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Involving parents and family environment for enhancing the mathematics education of children with Down syndrome

Elena Gil-Clemente[1] | **Ana Millán Gasca**[2] | **Rebeca Paricio Badías**[3]

[1]Universidad de Zaragoza, Zaragoza, Spain

[2]Roma Tre Università, Roma, Italy

[3]Sociedad de Estudios sobre el Síndrome de Down, Zaragoza, Spain

Correspondence
Elena Gil-Clemente, María Zambrano 6, 7B. 50018, Zaragoza, Spain.
Email: elenagil@unizar.es

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Abstract

A research project involving 16 children with Down syndrome aged 4–14 approaching mathematics at home was developed during the COVID-19-lockdown in Spain. In the research project a series of 14 short mathematical challenges were designed and delivered in a WhatsApp group used as a communication and documentation tool with the participating families. Videos recorded by them were subsequently analyzed with the purpose of describing and analyzing: (a) the experience children live in contact with informal situations underlying the concepts of number and shape and (b) the engagement of families. The choice of these informal situations is based upon recent research that shows the need to consider the wide range of occasional experiences with mathematical ideas children have in their family prior to schooling. In order to design productive mathematical activities we drew on our past research for the choosing of crucial mathematical issues (whole numbers, plane geometry, and solids). We discuss the evolution in parents from practical help to deep engagement, and the fact that notwithstanding diversity in family contexts, engagement and an especially joyful approach to mathematics emerged. The use of videos opens up a path to the possibility to online learning for children with Down syndrome.

KEYWORDS

Down syndrome, early mathematics, informal practice, intellectual disabilities, mathematics education

INTRODUCTION

It is commonly accepted that children with Down syndrome have difficulties handling numbers. Although they usually reach an adequate understanding of counting and cardinality (Abdelhameed, 2007; Bird & Buckley, 2001; Porter, 1998) it is hard for them to transition to more advanced topics such as algorithm processes and solving arithmetical problems (Bruno et al, 2011; Buckley, 2007). Monari Martinez and Pellegrini (2010) and Monari Martinez and Benedetti (2011) propose that calculation skills should not be the unique goal of mathematics with children and teenagers with Down syndrome. Elizabeta Monari has shown that teenagers with Down syndrome with scarce

arithmetical skills are able to understand and solve in other areas, namely algebra (equations connected to elementary geometry) and analytic geometry, provided that a suitable educational methodology is used. Faragher (2014) reformulated the dilemma between the numeracy goal and the general educational goal, suggesting the need to work on both as complementary goals of mathematics education for children with disabilities. We claim that Monari-Martinez and Faragher's shared pedagogical views on mathematics for children with intellectual disabilities can be linked with Francis Su's concept of "human flourishing" through mathematics (Su, 2020). Mathematics—Su argues—is linked to human aspirations (exploration, meaning, play, beauty, permanence, truth, struggle, power, justice, freedom, and community) and thus mathematical experiences help the development of virtues such as imagination, expectation of enchantment, concentration, perseverance, joyfulness, admitting error, resourcefulness, and self-reflection (Su, 2020). Such a concept is consistent with the classical tradition (derived from Plato's views) on the pedagogical effect of mathematics (Fried, 2018; Millán Gasca, 2016), and—we claim—implies a re-placement of mathematics in culture contrary to an only utilitarian view of mathematics as a mere tool, specially in daily lifethat is needed for educational and social inclusion or "mathematics for all" (Faragher & Gil-Clemente 2019; Israel & Millán Gasca, 2012; Gil-Clemente & Millán Gasca, 2021; Schubring, 2015).

Our research on mathematics educational activities for children with Down syndrome has adopted such a pedagogical approach and involves an integrated vision of arithmetic and geometry, starting from early childhood (Millán Gasca, 2016; Cogolludo-Agustín & Gil-Clemente 2019; Gil-Clemente 2020, 2022, 2023).

The experimental setting consists of regular-one h and a half leisure time workshops, to which children and teenagers diagnosed with Down Syndrome attend two/three times a month (on a Saturday). This setting has been designed and implemented since 2015 by a scholarly research group in collaboration with the association SEsDown (Society for the Study of the Down Syndrome) based at Zaragoza (since 2018 in a fixed venue), with ongoing modifications and improvements. Twenty-four children of 3–17 years old have taken part in the research in years 2015–2023 divided in three age groups (young children, children 8–12 years old, and teenagers). Each workshop flows through 3/4 activities (each of them 10–30 min long). SEsDown mathematical workshops:

- involve the educational use of mimesis (Egan, 1988; Scaramuzzo, 2016), play, music, and hands on experience.
- draw on leisure time pedagogy: "festivity" and fantasy (Cox, 1969; López Quintás, 2003), predilection (hobby), community of choice and friendship.
- take advantage of a deep relationship between educators and children as well as attention to the interaction among peers (collaboration and help, dialog, and explaining).

For participants, leisure time mathematical workshops are an educational leisure time opportunity intended to act as a bridge between informal mathematical experience (notably at home) and school mathematics, that offers at the same time a systematic learning and flourishing path.

The leisure time workshop research methodology (LTWM) (Cogolludo-Agustín et al., 2023) has been developed as a qualitative research methodology inspired by Max Van Manen's approach to research of lived experience (Van Manen, 1990), and by research for practice (Faragher & Clarke, 2014), as it is driven by the attention to developing materials to be applied in educational practice for inclusive mathematics. Raw data includes:

1. Reflexive observation notes by single educators associated to the workshops: mathematics school teachers, special needs school teachers (presently in Spain the so called therapeutical pedagogy (PT) school teachers), both in preschool and primary school; prospective teachers in those areas; mathematics and physics undergraduate students. They complete their notes after the session, based on an observation guide handled to them before the session, which contains some standard points as well as points specific to the single one and a half hour session, regarding both the group and a single child with which each of them is associated.
2. Reflexive observation and research notes by regular or associated researchers (mainly from the University of Zaragoza, Roma Tre University, and Public University of Navarra).
3. Audiovisual data: short videos, photographs.
4. Reports of anecdotes by educators and researchers.
5. Focus groups coordinated by the main researchers.
6. Interviews by researchers to single children, to single families, and to single educators.

Some quantitative tools have been applied, regarding performance tests.

Among the main results of the research: (1) our starting hypothesis on the crucial role of the geometrical objects and relations for the education of children with Down Syndrome has been validated by single child assessment of performance, understanding, and

human flourishing (enjoyment, expression, participation, perseverance, etc.); (2) the actual possibility of involving young children (3–5 years old) with Down Syndrome in mathematics activities has unexpectedly revealed itself; (3) the role of mathematical activities in enhancing oral expression has emerged and is currently under examination; (4) the working hypothesis on the feasibility of developing a wide learning path in the area of mathematics from an early age until the age corresponding to secondary compulsory education has been confirmed, and has been applied to curricular adaptations for ordinary schools; (5) the interest and involvement of families has emerged.

Goal of the study

The study discussed in the present paper regards the last aspect of our research, which has as its main goal an enquiry about the potential role of the family and the home environment for children with Down Syndrome approaching geometrical/arithmetical knowledge, as regards both engagement and performance (learning and flourishing).

SEsDown's families have always shown a great commitment to the mathematical workshops. First, having their sons and daughters attending them regularly: only five children have given up the workshops since 2015, thus strikingly an initial group of six children attended for 10 scholar courses. Moreover, families enquired about the activities including the engagement and performance of their children expressed their satisfaction with the work being done and with the ongoing research. Specific actions have been developed to cope with this, such as special meetings, letters to the families and time devoted to conversation before and after the workshop time.

The experimental setting for the present research design was offered by the exceptional circumstances of the lockdown in Spain from March to June 2020 in response to the COVID-19 pandemic. It consisted of a series of 14 short home sessions, to which children (and their respective families) currently taking part in the 2020 leisure time workshops at Zaragoza were invited, on a regular basis. Each session revolves around a short, simple challenge-activity on number, shape, or space. Single guided activities were suggested to families that could be done at home, with ordinary objects (toys, cooking tools, and clothing) and in different rooms or areas (storage, corridors, hall, etc.). An adaption of the already tested LTMW research methodology was developed for this enquiry, including some quantitative data collection to assess the families cooperative behavior and involvement.

Family math

Since the middle 1980s, educational and psychological research has identified and explored informal arithmetical knowledge that toddlers acquire before being taught and children continue to develop throughout schooling, and its interaction with school mathematics teaching and learning (Baroody, 1987; Duschl et al., 2007; Fuson, 1988; Hughes, 1986; Tolchinsky, 2003). Karen Fuson has described a network of values and use of numbers developed by children 2–8 years old, from which she underlined the relevance of occasional daily experiences, specially at home, to ensure a subsequent deep mathematical understanding. A conceptual network has also been studied for geometrical conceptions (Millán Gasca, 2016, Colella et al., 2017). Awareness of the wide range and diversity of these occasional experiences involving mathematical ideas that children have in family and other non-school contexts (sports, hobbies, scout groups, and other leisure time activities) allows for the design of productive mathematical activities that link the formal and non-formal learning and flourishing in these contexts.

Among informal contexts of mathematical experience and learning, home environment is receiving increasing attention. The so-called family math regards "activities that happen within the context of family relationships, the community, and everyday life that support young children and families to strengthen their math awareness, understanding and confidence" (Eason et al., 2020, p. 5). Two such examples of this are: a Family Math Movement, which has developed in the USA since 2016, involving several organizations (see https://www.education-first.com/familymath/) and Stanford University DREME's (Development and Research in Early Mathematics Education) project (https://familymath.stanford.edu/).

Three main insights from recent research in mathematics education in home environments were considered in our research design: (1) to provide guided activities on spatial skills and patterning (Cheng & Mix, 2014); (2) to build on parents' high expectations and beliefs about the possible role of early informal math experiences and children's later school success (Aunola et al., 2023; Kleemans et al., 2012); (3) to design inclusive strategies suitable for any cultural or socioeconomic family status (Greenman et al., 2011; Jordan & Levine, 2009).

No mention is made of children with learning disabilities and their home math experiences in this research. The study presented in this paper aims to fill this gap following the line of previous studies that highlight the relevance of an environment at home that supports a

good relationship with numbers and geometrical forms for children with Down syndrome (Bird & Buckley, 2011).

The three specific goals of this educational research design are: (1) to develop pseudo-informal mathematical experiences (number, shape, and space) involving home areas and objects to investigate their feasibility; (2) to explore the engagement of parents and siblings in the mathematical activities to identify the features of the attitudes and contributions of parents and siblings (both helpful to or hindering their success); and (3) elicit some aspects of the lived experience of children (Van Manen, 1990) when approaching the pseudo-informal mathematics activities at home, regarding mainly their "flourishing with mathematics" (see Section 1).

DESIGN AND METHODS

In this section, we describe the research design (context, participants, research team), the structure of the experimental setting, the targeted mathematical contents, the instruments developed to organize the 14-session path (including challenge cards, short videoclips as families' feedback and the communication channels, and WhatsApp) and the research methodology (collection of data and analysis).

Context

In March 2020, usual activities in Spain stopped due to the COVID-19 disease and current life changed radically. The population was forced into a hard lockdown and physical presence in every school or leisure time and outdoors activities center was forbidden—including the Spanish association SEsDown Saturday mathematics workshops. The schooling and social life of every child including those with disabilities was severely harmed. Pupils had difficulties in following math lessons online, specially in the case of children with intellectual disabilities (Berástegui, 2020; Daniels et al., 2020; Lillo-Navarro et al., 2022).

The staff of the association SEsDown decided to organize and offer a special proposal at home both to fill the training gap regarding children attending the mathematics workshops and to help families cope with the overwhelming difficulties faced to maintain contact with schools (schools were often almost unable to connect to children with intellectual disabilities). This complementary program or itinerary was named "SEsDown Little mathematicians at home". An effort was made to rely on and at the same time to enhance indoor communication and to recreate at home the relaxed and pleasant atmosphere children were used to in the ordinary workshops, thus offering support in the tough circumstances of lockdown.

Since the research project was conceived at a time of great uncertainty (it was not clear how long the school closures would last), its design was dynamic and not closed from the beginning. It took shape as the confinement lengthened.

Participants and families

The proposal was addressed to the families of 18 regular attendants to the SEsDown mathematical workshops (see below regarding the invitation letter). One family did not accept, and two families did not answer to the offering. Two families who accepted and received materials, did not produced any feedback. To respect their privacy, no further enquiry was made of these families. Thus, those children and their families are not included in the present study.

Data and analysis included in the present study involves 13 children and teenagers with Down Syndrome aged 4–13 (participants from now on), seven males and six females: six children 4–7 years old; five children 8–10 years old; and two teenagers (aged 12 and 13).

All the families consisted of father, mother, and some relatives: no siblings (one family); one sibling (six families); two siblings (three families); more than two siblings (three families with three, four and eight siblings, respectively). All the families lived in the Spanish town of Zaragoza.

All the participants had been regularly attending, throughout the year, the mathematics SEsDown workshops, therefore it was a convenience sample as opposed to a structured selection.

The participants had in the previous months and (most of them) in previous years been offered mathematical experiences involving (Gil-Clemente, 2023):

- Geometrical primordial object and relationships borrowed from Hilbert's (1902) axiomatic presentation of Euclidean geometry such as point, line, straight line, segment, angle, circle, sphere, surface, plane surface, betweenness, to pass through or to lie, part, and congruence.
- Arithmetical primordial objects and relationships borrowed from Giuseppe Peano axiomatic presentation of natural numbers: number 1 and the sequence of counting numbers being one after another, correspondence one to one between numbers and objects, and understanding of cardinality.
- Measurement (integrating arithmetic with geometry).

Both participants and their families understood those sessions as a temporary substitution of the ordinary

leisure time workshops that maintain their spirit. The proposal was thus supported by previous ties with families and participants, by their trust in the SEsDown researchers and educators, and by the need to cope with the extremely hard circumstances of the lockdown for their daughters and sons with intellectual disability.

Research team

The team that supported the research project consisted of: (1) two researchers responsible for the theoretical background and in charge of designing the mathematical challenge-activities and analyzing children's performance (the two first authors of the present paper); (2) a technical research assistant in charge of creating and maintaining the WhatsApp group (the third author of the present paper); and (3) a group of 15 volunteer educators currently volunteering in the SEsDown workshops, that received challenge cards, watched videos and encouraged participants and families from their homes, using WhatsApp messages. The first author led the implementation of the activities, worked closely with the technical professional, coordinated the team of volunteers and carried out the communication with the families firsthand.

Participants and families had a pre-existing acquaintance with the above team prior to the study.

Structure, flow, and mathematical contents

A series of 14 mathematical challenge-activities (designed for 10–20 min activity each) was designed for a 5-week span (Table 1).

The initial contact with the parents was planned through a letter invite via email.

The structure and flow of this itinerary "SEsDown Little mathematicians at home" was supported by a WhatsApp chat as a channel of communication.

Each activity was proposed by means of a challenge card shared by WhatsApp. The challenge cards were delivered almost daily from 23 March to 3 April 2020 (activities 1–9); the delivery was resumed after 12 days Easter holidays until 5 May (activities 10–14), with a slower pace, as the lockdown was eased, and children were allowed to go out for walks.

Parents were asked to record a short video that synthesized the child's work. The basic contribution of families was unavoidable and consisted of three aspects: maintaining the connection channel by carefully reading the challenge cards; organizing the activity at home; and producing videos showing the activity work.

TABLE 1 Little mathematicians at home.

Challenge	Mathematical content
1. Let us count to infinity!	Word number sequence
2. What are numbers for?	Transitive counting
3. Let us move forward!	Straight line
4. We walk around in circles!	Circumference
5. We look for circles	Circumference and circle
6. What can you do with a tube?	Cylinders
7. Earth and sea	Semi-plane
8. How many boxes can you find?	orthohedron
9. Let us count in pairs!	Word number sequence
10. Singing with numbers	Word number sequence Transitive counting
11. How many candies have I got?	Solving little oral problems
12. Add one, subtract one	Solving little oral problems
13. Forming rectangles	Integrating numbers and forms
14. Wool geometry	Plane shapes

Note: Flow of the series of mathematical challenges (March–May, 2020).

Targeted mathematical contents can be roughly divided in three areas:

1. Natural numbers: oral number, word sequence, and counting.
2. Geometrical primitive concepts: point, straight line, and plane.
3. Plane (2D) and solid (3D) shapes: rectangles, circles, and orthohedrons.

Several challenges required the use of house space and objects (the floor, a carpet, the tiles, etc.) and the collaboration of parents and siblings (for finding the objects, for helping them in the play, etc.).

Instruments

We give some details on the instruments and its evolution during the itinerary.

The invitation letter

The invitation letter (signed by the first author) was sent on March 23, 2020, 8 days after the beginning of the lockdown in Spain. It started with some comments on the need to accept the situation with hope and with the evocation of the venue of the previous in-presence workshops and moved on to propose the creation of a

WhatsApp chat to send daily mathematical activity to be done at home, as well as the possibility to record and share experiences. The first activity was put forward at the end of this letter:

"And here is the first activity: count up to the number we know!! It doesn't matter if we skip a number, the important thing is having fun... And if you record your little ones (which I'm sure they will love) and send it to the group or to me, we can encourage each other."

The challenge cards

Each challenge-activity was posed on a so-called challenge card with a common graphic design (see Figure 1): the logo of the association and a title in red that prompted action.

Instructions were brief and short and were addressed to children, suggesting closeness and joy. Nevertheless, since many of the children are not able to read, the complicity of the parents was a necessary condition.

WhatsApp short messages with hints for parents

Some mathematical background for parents was added to each challenge and delivered next to the card using short WhatsApp messages. These hints gave them quick orientation on the mathematical meaning of the activity (see Figure 2, for an example). Family math literature in fact suggests that parents' awareness of what is behind an activity improves the quality of their assistance and supervision.

Videos and messages from families, and circular communication

After acting at home together in reaction to the challenge ("solving" the little mathematical problem), families were asked to record and send back a short video showing the performance of their child. Due to the demanding conditions of the lockdown for families, no guidelines were proposed other than its short duration (from less to 1′ to 2′) and every kind of recording was gathered, also accepting more than one video for a challenge-activity. Videos were intended as the main source of raw data on participant's performance and on parents and siblings' involvement and role in participant's lived experience.

Further, families exploited the WhatsApp group to send their comments on the performance of other participants. This was done, impromptu, during or after their own activity, or after reading and watching messages and videos from other families. This material was considered as raw data. The first author and the educators took part in this exchange following the communication style and criteria of the ordinary workshops.

Research methodology

The LTWM research methodology (see Section 1) was applied for this project, with some modifications and restrictions.

Collected data included only:

- short videos;
- messages from families in the WhatsApp chat.

The following aspects were considered for qualitative analysis of videos sent in response to each challenge: (a) participant's body gestures, including expressions; (b) participant's words; (c) parents' and siblings' comments; (d) places where the videos were recorded; (e) presence or absence of family members in the challenge activity. This analysis was linked to the three goals of the research, and was the main tool for goal (3).

Moreover, a quantitative analysis of number and distribution of videos for single challenge-activity or for single family, as well as of percentage of interactions in WhatsApp messages was carried out. These tools were designed both to assess the itinerary and the single challenge-activities as experienced and felt by participants and families (goal 1), and to measure the families involvement (goal 2).

FIGURE 1 On the left, example of challenge card (in Spanish); on the right, our translation.

Ethical review and approval

Ethical review and approval were waived for this research, it being an educational study in the field of Didactics of Mathematics and Special Needs, developed by the SESDOWN Association, legally established according to Spanish legislation, whose statutes include, among others, the following objectives: (1) the research and development of studies on Down syndrome, intellectual disability and functional diversity from an integrated and interdisciplinary perspective; (2) the dissemination of studies and scientific advances made on Down syndrome, intellectual disability, and functional diversity.

The present study aligns with the Helsinki declaration: written informed consent is obtained from the parents of children participants, as their legal representatives at the beginning of each academic year. This consent includes the record and use of images of the development of the activities and research findings for both research and dissemination purposes and the subsequent publication in national and international journals.

RESULTS AND DISCUSSION

We describe evidence regarding two aspects: the performance of participants (learning and flourishing, see Section 1) and the involvement of parents and siblings and its impact on the family lived experience.

Families involvement

Videos

The total number of recorded and sent videos was 89 (Figures 3 and 4). Let us consider Figure 3 for assessment of the impact of single challenge-activities and the flow of the lived experience. A mean of 6.3 ($S = 2.4$) videos per challenge was received. The maximum number of videos was received for challenge-activity 10 (singing with numbers, 11 videos), no activity received only one video from each participant. Further, for challenge-activities 2 and 13 (respectively in the first and second phase of the itinerary) 10 videos were received. Challenge-activities 8 (How many boxes do you find?) and 14 (wood geometry) received the minimum of three videos, perhaps due to the date they were delivered.

The evolution in time shows some fatigue in the second phase, after the lockdown conditions became lighter, if compared with the regularity of numbers in the first phase (around eight videos for each challenge-activity), but with peaks for challenge-activities 10 and 13.

Let us consider Figure 4 for identifying levels of intensity in involvement.

The number of videos and their regular pace is helpful to explore the engagement and awareness of families. The group of youngest children (6 of 13 participants) showed the greatest involvement with a 65% of the videos sent. Three families of this group, sent more than

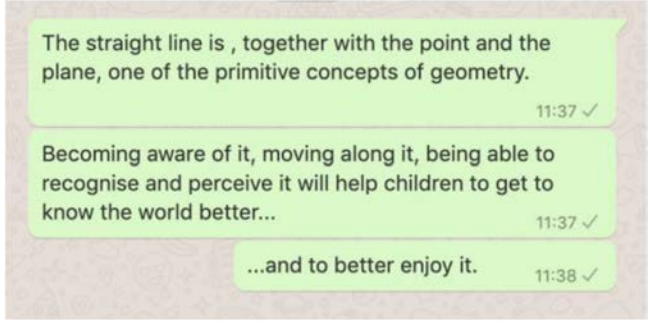

FIGURE 2 Mathematical hints for parents added to challenge 3 (in Spanish in the original, our translation).

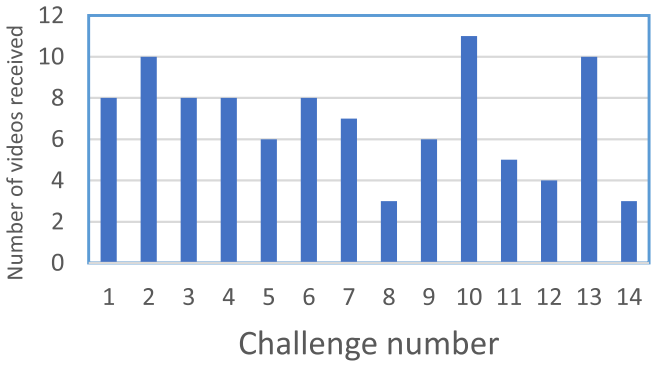

FIGURE 3 Number of videos received for each challenge.

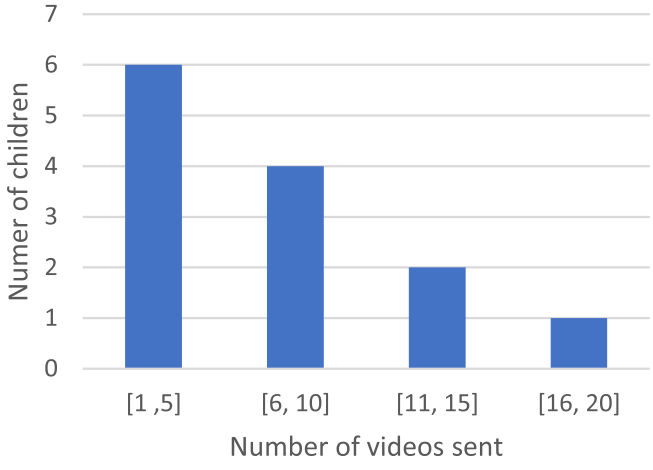

FIGURE 4 Total number of videos received from single families.

10 videos each, and the maximum of 19 videos was reached by the family of a 5-year old child. The families of children over 8 years age, except one, sent a lower-than-average number of videos.

The following remarks are derived from the qualitative and phenomenologically inspired analysis of the videos regarding the goal (1) and (2) of the research:

1. The use of home environment was a key for the success of activities, because it endowed the activities with an air of freedom and festivity. For instance when children are encouraged to count by themselves in challenges 1 and 2, it is frequent to see them in a sofa, or in their own bed and the choice of this place makes them more confident with the task, allowing them even to balance on the seat to accompany the reciting. It is also frequent to see children facing the challenges proposed in the floor, where they can feel free to extend their toys without fear. It is significant that in the challenge they are asked to walk in a straight line, all of them choose the corridor, the place in a house more similar to a linear path.
2. Families put their imagination in action to choose objects at home to perform these activities. The choice of everyday objects (card games, animal toys, sweets, fairy tales, even a calendar, etc.) is common when they are ask to count objects or to pose arithmetical problems. In this way these objects prompt them to perform these tasks. In the geometrical tasks (specifically those involving circles, rectangles, polygons, cylinders, and orthohedron) a great variety of daily objects appear too: ribbons to make polygons, buttons, and yoghurts configured as rectangles, toolboxes for counting corners, and at last biscuits, drums, clocks, and plates as examples of objects where a circle is found.
3. The challenge activities were suitable to be done at home because they were easily related to everyday actions: walking a straight path, standing in one location waiting for something, jumping for counting, dividing the floor of a room into two parts in order to represent a pirate's tale, etc. (Figure 5).
4. The participation of parents and siblings, as recorded in many videos (Figure 6), the expressions we can listen to in these videos, contributed to recreate the atmosphere of confidence and joy lived each Saturday in SEsDown mathematical workshops in every short session at home and reinforced a sense of community among educators, participants and families, that is crucial in those workshops (see Section 1).

   In many videos we can see siblings involved in the activities and in others we can listen the voice of parents, mostly mothers (never their image) showing their participation.

   There are several types of interventions: (1) interventions of encouragement (come on, well done, great, that's the way to do it sweetie, you have formed a triangle, ole...); (2) interventions of help and guidance (how many dinosaurs do you have? and if you put one more candy?, let's count again, look at your brother and keep an eye on him, take the ribbon with your hands, this drum is shaped as a, etc.); (3) interventions that make the child interact with the other children at distance (show your friends what you are doing, say goodbye to everyone, send them a kiss, show everyone your circles, etc.). It is noteworthy that it is never

FIGURE 5 Still images from videos recorded for challenges 3 and 7, respectively.

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FIGURE 6 Still images from videos recorded for challenges 8 (left) and 4 (right).

heard a family member making a negative assessment of the child's performance and that the common tone of the voices heard is soft, cheerful, and affectionate. For instance we can even listen to several siblings shouting with joy while walking together around the perimeter of a circular object.

Interactions in the WhatsApp group

Considering all the contributions to messages in the WhatsApp chat (the first author, educators, parents, and some of the eldest participants prompted to participate with some comments addressed to their friends), more than half of the interactions came from families (Figure 7).

Considering the evolution in time, the WhatsApp group was initially intended only as a communication channel

- to send the challenge cards and receive the videos;
- to facilitate an accompanying interchange between families and the research team for mainly practical purposes, soft introduction to a new scenery, and some encouragement.

The WhatsApp chat acquired many further uses, added to the research design, implemented by the first author (Figure 8, see Figure 2):

- mathematical and educational hints to parents aid understanding and acting in reaction to the challenge cards
- personal comments to each child who sent a video
- general comments when something interesting before, during or after the challenges was observed. Educators limited their interventions to encouraging and congratulating children's participation. The fact that the children knew them before was an important factor in this animation commitment

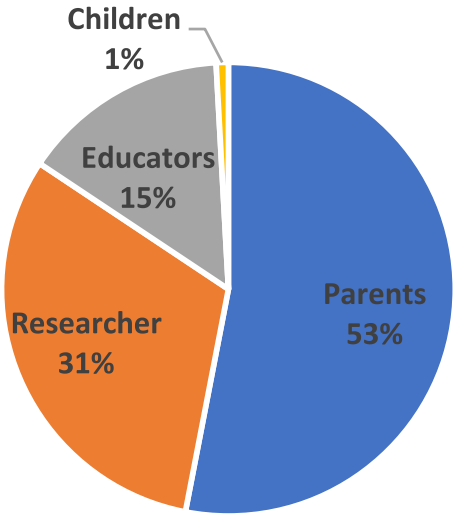

FIGURE 7 Percentages of messages in the WhatsApp group (March–May 2020).

Moreover, remarkably, although they were not asked to do so (but only to record and send videos), families progressively decided to encourage all the other participants and families with WhatsApp messages. We list the main areas:

- sharing daily experiences with the rest of the parents further than only through videos, thus with a more reflexive nuance
- joy and appreciation of other participants and families performance
- expression of gratitude for all the things they had learned with their children (see Figure 9) and confirmed the idea that family can support children's engagement in mathematics if they are appropriately supported

In time, the general evolution in the chat was from feelings of annoyance, because children did not want to participate in the challenges ("today he didn't want to hear about squares and rectangles"), to expressions of

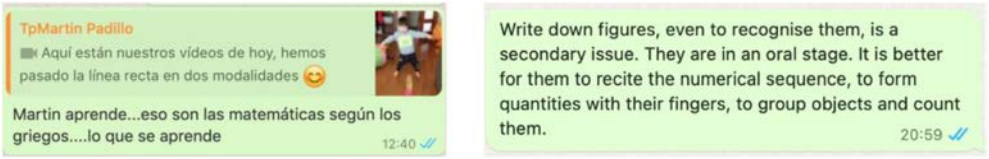

FIGURE 8 Example of individual comment (on the left) and a general comment (on the right) after receiving the videos corresponding to a challenge card. (The originals are in Spanish. Translation of the dialog on the right: Parents: "Here are our videos, today. We have walked the straight line in two different modes". Researcher: "Martin is learning... these are mathematics according to the Greeks, what it is learnt").

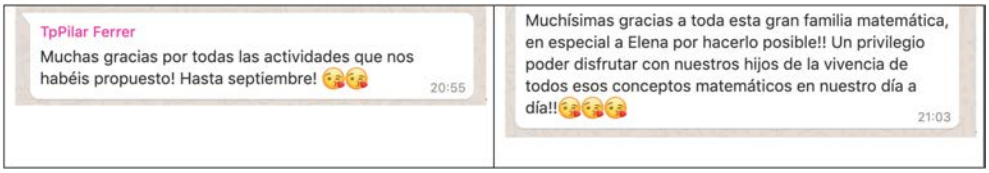

FIGURE 9 Parents express their gratitude at the end of the project: two examples. (Translation into English "Thank you very much for all the activities proposed. See you in September!" (on the left); "Thank you very much to all this big mathematical family, specially to Elena for making it possible! It is a privilege to be able to enjoy with our children the lived experience of all these mathematical concepts in our daily life..." (on the right, our emphasis)).

pride for their performance ("step by step we are getting results!"). It is current to read mothers were worried about the performance of their own children but encouraging other children. There were specifically two mothers who showed personally this group evolution.

Altogether, it is possible to notice a clear evolution in the role of families from a simple and ineludible help for following the home itinerary for participants and execution of the activities to a deep engagement, fascination and understanding. Parents were initially hidden behind their children as main stars in the videos, while in the messages parents came to the foreground with individual communication style and cultural or relationship approaches. The conditions of lockdown both were an obstacle and acted as motivation for parents.

Let us turn to the results regarding participants' performance, that can integrate the above-mentioned results, to consider the role of such an involvement, in combination with the home pseudo-informal activities, in enhancing the mathematical experience of children and teenagers with Down Syndrome.

Participants' performance

From the qualitative and phenomenologically inspired analysis of the videos by the research team, the following results can be listed, regarding the lived experience of the children (Van Manen, 1990) when approaching the pseudo-informal mathematics activities at home, regarding mainly their "flourishing with mathematics" (goal 3).

Notwithstanding diversity in family contexts (number and age of members, availability of spaces and objects, including toys), engagement emerged both in videos and in messages referring participant's words and graphical productions. The research team was aware of their expressive register (including facial expression, body gesture, and linguistic codes), as all of them were regular attendants to the ordinary workshops. The participants enjoyment in the videos was evident. Of course the video has a bias, as each family chose what images to share with the educator, the whole research team, and the other families (Figure 10)

1. Participant's behavior in the videos shows their approval of the challenge activities as suitable for them and for the lockdown situation, by taking the action very seriously.
2. A spirit of play is a constant in almost all the videos sent. Children play with objects, with parents, with siblings. These games are not devoid of challenges that children face with a sense of fun, and a certain amount of struggle as their bodily expressions, well known to researchers, show again. (Figure 11).

FINDINGS AND CONCLUSIONS

We present our findings and main conclusions regarding family math in the case of children with Down Syndrome, in four steps.

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FIGURE 10 Still images from videos recorded for challenges 1 and 2, respectively.

FIGURE 11 Still images from videos recorded for challenges 13 and 6, respectively.

Suitability of the home environment for the development of informal mathematical activities

We have reformulated and extended some of the established ideas about the role of the family in mathematics education. Firstly, children live and interpret life in the context of the family. Mathematics is a way in which human beings give meaning to their experiences with the world and families are a privileged place to do so because of the great range of opportunities it offers. Secondly, in a family context children face problems and solve them. Implicit within mathematics is the capacity to cope with situations that demand an answer and to respond to them and family is a good setting to practice, allowing children to act more confidently than in the school context. Thirdly, in a family context children communicate freely. Mathematics is also a language, in which words have their own meaning and the family is a place where members share a special way to communicate.

We have offered evidence that parents and siblings may have a role in helping children explore their surroundings confidently (their room, other places in the house, etc.), understand the requests (with their close explanations, using body language, etc.) and increase their awareness (the voice of parents is heard in many videos making children conscious of what they are doing). In our experience children with Down syndrome are eager to communicate their discoveries about numbers and shapes and look for the better way of doing it.

Guided informal mathematical experiences at home are a step to equity in providing access to formal education for children with intellectual disabilities

We confirmed the idea that family can support children with Down Syndrome's engagement in mathematics if they are appropriately supported. The study described

above is an example of an itinerary of guided activities for parents and siblings, involving mathematical contents beyond numeracy, in line with the suggestions made by the USA Family Math Roadmap (2018) (see Section 1).

The need for inclusive strategies to bridge the gap between families of different cultural contexts in accessing school is currently emphasized (also because of the circumstances of lockdown in many countries). Children that have not the opportunity to access informal mathematical experiences before schooling will be at a disadvantage, as much as children who have not had access to a rich linguistic environment. We have broadened this sense of inclusiveness to families with children with intellectual disabilities who may suffer from a lack of informal experiences with mathematics that can be compared with that experienced by children from disadvantaged backgrounds: in fact, as it is assumed that children with intellectual disabilities will have difficulties in understanding mathematical ideas, they are often deprived of access to them (Monari-Martinez, 2002). This is not the same for reading, because usually parents, not only those who have children with intellectual disabilities, relegate mathematics to a backseat compared to reading at home (Cannon & Ginsberg, 2008). The lack of informal experiences with mathematics acts as a kind of self-fulfilling prophecy (Zimpel, 2008) and is the first step towards a poor future relationship with the discipline (Skwarchuk et al., 2014). The experience described acts as a bridge between the informal knowledge children build at home and school tasks. Challenges proposed were on the one hand guided, but on the other hand they allowed a space of freedom to be developed by the child in his or her family and non-school environment.

A formative approach to mathematics is possible at home

Engagement and an especially joyful approach to mathematics emerged in the videos also as a consequence of focusing on challenges and play, instead of routine tasks, usually linked to school mathematics.

2020The general framework of the study (see Section 1) focuses on the educational value of mathematics. Math performance of children with Down syndrome is not pursued as a unique goal. The aim is to show how exposure to mathematics contributes to their personal growth or human flourishing. Exploration, understanding and awareness were also the educational goals of the itinerary "Little mathematicians at home". The inclusion of activities related to geometry also assured this broadening of the focus of mathematics. Mathematics has shown its power to engage children with intellectual disabilities despite the physical distance that separated the educators who designed the activities and the children.

For describing the achievements in terms of human flourishing, (Su, 2020) we pick four aspects: play, struggle, freedom and community. In the days of lockdown, participants and their families played. They played with math, driven by a great curiosity. Play is a genuinely human activity (Huizinga, 2004) and in this sense the itinerary contributes to the personal growth of the participant families. Participants struggled, because all the activities proposed were formulated as challenges. And they built further self-confidence through struggle. Activities were not easy for them sometimes, but with practice and a bit of help, they succeeded. And this success made them grow. It was a series of free, voluntary activities and this freedom made families and participants comfortable. No one was going to judge them, they could send the videos when they wanted, there was no test at the end. This freedom is quite the opposite to the anxiety and fear that mathematics raises on too many occasions. And at last, community. Mathematics is a world that welcomes everybody, also children with intellectual disabilities often excluded from this world. The activity has contributed to create a supportive community where they play and struggle together and thus they have experienced the real satisfaction of learning and sharing. The experience of our diversity transforms it in a value for society.

Qualitative data from parents show that they had developed an advanced outlook on the nature and educational role of primary mathematics beyond numeracy prior to this proposal of indoor mathematics. They also show high expectations from their daughters and sons. The open, curious attitude of families towards mathematics and their high expectations for their child's achievements made possible an atmosphere of play and enthusiasm during the activities, just as teacher's self-confidence supports a safe and productive learning environment in mathematics classrooms.

Some clues about the using of technology in the mathematical education of children with intellectual disabilities

Being recorded was essential for participants involvement, helping to save the distance among all of them. They all wanted to appear in a video showing their friends what they were doing. Videos made them feel closer in a moment when they were locked at home and this was invaluable. Videotaping the activities was a

success factor, both for the additional encouragement for participants and families and because of the help they provided for collective learning. This success opens the path to the possibility of using video as a tool of online learning for children with intellectual disabilities.

The possibility of using the material (text, video, and graphic production) in the training of prospective teachers has been an unexpected result of the experience. Short edited video clips capturing the mathematics underlying in children's everyday experience is a useful tool for the training of prospective teachers working with children with intellectual disabilities: looking at children enjoying mathematics and being conscious about the way in which mathematics has a role in their "being in the world" impacts in their training, because it both changes their views on mathematics and contributes to support the trust that it is possible to teach math in a pleasant and effective way.

This experience encourages our research team to continue the work exploring the possibility of using videos as a tool for online mathematical education of children with intellectual disabilities.

This paper presents an investigation regarding pseudo-informal mathematical experience in the home environment for children with Down syndrome developed during the lockdown lived in Spain in 2020 during the COVID-19 pandemic. The investigation was designed as part of an ongoing research project regarding mathematics in the upbringing of children with Down syndrome, based on LTMW (leisure time mathematics workshops) research methodology. Because the involvement of parents in the mathematical education of their children has emerged as a crucial aspect to be considered in our ongoing project, the lockdown resulting from the sanitary emergency make it possible to explore it further, by designing a series of challenge-activities to be "solved" at home. The structure, flow, and contents of the proposal was developed in continuity with leisure time math sessions that children and parents were familiar with. Moreover, we were inspired by recent research for practice on Family Math regarding informal mathematical experience.

The description we have made of the experience lived by children when approaching mathematics in the informal context of their homes, based on videos, shows that family is a suitable environment for mathematical experience for children with Down syndrome, possibly enhancing formal, and school mathematical learning.

The exploitation of home experiences regarding number, shape, and measure in activities involving body motion and use of daily materials—instead of routine tasks—appear key to the success of the simple and short challenge cards, and show the path for future work. The investigation confirms the efficacity of a change of perspective on the mathematical education for people with intellectual disabilities. Moreover, giving up an approach centered in functionality and utilitarian aspects and adopting a vision considering mathematics as a manifestation of our human kind, enhances family commitment. Working on the family environment can be helpful in easing the inclusion of children with intellectual disabilities in the school system and thus in society.

AUTHOR CONTRIBUTIONS

Elena Gil-Clemente and **Ana Millán Gasca** conceived the presented approach and developed the theoretical framework. **Elena Gil-Clemente** led the experimental activities and collected the data. **Rebeca Paricio Badías** contributed to the methodologies and use of software. All authors actively participated in the discussion of the results, reviewed and approved the final version of the work and agreed to the published version of the manuscript.

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

ETHICS STATEMENT

Ethical review and approval were waived for this study, because it is an educational study, in the field of Didactics of Mathematics and Special Needs, developed by the SESDOWN Association, legally established according to Spanish legislation, whose statutes include, among others, the following objectives: (1) The research and development of studies on Down syndrome, intellectual disability and functional diversity from an integrated and interdisciplinary perspective. (2) The dissemination of studies and scientific advances made on Down syndrome, intellectual disability and functional diversity. However, our research aligns with the Helsinki declaration and informed consent was obtained from all subjects involved in the study. Written informed consent has been obtained from the families participants to record and use of images of the development of this activity for both research and dissemination purposes. This permission includes the subsequent publication in national and international journals.

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