Taxonomy of Deformable Object Shape Control

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***Abstract*—Deformable object shape control involves the autonomous manipulation of deformable objects so that they acquire a desired target shape. This constitutes a vast challenge due to the wide variety of deformable object types and manipulation methods. The multidisciplinary nature of shape control spans fields such as deformable solid mechanics, control engineering, robotics and computer science. Therefore, existing approaches often focus on particular sub-problems and propose specific sets of definitions and constraints. This diversity complicates the analysis, characterisation, and comparison of methods, as existing works often differ in scope and terminology. In this paper, we present a practical taxonomy of shape control elements for effective identification and characterisation of shape control methods. Seeking consensus among relevant disciplines, our taxonomy aims to bring clarity to this rather complex problem. We illustrate our taxonomy's applicability and use it to classify characteristics of several representative shape control methods.**

***Index Terms*—Shape control, visual servoing, perception for grasping and manipulation.**

I. Introduction

A. Motivation for a Shape Control Taxonomy

SHAPE control of deformable objects (or *shape control*) solves the problem of autonomously manipulating deformable objects so that they acquire a desired target shape (Fig. 1). This definition, while straightforward and giving a valid general idea, is open-ended.

Mature disciplines, such as deformable solid mechanics, achieve the formalisation of some elements involved in the shape control problem (e.g. the characterisation of deformable materials from a physics point of view). However, these precise definitions stand in contrast to the wide variety of tasks and objects involved in many of the practical applications of shape control. Shape control tasks often arise in areas such as the manufacturing industry, food processing, or surgical procedures, where objects and shapes are complex and diverse, thus it becomes necessary to classify them in a more general and manageable manner. The same applies to the characterisation of other elements of the shape control problem, such as the object's available information (sensing), the nature of the employed control strategies or the shape error definition.

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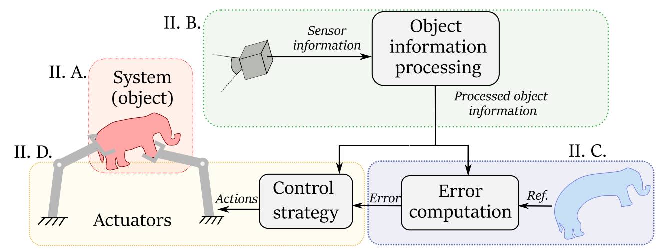

Fig. 1. Shape control scheme. The four main groups into which our taxonomy is divided are shown: deformable object characteristics (II-A), available object information (II-B), shape error computation (II-C), and control strategy (II-D).

This paper addresses the need for a taxonomy of elements in shape control. With a growing body of literature, confusion arises in many concepts and definitions. To name a few examples: regarding the problem's dimensionality, there exists ambiguity between whether *3D shape control* refers to dimensions of the workspace or dimensions of the object. When willing to compare methods, one faces numerous definitions of shape error, ranging from the comparison of discrete features to error definitions through surface point clouds or continuous curves. Other aspects such as the blending of mechanical and control concepts can lead to confusion, as seen with terms such as *model-free control*, which, from a control engineering point of view, often implies the use of a (non-mechanical) model, even if it is highly local or simplified.

B. Related Work on DOM Analysis

Surveys in the Deformable Object Manipulation (DOM) literature [1], [2], [3], [4], [5] consider shape control as one of the many specific tasks that can be performed on non-rigid objects (e.g. cutting, folding, knotting, etc.). We now summarise their classification criteria of DOM elements.

In [1], authors classify state-of-the-art approaches into several categories based on criteria such as **deformation models** (precomputed, learned or model-free), **object dimensionality** (1D, 2D and 3D), **control strategies** (classical, robust or adaptive control), **perception systems** (force-based, vision-based or a combination of both) and the **predominant actions** involved (deformation or transport actions). They provide a comprehensive review of the different methodologies, highlighting their advantages, limitations, and areas of application.

Authors in [2] classify deformable objects into four categories according to their **physical properties and geometry**: uniparametric with no compression strength (cables, ropes), biparametric (flat) with large strain or displacement, biparametric without compressive strength (cloth type), and triparametric objects (sponges, food). They further classify research approaches according to the **type of task performed** on the deformable object, such as sensing, manipulation and task-specific actions.

Survey [3] analyses challenges of robotic manipulation of deformable objects, covering design, sensing, modelling, planning, and control. It highlights the need to customise grippers to meet the specific requirements of deformable objects and classifies **sensing techniques** into visual, tactile and force methods. They distinguish between **modelling approaches** using frequently updated local models and complex global models using methods such as finite elements or neural networks. **Planning strategies** are also discussed, emphasising the need to take into account the high degrees of freedom of objects and advocating contact-centred planning. **Control strategies** are divided into model-based and model-free methods.

Focusing on the modelling of deformable objects, [4] presents a structured exploration of deformable object manipulation in robotics across four key areas: **shape representation**, **dynamics modelling**, **parameter learning and estimation**, and **manipulation strategies**. **Shape representation** is divided into four types of models: implicit curves and surfaces (e.g. algebraic curves, level set methods), explicit parameterised representations (e.g. splines), free-form, and discrete representations (e.g. meshes), which provide the mathematical basis for modelling deformable objects. **Dynamic modelling** is classified into particle-based models (particle systems, mass-spring systems), constitutive models (finite element, volume, difference methods, boundary element method, long element method) and their approximations (modal analysis, active contours), describing in detail how these models simulate deformations under external forces. **Parameter learning and estimation** methods are classified into direct estimation, error minimisation (exhaustive searches, iterative methods, genetic algorithms, neural networks) and probabilistic methods. Finally, **manipulation strategies** involve planning object manipulation and controlling deformation, using both model-based and real-time sensor-based approaches.

A classification system is outlined in [5], incorporating modelling approaches, perception techniques, and manipulation strategies. With focus on **physics-based modelling**, [5] classifies **deformation models** into mass-spring systems, position-based dynamics and continuous (mechanic) models. **Perception techniques** are described in detail, distinguishing between types of sensors (vision, force, touch) and the different perceptual tasks they address, such as state estimation, identification of material properties, object detection, segmentation, and classification. Their classification of **manipulation strategies** elaborates on methodologies for **planning**, **controlling** and **learning** the manipulation of deformable objects. Furthermore, within their classification tables, they consider the **dimensionality of the objects** (1D, 2D, 3D), the types of **sensors** being used and whether single or multiple **manipulators** are involved.

C. Contributions

The analysis of DOM studies [1], [2], [3], [4], [5] reveals an important gap in the literature. While general surveys [1], [2], [3], [5] provide a valuable general analysis of DOM, they fall short in offering the necessary depth and structure for shape control. In contrast, tutorials like [4] offer a meticulous and in-depth examination of DOM components, such as deformable object modelling techniques, yet they maintain a highly specific focus. Hence, the literature either discusses DOM broadly or concentrates on very specific elements, lacking a comprehensive and hierarchical analysis of shape control as a relevant subdomain within DOM.

In our proposed taxonomy of shape control, we fill a gap between broad DOM surveys and very detailed and specific studies. By focusing on the fundamentals, our analysis avoids redundancy by omitting details on well-covered topics, such as existing sensor types and the classic literature on control, as discussed in [1], or shape representations, explored in depth in [4]. On the other hand, it incorporates relevant elements overlooked in existing surveys on deformable object manipulation (DOM), such as the definition of shape error for shape comparison, the information acquisition source and acquisition phase, and the intrinsic and extrinsic dimensionalities of the object. These aspects represent novel analyses not covered in previous DOM surveys [1], [2], [3], [4], [5]. We also avoid unnecessary complexity by maintaining a holistic but practical format: our taxonomy follows a shape control system structure, with each main category serving as a fundamental building block. We ensure hierarchical coherence between elements of similar descriptive importance. The balance between generality and detail, the coherent hierarchy, the control-oriented modularity and the user-friendly figures and examples make this taxonomy accessible and suitable for characterising existing and future methods, serving as a valuable tool for shape control analysis.

II. Shape Control Problem: Taxonomy

We classify the elements of the shape control problem into 4 fundamental groups, each of which constitutes an independent control block: **deformable object characteristics** (Section II-A), **available object information** (Section II-B), **error definition** (Section II-C) and **control strategy** (Section II-D). See Fig. 1 for a general overview of the mentioned groups and their corresponding sections. Through the taxonomy, we apply our proposed classification criteria to various shape control strategies: Tables I to IV, which are introduced in Section II-A to II-D respectively, constitute a practical demonstration of our taxonomy. In the tables, some information items have been omitted when reasonable uncertainties existed (i.e. such items were not explicitly addressed in the corresponding paper and could not be easily inferred from the document).

A. Deformable Object: System Characteristics

Deformable objects constitute the system to be controlled. From the shape control perspective it is important to provide information about their geometry, their deformation properties and their suitability for perception and interaction. We propose three criteria for characterising deformable objects (See Fig. 2 and Table I).

1) Object's Geometry: The object's geometry is crucial for this taxonomy (criterion **A1** in Fig. 2). Regarding shape control dimensionality, some authors refer to the dimensions of the object's geometry whereas others refer to the dimensions of the space in which the object is being manipulated, thus leading to confusion. Adopting mathematical terminology, we propose the distinction between intrinsic and extrinsic frames.

The intrinsic reference frame is *mounted* on the object and allows describing the intrinsic object properties without referencing to a larger dimensional space. On the other hand, the extrinsic frame of reference can be defined within the object's dimensionality or refer to a larger dimensional space where the shape control process takes place (i.e., the object's embedding). This distinction is relevant as it allows to properly

TABLE I
EXAMPLE OF CRITERIA FROM SECTION II-A APPLIED TO THE CHARACTERISATION OF SEVERAL SHAPE CONTROL APPROACHES

Method	A1 Geometry	A2 Deformation properties	A3 Perception and interaction suitability
[6]	2D open intrinsic, 3D extrinsic	Bending (isometric), low stress limit	Rich visual texture
[7]	2D open intrinsic, 3D extrinsic	Strain and bending, low stress limit	Rich visual texture
[8]	1D closed intrinsic, 3D extrinsic	Strain and bending, composite objects	Textureless
[9]	2D open intrinsic, 3D extrinsic	Large bending, low stress limit	-
[10]	1D open and closed non-convex intrinsic, 3D extrinsic	Large strain and bending, medium stress	Textureless
[11]	2D open intrinsic, 3D extrinsic	Plastic behaviour (kinetic sand)	Textureless, cannot be grasped
[12]	1D closed convex and non-convex intrinsic, 3D extrinsic	Large strain and bending	-
[13]	3D intrinsic, 3D extrinsic	Large strain and bending, medium stress	Textureless
[14]	2D open intrinsic, 3D extrinsic	Large strain and bending	Textureless
[15]	2D open intrinsic, 2D extrinsic	Medium/low strain and bending	Textureless
[16]	2D open or closed intrinsic, 3D extrinsic	Medium strain and large bending	Textureless
[17]	1D open intrinsic, 3D extrinsic (DLOs)	Low strain and large bending	Textureless
[18]	2D open or closed intrinsic, 3D extrinsic	Medium strain and large bending	Textureless

TABLE II
CRITERIA IN SECTION II-B APPLIED TO SEVERAL SHAPE CONTROL APPROACHES

Method	B1 Information type	B2 Information resolution	B3 Information acquisition
[6]	Geom.: 2D open surf. embedded in 3D	Medium (discrete 3D mesh)	Estim. online, visual features with RGB camera.
[7]	Geom.: 2D open surf. embedded in 3D	Medium (discrete 3D mesh)	Estim. online, visual features with RGB camera.
[8]	Geom.: 2D open surf. embedded in 3D	High (1D closed contour)	Meas. online with monocular RGB camera
[9]	Geom.: 2D open surf. embedded in 3D	Low (few discrete 3D marker points)	-
[10]	Geom.: 2D open surf. embedded in 3D	High (1D open/closed contour)	Meas. online with RGBD camera.
[11]	Geom.: 3D volume	High (3D point cloud)	Meas. online with RGB-D camera.
[12]	Geom.: 2D open surf. embedded in 2D	High (1D closed contour)	-
[13]	Geom.: 3D volume	High (3D point cloud)	Meas. online with RGBD camera.
[14]	Geom.: 2D open surf. embedded in 3D	High (3D point cloud)	Meas. online with RGBD camera. Isotropy assumpt.
[15]	Geom.: 2D open surf. embedded in 3D	High (1D closed contour)	Meas. online with RGB camera
[16]	Geom.: 2D open or closed surf. embedded in 3D	High (3D point cloud)	Meas. online with RGBD camera. Isotropy assumpt.
[17]	Geom.: 1D open embedded in 3D; Physic.: def. param.	High (3D point cloud)	Geom.: Meas. online, 3D scanner; Physic.: estim. online
[18]	Geom.: 3D volumetric (dense)	Medium (3D tetrahedral mesh)	Mesh estim. online from few laser-reflective markers

Abbreviations: geom. (geometry), surf. (surface), physic. (physical), param. (parameter), def. (deformation), estim. (estimated), meas. (measured) and assumpt. (assumption).

TABLE III
CRITERIA FROM SECTION II-C APPLIED TO SEVERAL SHAPE CONTROL METHODS

Method	C1 Shape control scope	C2 Error dimensionality	C3 Reference's time dependence	C4 Error definition
[6]	Shape + scale + transport	2D intrinsic, 3D extrinsic	Fixed targ., variable ref.	Discrete features, RMS error
[7]	Shape + scale + transport	Discrete points pos.	Constant	Mesh point matching
[8]	Shape + scale + transport	Discrete points pos.	Constant	Homogeneous contour mapping
[9]	Shape + scale	Discrete points pos.	Constant	Discrete feature points alignment
[10]	Shape + scale	Discrete points pos.	Fixed targ., variable ref.	Homogeneous contour mapping
[11]	Shape	Image pixels	Constant	Point (pixel) alignment
[12]	Shape + scale + transport	Discrete points pos., 2D	Constant	Homogeneous contour mapping
[13]	Shape + scale	Discrete points pos., 30 feature desc.	Constant	Discrete features (FPFH) alignment
[14]	Shape + scale	1D intrinsic, 3D extrinsic	Fixed targ., variable ref.	Elastic contour point matching
[15]	Shape	1D intrinsic, 2D extrinsic	Constant	Elastic contour mapping
[16]	Shape + scale	2D intrinsic, 3D extrinsic	Fixed targ., variable ref.	Homogeneous surface mapping
[17]	Shape + scale + transport	1D intrinsic, 3D extrinsic	Fixed targ.	Homogeneous curve map
[18]	Shape + scale + transport	2D intrinsic, 3D extrinsic	Fixed targ., variable ref.	Homogeneous shape point matching

Abbreviations: pos. (position), targ. (target), ref. (reference) and desc. (descriptor).

TABLE IV
CRITERIA FROM SECTION II-D APPLIED TO SEVERAL SHAPE CONTROL APPROACHES

Method	D1 System modelling	D2 def. scale	D3 System stability	D4 Consider. and constr.	D5 Control actions (and object contact type)
[6]	Geometric (SfT)	Global	-	Robust to occlusions	3D pos. and orient. , small contact region, few grippers
[7]	ARAP Jacobian	Global	LAS	-	3D pos. and orient. (6 DoF), medium-sized contact
[8]	Deformation Jacobian for sliding control	Global	SGPFS	-	3D velocity, small contact region
[9]	Deformation Jacobian	Global	-	Collisions, strain avoidance	3D position, disc. contact, dual arm manip.
[10]	Local interaction matrix (online updated)	Global	LAS	-	3D position, disc. contact, single arm robot
[11]	Deep neural network	Local	-	-	3D position, small contact region, special tool
[12]	Consensus-based model	Global	GAS	Smooth deformation	2D position, disc. contact, few grippers
[13]	Deep neural network	Global	-	Robust to occlusions	3D velocity, disc. contact, few grippers
[14]	Quadratic mesh deformation energy	Global	LAS	-	3D velocity, disc. contact, few grippers
[15]	Curvature-based Energy Jacobian	Global	LAS	Seeks low deformations	2D trans. + 1D orient., disc. contact, few grippers
[16]	Procrustes-like multi-scale rigidity	Global	LAS	-	3D pos. and orient (6 DoF), small/medium contact region
[17]	Serial chain (axial springs)	Global	-	-	Disc. contact, variable gripper configuration, one gripper
[18]	Hyper-FE (Finite Element)	Global	-	Collisions, strain energy	3D pos., disc. contact, few grippers.

Abbreviations and acronyms: def. (deformation), SfT (Shape from Template), ARAP (As-Rigid-As-Possible), LAS (local asymptotic stability), SGPFS (semiglobal practical finite time stable), GAS (global asymptotic stability), pos. (position), orient. (orientation) and disc. (discrete).

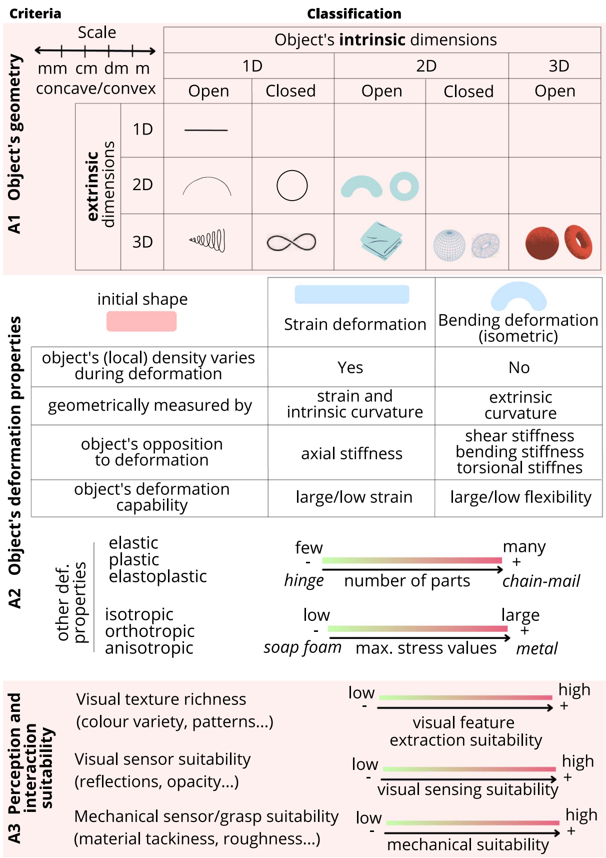

Fig. 2. Object characteristics according to: object's geometry (**A1**), object's deformation properties (**A2**) and object's suitability for interaction (**A3**).

define both the object and the dimensional scope of a shape control strategy. Controlling the shape of a 1D object like a cable (intrinsic coordinates) embedded in 3D space (extrinsic coordinates) might involve different approaches and strategies compared to controlling the same object in a 2D space (e.g. a cable lying on a table).

Objects with intrinsic 1D geometries, also known as Deformable Linear Objects (DLOs), are usually represented as open (with endpoints) or closed curves (forming continuous loops). Closed curves also serve to describe the shape of intrinsic 2D open geometries when they are embedded in 2D, as their boundary (i.e., their contour) constitutes a continuous loop. However, when embedded in 3D, open 2D geometries cannot be fully described through their contour as this representation leads to ambiguities. For example, a circular contour constitutes the boundary of both a cone and a hemisphere.

Closed 2D domains, regardless of their embedding, inherently lack boundaries and therefore have no contours. However, visual contours are often used to retrieve the object's shape from lower-dimensional projections of the object's shape (e.g., a circle as the visual contour of a sphere projected in a 2D image). We refer to these "visual boundaries" as *apparent contours*. Similarly to the contours of open 2D domains that are embedded in 3D, apparent contours lead to ambiguities. To ensure proper representation of intrinsic 2D or 3D geometries that are embedded in 3D, it is necessary to employ a surface representation, rather than relying on apparent contours.

The boundary of volumetric dense objects is defined by the 2D surface that encloses them. When the enclosed volume contains no internal holes, a closed 2D surface can fully represent the object's geometry. However, when willing to describe the shape of objects with holes, several curves or surfaces might be needed. To properly characterise the existence of holes in a shape, we propose using the topological term *genus*, which refers to the number of holes (e.g., a sphere is a genus 0 surface whereas a doughnut is a genus 1 surface).

Other important aspects regarding the object's geometry are its scale and its convexity as they are directly related to the object's suitability for grasping or gripper positioning (e.g. larger objects allow for a larger number of grippers to be placed, concave object's might be hard to grasp in certain points due to gripper-to-object collisions).

2) Object's Deformation Properties: Considering the object's deformation properties, we propose a classification with respect to the object's deformation capabilities, regarding both the type and the amount of deformation it can undergo (criterion **A2** in Fig. 2). In relation to the deformation type, in the context of deformable solid mechanics, materials are classified according to the mechanical or thermodynamic magnitudes and characteristics that are relevant to their deformation process (e.g. elastic, plastic, isotropic, etc.), amongst other criteria. This can be useful for those shape control applications that focus on object's made of known and well-characterised materials. However, seeking practicality, we propose classifying the object's deformation capability into two types:

Strain deformation: It causes a significant density variation on the object (locally or globally). The object's resistance to this kind of deformation is measured by its axial stiffness. The geometrical measurements of the amount of intrinsic deformation are the strain (which compares the object's increase or decrease in length, area, or volume along different directions) and, for intrinsic 2D and 3D objects, the changes in intrinsic curvature (e.g. Gaussian curvature).

Bending (isometric) deformation: It does not cause significant variations in the object's density (neither locally nor globally). The object's resistance to this deformation is measured by its shear, bending and torsional stiffness. The geometrical measurement of the amount of extrinsic deformation is the change in extrinsic curvature (e.g. mean curvature).

As an illustrative example and considering both types of deformation, a sheet of paper is a 2D object that cannot undergo strain deformations (very low strain) but can bend isometrically when embedded in 3D space (large flexibility). Deformations can be further categorized based on the object's mechanical properties into **elastic**, **plastic**, and **elasto-plastic**, which describe the material's behaviour under stress. Additionally, the material's structural characteristics can be **isotropic**, **orthotropic**, or **anisotropic**, each defining how the properties of the material vary in different deformation directions.

Certain deformable objects may consist of multiple components joined together mechanically. This is the case of agglomerates, which can be characterised through their physical properties (e.g., density, consistency index), and mechanisms. We classify mechanisms with respect to the number of parts that constitute them (as it provides an estimation on how the object behaves mechanically) into mechanisms with few parts (e.g. a hinge, that behaves like two solid objects attached together) and many parts (e.g. chain-mail, which behaves very much like cloth). Recall that proposed classification criteria allow for the

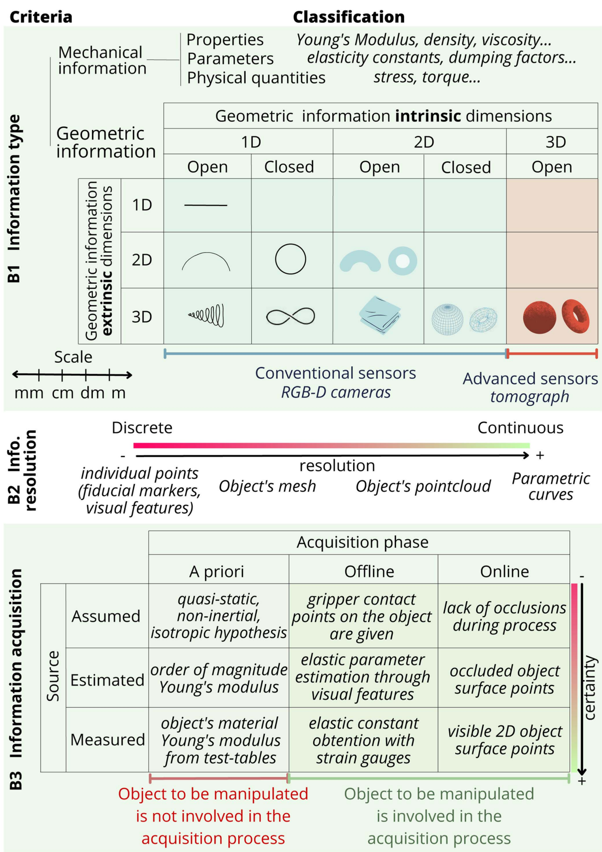

Fig. 3. Object available information classification: information type (**B1**), information resolution (**B2**), and information acquisition (**B3**). Cells in the information acquisition table contain illustrative examples.

combination of characteristics. For example, a mechanism could present both large or fine parts and those parts could present diverse deformable behaviours.

The last deformation property we include is the object's **elastic limit** and **breaking stress**. Some shape control applications need to be especially careful not to reach the breaking stress during the deformation process, and thus the shape control strategy can be strongly influenced by this constraint.

3) Object's Perception and Interaction Suitability: One of the main challenges in shape control is the perception and interaction/actuation process (criterion **A3** in Fig. 2). In the case of visual sensors, visual texture richness allows retrieving information (e.g., visual descriptors). The object's optical properties (e.g. the material's reflectivity) can determine the object's suitability for visual sensors such as RGB-D cameras. Other aspects such as the material's tackiness or roughness have a strong influence in the suitability of the grasping process or the use of mechanical sensors like strain gauges or tactile sensors. If a method strongly relies on specific sensors or actuators, such limitations should be properly stated for a proper characterisation of the approach.

B. Available Object Information Classification

In this section we classify the available object information with respect to three criteria: information type, information resolution and information acquisition (see Fig. 3 and Table II). We do not centre this analysis on the available sensor types as the same type of information can be acquired with different sensors. Rather, we focus on the elements that constitute the available information for the analysis of the object's behaviour (Section II-A) and the definition of the shape error (Section II-C).

1) Information Type: we define the mechanical information and the geometric information groups (criterion **B1** in Fig. 3). A shape control system might make use of information about mechanical properties (e.g. density), deformation parameters (e.g. mesh elasticity constants) and physical quantities (e.g. torque values). Similarly to the object's classification in Section II-A, we classify the available geometric information regarding its intrinsic and extrinsic dimensions. However, the information's dimensionality should not be confused with the real object's dimensionality and both should be properly specified. This distinction is relevant as the available geometric information has an important impact on the control strategy and the definition of error metrics. For example, a 2D object that constitutes a closed manifold embedded in 3D (e.g. an inflated balloon) being perceived with just one RGB-D camera. In this case, the camera provides a 2D open surface (partially perceived balloon surface) embedded in 3D. The object's scale information is also taken into account in this classification as it may not be available in some cases (e.g. when using a monocular camera with no scale-related visual references).

2) Information Resolution: We propose a characterisation (criterion **B2** in Fig. 3) ranging from discrete (e.g. torque values on grasped points) to higher resolution information (e.g. point clouds) and continuous forms such as parametric curves. This resolution criterion can also be applied to the information's frequency/latency, which becomes particularly relevant when inertial behaviours cannot be neglected.

3) Information Acquisition: We divide the information acquisition process (criterion **B3** in Fig. 3) into two correlated criteria: the information source and the information acquisition phase. We propose classifying information sources according to their certainty into **assumed** (e.g. assumption of quasi-static or non-inertial object behaviour), **estimated** (e.g. visually obtained elastic parameters) and **measured** (e.g. object surface points obtained with LiDAR sensor). On the other hand, information can be acquired at different stages over time. In the acquisition phase, we differentiate between information obtained **a priori** (where the object to be manipulated is not involved in the acquisition process), **offline** (where the object is involved, but acquisition occurs before control begins), and **online** (where information is obtained from the object during the shape control process). A shape control system may incorporate a combination of the described types of information sources and acquisition phases. Clearly specifying both the information source and acquisition phase is required for accurately characterising a shape control method.

C. Error Definition Criteria

The concept of *shape* lacks standard mathematical formalisation (see [4] for several shape-representation methods). This leads authors to use a variety of approaches in order to infer *shape error* between the manipulated and desired shapes of the object. This diversity underlines the need to include a classification of shape comparison and error definition in this taxonomy (see Fig. 4 and Table III).

Consider a shape representation (e.g., geometric descriptors, the contour's curvature, etc.) of the manipulated object

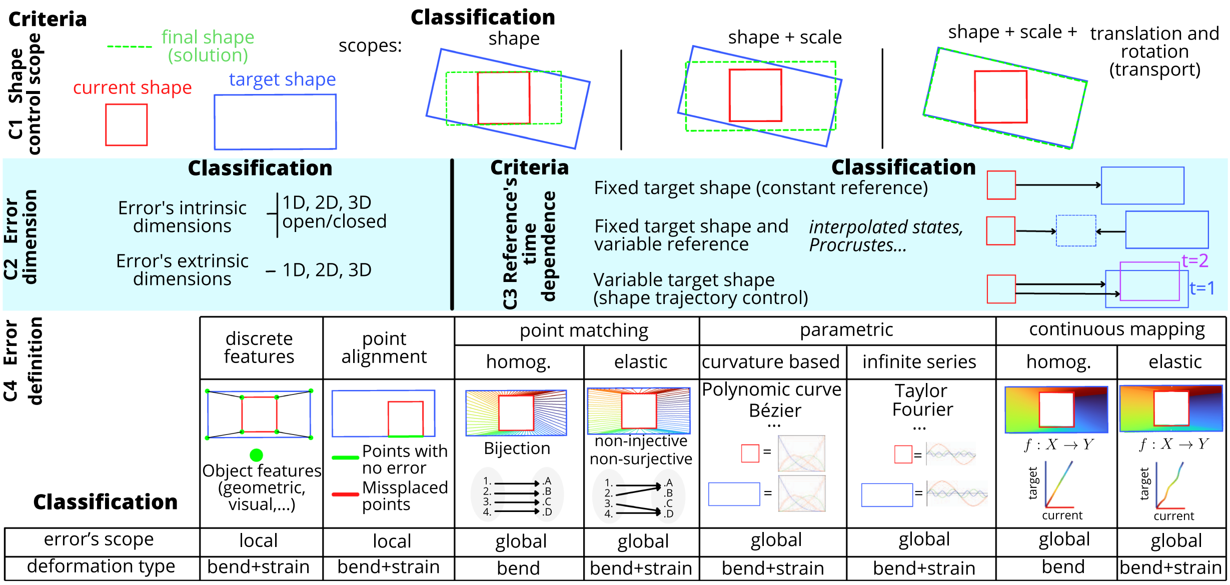

Fig. 4. Shape error characterisation: shape control scope (**C1**), error dimensionality (**C2**), reference time dependence (**C3**), and shape error definition (**C4**).

$\mathcal{S}(t, u(t))$, where $u(t)$ denotes manipulation actions, and the corresponding shape representation of the desired target shape $\mathcal{S}_{\mathrm{t}}(t)$. Omitting time dependence, let $\Pi(\mathcal{S}_{\mathrm{t}}) = \mathcal{S}$ denote the correspondence of elements of $\mathcal{S}_{\mathrm{t}}$ to elements of $\mathcal{S}$ (e.g., feature correspondences, a contour map, etc.). Additionally, consider a shape error metric $E(\mathcal{S}, \Pi(\mathcal{S}_{\mathrm{t}}))$ (for a formal definition of *shape metric* see [19]) that measures the similarity between shapes $\mathcal{S}$ and $\mathcal{S}_{\mathrm{t}}$ under the correspondences established by Π (e.g., Euclidean distance between matched discrete features, curvature differences between mapped contours, etc.). We formulate the fundamental shape control problem as:

$$\min_{u} E(\mathcal{S}(u), \Pi(\mathcal{S}_{\mathrm{t}})), \tag{1}$$

Formulation (1) is the fundamental starting point from which additional considerations and constraints can be defined (those are classified in Section II-D). Considering (1) we now categorise shape error definition with 4 criteria.

1) Shape Control Scope: Depending on the invariance scale and rigid transforms of the shape representation and the error metric $E(\mathcal{S}(u), \Pi(\mathcal{S}_{\mathrm{t}}))$, the formulation in (1) leads to three different scopes of the shape control problem (criterion **C1** in Fig. 4). The three scopes are: 1) controlling shape, scale, and translation and rotation (transport); 2) controlling shape and scale; and 3) controlling only shape. The shape control problem is solved when there exists a continuous bijection between the current and the target shape points such that the Euclidean distance between all mapped points is zero (for the former case); up to a Euclidean transform (for the second case), and up to a similarity transform (for the third case).

2) Error Dimensionality: This criterion analyses the error's dimensionality (criterion **C2** in Fig. 4), which may differ from the object's information dimensionality. We consider this distinction to be fundamental for shape control. For example, picture a pizza (i.e. 2D surface) perceived by an RGB-D sensor (2D information embedded in 3D). The pizza undergoes a deformation process such that its 1D boundary or *contour* (1D circle embedded in 3D) remains intact while its centre elevates, generating a cone-like shape. An error that relies on the pizza's 1D contour, even though the 2D surface information is available, will not be able to trace the change of shape experienced by the pizza and thus will not be suitable for that particular deformation process.

3) Reference's Time Dependence: The control reference $\mathcal{S}_{\mathrm{t}}(t)$ might remain constant $\mathcal{S}_{\mathrm{t}}(t) = \mathcal{S}_{\mathrm{t}}(t_0)$, $\forall t$ or change through time $\exists\, t > 0 \ni \mathrm{d}\mathcal{S}_{\mathrm{t}}/\mathrm{d}t \neq \mathbf{0}$ (criterion **C3** in Fig. 4). While keeping a constant target shape, new (variable) control references can be computed (interpolated intermediate states, Procrustes geometric optimisation, etc.) in order to favour the convergence of shape control strategies. On the other hand, the shape control reference can vary with time given a time dependent target shape, leading to shape trajectory control.

4) Error Definition Types: The last criterion we introduce for the error characterisation is the error definition through shape comparison, that is, how the correspondence $\Pi(\mathcal{S}_{\mathrm{t}}) = \mathcal{S}$ between shape representations is defined in order to compute $E(\mathcal{S}, \Pi(\mathcal{S}_{\mathrm{t}}))$ in (1). For this, we propose a set of 8 categories (criterion **C4** in Fig. 4). Note that our focus is not on the definition of specific shape representations $\mathcal{S}$ or $\mathcal{S}_{\mathrm{t}}$, something covered in depth in [4]), but rather on the methods that facilitate a proper comparison of such shape representations (i.e., Π in (1)).

Our error classification ranges from discrete to continuous methods, starting with discrete feature-based errors that compare sparse visual or geometric features of an object to its target shape, without fully representing the object's geometry. Errors based on point alignment focus on points that align with the target, and are effective for local but not global deformations. For global geometric errors, shape point matching techniques, classified into homogeneous (uniform distribution of points) and elastic (variable spatial density) matching, provide a holistic approach to shape errors for bending and strain deformations, respectively. In addition, parametric errors based on curves (e.g., Bézier curves) or infinite series (e.g., Fourier series or Laplace-Beltrami eigenfunctions) focus on the comparison of the mathematical parameters that define the shapes. We also present errors based on continuous maps, similar to their discrete counterpart (shape point matching). They are divided into homogeneous (e.g.,

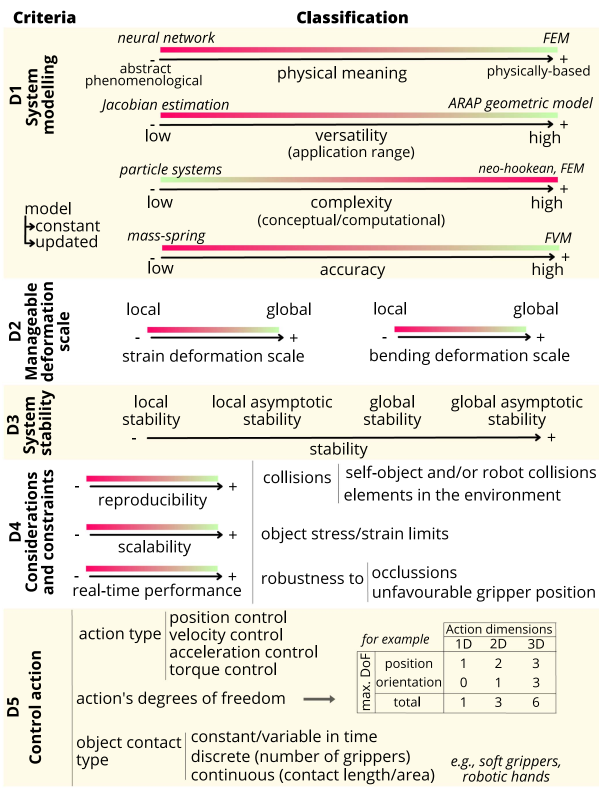

Fig. 5. Shape control strategy classification criteria: system modelling (**D1**), manageable deformation (**D2**), control system stability (**D3**), considerations and constraints (**D4**), and characterisation of the control action (**D5**). In the figure the following acronyms can be found: *ARAP* (As-Rigid-As-Possible), *FEM* (Finite Element Method), *FVM* (Finite Volume Method), and *DoF* (Degrees of Freedom).

functional maps) and elastic (e.g., Fast Marching Method based mapping). Elastic continuous maps are particularly suitable for strain deformation processes.

D. Control Strategy Characterisation

We propose classifying shape control strategies with respect to 5 criteria (Fig. 5 and Table IV): system modelling, manageable deformation scale, system's stability, considerations and constraints, and control action characteristics.

1) System Modelling: In the literature, the term *model* is used from two perspectives: mechanical and control engineering. The former refers to simulation models as low-abstraction (constitutive) models describing the system's physical behaviour (e.g., physics-based models [5] and dynamic representations [4]). The latter, more control-oriented, includes abstract models (e.g., data-driven Jacobian matrices or neural networks) for defining control laws. In the context of shape control, we propose merging both perspectives and classifying the object's model based on their physical meaning (criterion **D1** in Fig. 5). Physically-based models facilitate the consideration of other deformation aspects besides shape, e.g. stress constraints. On the other hand, they usually require complex computations than abstract or phenomenological models. In this criterion we also consider the model's versatility, i.e., whether, once computed, the model works in multiple scenarios or it is tailored to specific objects, materials, or deformation types. We also incorporate the complexity (both conceptual and computational) and accuracy criteria as quantifiable characteristics that facilitate the comparison of models.

2) Manageable Deformation Scale: this criterion involves the scale of the deformations that can be managed by the control strategy (criterion **D2** in Fig. 5). Regarding both types of deformations (strain and bending) a control strategy may be able to handle local and/or global deformations. Approaches that properly handles severe bending processes (global bending deformations, as in origami) might not be suitable for large stretching processes (global strain deformations, as in dough manipulation).

3) System Stability: Closed-loop system stability analysis has general relevance in control engineering, but given the non-linearity and the sub-actuation of deformable objects, it is of particular importance in the problem of shape control (criterion **D3** in Fig. 5). Likewise, controllability and reachability are included in this criterion.

4) Considerations and Constraints: Criterion **D4** in Fig. 5 focuses on considerations and constraints that are relevant regarding the feasibility and effectiveness of the shape control strategy. This encompasses reproducibility, highlighting the ease with which a method, such as an online updated Jacobian-based strategy, can be reproduced compared to, e.g., more data-intensive neural network approaches. Scalability, which assesses whether the strategy remains effective with variations in object size or number of actuators, is also relevant. Real-time performance, or the ability of the system to maintain the required closed-loop frequencies, is essential for dynamic control tasks. Practical constraints include collision avoidance or object strain limits.

5) Control Action: The last criteria are related to the output of the control block, i.e. the control actions (criterion **D5** in Fig. 5). Within this classification we distinguish between the action type, the action dimensions and DoF (Degrees of Freedom), and the object contact type. Another important aspect is the action's dimensions and DoF (both regarding position and orientation), closely related to the type of stress the grippers can transmit to the object (e.g. torsional stress transmitted to the object can only be directly controlled if the gripper has rotation DoF). We distinguish between constant and variable grasping, depending on whether the gripper grasping positions on the object remain constant or change over time. We propose classifying the gripper contact type into discrete contact points (an approximation for objects with a large size compared to those of the gripper's) or continuous contact (contact lengths or areas, used when the gripper's size cannot be neglected or when continuous contact is regarded in the control strategy, e.g., manipulation with soft grippers). Contact position and type has a direct impact on how actions are transmitted to the deformable object (e.g. a single contact point does not allow for proper transmission of gripper rotations).

III. Taxonomy Application

To facilitate the application of our taxonomy, we have designed a user guide.[1] This guide streamlines the identification and analysis of shape control methods, and includes user-friendly questions as well as metrics and example values for

[1] https://github.com/nachocz/Taxonomy-of-deformable-object-shape-control

quantitative analysis. While our taxonomy is exhaustive enough to cover existing and upcoming methods, the public repository format allows users to collaborate and provide updates that accommodate unforeseen future developments.

One of the main challenges in shape control is to fairly compare methods. This requires aligning their characteristics according to our taxonomy's main criteria. Specifically, aligning system characteristics (Section II-A) enables comparisons of methods with shared scope. Similarly, aligning error definition criteria (Section II-C) across methods establishes uniform metrics for measuring success or failure.

IV. Conclusion

In our exploration of the problem of shape control, we have introduced a practical taxonomy to enhance the characterisation and comparison of shape control methods. Through criteria covering system characteristics, available information, error definition and control strategies, we have established a clear and systematic framework that not only facilitates the examination of existing methods, but it is also designed to embrace future innovations.

This taxonomy enabled us to identify several promising research directions. Firstly, there is a need to move beyond the commonly applied quasi-static manipulation assumption, as demonstrated in recent work [20], particularly for tackling the unexplored problem of shape trajectory control. Another area for future research involves studying the use and positioning of soft grippers for shape control, exploiting continuous contact regions rather than the traditional single contact points [21]. Additionally, research may focus on the definition and feasibility analysis of target shapes, as current methods often define achievable targets by pre-acquiring shapes in experimental setups. Finally, reducing dependence on offline exploration of object behaviour (e.g., by exploiting more versatile models [4]) is essential for advancing practical applications.

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