after the end of the school year. The first session involved a hide-and-seek game with toys, requiring the children to count the number of toys hiding. The other two sessions occurred in a park during summer after the school year ended. The children played games with their friends and needed to count objects to provide the cardinal of collections (e.g., five balloons to play a certain game). In each generalization session, the instructor presented six how-many situations to the three children with concrete everyday objects.

Maintenance

Due to the proximity of the end of the school year and occasional participant absences, the number of maintenance sessions varied, ranging from one to three sessions over the four weeks following intervention. Additionally, two extra maintenance sessions were conducted after the summer holidays, 16 weeks after training concluded. In these sessions, six ‘how-many and label’ tasks, similar to the baseline sessions, were performed. The children were first asked how many objects there were and then to label the quantity.

Results

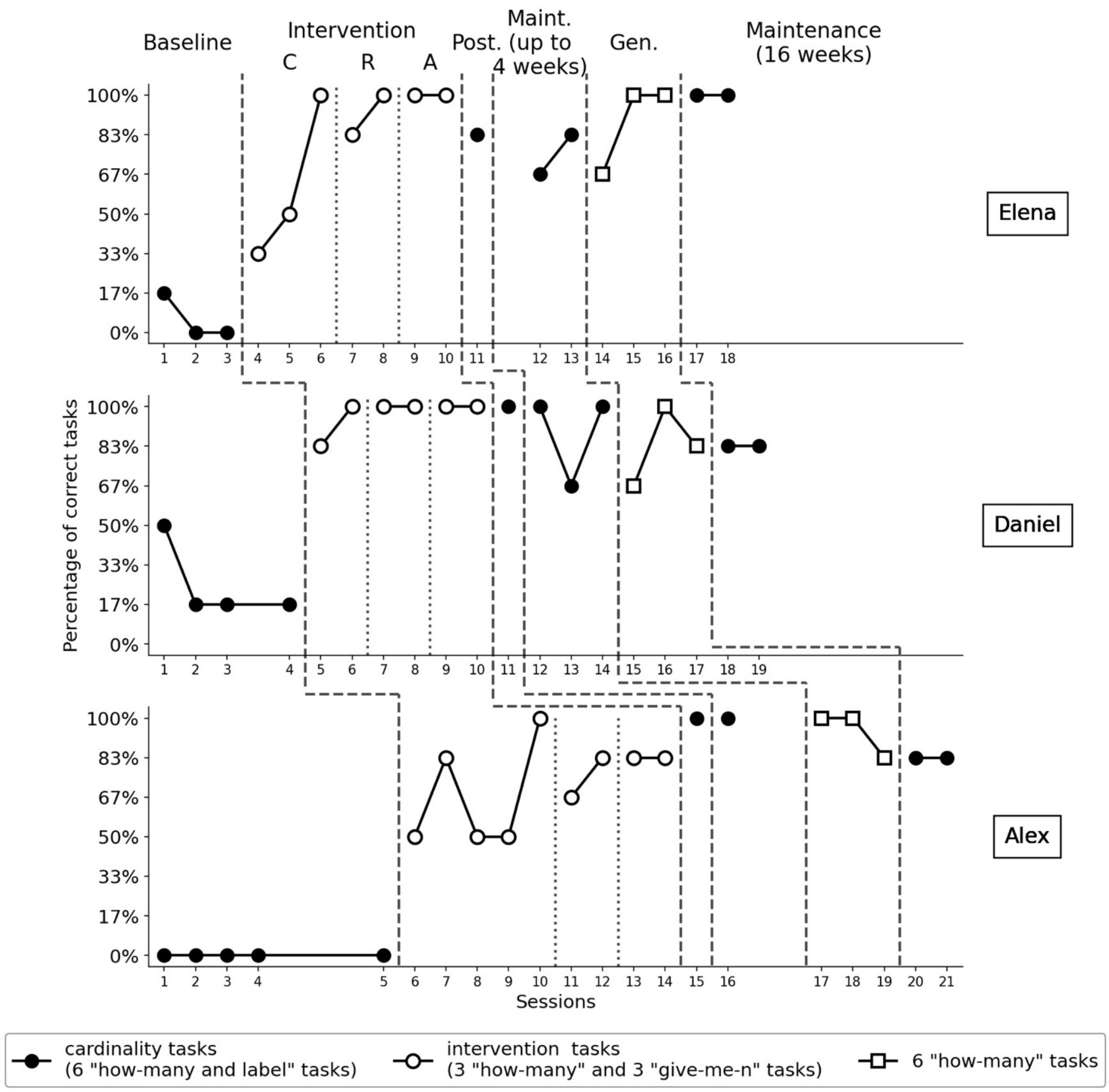
Following the guidelines available for single-case research studies (Kratochwill et al., 2013), we visually analyzed the data of each participant for: (a) level, (b) trend, (c) variability, (d) immediacy of effect, (e) overlap, and (f) consistency of data patterns. Visual analysis of the graph shows a functional relation between CRA sequence and the percentage of correct tasks performed independently correct. During the baseline condition, none of the children showed to have acquired the cardinal principle. All three maintained a non-increasing trend. After the beginning of intervention, the three participants showed a change in level and an increasing trend, with no overlap of data between total success at baseline and intervention. Immediate improvement was observed in the three children, who obtained a mean accuracy of 82% during the intervention, 89% during generalization and 89% during maintenance.

Elena had a mean baseline performance of 6% in ‘how many’ and label tasks. In particular, she solved correctly one task involving two objects, providing the cardinality by subitizing. Elena seemed unmotivated during these sessions, and when asked about the number of objects, she either did not answer or said numbers, apparently at random.

Upon introduction of the intervention and during the Concrete phase, Elena demonstrated an immediate increase and achieved a 100% of correct answers during the third intervention session of this phase. She obtained an mean of 81% correct responses during instruction. During the instructional sessions, emphasis was placed on working with topics of interest to capture the child's attention. Upon entering the representational phase, she showed some difficulty with the give-me-*n* (or draw-*n*) tasks, but by the second instruction session of this phase, she also achieved full success. She showed no difficulties in the abstract phase.

Figure 2

Percentage of Correct Tasks Solved by the Children.



As shown in Figure 2, the dependent variable was measured during posttest, obtaining 83% of success. Then, during maintenance up to four weeks after completion of intervention, Elena obtained an average of 75% correct tasks. There was no overlap of data between baseline and maintenance. Elena identified the number of objects almost always by subitizing. Errors were due in all occasions when labeling sets of four items with the symbol of number five, and vice versa. In maintenance after 16 weeks, she obtained 100% success in both sessions. During generalization, she obtained a mean score of 89%, failing only in the first session to identify the cardinal in sets of five elements.

Daniel scored a 25% success rate on the baseline pretests. Daniel seemed distracted, and at times annoyed. Correct answers were obtained by subitizing the cardinal in small sets. For larger quantities, he occasionally engaged in counting, making errors in sequence, stable-order, and one-to-one correspondence principles. Upon introduction of the intervention, Daniel showed an immediate increase obtaining a mean score of 97%. He showed a very good attitude towards the tasks, and a high concentration that led to a 100% success rate on all how-many tasks in all phases, showing difficulty only in the beginning on one give-me-*n* task.

As shown in Figure 2, Daniel obtained 100% of success in the posttest session. Then, during maintenance up to 4 weeks, he obtained an average of 89% success rate, and in the maintenance sessions after 16 weeks, 83% success rate, with no overlap of data with baseline data points. On most occasions, he solved the tasks by subitizing, and the errors were all due to the identification of the cardinal in sets of four and five elements. During generalization he obtained a success rate of 83%, showing difficulties in the hide-and-seek task, again when identifying 5-element sets.

Alex had a baseline performance of 0%. He showed not understanding the how-many question, repeating the question, or often answering ‘one’ and ‘three’. He did not engage in subitizing on any occasion, and sometimes made errors associated with stable-order and one-to-one principles. Upon the introduction of the intervention, Alex exhibited variability in his performance, achieving an average success rate of 72% in the instruction sessions. He demonstrated better performance in "give-me-n" tasks, although sometimes he tended to use all available objects or draw more than required. However, in "how-many" tasks, his main difficulties included stating a number without prior counting or counting but failing to provide the cardinality. It was crucial to reinforce his understanding of both types of tasks, especially during the concrete phase, by making use of the pictograms mentioned in the methodology.

As shown in Figure 2, Alex obtained a 100% success in the posttest, and maintained the acquired skills obtaining 100% accuracy in the maintenance sessions up to four weeks after instruction, and 83% 16 weeks after instruction, with no overlap of data with baseline data points. His errors occurred when identifying the cardinal of three-item sets, in which he made sequence errors (e.g., saying ‘one, two and two’) or coordination errors (e.g., ‘one and two’ while pointing at three objects). Alex generalized what he learned during the generalization sessions in the park outside the school context, obtaining 100% success.

Interobserver Agreement and Procedural Fidelity

Interobserver agreement was 100% during baseline, 88% in the intervention, 77% in maintenance and 86% during generalization. The mean interobserver agreement across all conditions was 86% for Elena, 92% for Daniel, and 83% for Alex. Procedural fidelity was 100% for Elena and 93% for Daniel and Alex.

Social Validity

The interviews were qualitatively analyzed by grouping similar answers, and illustrative examples provided by the interviewees were selected. In the initial interviews, both the tutors and families of the three children reported significant difficulties in aspects of basic arithmetic, as well as a lack of enjoyment in learning these skills. At the conclusion of the intervention based on the CRA sequence, the tutors reported observing improvement in all three children and noticed an increase in their initiative to count objects. The families reflected similar sentiments. Daniel's family expressed: 'At different times of the day, he counts objects such as stairs and things he is going to eat. We also play hide-and-seek, and he counts by himself. We are thrilled with the progress he has made'. Likewise, Elena's family expressed: 'Now we engage in various counting games because she enjoys it and no longer gets frustrated. She counts the days of the week on a calendar we have at home'. Finally, Adam's mother shared: 'My son has undergone a significant transformation. He communicates more and he is genuinely interested in counting objects. Previously, it felt like an obligation, and he mostly refused to do it. Additionally, counting helps us manage his restlessness and determine the duration of a time-out from a certain activity'.

Discussion and Conclusions

This study shows the existence of a functional relationship between the use of the CRA sequence and the acquisition of the cardinal principle in three children with ASD. This study represents a contribution to the field of early mathematical learning, particularly for students with ASD, by expanding the limited number of research studies focused on mathematical learning of this student population at early ages (e.g. Root et *al*., 2020; Titeca et *al*., 2014). Specifically, it extends research on the learning of the cardinal principle by providing evidence of the effectiveness of the CRA sequence, not previously utilized, to our knowledge, for learning this principle.

In our study, the CRA sequence allowed for advancing from concrete to abstract tasks, enhancing the skill to determine collection cardinality with up to five objects. Each child demonstrated individual progress in their initial numerical knowledge. Elena and Daniel tended to make errors in one-to-one correspondence, addressed by using templates to ensure accurate counting. Alex faced challenges in acquiring the cardinal principle, evident in both how-many and give-me-*n* tasks. In the former, Alex counted collections without indicating the cardinal, possibly reflecting a phase prior to cardinality acquisition. In the latter, he struggled to stop at the requested number, and emphasizing and repeating the last number word proved helpful (Paliwal and Baroody, 2018).

CRA has proven to be particularly suitable as it has allowed for the flexibility in the use of materials and representations, adapting them to the interests of the participants, aligning with previous studies involving students with ASD, such as Jowett et *al*. (2012) where topics of interest were effectively utilized. The design of the tasks (both 'how many' and 'give-me-*n*', Paliway and Baroody, 2018), has also proven to be suitable for learning the cardinality principle. Remarkably, 'give me-*n*' tasks were generally more understandable for the study participants, in contrast to what has been observed in previous studies with typically developing children (Johnson et *al.*, 2019). This improved understanding was facilitated by supporting actions like 'put' and 'give' with corresponding pictograms and gestures from the instructor addressing potential verbal comprehension difficulties.

It is important to note that in this study, the abstract phase of the CRA sequence has been adapted to combine symbols with either concrete manipulations or visual representations, which is essential for learning the cardinality principle. Previous research has explored similar adaptations of the CRA sequence to teach mathematical concepts to children with intellectual disabilities, such as counting skills (Kanellopoulou, 2020), fractions (Morano et al., 2020) or pre-addition skills to children with ASD (Kahveci et al., 2023). For example, Morano et al. (2020) employed CRA-I (CRA Integrated), which integrated manipulatives, representations, and abstract notation in each lesson, with results similar to traditional CRA methods. Kahveci et al. (2023) proposed a stage integrating graphical representations with abstract notations when teaching pre-addition skills to children with ASD, thereby accommodating the unique learning styles of individuals with ASD. The integration of CRA stages is considered beneficial, particularly during the challenging transition from the representational to the abstract phase for students with learning difficulties, especially those with ASD, aiding in the generalization and conceptualization of abstract concepts.

Finally, the generalization activities showed the quality of the learning acquired in a natural environment. While acquiring academic skills is crucial, it is imperative to focus on generalizing these skills to functional situations and everyday life, particularly for students with intellectual disabilities and/or autism. Hence, it is essential to explore meaningful ways to transfer acquired academic skills to non-school contexts and conduct studies to assess this skill transfer. The transfer was also evident in the parents' interviews as they reported instances of their children applying cardinality in everyday situations.

Limitations and Future Research

The limitations of the study stem from the fact that it was conducted in a period within a specific school year, which meant that it was not possible to extend the size of the collections subject to cardinality establishment beyond the number 5. Another aspect not worked on in this study has been the writing of numerical symbols, which could have been addressed in the abstract phase of CRA. Given that numerical writing can lead to the analysis of other types of skills related to fine motor skills, it was decided not to include it in the study.

Considering the positive impact observed in the children’s family environments, future research could involve families in working on early math skills as well. Parent-mediated interventions hold great potential for positively influencing interactions and developmental outcomes in children with ASD since they offer an intensive, naturalistic, and personalized approach to skill stimulation, which proves beneficial for children with ASD (Trembath et *al*., 2019).

This study demonstrates the suitability of the CRA instructional sequence for students with ASD, as previously suggested by other studies (Kaya and Yildiz, 2023; Polo-Blanco et *al*., 2019). Finally, and due to the limited research on foundational early childhood mathematics skills in children with ASD (Ingelin et *al*., 2023), it is crucial to continue studying the effectiveness of interventions targeting the development of these essential skills from a very young age.

Data Availability Statement (DAS). The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to their containing information that could compromise the privacy of research participants.

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