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Maintenance scheduling of a protection system subject to imperfect inspection and replacement

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Abstract. An inspection and replacement policy for a protection system is described in which the inspection process is subject to error, and false positives (false alarms) and false negatives are possible. We develop two models: one in which a false positive implies renewal of the protection system; the other not. These models are motivated by inspection of a protection system on the production line of a beverage manufacturer. False negatives reduce the efficiency of inspection. Another notion of imperfect maintenance is also modelled: that of poor installation of a component at replacement. These different aspects of maintenance quality interact: false alarms can, in a worst case scenario, lead to the systematic and unnecessary replacement of good components by poor components, thus reducing the availability of the system. The models also allow situations in which maintenance quality differs between alternative maintainers to be investigated.

1. Introduction

In this paper we model an inspection policy for a protection or stand-by system [1] with a single component. Our particular focus is upon the effect of imperfect maintenance, and this is the novelty of our approach. The protection system is composed of a component and a socket which together provide an operational function [2]. The protection system is required to function only on demand, for example, in the event of an emergency. The functional status (good or failed) of the protection system is established only by a test carried out at inspection. This test is imperfect; both a false positive (test says failed but protection system is good) and a false negative (test says good but protection system is failed) are possible. Such imperfect testing is analogous to that encountered in quality control [3] and screening procedures in medicine [4]. It may also arise in modern electronic systems, such as that used in the latest automotive technology to monitor oil levels, tyre pressures, and such like (eg. [5]).

To illustrate the ideas in our paper, we consider a complex machine that fills a flexible package with a non-carbonated beverage. The protection system is a safety device whose

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purpose is to prevent misalignment of the machine that is responsible for cutting, forming and sealing the flexible package. This latter machine (the carton maker) may be irreparably damaged if undue misalignment of its parts is not prevented by the safety device. Misalignments are events that occur randomly in time. The safety device is itself subject to failure, and we are concerned with maintenance planning for the safety device. The purpose of the maintenance is purely to support a high level of production output for the carton maker. Inspection of the safety device to determine its functional state (good or failed) is carried out by the equipment operator: the beverage manufacturer. We evaluate two situations. In the first, when the beverage manufacturer detects a fault with the protection system, the maintenance team of the original equipment manufacturer (OEM) of the carton maker is required to take appropriate action. In the second, all maintenance actions for the protection system are carried out by the beverage manufacturer "in-house"; the OEM merely supplies replacement components. Thus in the former situation, when a fault is "detected", the OEM is called out at significant expense to the beverage manufacturer. The first action of the OEM is to determine if the protection system is indeed failed. If it is not failed, a false positive has occurred and no further action is taken. In the second situation, when a fault is detected, the protection system is replaced by the maintenance team of the beverage manufacturer. The response to detection of a fault is then different in the two situations, and we develop a model for each situation. In the first, a false positive implies only an additional cost to the beverage manufacturer; the protection system reliability is not affected. In the second, a false positive implies a renewal of the protection system. These two models are described in the following sections. Notionally, the two situations reflect variation in the quality of maintenance between the OEM and the operator; the additional maintenance experience of the OEM has a direct cost premium. This idea of quality of maintenance has been developed by Scarf and Cavalcante [6]. Quality of maintenance is a notion that is also related to concepts in maintenance outsourcing [7-9]. We should note that it is not necessarily the case that "in-house" maintenance is of a lower quality than outsourced maintenance, although in the case study we describe there is the expectation that it is so. The models we develop are flexible in this respect however.

False negatives have a more straightforward effect in our models. The occurrence of a false negative has no direct effect on the reliability of the protection system or the cost of maintenance; at inspection the failed protection system remains failed so that the failure is unrevealed in spite of the inspection. There are however consequences for the cost of lost production if a misalignment event occurs that requires the protection system to operate. This cost is potentially very large.

In the standard manner, we suppose that a component (used in the protection system) deteriorates over time. On replacement of the system, the existing component in the socket is substituted by a new component; we describe this throughout as component replacement. However, we further suppose that a component arises from a heterogeneous population; that is, a component may be weak, with a short life, or strong, with long life. The source of such component heterogeneity may be variation in component manufacture or may be variation in the quality of installation; some installations may be poorly executed. Whatever the source of heterogeneity, the failure time (or lifetime) distribution is a mixture in our models. In this way, notionally, the lifetime of the protection system is also influenced by maintenance quality. When the component lifetime distribution is a mixture, it has been argued that a two phase inspection policy may be appropriate [10,11], with frequent inspections in early life and infrequent inspections in later life; although the case may reverse if the mixture is dominated by the wear phase of the good component sub-population. The papers [10,11] describe a perfect inspection policy that is a special case of that introduced by Barlow and Proschan [12], and an extension of the policy considered by Vaurio [13]. Badía *et al.* [14,15] consider pure inspection policies with a single phase (a single, constant inspection interval) and imperfect testing. In [14], failures are unrevealed, that is, they are detected only by inspection. We extend this work here by allowing for the possibility of renewal at a false positive, and by supposing that, at the final inspection, preventive replacement is carried out provided that the system is not renewed beforehand. A different model is considered in [15], in which failures can be either revealed (detected as soon as they occur) or unrevealed.

From a reliability point of view, mixture distributions have been well studied (e.g. [16-20]). Maintenance policies appropriate for such systems comprising components with mixed lifetimes have been less well studied; early work is described in [21], and more recently studies that describe extensions of age based replacement and block replacement to such systems have been published [22,23]. From these, the notion of quality of preventive maintenance has been developed [6,24]. Our paper here then develops this notion of quality of preventive maintenance further in the context of a protection system with unrevealed failures.

The structure of the paper is as follows. In the next section, we describe the system, the failure model, the general maintenance policy, and the cost structure. In section 3 we consider the situation in which a false positive at inspection incurs an additional cost. In section 4, an alternative model is developed in which the system is renewed at a false positive. Section 5 presents our application of the models. We then conclude with a short discussion.

Finally, we should point out that although we build models here for idealized situations, we would hope that these models will be useful for supporting decision-making about the maintenance management of real systems and contexts that are inherently more complex.

2. General considerations

For the two models, some elements of the model are common, in particular the inspection schedule. We also have some common notation. This is listed below. Notation that is model specific is listed in the corresponding sections.

Common notation

X	time to failure of a component
$R(x)$	reliability function of a component, a mixture: $R(x) = P(X > x) = pR_1(x) + (1-p)R_2(x)$
M_1	number of inspections in phase 1
M_2	number of inspections in phase 2
T_1	time between scheduled inspections in phase 1
T_2	time between scheduled inspections in phase 2

α	probability of a false positive (false alarm)
β	probability of a false negative i.e. of not detecting a failure at an inspection when it is present
c_0	unitary cost of inspection
c_{r}	cost of renewal of a failed system ≡ replacement of a failed component
c_{m}	cost of renewal of an unfailed system ≡ replacement of an unfailed component
μ	rate of occurrence of demands for the protection system
c_{d}	cost of a single unmet demand
t_0	time duration of an inspection
t_{r}	time duration of renewal of a failed system (replacement of a failed component)
t_{m}	time duration of renewal of an unfailed system (replacement of an unfailed component)

We consider a single-component protection system that undergoes an inspection or test to detect failure. If the system fails between inspections, then the failure is unrevealed, so that an inspection provides the only opportunity at which to detect a failure. Inspections are scheduled for when the system reaches ages jT_1; $j = 1,2,...,M_1$ (phase 1) and ages $M_1T_1 + jT_2$; $j = 1,2,...,M_2$ (phase 2). There are thus $j = 1,2,...,M_1$ inspections in phase 1 and $j = 1,2,...,M_2$ in phase 2. At age $M_1T_1 + M_2T_2$ (the final scheduled inspection), a replacement of the component is also scheduled. At component replacement, the system is renewed and system age is reset ($t = 0$).

The decision variables in the maintenance policy are M_1, T_1, M_2 and T_2 and we determine the long-run cost per unit time and average availability of a (M_1, T_1, M_2, T_2) policy.

Testing is imperfect. A false positive (false alarm) can occur on inspection, that is if the system is considered to have failed when it is actually in the good functional state (test=failed, system=good). Furthermore the test can fail to detect a real failure of the system (test=good, system=failed); this is a false negative. This classification and the associated probabilities are summarized in table 1. We assume that on inspection, if the test says failed and the system is failed (test=failed, system=failed), then the component is replaced and the system is renewed. With respect to false positives, we consider two models. In the first model, the consequence of a false positive is that a cost is incurred, but the component is not replaced so that a false positive does not change the system reliability. For example, on alarm (test=failed), engineers prepare to carry out a replacement, but on further investigation, the component is found to be good and is not replaced. Such a maintenance action might incur significant cost. In the second model, the consequence of a false positive is that the component is replaced and consequently the system is renewed. Here for example, engineers may not have the capability to investigate an alarm or the cost of component replacement might be such that investigation of the alarm may not be economic.

We take into account the costs derived from inspections, component replacement, false positives, and those due to the downtime incurred while a failed system remains undetected. For this latter cost of downtime, we assume that the demand for the function of the protection system occurs according to a Poisson process with rate μ and that c_{d} is the cost of a single unmet demand (a constant). This implies that if the protection system is down for time τ, the cost of the downtime is $(\mu \times c_{\mathrm{d}})\tau$. The cost-rate for unmet demand is thus $\mu \times c_{\mathrm{d}}$. This implies that a high cost of unmet demand and a low rate of occurrence of demands is equivalent to a low cost of unmet demand and a high rate of occurrence of demands. Whether the maintenance policy should be the same in each of these cases is another matter. As is customary in the literature, we assume that unmet demands do not impact upon maintenance of the protection system, i.e. that μ is small. To do otherwise is beyond the scope of this paper; nonetheless, the development of a model in which unmet demand events impacted upon maintenance would make an interesting study.

Table 1: Classification of system and inspection status with associated probabilities.

System status / Inspection outcome	System good	System failed
Test says system good	True positive, 1-α	False negative, β
Test says system failed	False positive, α	True negative, 1-β

We consider two different time durations for replacements. The first, t_{r}, corresponds to a replacement of a failed component when test=failed and system=failed. The second, t_{m}, corresponds to a replacement of an unfailed component that occurs at age $M_1T_1 + M_2T_2$ if the system reaches this age and is unfailed at this age or following a false positive in model 2. The corresponding costs are c_{r} and c_{m}. Downtime due to inspection is assumed to be zero.

This distinction between costs (and times) of replacement of a failed and an unfailed system may exist in practice because system failure may involve additional action on the part of the maintainer. Alternatively, unfailed components may be recycled by the maintainer as part of their spares provisioning policy. One might formulate our models by considering a distinction between planned and unplanned replacements, so that the only replacement that is planned can occur at age $M_1T_1 + M_2T_2$. Planned and unplanned replacements being different allows one to take into account possible delays to replacement when an inspection reveals a failed item. Such a delay might be due to an unavailable part. However a planned replacement would not be likely to experience such a delay. In this case, there is no sense in scheduling an inspection at age $M_1T_1 + M_2T_2$. The cost and availability functions that we derive in models 1 and 2 below then change to a small extent. Further, we might suppose that $c_{\mathrm{r}} = c_{\mathrm{m}}$ and $t_{\mathrm{r}} = t_{\mathrm{m}}$, so there is no distinction between replacement of failed and unfailed components or between planned and unplanned replacements. In this case, the inspection works like a setup with the maintainer prepared to make a routine component replacement if required.

We assume that in practice the costs thus discussed are quantifiable and known. Where they are not, and indeed the estimation of μ in the cost rate of unmet demand may be difficult, receiver operating characteristic analysis is a useful tool for the optimization of inspection policies. For a recent interesting paper on this topic and which also considers the existence of sub-populations of weak and strong components, albeit in a burn-in maintenance context, see [25].

Finally, in our models, the existence of an inspection at $M_1T_1 + M_2T_2$ at the time of replacement can be justified in terms of cause identification or the benefit of component recycling. The maintainer may be prepared to pay for additional information about the system state at this time.

3. Model 1: no renewal at a false positive inspection

In this first model, false positives (false alarms) result in a cost but have no effect on system reliability. We effectively model the case in which, when an alarm occurs, its cause is investigated further at additional cost and this further investigation reveals the true system state. Thus, in the case of a false positive, the component is found to be good and is not replaced. Such a maintenance action might incur significant cost, which we denote by c_{I}. This cost in our formulation is not incurred at the final inspection at $M_1T_1 + M_2T_2$. As we describe above, for a given inspection schedule, false negatives only have an effect upon the availability of the protection system.

For the two phase inspection policy, we have the following additional notation.

Specific notation for model 1

- c_{I}: cost of a false positive (false alarm)
- K_1: number of inspections in phase 1 previous to failure or to the beginning of phase 2 whichever comes first.
- K_2: number of inspections in phase 2 (from M_1T_1 onwards) previous to failure.
- K_3: number of inspections in phase 1 after failure until its detection or to the beginning of phase 2 whichever comes first.
- K_4: number of inspections in phase 2 after failure until its detection or the system is renewed at $M_1T_1 + M_2T_2$ whichever comes first.
- n_{I}: number of false positives in a cycle.

First we consider the case $M_1, M_2 > 0$, that is, there is at least one inspection in each phase.

The range of K_1 is $\{0,1,...,M_1\}$. For $i = 0,1,...,M_1 - 1$,

$$P(K_1 = i) = P(iT_1 \le X < (i+1)T_1) = R(iT_1) - R((i+1)T_1).$$

For $i = M_1$,

$$P(K_1 = M_1) = P(X \ge M_1T_1) = R(M_1T_1).$$

Next, the expected value of K_1 is obtained:

$$E(K_1) = \sum_{i=1}^{M_1-1} i\{R(iT_1) - R((i+1)T_1)\} + M_1R(M_1T_1) = \sum_{i=1}^{M_1} R(iT_1). \qquad (1)$$

The number of inspections in phase 2 previous to failure, K_2, takes a value in $\{0,1,...,M_2\}$ with corresponding probabilities:

$$P(K_2 = 0) = P(X < M_1T_1 + T_2) = 1 - R(M_1T_1 + T_2),$$

$$P(K_2 = i) = P(M_1T_1 + iT_2 \le X < M_1T_1 + (i+1)T_2) = R(M_1T_1 + iT_2) - R(M_1T_1 + (i+1)T_2),$$

($i = 1,...,M_2 - 1$), and

$$P(K_2 = M_2) = P(X \geq M_1T_1 + M_2T_2) = R(M_1T_1 + M_2T_2).$$

The mean value of K_2 is therefore given by

$$E(K_2) = \sum_{i=1}^{M_2-1} i\{R(M_1T_1 + iT_2) - R(M_1T_1 + (i+1)T_2)\} + M_2R(M_1T_1 + M_2T_2)$$
$$= \sum_{i=1}^{M_2} R(M_1T_1 + iT_2). \quad (2)$$

The range of K_3 (number of inspections in phase 1 after failure until it is detected or until inspections in the phase 2 begin whichever occurs first) is $\{0,1,...,M_1\}$. Its probability function is as follows:

$$P(K_3 = 0) = P(X > M_1T_1) = R(M_1T_1).$$

For $i = 1,...,M_1$,

$$P(K_3 = i) = \{R(0) - R((M_1 - i)T_1)\}\beta^{i-1}(1-\beta) + \{R((M_1 - i)T_1) - R((M_1 - i + 1)T_1)\}\beta^{i-1}.$$

The first term in the equation above corresponds to a failure that is detected at the *i*th inspection after it occurs. The second term is the probability of a failure that takes place in the first phase and the failure is not detected in the first *i*-1 inspections and the *i*th inspection takes place at M_1T_1. In this latter case $K_3 = i$ regardless of whether the failure is detected or not; if not, phase 2 begins. The expression then simplifies to:

$$P(K_3 = i) = \beta^{i-1}(1-\beta) + R((M_1 - i)T_1)\beta^i - R((M_1 - i + 1)T_1)\beta^{i-1}, \ (i = 1,...,M).$$

Then

$$E(K_3) = \frac{1 - M_1\beta^{M_1} + M_1\beta^{M_1+1} - \beta^{M_1}}{1-\beta} + M_1\beta^{M_1} - \sum_{i=1}^{M_1} \beta^{M_1-i}R(iT_1)$$
$$= \frac{1-\beta^{M_1}}{1-\beta} - \sum_{i=1}^{M_1} \beta^{M_1-i}R(iT_1). \quad (3)$$

Let A denote the following event: a failure occurs in $(0, M_1T_1)$ and it is not detected in this interval. Then

$$P(A) = \sum_{i=1}^{M_1} \{R((i-1)T_1) - R(iT_1)\}\beta^{M_1-(i-1)} = S(T_1, M_1, \beta).$$

In addition let B represent the event that a failure occurs in $(0, M_1T_1)$ and is detected in that interval. Then

$$P(B) = \sum_{i=1}^{M_1} \{R((i-1)T_1) - R(iT_1)\}(1 - \beta^{M_1-(i-1)}).$$

These probabilities are used in the consideration of the distribution of K_4, the number of inspections in phase 2 after failure until it is detected or the preventive replacement is carried out at $M_1T_1 + M_2T_2$, whichever comes first. The range of K_4 is $\{0,1,...,M_2\}$. Consequently

$$P(K_4 = 0) = \sum_{i=1}^{M_1} \{R((i-1)T_1) - R(iT_1)\}(1 - \beta^{M_1-(i-1)}) + R(M_1T_1 + M_2T_2),$$

$$P(K_4 = i) = S(T_1, M_1, \beta)\beta^{i-1}(1-\beta) + \{R(M_1T_1) - R(M_1T_1 + (M_2 - i)T_2)\}\beta^i(1-\beta)$$
$$+ \{R(M_1T_1 + (M_2 - i)T_2) - R(M_1T_1 + (M_2 - (i-1))T_2)\}\beta^{i-1}, \quad (i = 1, \ldots, M_2 - 1).$$

The first term above corresponds to a failure occurring in (0, M_1T_1) and not detected in that interval but after i inspections in the second phase. The second term represents a failure that occurs in the second phase and detected after i inspections. The third term is the probability of a failure that takes place in the second phase and not detected in the following i-1 inspections and the ith inspection happens to coincide with the preventive replacement at $M_1T_1 + M_2T_2$. The expression then simplifies to:

$$P(K_4 = i) = \{S(T_1, M_1, \beta) + R(M_1T_1)\}\beta^{i-1}(1-\beta) + R(M_1T_1 + (M_2 - i)T_2)\beta^i - R(M_1T_1 + (M_2 - (i-1))T_2)\beta^{i-1},$$

($i = 1, \ldots, M_2 - 1$), and

$$P(K_4 = M_2) = \{S(T_1, M_1, \beta) + R(M_1T_1) - R(M_1T_1 + T_2)\}\beta^{M_2-1}.$$

The expectation of K_4 turns out to be

$$E[K_4] = \{S(T_1, M_1, \beta) + R(M_1T_1)\}\sum_{i=1}^{M_2-1} i\beta^{i-1}(1-\beta) + M_2\beta^{M_2-1}\{S(T_1, M_1, \beta) + R(M_1T_1)\} - \sum_{i=1}^{M_2}\beta^{i-1}R(M_1T_1 + (M_2 - (i-1))T_2).$$

The following expression can be derived from this:

$$E[K_4] = \frac{1-\beta^{M_2}}{1-\beta}\{S(T_1, M_1, \beta) + R(M_1T_1)\} - \sum_{i=1}^{M_2}\beta^{i-1}R(M_1T_1 + (M_2 - (i-1))T_2). \quad (4)$$

By using the expressions in (1), (2), (3) and (4), the expected length of a renewal cycle is obtained:

$$E[\tau] = (T_1 + t_0)(E[K_1] + E[K_3]) + (T_2 + t_0)(E[K_2] + E[K_4]) + t_{\mathrm{r}}F(M_1T_1 + M_2T_2) + t_{\mathrm{m}}R(M_1T_1 + M_2T_2). \quad (5)$$

The expected uptime in a cycle is

$$U(M_1, T_1, M_2, T_2) = \int_0^{M_1T_1 + M_2T_2} R(t)dt.$$

Hence the expected downtime is

$$D(M_1, T_1, M_2, T_2) = E[\tau] - U(M_1, T_1, M_2, T_2).$$

The mean cost of inspections in a cycle is

$$C_{in} = c_0(E[K_1] + E[K_3] + E[K_2] + E[K_4]).$$

Let the number of false positives in a cycle be n_{I}. Then n_{I} conditional on $K_1 + K_2$ inspections previous to failure, $n_{\mathrm{I}} \mid K_1 + K_2$, has a binomial distribution with parameters $K_1 + K_2$ and α. Therefore $E[n_{\mathrm{I}} \mid K_1 + K_2] = (K_1 + K_2)\alpha$, so that

$$E[n_{\mathrm{I}}] = \alpha E[K_1 + K_2] = \alpha\{\sum_{i=1}^{M_1} R(iT_1)\} + \sum_{i=1}^{M_2} R(M_1T_1 + iT_2)\}.$$

The expected cost of a renewal cycle including the cost of inspection, the cost of false positives (noting that a false positive if it occurs at $M_1T_1 + M_2T_2$ does not incur a cost), the cost of replacement, and the expected cost of unmet demand is given by

$$E[C(\tau)] = C_{in} + c_{\mathrm{I}}\alpha\{\sum_{i=1}^{M_1} R(iT_1) + \sum_{i=1}^{M_2-1} R(M_1T_1 + iT_2)\}$$
$$+ c_{\mathrm{r}}F(M_1T_1 + M_2T_2) + c_{\mathrm{m}}R(M_1T_1 + M_2T_2) + c_{\mathrm{d}}D(M_1, T_1, M_2, T_2). \qquad (6)$$

The long run cost per unit time is given by

$$Q(M_1, T_1, M_2, T_2) = E[C(\tau)] / E[\tau]. \qquad (7)$$

In addition, the average availability is

$$A(M_1, T_1, M_2, T_2) = \frac{E[U]}{E[U] + E[D]} = \frac{\int_0^{M_1T_1 + M_2T_2} R(t)dt}{E[\tau]}.$$

The long-run cost per unit time and the average availability are the measures we use to compare policies.

Some preliminary work on this model was developed in Berrade et al. [26]. The model in Cavalcante *et al.* [10] constitutes a special case of the model when $\alpha = \beta = 0$, and the model in Berrade [27] is another special case with $M_2 = \infty$.

The single phase inspection and replacement policy $M_1 = 0$, $M = M_2 > 1$ can be obtained by setting $T_1 = T_2$ and $M_1 + M_2 = M > 1$ in the expressions for the general policy. This is because these cases are equivalent.

The expressions above simplify when $M_1 = 0$, $M_2 = 1$. In this case, the policy is effectively a pure replacement policy with a single phase, with preventive replacement at T_2 (although as we state in section 2 we suppose that there is an inspection at preventive replacement). The expected length of a renewal cycle is

$$E[\tau] = (T_2 + t_0) + t_{\mathrm{r}}F(T_2) + t_{\mathrm{m}}R(T_2)$$

The expected downtime in a cycle is

$$D(T_2) = E[\tau] - \int_0^{T_2} R(t)dt.$$

The expected cost of a cycle is

$$c_0 + c_{\mathrm{r}}F(T_2) + c_{\mathrm{m}}R(T_2) + c_{\mathrm{d}}D(T_2),$$

and the long run average cost per unit time is

$$Q(T_2) = c_{\mathrm{d}} + \frac{c_0 + c_{\mathrm{r}}F(T_2) + c_{\mathrm{m}}R(T_2) - \int_0^{T_2} R(t)dt}{(T_2 + t_0) + t_{\mathrm{r}}F(T_2) + t_{\mathrm{m}}R(T_2)}.$$

Notice that this expression does not involve the false positive parameter, α. This is because the only inspection is the final inspection, and the false alarm cost cannot be incurred at the final inspection.

We do not consider the case $M_1 = 1,\ M_2 = 0$. This is because if we formulate the general policy such that preventive replacement immediately follows the final inspection, then this case is the same as $M_1 = 0,\ M_2 = 1$. If we formulate the general policy such that the preventive replacement follows the final inspection in the second phase, but not the first phase, then as there is no second phase if $M_1 = 1,\ M_2 = 0$; the policy consists of a single inspection at T_1 only. If the component fails before this time and there is no false negative at T_1, then the component is replaced and the cycle length is T_1; otherwise the system is never replaced and the cycle length is non-finite and the average availability is zero, and thus the policy, in this case, has no practical relevance. Finally, the policy is degenerate for $M_1 = M_2 = 0$.

4. Model 2: renewal at a false positive inspection

Model 2 is as model 1 except in one key aspect. In model 1, the consequence of a false positive is that a cost is incurred but the component is not replaced, so that a false positive does not change the system reliability. In model 2, if a false positive (false alarm) occurs then the component is replaced and the system is renewed. Here engineers are not interested to investigate an alarm (when the test says the protection system is failed), and carry out component replacement regardless of the real state of the protection system. The cost of a false positive is then c_{m}, the cost of replacement of an unfailed component.

The inspection policy is the same as in model 1: inspect at ages jT_1; $j = 1,2,...,M_1$ (phase 1) and at ages $M_1T_1 + jT_2$; $j = 1,2,...,M_2$ (phase 2). Additional notation is as follows.

Specific notation for model 2

- I_1: number of inspections in phase 1 previous to failure, to a false positive, or to the beginning of phase 2 whichever comes first
- I_2: number of inspections in phase 2 (from M_1T_1 onwards) previous to failure, to a false positive, or to preventive replacement at $M_1T_1 + M_2T_2$ whichever comes first.
- I_3: number of inspections in phase 1 after failure until its detection or to the beginning of phase 2 whichever comes first.
- I_4: number of inspections in phase 2 after failure until its detection or the system is renewed at $M_1T_1 + M_2T_2$ whichever comes first.

Again, we consider first the case $M_1, M_2 > 0$. When $M_1 = 0,\ M_2 = 1$, the policy under model 2 is the same as under model 1, because renewal follows the only inspection in both models. The policy $M_1 = 1,\ M_2 = 0$ has no practical relevance so we do not consider it. The case $M_1 = M_2 = 0$ is degenerate.

For $M_1, M_2 > 0$, the range of I_1 is $\{0,1,...,M_1\}$. For $i = 0$,

$$P(I_1 = 0) = 1 - R(T_1).$$

For $i = 1,...,M_1 - 1$,

$$P(I_1 = i) = \{R(iT_1) - R((i+1)T_1\}(1-\alpha)^i + R(iT_1) - R(iT_1\}(1-\alpha)^{i-1}\alpha .$$

For $i = M_1$,

$$P(I_1 = M_1) = R(M_1 T_1)(1-\alpha)^{M_1 - 1}.$$

The expected value of I_1 is obtained:

$$E(I_1) = \sum_{i=1}^{M_1} R(iT_1)(1-\alpha)^{i-1}.$$

Next, I_2, takes a value in $\{0,1,...,M_2\}$ with corresponding probabilities

$$P(I_2 = 0) = 1 - R(M_1 T_1 + T_2)(1-\alpha)^{M_1},$$

$$P(I_2 = i) = \{R(M_1 T_1 + iT_2) - R(M_1 T_1 + (i+1)T_2)\}(1-\alpha)^{M_1 + i} + R(M_1 T_1 + iT_2)(1-\alpha)^{M_1 + i - 1}\alpha,$$

for $i = 1,...,M_2 - 1$, and

$$P(I_2 = M_2) = R(M_1 T_1 + M_2 T_2)(1-\alpha)^{M_1 + M_2 - 1}.$$

Therefore

$$E[I_2] = \sum_{i=1}^{M_2} R(M_1 T_1 + iT_2)(1-\alpha)^{M_1 + i - 1}. \tag{9}$$

Next, I_3 takes values on $\{0,1,...,M_1\}$ with

$$P(I_3 = 0) = \sum_{i=1}^{M_1 - 1} R(iT_1)(1-\alpha)^{i-1}\alpha + R(M_1 T_1)(1-\alpha)^{M_1 - 1},$$

$$\begin{aligned} P(I_3 = i) = \sum_{k=1}^{M_1 - i} &\{R((k-1)T_1) - R(kT_1)\}(1-\alpha)^{k-1}\beta^{i-1}(1-\beta) \\ &+ \{R((M_1 - 1)T_1) - R((M_1 - i + 1)T_1)\}(1-\alpha)^{M_1 - i}\beta^{i-1}, \end{aligned},$$

for $i = 1,...,M_1 - 1$, and

$$P(I_3 = M_1) = (1 - R(T_1))\beta^{M_1 - 1},$$

so that

$$E(I_3) = \sum_{i=1}^{M_1} \{R((i-1)T_1) - R(iT_1)\}(1-\alpha)^{i-1} \frac{1-\beta^{M_1 - i + 1}}{1-\beta}. \tag{10}$$

Next, I_4 takes values on $\{0,1,...,M_2\}$. In what follows, we consider the function:

$$S_2(T_1, M_1, \alpha, \beta) = \sum_{i=1}^{M_1} (1-\alpha)^{i-1}\beta^{M_1 - i + 1}\{R((i-1)T_1) - R(iT_1)\},$$

which represents the probability of a failure occurring in $(0, M_1 T_1)$ and not detected in that interval and in addition no false positive occurrence in $(0, M_1 T_1)$. Now

$$\begin{aligned} P(I_4 = 0) = &\sum_{i=1}^{M_1} (1-\alpha)^{i-1}(1-\beta^{M_1 - i + 1})\{R((i-1)T_1) - R(iT_1)\} \\ &+ \sum_{i=1}^{M_1} (1-\alpha)^{i-1}\alpha R(iT_1) + \sum_{i=1}^{M_2 - 1} (1-\alpha)^{M_1 + i - 1}\alpha R(M_1 T_1 + iT_2) \\ &+ (1-\alpha)^{M_1 - M_2 - 1} R(M_1 T_1 + M_2 T_2). \end{aligned}$$

The first term represents a failure occurring in $(0, M_1 T_1)$ which is detected in that interval and no false positive occurrence therein. The second term and the third terms are respectively the probabilities of a false positive in $(0, M_1 T_1)$ and $(M_1 T_1, M_1 T_1 + (M_2 - 1)T_2)$. The last term corresponds to the probability of a cycle that ends with a preventive maintenance at $M_1T_1 + M_2T_2$ provided that no false positive and no failure has occurred before.

For $j = 1,..., M_2 - 1$,

$$P(I_4 = j) = \sum_{i=1}^{M_1} \beta^{j-1}(1-\beta) S_2(T_1, M_1, \alpha, \beta)$$
$$+ \sum_{i=1}^{M_2-j} (1-\alpha)^{M_1+i-1} \beta^{j-1}(1-\beta)\{R(M_1T_1 + (i-1)T_2) - R(M_1T_1 + iT_2)\}$$
$$+ (1-\alpha)^{M_1+M_2-j} \beta^{j-1}\{R(M_1T_1 + (M_2 - j)T_2) - R(M_1T_1 + (M_2 - j + 1)T_2)\}.$$

The first term corresponds to a failure that occurs in $(0, M_1T_1)$ not detected in that interval but after j inspections in the second phase. The second term is a failure that takes place in $(M_1T_1, M_1T_1 + (M_2 - j)T_2)$ and detected after j inspections. The third term represents a failure in the second phase which is corrected at $M_1T_1 + M_2T_2$ after j inspections. In all the cases the probability of no false positive is also included.

Finally, we have

$$P(I_4 = M_2) = \beta^{M_2-1}[S_2(T_1, M_1, \alpha, \beta) + (1-\alpha)^{M_1}\{R(M_1T_1) - R(M_1T_1 + T_2)\}].$$

The first term is the probability of a failure occurring in $(0, M_1T_1)$ not detected in that interval and no false positive occurrence. The second term indicates a failure in $(M_1T_1, M_1T_1 + T_2)$ and no false positive. In both cases the detection after M_2 inspections of the phase 2 implies that the renewal occurs at $M_1T_1 + M_2T_2$.

After some algebra we have

$$E[I_4] = \frac{1-\beta^{M_2}}{1-\beta} S_2(T_1, M_1, \alpha, \beta)$$
$$+ \sum_{i=1}^{M_2} \frac{1-\beta^{M_2+1-i}}{1-\beta} (1-\alpha)^{M_1+i-1}\{R(M_1T_1 + (i-1)T_2) - R(M_1T_1 + iT_2)\}. \quad (11)$$

The replacement time takes values as follows:

$$\begin{cases} t_r, & \text{if } X < M_1T_1 +, M_2T_2 \text{ and there are no false positives,} \\ t_m, & \text{otherwise.} \end{cases}$$

The probability that the replacement time takes the value t_r is

$$P(t_r) = \sum_{i=1}^{M_1} (1-\alpha)^{i-1} \int_{(i-1)T_1}^{iT_1} dF(t) + \sum_{i=1}^{M_2} (1-\alpha)^{M_1+i-1} \int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2} dF(t).$$

The foregoing expression can also be expressed as

$$P(t_r) = 1 - \alpha\left(\sum_{i=1}^{M_1} (1-\alpha)^{i-1} R(iT_1) + \sum_{i=1}^{M_2-1} (1-\alpha)^{M_1+i-1} R(M_1T_1 + iT_2)\right)$$
$$- (1-\alpha)^{M_1+M_2-1} R(M_1T_1 + M_2T_2). \quad (11a)$$

The probability that the replacement time takes the value t_m is

$$P(t_m) = 1 - P(t_r). \quad (11b)$$

Therefore, the expected length of a renewal cycle is

$$E[\tau]=(E[I_1]+E[I_3])(T_1+t_0)+(E[I_2]+E[I_4])(T_2+t_0)+t_{\mathrm{r}}P(t_{\mathrm{r}})+t_{\mathrm{m}}P(t_{\mathrm{m}})\,. \quad (12)$$

with $E[I_1]$, $E[I_2]$, $E[I_3]$, and $E[I_4]$ given by (8)-(11) respectively.

The following calculation gives the expected uptime:

$$\begin{aligned}U(M_1,T_1,M_2,T_2)=&\sum_{i=1}^{M_1}(1-\alpha)^{i-1}\{\int_{(i-1)T_1}^{iT_1}tdF(t)+\alpha iT_1R(iT_1)\}\\&+\sum_{i=1}^{M_2}(1-\alpha)^{M_1+i-1}\int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2}tdF(t)\\&+\sum_{i=1}^{M_2-1}(1-\alpha)^{M_1+i-1}\alpha(M_1T_1+iT_2)R(M_1T_1+iT_2)\\&+(1-\alpha)^{M_1+M_2-1}(M_1T_1+M_2T_2)R(M_1T_1+M_2T_2).\end{aligned}$$

After additional calculations we obtain

$$U(M_1,T_1,M_2,T_2)=\sum_{i=1}^{M_1}(1-\alpha)^{i-1}\int_{(i-1)T_1}^{iT_1}R(t)dt+\sum_{i=1}^{M_2}(1-\alpha)^{M_1+i-1}\int_{M_1T_1+(i-1)T_2}^{M_1T_1+iT_2}R(t)dt\,. \quad (13)$$

Hence the expected downtime is

$$D(M_1,T_1,M_2,T_2)=E[\tau]-U(M_1,T_1,M_2,T_2)\,. \quad (14)$$

The cost derived from the replacement of the system takes two possible values:

$$\begin{cases}c_{\mathrm{r}}, & \text{if } X<M_1T_1+,M_2T_2 \text{ and there are no false positives,}\\ c_{\mathrm{m}}, & \text{otherwise.}\end{cases}$$

so that $P(c_{\mathrm{m}})=P(t_{\mathrm{m}})$ and $P(c_{\mathrm{r}})=P(t_{\mathrm{r}})$ in the notation of equations (11a&b). Note $P(c_{\mathrm{m}})=1-P(c_{\mathrm{r}})$, and $P(c_{\mathrm{m}})\neq P(I_4=M_2)$ because c_{m} is the associated cost not only for the preventive replacement at $M_1T_1+M_2T_2$ but also when a false positive occurs.

The expected total cost of a renewal cycle turns out to be

$$\begin{aligned}E[C(\tau)]=&c_0(E[I_1]+E[I_2]+E[I_3]+E[I_4])\\&+c_{\mathrm{r}}P(c_{\mathrm{r}})+c_{\mathrm{m}}P(c_{\mathrm{m}})+c_{\mathrm{d}}D(M_1,T_1,M_2,T_2).\end{aligned} \quad (15)$$

In the particular case $c_{\mathrm{r}}=c_{\mathrm{m}}=c$, the previous expression becomes

$$E[C(\tau)]=c_0(E[I_1]+E[I_2]+E[I_3]+E[I_4])+c+c_{\mathrm{d}}D(M_1,T_1,M_2,T_2)\,.$$

The long-run cost per unit time and the average availability can then be obtained from expressions (15) and (12), and (13) and (14), respectively.

Again, the case $M_1=0,\ M=M_2>1$ can be obtained by setting $T_1=T_2$ and $M_1+M_2=M>1$ in the expressions for the general policy. This is because they are equivalent.

5. Case study: inspection of a production line protection system

5.1 *Specification of parameter values*

Our example looks at a complex machine that fills a flexible package with a non-carbonated beverage. The protection system is a safety device whose purpose is to prevent misalignment of the carton maker; misalignments occur randomly in time; if the safety device fails to operate during a misalignment event then the carton maker may be irreparably damaged. The cost of failure of the carton maker due to misalignment has been estimated by the beverage manufacturer. This cost is based upon the cost of lost production time (72 hours), and the cost of replacement of the carton maker (more precisely the jaws that are responsible for cutting, forming and sealing the packaging). For reasons of confidentiality, all costs are given with respect to an unspecified unit cost. The beverage manufacturer inspects the protection system (safety device) to determine if it is good or failed. In fact at inspection, cleaning and lubrication tasks are also carried out. Inspection time is 2 hours. In terms of the unit of cost, inspection costs 5 units. Further, this inspection takes place during a scheduled shutdown, so there is no lost production arising due to inspection. (This is something of a simplification because the carton maker operates for 2 or 3 shifts a day, and the number of shifts used is dependent to some extent on demand for the beverage.) The cost of an unmet demand, c_d, is 27,000 units. The demand rate has been estimated to be of the order of 1 demand every 4 years.

We evaluate two situations. In the first, when the beverage manufacturer detects a fault with the protection system, the OEM of the carton maker takes appropriate action. In the second, all maintenance actions for the protection system are carried out by the beverage manufacturer "in-house"; the OEM supplies replacement components. The situations then broadly correspond to the two policies: OEM maintenance; and "in-house" maintenance.

Thus in the first situation (model 1), when a fault is detected, the OEM is called out at significant expense to the beverage manufacturer. The call-out charge is 50 units; this is essentially a set-up cost that includes the cost of travel for the OEM maintenance engineer. The first action of the OEM engineer is to determine if the protection system is indeed failed. If it is unfailed, a false positive has occurred and no further action is taken and the call-out charge is incurred: that is, the cost of a false positive, c_I, is 50 units. If the protection system is failed then it is replaced at cost c_r=150. This implies that the cost of the component spare part is 100 units. We further suppose that there is no lost production during these activities because they are scheduled during a production stoppage. Finally if the device reaches its critical age for replacement then the OEM replaces it with cost $100=c_m< c_r=150$. The difference between c_m and c_r here (50 units) we attribute to the value of the unfailed component part which may be reconditioned and returned to the stock of spare parts.

In the second situation (model 2), when a fault is detected, the protection system is replaced by the maintenance team of the beverage manufacturer. The cost incurred is now c_r=105 if the device is failed (the cost of the spare part, plus the set up cost for the beverage manufacturer which we set equal to the cost of inspection i.e. 5 units), and c_m=55 if the device is unfailed (the cost of the component minus its value as a spare plus the set up cost for the beverage manufacturer). If the device reaches its critical age for replacement then the beverage manufacturer replaces it with cost c_m=55.

These are the base case values for the cost parameters in the two situations. We will in what follows also consider the effect on the optimal policies of variations in these costs.

Notionally, the two situations reflect variation in the quality of maintenance between the OEM and the operator; the additional maintenance experience of the OEM has a direct cost premium. This variation in quality is also reflected in the value of the probability of a false positive, α. In practice, if the first situation applies then this is the percentage of unnecessary calls to the OEM following inspection and should be straightforward to estimate in practice. We take this to be 0.2 in our base case. The probability, β, of not detecting a failure present at an inspection (false negative) depends on the skill level of the maintainer and the complexity of the device and is more difficult to evaluate. We therefore use a value similar to α.

The downtime for preventive replacement of a failed component, t_r, is taken to be 6 hours. The downtime for preventive replacement of an unfailed component, t_m, is taken to be the same.

Finally, we specify the failure model parameters. The mean time to failure of the device is estimated to be of the order of 5000 hrs; the variance is unknown. There are some early failures in approximately 10% of the cases. Therefore we set p=0.1. The main reason for early failure is unknown. Weibull distributions are used for the two mixture distribution components with characteristic lives and shape parameters η_1=500, β_1=2.5 and η_2=500, β_2=4.5, respectively. These values imply a mean time to failure of 5800 hrs with standard deviation 2350 hours.

5.2 *Results*

Our results are shown in Table 2 (for situation 1) and in Table 3 (for situation 2). The base case is in the top row of each table. Where parameter values are varied from the base case, the values are emboldened for ease of inspection of the tables. In our discussion of these results, we first consider the effect of the imperfect inspection parameters for both situations. Then we consider the effect of the failure model parameters; then the cost parameter effects. We finish with a comparison of the two situations: OEM maintenance versus "in-house" maintenance.

As the probability of a false positive, α, increases in both situations both T_1^* and T_2^* increase and M_1^* and M_2^* decrease such that $M_1^*T_1^* + M_2^*T_2^*$ stays broadly constant. Recalling that $M_1 T_1 + M_2 T_2$ is the time to preventive replacement, this implies that one should do less inspection as α increases but keeping the frequency of preventive replacement constant. This is as we might have expected. As α increases the long-run cost increases and the average availability decreases accordingly. As the probability of a false negative, β, increases there is a tendency to do more inspections, but more so in situation 1 than in situation 2. One can afford to do more inspections to mitigate against false negatives only if false positives are unlikely. Again the long-run cost increases with β and the rate of increase with respect to β appears to be similar to that with respect to α. Therefore, false negatives and false positives are comparable issues economically.

Regarding failure model parameter effects, the effect of η_2 on cost and availability in both situations is as expected; also as η_2 decreases T_2^* decreases, even to the extent that, in situation 2, $T_2^* < T_1^*$ for small η_2. The effect of η_1 on T_1^* is similar, supporting the idea for two phases. The effects of β_1 and β_2 are as expected; as they increase failures become more predictable and inspection intervals can be extended. The effect of p is dramatic and is again as expected; as the probability of early failure increases the number of inspections increases and the interval between inspections in the first phase decreases markedly, while the time to preventive replacement stays broadly static. This further supports our case for a two phase policy.

The effect of the cost of inspection is greater on T_2^* than on T_1^* in situation 1 and similar in situation 2; this is perhaps because in situation 2 the inspection cost is a greater proportion of the maintenance costs. In situation 2, decreasing c_m leads to more frequent preventive replacement and vice versa. In situation 1 the effect is not so large but the proportional change in c_m is not so large here. The effect of c_r on the optimum policy in both models is very small; c_r does not appear to be driving the optimum policy at all, although the long-run cost increases with c_r. This is not unexpected; in a simple maintenance policy such as age based replacement if the cost of failure replacement is merely 1.5 times the cost of preventive replaced then preventive replacement is only marginally cost-efficient with respect to failure based replacement. The effects of replacement downtimes, t_r and t_m, appear to be negligible.

With regard to the cost of unmet demand, we see that as $\mu \times c_d$ increases one should do more inspections and more frequent replacements in both situations.

The similarity between the two situations is reassuring; cost parameter effects are comparable, as are imperfect inspection parameter effects. However, it is the effect on the long-run cost in the two situations that is interesting; in particular the effect of the imperfect inspection parameters on the long-run cost is greater in situation 2 than in situation 1. This means that while in the base case and the perfect inspection case ($\alpha=\beta=0$), situation 1 is a more expensive policy than situation 2, when either of the false imperfect inspection probabilities reaches 0.4 then situation 1 becomes a cheaper policy than situation 2. The long-run cost of OEM maintenance is in fact approximately 25% higher than "in-house" maintenance in the perfect inspection case. This cost advantage is lost when α and β are of the order of 0.3. Furthermore, if imperfect inspection probabilities are at these values, the average availability is also significantly lower with "in-house" maintenance.

6. Discussion

In this paper we compare two broad maintenance policies: maintenance carried out by the original equipment manufacturer (OEM); and maintenance carried out "in-house". This comparison is made in the context of a protection system that has to operate on demand and that is subject to imperfect inspection of its status. We develop two models, each to describe an idealized situation. In the first, imperfect inspection is manifest in that a false alarm implies an additional cost to the system owner; in the second, a false alarm implies renewal of the protection system. In both cases, there can occur false negative inspections, in which the system is regarded as good when it is in fact failed. A further complicating factor is that on replacement of the system, the lifetime of "new" system has a mixed distribution with two elements: one with a short characteristic life representing a poor replacement (with a weak component, say); the other with a long characteristic life representing a good replacement (with a strong component, say); thus, we also consider imperfect replacement in the models.

The models we describe might be further developed in order to consider a system in which failures are immediately revealed but are preceded by a defective state. The effect of false positive and false negative inspections upon the cost and reliability of a critical system may then be investigated. Berrade et al. [28] outline such a development. Following [29], one might also consider these models over a finite planning horizon, so that there is an operational requirement for the system only up to some time S. Such a “stopping time” S might itself be a random variable, with the stoppage caused by an unmet demand.

The ideas in the paper here are illustrated using a case study that describes a protection system used on the production line of a beverage manufacturer. The case study provides the motivation for the models of the two maintenance policies. The comparison of the long-run costs and average availabilities is carried out for values of model parameters that relate to the case study. When OEM maintenance costs are significantly higher than “in-house” maintenance costs, if inspection is imperfect to a large degree then we find that it is optimal to use the OEM for maintenance.

The models we develop cannot exactly mirror the problem contexts because our models are approximations to the reality. Therefore, our principle message here is not that one policy is better than another. Instead, our message is that it is possible to make comparisons using these models, and that such comparisons can inform policy about maintenance planning, provided one has reasonable information about maintenance costs and the failure behaviour of the protection system. Furthermore, this paper highlights the importance of considering some aspects of maintenance that are commonly neglected, such as the judgment errors in maintenance actions. In some circumstances, a maintenance plan, under a high level of false alarms, can impose more harmful effects than beneficial ones. That is, false alarms can lead to the systematic and unnecessary replacement of strong components by weak ones, thus reducing the reliability and availability of the system.

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References

[1] Jorgensen DW, McCall JJ, Radner R. Optimal replacement policy. North Holland: Amsterdam, 1967.

[2] Ascher H, Feingold H. Repairable Systems Reliability. New York: Marcel Dekker, 1984.

[3] Schilling EG, Neubauer DV. Acceptance Sampling in Quality Control 2^{nd} Edition. CRC Press, New York, 2009.

[4] Altman DG. Practical Statistics for Medical Research. Chapman and Hall, New York, 2010.

[5] Liu B, Makis V. Gearbox failure diagnosis based on vector autoregressive modelling of vibration data and dynamic principal component analysis. IMA Journal Management Mathematics 2008; 19, 39-50.

[6] Scarf PA, Cavalcante CAV. Modelling quality in replacement and inspection maintenance. International Journal of Production Economics, doi: 10.1016/j.ijpe.2011.08.011.

[7] Murthy DNP, Asgharizadeh E. Optimal decision making in a maintenance service operation. European Journal of Operational Research 1999; 116, 259–273.

[8] Asgharizadeh E, Murthy DNP. Service contracts: A stochastic model. Mathematical and Computer Modelling 2000; 31, 11–20.

[9] Tarakci H, Tang K,Teyarachakul S. Learning effects on maintenance outsourcing, European Journal of Operational Research 2009; 192, 138-150,
[10] Cavalcante CAV, Scarf PA, de Almeida A, Cosmo de Silva AT. A two-phase inspection policy for a single component preparedness system with a mixed time to failure distribution. In Reliability, Risk and Safety: Theory and Applications, Bris et al. (eds.) Taylor and Francis, London, 2009, pp. 525-530.
[11] Cavalcante CAV, Scarf PA, de Almeida A. A study of a two-phase inspection policy for a preparedness system with a defective state and heterogeneous lifetime. Reliability Engineering and System Safety 2011; 69, 627-635.
[12] Barlow RE, Proschan, F. Mathematical Theory of Reliability. New York: Wiley, 1965.
[13] Vaurio JK. Availability and cost functions for periodically inspected preventively maintained units. Reliability Engineering and System Safety 1999; 63, 133-140.
[14] Badía FG, Berrade MD, Campos CA. Optimization of inspection intervals based on cost. Journal of Applied Probability 2001; 38, 872-881.
[15] Badía FG, Berrade MD, Campos, CA. Optimal inspection and preventive maintenance of units with revealed and unrevealed failures. Reliability Engineering and System Safety 2002; 78, 157-163.
[16] Dhillon BS. A hazard rate model. IEEE Transactions on Reliability 1979; 28, 150-154.
[17] Jiang R, Murthy DNP. Modeling failure-data by mixture of 2 Weibull distributions: a graphical approach. IEEE Transactions on Reliability 1995; 44:477-488.
[18] Proschan F. Theoretical explanation of observed decreasing failure rate. Technometrics 1963; 5, 375-383.
[19] Gupta PL, Gupta RC. Ageing Characteristics of the Weibull Mixtures. Probability in the Engineering and Informational Sciences 1996; 10, 591-600.
[20] Finkelstein MS, Esaulova V. Modeling a Failure Rate for a Mixture of Distribution Functions. Probability in Engineering and Informational Sciences 2001; 15, 383-400.
[21] Murthy DNP, Maxwell MR. Optimal age replacement policies for items from a mixture. IEEE Transactions on Reliability 1981; 30, 169–170.
[22] Scarf PA, Cavalcante CAV, Dwight R, Gordon P. An age-based inspection and replacement policy for heterogeneous components. IEEE Transactions on Reliability 2009; 58, 641-648.
[23] Scarf PA, Cavalcante CAV. Hybrid block replacement and inspection policies for a multi-component system with heterogeneous component lives. European Journal of Operational Research 2010; 206, 384-394.
[24] Scarf PA, Cavalcante, CAV. A model of quality in preventive maintenance. In Reliability, Risk and Safety, Ale et al. (eds.) Taylor and Francis, London, 2010, pp. 1424-1429.
[25] Wu, S., Xie, M. Classifying weak, and strong components using ROC analysis with application to burn-in. IEEE Transactions on Reliability 2007; 56, 552-561.
[26] Berrade MD, Scarf PA, Cavalcante CAV. Imperfect testing, false alarms and maintenance scheduling. In Reliability, Risk and Safety, Ale et al. (eds.) Taylor and Francis, London, 2010, pp. 1361-1366.
[27] Berrade MD. A two-phase inspection policy with imperfect testing. Applied Mathematical Modelling 2012; 36, 108-114.
[28] Berrade MD, Scarf PA, Cavalcante CAV. A study of the effect of imperfect inspection on the efficacy of maintenance for a non-repairable system with a defective state. Submitted to ESREL 2011.
[29] Nakagawa T, Mituzani S. (2009) A summary of maintenance policies for a finite interval. Reliability Engineering and System Safety 2009; 94, 89-96.

Table 2. Situation (policy) 1: OEM maintenance. Top row is the base case. Parameter values are emboldened where they vary from the base case.

Mixed failure distribution parameters					cost parameters						downtimes		false alarm parameters		optimum values of decision variables, long run cost (*Cost*), and average availability						
β_1	η_1	β_2	η_2	p	c_0	c_m	c_r	μ	c_d	c_I	t_r	t_m	β	α	M_1	M_2	T_1	T_2	$Cost$	A	
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	2	360	1356	0.0744	0.981	1
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	**0**	5	6	193	533	0.0591	0.988	2
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	**0.4**	2	2	555	1377	0.0839	0.975	3
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	**0.6**	2	2	571	1406	0.0914	0.975	4
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	**0**	**0**	3	7	272	508	0.0546	0.989	5
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	**0**	0.2	2	2	456	1424	0.0669	0.983	6
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	**0.4**	0.2	4	2	304	1302	0.0838	0.976	7
2.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	**0.6**	0.2	6	2	227	1259	0.0978	0.971	8
1.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	3	379	950	0.0761	0.980	9
3.5	500	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	2	332	1391	0.0741	0.981	10
2.5	**400**	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	2	305	1427	0.0734	0.982	11
2.5	**600**	4.5	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	2	413	1287	0.0753	0.980	12
2.5	500	**3**	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	4	352	658	0.0919	0.976	13
2.5	500	**6**	7000	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	2	360	1577	0.0645	0.981	14
2.5	500	4.5	**5000**	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	2	346	925	0.0954	0.976	15
2.5	500	4.5	**9000**	0.1	5	100	150	0.00005	27000	50	6	6	0.2	0.2	3	2	375	1766	0.0617	0.983	16
2.5	500	4.5	7000	**0.01**	5	100	150	0.00005	27000	50	6	6	0.2	0.2	1	2	814	1345	0.0533	0.990	17
2.5	500	4.5	7000	**0.05**	5	100	150	0.00005	27000	50	6	6	0.2	0.2	2	2	577	1263	0.0647	0.984	18
2.5	500	4.5	7000	**0.15**	5	100	150	0.00005	27000	50	6	6	0.2	0.2	4	2	273	1401	0.0826	0.978	19
2.5	500	4.5	7000	**0.25**	5	100	150	0.00005	27000	50	6	6	0.2	0.2	5	3	211	1059	0.0973	0.974	20
2.5	500	4.5	7000	0.1	**2**	100	150	0,00005	27000	50	6	6	0.2	0.2	3	3	335	969	0.0701	0.982	21
2.5	500	4.5	7000	0.1	**10**	100	150	0,00005	27000	50	6	6	0.2	0.2	3	2	367	1387	0.0811	0.980	22
2.5	500	4.5	7000	0.1	5	**70**	150	0,00005	27000	50	6	6	0.2	0.2	3	2	354	1303	0.0670	0.982	23
2.5	500	4.5	7000	0.1	5	**130**	150	0,00005	27000	50	6	6	0.2	0.2	3	3	345	1018	0.0815	0.981	24
2.5	500	4.5	7000	0.1	5	100	**110**	0,00005	27000	50	6	6	0.2	0.2	3	2	361	1361	0.0726	0.980	25
2.5	500	4.5	7000	0.1	5	100	**190**	0,00005	27000	50	6	6	0.2	0.2	3	2	359	1350	0.0762	0.981	26
2.5	500	4.5	7000	0.1	5	100	150	**0,0001**	27000	50	6	6	0.2	0.2	4	3	251	874	0.0961	0.986	27
2.5	500	4.5	7000	0.1	5	100	150	**0,000005**	27000	50	6	6	0.2	0.2	1	2	837	2203	0.0385	0.941	28
2.5	500	4.5	7000	0.1	5	100	150	0,00005	**10000**	50	6	6	0.2	0.2	2	2	619	1568	0.0540	0.970	29
2.5	500	4.5	7000	0.1	5	100	150	0,00005	**50000**	50	6	6	0.2	0.2	4	3	252	885	0.0933	0.986	30
2.5	500	4.5	7000	0.1	5	100	150	0,00005	27000	50	**2**	**2**	0.2	0.2	3	2	359	1348	0.0730	0.982	31
2.5	500	4.5	7000	0.1	5	100	150	0,00005	27000	50	**10**	**10**	0.2	0.2	3	2	361	1363	0.0759	0.979	32
2.5	500	4.5	7000	0.1	5	100	150	0,00005	27000	**10**	6	6	0.2	0.2	4	5	240	630	0.0633	0.986	33
2.5	500	4.5	7000	0.1	5	100	150	0,00005	27000	**100**	6	6	0.2	0.2	2	2	555	1377	0.0839	0.975	34

Table 3. Situation (policy) 2: “in-house” maintenance. Top row is the base case. Parameter values are emboldened where they vary from the base case.

Mixed failure distribution parameters					cost parameters					downtimes		false alarm parameters		optimum values of decision variables, long run cost (*Cost*), and average availability						
β_1	η_1	β_2	η_2	p	c_0	c_m	c_r	μ	c_d	t_r	t_m	β	α	M_1	M_2	T_1	T_2	*Cost*	A	
2.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	0.2	0.2	2	3	497	975	0.0683	0.975	1
2.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	0.2	**0**	4	5	229	580	0.0468	0.988	2
2.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	0.2	**0.4**	2	2	616	1386	0.0894	0.965	3
2.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	**0**	**0**	3	5	278	612	0.0425	0.989	5
2.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	**0**	0.2	2	2	457	1395	0.0569	0.982	6
2.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	**0.4**	0.2	3	3	408	901	0.0842	0.967	7
1.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	0.2	0.2	2	5	440	693	0.0707	0.975	9
3.5	500	4.5	7000	0.1	5	55	105	0.00005	27000	6	6	0.2	0.2	2	2	609	1246	0.0668	0.974	10
2.5	500	4.5	**5000**	0.1	5	55	105	0.00005	27000	6	6	0.2	0.2	4	3	597	288	0.0819	0.970	15
2.5	500	4.5	**9000**	0.1	5	55	105	0.00005	27000	6	6	0.2	0.2	2	4	489	1042	0.0594	0.977	16
2.5	500	4.5	7000	**0.01**	5	55	105	0.00005	27000	6	6	0.2	0.2	2	2	1360	442	0.0376	0.990	17
2.5	500	4.5	7000	**0.25**	5	55	105	0.00005	27000	6	6	0.2	0.2	4	3	253	1130	0.1059	0.961	20
2.5	500	4.5	7000	0.1	**2**	55	105	0.00005	27000	6	6	0.2	0.2	3	1	449	788	0.0638	0.976	21
2.5	500	4.5	7000	0.1	**10**	55	105	0.00005	27000	6	6	0.2	0.2	2	2	565	1326	0.0748	0.972	22
2.5	500	4.5	7000	0.1	5	**25**	105	0.00005	27000	6	6	0.2	0.2	3	2	348	1239	0.0551	0.978	23
2.5	500	4.5	7000	0.1	5	**85**	105	0.00005	27000	6	6	0.2	0.2	2	3	564	1006	0.0795	0.972	24
2.5	500	4.5	7000	0.1	5	55	**70**	0.00005	27000	6	6	0.2	0.2	2	3	498	977	0.0664	0.974	25
2.5	500	4.5	7000	0.1	5	55	**140**	0.00005	27000	6	6	0.2	0.2	2	3	496	973	0.0701	0.974	26
2.5	500	4.5	7000	0.1	5	55	105	**0.0001**	27000	6	6	0.2	0.2	3	3	324	902	0.0976	0.979	27
2.5	500	4.5	7000	0.1	5	55	105	**0.000005**	27000	6	6	0.2	0.2	1	1	908	3682	0.0282	0.940	28