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BRIEF REPORT

Coevolution and dynamic processes: an introduction to this issue and avenues for future research

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Abstract
In this paper, drawing upon previous contributions to modern Schumpeterian economics, we argue that coevolution in economic systems operates when considering several evolving populations within a socio-economic system, these populations mutually shape their respective selection, learning, and/or novelty generation mechanisms. The properties that arise from coevolution should be analyzed as emerging from multiple populations in co-determination. The notion of coevolution appears not only in Schumpeterian economics but, in general, in many branches of heterodox thought. Likewise, it can also be found in Biology, Sociology, Political Science, History, Philosophy, Law, and Computational studies. In this introduction, after providing a neat definition of coevolution, we illustrate with formal examples how coevolution can be represented and, potentially, empirically tested. Finally, we present the contributions to the SI and suggest avenues for future research.

Keywords Coevolution · Evolving systems · Technology · Institutions · Economic thought

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1 Introduction

In this Special Issue (SI) we, and the authors in the issue, deal with the concept of coevolution in dynamic systems and its applications. Together with the contributors to this endeavour, we point to coevolution as a force operating within dynamic systems of different types. Following a well-stablished tradition in modern Schumpeterian economics that spans from Nelson and Winter (1982) or Dosi et al. (1988), all the way to Dosi (2023) or Dopfer et al. (2024), we argue in this introduction that considering socio-economic systems as developing in an evolving complex manner, and being composed of market and non-market realms in co-determination, is the best way of characterizing their innovative dynamics. In fact, we can focus on representing societies as continuously changing from within, through processes involving selection of heterogeneous (boundedly-rational) competing organizations and agents, displaying learning processes that entail the uneven replication of embodied technological and human traits (routines, habits, values) and showing an ongoing generation of novelties in complementary realms (Metcalfe 1998; Foster and Metcalfe 2001; Dopfer and Potts 2008; Nelson 2018).

Drawing upon the aforementioned conception of economies as evolving complex systems, we initiate our discussion of coevolution in sections "2" and "3." Before doing that, we want to anticipate in this introduction that, although our departure point is the Schumpeterian tradition, the importance of coevolution in dynamic processes has been pointed out by a wide variety of authors who have approached the phenomenon from different perspectives. These complementary views coming from distinct schools of thought (Hayekian, post-Keynesian, Analytic Marxism, Institutionalism, Historicist school, Evolutionary-naturalistic approach, Complexity economics) consider other driving forces different from technological change (such as class relations, gender relations, human-nature relations, State interventions, institutional settings, private agency, path-dependency within spontaneous interactions), and they draw upon these aspects to explain socio-economic change (Davidson 1972; Roemer 1986; Heilbroner 1987; Gowdy 1994; McKelvey and Baum 1999; Witt 2003; Van den Berg and Stagl 2003; North 2005; Elsner and Hanappi 2008; Hidalgo and Hausmann 2009; Gual and Norgaard 2010; Hodgson and Knudsen 2010; Hanappi 2020; Liu et al. 2021).

The existence of this rich variety of approaches stresses the importance of understanding what coevolution really means in economics and social sciences, and how these processes bring about the structural changes observed at multiple levels of human systems. In this SI, the modern Schumpeterian tradition will appear as being complementary to other heterodox frames, and this is the reason why the collection of papers includes (besides Schumpeterian economics) papers on Institutional theory, Complexity, post-Keynesian thinking in Stock flow consistent ABMs, Innovation systems theory, Austrian economics, and authors working in History, Political Science, Computational thinking, Biology, Philosophy and Cultural studies. As we will see, the variegated composition of this SI on coevolution represents shared interests and a proximate common perception on the relevance of coevolutionary processes operating through multiple layers in reality. It will also be shown how, although the specific concept of coevolution in this paper (Almudi and Fatas-Villafranca 2021, 2022) is inspired by the modern Schumpeterian tradition and tries to follow the initial project suggested by Chris Freeman, given its generality, it can trespass on other heterodox schools of economic thought.

From the Schumpeterian tradition, the notion of coevolution appears (more or less explicitly mentioned) as a fundamental mechanism to understand innovation-driven economic change in many contributions, for several decades (Nelson and Winter (1982), Nelson (1990), Dosi and Coriat (1998), Foster and Metcalfe (2001), Murmann (2003, 2013), Freeman (2019), Dosi and Nuvolari (2020), Dosi and Roventini (2019), Almudi and Fatas-Villafranca (2024)). These authors claim that technological change is the fundamental driving force behind socioeconomic transformation, but they emphasize that technology is inseparable from institutional and organizational dynamics, and it is always shaped by market transformations and the State. Thus, in Nelson (2005, 2018) we see how science and technology are related through subtle bidirectional effects in virtually all realms of economic activity. This becomes clear in the coevolution of industries and regulations, in how the dynamics of national university systems support national innovative firms, in public procurement programs that shape even the social ethos, or in how market selection processes in vertically integrated sectors are coupled and the sectors coevolve. We, the authors contributing to this SI want to go even further, and explore how these effects operate not only in all productive (market-related) sectors (manufacturing, services, construction, primary activities), but also in absolutely all realms of human activity (including populations of public agencies, civil and political organizations and agents seeking to reach power and shape the social values, in cultural, environmental and even intimate personal realms).

In this SI, we and the authors in the issue aim at extending the traditional approach in modern Schumpeterian economics, and we bring to the center of the debate the perception that, although innovative firms remain at the core of capitalist processes, the internal corporate capabilities and their dynamics are enhanced and supported by a constellation of norms, social rules, non-market bodies (science and knowledge producing centers, law, regulatory agencies) and individuals that conform the external framework of the firm (Metcalfe 2001). Likewise, there are agents aiming to change values, and civil organizations seeking to shape culture, market demand, and the institutional surroundings (Witt 2003).

Clearly, this interlinked-realm envision was strongly stated and developed in pioneering works by Chris Freeman. According to Freeman (2019), capitalist economic transformations should be analyzed as emerging from processes of coevolution involving the development of science and institutions, technology, market-related corporate-driven changes, political transformations, and cultural evolution. But we believe that the Freeman approach demands from us a much wider and proper attempt to understand the mutual interactions between modern institutional and political set-ups, firms and markets, and evolving sets of cultural values. In fact, it seems clear that the Freeman project (Dosi et al. (1988), Dosi and Nuvolari (2020)) aspires to transform the manner in which we do economics as a whole, moving far from the full rationality cum equilibrium framework, and it cannot be said to be accomplished just by developing innovation studies as an additional branch of applied economics.

Then, in this SI, we try to advance along the lines of this grand coevolution program by proceeding through the following *sequence of steps*, which clearly trespasses many heterodox schools of thought and fields of application:

(1) We should provide a neat and general definition of coevolution (Almudi and Fatas-Villafranca 2021) and we must specify all the elements potentially involved in these processes (Sect. 2). Stating a precise definition of coevolution will allow us to achieve certain goals: (i) to limit the scope of the coevolution phenomenon in dynamic settings; (ii) to expand the possibilities for its detection and exploration in many fields of study and from many different analytical angles; (iii) to focus on the debates and open issues that arise when we address dynamic phenomena from a coevolution perspective (think about the notion of causality, its empirical tractability, the differences between multi-population thinking and other mechanisms such as feedbacks and loops).

(2) New techniques are required to test empirically if coevolution is operating in specific situations. This is a big challenge; it is an open issue which exceeds the scope of this volume. Nevertheless, with some limitations on the generality of our findings, in this paper (and in the SI) we offer approximations by developing tailored models (section “3” and the Appendix), and we obtain coevolution equations suitable for testing.

(3) We should try to revise how coevolution appears not only in modern Schumpeterian economics but also in other fields of inquiry (History, Biology, Philosophy), and of course in complementary lines of heterodox thinking interested in the dynamics of power, sustainability, and complex economic change (post-Keynesian thinking, Complexity theory, Institutional economics, Computational economics).

(4) We should indicate—as a distillation of previous literature and points (1), (2), and (3)—operational applications of the coevolution notion. This can stimulate future avenues of theoretical and empirical work. We try to anticipate in this paper that a neat core along this sequence of steps will emerge from the different contributions in this volume.

The rest of our paper is organized as follows. In section “2,” we deal with the concept of coevolution. In section “3” (and in Appendix), we develop alternative and illustrative formalizations that can make operational and contribute to develop the formal analysis and testing of the coevolution idea. We close the paper with section “4,” in which we anticipate briefly some ideas to come in the many contributions to the issue, suggesting avenues for future research.

2 Evolving systems, coevolution, and how to identify it

We devote this section to explain how our specific definition of coevolution (Almudi and Fatas-Villafranca 2021) can become operational in many different realms, as long as, at least two evolving populations are interacting. Indeed, as we point out later on, if coevolution is detected in a complex system, it may act as a coordinating (or de-coordinating) mechanism among the populations involved. To understand how coevolution operates among populations, we have to be able to explain how systemic endogenous properties may emerge.

In what follows, we characterize coevolutionary systems as compositions of multiple evolving populations in mutual interaction. Thus, for coevolution to take place, a dynamic system must be composed of, at least, two evolving populations of any kind. Let us define an evolving population in a socio-economic frame as follows: (i) a set of heterogeneous agents operating in a specific realm (firms within a sector, consumers in a market, public agencies managing activities), with these agents being boundedly-rational but, at the same time, holding specific intentions; (ii) these heterogeneous and boundedly-rational agents are embedded within specific contexts in which they can learn potentially replicable traits; (iii) within the realm in which these agents operate, some domain-specific selection mechanisms take place, and novelties come about in both, the replicable traits, and the entities subject to ex-post selection. In dynamic terms, the characterization of population(s) that we propose implies that the agents must try to adapt and survive and even grow in changing environments. Some agents will gain relative presence, with their replicable traits becoming prevalent, while others will lose it and their replicable traits will become rare. Moving a step further, we can say that when several evolving populations appear as composed in a specific coevolutionary setting, then the changing agents in the different populations will operate as intra- and inter-population shapers of the interconnected environments. These environments become gradually co-determined.

As it is shown in Table 1 below, we can apply the characterization of populations as complex evolving systems (that may eventually coevolve) to a wide variety of realms, from markets to public agencies and civil society.

In Table 1, the first column refers to the specific realm in which heterogeneous agents (second column), endowed with potentially replicable traits (third column) operate, compete, and try to adapt, survive, and grow. In any of those realms, some kind of selection and/or innovation process is taking place. For example, in the market realm, if demand is price oriented, as time goes by, those firms whose strategies are more focussed on reducing costs than on improving performance may be selected, gaining presence. In the individual or self-realm, it could be the case that some individuals are high-skilled workers while others are low-skilled. In this case, which workers are going to be selected by firms may depend on the predominant type of sector (high tech vs. low tech). Workers can try to adapt by training and getting new skills at the proper institutions. In turn, the evolution of the population of workers will influence innovation, firm capabilities, and even market demand. Finally, in the civil society realm, for an organization to get funding, survive, and make a social effect, it can be crucial the profile of members (individual, workers) that it attracts. The ability to search, convince people, and properly fulfil the requirements to be funded can be very different from one organization to other. Civil organizations can improve their abilities to get funded by learning, attracting good members, and by promoting values that fit the ethos of certain groups.

Table 1 Entities and mechanisms in a socio-economic process of coevolution

Realm	Heterogenous agents	Replicable traits	Selection processes	Novelty sources
Market	Firms	Routines, strategies, techniques	Market selection (e.g., price vs. Performance oriented demand)	Inner innovation
Individual	Individuals	Habits, values, skills, ideas	Individual selection (e.g., high-skilled vs. low-skilled workers)	Training and education
Civil society	Civil organizations	Routines, ideas, social proposals, strategies	Civil society selection (e.g., high vs. low ability to get funding)	Creative learning

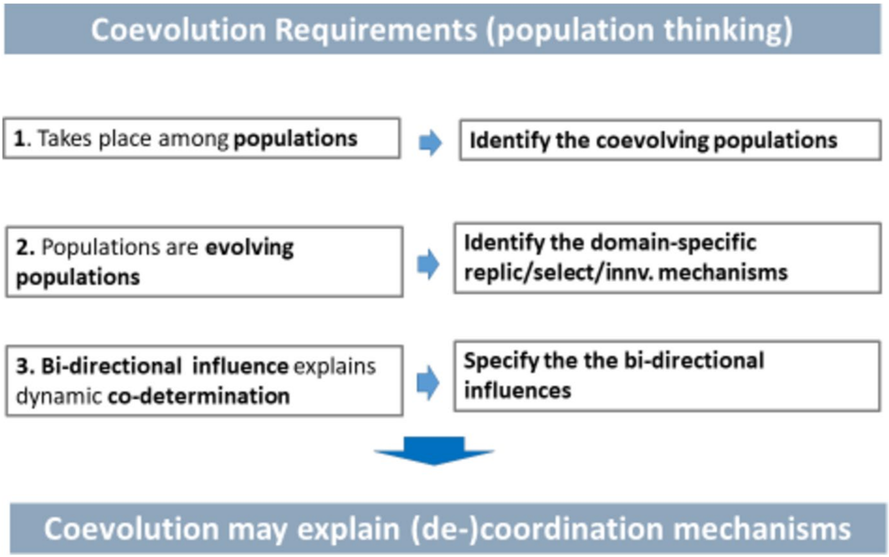

Fig. 1 A descriptive and synthetic presentation of coevolution

Now, once we have seen what an evolving population in a socio-economic system is, and how to identify its elements, we can state our definition of coevolution (Almudi and Fatas-Villafranca 2021).

Coevolution: Let us state that two (or more) evolving populations coevolve, if these populations causally influence each other in such a way that the multidirectional influences shape the innovation, learning/imitation, and/or the selection processes that are specific to each domain. In this way, the multiple evolving populations linked by coevolution will become dynamically codetermined.

According to this definition, to detect coevolution the following requirements must be fulfilled: (i) at least two evolving populations have to dynamically interact; (ii) such an interaction must result in some (or each) evolving population affecting replication, selection, or novelty mechanisms of the other populations; (iii) this mutual effects will bring about emergent properties (see Fig. 1).

We would like to emphasize that our coevolution notion draws on the idea that reality is a plural composition of domains (in the sense of interlinked populations of entities connected in complicated topological manners) that, despite of being to some extent self-referential, are transversally (vertically and horizontally) intertwined in a dynamic manner. This notion of coevolution reflects the idea that coevolution does not refer merely to feedbacks, but it involves multilayer and multi-population vertical and horizontal shaping interlinks, operating in diverse agent-to-agent intra (and inter) population directions, that condition selection, learning, and novelty generation processes among realms (markets, institutions, the State, civil society, nature). From these processes, different types of properties may emerge (such as the determinants of the overall growth rate in coevolution models, or the coevolving trajectories of multiple sectors in industrial dynamics).

In the next section (and in the Appendix), we develop two formal proposals to illustrate how coevolution can be formally explored when markets and institutions are involved. We conceive institutions in a broad sense involving not only systems of social rules, but also non-market organizations and agencies (Nelson 2018). In the model in section "3," we obtain the specific *coevolution equations* suitable for empirical testing. In the model in Appendix, *sectoral paths and distributions* are shown. Both are illustrative examples of emergent properties from coevolution frameworks. The aim of these exercises is to show that coevolution can be empirically tested, but the way in which this has to be done is different from other empirical validation exercises. This is a very important open issue emerging from the concept of coevolution, and we hope that these particular modelling exercises highlight ways to advance in this novel research line.

3 Technology, institutions, and implications for economic geography in a multisectoral model of coevolution and growth

In this section, we illustrate how the coevolution notion can be implemented to deal with several phenomena: rural to urban migrations in times of large technological transformations, and/or growth in stylized multisector economies driven by institutions and technical advance. In Appendix, we extend the formal implementation to tackle the coevolutionary dynamics of vertically integrated sectors in ABMs. For applications of the model in Appendix, we can think about an upstream sector devoted to produce bio-machine tools, that are then used downstream by applied bio-firms doing sequencing, genomics, and which can carry out diagnostics for customers seeking to discard genetic diseases, cancer, and fertility issues. Both, the model in this section and the one in Appendix, illustrate ways for future research on coevolution.

In this section, we present a growth model in which two populations of locations in which technical practice, production, and innovation occur (cities in two regions, firms in two different sectors), and a third population at a different level in which we place competing agencies or institutions coevolve. Institutions (public agencies, regulations and norms, research institutes) provide, either needed infrastructure and laws, or new scientific knowledge (Universities, National Institutes). The institutional outcomes are essential for production and technological advance in practice. To keep the math simple, we analyze the coevolution between two populations in one layer (regions or sectors as realms of practice), and a population of institutions in a different level. The evolution of sectors/regions feeds on the institutional dynamics, and institutional change feeds on the two populations carrying out practice.

Let us denote by $I = \{1,2, \dots, n\}$ the set of locations of practice in sector I (firms in sector I, cities in a region), with $i \in I$ denoting the i-location of practice, and a_{it} denoting the quality level of that practice in i (labor productivity in firms or cities, which can be related to wages and living possibilities that attract workers/citizens) within I at t, $t \in R$. Besides this, we denote the same aspects for population J

by $J = \{1,2, \dots, m\}$, with $j \in J$ being now the j-location in J, with levels of performance at t given by b_{jt}.

We consider a constant and continuous mass of moving agents (citizens, workers) in the economy. The mass is equally divided between both populations of practice. We can normalize both sectoral masses to one and assume full employment and neither demand-shortcomings nor rationings in the markets. Within each population at the level of practice, we denote by $e_{\mathrm{it}}, \varepsilon_{\mathrm{jt}}$ the shares of sectoral employment (or citizens in a region) that work/live at t in each $i \in I$, and $j \in J$. Clearly, $\sum_{i=1}^{n} e_{\mathrm{it}} = 1$ and $\sum_{j=1}^{m} \varepsilon_{\mathrm{jt}} = 1$. In the model, we consider that production functions at the locations of practice are linear labour-technology functions.

Apart from populations I and J, we consider that there exists a third population S (rival institutions) which includes two objects: two universities, two scientific fields, two public agencies providing infrastructure, regulation. We have $S = \{A, B\}$, with A and B being rival institutions which produce essential outcomes for I and J, respectively. The advances in A are of help for I; the advances in B are useful for J. Likewise, we will denote by $s_{\mathrm{At}}, s_{\mathrm{Bt}}$ the share of total budget (or power) under control of A and B, respectively, at t, so that $s_{\mathrm{At}} + s_{\mathrm{Bt}} = 1$. We assume that the higher the level of s_{At} or s_{Bt}, the stronger and more intense (relatively) the quality of practice in locations in I (fed and related to A), or in J (related and dependent from B).

We represent in the model that the higher the amount of funding or power allocated by society to a certain institution (A or B), the stronger the flow of support that this institution provides to its related location of practice (I or J) to improve performance and to advance. In turn, as the spots of practice within I or J improve performance and production, they gain relevance/power as population (region, sector), and promote the expansion of the related institution in S.

3.1 Dynamics within the realms of technical practice

We present hereby additional assumptions:

(a) We assume linear labour-knowledge production technologies in I and J, so that the corresponding average levels of technology (labour productivity), and the growth rates are as follows:

$$a_t = \sum_{i=1}^{n} e_{\mathrm{it}} a_{\mathrm{it}}, \; b_t = \sum_{j=1}^{m} \varepsilon_{\mathrm{jt}} b_{\mathrm{jt}} \tag{1}$$

$$\hat{a}_t \equiv \frac{\dot{a}}{a_t} = \frac{d}{\mathrm{dt}} \ln a_t, \; \hat{b}_t \equiv \frac{\dot{b}}{b_t} = \frac{d}{\mathrm{dt}} \ln b_t$$

(b) It can be shown from what we have stated above that the overall rate of technological advance is given by the following:

$$\hat{q}_t = \left(1 + b_t / a_t\right)^{-1} \hat{a}_t + \left(1 + a_t / b_t\right)^{-1} \hat{b}_t \tag{2}$$

(c) Location (firm or city) i: innovation in $I = \{1,2, \dots, n\}$. We consider that for each $i \in I$ the performance (labour productivity) grows depending on a cumulative effect, and on the scope of institutional support that i reaches from A. Since, firstly, we want to introduce coevolution in the smoother and neater way, we propose that:

$$\frac{d}{dt}\ln a_{it} = s_{At} \tag{3}$$

(d) Location *j:* innovation across $J = \{1,2,\ldots,m\}$. We consider that for $j \in J$, technology (labor productivity) grows driven by the following technical innovation function:

$$\frac{d}{dt}\ln b_{jt} = s_{Bt} \tag{4}$$

(e) Intra-population $I = \{1,2,\ldots,n\}$ selection:

Within the population I, we can assume different types of selection processes. Here, for clarity, we propose competition among locations of practice (firms/cities), seen in locations being able to attract highly skilled workers, or more intense flows of migrants, depending on location performance (technology, productivity, living standards) which may be also a proxy of wages. The rationale is that the relationship between location-specific salary is proxied by productivity a_{it}, and, additionally, the interest of skilled migrants—within I—to develop their careers or to live in good conditions, is conditioned by having access to technologically-progressive locations. More precisely, we assume random matching among skilled workers/citizens in I, and a flow of revision (learning by boundedly rational workers/migrants) which gradually leads people to try to get placement in promising locations.

If this is so, we can assume that workers employed in i and k within I meet with size-proportional probabilities, so that probabilities of random meetings are proportional to employment shares, more precisely they are $(\alpha e_{it} e_{kt})$, $0 < \alpha < 1$. Furthermore, we may represent workers/migrants tending to change their employment towards better locations by $f_{ikt} = \max\{(a_{it} - a_{kt});0\}$—the switching flow from k to i at any period of time; thus, we have a revision protocol (learning, migration):

$$f_{ikt} - f_{kit} = (a_{it} - a_{kt})$$

If we take together the pairwise-random matchings proposed, and the revision protocol of learning, it follows that the share of location $i \in I$, evolves driven by the following replicator dynamics system (Almudi et al. 2017):

$$\dot{e}_{it} = \alpha e_{it}(a_{it} - a_t)\cdots,\ \cdots 0 < \alpha < 1 \tag{5}$$

(f) Intra-population $J = \{1,2,\ldots,m\}$ selection:

Within the population of heterogeneous spots of technological practice in J, we can assume a similar argument. Thus, we arrive at the replicator dynamics in eq. (6):

$$\dot{\varepsilon}_{jt} = \alpha\varepsilon_{jt}(b_{jt} - b_t)\cdots,\ \cdots 0 < \alpha < 1 \tag{6}$$

3.2 Coevolution

We close the model with the assumptions on the realm of institutions $S = \{A, B\}$, which control and manage, respectively, shares of budget/power and support $s_{At}, s_{Bt}, \ s_{At} + s_{Bt} = 1, \forall t$. In this bi-dimensional case, it is clear that $s_{Bt} = 1 - s_{At}$. We pose a relative-fitness approach in this institutional-layer of the model. Thus, we consider $g_B = 1$ as being the normalized fitness of institution B, with $g_A = (1 + a_t/b_t)$ being the relative fitness of A. Notice that we are considering that institution A is incorporating something perceived as superior in some sense (e.g., organic chemistry vs. knowledge on natural techniques to produce dyes, or new infrastructure vs. old things in technical transitions), so that the relative fitness in A is identified as superior to g_B. This can be the case in situations of large technological transformations (new vs. old activities in multisector economies, the development of progressive geographical placements, both needing new vs. old knowledge, or new vs. old regulations and infrastructures).

Let us note—and this is crucial in our coevolution argument—that we are assuming that, the higher the level of technological advance and performance in population (sector or region) I (the one which benefits from advances in A) with respect to the technical level in J, —that is, the higher (a_t/b_t), the higher the relative fitness of A with respect to B. To simplify the math, we propose a bi-dimensional replicator according to which the institution with higher than average relative fitness—which will be higher the greater the value (a_t/b_t) dependent on the evolution of the populations of practice—captures an increasing share of resources (budget) and political/social support and power. Both A and B generate flows of useful outcomes for the related field of practice. That is to say, eventually, the process will feed back into sectors. Formally, since $s_{Bt} = 1 - s_{At}$, we are going to focus on the analysis of s_{At} drawing on the following differential equation which captures all the reflections above:

$$\dot{s_A} = s_{At}\left[g_{At} - \left(s_{At}g_{At} + s_{Bt}g_{Bt}\right)\right] = s_{At}\left(1 - s_{At}\right)\left(a_t/b_t\right) \tag{7}$$

Note that eqs. (1) to (7) fully drive the dynamics of our coevolution model. As we will show in the following subsection (which conveys the first result that we obtain in the model), the evolving populations of practice I and J coevolve with (and through) the realm of institutions S. The result 1 in the next subsection shows interesting properties of this coevolutionary process in its smooth-variant. Later on, we will see the case of non-smooth coevolution, in which blockages and imperfections in the perception of new opportunities, in absorptive capacity or in the plasticity of the locations of practice take place.

3.3 Result 1

Proposition: *From Eqs.* (1) *to* (7), *if we observe the coevolving dynamics in the three populations I, J and S, we can prove the following statements [$V(a)_t$ and $V(b)_t$ are the variances on technical practice within I and J, respectively]:*

1) *The instantaneous rate of technological change (or rate of average productivity growth) in population of practice I is given by the following*:

$$\widehat{a}_t = \alpha \frac{V(a)_t}{a_t} + s_{\text{At}} \tag{8}$$

2) *The instantaneous rate of technological change in population of practice J is given by the following*:

$$\widehat{b}_t = \alpha \frac{V(b)_t}{b_t} + 1 - s_{\text{At}} \tag{9}$$

3) *The overall rate of technological change (overall rate of productivity growth) is as follows*:

$$\widehat{q}_t = \left(\frac{1}{1 + {}^{b_t}\!/_{a_t}}\right)\left(\alpha \frac{V(a)_t}{a_t} + s_{\text{At}}\right) + \left(\frac{1}{1 + {}^{a_t}\!/_{b_t}}\right)\left(\alpha \frac{V(b)_t}{b_t} + 1 - s_{\text{At}}\right) \tag{10}$$

4) *Considering the results in* (1), (2), *and* (3), *and keeping in mind that* $s_{\text{Bt}} = 1 - s_{\text{At}}$, *the following dynamic path for the flow of outcomes from institution A, closes coevolution in the model*:

$$s_{\text{At}} = \frac{1}{1 + \left(s_{A0}^{-1} - 1\right)\exp - \left({}^{a_t}\!/_{b_t} \bullet t\right)} \tag{11}$$

Proof. (1) Drawing on expression (1) for a_t, if we take time derivatives and we consider eqs. (3) and (5), we obtain the following:

$$\dot{a} = \sum_i \dot{e}_i a_{\text{it}} + \sum_i e_{\text{it}}(a_{\text{it}} s_{\text{At}}) = \alpha \sum_i e_{\text{it}} a_{\text{it}}(a_{\text{it}} - a_t) + a_t s_{\text{At}} \rightarrow$$

$$\widehat{a}_t = \alpha \frac{V(a)_t}{a_t} + s_{\text{At}}$$

(2) From (1) for b_t and considering eqs. (2) and (6) in the time derivatives, we take $s_{\text{Bt}} = 1 - s_{\text{At}}$, and we obtain $\widehat{b}_t = \alpha \frac{V(b)_t}{b_t} + 1 - s_{\text{At}}$.

(3) We consider Eq. (2) and we substitute eqs. (8) and (9) in it. We have Eq. (10).

(4) Eq. (7) can be written as follows: $\dot{s_A} = \left(\frac{a_t}{b_t}\right) s_{\text{At}}\left(1 - s_{\text{At}}\right)$.

This is a logistic differential equation with coefficient $\left(\frac{a_t}{b_t}\right)$. It can be integrated leading to Eq. (11).

Now, if we observe the results in proposition 1, we get that in ideal coevolutionary conditions:

$$\lim_{t \to \infty} s_{\text{At}} = 1,$$

(full support to the superior institution A in the limit), with a speed given by the relative improvement in technical practice in the related population of practice I as compared with the other $\left(\frac{a_t}{b_t}\right)$; in turn, closing the coevolution argument, we see how s_{At} affects in opposite directions technical practice in I and J (see eqs. (8) and (9); also in Eq. (10)). In fact, as replicators (2), (3) cum (5), and (6) develop in "ideal" conditions, selection ends up concentrating activity in the frontier-level of activity in both I and J, so that in the limit, the variances tend to vanish, productivity stabilizes in population J at a high level, but it keeps growing in (the technologically superior) population I, at a rate which fully feeds the limit trajectory of aggregate productivity growth $\widehat{q}$. Coevolution operates without blockages in the direction of improvement, intra-population selections works relatively smoothly and feeds back in the "right" direction the realm of institutions. This situation leads to a maximum performance in terms of long-run overall productivity growth, Eq. (10).

Now, it is interesting to see how these results get "worse" when we incorporate "imperfections" in coevolution.

Thus, let us focus in the mutualistic coevolution effect linking I and institution A, and consider the following variations on the basic assumptions (we leave population J and its working mechanisms with no changes, as before):

(1) Let us assume now that technology absorptive capabilities, and/or the strength and propelling role of A are not equally distributed (at top levels) across population I, so that we change expression (3) by (12) (where parameter λ_i is a blocking factor in the operational assimilation -in practice- of outcomes from institution A):

$$\frac{d}{\mathrm{dt}}\ln a_{it} = \lambda_i s_{\mathrm{At}}, 0 < \lambda_i < 1 \tag{12}$$

(2) Let us consider that, even if the outcomes being produced in A are potentially superior to those in B, the perception of the relative fitness may be imperfect (as measured by an opacity parameter $0 < \gamma < 1$ in the population dynamics within S). The simplest way to introduce this change in the model is by reformulating in Eq. (7) as follows:

$$\dot{s_A} = s_{\mathrm{At}}\left[g_{At} - \left(s_{\mathrm{At}}g_{\mathrm{At}} + s_{\mathrm{Bt}}g_{\mathrm{Bt}}\right)\right], g_{\mathrm{At}} = \gamma(1 + a_t/b_t), g_{\mathrm{Bt}} = 1 \tag{13}$$

In this new version of the model, it is clear that eq. (11) can be re-written as follows:

$$s_{\mathrm{At}} = \frac{1}{1+\left(s_{\mathrm{A0}}^{-1}-1\right)\exp\left(1-\gamma(1+a_t/b_t)\right)\bullet t}, \quad s_{\mathrm{Bt}} = 1 - s_{\mathrm{At}} \tag{14}$$

The analysis of this version shows an interesting result.

3.4 Result 2

Proposition: *From Eqs.* (1), (2), (4), (5), (6), (12), (13), *and* (14), *if we combine the dynamics in the three populations I, J, and S, we can proof the following:*

1) *The rates of productivity growth* (*advance in performance*) *in I and J are now*:

$$\widehat{a}_t = \alpha \frac{V(a)_t}{a_t} + s_{At}\left(\frac{C(a, \lambda)_t}{a_t} + \lambda_t \right) \tag{15}$$

$$\widehat{b}_t = \alpha \frac{V(b)_t}{b_t} + 1 - s_{\mathrm{At}} \tag{16}$$

2) *The overall rate of productivity growth is given by the following*:

$$\widehat{q}_t = \left(\frac{1}{1 + {}^{b_t}/_{a_t}} \right)\left(\alpha \frac{V(a)_t}{a_t} + s_{\mathrm{At}}\left(\frac{C(a, \lambda)_t}{a_t} + \lambda_t \right) \right) + \left(\frac{1}{1 + {}^{a_t}/_{b_t}} \right)\left(\alpha \frac{V(b)_t}{b_t} + s_{\mathrm{Bt}} \right) \tag{17}$$

3) *From eqs.* (13) *and* (14) *and considering* $s_{\mathrm{Bt}} = 1 - s_{\mathrm{At}}$, *the following dynamic pattern for the flow of outcomes from A closes coevolution in our model*:

$$s_{\mathrm{At}} = \frac{1}{1 + \left(s_{A0}^{-1} - 1 \right) \exp\left(1 - \gamma(1 + a_t/b_t) \right) \bullet t} \tag{18}$$

Proof. The proof is analogous to that in the first proposition.

If we observe the new result in the imperfect coevolution version of the model, we perceive new aspects. Thus, firstly, two very interesting terms appear now in the intra-dynamics within population *I*. This new factors are $C(a, \lambda)_t$, the instantaneous co-variance (within *I*) between location-current technical practice and absorptive capacity (notice that it can be negative, thus eroding progress), and λ_t which measures the average absorptive capacity of new institutional outcomes which come from *A*, and fuel sector *I*. Secondly, in Eq. (18), we see that the dynamics of s_{At}, depends of the sign of $1 - \gamma\left(1 + \frac{a_t}{b_t} \right)$. This is positive iff $\frac{a_t}{b_t} > \frac{1-\gamma}{\gamma}$. This imperfect coevolution process is perceived in historical cases.

We can find examples for these different cases of smooth/no-smooth coevolution in the economic geography dynamics which often accompanies growth during intense processes of technological transformation (British industrial revolution, post-WWII dynamics in Europe). In the British case, although the industrial revolution promoted a general migration from rural areas to cities in a coevolutionary way, there were cases in which the interaction between technical and geographical conditions favored migrations, whereas in other cases these mechanisms did not operate smoothly. The flat landscape of London or Bristol favored the settlement of new workers, while the difficult geography of the Highlands prevented this territory from such an expansion. The provision of infrastructures, institutional support, migrations, and technical advance coevolved underlying the geographical migrations and urban dynamics which led to the emergence of the large British cities in the nineteenth century; it can also explain the contention of other geographical locations. Note that the model results not only clarify and develop the coevolution argument, but also highlight roles for policy-makers in trying to remove blockages of coevolution and/or in promoting coevolution engines. In more complex systems (with randomness, entry/exit, externalities, and additional feed-backs, see the Appendix), the role of the policy-maker becomes more problematic. But this is just an introductory paper with illustrative purposes. Thus, instead of extending our tentatively proposed models, we prefer to devote the final section—section "4"—to present some of the many ideas, and applications that appear in the set of contributions to this issue. We will also sum up lines for future research.

4 The contributions to this SI and ways ahead

In this SI, we deal with the concept of coevolution in dynamic systems and its applications. Together with the contributors to this joint adventure, we highlight coevolution as a force operating within complex systems of different types. We briefly summarize below how new models, philosophical views, historical approaches, legal issues, policy implications, innovation economics, institutional thought, geography, and anthropology can be inspired by the coevolution notion that we have presented above.

The paper of M. Novak (2024) in this SI *"Sociologically influenced coevolutionary dynamics"* revolves around some sociological and political factors that can enrich previous studies (as those modelling ideology competition in Almudi et al. 2017). Whereas previous works draw on socio-economic determinants to explain the dynamic processes underlying the diffusion of ideological utopias in society, Novak extends the coevolution notion to the creative ideation of these worldviews. These processes are driven by coevolution mechanisms which play a fundamental role in explaining how different views on how to organize society (market-oriented, cultural, state, civil society), compete to shape the social ethos. In Almudi et al. (2017), the dynamic processes underlying each societal configuration would be driven by the efforts of individual agents (citizen commitment of resources to persuade others) and the discussion and learning processes involved. In her new context, Novak (2024) proposes to extend the coevolution analysis to introduce new factors: (i) the very infant phase of ideas/utopias creation with individual politically creative actions; (ii) how to identity the specific power dynamics that can shape, catalyze, and block the diffusion of ideas.

In the papers by Ongay (2024) and Pérez-Jara (2024), the coevolution notion is explored from the perspectives of biology, philosophy, and natural sciences. In Ongay's *"Cause and effect in biology, culture and the (extended) mind: a coevolutionary approach,"* it is discussed how coevolutionary characterizations of causation can help to embrace a pluralistic view of the structure of reality. The author explores, within different scientific realms (from biology to ethology, culture and mind), how the one-causation approach can lead to a narrow construction and application of science, which may be misleading. In that sense, the coevolution notion and the idea of multiple relations among processes can be a suitable way out of insufficient representations.

The contribution by Pérez-Jara (2024), *"The ontology of coevolution beyond economic systems"* claims for the existence of dynamic codetermination processes—with vertical and horizontal complex interactions among multiple domains—in almost all fields of reality. It is necessary to recognize the complex interplay of continuities and discontinuities that shape reality for a real understanding of complex phenomena. In this sense, the coevolutionary approach is not only useful to address socio-economic issues, but also to approach a wide variety of natural and human problems (from the emergence of increasingly complex states of matter-energy, to the origins, and organization of living systems).

From a historical coevolutionary viewpoint, Kitsikopoulos (2024) deals with the relationships between technological change, organizational adaptations, and growth (fabric size, the integration of novel techniques and transformations in the locus of production by the end of the eighteenth century) during the British industrial revolution. As compared with pre-industrial periods, we see in the paper how capital and labour requirements attached to very significant new technologies at that time (steam power, textiles, cotton industries, woollen activities) coevolved with standards of living, organizational patterns of production, markets and conditions of trade. All these aspects transformed the whole landscape of cities, firms, rural activities, and the scale of action. This coevolution processes unchained profound turbulences leading to modern economic growth.

Felix-Fernando Muñoz (2024)—in his *"coevolution of technology, markets and culture: the case of AI"*—poses a deep and original characterization of agents (human and organizational economic agents with sophisticated cognitive and ethical abilities) in coevolution, to explore the co-determined dynamics of digital technologies and AI, markets, and cultural transformations. The contemporary significance of this work is clear, and the properties obtained by Muñoz in his paper can be surprising to many: from his discussion of rationality, intelligence, the processes of technological and institutional coevolution, and certain fundamental principles of human action, he distinguishes between realistic expectations and (vs.) utopian beliefs on the possibilities of current technological advances.

The paper by Moreno-Casas (2024), *"A coevolutionary approach to institutional lock-in"* analyzes the potential coevolution between mental models and intuitional settings in alternative overall systems of socio-economic organization (socialism, capitalism, mixed economies). The author argues that in most (if not all) of the existing literature, there exists a unidirectional causation explanation for institutional change which only goes from mental models to institutional settings. This unidirectional link cannot explain institutional lock-in when mental models deviate from institutional settings. To deal properly with this type of unexplained institutional blockages, the author sets up a coevolutionary approach between mental models and institutional settings in which both aspects can change and become co-determined. This framework is deployed to heuristically analyze under which circumstances the institutional settings characterizing free markets, welfare state, and socialism can end up in societal lock-ins.

Vetsikas et al. (2024) uses the notion of coevolution to explore aspects related to the literature on national innovation systems. In their "*Exploring the coevolution of heterogeneous actors in national innovation systems,"* they study the interactions of the government, industry, academia, society, and finance in a dynamic systems approach for the case of Finland. They simulate results with alternative policy mixes in different regime parameters and detect interesting effects in multiple parts of the Finish economy. It is a suggestive application which connects in a very natural way the notion of coevolution with the stream of thinking on innovation systems.

The contribution by Yoguel and Lepratte (2024), *"Co-production, artificial intelligence and replication: the path of routines dynamics"* deals with replication processes of routines and their connections to technological solutions and the formation of dynamic capabilities at the corporate and, in general, at the organizational level in systems of production. They study coevolution involving these aspects in the case of AI advances. The authors explore how the three main elements (replication, technical solutions, and dynamic capabilities) coevolve in specific circumstances by conducting two different case studies: IMAGEAI, which is a case based on inter-organizational replication processes to offer technical solutions for AI-based neurological image analysis; and GEN-IT + HOSP-AI, a case-study which deals with intra-organizational replication processes to offer technical solutions for AI-based genomic analysis in the Electronic Health Records (HER) of health institutions. The results obtained shed new light into the debate about replication processes and how they coevolve with novel technological solutions and institutional and organizational frames in the private and public spheres.

We also incorporate in the collection of papers two articles connecting coevolution with the work on computational ABMs (growth with rapidly evolving industrial dynamics in some sectors—the Lobet, Llerena, and Lorentz's paper, and besides this, a micro-to-macro paper by Danilo Spinola and colleagues with strong implications for development policy).

Apart from these studies, there is also a paper by Roger Koppl, "*Of thoughts and things,"* that studies the coevolution of technology and culture for technical advances and historical cases related to anthropological records. Finally, the coevolution of law and technology is studied by Eckardt (2024) in her article on digital legislation within the digital platform economy in the EU. In her "*EU digital law and the digital platform economy,"* she shows how digital ecosystems coevolve at the basis of the platform economy and highlights the role of telecommunication monopolies, and oligopolistic positions. Eckardt presents empirical and conceptual tools to understand regulations, legislation, and competition policy aspects in the EU.

With this anticipation of the contents in the collection of papers, we close the introduction and open the discussion to the authors. A wide bundle of potential developments follows from the papers. Apart from the mathematical and computational illustrative applications of the coevolution notion, there are philosophical and sociological questions with implications for science and policy. The literatures on Institutional theory, History, Technology and Law studies, Anthropology, as well as

Micro and Macro-economic theory are clearly involved in this SI. Finally, it is clear that research on Innovation Systems, Geography, Industrial dynamics, and Growth can benefit from future developments around the coevolution notion. We believe that advances along these lines will come from complementary angles of heterodox economic thought.

Appendix

In this Appendix, we present an ABM for a two-sector economy in which a machine-producer Sector 1 coevolves with a downstream user industry (Sector 2). The complete analysis of a similar model can be seen in Almudi and Fatas-Villafranca (2021). In this Appendix, we seek to illustrate how the coevolution notion can be implemented for ABMs with random sources and radical complexity.

A coevolutionary two-sector ABM with Schumpeterian dynamics

We propose a model in which two sectors coevolve. In Sector 1 distinct and gradually improved varieties of machines are produced and sold to Sector 2. In Sector 2, different varieties of a consumption good are produced (and sold to a final market) by firms which buy machines to Sector 1. At time "0" the number of firms in both sectors is: Sector 1 $\left(i = 1, \dots, n_0\right)$, Sector 2 $\left(j = 1, \dots, m_0\right)$. We assign to each firm in Sector 1 an initial machine performance (in normalized terms and changing with time). The initial distribution of performances is $\left(x_{10}, \dots, x_{n_0 0}\right)$. Likewise, we set for each firm in Sector 2 an initial level of knowledge related to the use of machines. The evolving distribution of firm's knowledge -in relative and normalized terms- has at time "0" the vector $\left(X_{10}, \dots, X_{m_0 0}\right)$. We seed the distributions with a uniform distribution (0,1).

Sector 1

Prices and performance

At any time, we have a changing number of firms $\left(i = 1, \dots n_t\right)$ in Sector 1. These firms produce different varieties of a good that we call "machines". We assume boundedly-rational profit-seeking firms which compete in price p_{it} and machine performance x_{it}. Regarding prices, we will assume that firms in Sector 1 set prices by applying a mark-up ($\mu_{\text{it}} > 1$) over their expected unit cost. The price set up by firm i at t is as follows:

$$p_{\text{it}} = \mu_{\text{ti}} c^{e}_{\text{it}} \tag{19}$$

We propose a pricing routine in which the perceived elasticity of demand evolves. More precisely, each firm i establishes at any time the set of "relevant rivals" (depending on performance distance) drawing upon data from t-1. We can define this set as follows:

$$\Lambda_{\text{it}-1} = \left\{ \ell : \left| x_{\ell t-1} - x_{i-1} \right| < \sigma_i x_t^{max} \right\} \ \sigma_i \epsilon \ (0{,}1) \tag{20}$$

The intensity of direct competition is calculated by adding up the market share of the close rivals: $\sum_{\ell \in \Lambda_{\text{it}-1}} s_{\ell t-1}$. If we consider this aspect as being a determinant of demand elasticity, we can suppose that the mark-up fixed by firm i at t is as follows:

$$\mu_{\text{it}} = \frac{\eta + \sum_{\ell \in \Lambda_{it-1}} s_{\ell t-1}}{\eta + \sum_{\ell \in \Lambda_{it-1}} s_{\ell t-1} - s_{it-1}}, \ \eta > 1 \tag{21}$$

We will set up later on, how firms improve their machines through R&D.

Demand, production, and costs

We assume demand-driven production in Sector 1 so that $q_{\text{it}} = q_{\text{it}}^d$. Likewise, we consider that total costs include production costs and R&D costs. We assume that we have constant unit production costs (c). In order to set prices (see (19)), firms use expected unit costs. Let us introduce naive expectations so that the expected unit cost is as follows:

$$c_{\text{it}}^e = c + \frac{R_{\text{it}}}{q_{\text{it}}^e} = c + \frac{R_{\text{it}}}{q_{\text{it}-1}}, \ c > 0 \tag{22}$$

As we will see, once the structure of demand has been formed, and the exchanges between Sectors 1 and 2 have taken place, firms will know the effective production and unit costs. Then, the real profit for firm i at t will be as follows:

$$\pi_{\text{it}} = \left(p_{\text{it}} - c_{\text{it}} \right) q_{\text{it}} \ \ c_{\text{it}} = c + \frac{R_{\text{it}}}{q_{\text{it}}} \tag{23}$$

We consider that only profitable firms remain in the market. Then, the profit at a sector level will be as follows:

$$\pi_t^1 = \sum \pi_{\text{it}} \ \ \pi_{\text{it}} > 0$$

Now, we asume that firms devote to R&D a proportion of their profits with a lag:

$$R_{\text{it}} = r_i \pi_{\text{it}-1}, \ r_i \ \in (0{,}1) \tag{24}$$

Regarding demand, we assume that every firm in Sector 2 demands, at most, one unit of a specific variety of the "machines," and it uses it for producing her variety of the consumption good. For simplicity, we assume that every unit of capital totally depreciates at no cost in one period.

When selecting a specific capital-good firm at t, user-firms examine the levels of price and performance in Sector 1. Later on, we will set up the buying process. Now, if we define the set of users for firm i at t as Ω_{it}, we have $q_{\text{it}}^d = \#\Omega_{\text{it}}$

Innovation

Expression (25) is a performance improvement equation where R_{it} is R&D spending:

$$\frac{x_{it+1} - x_{it}}{x_{it}} = \gamma_{it} - \overline{\gamma}_t \tag{25}$$

where

$$\gamma_{it} = \phi\left(\frac{x_t^{max} - x_{it}}{x_t^{max}}\right) + (1 - \phi)\frac{R_{it}}{R_t^{max}} + u_{it}$$

$$\overline{\gamma}_t = \sum\nolimits_i x_{\text{it}}\gamma_{\text{it}},$$

$$u_{it} \sim \Gamma\left(u_{min}, u_{mode}, u_{Max}\right)$$
$$\phi \in [0,1]; u_{min} = u_{mode} = 0; u_{Max} \in [0,1]$$

The expression $\Gamma\left(u_{min}, u_{mode}, u_{Max}\right)$ is a density function of a triangular probability distribution. The parameter u_{Max} represents the span of the base of technical opportunities.

In eq. (25), we are assuming that the productivity of R&D depends on new knowledge which comes from two engines: R&D lab research activities; and imitation, represented through the gap in (25).

Entry-exit

Firms in Sector 1 whose profits become $\pi_{\text{it}} \leq 0$ exit the market. Likewise, at every t, one new firm enters the sector. With a small probability, the entrant will be a novel mutation with (random) fully-new characteristics. Otherwise, the new entrant will copy one of the incumbents with probability proportional to market shares.

Sector 2

The number of firms in Sector 2 changes ($j = 1 =, \dots, m_t$). Firms in Sector 2 produce varieties of a consumption-good (prices p_{jt}, qualities y_{jt}). These variables are mediated by the technical characteristics of the providers (Sector 1). Attending to the distribution of performances in Sector 1 $\left(x_{1t}, \dots, x_{n_t t}\right)$, and depending on the cognitive endowments in Sector 2 $\left(X_{1t}, \dots, X_{m_t t}\right)$ each firm j decides its provider. It buys one machine. We suppose that the overall production in Sector 2 is normalized to 1 and fully sold, with market shares arising from replicator dynamics.

We propose the following process of choice for each firm j, ($j = 1, \dots, m_t$):

(1) Firm j delimits the set of (cognitively) feasible options which is constrained by the firm cognitive capabilities $\rho_j \in (0, 1)$. This is an understanding radius (a firm-specific trait). The set of feasible providers for firm j is as follows: $\Xi_{\text{jt}} = \left\{ i \middle| \left|X_{\text{jt}} - x_{\text{it}}\right| < \rho_j \cdot x_t^{\text{Max}} \right\}$.

(2) Firm j chooses one of the feasible providers with a probability which is proportional to $\left\{ \alpha_1 x_{\text{it}} + (1 - \alpha_1)\left(1 - \frac{p_{\text{it}}}{\sum_i p_{\text{it}}}\right)\right\}$ $\alpha_1 \epsilon (0,1)$

(3) If the option implies non-positive profits, then firm j leaves the market. Otherwise, the quality of firm j becomes $y_{jt} = x_{jt}$.

Since this process works for all the firms in Sector 2, we can define the set of customers for every single firm in Sector 1: $\Omega_{it} = \{i - \text{customers}\}_t$.

Likewise, as long as a firm uses one specific variety of capital-good (with a specific technology level), we assume that this level of performance becomes the firm's cognitive endowment for the next period, $X_{jt+1} = x_{jt}$.

The market

We state market competition in Sector 2. We have defined how to get the quality of firm j, y_{jt}. Regarding price, we assume that firms apply a pricing rule. Then, we have as follows:

$$p_{jt} = \left(\frac{\delta}{\delta - s_{jt-1}}\right) p_{\text{mach}} \tag{25}$$

where p_{mach} is the cost of the chosen machine and δ the elasticity of demand (higher than one). We define a fitness level for each firm j, which combines quality and price so that:

$$f_{jt} = \alpha_2 \frac{y_{jt}}{y_t^{\max}} - (1 - \alpha_2) \frac{p_{jt}}{p_t^{\max}}, \qquad \alpha_2 \in (0,1)$$

It is clear that we are representing both dimensions as related to maximum quality and price in Sector 2 at t. Now, from this fitness variables, we fix the market process in Sector 2 as follows:

$$\frac{s_{jt+1} - s_{jt}}{s_{jt}} = f_{jt} - \bar{f}_t, \ \bar{f}_t = \sum_j s_{jt} f_{jt}. \tag{26}$$

Entry-exit

Firms in Sector 2 with a share lower than 0.01 leave the market, and we assume that one new firm enters the sector at any time.

With a small probability, the new entrant will be a fully new mutation. Otherwise, the new entrant will copy one incumbent, with probability proportional to market shares.

An illustrative result

For a plausible and representative parametric run simulated from the model, we present in Fig. 2 the emergent paths of the Herfindhal concentration index, depicted for both sectors (sectors one and two in coevolution). We see (Fig. 2) how a more concentrated structure endogenously emerges in the machine-producer sector (Sector 1), and a more atomistic structure emerges in the consumption industry (Sector 2).

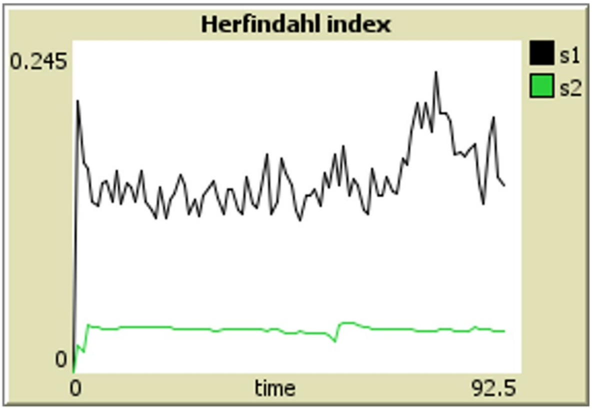

Fig. 2 Herfindhal index paths with Sectors 1 and 2 in coevolution

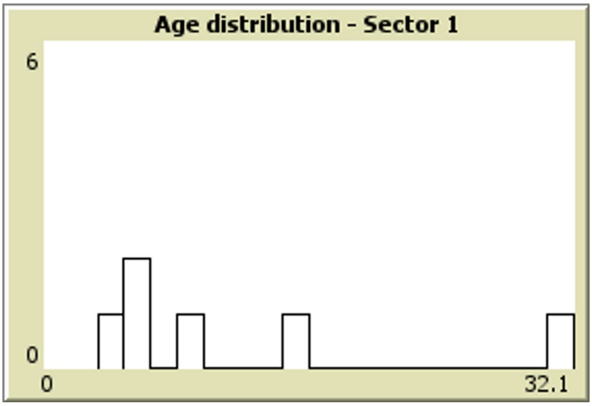

Fig. 3 Distribution of firm ages at $t = 89$ for the run in Fig. 2, Sector 1

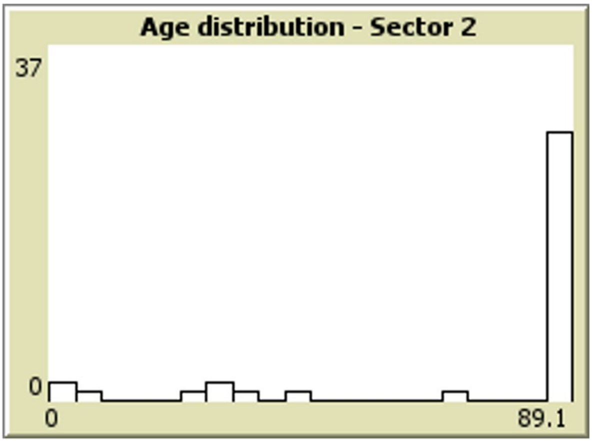

Fig. 4 Distribution of firm ages at $t = 89$ for the run in Fig. 2, Sector 2

Likewise, in Figs. 3 and 4, we can see the heterogeneous distributions of firm ages in both sectors at t = 89, the last period in the run, corresponding to the simulation in Fig. 2.

In Figs. 3 and 4, we represent the time steps of life, that is to say the firms' life-span, in the horizontal axis, and the corresponding number of firms with the specific ages (life-spans) at t = 89 in the vertical axis. We do not seek to explain anything exhaustively, just we try to show how the coevolution notion explained in the paper can be used in ABM settings. In any case, attending to Fig. 3 and Fig. 4 we see that, from the simulation of our coevolution model, very different age distributions emerge for each sector. We can think about supplier-driven innovation sectors and downstream (coevolving) sophisticated users as potential fields of application for this framework.

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Data availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

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