Universidad Zaragoza

Academic Year/course: 2022/23

27041 - Differentiable Manifolds

Syllabus Information

Academic Year: 2022/23
Subject: 27041 - Differentiable Manifolds
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 6.0
Year: 4
Semester: First semester
Subject Type: Optional
Module:

1. General information

1.1. Aims of the course

This subject and its syllabus have the following goals:

Introduce the student to the main results and tools in differential geometry that allow the study of a large class of spaces, which have also applications to other areas of mathematics and physics. The main objects to study are the differentiable manifolds and the smooth maps.

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (https://www.un.org/sustainabledevelopment/es/), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

1.2. Context and importance of this course in the degree

This subject is part of the módulo Ampliación de Geometría y Topología (Higher Geometry and Topology).

It is recommended that the student is familiar with results and techniques from algebra, analysis and topology, mainly those provided in Topología General and Geometría de Curvas y Superficies.

1.3. Recommendations to take this course

As mentioned in section 1.2, students are recommended to have acquired the competencies associated with the módulo Fundamentos de Geometría y Topología (Fundamentals in Geometry and Topology).

2. Learning goals

2.1. Competences

Upon succesfully completion of this subject the student will improve the following abilities:

- Carry out the goals described in section 2.2.
- CG3. To have the ability to gather and interpret the relevant data, particularly in the field of Mathematics, in order to make statements using analytical methods as well as abstraction, containing insights on relevant topics, be it of a social, scientific, or ethical nature.
- CG5. To develop learning skills that will be necessary to continue studies in Mathematics with a high degree of autonomy.
- CT1. Be able to clearly state, both orally and in writing, the student's reasoning, problem solving techniques, reports, etc.
- CE1. Understand and apply both mathematical language and methods. Learn rigorous proofs of the basic theorems in the different areas of Mathematics.

2.2. Learning goals

In order to pass this class, the student should be able to show the following skills:

- Understand the notions of differentiable manifold and smooth map between manifolds.
- Be able to make computations in local coordinates.
- Recognize and construct new manifolds as submanifolds of other given manifolds.
- Determine properties of manifolds endowed with metric and/or group structure.

2.3. Importance of learning goals

The learning objectives provide basic skills within the degree. (See Context and importance of this course in the degree.)

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The students must demonstrate that they have achieved the learning objectives by means of the following evaluation activities. Along the course, the students are asked to solve different activities (mostly exercises and problems) and to give an oral presentation about a complementary subject related to the program of the course.

These activities (NC) correspond to the 70% of the final grade.

The other 30% will come from a written final exam (EF) after the end of the classes.

According to current bylaws, a student also has the right to show up to the final exam and complete the class upon passing the test.

Therefore, the final grade will be the greater of the following two quantities: (EF) and 0,7(NC)+0,3(EF).

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives.

A wide range of teaching and learning tasks are implemented, such as lectures, problem-solving sessions, tutorials and autonomous work and study. The proposed projects about complementary aspects of the subject will be supervised by the teacher along the course.

4.2. Learning tasks

This course is organized as follows:

- Lectures (three weekly sessions).
- Problem-solving sessions (one weekly session; oral presentations of problems).
- Tutorials (including the supervision of the proposed projects).
- Autonomous work and study.

These tasks will take place in-person at the classroom, unless the University of Zaragoza stablishes that, because of the public health situation, they should be done online.

4.3. Syllabus

- Topic 1. Differentiable manifolds.
- Topic 2. Manifolds and smooth maps.
- Topic 3. Topological properties of manifolds. Partitions of unity.
- Topic 4. Tangent space. Differentiation on a manifold.
- Topic 5. Submersions, immersions and embeddings.
- Topic 6. Submanifolds.
- Topic 7. Lie group actions.
- Topic 8. Integral curves and flows.
- Topic 9. The Lie derivative.
- Topic 10. One-parameter subgroups of a Lie group.
- Topic 11. The exponential map.
- Topic 12. The closed subgroup theorem.

4.4. Course planning and calendar

Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or please refer to the Faculty of Sciences website and Moodle.

4.5. Bibliography and recommended resources

- Lee, J.M.: Introduction to smooth manifolds. Second edition. Graduate Texts in Mathematics, 218. Springer, New York, 2013.
- Lee, J.M.: Introduction to Riemannian manifolds. Second edition. Graduate Texts in Mathematics, 176. Springer, Cham, 2018.
- Auslander, L; Mackenzie, R.E.: Introduction to Differentiable Manifolds. Mc.Graw-Hill. 1963.
- Boothby, W.M.: An introduction to Differentiable Manifolds and Riemannian Geometry . Ac. Press. 1975.
- Brickell, F.; Clark, R.S.: Differentiable Manifolds . Van Nostrand, 1970.
- Warner, F.W.: Foundations of differentiable manifolds and Lie groups. Corrected reprint of the 1971 edition. Graduate Texts in Mathematics, 94. Springer-Verlag, New York-Berlin, 1983.
- Burns, K; Gidea, M.: Differentiable Geometry and Topology. Chapman & Hall /CRC. 2005.
- Conlon, L.: Differentiable Manifolds. A First Course. Birkhäuser, 1993.
- Gamboa, J.M.; Ruiz J.M.: Iniciación al estudio de las Variedades Diferenciables. Sanz y Torres 2016.
- Lee, J.M.: Introduction to smooth manifolds. Springer-Verlag 2002.
- Outerelo, E.; Ruiz, J.M; Rojo, J.A.: Topología Diferencial. Sanz y Torres 2014.

http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27041