Universidad Zaragoza

Academic Year/course: 2023/24

27013 - Geometry of Curves and Surfaces

Syllabus Information

Academic year: 2023/24
Subject: 27013 - Geometry of Curves and Surfaces
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 10.5
Year: 3
Semester: Annual
Subject type: Compulsory
Module:

1. General information

This course introduces the basic concepts of differentiable curves and surfaces in the plane R^2 plane and the space R^3. We will study properties that depend on their inclusion in the ambient space, such as curvature, torsion, or Frénet reference systems. Intrinsic properties such as curve length, surface area, fundamental forms, Gaussian curvature, and Euler characteristic will also be studied.

The approaches and objectives of this module are aligned with the Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda; the learning activities could contribute to some extent to the achievement of the goals 4 (quality education), 5 (gender equality), 8 (decent work and economic growth), and 10 (reducing inequality).

2. Learning results

- Recognize the nature of the points of a curve in R^2 and R^3.
- Calculation of the dihedron and the Frenet trihedron and the curvature and torsion. Fundamental theorems.
- Use the first fundamental form of a surface to solve problems about lengths, angles, and areas.
- Use the second fundamental form of a surface to recognize the nature of its points. Know its relation with the Gauss map. Know how to calculate, apply and interpret the principal, Gaussian and mean curvatures.
- Understand intrinsic geometric properties: covariant derivative, Gauss's Theorem Egregium, geodesics and Gauss-Bonnet Theorem.
- Understand the difference between local and global problems.

3. Syllabus

1. Regular plane curves. Frénet's frame, tangent and normal vector fields along a curve, curvature, arc length. Contact theory. Fundamental theorem for plane curves.
2. Biregular spatial curves, Frénet frame (tangent, normal and binormal fields), arc length, torsion, curvature, evolute. Fundamental theorem for spatial curves. Local canonical form.
3. Regular surfaces. Local theory: 2-function graphs, charts and regular values of 3-functions. Examples. Parametrized surfaces. Curves in surfaces and tangent plane. Charts, coordinate vector fields, change of charts.
4. Differentiable functions and maps. First fundamental form: lengths, angles and areas. Orientations.
5. Geodesic and normal curvature. Second fundamental form and Gauss map. Types of points, principal, normal and Gauss curvature. Principal directions, asymptotic curves, umbilic points.
6. Intrinsic geometry. Covariant derivative and Gauss Theorema Egregium. Isometries, conformal maps and isothermal coordinates. Geodesics and exponential map: distance and convexity. Gauss-Bonnet theorems.

4. Academic activities

Master classes: 75 hours.
Problem solving: 20 hours.
Computer classes: 10 hours.
Study: 150 hours.
Assessment tests: 7.5 hours.

5. Assessment system

- The written tests have a weight of 60% and a minimum grade of 4/10 is required to pass.
- Computer labs have a weight of 20%.
- The continuous evaluation will have a weight of 20%.
- The student will be able to take an partial test at the end of the first semester. A minimum of 4/10 is required in order to allow averaging with the second partial test of the class.
- In the official calls, the student can decide whether to take a partial test (for the second part of the class) or a final test (both parts of the class). In any case, a minimum of 4/10 is required for each partial test in order to allow averaging.
- The evaluation of the labs will be carried out as follows:
    - a 25% (5% of the total) for attendance and completion of computer labs in class;
    - another 25% (5% of the total) for the evaluation of the problems to be collected;
    - a 50% (10% of the total) for the evaluation of the computer lab's exam.
    - Students who have passed the labs in previous years may choose between maintaining the previous grades or completing the labs again.
- The continuous evaluation will consist of the presentation of problems in class, the completion of assigments or turning in work requested by groups of students on particular topics.

Students who wish to do so may only take a global test in order to assess the acquisition of the class requirements.