

Geographical proximity, nonlinearities and financial behaviour of firms.

Does firm size matter?

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Abstract

The paper highlights the role played by nonlinearities and geographical proximity in an attempt to better understand the financial behaviour of firms. Our study focuses on three main financial dimensions: profitability, indebtedness and liquidity. A classical partial adjustment model is specified in order to capture the movements produced in each dimension. Using a large sample of Spanish industrial companies, located along the Mediterranean Basin, we evaluate the impact of nonlinearities, the heterogeneous behaviour of companies, and the importance of local networks. The impact of physical proximity is greater for small firms, which are more dependent on what happens in their neighbourhood. Moreover, the impacts are not homogeneous for the three financial ratios: we find that the effect of proximity is stronger for the profitability ratio than for indebtedness and liquidity.

Keywords:

Financial ratios, Partial adjustment model, Geographical proximity, Spatial interactions, Spanish industrial companies.

JEL classification: G30, M21, R12, R32.

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Proximidad geográfica, no linealidad y comportamiento financiero de las empresas. ¿Importa el tamaño de la empresa?

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Resumen

El artículo pone de manifiesto la importancia de la proximidad geográfica y de las no linealidades para comprender correctamente el comportamiento financiero de las empresas. Nuestro trabajo se centra en tres dimensiones financieras principales, como son la rentabilidad, el endeudamiento y la liquidez especificando un modelo de ajuste parcial con el objetivo de capturar los movimientos producidos en cada dimensión. Usamos una gran muestra de compañías industriales españolas, localizadas a lo largo del litoral mediterráneo y evaluamos el impacto de las no linealidades, el comportamiento heterogéneo de las empresas así como la importancia de las redes locales. El impacto de la proximidad geográfica es mayor para las empresas de pequeño tamaño, que son más dependientes de lo que ocurre en sus inmediaciones. Además, estos impactos no son iguales para los tres ratios financieros; en concreto, obtenemos que el efecto de la proximidad es más relevante para la rentabilidad que para los casos del endeudamiento y la liquidez.

Palabras clave:

Ratios financieros, modelo de ajuste parcial, proximidad geográfica, interacción espacial, empresas españolas.

1. Introduction

The literature on corporate finance provides substantial empirical evidence about the relevance of proximity as well as the impact of nonlinearities. For example, the financial entities located nearby a company have more and better information about its financial situation, as it is a potential client (Carbó *et al.*, 2003; Degryse and Ongena, 2005). It is clear that problems stemming from the existence of asymmetric information can be reduced through physical proximity. This observation can be extended to other aspects of finances, such as the relationship between firms and investors and capital providers (O'Brien and Tan, 2015; Brown *et al.*, 2014), as well as with other financial entities including local suppliers (Rao *et al.*, 2015; Knyazeva and Knyazeva, 2012; Dass and Massa, 2011). This part of the discussion is well documented in the literature. The effect on a firm's financial decisions of geographical proximity to other similar companies has been examined in the large literature devoted to clusters; however, there are few studies considering interaction effects from peers in a purely geographical analysis. Filling that gap is one of the aims of this paper.

The second aim is an attempt to disentangle the intricate relationship between proximity and nonlinearities. Different authors, such as Davis and Peles (1993) or Wu and Ho (1997), have concluded that firms' reactions to shocks in the environment are not homogeneous, but rather specific to the characteristics of each firm. Size, sector of activity, risk of imbalances, etc. are factors usually identified as potential sources of nonlinearities in the processes of adjustment. Often both elements, spatial proximity and nonlinearities, are intermingled and it is difficult to clearly identify the impact of each one. In order to address this issue, we believe that a proper specification of each element is a prerequisite.

The existence of intangible cross-sectional interdependencies among companies, understood as "*the transmission of an idea, practice or behaviour through the influence of other agents*" (Reppenhausen, 2010), has been reported in a large collection of papers. This finding is based on the assumption that geographical proximity favours social connections among companies, thus overcoming barriers to knowledge exchange (Granovetter, 1985). In this sense, Leary and Roberts (2014) find dependence in the financial structures between peer companies. According to these authors, there is ample empirical evidence supporting the idea that a company's financial decisions are influenced by similar decisions taken by its peers. Therefore, it is clear that proximity improves the transmission of information impacting firms' financial decisions. However, the impact may not be the same for all types of companies. For example, large corporations working in national or international markets are likely to be less sensitive to what happens nearby their headquarters than they are to the

decisions taken by other large rival corporations. These companies have enough internal resources to analyse markets and competitors by themselves (Reppenhagen, 2010). In the same vein, hi-tech companies are more connected to research and innovative centres than to their local neighbourhood. This causes differences in financial behaviour depending on the industrial sector (Gallizo *et al.*, 2008). These are simple examples of certain unavoidable impacts of nonlinearities.

Our main hypothesis is that geographical proximity is relevant when seeking to obtain information from peers. Moreover, this factor should be more important for certain groups of companies, such as small firms which have less access to global information and are more dependent on their local environments (Beck *et al.*, 2011). The objective of our work is to evaluate whether firms' financial behaviour is influenced by similar decisions taken by nearby companies. We then seek to determine whether the proximity effect varies depending on certain characteristics of the company, especially size, and to test for nonlinearities.

We contribute to current literature in several ways. Firstly, we have tried to better characterize the factors which influence firms' financial behaviour. Previous literature has also considered the results of other companies, but as a mere additional element such as common industrial trends or the like; however, physical proximity has received little attention. Secondly, our analysis focuses specifically on the case of small companies, which have not generally been the focus of attention. Thirdly, in line with previous studies, we show that, in order to understand the financial behaviour of (small) companies, it is not enough to simply consider internal factors: the characteristics of neighbouring companies are also critical. Finally, we propose a general approach so that nonlinearities can interact with the geographical effects.

The structure of the paper is as follows. Section 2 discusses different mechanisms through which proximity may affect the financial behaviour of firms. Section 3 extends the well-known partial adjustment model by adding a spatial dimension to the adjustment process. Sections 4 and 5 contain an application to the case of a large sample of Spanish companies located along the so-called Mediterranean axis. Section 6 summarizes the main findings and future developments.

■ 2. The effects of geographical proximity on the financial behaviour of companies

There is abundant evidence highlighting the importance of physical proximity to other firms on the financial decisions taken by companies. It is clear that companies are connected with other companies and that managers' financial decisions are

conditioned by the reaction of other companies, especially in situations of uncertainty (Granovetter, 1985). Geographical proximity fosters a kind of a contagion effect that promotes the exchange of information (Leary and Roberts, 2014). The spreading of this chain of exchanges is limited by distance, so the greater the distance, the weaker the exchange. In other words, companies' financial management relies not only on the characteristics of the company, but also on the decisions taken in their neighbourhood.

Regarding previous literature, we find different explanations for the proximity effect on finance. Several studies have focused on economic factors that promote mimicking strategies, relying on agency theory and the existence of asymmetric information. In this context, proximity between agents would alleviate problems caused by the lack of information, increasing the propensity of the managers to undertake local operations. Fernández and Maudos (2009) and Palacín-Sánchez and Di Pietro (2016) conclude that bank branches tend to act in local markets in order to reduce costs and increase earnings, while Uysal *et al.* (2008), and Massa and Simonov (2006) find similar results in relation to investors. The adoption of such strategies by banks and capital providers may lead, in practice, to a geographical segmentation of the financial markets, where different economic conditions apply to different areas. This also means that companies producing in the same area will face similar financial conditions, and that these conditions may change, slightly, between areas.

From another perspective, different authors have tried to characterize the nature of the non-monetary factors related to geographical proximity. Pirinsky and Wang (2006) find a “*co-movement in the stock returns of firms headquartered in the same area*”; this co-movement should be attributed to a kind of regional culture, as can be measured by a set of local indicators. Gao *et al.* (2008) conclude that firms' location partly explains their capital structures and payout policies, showing that companies located in the same area exhibit similar leverage and liquidity ratios. The similarity is explained by regional practices common to all the companies located in the neighbourhood, which is reinforced by the interaction among companies.

Managers working in the same area normally have good opportunities to engage in valuable social relationships with their peers, exchanging ideas and learning from their experience. Sharing a local environment also facilitates managerial interactions among firms through different channels. For example, managers may attend the same professional clubs or meetings (Davis and Greve, 1997). The spread of the practice of board members serving on the boards of several corporations is another form of (stronger) interaction, which means that managers have direct access to the decision-making process of other companies through board interlock (Mizruchi, 1996). Finally, sharing external agents, such as clienteles, suppliers or financial en-

tities, is an additional way to get information about other companies (Greve, 2005). In particular, banks play an important role in spreading information in the area. Financial entities have their own credit score models, which are applied to the loan evaluation process (Treacy and Carey, 2000). The application of these scores gives different pricing of capital for different companies and, often, the final results have a clear spatial pattern, which reveals the existence of practices of discrimination between areas (in the sense that financial conditions offered by banks are different as a function of the environmental characteristics). This mechanism offers implicit information about the financial situation of other companies. Another example of informal information comes from suppliers providing, on certain occasions, sales prices determined according to the conditions of the local markets. Storper and Venables (2004) highlight the relevance of face-to-face contacts in strengthening communication between firms, when there is imperfect information and changing external conditions. Ter-Wal and Boschma (2009) examine the connectedness between geographically close companies, where labour mobility facilitates social interrelationships with former colleagues, giving rise to knowledge networks between these firms.

■ 3. The partial adjustment model with nonlinearities and spatial effects

This section is devoted to the well-known partial adjustment model (PAM hereafter), which is our chosen approach for evaluating nonlinearities and geographical proximity effects in firms' financial practices.

The PAM model assumes that the dynamics in firms' financial ratios follow an adjustment process towards target values, as represented by the industrial averages (Lev, 1969). Given an external shock that disturbs previous equilibriums, managers should reconsider their current financial objectives. The lack of perfect information leads the companies to assume the average value of their industry as a benchmark value. The main assumption underlying the PAM model revolves around the idea that financial ratios cannot deviate too much from their equilibrium values. Companies with financial ratios far from the industrial average (as a proxy for equilibrium) incur costs as a result of being unbalanced, which are usually higher than the adjustment costs. That means that managers attempt to change accounting and/or commercial practices (e.g. inventory evaluation methods) and to readjust their financial ratios (Lev, 1969). In addition, external market forces exert similar pressures on financial ratios. For example, the expectation of high return assets will encourage new companies to enter into the industry, driving profitability ratios towards the average values (Peles and Schneller, 1988).

3.1. The Partial Adjustment model (PAM)

The importance of company finances has attracted much attention, with many papers aiming to model the dynamic nature of financial ratios. In this paper, we use the PAM based on Lev (1969) and Chen and Ainina (1994). In essence, this model assumes that the change in the output of agent i in period t , y_{it} , adjusts proportionally to the difference between the optimal amount in period t y_{it}^* and the observed amount in $(t-1)$, y_{it-1} . We can write:

$$y_{it} - y_{it-1} = \delta (y_{it}^* - y_{it-1}) + e_{it} \quad (1)$$

where δ is a parameter that measures the speed of adjustment and e_{it} a white noise. According to (1), the speed of adjustment δ is the ratio between the optimal change, $(y_{it}^* - y_{it-1})$, and the observed change $(y_{it} - y_{it-1})$. To confirm the existence of an adjustment process, δ should lie between 0 and 1.

Parameter δ reflects the limited capacity of the firm to accomplish the required adjustments, due to technological and institutional constraints. In the case that $\delta = 0$, there is no adjustment and output in t coincides with the output in $(t-1)$. At the other extreme, if $\delta = 1$ the gap is corrected instantaneously, so that $y_{it}^* = y_{it}$. According to previous evidence (i.e., Chen and Ainina, 1994) δ is usually an interior point in the interval $[0,1]$, so that the adjustment is only partially completed in the period, giving rise to the name to the model.

Let us note that the objective, y_{it}^* , is not directly observable and must be estimated beforehand. There are several proposals in the literature for doing so, such as Lev (1969, note 2), who suggests estimating this optimal value through the industrial average in period $t-1$.

The assumption of a homogeneous speed of adjustment, as appears in (1), is a reasonable hypothesis for a homogeneous set of companies. However, when using a heterogeneous group, this hypothesis should be relaxed. The seminal paper of Lev (1969) highlights the relevance of the problem: “*in such a large and heterogeneous sample, there is no way to identify specific techniques which probably differ from firm to firm*” (p. 299). Taking into account this limitation, several authors have dealt with the heterogeneity problem by examining the adjustment processes of financial ratios in function of firm characteristics (Lee and Wu, 1988; Lee, 1985; Fieldsend *et al.*, 1987; Lev and Sunder, 1979; Seay *et al.*, 2004, Aybar-Arias *et al.*, 2012). The size of the firm and the economic sector it belongs to are factors commonly used to introduce heterogeneity in (1), but are not the only ones to consider, as discussed below.

3.2. Distortion factors in a PAM equation

Firm size is an important factor that conditions the speed of adjustment of the financial ratios towards the objective; indeed, previous researches have considered the size of the company as a specific source of instability. The heterogeneity hypothesis appears reasonable, but we should point out that there is no consensus in the applied literature. This relationship is not obvious, even from a purely theoretical point of view.

Heterogeneity may arise because of characteristics that are specific to small companies rather than large ones. Managers in SMEs face severe restrictions in securing equity and debt (Brown *et al.*, 2005). Besides, they have limited access to market information and their decisions are strongly conditioned by the mutual influence between the company and its immediate neighbourhood (Palacin *et al.*, 2013). Under these circumstances, it seems unlikely that there is a general adjustment process, common to all the companies in the area, which drives the financial ratios of each firm towards similar target values. On the contrary, it is reasonable to suppose that the adjustment processes will differ in intensity for different groups of companies in function of, for example, their size.

Large firms have more resources and better access to capital markets and information. Therefore, their size allows them to adjust their financial ratios faster and better than small companies do. Moreover, small companies have stronger incentives for attempting to achieve equilibrium given the (relatively) high costs resulting from disequilibrium (Davis and Peles, 1993). Wu and Ho (1997) show that smaller firms are more vulnerable to industrial cycles, leading to greater fluctuations in their financial ratios. They suggest that smaller firms must quickly adjust their ratios to the optimal value if they want to survive. The size of the firm appears to be one of the main sources of heterogeneity but it is not the only one. For example, Chen and Ainina (1994) allow for differences in the partial adjustment model of (1) depending on the activity of the company. Lee and Wu (1988) show that there are strong differences in the financial ratio adjustment patterns in different industrial sectors. In the same vein, Gallizo and Salvador (2003) and Gallizo *et al.* (2008) examine the financial ratio adjustment after aggregating the companies by productive sector. In general, all papers dealing with the nature of the activity find significant differences in the adjustment process when different subsectors are considered.

Lee and Wu (1988), Aybar-Arias *et al.* (2012) and Naveed *et al.* (2015) find that the distance to the objective (that is, the extent of the unbalance affecting the company) is an important element in determining the speed of adjustment. From a theoretical point of view, we should expect that companies in a worse situation (in the sense of being farther from the optimum) must make greater efforts to approach the average value of

their sector. However, the pressure will be significantly weaker for companies with financial ratios close to the objective. These results point to the existence of nonlinearities in the financial ratio adjustment process stemming from companies' incentives.

3.3. A PAM model with discrete breaks and spatial effects

As stated above, previous studies have detected asymmetric behaviour in the adjustment coefficients that control the dynamics of several financial variables, depending on the characteristics of the firm (Drobetz *et al.*, 2015). These results underline the fact that heterogeneity “*should be incorporated in empirical studies of corporate leverage*” (Faulkender *et al.*, 2012). To that end, we introduce nonlinearities in the PAM process (1), using a set of simple dummy variables, in order to capture the factors discussed above. First, let us extend equation (1) as:

$$y_{it} - y_{it-1} = \delta (y_{it}^* - y_{it-1}) + \sum_{k=1}^K \delta_k (y_{it}^* - y_{it-1}) f_{kit} + e_{it} \quad (2)$$

where f_{kit} is a binary variable which takes the value 1 if company i belongs to category k ($k=1, \dots, K$). The coefficient δ measures a common adjustment speed for the companies in the sample. This common rate is corrected by the value of the parameter δ_k , in the case where company i belongs to the k -th category.

The geographical distribution of the firms is another factor that should be taken into account. Companies can be located near to or far from other companies, but they are not isolated entities. On the contrary, Location Theory shows that they tend to create networks with their neighbours, to which they are linked through different channels. This interaction has a non-negligible impact on how companies are managed, including their finances, as has been widely recognized in the literature reviewed in Section 2. In short, there are also spatial factors acting in the PAM equation (2), especially in the case of SMEs, where the group of nearest neighbours is a key point of reference.

The interaction among neighbours may take place as a kind of imitation effect: if my neighbours react quickly to shocks in the environment, I will probably do the same. This leads to what has been termed the Spatial Lag Model (SLM). The significance of the spatial interaction coefficient in the SLM highlights the interconnection among financial practices of geographically close companies. From an empirical perspective, this result is in line with previous analyses based on surveys to CEOs of different companies, which show that companies adopt financial practices in response to the financial values of their closest neighbours (Graham and Harvey, 2001). Another source of interaction operates through the errors of the PAM equation (perception errors, overreactions, misinterpretations, etc.) which are

common to most of the companies located in a certain area; this results in the so-called Spatial Error Model, SEM. Other forms of interaction are possible (Lesage and Pace, 2009), such as the Spatial Durbin Model, SDM, introduced below.

For the spatial modelling of the PAM, a more precise definition of the notion of neighbourhood is needed (i.e., who are and where are located the neighbours of every company in the sample). The literature on spatial econometrics solves this question by using the so-called weighting matrix (of order $N \times N$, with N being the number of companies in the sample), denoted by \mathbf{W} . This matrix is usually built exogenously. Each row describes the neighbourhood structure for a given company, where each of the companies in the sample appears in a column of the matrix. The terms in the diagonal are set to zero, for identification purposes, while there is ample flexibility for defining the other weights. A simple solution consists in assigning a 1 to the (i, j) term of the matrix if companies i and j are defined as neighbours on the basis, for example, that they are (less than) a certain distance apart (other approaches used to build the matrix can be found in Harris *et al.*, 2011). It is standard practice to subsequently row-standardize the matrix so that the sum of every row is equal to one.

On this basis, there are three families of spatial models that provide a good fit to our PAM models, with breaks, (2). They are:

■ The spatial lag model, SLM:

$$y_{it} - y_{it-1} = \delta(y_{it}^* - y_{it-1}) + \sum_{k=1}^K \delta_k (y_{it}^* - y_{it-1}) f_{kit} + \rho \sum_{j \neq i}^N \omega_{ij} (y_{it} - y_{it-1}) + e_{it} \quad (3)$$

■ The spatial error model, SEM:

$$y_{it} - y_{it-1} = \delta(y_{it}^* - y_{it-1}) + \sum_{k=1}^K \delta_k (y_{it}^* - y_{it-1}) f_{kit} + u_{it}; \quad u_{it} = \rho \sum_{j \neq i}^N \omega_{ij} u_{it} + e_{it} \quad (4)$$

■ The spatial Durbin model, SDM:

$$y_{it} - y_{it-1} = \delta(y_{it}^* - y_{it-1}) + \sum_{k=1}^K \delta_k (y_{it}^* - y_{it-1}) f_{kit} + \rho \sum_{j \neq i}^N \omega_{ij} (y_{it} - y_{it-1}) + \sum_{k=1}^K \theta_k \sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) f_{kit} + e_{it} \quad (5)$$

In all cases, ω_{ij} is the (i, j) -th element of matrix \mathbf{W} . The SDM adds a spatial lag of the regressors, $\sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) f_{kit}$, to the right-hand side of the SLM equation (3). In fact, the SDM can be seen as an intermediate between the SLM and SEM equations, which combines features of the two (Lesage and Pace, 2009, for the details). The three equations can be estimated by maximum likelihood using of widely-used codes.

3.4. Instability in the spatial dependence mechanisms

As with the adjustment speed ratio, the spatial interaction parameter (ρ) may be unstable, implying that different groups of companies (i.e., small and large companies) may have different interactions with their neighbours. This is a reasonable assumption given that small firms are very sensitive to their local environment, whereas large companies are more involved in national or international markets. Instability in the spatial dependence parameter has been addressed in Mur *et al.* (2008) and (2010), who develop several tests and corresponding estimation techniques.

Therefore, the question is: *Does the effect of proximity on financial contagion have the same intensity for large companies as for small ones?* If the answer is NO, equation (3) should be rewritten as follows:

$$y_{it} - y_{it-1} = \delta(y_{it}^* - y_{it-1}) + \sum_{k=1}^K \delta_k(y_{it}^* - y_{it-1}) f_{kit} + \rho \sum_{j \neq i}^N \omega_{ij}(y_{it} - y_{it-1}) + \rho^* \sum_{j \neq i}^N \omega_{ij}^*(y_{it} - y_{it-1}) + e_{it} \quad (6)$$

ρ is the spatial dependence coefficient, which is common to every company in the sample, whereas ρ^* is the differential effect corresponding to SME firms. ω_{ij}^* are the weights of a second weighting matrix, \mathbf{W}^* , which, according to Mur *et al.* (2008), must have the same elements as the original \mathbf{W} matrix in the rows and columns corresponding to companies belonging to the differential group (i.e., SME firms); the other \mathbf{W}^* terms are zeros. Similar extensions apply for the cases of the SEM and SDM models.

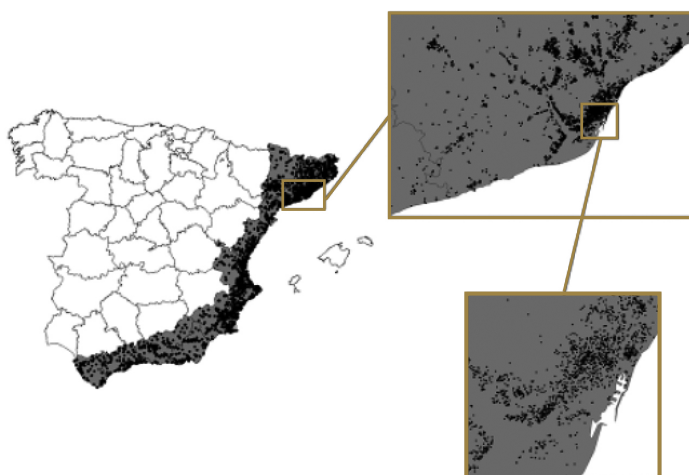
Finally, in order to test if there is indeed a break in the spatial dependence coefficients in the PAM equations, we extend the discussion in Mur *et al.* (2010), developed for a single cross-section, to a panel data framework. The Appendix describes how we obtain the $LM_{SLM/FE}^{break}$ (eq. A9) and $LM_{SEM/FE}^{break}$ (eq. A15) tests used in Section 5.

4. Financial ratios and sampling information

The study presented in the next section uses a sample of Spanish industrial companies located in the 12 provinces (NUTS 3 units) situated along the Mediterranean Basin. Data come from the SABI (*Sistema de Balances Ibéricos*) database, which provides accounting and financial information for each company in the database, as well as other useful information such as geographical location, sector of activity (NACE code), etc. The location of a company is taken as the location of its headquarters, which are the centres where the company's main financial decisions are taken and where interaction with other agents occurs (Pirinsky and Wang, 2010).

The initial sample contained 38,323 industrial companies (NACE codes 1000 to 4100) for the period 2006-2012. However, we decided to use only companies with complete information for the whole period, thus eliminating failed businesses, newcomers and other companies with anomalies in their financial records. Thus, the final sample comprised 12,420 companies. Their geographical distribution appears in Figure 1. This sample represents approximately 6% of all industrial companies registered in Spain (214,992), according to the official register DIRCE (*Instituto Nacional de Estadística*, INE, 2012), in 2012. We are aware that this period, 2006-2012, is problematic due to the economic crisis that rocked the Spanish economy.

■ **Figure 1. Location of the companies in the sample**



Three ratios were defined for each company, to represent its main financial features. According to Soboh *et al.* (2009), financial dimensions can be classified into two categories. The first category includes the liquidity and indebtedness dimensions, which measure the capacity of a firm to pay its current obligations as they arise and the nature of any financing equity. The second category is related to profitability, and evaluates the capacity of the company to generate earnings. In our case, the liquidity dimension has been measured by the current ratio (CU, hereafter) calculated as short-term assets divided by short-term liabilities. Indebtedness is evaluated through the debt equity ratio (DE, hereafter) calculated as total liabilities over total assets. The profitability dimension is evaluated by the profitability ratio (PR, hereafter) which is net operating income divided by total assets. The three ratios are log transformed. Finally, in line with Lev (1969), we evaluate the objective y_{it}^* in the PAM as the sectoral average value for each financial ratio in the previous year.

Using this information, a collection of factors that impact on firms' adjustment process appear in Table 1.

● **Table 1. Factors influencing the adjustment process in the PAM equation**

| | |
|----------|--|
| d_{it} | Distance to the objective (in the corresponding ratio) |
| | $d_{it,1} = 1$ if $L_t < (y_{it}^* - y_{it-1}) < U_t$; 0 otherwise. |
| | $d_{it,2} = 1$ if $(y_{it}^* - y_{it-1}) \leq L_t$; 0 otherwise. |
| | $d_{it,3} = 1$ if $(y_{it}^* - y_{it-1}) \geq U_t$; 0 otherwise. |
| s_{it} | Company size ¹ |
| | $s_{it,1} = 1$ if it is a micro company (fewer than 10 employees); 0 otherwise. |
| | $s_{it,2} = 1$ if it is a small company (between 11 and 50 employees); 0 otherwise. |
| | $s_{it,3} = 1$ if it is a medium-to-large company (more than 51 employees); 0 otherwise. |
| t_{it} | Technological Intensity (TI) ² |
| | $t_{it,1} = 1$ if the company belongs to a low TI index sector; 0 otherwise. |
| | $t_{it,2} = 1$ if the company belongs to a medium-low TI index sector; 0 otherwise. |
| | $t_{it,3} = 1$ if the company belongs to a medium-high TI index sector; 0 otherwise. |
| | $t_{it,4} = 1$ if the company belongs to a high TI index sector; 0 otherwise. |

L_t and U_t indicate the percentiles of 10% and 90%, respectively, in the distribution of gaps to the optimum in the corresponding ratio, in period t .

¹ According to the European Commission (2003), a company is considered medium size if the number of employees is between 51 and 250, and large if it has more than 250 employees. We have merged these two groups in our analysis.

² According to the Statistical Classification of Economic Activities of the European Community.

d_{it} represents the distance to the financial target. In this case, we consider companies with financial ratios far from the objective targets (below and above the 10th and 90th percentile of the d_{it} distribution respectively). s_{it} indicates firm size following the European Commission (2005) classification based on the number of employees. t_{it} represents the technological intensity of each company, applying NACE codes classification¹.

■ 5. Results

A summary of statistics for the main variables in the study appears in Table 2. According to the national official census of companies (INE, 2014), the Spanish industrial production system is characterized by the small size of the firms: 38.4% have no employees, 78.4% have 5 employees or fewer and only 7% have more than 20 employees. That is, our sample, taken from SABI, coincides roughly with the composition of the national industry. Regarding technological intensity, TI, we find that almost 80% of the industrial companies come from low or low-medium TI sectors. Lastly, let us add that we have computed the financial ratios (CU, DE, PR) for the different categories of size and TI, and these are shown in Table 2. The ANOVA F -test in the last column confirms the strong heterogeneity among the different categories of companies: the null hypothesis of equal means is rejected in all cases except for the profitability ratio by size of the company).

¹ Statistical Classification of Economic Activities in the European Community (see <http://ec.europa.eu/eurostat> for further details).

● Table 2. Companies by categories. Descriptive analysis of the financial ratios

| | Companies | | | CU | | | DE | | | PR | | | | | | | | |
|---|-------------|-------------|-------------|-------------|-------------|-------------|---------|-------------|-------------|-------------|-------------|-------------|-----------|-------------|-------------|-------------|-------------|-----------|
| | N | % | | Min | Max | Mean | SD | F | Min | Max | Mean | SD | F | | | | | |
| by Distance to objective[‡] | | | | | | | | | | | | | | | | | | |
| $Y_{it} - Y_{it-1} < P_{10}$ | 845 | 10.0% | -3.29 | -1.21 | -1.86 | -0.53 | 0.53 | 8156.2*** | -0.41 | -0.27 | -0.32 | 0.04 | 5125.3*** | -3.59 | -0.91 | -1.54 | 0.58 | 7768.8*** |
| $P_{10} < Y_{it} - Y_{it-1} < P_{90}$ | 6764 | 80.0% | -1.21 | 0.86 | 0.10 | 0.52 | | | -0.27 | 0.28 | -0.00 | 0.15 | | -0.91 | 0.77 | 0.08 | 0.42 | |
| $Y_{it} - Y_{it-1} > P_{90}$ | 845 | 10.0% | 0.86 | 1.66 | 1.08 | 0.18 | | | 0.28 | 0.59 | 0.35 | 0.05 | | 0.77 | 1.36 | 0.93 | 0.12 | |
| by Size[‡] | | | | | | | | | | | | | | | | | | |
| Micro | 3004 | 35.5% | 0.14 | 4.36 | 1.62 | 0.68 | 3.7** | 0.01 | 0.98 | 0.63 | 0.19 | 0.19 | 67.7*** | 0.08 | 4.35 | 1.50 | 0.66 | 1.32 |
| Small | 4533 | 53.6% | 0.08 | 4.25 | 1.67 | 0.68 | | 0.11 | 0.99 | 0.59 | 0.18 | 0.18 | | 0.06 | 4.39 | 1.50 | 0.64 | |
| Medium | 917 | 10.8% | 0.13 | 4.08 | 1.61 | 0.68 | | 0.12 | 0.99 | 0.55 | 0.17 | 0.17 | | 0.27 | 4.67 | 1.46 | 0.61 | |
| by Technological Intensity[‡] | | | | | | | | | | | | | | | | | | |
| Low TI | 3797 | 44.9% | 0.08 | 4.25 | 1.59 | 0.69 | 31.2*** | 0.14 | 0.99 | 0.61 | 0.19 | 0.19 | 13.7*** | 0.06 | 4.67 | 1.52 | 0.70 | 11.0*** |
| Medium-Low TI | 2954 | 34.9% | 0.20 | 4.18 | 1.66 | 0.67 | | 0.01 | 0.99 | 0.63 | 0.18 | 0.18 | | 0.22 | 3.82 | 1.44 | 0.59 | |
| Medium High TI | 1545 | 18.3% | 0.25 | 4.36 | 1.76 | 0.67 | | 0.16 | 0.99 | 0.58 | 0.18 | 0.18 | | 0.21 | 4.39 | 1.50 | 0.59 | |
| High TI | 158 | 1.9% | 0.51 | 4.09 | 1.89 | 0.74 | | 0.19 | 0.98 | 0.53 | 0.17 | 0.17 | | 0.26 | 3.42 | 1.46 | 0.58 | |
| TOTAL | 8454 | 100% | 0.08 | 4.36 | 1.65 | 0.68 | | 0.01 | 0.99 | 0.63 | 0.18 | 0.18 | | 0.06 | 4.67 | 1.50 | 0.64 | |

‡ Descriptive statistics for $Y_{it} - Y_{it-1}$ in the year 2009

‡ for average value 2006-2012

SD=standard deviation.

P-value in brackets. (***), (**), (*) mean significant at 1%, 5% and 10%, respectively.

As mentioned above, spatial or cross-sectional models depend on the so-called *spatial weight matrix*. In this case, we have built a binary weighting matrix \mathbf{W} based on the 240 nearest neighbours, which means that for each company i , the 240 geographically nearest companies are considered its neighbours². It is clear that this matrix, based on geographical distance, is strictly exogenous.

Table 3 reports PAM estimates using several spatial econometric panel equations. We begin with the nonlinear, non-spatial PAM equations that appear in the first three columns as:

$$\begin{aligned}
 y_{it} - y_{it-1} = & \delta(y_{it}^* - y_{it-1}) + \sum_{k=2}^3 \delta_{dk} (y_{it}^* - y_{it-1}) d_{itk} + \sum_{k=2}^3 \delta_{sk} (y_{it}^* - y_{it-1}) s_{itk} + \\
 & + \sum_{k=2}^4 \delta_{tk} (y_{it}^* - y_{it-1}) t_{itk} + v_{it}
 \end{aligned} \tag{7}$$

The descriptions of the variables appear in Table 1. v_{it} is a composed error term, $v_{it} = \eta_i + e_{it}$, where e_{it} is a pure idiosyncratic white noise error, and η_i is an unobservable individual effect (we found no evidence of unobservable time effects). Overall, the results from this specification are reasonable, with a moderate speed of adjustment for the three ratios. Unbalanced companies seem to be more motivated to quickly restore equilibrium, as are large and hi-tech firms. Let us note that a standard F -test (not in the Table) confirms the existence of unobserved individual effects in the sample (as stated above, time effects are not present) and the Hausman test rejects the null hypothesis of random effects. So, the estimates in the first three columns refer to the fixed effects specification.

The residuals of the three estimations suffer from a severe problem of omitted spatial dependence. In particular, we obtain very high values for the LM-err and LM-lag tests, for the three financial ratios. The Lagrange Multipliers at the bottom of Table 3 point to substantive spatial dependence in all three cases. Following Elhorst (2014), we specify an SDM model to handle this problem:

$$\begin{aligned}
 y_{it} - y_{it-1} = & \delta(y_{it}^* - y_{it-1}) + \sum_{k=2}^3 \delta_{dk} (y_{it}^* - y_{it-1}) d_{itk} + \sum_{k=2}^3 \delta_{sk} (y_{it}^* - y_{it-1}) s_{itk} + \\
 + \sum_{k=2}^4 \delta_{tk} (y_{it}^* - y_{it-1}) t_{itk} + & \rho \sum_{j \neq i}^N \omega_{ij} (y_{it} - y_{it-1}) + \sum_{k=1}^2 \theta_{dk} \sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) d_{itk} + \\
 \sum_{k=1}^2 \theta_{sk} \sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) s_{itk} + & \sum_{k=1}^3 \theta_{tk} \sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) t_{itk} + v_{it}
 \end{aligned} \tag{8}$$

The main results appear in columns four to six. The LR tests confirm that the SDM outperforms both the SLM (LR SLM vs. SDM), and the SEM (LR SEM vs. SDM)³. Overall, the estimated coefficients have the expected sign. We obtain significant and positive adjustment coefficients for every financial ratio (δ), which are roughly in line with previous literature (Gallizo *et al.*, 2008; Mate *et al.*, 2012, 2017).

² We have selected this matrix because it provides better results than other matrices. A total of 240 neighbours roughly translates to a circle with a 40 km radius around each company, so it is reasonable to assume that companies within this radius are subject to similar economic conditions. Similar results were found with a different number of neighbours.

³ The results for SLM and SEM are not included in the paper but are available from the authors upon request.

Moreover, the adjustment process has strong nonlinearities. For example, the speed of adjustment is greater for companies with financial ratios far from the target ($\hat{\delta}_{d2}$) than for companies whose financial ratios are near to, or above, the target ($\hat{\delta}_{d3}$). Faulkner *et al.* (2010) and Aybar-Arias *et al.* (2012) obtained similar results. Their reasoning can also be applied to our case: companies with financial ratios far from the objective face higher costs because of the disequilibrium and, therefore, they have additional incentives to quickly adjust their financial structure.

● **Table 3. Estimation results of the PAM model**

| | FE Model | | | SDM with FE | | | SDM with spatial break + FE | | |
|--|-----------|-----------|-------------|-------------|-----------|-----------|-----------------------------|-----------|-----------|
| | Coef. | Coef. | Coef. | Coef. | Coef. | Coef. | Coef. | Coef. | Coef. |
| δ (distance) | 0.530*** | 0.467*** | 0.563*** | 0.532*** | 0.469*** | 0.582*** | 0.485*** | 0.432** | 0.625*** |
| δ_{d2} (gap below 10%) | 0.153*** | 0.007 | 0.097*** | 0.151*** | 0.010* | 0.089*** | 0.201*** | 0.012* | 0.091*** |
| δ_{d3} (gap above 90%) | -0.012 | 0.038*** | -0.016** | -0.010 | 0.038*** | -0.008 | -0.018 | 0.043*** | -0.007 |
| δ_{s2} (size small-med) | -0.041*** | -0.013*** | -0.017*** | -0.041*** | -0.013*** | -0.022*** | -0.038*** | -0.009*** | -0.023*** |
| δ_{s3} (size large-med) | -0.029** | -0.015 | -0.026** | -0.029** | -0.011 | -0.050*** | -0.027** | -0.015 | -0.052*** |
| δ_{t2} (tech low-med) | 0.010 | 0.021*** | 0.045*** | 0.009 | 0.016** | 0.029*** | 0.006 | 0.021** | 0.035*** |
| δ_{t3} (tech med-high) | 0.026** | 0.045*** | 0.137*** | 0.024** | 0.039*** | 0.113*** | 0.021** | 0.037*** | 0.115*** |
| δ_{t4} (tech high) | -0.038* | 0.045** | 0.066*** | -0.038* | 0.042** | 0.059** | -0.042* | 0.047** | 0.063** |
| θ | -- | -- | -- | -0.480*** | -0.536*** | -0.752*** | -0.473*** | -0.531*** | -0.754*** |
| θ_{d2} (gap below 10%) | -- | -- | -- | 0.084 | -0.216*** | 0.075 | 0.089 | -0.210*** | 0.074 |
| θ_{d3} (gap above 90%) | -- | -- | -- | -0.117 | -0.052 | -0.046 | -0.128 | -0.052 | -0.048 |
| θ_{s2} (size small-med) | -- | -- | -- | 0.093 | 0.069 | 0.063 | 0.061 | 0.055 | 0.078 |
| θ_{s3} (size large-med) | -- | -- | -- | 0.352* | 0.040 | 0.385*** | 0.330* | 0.016 | 0.388*** |
| θ_{t2} (tech low-med) | -- | -- | -- | 0.030 | 0.448*** | 0.170** | 0.041 | 0.443** | 0.174** |
| θ_{t3} (tech med-high) | -- | -- | -- | 0.428*** | 0.525*** | 0.273** | 0.434*** | 0.499*** | 0.283** |
| θ_{t4} (tech high) | -- | -- | -- | 0.753** | 0.683* | -0.032 | 0.786** | 0.681* | -0.052 |
| ρ | -- | -- | -- | 0.352*** | 0.482*** | 0.856*** | 0.474*** | 0.610*** | 0.785*** |
| ρ^* | -- | -- | -- | -- | -- | -- | -0.164** | -0.137* | 0.089*** |
| pseudo R^2 | 0.302 | 0.2415 | 0.320 | 0.366 | 0.348 | 0.452 | 0.375 | 0.353 | 0.461 |
| Log-Lik | -39929 | -93307 | -14008 | -39821 | -93527 | -15007 | -39819 | -93535 | -15004 |
| Specification tests | | | | | | | | | |
| LR SEM vs SDM | | | 32.34*** | 64.59*** | 31.15*** | | | | |
| LR SLM vs SDM | | | 63.46*** | 109.80*** | 498.34*** | | | | |
| LR SEM vs SDM | | | 197.0*** | 440.20*** | 513.69*** | | | | |
| LR SDM vs SDM ^{break} | | | | | | 4.04*** | 3.27* | 5.95*** | |
| Diagnostic Test | | | | | | | | | |
| LM-err | 226.8*** | 1051.4*** | 72720.0*** | | | | | | |
| LM-EL | 3.5* | 5.3** | 10.9** | | | | | | |
| LM-lag | 338.0*** | 1382.2*** | 143997.4*** | | | | | | |
| LM-LE | 114.7*** | 336.1*** | 71287.7*** | | | | | | |
| Break Tests (break: size of the company) | | | | | | | | | |
| LM ^{break} _{SML/FE} | | | | 4.95** | 4.79* | 6.69*** | | | |
| size<10 employees | | | | 0.25 | 2.69* | 26.07*** | | | |
| LM ^{break} _{SML/FE} | | | | | | | | | |
| size>50 employees | | | | | | | | | |

(***), (***) and (*) mean significant at 1%, 5% and 10% respectively.
FE means Fixed Effects

The size of the companies ($\hat{\delta}_{s,2}$ and $\hat{\delta}_{s,3}$) is also significant and the estimates appear with a negative sign for the three ratios (we should recall that the reference group is small companies). Consequently, our estimates indicate that the adjustment speed decreases as the size of the company increases. Davis and Peles (1993) and Wu and Ho (1997) obtained a similar result, which highlights the importance, for small companies, of being as near as possible to the optimum, due to the higher cost of disequilibrium. There is less pressure on large companies so they react more slowly. Finally, similar to Gallizo *et al.* (2002), we obtain positive estimates for the technological intensity ($\hat{\delta}_{ij}, j = 2,3,4$): the speed of adjustment increases as the technological content of the company rises.

The SDM also includes the spatial lag of the endogenous variable, whose estimated coefficients ($\hat{\rho}$) are highly significant, showing the extent of the cross-sectional dependence in the sample. The dependence is stronger for the case of the profitability ratio, PR, and weaker for the current (CU) and debt (DE) ratios, which can be attributed to the financial constraints that limit the capacity of the companies to adjust their structures.

What does spatial dependence mean in this context? Regarding the CU, it means that a 1% average improvement in the unbalance of the CU in nearby companies, $\sum_{j \neq i}^N \omega_{ij} (y_{it} - y_{it-1})$, will contribute to a 0.352% increase in the effort made by company i , $(y_{it} - y_{it-1})$ towards achieving its own goal. Therefore, the adjustment process in company i depends not only on its own characteristics but also on the financial results of its neighbours. An analogous interpretation can be applied to DE and PR.

As stated above, proximity facilitates imitation among companies, which strengthens the role of local networks. Let us recall that PR is strongly influenced by market conditions, which are beyond the control of the company. Therefore, a quick reaction to changes in PR of nearby companies can be considered a reaction to changes occurring in the local market. In other words, the strong interdependence in the case of PR can be attributed to the need to quickly react to changes in the local markets, which is critical for small and medium sized firms. In the case of DE and CU, these external forces are not as relevant, in the short-term.

Next, we check whether the endogenous spatial effects vary for different groups in the sample; we focus on *size*. From a technical perspective, this is a problem of breaks in the coefficients of spatial dependence. The corresponding test equation and null hypothesis are:

$$\begin{aligned}
 y_{it} - y_{it-1} = & \delta (y_{it}^* - y_{it-1}) + \sum_{k=2}^3 \delta_{dk} (y_{it}^* - y_{it-1}) d_{itk} + \sum_{k=2}^3 \delta_{sk} (y_{it}^* - y_{it-1}) s_{itk} + \\
 & + \sum_{k=2}^4 \delta_{sk} (y_{it}^* - y_{it-1}) t_{itk} + \rho \sum_{j \neq i}^N \omega_{ij} (y_{it} - y_{it-1}) + \sum_{k=1}^2 \theta_{dk} \sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) d_{itk} + \\
 & \sum_{k=1}^2 \theta_{sk} \sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) s_{itk} + \sum_{k=1}^3 \theta_{tk} \sum_{j \neq i}^N \omega_{ij} (y_{it}^* - y_{it-1}) t_{itk} + v_{it}
 \end{aligned} \tag{9}$$

$$\left. \begin{array}{l} H_0: \rho^* = 0 \\ H_A: \rho^* \neq 0 \end{array} \right\}$$

To proceed with (9), we compute the $LM_{SLM/FE}^{break}$ tests⁴ considering, respectively, large (more than 50 employees) and small (fewer than 10 employees) companies. The results appear at the bottom of the Table, in columns four to six, showing that the null hypothesis of stability in the spatial dependence for the three ratios must be rejected (the exception is the CU ratio for large companies).

Columns seven to nine in Table 3 display the estimation of the SDM model with a break in the spatial dependence parameter for the group of small companies. If there is a break in the spatial interaction mechanism that is not accounted for, the estimates will be biased. This flaw is corrected in our final specification, in which we find a spatial interaction effect common to all the companies in the sample ($\hat{\rho}$), plus a spatial interaction term that applies only to the group of small companies ($\hat{\rho}^*$). For example, in the case of CU, the common spatial interaction coefficient decreases to 0.310 for the group of small companies ($\hat{\rho} + \hat{\rho}^* = 0.474 - 0.164 = 0.310$). For DE, we find a similar impact: the interaction coefficient decreases to 0.473 ($\hat{\rho} + \hat{\rho}^* = 0.610 - 0.137$). However, for PR, the spatial effects are stronger ($\hat{\rho} + \hat{\rho}^* = 0.874$)⁵.

The results indicate that, for the group of small companies, spatial dependence is weaker for the liquidity and indebtedness dimensions, but it is quite strong for the case of profitability. Our findings offer additional support to the hypothesis that large and small companies behave differently in the case of external shocks to the market. Previous studies are not conclusive on this point. Some of them underline the role of informational asymmetries for small companies, which result in a greater willingness to imitate the financial practices of other nearby companies (Carreira and Silva, 2010). Other studies consider that large companies have more resources for monitoring competitors' financial practices and, therefore, are in better position to identify and implement better financial practices (Reppenhausen, 2010, Greve, 2005).

It is clear that small and large companies differ not only in relation to their ability to react to unbalances, but also in their responsiveness to the environment with respect to the main financial variables. Our findings show that external financial practices, related to DE and CU, have a weaker impact for small companies

⁴ Note that the SDM model encompasses the SLM model, which means that the extends immediately to the SDM model with obvious changes in notation.

⁵ A similar analysis can be carried out for the group of large companies. It is not included in the paper but details are available from the authors upon request.

than for large ones: if company i is surrounded by other companies, which are adjusting their CU and/or DE, the impact of these adjustments will be more effective if i is a large company.

This result may relate to the financial constraints affecting small companies (Beck and Demirgüç-Kunt, 2006). In general, a small company has few opportunities to adjust its CU or DE given that banks and investors are highly reluctant to provide funding for this group, because of higher risks and information asymmetries (Acharya *et al.*, 2007).

The situation regarding the PR is different given that small companies are now more reactive to the environment. The reason is clear according to Beck *et al.* (2011): small companies are more dependent on external conditions than medium-to-large companies. Consider a small company located in a region where profitability ratios are decreasing due to a negative shock in the regional market; in such a case, the strong dependence of small companies on their local neighbourhood will quickly lead to reduced earnings, thus diminishing their PR at a faster rate than for a large company (which is diversified in other markets).

Regarding the spatially-lagged explanatory variables (θ 's parameters), the extended SDM model shows that only five out of seven are significant for at least one financial ratio, indicating that their contribution is quite limited. These variables measure the changes in the adjustment process of company i due to changes in the gap of its neighbours. The estimate of θ is negative and highly significant, which points to the fact that if most companies located in a given area have problems (i.e., their gap increases), the situation of company i will be somewhat alleviated, thus reducing the pressure to adjust its own financial ratios. Overall, the results coincide with previous literature, which concludes that the impact of large and technologically-advanced firms is greater than that of other companies (O'Brien and Tan, 2015).

■ 6. Conclusions

This paper examines the impact of geographical proximity on the financial dynamics of companies, highlighting differences between small and large companies. With this aim, we evaluate firms' financial behaviour through three key financial ratios (profitability, indebtedness and liquidity), specifying a typical PAM model, extended to allow for nonlinearities and cross-sectional dependence among neighbouring companies. The equations have been estimated using a large sample of Spanish industrial companies located in the Mediterranean Axis.

Our results confirm the importance of geographical proximity between companies. We conclude that the financial adjustment process is not only determined by external market shocks or by firms' managerial decisions; the financial situation of neighbouring companies and the decisions they take are key to understanding the decisions of a specific company. This effect is accentuated for small companies. We find additional differences between small and large companies with respect to their responsiveness to changes in the environment. This result could be related to the capacity of these companies to react to external shocks. In general, small companies are more influenced by external characteristics, as they face greater market barriers related to informational asymmetries and financial restrictions.

Regarding differences between financial dimensions, we find that the interaction effect arising from firms' geographical proximity is stronger for profitability than for indebtedness or liquidity ratios. This result reflects the varying ability of managers to control for the different financial dimensions. The profitability dimension thus depends, to a great extent, on local market conditions over which managers have little control. On the other hand, managers take decisions on debt and current ratios which are under their sphere of influence. Another interesting result is that nonlinearities and spatial effects reinforce each other. In this sense, we find that the profitability ratios of small companies, located nearby, adjust more quickly than those of large companies, which are less involved in the local networks. Something similar happens with debt and current ratios.

In sum, our study provides additional evidence about the financial ratio adjustment process, recognizing the relevant role of geographical proximity between peer companies. The impact of this variable yields useful insights into the financial strategies of different groups of companies. Our results suggest that neighbouring companies' financial decisions should not be taken in isolation but rather jointly with the characteristics of the firms, especially firm size. Future research in this area should reconsider financial analysis by including the geographical proximity between peer companies as an additional indicator that can explain firms' financial behaviour.

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■ APPENDIX: The Lagrange Multipliers used to test for a structural break in the spatial coefficient of a panel data model with fixed effects

This Appendix contains additional results on the LM-break test in a spatial panel data framework, introduced in Section 4. Specifically, we focus on the likelihood function, the score vector and the information matrix required to obtain the statistic.

Section A.1. The SLM fixed effects panel data model

The model that we are considering has the following expression:

$$y_{it} = \mu_i + \gamma_0 \sum_{j=1}^N \omega_{ji} y_{jt} + \gamma_1 \sum_{j=1}^N \omega_{ji}^* y_{jt} + x_{it}' \beta + u_{it} \quad (A1)$$

with $\{\omega_{ji}, i, j=1, \dots, N\}$ and $\{\omega_{ji}^*, i, j=1, \dots, N\}$ being weights corresponding to the weighting matrices \mathbf{W} and \mathbf{W}^* ; $\{\mu_i, i=1, \dots, N\}$ is a sequence of unobserved fixed effects. Assuming that the error term u_{it} is normally distributed, the log-likelihood function is:

$$l(\theta) = -\frac{NT}{2} \ln(2\pi\sigma^2) + T \ln |\mathbf{B}| - \frac{1}{2\sigma^2} \sum_{j=1}^N \left(y_{it} - \mu_i - \gamma_0 \sum_{j=1}^N \omega_{ji} y_{jt} - \gamma_1 \sum_{j=1}^N \omega_{ji}^* y_{jt} - x_{it}' \beta \right)^2 \quad (A2)$$

\mathbf{B} is the so-called diffusion matrix, $\mathbf{B} = \mathbf{I}_T \otimes \mathbf{B} = \mathbf{I}_T \otimes (\mathbf{I} - \gamma_0 \mathbf{W} - \gamma_1 \mathbf{W}^*)$ and θ is a vector of $(N+k+3)$ parameters. Using a more compact notation, the likelihood can be written as:

$$l(\theta) = -\frac{NT}{2} \ln(2\pi\sigma^2) + T \ln |\mathbf{B}| - \frac{1}{2\sigma^2} (\mathbf{By} - \mathbf{X}\beta - \boldsymbol{\tau} \otimes \boldsymbol{\mu})' (\mathbf{By} - \mathbf{X}\beta - \boldsymbol{\tau} \otimes \boldsymbol{\mu}) \quad (A3)$$

where $\boldsymbol{\tau}$ is a $(T \times 1)$ vector of ones and $\boldsymbol{\mu}$ is the $(N \times 1)$ vector of fixed unobserved effects. According to Elhorst (2014), we first have to demean the data to eliminate the unobserved fixed effects. Let us denote the demeaned data as:

$$\mathbf{y}^* = \begin{bmatrix} \mathbf{y}_1^* \\ \mathbf{y}_2^* \\ \vdots \\ \mathbf{y}_N^* \end{bmatrix}; \mathbf{y}_i^* = \begin{bmatrix} y_{i1} - \bar{y}_i \\ y_{i2} - \bar{y}_i \\ \vdots \\ y_{iT} - \bar{y}_i \end{bmatrix}; \mathbf{X}^* = \begin{bmatrix} \mathbf{X}_1^* \\ \mathbf{X}_2^* \\ \vdots \\ \mathbf{X}_N^* \end{bmatrix}; \mathbf{X}_i^* = \begin{bmatrix} x_{i11} - \bar{x}_{i1} & x_{i21} - \bar{x}_{i2} & \dots & x_{ik1} - \bar{x}_{ik} \\ x_{i12} - \bar{x}_{i1} & x_{i22} - \bar{x}_{i2} & \dots & x_{ik2} - \bar{x}_{ik} \\ \vdots & \vdots & \ddots & \vdots \\ x_{i1T} - \bar{x}_{i1} & x_{i2T} - \bar{x}_{i2} & \dots & x_{ikT} - \bar{x}_{ik} \end{bmatrix}; \mathbf{u}^* = \begin{bmatrix} \mathbf{u}_1^* \\ \mathbf{u}_2^* \\ \vdots \\ \mathbf{u}_N^* \end{bmatrix}; \mathbf{u}_i^* = \begin{bmatrix} u_{i1} - \bar{u}_i \\ u_{i2} - \bar{u}_i \\ \vdots \\ u_{iT} - \bar{u}_i \end{bmatrix}$$

The means refer to the individual means, that is: $\bar{y}_i = \frac{\sum_{t=1}^T y_{it}}{T}$ (the same for the other variables). Note that after demeaning the number of unknowns in the model decreases from $(N+k+3)$ in θ to only $(k+3)$ in θ^* . The score vector for the log-likelihood of (A3) is:

$$\mathbf{g}(\boldsymbol{\theta}^*) = \begin{bmatrix} \frac{\partial l}{\partial \boldsymbol{\beta}} \\ \frac{\partial l}{\partial \gamma_0} \\ \frac{\partial l}{\partial \gamma_1} \\ \frac{\partial l}{\partial \sigma^2} \end{bmatrix} = \frac{1}{\sigma^2} \begin{bmatrix} \mathbf{X}'' \mathbf{u} \\ -T\sigma^2 \text{tr}[\mathbf{B}^{-1} \mathbf{W}] - \frac{1}{2} \{ \mathbf{y}'' (\mathbf{I}_T \otimes \mathbf{W}) \mathbf{u}^* + \mathbf{u}'' (\mathbf{I}_T \otimes \mathbf{W}) \mathbf{y}^* \} \\ -T\sigma^2 \text{tr}[\mathbf{B}^{-1} \mathbf{W}^*] - \frac{1}{2} \{ \mathbf{y}'' (\mathbf{I}_T \otimes \mathbf{W}^*) \mathbf{u}^* + \mathbf{u}'' (\mathbf{I}_T \otimes \mathbf{W}^*) \mathbf{y}^* \} \\ -\frac{NT}{2} + \frac{\mathbf{u}'' \mathbf{u}^*}{2\sigma^2} \end{bmatrix} \quad (\text{A4})$$

The expected value of the second derivatives of the score terms under the null hypothesis that there is no break in the spatial dependence parameter are:

$$\begin{aligned} -E \left[\frac{\partial^2 l}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}'} \right] &= \frac{1}{\sigma^2} \mathbf{X}'' \mathbf{X}^* \\ -E \left[\frac{\partial^2 l}{\partial \boldsymbol{\beta} \partial \gamma_0} \right] &= \frac{1}{\sigma^2} \boldsymbol{\beta}' \mathbf{X}'' \{ \mathbf{I}_T \otimes (\mathbf{B}_0^{-1} \mathbf{W}) \} \mathbf{X}^* \\ -E \left[\frac{\partial^2 l}{\partial \boldsymbol{\beta} \partial \gamma_1} \right] &= \frac{1}{\sigma^2} \boldsymbol{\beta}' \mathbf{X}'' \{ \mathbf{I}_T \otimes (\mathbf{B}_0^{-1} \mathbf{W}^*) \} \mathbf{X}^* \\ -E \left[\frac{\partial^2 l}{\partial \boldsymbol{\beta} \partial \sigma^2} \right] &= \mathbf{0} \\ -E \left[\frac{\partial^2 l}{\partial \gamma_0 \partial \gamma_0} \right] &= T \text{tr} [\mathbf{B}_0^{-1} \mathbf{W} \mathbf{B}_0^{-1} \mathbf{W}] + T \text{tr} [\mathbf{W} \mathbf{W} \mathbf{B}_0^{-1} \mathbf{B}_0^{-1}] - \frac{1}{\sigma^2} \text{tr} \{ \mathbf{I}_T \otimes (\mathbf{W} \mathbf{B}_0^{-1}) \} \\ &\quad \{ \mathbf{X}^* \boldsymbol{\beta} \boldsymbol{\beta}' \mathbf{X}^* \} \{ \mathbf{I}_T \otimes (\mathbf{W} \mathbf{B}_0^{-1}) \} \\ -E \left[\frac{\partial^2 l}{\partial \gamma_0 \partial \gamma_1} \right] &= T \text{tr} [\mathbf{B}_0^{-1} \mathbf{W} \mathbf{B}_0^{-1} \mathbf{W}^*] + T \text{tr} [\mathbf{W} \mathbf{W}^* \mathbf{B}_0^{-1} \mathbf{B}_0^{-1}] - \frac{1}{\sigma^2} \text{tr} \{ \mathbf{I}_T \otimes (\mathbf{W} \mathbf{B}_0^{-1}) \} \\ &\quad \{ \mathbf{X}^* \boldsymbol{\beta} \boldsymbol{\beta}' \mathbf{X}^* \} \{ \mathbf{I}_T \otimes (\mathbf{W} \mathbf{B}_0^{-1}) \} \\ -E \left[\frac{\partial^2 l}{\partial \gamma_0 \partial \sigma^2} \right] &= \frac{T}{\sigma^2} \text{tr} (\mathbf{W} \mathbf{B}_0^{-1}) \\ -E \left[\frac{\partial^2 l}{\partial \gamma_1 \partial \gamma_1} \right] &= T \text{tr} [\mathbf{B}_0^{-1} \mathbf{W}^* \mathbf{B}_0^{-1} \mathbf{W}^*] + T \text{tr} [\mathbf{W}^* \mathbf{W}^* \mathbf{B}_0^{-1} \mathbf{B}_0^{-1}] - \frac{1}{\sigma^2} \text{tr} \{ \mathbf{I}_T \otimes (\mathbf{W}^* \mathbf{B}_0^{-1}) \} \\ &\quad \{ \mathbf{X}^* \boldsymbol{\beta} \boldsymbol{\beta}' \mathbf{X}^* \} \{ \mathbf{I}_T \otimes (\mathbf{W}^* \mathbf{B}_0^{-1}) \} \\ -E \left[\frac{\partial^2 l}{\partial \gamma_1 \partial \sigma^2} \right] &= \frac{T}{\sigma^2} \text{tr} (\mathbf{W}^* \mathbf{B}_0^{-1}) \\ -E \left[\frac{\partial^2 l}{\partial \sigma^2 \partial \sigma^2} \right] &= \frac{NT}{2\sigma^4} \end{aligned} \quad (\text{A5})$$

We partition the information matrix (evaluated in the null hypothesis) as follows:

$$\mathbf{I}(\boldsymbol{\theta}^*)_{H_0} = \begin{bmatrix} \mathbf{I}_{\beta\beta} & \mathbf{I}_{\beta\gamma_0} & \mathbf{I}_{\beta\sigma^2} & \mathbf{I}_{\beta\gamma_1} \\ (kxk) & (kx1) & (kx1) & (kx1) \\ \mathbf{I}_{\gamma_0\gamma_0} & \mathbf{I}_{\gamma_0\sigma^2} & \mathbf{I}_{\gamma_0\gamma_1} \\ (1x1) & (1x1) & (1x1) \\ \mathbf{I}_{\sigma^2\sigma^2} & \mathbf{I}_{\sigma^2\gamma_1} \\ (1x1) & (1x1) \\ \mathbf{I}_{\gamma_1\gamma_1} \\ (1x1) \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{I}_{\beta\beta} & \mathbf{I}_{\beta\gamma_0} & \mathbf{I}_{\beta\sigma^2} \\ (kx1) & (kx1) & (kx1) \\ \mathbf{I}_{\gamma_0\gamma_0} & \mathbf{I}_{\gamma_0\sigma^2} \\ (1x1) & (1x1) \\ \mathbf{I}_{\sigma^2\sigma^2} \\ (1x1) \end{bmatrix} \quad \begin{bmatrix} \mathbf{I}_{\beta\gamma_1} \\ (kx1) \\ \mathbf{I}_{\gamma_1\gamma_1} \\ (1x1) \\ \mathbf{I}_{\sigma^2\gamma_1} \\ (1x1) \end{bmatrix} \quad (\text{A6})$$

$$\begin{bmatrix} \mathbf{M}_{21} = \mathbf{M}'_{12} \\ \mathbf{M}_{22} = \begin{bmatrix} \mathbf{I}_{\gamma_1\gamma_1} \\ (1x1) \end{bmatrix} \end{bmatrix}$$

where $\mathbf{I}_{ab} = -E \left[\frac{\partial^2 l}{\partial a \partial b} \right]$. As stated above, the null hypothesis is that there is no break in the spatial dependence parameter of the SLM model:

$$\left. \begin{aligned} H_0: \gamma_1 = 0 \\ H_0: \gamma_1 \neq 0 \end{aligned} \right\} \quad (A7)$$

Then, the score of (A3) becomes:

$$\mathbf{g}(\boldsymbol{\theta}^*)_{H_0} = \begin{bmatrix} \frac{\partial l}{\partial \boldsymbol{\beta}} \\ \frac{\partial l}{\partial \gamma_0} \\ \frac{\partial l}{\partial \gamma_1} \\ \frac{\partial l}{\partial \sigma^2} \end{bmatrix}_{H_0} = \begin{bmatrix} \mathbf{0} \\ 0 \\ \frac{1}{2\sigma^2} \left\{ \mathbf{y}'' (\mathbf{I}_T \otimes \mathbf{W}'') \hat{\mathbf{u}}^* + \hat{\mathbf{u}}'' (\mathbf{I}_T \otimes \mathbf{W}'') \mathbf{y} \right\} - Ttr [\mathbf{B}_0^{-1} \mathbf{W}'] \\ 0 \end{bmatrix} \quad (A8)$$

$\hat{\mathbf{u}}^*$ is the $(TN \times 1)$ vector of ML demeaned residuals of the SLM model, estimated under the assumption that there is no break in the spatial coefficient, $\hat{\sigma}^2$ the estimated ML variance and \mathbf{B}_0 the estimated diffusion matrix under the null hypothesis, $\mathbf{B}_0 = (\mathbf{I} - \hat{\gamma}_0 \mathbf{W})$.

Finally, the Lagrange Multiplier needed for the hypothesis of (A7), in the case of a Spatial Lag Model with Fixed Effects, is:

$$\begin{aligned} LM_{SLM/FE}^{break} &= \left[\mathbf{g}(\boldsymbol{\theta}^*)_{H_0} \right]' \left[\mathbf{I}(\boldsymbol{\theta}^*)_{H_0} \right]^{-1} \left[\mathbf{g}(\boldsymbol{\theta}^*)_{H_0} \right]_{as} \chi^2_{(1)} \\ \Rightarrow LM_{SLM/FE}^{break} &= \frac{\left(\frac{\left\{ \mathbf{y}'' (\mathbf{I}_T \otimes \mathbf{W}'') \hat{\mathbf{u}}^* + \hat{\mathbf{u}}'' (\mathbf{I}_T \otimes \mathbf{W}'') \mathbf{y} \right\} - Ttr [\mathbf{B}_0^{-1} \mathbf{W}']}{2\hat{\sigma}^2} \right)^2}{\left(M_{22} - M_{21} M_{11}^{-1} M_{12} \right)} \end{aligned} \quad (A9)$$

Section A.II. The SEM fixed effects panel data model

The equation corresponding to the SEM case is:

$$\left. \begin{aligned} y_{it} &= \mu_i + x_{it}' \boldsymbol{\beta} + u_{it} \\ \mu_{it} &= \gamma_0 \sum_{j=1}^N \omega_{ji} u_{jt} + \gamma_1 \sum_{j=1}^N \omega_{ji}^* u_{jt} + \varepsilon_{it} \end{aligned} \right\} \quad (A10)$$

The log-likelihood function, assuming that the random terms, ε_{it} , are normally distributed is:

$$l(\boldsymbol{\theta}) = -\frac{NT}{2} \ln(2\pi\sigma^2) + T \ln |\mathbf{B}| - \frac{1}{2\sigma^2} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta} - \boldsymbol{\tau} \otimes \boldsymbol{\mu})' (\mathbf{B}' \mathbf{B}) (\mathbf{y} - \mathbf{X}\boldsymbol{\beta} - \boldsymbol{\tau} \otimes \boldsymbol{\mu}) \quad (A11)$$

\mathbf{B} is the $(NT \times NT)$ matrix $\mathbf{B} = \mathbf{I}_T \otimes \mathbf{B} = \mathbf{I}_T \otimes (\mathbf{I} - \gamma_0 \mathbf{W} - \gamma_1 \mathbf{W}^*)$, and $\boldsymbol{\theta}$ the vector of $(N+k+3)$ parameters to be estimated. The hypothesis of interest remains the same:

$$\left. \begin{aligned} H_0: \gamma_1 &= 0 \\ H_0: \gamma_1 &\neq 0 \end{aligned} \right\} \quad (A12)$$

Like the SLM case, the data have to be demeaned to eliminate the unobserved effects. Under these circumstances, the score vector for the null of (A12) has a compact expression:

$$\mathbf{g}(\boldsymbol{\theta}^*)_{H_0} = \begin{bmatrix} \frac{\partial l}{\partial \boldsymbol{\beta}} \\ \frac{\partial l}{\partial \gamma_0} \\ \frac{\partial l}{\partial \gamma_1} \\ \frac{\partial l}{\partial \sigma^2} \end{bmatrix}_{H_0} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \frac{1}{2\sigma^2} \hat{\mathbf{u}}^* \left\{ \mathbf{I}_T \otimes (\mathbf{B}_0' \mathbf{W}^* + \mathbf{W}^* \mathbf{B}_0) \right\} \hat{\mathbf{u}}^* - \text{Tr}[\mathbf{B}_0^{-1} \mathbf{W}^*] \\ \mathbf{0} \end{bmatrix} \quad (A13)$$

Similarly, we can obtain the elements of the information matrix under the null:

$$\begin{aligned} \mathbf{I}_{\beta\beta} &= \frac{1}{\sigma^2} \mathbf{X}' \left\{ \mathbf{I}_T \otimes (\mathbf{B}_0' \mathbf{B}_0) \right\} \mathbf{X} \\ \mathbf{I}_{\beta\gamma_0} &= \mathbf{0} \\ \mathbf{I}_{\beta\sigma^2} &= \mathbf{0} \\ \mathbf{I}_{\beta\gamma_1} &= \mathbf{0} \\ \mathbf{I}_{\gamma_0\gamma_0} &= \text{Tr}[\mathbf{B}_0^{-1} \mathbf{W} \mathbf{B}_0^{-1} \mathbf{W}] + \text{Tr}[\mathbf{W} \mathbf{W} \mathbf{B}_0^{-1} \mathbf{B}_0^{-1}] \\ \mathbf{I}_{\gamma_0\sigma^2} &= \frac{T}{\sigma^2} \text{tr}(\mathbf{W} \mathbf{B}_0^{-1}) \\ \mathbf{I}_{\gamma_0\gamma_1} &= \text{Tr}[\mathbf{B}_0^{-1} \mathbf{W} \mathbf{B}_0^{-1} \mathbf{W}^*] + \text{Tr}[\mathbf{W} \mathbf{W}^* \mathbf{B}_0^{-1} \mathbf{B}_0^{-1}] \\ \mathbf{I}_{\sigma^2\sigma^2} &= \frac{NT}{2\sigma^4} \\ \mathbf{I}_{\sigma^2\gamma_1} &= \frac{T}{\sigma^2} \text{tr}(\mathbf{W}^* \mathbf{B}_0^{-1}) \\ \mathbf{I}_{\gamma_1\gamma_1} &= \text{Tr}[\mathbf{B}_0^{-1} \mathbf{W}^* \mathbf{B}_0^{-1} \mathbf{W}^*] + \text{Tr}[\mathbf{W}^* \mathbf{W}^* \mathbf{B}_0^{-1} \mathbf{B}_0^{-1}] \end{aligned} \quad (A14)$$

Using the same partition, of $\mathbf{I}(\boldsymbol{\theta})_{H_0}$ as in (A6), the Lagrange Multiplier for this case is:

$$\begin{aligned} \text{LM}_{\text{SEM/FE}}^{\text{break}} &= \left[\mathbf{g}(\boldsymbol{\theta})_{H_0} \right]' \left[\mathbf{I}(\boldsymbol{\theta})_{H_0} \right]^{-1} \left[\mathbf{g}(\boldsymbol{\theta})_{H_0} \right]_{as} \chi^2(1) \\ \Rightarrow \text{LM}_{\text{SEM/FE}}^{\text{break}} &= \frac{\left(\frac{\left\{ \hat{\mathbf{u}}^* \left\{ \mathbf{I}_T \otimes (\mathbf{B}_0' \mathbf{W}^* + \mathbf{W}^* \mathbf{B}_0) \right\} \hat{\mathbf{u}}^* \right\}}{2\hat{\sigma}^2} - \text{Tr}[\mathbf{B}_0^{-1} \mathbf{W}^*] \right)^2}{\left(\mathbf{M}_{22} - \mathbf{M}_{21} \mathbf{M}_{11}^{-1} \mathbf{M}_{12} \right)} \end{aligned} \quad (A15)$$

