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Inspection and maintenance of a system with a bypass component

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ABSTRACT

We present an inspection and maintenance model for a two-component lubrication system, filter and bypass valve, with applications to centralized lubrication systems. It presents significant differences from redundant systems in previous studies on cold, warm or hot stand-by components. These are the dissimilarity between the filter and the bypass, as the latter can induce catastrophic damage after a long working period, and the stochastic dependence between the filter and the bypass valve. Inspection and testing is focused on the valve, and only if it fails to open on inspection, or it is found to be open, is the filter inspection triggered. Preventive maintenance is mainly concerned with the filter, which is replaced periodically and also when an inspection detects an open valve or a clogged filter. The sensitivity analysis reveals that the optimum policy depends more on the parameters defining the lifetime of the filter than on those of the valve.

1. Introduction

A bypass is an alternative channel for fluids to flow when the main way is obstructed or fails. These systems can be found in cars, for example, to redirect brake fluid to prevent leaks, which are particularly undesirable while driving. When pipelines are blocked or undergoing maintenance, a bypass is used to divert their contents, namely, water, petroleum or gas, so that the supply is not interrupted. Bypasses are installed not only as a back-up to guarantee a continuous flow of fluid, but also to reduce excessive flow pressures. Where the fluid flow is essential for a system to work, the bypass is a safeguard against a catastrophic failure.

Filters are designed to prevent harmful particles in fluids from entering a main unit. They are therefore critical elements in many applications [1]. Heiden et al. [2] remark the systemic impurities in biodiesel fuels and also the tighter filtration required in the latest generation of high pressure common rail diesel engines. Oil filters protect lubricated components from wear and damage caused by contaminants. The work in [3] analyzes condition monitoring and oil filtration in wind turbines. This study enhances the need for maintenance as a large number of them are out of warranty, replacement costs of most subsystems are very high and early detection of deterioration prevents catastrophic failure.

When the filter is ok, there is a full oil flow through it. This protection is reinforced by a bypass valve which allows the oil to flow if the filter is clogged due to the accumulated particles. The pressure difference between both sides of a clogged filter opens a safety valve, allowing the oil to bypass the filter and reach the main system, preventing dry running. The unfiltered oil can cause a permanent and catastrophic damage to the engine if the filter condition remains undetected for a long enough period. However, dirty oil with particles is preferable to no oil at all [4] . If the valve does not open, the pressure difference increases dramatically over time, as particles continue to fill the filter, which will eventually break or burst. From that moment on, the main system (e.g. an engine) operates without lubrication and therefore its catastrophic failure will occur (Gomes et al. [5]), unless the maintainer notices the burst, intervening in time to shut the engine down.

This work focuses on the inspection and maintenance of a two-component lubricating system: the filter and the bypass valve. The case study refers to centralized lubrication systems present in heavy-duty equipment (petrochemical plants, wind or hydraulic turbines, marine engines, etc.). The time to filter failure is a two-stage process: The first stage lasts from the moment that a clean filter starts working until it becomes clogged. The second stage covers from that time until the clogged filter bursts. When this catastrophic failure occurs, there is an additional risk that the maintainer cannot intervene in time to stop the dry run. It is important to note that a catastrophic failure of the filter cannot occur if the valve works, since the bypass relieves the pressure on the filter, i.e., the second stage can be infinite. Inspection and maintenance of filters and valves are therefore important. The studies in Bhandari et al. [6] and Trotta et al. [7] indicate that proper scheduling of maintenance in valve systems is essential for facilities to remain safe. Previous papers have highlighted inspection models in the context of valves. Alfares [8] points out the balance between safety

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and cost when inspecting valves in a petrochemical plant, and thus the interest in obtaining cost optimal policies. Cavalcante et al. refer to a natural gas supply network in [9] and to isolation valves in protection system in [10] . Flage [11] considers safety critical valves as used in offshore oil and gas production and transportation systems.

The bypass can initially be seen as a redundant component. However, the model in this paper presents significant differences with respect to the cold stand-by systems in [12,13] since the bypass may fail during idle periods and thus, there is no zero failure rate. This study also introduces important variations with respect to previous research on warm stand-by systems. The switch from the warm stand-by mode to the operational mode as well as the inspection procedure, is different from those in [14,15]. The study in [15] focuses on warm stand-by systems but does not include inspection. The bypass only works on demand when the filter is clogged, causing the valve to open. The assumptions in [16] present a stand-by unit that switches on when the primary unit in a cooling equipment fails. However, there is also a strong distinction in this case, as both components can develop the cooling function in the same way. In contrast, lubricating through the bypass with dirty oil can damage the main system in the long run. Other papers on warm stand-by units have studied identical components [17] with the standby unit operating under milder conditions than the active one [18]. The works in [19–21], assume that the active and stand-by units alternate the working state to reduce the probability of failure. This type of switch does not apply to a bypass, which is a temporary solution.

Maintenance of multi-unit systems has to assess not only the risk of each unit failing separately, but also possible interactions between them. Dependence between units is reported in many engineering systems: critical and auxiliary parts [22], a gearbox system [23] or in bridge maintenance [24]. If it is ignored, the possibility of failure may be underestimated, making maintenance procedures less effective, which in turn can be responsible for missed project deadlines. The works in [25,26] take into account the stochastic dependence between components although not in the context of stand-by components. The interaction between the filter and the bypass is assumed in this paper.

We present a new model for inspection and maintenance of redundant systems not covered by previous studies on cold, warm or hot stand-by components. The differences are as follows:

1. The bypass valve and the filter are different units in terms of their state space and lubrication effects:

   Regarding the state space, the bypass valve presents two possible states, good or failed. In the first case it works on demand, but in the second case it does not open when needed. The filter can be in one of three states, good, clogged (defective) and blown-out (failed). The defective state triggers the activation of the bypass. The bypass activation prevents the filter from blowing out. The operational effects of both units are also dramatically different, as the use of a bypass is a temporary solution, which in turn may have a negative long-term impact on the lubricated system. Thus, we model that a long enough period of lubrication with dirty oil through the bypass can damage the main system being lubricated.

2. The inspection procedure. We define a periodic inspection of the bypass valve and an opportunity-based inspection of the filter. Only if the valve is found to be open or failed on inspection is the filter also inspected.

3. The type of stochastic dependence between the filter and the bypass:

   When the filter is in the good state, it is not affected by valve failures as there is no need to use the latter. If the filter is clogged, it will not burst if the valve opens on demand. However, if the valve does not open, the filter will explode when the pores of the filter are filled with enough particles. A defective distribution models the time from filter clogging to bursting. Defective distributions have been used in [27] to describe heterogeneous populations. As far as we know, this is the first time they are used to model interactions between components.

We model the two-stage failure process of the filter by means of the delay-time concept (Christer [28]) which has been used in previous references as [9–11]. The defective distribution of this paper is a new approach to the delay time which is infinite when the filter is clogged and the valve works. To our knowledge, this idea has not been considered in previous literature. The text by Feller [29] is a good reference for a deeper study of defective distributions.

This paper is structured as follows: The hybrid inspection and maintenance model and the cost function are presented in Section 2. They are also compared with the pure maintenance model without inspections. The latter is also of interest for real applications. Section 3 is devoted to the numerical study, which aims at providing useful information to maintainers based on the range of application of the new model with respect to the values of the parameters. In particular, we analyze the effect of filter variability. As the random times involved in this problem are sometimes not directly observable, we give in this section some guidelines based on reasonable assumptions for their evaluation. The conclusions are in Section 4.

2. The model

Assumptions:

- The valve only has one type of failure when a closed valve does not open if there is a demand of use. We neglect the possibility that once the valve is open and the bypass starts working, it will close later as this is very unusual because the overpressure keeps the bypass open.
- The filter can present three states, good, defective and failed. The defective state occurs when the filter is clogged and the failed state when it bursts. The latter only happens if the valve does not open when it is required to do so.
- The valve failure and the defective state of the filter are only detected by inspection.
- The failure of the filter (when it bursts) is unrevealed with probability q. This condition refers to those cases where the main system cannot be stopped in time from working in dry condition.
- If the valve does not open, the clogged filter lubricates the engine with clean oil until the clog is detected on inspection or the filter bursts, whichever comes first.
- The catastrophic failure of the engine is detected as soon as it occurs. It can happen under the following events:
  1. There is a lack of lubrication because the valve does not open, then the filter bursts and this state of the filter is unrevealed.
  2. The valve works on demand but the time that the dirty oil lubricates the engine before the clogged filter is detected is too long.

Notation:

- c_1: The valve, also called bypass or bypass-valve.
- c_2: The filter.
- X: time to failure of c_1 with $f_X(x)$, $F_X(x)$ and $\bar{F}_X(x)$ the corresponding density, cumulative distribution and reliability functions, respectively.
- Y: time span until the filter is clogged with $f_Y(y)$, $F_Y(y)$ and $\bar{F}_Y(y)$ the density, cumulative distribution and reliability functions.
- Z_0: delay time of c_2, that is, from the moment when the filter is clogged until it bursts.
- H_0: random time until the engine fails catastrophically when it is lubricated by the bypass with dirty oil.

- q: probability that the filter failure not be revealed when it bursts.
- R_j: indicator that the valve is replaced at jT in the lubricating system in use.
- c_0: cost of valve inspection.
- c_r: cost of replacing the valve and inspecting the filter.
- cd_1: cost rate per unit of time while c_2 (filter) is defective and c_1 (bypass valve) is failed (the bypass does not work).
- cd_2: cost rate per unit of time while c_2 is defective and the bypass works.
- c_{F_1}: cost of replacement of the system (valve and filter) when the engine which is being lubricated is not catastrophically damaged.
- c_{F_2}: cost of replacement of the system (valve and filter) and repair of the engine when it is catastrophically damaged.
- c_{PM}: cost of preventive replacement of the filter-valve system.
- τ: length of a cycle. Period until the filter-valve system is replaced.
- $C(\tau)$: cost of a cycle.

Decision variables:

- T: inspection interval.
- M: maximum number of inspections in a renewal cycle.

Both the delay time, Z_0, and H_0 are defective distributions. A defective distribution is given by a mixture that presents a non-zero probability of being infinite. It can be used to model life lengths in the case of immune individuals, or systems free of a particular defect so they cannot fail due to that defect.

$$Z_0 = \begin{cases} Z, & \text{the valve does not open when there is a demand of use} \\ \infty, & \text{otherwise} \end{cases}$$

with $f_Z(z)$, $F_Z(z)$ and $\bar{F}_Z(z)$ the density, cumulative distribution and reliability functions of Z. In other words, Z_0 is given by the following mixture:

$$P(Z_0 > z) = pP(Z > z) + (1-p), \quad z > 0$$

with p being the probability that the valve not open when it is required to do so. The value of p is obtained for each inspection interval. Thus, the probability that the valve in use, installed at jT, not be available when needed in the interval $(iT, (i+1)T)$, $i \geq j$, is

$$\int_{iT}^{(i+1)T} f_X(x - jT) \int_{x}^{(i+1)T} f_Y(y) dy dx$$

Z_0 models a particular stochastic dependence between the filter and the valve, since the failure of the latter only affects the filter when it is clogged (defective).

H_0 is also represented by a defective distribution:

$$H_0 = \begin{cases} H, & \text{the valve opens when there is a demand of use} \\ \infty, & \text{otherwise} \end{cases}$$

$$P(H_0 > h) = (1-p)P(H > h) + p, \quad h > 0$$

$f_H(h)$ and $\bar{F}_H(x)$ are the corresponding density and reliability functions of H.

The following inspection policy is proposed: The valve is inspected at times jT, $j = 1, 2, \ldots, M$, checking if it works. As in Berrade et al. [30], we consider the following conditional inspection of the filter, depending on the state of the valve:

- The valve does not open on inspection, then the filter is inspected too. If it is in the defective state (clogged), both components are replaced. If the filter is ok, it is left as it was and only the valve is replaced.
- The valve is found to be open on inspection, which is an indicator of a clogged filter. The pressure of the oil when the filter is clogged causes the valve to open. As in the previous case, the filter and the valve are replaced.
- If the valve is found to be ok on inspection, then the filter is not inspected and both components are left as they are. The maintainer has to wait until the following inspection time and repeat the procedure.

A cycle is completed when the two-component system (valve and filter) is replaced. This occurs whichever of the following three events comes first:

- The filter bursts (corrective replacement).
- The filter is detected to be clogged on inspection (preventive replacement).
- At MT (preventive replacement).

This paper aims at modeling the inspection and maintenance of the lubrication system consisting of the filter and the valve. Nevertheless, lack of oil or unfiltered oil can also catastrophically damage the engine which will have to be repaired. The induced cost is taking into account in the model. Other engine maintenance is not considered.

According to the inspection policy, the valve can be replaced several times before the whole system (filter and valve) is changed. In what follows, R_j denotes that the valve is replaced at jT ($j = 0, 1, 2, \ldots, M-1$) in the system in use and $p_j(T)$ the corresponding probability:

$$P(R_j) = p_j(T)$$

Next, the strategy for obtaining $p_j(T)$ is outlined:

The probability that the valve is found to be failed at $t = T$:

$$p_1(T) = F_X(T)$$

The probability that the valve is found to be failed at $t = 2T$:

$$p_2(T) = (F_X(2T) - F_X(T)) + F_X(T)F_X(T) = (F_X(2T) - F_X(T)) + p_1^2(T)$$

In the first term, the valve installed at $t = 0$ (first valve) fails in $(T, 2T)$. In the second term, the first valve fails in $(0, T)$, it is replaced on inspection at T, and the replacement fails again in $(T, 2T)$. That is, the valve that was new at T does not survive an interval of T time units.

The probability that the valve is found to be failed at $t = 3T$:

$$p_3(T) = (F_X(3T) - F_X(2T)) + (F_X(2T) - F_X(T))p_1(T) + F(T)p_2(T)$$

In the first term, the first valve fails $(2T, 3T)$. In the second term, the first valve fails in $(T, 2T)$ and after being is replaced at $2T$, this new valve does not survive an interval of T time units, implying that the inspection at $3T$ will detect a failed valve. In the last term, the first valve is replaced at T and the probability of detecting a valve that fails $2T$ time units later is $p_2(T)$.

Then, the following recursive formula applies:

$$p_j(T) = \sum_{k=1}^{j} (F_X(kT) - F_X((k-1)T))p_{j-k}(T) \quad (1)$$

In (1) it is assumed that

$$p_0(T) = 1$$

which is consistent with a new valve being installed at $t = 0$.

Fig. 1 describes the two-component system, valve and filter, as well as the different events that lead to the completion of a renewal cycle due to filter failure. The valve is represented to be in good state after the inspection at iT, although it is not necessarily the one installed at $t = 0$, but it could have been replaced on previous inspections at jT, $j = 1, 2, \ldots, i$.

There are two scenarios, whether the valve opens on demand or not:

- Scenario 1: The valve fails after a random time X and the filter is clogged at Y with $Y > X$. The valve does not open when the filter is clogged in $(iT, (i+1)T)$: in this case, the oil lubricates through the filter until the filter bursts (failed state) after a random time Z. This type of failure is denoted $F_1^{(i)}$. Two sub-cases can then occur: (i) The failed state of the filter is detected immediately (with probability $1-q$), preventing the catastrophic failure of the engine and incurring a cost c_{F_1} which accounts for the replacement of the valve and the filter; (ii) The failure of the filter is not observed (with probability q) and shortly after the filter fails, the engine is catastrophically damaged. The derived cost is c_{F_2} ($> c_{F_1}$) since, apart from replacing the valve and the filter, the engine has to be repaired.

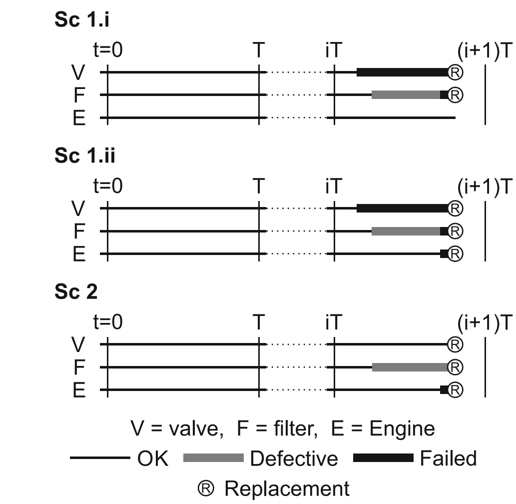

Fig. 1. Different scenarios for replacement on failure.

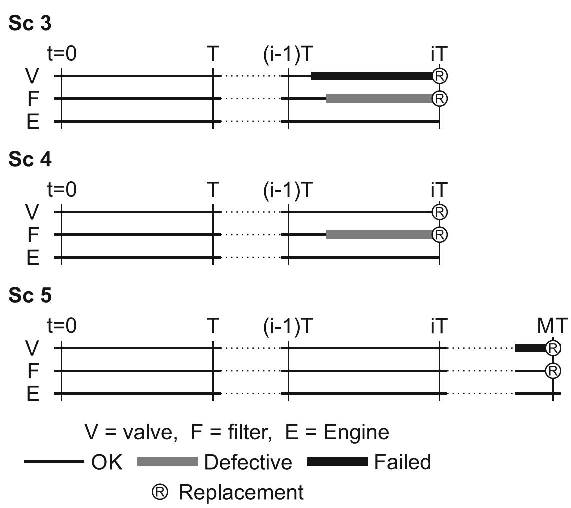

Fig. 2. Different scenarios for preventive replacement.

- Scenario 2: The valve opens when the filter is clogged (defective state) in $(iT, (i+1)T)$: in this case, the dirty oil enters the engine, which is catastrophically damaged after a random time, H, of dirty lubrication. $F_2^{(i)}$ denotes this type of failure. The cost is c_{F_2} as in the previous case.

Fig. 2 presents two additional scenarios corresponding to a cycle completed on preventive replacement. They occur depending on whether or not the valve opens when the filter is clogged. Several replacements of the valve can occur on inspections at jT, $j = 1, 2, \ldots, i-1$. Hence, the valve is represented in good state after the inspection at $(i-1)T$.

- Scenario 3: The valve does not open when the filter is clogged in $((i-1)T, iT)$. The failure of the valve and the clogged filter are detected on inspection at iT. This event is denoted $S_1^{(i)}$.
- Scenario 4: The valve opens when the filter is clogged in $((i-1)T, iT)$. Inspection at iT reveals the open valve and the clogged filter. The period of dirty lubrication is not been long enough to damage the engine. This event is $S_2^{(i)}$.

Fig. 2 also presents the following scenario for preventive replacement:

- Scenario 5: The filter has not become clogged after M inspections. The valve can be either failed, as shown in Fig. 2, or good.

Both the filter and the valve are replaced in either Scenario 3, Scenario 4 or Scenario 5. The cost of preventive replacement in the last three scenarios is c_{PM} and usually $c_{PM} < c_{F_1}$.

Section 2.1 presents the algebra for scenarios 1 and 2, and Section 2.2 for scenarios 3, 4 and 5 with piece-wise expressions of the expected cycle length and the expected downtime. Section 2.3 contains the full expression of both the expected cycle length and the cost function.

2.1. Replacement on failure and downtime

The following scenarios lead to the replacement of the system (filter and valve) in $(iT, (i+1)T)$ for $i = 0, 1, 2, \ldots, M-1$:

- **Scenario 1** (Failure $F_1^{(i)}$): c_1 (the bypass valve) fails in $(iT, (i+1)T)$ before c_2 (the filter) enters the defective state. The valve does not open when it is required to do so. The clogged filter bursts due to the increasing pressure of the oil.

The corresponding probabilities conditional to R_j, that is to the event that a new valve was installed at jT, $j = 0, 1, 2, \ldots, i$:

$$P(F_1^{(i)}|R_j) = \int_{iT}^{(i+1)T} f_X(x - jT) \int_{x}^{(i+1)T} f_Y(y) F_Z((i+1)T - y)\, dy\, dx \tag{2}$$

The first integral in (2) indicates that the valve installed at jT remains in good state until it fails at x in $(iT, (1+1)T)$. The second integral means that the filter becomes clogged at y in the same interval after x and therefore the valve is not available when needed. The term $F_Z((i+1)T-y)$ describes that the filter bursts before inspection at $(i+1)T$.

The replacement in $(iT, (i+1)T)$ involves only filter and valve with probability $(1-q)P(F_1^{(i)}|R_j)$. In addition, the engine is also repaired or replaced when the maintainer cannot prevent its catastrophic damage with probability $qP(F_1^{(i)}|R_j)$.

- **Scenario 2** (Failure $F_2^{(i)}$): c_1 has not failed by the time that c_2 enters the defective state in $(iT, (i+1)T)$. The oil starts to flow through the valve which opens when the filter becomes defective. The engine is catastrophically damaged after a period, H, of lubrication with dirty oil.

$$P(F_2^{(i)}|R_j) = \int_{iT}^{(i+1)T} f_Y(y) \bar{F}_X(y - jT) F_H((i+1)T - y)\, dy \tag{3}$$

The integral in (3) describes with $f_Y(y)$ that the filter becomes clogged at time y in $(iT, (i+1)T)$. The expression $\bar{F}_X(y - jT)$ indicates that the valve installed at jT remains good at y and hence the bypass is available when the filter is defective. The expression $F_H((i+1)T - y)$ means that the catastrophic failure of the engine occurs due to dirty oil lubrication before the open valve is detected at $(i+1)T$.

The parts of the expected downtime conditional to R_j, derived from scenarios 1 and 2:

$$E[D_{F_1^{(i)}}|R_j] = \int_{iT}^{(i+1)T} f_X(x - jT) \int_{x}^{(i+1)T} f_Y(y) \left(\int_0^{(i+1)T-y} z f_Z(z) dz \right) dy\, dx \tag{4}$$

$$E[D_{F_2^{(i)}}|R_j] = \int_{iT}^{(i+1)T} f_Y(y) \bar{F}_X(y - jT) \left(\int_0^{(i+1)T-y} h f_H(h) dh \right) dy \tag{5}$$

The downtime in (4) is incurred while the filter is in the defective state until it bursts whereas in (5) occurs since the moment that the valve opens until the time that the catastrophic damage of the engine occurs due to lubrication with dirty oil.

The parts of the expected cycle length in scenarios 1 and 2 are respectively:

$$E[\tau_{F_1^{(i)}}|R_j] = \tag{6}$$

$$\int_{iT}^{(i+1)T} f_X(x-jT)\int_{x}^{(i+1)T} f_Y(y)\left(\int_0^{(i+1)T-y}(y+z)f_Z(z)dz\right)dydx$$

$$E[\tau_{F_2^{(i)}}|R_j] = \int_{iT}^{(i+1)T} f_Y(y)\bar{F}_X(y-jT)\left(\int_0^{(i+1)T-y}(y+h)f_H(h)dh\right)dy \tag{7}$$

2.2. Preventive replacement and downtime

The system (filter and valve) is preventively replaced at iT ($i = 1,2,\dots,M$) if the filter is found to be defective on inspection at iT and this occurs under the following two events:

- **Scenario 3** (Event $S_1^{(i)}$): c_1 fails in $((i-1)T, iT)$ before c_2 enters the defective state. Inspection at iT reveals the failure of the valve and also the defective state of the filter in time to prevent the filter bursting.
- **Scenario 4** (Event $S_2^{(i)}$): c_1 has not failed before c_2 enters the defective state in $((i-1)T, iT)$. Therefore the bypass works and the valve is found to be open on inspection, implying that the filter is clogged. Thus $Z_0 = \infty$. The operating time of the bypass does not induce a catastrophic damage in the engine.

Next, some calculations conditional to R_j, $j = 0,1,2,\dots,i-1$ are obtained.

The probabilities of scenarios 3 and 4:

$$P(S_1^{(i)}|R_j) = \int_{(i-1)T}^{iT} f_X(x-jT)\left(\int_x^{iT} f_Y(y)\bar{F}_Z(iT-y)dy\right)dx \tag{8}$$

$\bar{F}_Z(iT-y)$ in (8) indicates that the filter has not burst before inspection at iT.

$$P(S_2^{(i)}|R_j) = \int_{(i-1)T}^{iT} f_Y(y)\bar{F}_X(y-jT)\bar{F}_H(iT-y)dy \tag{9}$$

The expression $\bar{F}_H(iT-y)$ in (9) indicates that the engine has not been catastrophically damaged due to lubrication through the bypass valve when the latter is found to be open on inspection at iT.

The corresponding part of the expected downtime from scenario 3:

$$E[D_{S_1^{(i)}}|R_j] = \int_{(i-1)T}^{iT} f_X(x-jT)\left(\int_x^{iT}(iT-y)f_Y(y)\bar{F}_Z(iT-y)dy\right)dx \tag{10}$$

In scenario 4, the valve works when it is required to do so and remains opens until inspection at iT. Although the main system is lubricated, it can be affected by the dirty oil. The longer it remains in this condition the worst and hence, we also assume a downtime during this period. The term of the expected downtime from scenario 4:

$$E[D_{S_2^{(i)}}|R_j] = \int_{(i-1)T}^{iT}(iT-y)f_Y(y)\bar{F}_X(y-jT)\bar{F}_H(iT-y)dy \tag{11}$$

The parts of the expected cycle length derived from scenarios $S_1^{(i)}$ and $S_2^{(i)}$:

$$E[\tau_{S_1^{(i)}}|R_j] = E[\tau_{S_2^{(i)}}|R_j] = iT \tag{12}$$

- **Scenario 5**: Replacement at MT occurs if the filter remains good at MT. The corresponding probability is given by $\bar{F}_Y(MT)$. No downtime is derived in this case, as the filter remains unclogged.

Observe that replacement at MT also happens under the events $S_1^{(M)}$ and $S_2^{(M)}$ corresponding to scenarios 3 and 4, respectively. In both cases the filter is clogged at MT.

2.3. Cost function

This model focuses on the maintenance of the lubrication system of an engine, which in some cases will also be repaired. All other engine maintenance is out of the scope of this model. Therefore, a cycle τ is completed when the two components, filter and valve, are replaced.

The expected length of a cycle is

$$E[\tau] = \sum_{i=0}^{M-1}\sum_{j=0}^{i}\left(E[\tau_{F_1^{(i)}}|R_j] + E[\tau_{F_2^{(i)}}|R_j]\right)p_j(T) + \sum_{i=1}^{M} iT\sum_{j=0}^{i-1}\left(P(S_1^{(i)}|R_j) + P(S_2^{(i)}|R_j)\right)p_j(T) + MT\bar{F}_Y(MT) \tag{13}$$

The number of inspections in a cycle, K, verifies:

$$P(K=0) = P(F_1^{(0)})$$

For $K = 1,2,\dots,M-1$

$$P(K=i) = \sum_{j=0}^{i}\left(P(F_1^{(i)}|R_j) + P(F_2^{(i)}|R_j)\right)p_j(T) + \sum_{j=0}^{i-1}\left(P(S_1^{(i)}|R_j) + P(S_2^{(i)}|R_j)\right)p_j(T)$$

For $K = M$:

$$P(K=M) = \sum_{j=0}^{M-1}\left(P(S_1^{(M)}|R_j) + P(S_2^{(M)}|R_j)\right)p_j(T) + \bar{F}_Y(MT)$$

Inspection at iT results in replacement of the valve if it is found to be failed on inspection and the filter has not entered the defective state. Let I_i ($i = 1,2\dots,M-1$) denote this event. In case that c_2 has entered the defective state before iT, then the whole system is preventively replaced.

$$P(I_i|R_j) = \int_{(i-1)T}^{iT} f_X(x-jT)\overline{F}_Y(iT)dx$$

Then

$$P(I_1) = \int_0^T f_X(x)\overline{F}_Y(T)dx$$

For $i = 2\dots,M-1$, it follows that

$$P(I_i) = \sum_{j=0}^{i-1} P(I_i|R_j)p_j(T)$$

Denoting by J the number of replacements of c_1, ($J = 0,1,2,\dots,M-1$), it follows that

$$J = \sum_{i=1}^{M-1} I_i$$

Thus, the mean number of replacements of c_1 in a renewal cycle, excluding that at MT:

$$E[J] = \sum_{i=1}^{M-1} E[I_i] = \sum_{i=1}^{M-1} P(I_i)$$

The filter (c_2) is inspected at iT if the valve fails to open on that inspection and also if it is found to be open. This second case occurs when the filter is clogged, leading to the replacement of the system. Therefore the number of replacements of c_1, excluding those that imply the replacement of the whole system follows the same distribution than J in model 1.

The expected downtime in a cycle:

$$E[D] = \sum_{i=0}^{M-1}\sum_{j=0}^{i}\left(E[D_{F_1^{(i)}}|R_j] + E[D_{F_2^{(i)}}|R_j]\right)p_j(T) + \tag{14}$$

$$\sum_{i=0}^{M}\sum_{j=0}^{i-1}\left(E[D_{S_1^{(i)}}|R_j]+E[D_{S_2^{(i)}}|R_j]\right)p_j(T)$$

The expected cost derived from downtime in a cycle:

$$E[C_d]= \quad (15)$$
$$\sum_{i=0}^{M-1}\sum_{j=0}^{i}\left(cd_1E[D_{F_1^{(i)}}|R_j]+cd_2E[D_{F_2^{(i)}}|R_j]\right)p_j(T)+$$
$$\sum_{i=0}^{M}\sum_{j=0}^{i-1}\left(cd_1E[D_{S_1^{(i)}}|R_j]+cd_2E[D_{S_2^{(i)}}|R_j]\right)p_j(T)$$

The expected cost of a cycle:

$$E[C(\tau)]= \quad (16)$$
$$c_0E[K]+c_rE[J]+$$
$$\sum_{i=0}^{M-1}\sum_{j=0}^{i}\left((1-q)c_{F_1}P(F_1^{(i)}|R_j)+qc_{F_2}P(F_1^{(i)}|R_j)+c_{F_2}P(F_2^{(i)}|R_j)\right)p_j(T)+$$
$$c_{PM}\sum_{i=1}^{M}\sum_{j=0}^{i-1}(P(S_1^{(i)}|R_j)+P(S_2^{(i)}|R_j))p_j(T)+c_{PM}\bar{F}_Y(MT)+E[C_d]$$

with $E[C_d]$ in (15). The previous equation shows that the cost of catastrophic engine damage (c_{F_2}) is incurred if the filter failure remains undiscovered and also if lubrication through the bypass is too long.

The objective cost function, $Q(T,M)$, corresponds to that of the key theorem of the renewal–reward processes [31]. $Q(T,M)$ is given by the ratio of the expected cost of a cycle to its expected length

$$Q(T,M)=\frac{E[C(\tau)]}{E[\tau]}$$

with $E[\tau]$ in (13) and $E[C(\tau)]$ in (16).

The following step is to find the optimum policy, $(T^\star,M^\star)$, that is

$$(T^\star,M^\star)=\arg\min_{T,M}\ Q(T,M)$$

In the numerical examples $Q^\star$ will denote the optimum cost, thus $Q^\star=Q(T^\star,M^\star)$.

2.4. Case $M=1$

The system described so far with a bypass being a separate unit from the filter, is usual in heavy-duty engines [32]. However, many small engines have oil filters with a built-in bypass. In these cases, neither the valve nor the filter is inspected, but the joint system is replaced at age T or when an indicator of its use (working hours, kilometers) reaches a given threshold. In what follows, we focus on the particular case of no inspection, that is, a pure maintenance policy.

- F_1: The valve fails in $(0,T)$ before the filter enters the defective state. Hence, the valve does not open when it is required to do so.
- F_2: The valve has not failed by the time that the filter enters the defective state in $(0,T)$. The valve opens when the filter becomes defective. The unfiltered oil damages the main system before detecting the filter condition.

$$P(F_1)=\int_0^T f_X(x)\int_x^T f_Y(y)F_Z(T-y)dydx$$

$$P(F_2)=\int_0^T f_Y(y)\bar{F}_X(y)f_Y(y)F_H(T-y)dy$$

The corresponding terms of the expected downtime derived from F_1 and F_2:

$$E[D_{F_1}]= \quad (17)$$
$$\int_0^T f_X(x)\int_x^T f_Y(y)\left(\int_0^{T-y}zf_Z(z)dz\right)dydx$$

$$E[D_{F_2}]=\int_0^T f_Y(y)\bar{F}_X(y)\left(\int_0^{T-y}hf_H(h)dh\right)dy \quad (18)$$

The terms of the expected cycle length on events F_1 and F_2:

$$E[\tau_{F_1}]= \quad (19)$$
$$\int_0^T f_X(x)\int_x^T f_Y(y)\left(\int_0^{T-y}(y+z)f_Z(z)dz\right)dydx$$

$$E[\tau_{F_2}]=\int_0^T f_Y(y)\bar{F}_X(y)\left(\int_0^{T-y}(y+h)f_H(h)dh\right)dy \quad (20)$$

The following two events correspond to preventive replacement for $M=1$.

- S_1: c_1 fails in $(0,T)$ before c_2 enters the defective state. Inspection at T reveals the failure of the valve and also the defective state of the filter.
- S_2: c_1 has not failed before c_2 enters the defective state in $(0,T)$. Therefore the bypass works and the valve is found to be open on inspection, implying that the filter is clogged. Thus $Z=\infty$. The operating time of the bypass does not induce a catastrophic damage in the main system.

The corresponding probabilities, expected downtimes, and expected length of a cycle are given as follows:

$$P(S_1)= \quad (21)$$
$$\int_0^T f_X(x)\left(\int_x^T f_Y(y)\bar{F}_Z(T-y)dy\right)dx$$

$$P(S_2)=\int_0^T f_Y(y)\bar{F}_X(y)\bar{F}_H(T-y)dy \quad (22)$$

The parts of the expected downtime derived from S_1 and S_2:

$$E[D_{S_1}]=\int_0^T f_X(x)\left(\int_x^T(T-y)f_Y(y)\bar{F}_Z(T-y)dy\right)dx \quad (23)$$

In case S_2, the valve works when it is required to do so and remains opens until inspection at T. Although the main system is lubricated, it can be affected by the dirty oil. The longer the equipment remains in this condition the worst and hence, we also assume a downtime during this period.

$$E[D_{S_2}]=\int_0^T(T-y)f_Y(y)\bar{F}_X(y)\bar{F}_H(T-y)dy \quad (24)$$

The downtime in (24) is incurred if the catastrophic damage does not occur before an inspection reveals that the valve is open.

The parts of the expected cycle length in scenarios S_1 and S_2:

$$E[\tau_{S_1}]=E[\tau_{S_2}]=T \quad (25)$$

Both events S_1 and S_2 lead to replacement at T and also if the filter remains good at T. The corresponding probability is given by $\bar{F}_Y(T)$ and no downtime is derived.

The foregoing functions apply for $M=1$, with zero replacements of c_1 ($J=0$) and zero inspections ($K=0$). Nevertheless, we can also model an inspection of the system filter-valve at T, if there has been no failure before, just for reliability estimation purposes. Next, the corresponding expression of the mean number of inspections is obtained:

If the system is inspected at T, then the number of inspections in a cycle only takes two values, 0 and 1:

$$P(K=0)=P(F_1)+P(F_2)$$

and, thus

$$E[K]=P(K=1)=1-P(F_1)-P(F_2)=P(S_1)+P(S_2)+\bar{F}_Y(T)$$

The expected length of a cycle:

$E[\tau] = E[\tau_{F_1}] + E[\tau_{F_2}] + T(P(S_1) + P(S_2)) + T\bar{F}_Y(T)$

The expected downtime cost

$E[C_d] = cd_1\left(E[D_{F_1}] + E[D_{S_1}]\right) + cd_2\left(E[D_{F_2}] + E[D_{S_2}]\right)$

The expected cost of a cycle

$E[C(\tau)] =$
$c_0 E[K] + (1-q)c_{F_1}P(F_1) + qc_{F_2}P(F_1) + c_{F_2}P(F_2) +$
$c_{PM}(P(S_1) + P(S_2) + \bar{F}_Y(T)) + E[C_d]$

3. Numerical study

The analysis in this Section deals with the following two issues:

A. The scope of inspection, that is, an approximation to the range of the parameters in the model under which inspection is profitable. Thus, we compare the general model with that where inspection is dropped or only one inspection is carried out for reliability estimation when the system is replaced.

B. Subjective appraisal of the parameters defining the times to valve failure, to filter clogging and the delay time.

There is not always enough data available to estimate the foregoing parameters. This is so for example, due to the lack of experiments, for confidentiality reasons, or because those times are not directly observable. Wang [33] analyzes the delay time estimation based on subjective opinions of experts. The use of a meaningful scale and order of magnitude that can be suitably encoded when required in a particular problem is a golden rule. In the case study, clogging occurs when a critical number of particles, P, reaches the filter. The following assumptions hold for the time until the filter becomes clogged, Y:

B.1 P takes very high values. The order of magnitude in car oil filters is 10^8.

B.2 Given the high quality of current manufacturing, small variations of P between filters can be expected. This is represented by a random variable Y with small standard deviation σ_Y.

B.3 The arrival of particles can be speeded up or slowed down under changes of the customary working conditions leading either to earlier or delayed replacement of filters. Therefore, huge values of σ_Y must also be analyzed.

In the following examples, the random variables, X, Y, Z and H are Weibull distributions. In particular, Z and H refer to the finite part of the mixture as defined in the notation. Points B.2 and B.3 lead to study the effect of changes in the mean and the standard deviation rather than the shape and the scale parameters. Table 1 contains the values of parameters in the base case. They are the result of engineering experience and dimensional analysis of the magnitudes that provide a plausible scenario. It is important to note that when the filter is clogged and the valve does not open, the former keeps on lubricating the main system with clean oil under an increasing pressure, which can result in the filter bursting. If the valve works on demand, the filter is no longer at risk of explosion, but the main equipment is poorly lubricated with dirty oil. Therefore, the downtime cost is assumed to be lower in the first case ($cd_1 < cd_2$).

Table 2 contains the sensitivity analysis under changes of the parameters of the time until the filter is clogged, Y, and from that moment until it bursts, Z. As σ_Y increases, so does the frequency of inspections of the valve, since inspections of the filter are triggered by those of the valve if it is found to be open. Therefore, the decreasing $T^\star$ aims at detecting those filters that become clogged at an earlier stage. It can be observed that the first four cases in Table 2 corresponding to the smaller values of σ_Y, lead to $M^\star = 1$, that is no inspection. The variation between filters is so small that the time when they will be clogged can be predicted with little error, making the bypass less relevant. Note that the valve which only has to work when the filter is clogged, would be unnecessary if the clogging time was deterministic ($\sigma_Y = 0$). Thus, a high homogeneity of the clogging time, Y, leads to no inspection of the valve ($M^\star = 1$). Therefore, valve inspections are no longer used to trigger filter inspections. Summarizing, preventive replacement at T as a unique maintenance turns out to be a useful strategy the lower σ_Y is, because the risk of a filter becoming clogged before replacement is negligible, making costly inspections unnecessary. This idea also explains the noticeable results in rows 4 and 5. They present the limiting case of σ_Y when $M^\star$ changes to values greater than 1. For $\sigma_Y \geq 0.3776$, filters present a higher variability and hence, they are less predictable. Then, the role of the valve and its inspection makes more sense. A second transition from a finite $M^\star$ to $M^\star = \infty$ occurs in cases 9 and 10 when the values of σ_Y are even higher and, at the same time, $T^\star$ is very small. Due to this high inspection frequency, clogged filters are very likely to be detected, leading to a complete renewal of the system. Therefore, the scheduled preventive maintenance at $M^\star T^\star$ is less necessary.

As expected, increasing values of μ_Z lead to relax inspection and postpone preventive maintenance. However, the results show that neither $T^\star$ nor $M^\star T^\star$ increase to infinity, but there is a threshold for both. Moreover, even for the smallest values of σ_Z in cases 18 and 19 inspection is still profitable. The behavior is therefore the opposite to σ_Y. No matter how long it takes for a clogged filter to burst, or whether that time is close to a constant, the use of a bypass is profitable because it is crucial to avoid the lack of lubrication in the main

Table 1
Parameters of the model in the base case.

$\mu_X = 2$	$\sigma_X = 0.3$	$\mu_Y = 1$	$\sigma_Y = 0.5$	$\mu_Z = 0.5$	$\sigma_Z = 0.25$	$\mu_H = 2$	$\sigma_H = 1$
$c_0 = 0.05$	$c_r = 0.2$	$cd_1 = 0.5$	$cd_2 = 5$	$c_{F_1} = 1.2$	$c_{F_2} = 30$	$c_{PM} = 1$	$q = 0.2$

Table 2
Effect of changes in the parameters of X, Y and Z on the optimum policy.

Case	σ_Y	μ_Z	σ_Z	μ_X	$M^\star$	$T^\star$	$Q^\star$	$M^\star T^\star$
1	**0.15**	0.5	0.25	2	1	0.9313	1.2300	0.9313
2	**0.28**	0.5	0.25	2	1	0.8668	1.4474	0.8668
3	**0.3642**	0.5	0.25	2	1	0.8270	1.6025	0.8270
4	**0.3776**	0.5	0.25	2	1	0.8210	1.6278	0.8210
5	**0.3776**	0.5	0.25	2	8	0.1681	1.6278	1.3444
6	**0.43**	0.5	0.25	2	10	0.1597	1.6374	1.5970
7	**0.5**	0.5	0.25	2	12	0.1584	1.6447	1.9005
8	**0.5382**	0.5	0.25	2	14	0.1559	1.6483	2.1831
9	**0.5577**	0.5	0.25	2	15	0.1560	1.6501	2.3395
10	**0.5577**	0.5	0.25	2	∞	0.1548	1.6501	∞
11	**0.67**	0.5	0.25	2	∞	0.1546	1.6559	∞
12	**0.9**	0.5	0.25	2	∞	0.1528	1.6689	∞
13	0.5	**0.22**	0.25	2	11	0.1579	1.6523	1.7370
14	0.5	**0.28**	0.25	2	11	0.1599	1.6494	1.7585
15	0.5	**0.9**	0.25	2	12	0.1588	1.6443	1.9057
16	0.5	**1.6**	0.25	2	12	0.1588	1.6443	1.9058
17	0.5	**2.9**	0.25	2	12	0.1588	1.6443	1.9058
18	0.5	0.5	**0.077**	2	12	0.1588	1.6443	1.9058
19	0.5	0.5	**0.14**	2	12	0.1588	1.6443	1.9055
20	0.5	0.5	**0.25**	2	12	0.1584	1.6447	1.9005
21	0.5	0.5	**0.45**	2	12	0.1564	1.6472	1.8767
22	0.5	0.5	**0.6**	2	11	0.1598	1.6496	1.7582
23	0.5	0.5	0.25	**0.34**	1	0.7822	1.8055	0.7822
24	0.5	0.5	0.25	**0.5638**	1	0.7828	1.8267	0.7828
25	0.5	0.5	0.25	**0.5721**	1	0.7827	1.8275	0.7827
26	0.5	0.5	0.25	**0.5721**	10	0.1826	1.8275	1.8260
27	0.5	0.5	0.25	**0.6453**	11	0.1753	1.7989	1.9282
28	0.5	0.5	0.25	**1.1**	12	0.1646	1.7015	1.9750
29	0.5	0.5	0.25	**3.6**	14	0.1547	1.6404	2.1659
30	0.5	0.5	0.25	**6.5**	14	0.1547	1.6404	2.1659

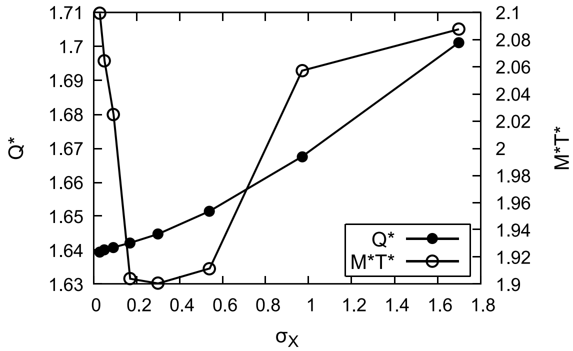

Fig. 3. Effect of the variability between valves.

Table 3
Effect of changes in the parameters of the bypass working time and q.

Case	q	μ_H	σ_H	$M^\star$	$T^\star$	$Q^\star$	$M^\star T^\star$
1	**0.062**	2	1	12	0.1587	1.6445	1.9040
2	**0.11**	2	1	12	0.1586	1.6446	1.9028
3	**0.2**	2	1	12	0.1584	1.6447	1.9005
4	**0.36**	2	1	12	0.1580	1.6451	1.8966
5	**0.65**	2	1	12	0.1575	1.6457	1.8897
6	0.2	**1.1**	1	17	0.0755	2.4159	1.2835
7	0.2	**1.5**	1	16	0.1139	1.8329	1.8225
8	0.2	**3.6**	1	11	0.1765	1.6105	1.9414
9	0.2	**6.5**	1	11	0.1765	1.6105	1.9414
10	0.2	2	**0.17**	11	0.1765	1.6105	1.9415
11	0.2	2	**0.31**	11	0.1765	1.6105	1.9415
12	0.2	2	**0.56**	11	0.1763	1.6107	1.9396
13	0.2	2	**1.3**	14	0.1305	1.7513	1.8266
14	0.2	2	**1.8**	16	0.0940	2.1079	1.5042
15	0.2	2	**2.1**	15	0.0829	2.3933	1.2440
16	0.2	2	**2.4**	14	0.0754	2.6970	1.0555

system. However, the poor quality of dirty lubrication compared to that provided by the filter makes it advantageous an earlier preventive replacement at $M^\star T^\star$, the greater the value of σ_Z is.

The effect of changes in the mean time to failure of the valve, μ_X, indicate that inspection is profitable as long as it is large enough. If not, as in cases 23, 24 and 25, $M^\star = 1$ indicates that the optimum policy is no inspection, but only a preventive replacement of the system filter-valve at $T^\star \simeq 0.78$ for all three. Since the filter is neither inspected, there is no reason to extend $T^\star$, which is actually determined by the parameters of the filter corresponding to the base-case, to prevent it from bursting. It is interesting to highlight the results in rows 25 and 26 which, as in 4 and 5, present the limiting case where either inspecting or not leads to the same optimum cost. The optimum policy is the same in cases 29 and 30 with the larger values of μ_X. No matter how good the valve is, it makes sense not to use the filter anymore, but to replace it, taking into account the parameters of its lifetime.

Fig. 3 summarizes the sensitivity analysis for σ_X. Neither $T^\star$ nor $M^\star T^\star$ are strictly monotonic, but both show little variation. As for μ_X, it seems that the optimum policy is determined by the characteristics of the filter.

Table 3 shows that the inspection frequency and the time for preventive maintenance are robust to changes in q. This is the probability that the maintainer is unaware of stopping the main equipment in time once the filter has burst, causing catastrophic damage to the former. This pattern in the optimum policy makes sense since q only applies if the maintenance does not succeed to detect the failed valve and the clogged filter. Hence, the scheduled maintenance $(T^\star, M^\star)$ does not change whatever the value of q is. Table 3 also reveals that inspection and preventive maintenance are relaxed as μ_H increases, but a finite threshold is observed. High values of μ_H correspond to tough equipments that are not damaged after a long period of unfiltered lubrication through the bypass. Hence, there is no need for larger postponements of the preventive maintenance, since it is very likely that both the filter and the valve will be replaced before $M^\star T^\star$.

Table 4
Effect of changes in the costs on the optimum policy.

Case	c_r	cd_1	cd_2	c_{F_1}	c_{F_2}	$M^\star$	$T^\star$	$Q^\star$	$M^\star T^\star$
1	**0.062**	0.5	5	1.2	30	14	0.1551	1.6409	2.1718
2	**0.11**	0.5	5	1.2	30	13	0.1564	1.6423	2.0333
3	**0.2**	0.5	5	1.2	30	12	0.1584	1.6447	1.9005
4	**0.36**	0.5	5	1.2	30	11	0.1615	1.6484	1.7760
5	**0.65**	0.5	5	1.2	30	11	0.1590	1.6533	1.7488
6	0.2	**0.086**	5	1.2	30	12	0.1585	1.6446	1.9020
7	0.2	**0.15**	5	1.2	30	12	0.1585	1.6446	1.9017
8	0.2	**0.28**	5	1.2	30	12	0.1584	1.6447	1.9013
9	0.2	**0.9**	5	1.2	30	12	0.1583	1.6449	1.8991
10	0.2	**1.6**	5	1.2	30	12	0.1581	1.6451	1.8967
11	0.2	0.5	**0.5**	1.2	30	14	0.2950	1.2270	4.1299
12	0.2	0.5	**0.86**	1.2	30	14	0.2756	1.2712	3.8580
13	0.2	0.5	**1.5**	1.2	30	14	0.2456	1.3439	3.4386
14	0.2	0.5	**2.8**	1.2	30	11	0.2010	1.4714	2.2110
15	0.2	0.5	**9**	1.2	30	14	0.1191	1.8837	1.6680
16	0.2	0.5	**16**	1.2	30	15	0.0924	2.1884	1.3854
17	0.2	0.5	5	**0.21**	30	12	0.1584	1.6447	1.9012
18	0.2	0.5	5	**0.37**	30	12	0.1584	1.6447	1.9011
19	0.2	0.5	5	**0.67**	30	12	0.1584	1.6447	1.9009
20	0.2	0.5	5	**1.2**	30	12	0.1584	1.6447	1.9005
21	0.2	0.5	5	**2.2**	30	12	0.1583	1.6448	1.8998
22	0.2	0.5	5	**3.9**	30	12	0.1582	1.6449	1.8987
23	0.2	0.5	5	1.2	**5.1**	11	0.1748	1.6152	1.9229
24	0.2	0.5	5	1.2	**9.3**	11	0.1723	1.6206	1.8955
25	0.2	0.5	5	1.2	**17**	12	0.1641	1.6301	1.9689
26	0.2	0.5	5	1.2	**54**	13	0.1471	1.6688	1.9121
27	0.2	0.5	5	1.2	**97**	14	0.1345	1.7053	1.8834

The effect of changes in the costs is presented in Table 4. When the cost of replacing the valve and inspecting the filter, c_r, increases, the advantage of inspection and replacement of only one element of the system becomes less advantageous compared to preventive replacement of both at $M^\star T^\star$. Therefore, $M^\star$ decreases and so does $M^\star T^\star$. The optimum policy is robust when cd_1 increases because this is a minor cost compared to that of a catastrophic failure of the main system, c_{F_2} and the maintenance is mainly focused on preventing it. A similar behavior is observed when c_{F_1} increases, since this cost is incurred when the fatal damage does not occur and c_{F_1} is also much lower than c_{F_2}. However, changes in cd_2 have a deep impact on the optimum policy, since the higher cd_2, the more the engine is damaged. Hence, $T^\star$ decreases and the preventive replacement at $M^\star T^\star$ occurs earlier. The inspection frequency also increases with the cost of catastrophic damage in the main equipment, c_{F_2}.

The model presented in this paper adds the periodic inspection of the filter, depending on the condition of the valve, to the usual scheduled replacement. Fig. 4 shows the comparative analysis of this model and that where inspection is dropped for different values of the inspection cost, c_0, and the rest of the parameters as in the base case. The dashed and dotted lines in the upper graph represent the optimum cost, $Q^\star$, for $M = 1$ (no inspection) and $M \geq 2$ (inspection), respectively. In the range $c_0 < 0.1$, the latter provides the optimum policy with more frequent inspections and larger times to preventive replacement at $M^\star T^\star$. For $c_0 > 0.1$, the benefit of inspection does not counterbalance its cost and the optimum policy is given by $M = 1$. In these cases, $T^\star$ is determined by the parameters of the filter.

4. Conclusions

This paper presents a model for inspection and maintenance of a lubrication system consisting of a filter and a bypass which is intended to prevent the catastrophic failure of the filter after it becomes defective. Valves that relieve pressure in pipelines or divert their contents when the main channel is blocked are usual examples of bypasses. The system filter-valve in heavy-duty equipment is the case study of this

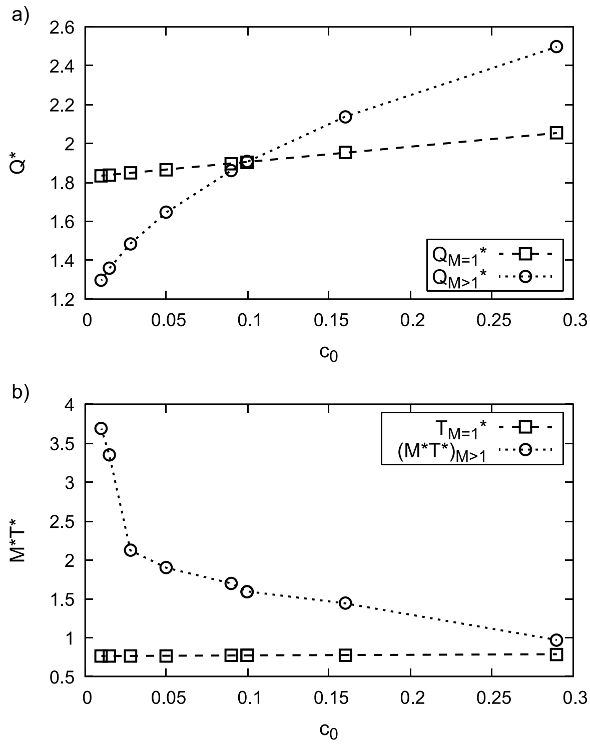

Fig. 4. Comparison of models with and without inspection.

research. Significant differences with systems with warm stand-by units justify this new model. The first one is that the filter and the bypass are not fully interchangeable since the quality of lubrication through the bypass is much lower than through the filter. Thus, the lubricated system can be damaged if the bypass operates for a long time. The new contributions of this model also cover the inspection procedure as well as the type of interaction between the filter and the valve. In general, filters are replaced within safe intervals when they can still carry out their function. In most cases, therefore, the time when the filter is clogged is not directly observed. Following this idea, this study also includes an approach to assessment of these random times based on reasonable assumptions. The main advantage is that noticeable changes in the physical conditions under which the system operates can be taken into account to modify the distributions in the model.

The numerical study reveals that the use of a bypass could be ignored in those cases where the randomness of the clogging time is small. Otherwise, it is worthwhile even if the time for a clogged filter until burst is long. Preventing the main system from running out of lubrication is essential, and so is the scheduled preventive maintenance to avoid a major damage caused by long periods of unfiltered oil. Moreover, a higher variability of the delay time between filters induces an earlier preventive replacement to avoid the shortest times to burst. In general, the optimum policy is more dependent on the parameters defining the filter than on those of the valve. Regarding costs, those incurred when the lubricated system undergoes a total failure are the most significant to explain the changes in the optimum policy. In short, maintenance is driven by the objective of keeping the filter unclogged most of the working time of the lubrication system.

CRediT authorship contribution statement

M.D. Berrade: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **E. Calvo:** Writing – review & editing, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **F.G. Badía:** Writing – review & editing, Visualization, Validation, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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