Universidad Zaragoza

Academic Year/course: 2022/23

27034 - Functional Analysis

Syllabus Information

Academic Year: 2022/23
Subject: 27034 - Functional Analysis
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 6.0
Year: 4
Semester: First semester
Subject Type: Optional
Module:

1. General information

1.1. Aims of the course

It is an optional course in the degree. The aims are the knowledge and control of the techniques in analyisis which are deeply related to algebra and topology (and, partially, to geometry) that allow a projection into many other areas of study in mathematics and other disciplines.

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (https://www.un.org/sustainabledevelopment/es/), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

1.2. Context and importance of this course in the degree

This is a course in the first semester in the fourth course of the Mathematics degree, which belongs to the module *Widening of mathematical analysis* together with the mandatory course in the first semester *Lebesgue integral* and the optional course in the second semester *Fourier analysis*. The three courses are deeply related.

The students will take this module after having taken the module *Initiation to mathematical analysis*, which includes the courses *Mathematical analysis I* in the first course, *Mathematical analysis II* in the second course and *Complex analysis* in the third course.

The course Functional analysis, as well as the other courses in the module *Widening of mathematical analysis,* represent a last stage in the basic and general training of a graduate in Mathematics, in the topics related to mathematical analysis.

1.3. Recommendations to take this course

It is specially urged to have passed the module *Initiation to mathematical analysis* and be enroled in the course *Lebesgue integra*l. Besides, it is convenient to have taken *Functional analysis* in the first semester if one wishes to take the course *Fourier analysis* in the second semester.

Attend continuously, and paying attention, to the theoretical and practical lectures.

Work with the material delivered by the instructors in a continuous way.

Make good use of the office hours, whose exact schedule will be delivered at the beginning of the course.

Students who cannot attend the lectures should communicate their situation to the instructors.

2. Learning goals

2.1. Competences

BASIC AND GENERAL:

- CG1 - Possess and comprehend knowledge in the area of mathematics on a level that, starting in the training acquired in the general secondary education, relies on advanced texts and includes some aspects that imply knowledge coming from the vanguard in the study of mathematics.

- CG2 - Know how to apply the mathematical knowledge to work in a professional way and own the competences that are shown by the resolution of problems in the area of mathematics and its applications.
- CG3 - Have the capacity to gather and interpret relevant data, particularly in the area of mathematics, in order to express judgments, using the capacity of analysis and abstraction, which include a consideration about relevant topic of a social, scientific, or ethic nature.
- CG4 - Have the ability to communicate, in an oral or written manner, information, ideas, problems, and solutions in the mathematical scope to both a specialized and onn-specialized audience.
- CG5 - Have developed those learning skills needed to pursue further studies in mathematics with a high degree of autonomy.

TRANSVERSAL:

- CT1 - Know how to express clearly, both in an oral and written manner, reasonings, problems, reports, and so on.
- CT2 - Learn new knowledge and techniques in an aoutonomous way.
- CT3 - Distinguiss, when facing a problem, the substantial to the accessory, formulate conjectures and reason to confirm or disprove them, identify mistakes in incorrect reasonings, and so on.
- CT4 - Work in teams, both interdisciplinary and restricted to the scope of mathematics, taking part in the discussions that arise.
- CT5 - Know how to obtain effective information through bibliographic and informatic resources.

SPECIFIC:

- CE1 - Comprehend and use the language and mathematical methods. Know rigurous proofs of the basic theorems in the different branches of mathematics.
- CE2 - Propose, analyse, validate, and interpret real and simple situations models, using the most adequate mathematical tools to the pursued ends.
- CE3 - Solve mathematical problems by means of basic calculus and other techniques.
- CE6 - Use search tools of bibliographic resources in mathematics and use those resources in modern languages, specially English.

2.2. Learning goals

Students will reach a good comprehension of mathematical analysis in its deep connection with algebra and topology, culminating in this way the vision of analysis in the degree in mathematics. In particular, they will get to

- Know the analytic and geometric forms of the Hahn-Banach theorem and its main consequences.
- Comprehend what completeness implies in relation to normed spaces, continuous and linear maps in this kind of spaces, and the spaces with a scalar product.

2.3. Importance of learning goals

They provide a basic training to deepen into topics in mathematical analysis. They allow to comprehend the disciplines that motivated, mainly, the development of functional analysis. quantum mechanics and integral and differential equations.

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The assesment will be made by a continuous evaluation system, which will consist of 4 tests, which will be rated, each one of them, over 25 points. The final mark, over 100 points, will be the sum of the marks obtained in these tests. The course will be passed with a final mark of 50 or highher.

The date of each one of these 4 tests will be fixed early enough and, in case that they need to be done out of the regular lecture hours, it will be guaranteed that all the students can take them.

Each one of these tests will consist both in theoretical questions, which will consist on questions about definitions or proofs of results seen in class, as well as practical exercises, which will consist on the resolution of exercises similar to those treated in the lecture room and in the material provided by the instructor.

The criteria in the assesment will take into account the ability to provide precise definitions and correct proofs of the main results treated in the course, as well as the ability to solve different problems, in a correct way and relying on the results and definitions seen in the course.

The students will have the right of taking a global exam, in the dates of the official convocations, fixed by the Faculty of Science.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

Master classes with theoretical concepts and results, and model exercises.

Problem sessions to practice and settle theoretical results and model exercises.

Proposed problems for personal student's work.

Use of Moodle to provide material and ease comminication.

Volunteer individual tutoring.

4.2. Learning tasks

This course will include the following learning tasks:

- Master classes with theoretical concepts and results and model exercises.
- Problem lectures in order to practice and settle the theoretical concepts and results.
- Proposed problems for personal student's work.
- Volunteer individual tutoring.
- Assesment tasks. Several continuous evaluation exams will be taken during the lectures period as well as a final global exam for those students who will not pass the course by means of the continuous evaluation.
- Use of the platform Moodle (https://moodle.unizar.es/add/) to provide the material.

The teaching activities and assessment tasks will take place in a face-to-face mode, except in the case that, due to the health situation, the dispositions emitted by the competent authorities and by the University of Zaragoza compel to take them to a greater or lesser extent in a telematic form.

4.3. Syllabus

1. Normed and banach spaces.
2. $L_p(\mu)$ spaces.
3. Modes of convergence of functions sequences.
4. Hilbert spaces.
5. Spectral theory of compact self-adjoint operators in Hilbert spaces on C.
6. The fundamental theorems of functional analysis: The Hahn-Banach theorem, the open mapping theorem, and the Banach-Steinhaus theorem.

4.4. Course planning and calendar

Four weekly hours of face-to-face lecture will be taught during the whole semester.

Four continuous evaluation tests will be taken during the semester. The dates will be fixed, together with the students, early enough.

There will be a final global exam for those students who did not pass the course by means of the continuous evaluation, whose date will be fixed by the Faculty od Sciences.

The final exams period, and the precise dates of those,as well as the general academic timetable, can be consulted in the Faculty of Sciences webpage: https://ciencias.unizar.es/calendario-y-horarios.

4.5. Bibliography and recommended resources

- Análisis funcional / Bernardo Cascales Salinas... [et al.] Murcia : Electrolibris ; [Madrid] : Real Sociedad Matemática Española, D.L. 2013.
- Rudin, Walter: Análisis real y complejo / Walter Rudin ; traducción José María Martinez Ansemil . - 3a. ed. Madrid[etc] : McGraw-Hill, cop.1987.
- Conway, John B.: A course in functional analysis / John B. Conway New York : Springer, 1985.
- Rudin, Walter: Functional Analysis, McGraw-Hill, 1973.
- Meise, R. y Vogt, D.: Introduction to Functional Analysis, Oxford Sci. Pub.,Clarendon Press, 1997.
- Horvath, J.: Topological Vector Spaces and Distributions, Addison Wesley, 1966.

http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27034