Universidad Zaragoza

Academic Year/course: 2022/23

27010 - Linear Geometry

Syllabus Information

Academic Year: 2022/23
Subject: 27010 - Linear Geometry
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 6.0
Year: 2
Semester: First semester
Subject Type: Compulsory
Module:

1. General information

1.1. Aims of the course

This subject and its syllabus have the following goals:

Basic notions and results of a geometric nature are introduced using the previously acquired knowledge of linear algebra. The fundamental concepts of affine and Euclidean geometry are introduced, with a special focus on the plane and space. This will be especially useful in order to recognize geometric objects and their properties in other areas of Mathematics and Sciences, as well as in various areas of everyday life, art, architecture, etc.

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (https://www.un.org/sustainabledevelopment/es/), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

1.2. Context and importance of this course in the degree

This subject belongs in the linear algebra section. This class will provide the student a geometrical interpretation of the abstract concepts from linear álgebra. This will be useful in a variety of other subjects throughout the degree.

1.3. Recommendations to take this course

It is recommended to have completed the module *Algebra Lineal* (27000).

The student is encouraged to attend and actively participate in the lectures, problem sessions, and labs, as well as to complete the individual and group work load. The use of office hours is also highly recommended. Their schedule will be announced at the beginning of the course.

2. Learning goals

2.1. Competences

Upon succesfully completion of this subject the student will improve the following abilities:

- Carry out the goals described in section 2.1.
- CT1. Be able to clearly state, both orally and in writing, the student's reasoning, problem solving techniques, reports, etc.
- CT3. Differentiate between essential and non-essential aspects of a problem, state conjectures and be able to provide reasons to either prove or refute them, identify errors or gaps in incorrect argumentations, etc.

2.2. Learning goals

In order to pass this class, the student should be able to show the following skills:

- Operate with points, vectors, distances, and angles both in affine and Euclidean spaces, as well as the

corresponding reference systems, subspaces, and their transformations.

- Know how to solve geometrical problems on the plane and space.
- Classify plane and space isometries determining their type and characteristic elements.
- Classify conics and quadrics and determine their intrinsic elements.

2.3. Importance of learning goals

The learning objectives provide basic skills within the degree. (See Context and reasons behind the subject area in the degree).

The fundamental concepts of affine and Euclidean geometry are introduced, with a special focus on the plane and space.

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The students must demonstrate that they have achieved the learning objectives by means of the following evaluation activities:

Evaluation throughout the course (10%): classwork, lab exercises and individual or group projects.

Final exam (90%).

In addition, according to current bylaws, a student also has the right to show up to a final exam and complete the class upon passing the test.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, problem-solving sessions, laboratory sessions, tutorials and autonomous work and study.

4.2. Learning tasks

This course is organized as follows:

- **Lectures.** Three weekly hours. Notes, solutions to problems, and other forums are available for students to interact with their instructors in Moodle.
- **Problem-solving sessions.** One weekly hour in small groups. Students are afforded the opportunity to submit individual homework assignments on a weekly basis. These assignments are checked by the instructor and returned on a regular basis. This process allows students to pinpoint strengths/weaknesses and helps their learning process.
- **Laboratory sessions.** Involving the use of computers. SageMath will be used (https://sage-mtm.unizar.es/)
- **Tutorials.** Students will be attended by the teacher at office hours.
- **Autonomous work and study.**
- **Final exam.**

The teaching activities and assessment tasks will take place in a face-to-face mode, except in the case that, due to the health situation, the dispositions emitted by the competent authorities and by the University of Zaragoza compel to take them to a greater or lesser extent in a telematic form.

4.3. Syllabus

- **Topic 1: Affine spaces.** Definition of affine spaces. Examples. First properties. Linear varieties and affine subspaces. Relative position of affine subspaces. New affine spaces from old. Grassmann formulas. Affine references and coordinates. Barycenter. Simple ratio. Thales, Ceva, and Menelao Theorems. Exercises.
- **Topic 2: Affine morphisms.** Definition of affine morphisms. Properties of affine morphisms. Examples. Homologies on the plane. Affine morphisms in coordinates. Invariant affine subspaces. Exercises.
- **Topic 3: Euclidean affine spaces and motions.** Euclidean affine space. Pythagorean Theorem. Distance between linear varieties. Definition of Euclidean motions. Examples. Gliding vector. Classification of Euclidean motions. Motions on the plane and three-dimensional space. Geometric interpretation. Exercises.
- **Topic 4: Quadrics and affine classification.** Equivalent quadratic polynomials. Invariants and classification

theorems. Affine classification of conics. Three dimensional quadrics. Quadrics with center. Exercises.

4.4. Course planning and calendar

Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or please refer to the Faculty of Sciences website (http://ciencias.unizar.es/) and Moodle.

4.5. Bibliography and recommended resources

Basic reference:

- A. Reventós, Affine maps, Euclidean motions and quadrics, Springer 2011.

Complementary references:

- Manuel Castellet, Irene Llerena. Algebra lineal y geometría. Reverté, D.L. 2000
- Luis M. Merino González, Evangelina Santos Aláez. Álgebra lineal: con métodos elementales. Los autores, D.L. 1999
- Hernández Rodríguez, Eugenio. Algebra y geometría. Wilmington, Delaware; Madrid. Addison-Wesley Iberoamericana. Universidad Autónoma, cop. 1994

http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27010