

# Microcausality and quantization of the fermionic Myers–Pospelov model

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**Abstract** We study the fermionic sector of the Myers and Pospelov theory with a general background  $n$ . The space-like case without temporal component is well defined and no new ingredients came about, apart from the explicit Lorentz invariance violation. The lightlike case is ill defined and physically discarded. However, the other case where a nonvanishing temporal component of the background is present, the theory is physically consistent. We show that new modes appear as a consequence of higher time derivatives. We quantize the timelike theory and calculate the microcausality violation which turns out to occur near the light cone.

## 1 Introduction

The need for a more fundamental theory at high energies has been justified in many different contexts. Divergences in quantum field theory, singularities in gravity and the lack of a unified quantum framework for all forces, are some of them. A consequence arising from this consideration, which has been extensively studied, is the possibility of having Lorentz invariance violation in the form of effective corrections [1–4]. This idea naturally leads to new extensions of the standard model and modified dispersion relations for particles. Today experimental searches for Lorentz invariance violation are being carried in diverse frontiers [5, 6].

In this context the Myers–Pospelov theory is a model that introduces Lorentz invariance violation through dimension five operators [7, 8]. The breakdown of Lorentz symmetry takes place in the scalar, fermion and gauge sectors and is characterized by an external timelike four-vector  $n_\mu$  defining a preferred reference frame. Experimental bounds for this model have been studied in several

phenomena, such as synchrotron radiation [9, 10], gamma ray bursts [11, 12], neutrino physics [13–15], radiative corrections [16–19], generic backgrounds [20, 21], and others [22, 23]. Typically, these phenomenological studies assume  $n$  to lie purely in the temporal direction [24]. In this work we will take  $n$  as general as possible and eventually we will consider some special choices.

In recent years, theories with higher time derivatives have been proposed as extensions of the standard model of particles [25–27]. One of the main advantages is that these theories soften the ultraviolet behavior of the quantum field theory, and hence problems like the hierarchy puzzle seem to be solved. Although they contain negative norm states [28, 29] the theoretical consistency was established many years ago [30, 31]. It can be shown that although unitarity is maintained, the price to pay is the lost of causality [32].

The new negative norm modes are relevant at high energies screening the ultraviolet effects of any standard quantum field theory leading to a low energy limit which is not sensitive to the details of the effective theory at microscopic scales (see, however, [33, 34]). The Myers and Pospelov theory has these ingredients when  $n$  has a nonvanishing temporal component. Hence, it is interesting to investigate the role of these new modes in order to check the behavior of the low energy limit of Myers–Pospelov theory. In this work we will analyze how these new modes affect the quantization of the theory, because it is the first step to study such low energy limit.

Moreover, interacting theories with higher time derivatives lose causality at the microscopic level if we want to maintain unitarity. An effect of this acausal behavior is for instance the negativity of certain decay rates. But also the Lorentz violating Myers and Pospelov theories have a natural violation of the microcausality principle, even without interactions [35, 36]. Since in this work we will not deal with interactions we will focus on the study of the last source of violation of microcausality.

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The layout of this work is the following. In Sect. 2 we introduce the fermionic Myers and Pospelov model where we find the dispersion relation in an arbitrary background. For special choices of the preferred four-vector we analyze the causality and stability of the different theories. In Sect. 3 we review the main aspects of a higher time derivative theory like the fermionic Lee–Wick model which will help us to understand the remaining sections. In Sect. 4 we quantize the timelike Myers and Pospelov theory by performing a decomposition of the theory into four individual fermionic oscillators. In Sect. 5 we discuss violations of microcausality where a perturbative computation of the anticommutator function is given. In the last section we give the conclusions and final comments. In Appendix we characterize the general solutions and dispersion relations.

## 2 Fermionic Myers–Pospelov model

The fermionic sector of the Myers–Pospelov theory is given by the Lagrangian

$$\mathcal{L} = \bar{\psi}(i \not{\partial} - m)\psi + \bar{\psi} \not{n}(g_1 + g_2 \gamma_5)(n \cdot \partial)^2 \psi, \tag{1}$$

where  $g_1$  and  $g_2$  are inverse Planck mass dimension couplings constants and  $n$  is a dimensionless four-vector defining a preferred reference frame with  $n^2 = +1, -1$ , or  $0$ .

The variation of the Lagrangian (1) produces the equations of motion

$$[i \not{\partial} - m + g_1 \not{n}(n \cdot \partial)^2 + g_2 \not{n} \gamma_5 (n \cdot \partial)^2] \psi(x) = 0. \tag{2}$$

In momentum space,  $\psi(x) = \int d^4 p e^{-ip \cdot x} \psi(p)$ , we obtain an algebraic equation,

$$[\not{p} - m - g_1 \not{n}(n \cdot p)^2 - g_2 \not{n} \gamma_5 (n \cdot p)^2] \psi(p) = 0. \tag{3}$$

The dispersion relation is given by

$$(p^2 - m^2 - 2g_1(n \cdot p)^3 + n^2(g_1^2 - g_2^2)(n \cdot p)^4)^2 - 4(n \cdot p)^4 g_2^2((n \cdot p)^2 - p^2 n^2) = 0. \tag{4}$$

In general (4) is an eighth order polynomial in  $\omega$  and it would yield at most eight real solutions. However, if  $n_0 = 0$  the order of the polynomial in  $\omega$  is four corresponding to particles and antiparticles of spin 1/2. The negative solutions correspond to antiparticles modes while the positive ones are particles modes. The situation for  $n_0 \neq 0$  is to obtain twice the number of solutions than in the standard case. This is due to the fact that we are dealing with a theory with higher time derivatives as can be seen from the equation of motion (2). In the next subsection we will discuss in more detail the nature of these extra solutions.

A derivation of Eq. (4) and the eigenspinor solutions are given in the Appendix. In what follows we will consider the case  $g_2 = 0$ . The case of a nonvanishing  $g_2$  introduces very complicated parameterizations as can be seen in the Appendix. However, it does not contribute to new relevant features and renders the calculations cumbersome. The reader interested in this case can go through the Appendix.

### 2.1 The timelike model

We start to analyze the purely timelike case by taking  $n = (1, 0, 0, 0)$  and as mentioned above setting  $g_2 = 0$ . In this case the dispersion relation (4) reduces to

$$\omega^2 - \mathbf{p}^2 - m^2 - 2g_1 \omega^3 + g_1^2 \omega^4 = 0, \tag{5}$$

from where we obtain the four solutions

$$\begin{aligned} \omega_{(a=1,2)} &= \frac{1 - \sqrt{1 - 4(-1)^a g_1 E_{\mathbf{p}}}}{2g_1}, \\ \omega_{(a=3,4)} &= \frac{1 + \sqrt{1 + 4(-1)^a g_1 E_{\mathbf{p}}}}{2g_1}, \end{aligned} \tag{6}$$

with  $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ .

The solutions  $\omega_{1,2}$  in the limit  $g_1 \rightarrow 0$  tend to the usual solutions  $\mp E$  while the solutions  $\omega_{3,4}$  go to infinity. These singular solutions are called Lee–Wick modes [30, 31] and will be explained in more detail in the next section.

In order to see the qualitative behavior of the solutions let us define the two functions  $f(\omega) = \omega^2 - m^2 - 2g_1 \omega^3 + g_1^2 \omega^4$  and  $g(\omega) = \mathbf{p}^2$ , and plot these functions of  $\omega$  in Fig. 1. The solutions are the intersection points of the curve  $f$  and the horizontal straight line corresponding to the fixed value of the momentum square, i.e.  $g(\omega) = \mathbf{p}^2$ . Hence, for small values of  $|\mathbf{p}|$  we find four solutions, one negative frequency which corresponds to an antiparticle and three positive frequencies. Among the positive frequencies the smallest one is the normal particle frequency and the other two correspond

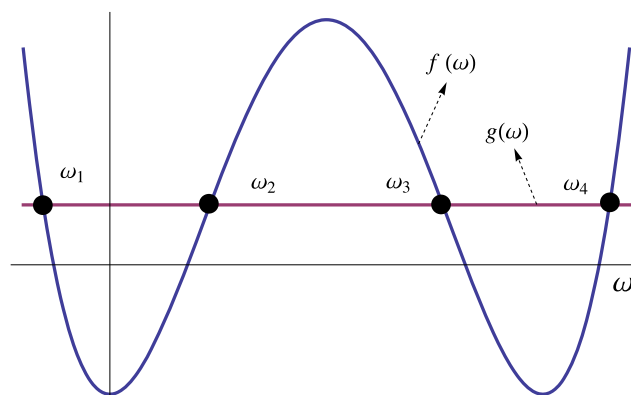


Fig. 1 The intersection of the horizontal straight line  $g(\omega) = \mathbf{p}^2$  with the curve  $f(\omega)$  corresponds to the solutions  $\omega_a$  given in (6)

to Lee–Wick modes. It is peculiar the behavior of the Lee–Wick solution whose frequency decreases with momentum, this will continue until the momentum reaches the value of  $|\mathbf{p}|_{\max} = \sqrt{\frac{1}{16g_1^2} - m^2}$  where it collapses with the normal particle mode. Above these values the solutions  $\omega_2$  and  $\omega_3$  become complex introducing stability problems. Furthermore, it is worth noting the differences in energy between particles and antiparticles which in the limit  $mg_1 \ll 1$  turns out to be  $4|g_1|m^2$ .

Some insight can be gained into the possible violations of microcausality in the model by looking at the group velocities [37]. The magnitude of the group velocities are

$$v_{(a=1,4)}^{(1)} = (-1)^a \frac{|\mathbf{p}|}{E_{\mathbf{p}}\sqrt{1 + 4g_1 E_{\mathbf{p}}}}, \tag{7}$$

$$v_{(a=2,3)}^{(2)} = (-1)^a \frac{|\mathbf{p}|}{E_{\mathbf{p}}\sqrt{1 - 4g_1 E_{\mathbf{p}}}},$$

and they are plotted in Fig. 2. According to the criteria of [37] we should expect small violations of microcausality in the theory since the velocities  $v_{(a=3,4)}^{(2)}$  can exceed normal signal propagation at high momenta. In Sect. 5 we give a detailed computation of microcausality.

### 2.2 The lightlike model

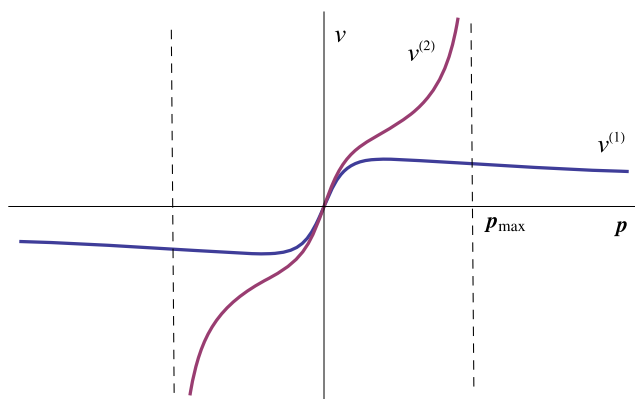
In the lightlike case and for simplicity taking  $n_0 = 1$  the dispersion relation reads

$$\omega^2 - \mathbf{p}^2 - m^2 - 2g_1(\omega - |\mathbf{p}| \cos \theta)^3 = 0, \tag{8}$$

where  $\theta$  is the angle between  $\mathbf{n}$  and  $\mathbf{p}$ . The solutions are

$$\omega_1 = \frac{1}{6g_1} + |\mathbf{p}| \cos \theta - A,$$

$$\omega_2 = \frac{1}{6g_1} + |\mathbf{p}| \cos \theta + B, \tag{9}$$



**Fig. 2** The magnitude of the group velocities  $v^{(1)}$  and  $v^{(2)}$  given in (7)

$$\omega_3 = \frac{1}{6g_1} + |\mathbf{p}| \cos \theta + B^*,$$

with

$$A = \frac{1 + 12g_1|\mathbf{p}| \cos \theta}{6g_1 K^{1/3}} + \frac{K^{1/3}}{6g_1}, \tag{10}$$

$$B = \frac{(1 + i\sqrt{3})(1 + 12g_1|\mathbf{p}| \cos \theta)}{12g_1 K^{1/3}} + \frac{(1 - i\sqrt{3})K^{1/3}}{12g_1}, \tag{11}$$

and

$$K = -1 + 54E_{\mathbf{p}}^2 g_1^2 - 18g_1|\mathbf{p}| \cos \theta \times (1 + 3g_1|\mathbf{p}| \cos \theta) + 3\sqrt{3}|g_1|\sqrt{-\Delta} \tag{12}$$

where  $\Delta$  is the discriminant of the third order polynomial (8).

Here, the roots can be real or complex depending on whether the discriminant is greater or less than zero, respectively. Therefore, the quantization of this model presents an extra complication of instability due to complex solutions. To see this more clearly consider the discriminant up to the linear order

$$\Delta \approx 4E_{\mathbf{p}}^2(1 + 18g_1|\mathbf{p}| \cos \theta). \tag{13}$$

For example we see that for momenta higher than  $|\mathbf{p}|_{\max} = \frac{1}{18g_1|\cos \theta|}$  the solutions in the anti-parallel direction can be imaginary. For these very high momenta the theory can violate causality since the retarded Green function gives a contribution at times  $t < 0$ . This is very similar to what occurs in the timelike model for  $\omega_{2,3}$ , see Fig. 3 of Sect. 5, however, here the instabilities are not controllable by restricting to lower momenta or introducing a cutoff [16, 17].

### 2.3 The spacelike model

Without loss of generality we can take the preferred vector as  $\mathbf{n} = (0, 0, 1)$ . The dispersion relation for this case is

$$\omega^2 - E_{\mathbf{p}}^2 + 2g_1 p_z^3 - g_1^2 p_z^4 = 0. \tag{14}$$

The frequency solutions are

$$\omega_{\pm} = \pm \sqrt{p_x^2 + p_y^2 + (p_z - g_1 p_z^2)^2 + m^2}. \tag{15}$$

Note that these solutions are always real and that we recover the usual dispersion relation when the preferred vector is orthogonal to the propagation, called blind momenta directions.

To discuss the causal structure of the theory let us compute the retarded Green function. We must check that it vanishes for times before the interaction is turned on, that is to

say, before the time  $t = 0$ . The retarded Green function in this case is

$$iS_R(x) = (i\cancel{\partial} - g_1\gamma^z\partial_z^2 + m) \times \int_{C_R} \frac{d^4p}{(2\pi)^4} \frac{e^{-ip \cdot x}}{(p_0^2 - \omega^2)}, \tag{16}$$

where the poles are given by the solutions (15) and  $C_R$  is the contour above the real axis as depicted in Fig. 3 of Sect. 5. The argument that causality is preserved is rather simple and goes as follows. For times  $t < 0$  the contour  $C_R$  must be closed from above and therefore does not enclose any pole. Recall that the poles lie on the real axis even for arbitrary high momenta. In this way there are no violations of causality in the spacelike model.

### 3 Lee–Wick theories

Before facing the problem of quantization, let us review some general aspects concerning higher derivative theories which may not be familiar for some readers. These kind of theories were studied by Lee and Wick and others some decades ago [29–32] and recently there has been a growing interest in them regarding the hierarchy problem in the standard model [25–27]. Unlike the theory we are considering, the Lee–Wick models are Lorentz invariant theories, however, they have in common the higher order time derivatives. We will devote this section to summarize the main features of the fermionic sector of a Lee–Wick model which will be important for our subsequent analysis.

In particular let us consider the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\cancel{\partial} - m)\psi - \frac{g}{\Lambda} \bar{\psi} \square \psi, \tag{17}$$

where  $g$  is a dimensionless positive coupling constant and  $\Lambda$  is an ultraviolet energy scale.

By defining the new fields

$$\begin{aligned} \psi_+ &= \beta(i\cancel{\partial} + m_-)\psi, \\ \psi_- &= \beta(i\cancel{\partial} - m_+)\psi, \end{aligned} \tag{18}$$

with  $\beta = (\frac{g/\Lambda}{m_+ + m_-})^{\frac{1}{2}}$  and

$$m_{\pm} = \frac{\mp 1 + \sqrt{1 + 4g\frac{m}{\Lambda}}}{2g} \Lambda, \tag{19}$$

the Lagrangian (17) can be written in terms of these fields as

$$\mathcal{L} = \bar{\psi}_+(i\cancel{\partial} - m_+)\psi_+ - \bar{\psi}_-(i\cancel{\partial} + m_-)\psi_-. \tag{20}$$

Here we have written a higher time derivative theory in terms of two decoupled standard fermions. However, the

second mode has the wrong sign in front of its Lagrangian density.

The non vanishing anticommutators will be

$$\begin{aligned} \{\psi_+^\alpha(\mathbf{x}, t), \psi_+^{\dagger\beta}(\mathbf{y}, t)\} &= -\{\psi_-^\alpha(\mathbf{x}, t), \psi_-^{\dagger\beta}(\mathbf{y}, t)\} \\ &= \delta^{\alpha\beta} \delta^3(\mathbf{x} - \mathbf{y}). \end{aligned} \tag{21}$$

Note that the minus sign of the anticommutators of the minus fields is responsible for the negative norm states.

Now, decomposing the new fields in terms of plane wave solutions we find

$$\begin{aligned} \psi_+(\mathbf{x}, t) &= \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_+}} \\ &\times [b_+^s(\mathbf{p})e^{-ip_+ \cdot x} u_+^s(\mathbf{p}) + d_+^{s\dagger}(\mathbf{p})e^{ip_+ \cdot x} v_+^s(\mathbf{p})], \end{aligned} \tag{22}$$

$$\begin{aligned} \psi_-(\mathbf{x}, t) &= \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_-}} \\ &\times [b_-^s(\mathbf{p})e^{-ip_- \cdot x} u_-^s(\mathbf{p}) + d_-^{s\dagger}(\mathbf{p})e^{ip_- \cdot x} v_-^s(\mathbf{p})], \end{aligned} \tag{23}$$

where  $p_{\pm} = (\omega_{\pm}, \mathbf{p})$  and  $E_{\pm} = \sqrt{\mathbf{p}^2 + m_{\pm}^2}$  and  $u, v$  are the eigenspinors satisfying the orthogonality relations

$$u_{\pm}^{\dagger s} u_{\pm}^r = v_{\pm}^{\dagger s} v_{\pm}^r = 2E_{\pm} \delta^{sr}. \tag{24}$$

The Hamiltonian of the theory can be written in terms of the standard creation and annihilation operators for the fields  $\psi_{\pm}$  as

$$\begin{aligned} H &= \sum_s \int d^3p (E_+ (b_+^{s\dagger}(\mathbf{p})b_+^s(\mathbf{p}) + d_+^{s\dagger}(\mathbf{p})d_+^s(\mathbf{p})) \\ &+ E_- (b_-^{s\dagger}(\mathbf{p})b_-^s(\mathbf{p}) + d_-^{s\dagger}(\mathbf{p})d_-^s(\mathbf{p}))), \end{aligned} \tag{25}$$

and,

$$\begin{aligned} \{b_{\pm}^s(\mathbf{p}), b_{\pm}^{r\dagger}(\mathbf{k})\} &= \pm(2\pi)^3 \delta^{sr} \delta^3(\mathbf{p} - \mathbf{k}), \\ \{d_{\pm}^s(\mathbf{p}), d_{\pm}^{r\dagger}(\mathbf{k})\} &= \pm(2\pi)^3 \delta^{sr} \delta^3(\mathbf{p} - \mathbf{k}), \end{aligned} \tag{26}$$

are the nonvanishing anticommutators of creation and annihilation operators for particles (b) and antiparticles (d) of spin  $s$  and  $r$ . Here the positivity of the energy spectrum and the indefiniteness of Fock space are evident. The propagators are

$$S_{\pm}(p) = \frac{\pm i(\cancel{p} \pm m_{\pm})}{p^2 - m_{\pm}^2}. \tag{27}$$

By introducing interactions the wrong sign may cause the loss of unitarity. However, it has been shown that with a suitable prescription for the propagators it is possible to maintain unitarity [32]. Although unitarity is kept, causality is

lost at a microscopic scale, as can be seen by the occurrence of negative decay rates.

Summarizing, theories with higher time derivatives have the following important features (see also [38, 39]).

- The theory doubles the number of modes.
- The new modes correspond to negative norm states.
- The theory can always be defined with positive energies and unitary S matrix.
- Causality is lost at a microscopic scale.

#### 4 Quantization

In this section we will proceed to quantize the free Myers–Pospelov theory for the special case of  $n$  purely timelike and  $g_2 = 0$ . As we mentioned above this case corresponds to a higher time derivative theory and it will have many features in common with the model reviewed in the previous section. However, we will take a different strategy for quantizing the theory because our present theory lacks Lorentz covariance.

In this case the Lagrangian is

$$L = \int d^3x \bar{\psi} (i \not{\partial} - g \gamma^0 \partial_t^2 - m) \psi, \tag{28}$$

$$= \int d^3x \psi^\dagger (i \partial_t - g \partial_t^2 - \hat{h}_D) \psi,$$

where  $\hat{h}_D = -i\boldsymbol{\alpha} \cdot \nabla + m\beta$  is the standard Dirac Hamiltonian operator and we have considered without loss of generality  $g = -g_1$  to make the contact with the previous section more transparent. Now let us write the field in terms of the standard solutions of the Dirac Hamiltonian operator,

$$\psi(\mathbf{x}, t) = \sum_{s,i} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} u_i^s(\mathbf{p}) \psi_i^s(\mathbf{p}, t) e^{i\varepsilon_i \mathbf{p} \cdot \mathbf{x}}, \tag{29}$$

where  $s$  is a spin index,  $i$  is the particle and antiparticle index, i.e.,  $u_1^s = u^s$  and  $u_2^s = v^s$  being  $u^s$  and  $v^s$  the standard spinors and  $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ . Remembering

$$\hat{h}_D u_i^s(\mathbf{p}) e^{i\varepsilon_i \mathbf{p} \cdot \mathbf{x}} = \varepsilon_i E_{\mathbf{p}} u_i^s(\mathbf{p}) e^{i\varepsilon_i \mathbf{p} \cdot \mathbf{x}}, \tag{30}$$

with the normalization convention

$$u_i^{s\dagger}(\mathbf{p}) u_j^r(\mathbf{p}) = 2E_{\mathbf{p}} \delta^{sr} \delta_{ij}, \tag{31}$$

where  $\varepsilon_1 = +1$  and  $\varepsilon_2 = -1$ , we have

$$L = \sum_{s,i} \int d^3p \psi_i^{s\dagger}(\mathbf{p}, t) (-g \partial_t^2 + i \partial_t - \varepsilon_i E_{\mathbf{p}}) \psi_i^s(\mathbf{p}, t). \tag{32}$$

Now it is clear we have reduced the quantum field theory problem to a set of four quantum mechanical systems at a given momentum.

These quantum mechanical systems have higher time derivatives and their quantization can be realized in a similar way as it was done in the previous section, but for a  $0 + 1$  quantum field theory. In other words for each index  $i$  and  $s$  we define the following fields:

$$\psi_{\pm,i}^s(\mathbf{p}, t) = \beta_i (i \partial_t \pm \omega_{\mp}^{(i)}(\mathbf{p})) \psi_i^s(\mathbf{p}, t), \tag{33}$$

where  $\beta_i = (\frac{g}{\omega_+^{(i)} + \omega_-^{(i)}})^{\frac{1}{2}}$  and

$$\omega_{\pm}^{(i)} = \frac{\mp 1 + \sqrt{1 + 4g\varepsilon_i E_{\mathbf{p}}}}{2g}. \tag{34}$$

The Lagrangian in terms of these fields is

$$L = \sum_{s,i} \int d^3p \psi_{+,i}^{s\dagger} (i \partial_t - \omega_+^{(i)}(p)) \psi_{+,i}^s - \sum_{s,i} \int d^3p \psi_{-,i}^{s\dagger} (i \partial_t + \omega_-^{(i)}(p)) \psi_{-,i}^s, \tag{35}$$

the equations of motion in terms of these fields are

$$(i \partial_t \mp \omega_{\pm}^{(i)}) \psi_{\pm,i}^s = 0, \tag{36}$$

whose solution are

$$\psi_{\pm,i}^s = C_{\pm,i}^s(\mathbf{p}) e^{\mp i \omega_{\pm}^{(i)} t}, \tag{37}$$

$$\psi_{\pm,i}^{\dagger s} = C_{\pm,i}^{\dagger s}(\mathbf{p}) e^{\pm i \omega_{\pm}^{(i)} t}. \tag{38}$$

Now it is straightforward to quantize this system by promoting the coefficients  $C$  and  $C^\dagger$  to operators and taking into account the minus sign of the second part of (35) which produces the minus sign in the anticommutation relations of the minus modes, i.e.,

$$\{C_{\pm,i}^s(\mathbf{p}), C_{\pm,j}^{\dagger r}(\mathbf{q})\} = \pm (2\pi)^3 \delta^{rs} \delta^3(\mathbf{p} - \mathbf{q}). \tag{39}$$

To make contact with the standard theory note that  $C_{+,1}^s$  corresponds to  $b^s$  which destroys standard fermion and  $C_{+,2}^s$  corresponds to  $d^{\dagger s}$  which creates standard antifermions, i.e.,

$$C_{+,1}^s(\mathbf{p}) \equiv b^s(\mathbf{p}), \tag{40}$$

and

$$C_{+,2}^s(\mathbf{p}) \equiv d^{\dagger s}(\mathbf{p}). \tag{41}$$

This correspondence is valid only for the plus modes because the standard theory is recovered when  $g$  goes to zero.

However, the minus modes have not a defining limit and we cannot refer to them as particle and antiparticle pairs.

The original field can be written as

$$\psi(\mathbf{x}, t) = \psi_+(\mathbf{x}, t) - \psi_-(\mathbf{x}, t), \tag{42}$$

with

$$\begin{aligned} \psi_+(\mathbf{x}, t) = \sum_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \\ \times e^{i\mathbf{p}\cdot\mathbf{x}} \left[ \frac{b^s(\mathbf{p})u^s(\mathbf{p})e^{-i\omega_+^{(1)}\cdot x}}{(1 + 4gE_{\mathbf{p}})^{1/4}} \right. \\ \left. + \frac{d^{s\dagger}(-\mathbf{p})v^s(-\mathbf{p})e^{-i\omega_+^{(2)}\cdot x}}{(1 - 4gE_{\mathbf{p}})^{1/4}} \right], \tag{43} \end{aligned}$$

$$\begin{aligned} \psi_-(\mathbf{x}, t) = \sum_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \\ \times e^{i\mathbf{p}\cdot\mathbf{x}} \left[ \frac{C_{-,1}^s(\mathbf{p})u^s(\mathbf{p})e^{i\omega_-^{(1)}\cdot x}}{(1 + 4gE_{\mathbf{p}})^{1/4}} \right. \\ \left. + \frac{C_{-,2}^{s\dagger}(-\mathbf{p})v^s(-\mathbf{p})e^{i\omega_-^{(2)}\cdot x}}{(1 - 4gE_{\mathbf{p}})^{1/4}} \right]. \tag{44} \end{aligned}$$

Putting it all together from Eq. (35) it is easy to arrive at the expression for the Hamiltonian

$$\begin{aligned} H = \sum_s \int d^3 p \left( (\omega_+^{(1)} b^{s\dagger}(\mathbf{p})b^s(\mathbf{p}) - \omega_+^{(2)} d^{s\dagger}(\mathbf{p})d^s(\mathbf{p})) \right. \\ \left. - (\omega_-^{(1)} C_{-,1}^{s\dagger}(\mathbf{p})C_{-,1}^s(\mathbf{p}) + \omega_-^{(2)} C_{-,2}^{s\dagger}(\mathbf{p})C_{-,2}^s(\mathbf{p})) \right). \tag{45} \end{aligned}$$

The first line of this expression is the standard Hamiltonian in the limit  $g$  goes to zero because  $\omega_+^{(1)} = -\omega_+^{(2)} = E_{\mathbf{p}}$ . This Hamiltonian is actually positive if we define the vacuum as the state which is annihilated by  $b, d, C_{-,1}$  and  $C_{-,2}$ . However, in the second line we must use the negativity of the anticommutators of  $C_-$  and the positivity of the  $\omega_-$  to check this statement.

Now it is clear that the spectrum of the theory is the following: fermions of spin one half and energy

$$E_f = \omega_+^{(1)}(\mathbf{p}) \approx E_{\mathbf{p}} - gE_{\mathbf{p}}^2, \tag{46}$$

and antifermions of spin one half and energy

$$E_{\bar{f}} = -\omega_+^{(2)}(\mathbf{p}) \approx E_{\mathbf{p}} + gE_{\mathbf{p}}^2, \tag{47}$$

and negative norm particles of spin one half and energies

$$E_c = \omega_-^{(1)}(\mathbf{p}) \approx \frac{1}{g} + E_{\mathbf{p}} - gE_{\mathbf{p}}^2, \tag{48}$$

and

$$E_{\bar{c}} = \omega_-^{(2)}(\mathbf{p}) \approx \frac{1}{g} - E_{\mathbf{p}} - gE_{\mathbf{p}}^2, \tag{49}$$

respectively. They sum up for particles of spin one half i.e. eight modes. This analysis agrees with the discussion in the subsection (2.1) restoring  $g \rightarrow -g_1$  and making the identification  $\omega_+^{(1)} \rightarrow \omega_2, \omega_+^{(2)} \rightarrow \omega_1, \omega_-^{(1)} \rightarrow -\omega_3$  and  $\omega_-^{(2)} \rightarrow -\omega_4$ .

### 5 Microcausality

In this section we will study the source of microcausality violation due to the noncovariant terms in the model. For this let us compute the anticommutator of free fermionic fields

$$iS(x - x') = \{\psi(x), \bar{\psi}(x')\}. \tag{50}$$

It is clear from Eqs. (39) and (42) that the plus and minus fields do not mix. Hence, with  $x' = 0$  we have

$$iS(x) = \{\psi_+(x), \bar{\psi}_+(0)\} + \{\psi_-(x), \bar{\psi}_-(0)\}. \tag{51}$$

Again restoring  $g \rightarrow -g_1$  the anticommutators can be shown to be

$$\{\psi_{\pm}(x), \bar{\psi}_{\pm}(0)\} = (i\not{\partial} + m)i\Delta_{\pm}, \tag{52}$$

with

$$\begin{aligned} \Delta_+(x) = \int \frac{d^3 p}{(2\pi)^3 2E_{\mathbf{p}}} e^{i\mathbf{p}\cdot\mathbf{x}} \left( \frac{e^{-i\omega_1 t}}{\sqrt{1 + 4g_1 E_{\mathbf{p}}}} \right. \\ \left. - \frac{e^{-i\omega_2 t}}{\sqrt{1 - 4g_1 E_{\mathbf{p}}}} \right), \tag{53} \end{aligned}$$

and

$$\begin{aligned} \Delta_-(x) = \int \frac{d^3 p}{(2\pi)^3 2E_{\mathbf{p}}} e^{i\mathbf{p}\cdot\mathbf{x}} \left( \frac{e^{-i\omega_3 t}}{\sqrt{1 - 4g_1 E_{\mathbf{p}}}} \right. \\ \left. - \frac{e^{-i\omega_4 t}}{\sqrt{1 + 4g_1 E_{\mathbf{p}}}} \right), \tag{54} \end{aligned}$$

where we have used the usual spin sum  $\sum_s u^s(\mathbf{p})\bar{u}^s(\mathbf{p}) = \gamma \cdot p + m$  and  $\sum_s v^s(\mathbf{p})\bar{v}^s(\mathbf{p}) = \gamma \cdot p - m$ . Let us combine terms with the same denominator, consider thus

$$i\Delta(x) = i\Delta_1(x) - i\Delta_2(x), \tag{55}$$

where

$$i\Delta_1(x) = \frac{-ie^{-it/2g_1}}{2\pi^2 r} \int_0^{|\mathbf{p}|_{\max}} d|\mathbf{p}| |\mathbf{p}| \sin(|\mathbf{p}|r)$$

$$\times \left( \frac{e^{\frac{i\sqrt{1-4g_1 E_p} t}{2g_1}}}{2E_p \sqrt{1-4g_1 E_p}} - \frac{e^{-\frac{i\sqrt{1-4g_1 E_p} t}{2g_1}}}{2E_p \sqrt{1-4g_1 E_p}} \right), \tag{56}$$

and

$$i \Delta_2(x) = \frac{-i e^{-it/2g_1}}{2\pi^2 r} \int_0^\infty d|\mathbf{p}| |\mathbf{p}| \sin(|\mathbf{p}|r) \times \left( \frac{e^{-\frac{i\sqrt{1+4g_1 E_p} t}{2g_1}}}{2E_p \sqrt{1+4g_1 E_p}} - \frac{e^{\frac{i\sqrt{1+4g_1 E_p} t}{2g_1}}}{2E_p \sqrt{1+4g_1 E_p}} \right), \tag{57}$$

where  $r = |\mathbf{x}|$  and we have performed the angular integration.

To proceed further let us make the change of variables  $d|\mathbf{p}| |\mathbf{p}| = dE_p E_p$  followed by  $z = g_1 E_p$  to arrive at

$$i \Delta_1(x) = \frac{e^{-it/2g_1}}{2\pi^2 r g_1} \int_\varepsilon^{1/4} dz \frac{\sin\left(\frac{r\sqrt{z^2-\varepsilon^2}}{g_1}\right) \sin\left(\frac{t\sqrt{1-4z}}{2g_1}\right)}{\sqrt{1-4z}}, \tag{58}$$

and

$$i \Delta_2(x) = \frac{e^{-it/2g_1}}{2\pi^2 r g_1} \int_\varepsilon^\infty dz \frac{\sin\left(\frac{r\sqrt{z^2-\varepsilon^2}}{g_1}\right) \sin\left(\frac{t\sqrt{1+4z}}{2g_1}\right)}{\sqrt{1+4z}}. \tag{59}$$

where  $\varepsilon = mg_1$ .

Alternatively, we could have started with the four momentum integral representation

$$i \Delta(x) = \oint_C \frac{d^4 p}{(2\pi)^4} \frac{e^{-ip \cdot x}}{g_1^2 (p_0 - \omega_1)(p_0 - \omega_2)(p_0 - \omega_3)(p_0 - \omega_4)}, \tag{60}$$

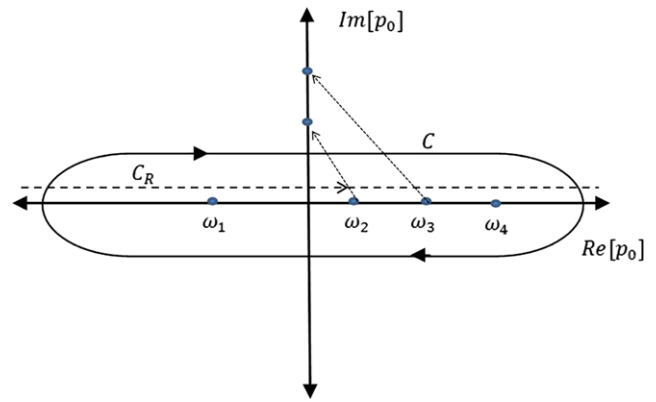
where  $C$  is the contour encircling all the poles in the clockwise direction and which satisfies

$$i S(x) = (i \gamma^\mu \partial_\mu + g_1 \gamma^0 \partial_t^2 + m) i \Delta(x), \tag{61}$$

arriving at the same result as in Eqs. (58), (59). One advantage, however, is that in this way it is more clear to see that for momenta higher than  $|\mathbf{p}|_{\max}$  both poles  $\omega_2$  and  $\omega_3$  move out from the region enclosed by the contour  $C$  and eventually become purely imaginary, see Fig. 3. Hence, they do not contribute to the integral when  $|\mathbf{p}| > |\mathbf{p}|_{\max}$  producing a natural cutoff in the integral (56).

To the lowest order in  $\varepsilon$  it is possible to solve the integrals; these are

$$i \Delta_1(x) = -\frac{e^{-it/2g_1}}{4(\pi r)^{3/2} \sqrt{2g_1}} \left( \cos\left(\frac{t^2+r^2}{4g_1 r}\right) N_1(x, g_1) \right.$$



**Fig. 3** For momenta above  $|\mathbf{p}|_{\max}$  both poles  $\omega_2$  and  $\omega_3$  move out from the region enclosed by  $C$  to the imaginary axis. The dotted contour corresponds to the usual prescription  $C_R$  to be closed from above when  $t < 0$

$$+ \sin\left(\frac{t^2+r^2}{4g_1 r}\right) N_2(x, g_1), \tag{62}$$

where

$$N_1(x, g_1) = C\left(\frac{\alpha r - t}{\sqrt{2\pi g_1 r}}\right) + 2C\left(\frac{t}{\sqrt{2\pi g_1 r}}\right) - C\left(\frac{\alpha r + t}{\sqrt{2\pi g_1 r}}\right), \tag{63}$$

$$N_2(x, g_1) = S\left(\frac{\alpha r - t}{\sqrt{2\pi g_1 r}}\right) + 2S\left(\frac{t}{\sqrt{2\pi g_1 r}}\right) - S\left(\frac{\alpha r + t}{\sqrt{2\pi g_1 r}}\right). \tag{64}$$

Above we have introduced the Fresnel integrals

$$C(y) = \int_0^y \cos\left(\frac{\pi z^2}{2}\right) dz, \tag{65}$$

$$S(y) = \int_0^y \sin\left(\frac{\pi z^2}{2}\right) dz, \tag{66}$$

and defined  $\alpha = \sqrt{1-4mg_1}$  and  $\beta = \sqrt{1+4mg_1}$ . Similarly, the other part is

$$i \Delta_2(x) = -\frac{e^{-it/2g_1}}{4(\pi r)^{3/2} \sqrt{2g_1}} \left( \cos\left(\frac{t^2+r^2}{4g_1 r}\right) N_3(x, g_1) + \sin\left(\frac{t^2+r^2}{4g_1 r}\right) N_4(x, g_1) \right), \tag{67}$$

with

$$N_3(x, g_1) = C\left(\frac{\beta r - t}{\sqrt{2\pi g_1 r}}\right) - C\left(\frac{\beta r + t}{\sqrt{2\pi g_1 r}}\right), \tag{68}$$

$$N_4(x, g_1) = S\left(\frac{\beta r - t}{\sqrt{2\pi g_1 r}}\right) - S\left(\frac{\beta r + t}{\sqrt{2\pi g_1 r}}\right).$$

For spacelike separations  $r^2 > t^2$  and making the approximation for small  $g_1$  in order to have  $\alpha = \beta \approx 1$  we find

$$i \Delta_1(x) \rightarrow -\frac{e^{-it/2g_1}}{4(\pi r)^{3/2}\sqrt{2g_1}}\varepsilon(t) \times \left( \cos\left(\frac{t^2+r^2}{4g_1r}\right) + \sin\left(\frac{t^2+r^2}{4g_1r}\right) \right), \tag{69}$$

$$i \Delta_2(x) \rightarrow 0. \tag{70}$$

Adding the contributions we have

$$i \Delta(x) = -\frac{\varepsilon(t)}{8(\pi r)^{3/2}\sqrt{2g_1}} \times \left( e^{\frac{i(r-t)^2}{4g_1r}}(1-i) + e^{-\frac{i(r+t)^2}{4g_1r}}(1+i) \right), \tag{71}$$

where  $\varepsilon(t) = \pm 1$  for the corresponding positive and negative values of  $t$ . The spacelike regions where microcausality is violated are the regions where the phase changes slowly:

$$\frac{(r-t)^2}{4g_1r} < 0. \tag{72}$$

This is very similar to what occurs in the photon sector of the Myers–Pospelov theory where the small violations of microcausality occur near the light cone [16, 17, 21].

### 6 Discussions and conclusions

In this work we have analyzed some aspects of the fermionic Myers and Pospelov model: Firstly we have found the general dispersion relations and solutions of the equation of motion. Secondly we have analyzed the consistency conditions for the cases purely timelike, lightlike and purely spacelike. Thirdly we explicitly quantized the pure time theory and finally we computed the microcausality violation.

In the purely spacelike case no inconsistencies were found. However, for the other two cases the theory is consistent for momenta below a natural cutoff. Furthermore, these cases show higher time derivatives features which double the number of degrees of freedom. The additional modes are negative norm states which might be controlled by suitable prescriptions studied in the known Lee–Wick theories. Microcausality was computed explicitly in the pure time case, leading to suppressed violations near lightlike four momenta.

In the quantization of the negative norm states appearing in the theory we have assumed that the Cutkosky prescription should work for the theory under consideration. However, this is quite far from being clear, because that procedure was introduced to maintain unitarity and covariance of Lee–Wick theories. We are not restricted to fulfill the covariance of the theory but we need to keep unitarity. This aspect

should be studied in future works to complete the analysis. After this, we would be ready to study new features due to interaction terms like radiative corrections, the low energy limit of the theory, and the violation of causality owed to negative norm states contained in the theory.

The success of the complete answer to these questions would give us a criteria to establish the validity of the Myers–Pospelov theory as a consistent effective theory containing possible effects of quantum gravity.

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### Appendix: General solutions and dispersion relations

In this appendix we will characterize the general solutions and dispersion relations of the equation of motion for the general fermionic Myers and Pospelov theory. This characterization is not essential for the understanding of the body of the work apart from some particular aspects concerning the dispersion relation. However, we include it for the sake of completeness.

Consider the equation of motion

$$(\not{a} - \not{b}\gamma^5 - m)\psi = 0, \tag{A.1}$$

where  $a_\mu \equiv p_\mu - g_1 n_\mu (n \cdot p)^2$  and  $b_\mu \equiv g_2 n_\mu (n \cdot p)^2$  are four-vectors which will help us to clear up the notation.

Let us define the following matrices:

$$\hat{M} \equiv \not{a} - \not{b}\gamma^5 - m, \quad \hat{h} \equiv [\not{a}, \not{b}]\gamma^5. \tag{A.2}$$

do not confuse the  $\hat{h}$  operator here with the Dirac Hamiltonian  $\hat{h}_D$  in the text. These operators satisfy the relations,

$$[\hat{M}, \hat{h}] = 0, \tag{A.3}$$

$$(\hat{M} + 2m)\hat{M} = a^2 - b^2 - m^2 - \hat{h}.$$

This means that the solutions of the equation of motion can be expressed in terms of the eigenvectors of  $\hat{h}$ .

By noticing that

$$\hat{h}^2 = 4[(a \cdot b)^2 - a^2 b^2], \tag{A.4}$$

the general dispersion relation is given by

$$(a^2 - b^2 - m^2)^2 - 4((a \cdot b)^2 - a^2 b^2) = 0, \tag{A.5}$$

or by Eq. (4) in terms of  $p$ . In the case  $b^\mu = 0$ , we have the simplified dispersion relation

$$a^2 - m^2 = 0. \tag{A.6}$$

Now, we will calculate the solutions of the equations of motion. As we pointed out above, we can find these solutions among the eigenvectors  $\psi_i$  satisfying

$$\hat{h}\psi_i = h_i\psi_i, \tag{A.7}$$

for the eigenvalues  $h_i$ . Then, let us find those eigenvectors. To do so, we notice that the  $\hat{h}$  operator can be written in terms of a rank two antisymmetric tensor,  $T_{\mu\nu} \equiv a_\mu b_\nu - a_\nu b_\mu$ , that is,

$$\hat{h} \equiv T_{\mu\nu} \varepsilon^{\mu\nu\sigma\rho} \mathcal{S}_{\sigma\rho}. \tag{A.8}$$

with  $\mathcal{S}_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$  and the convention  $\varepsilon^{0123} = 1$ .

From this tensor, we define two orthogonal three-vectors,

$$(\mathbf{u})^i \equiv T^{0i} = a^0(\mathbf{b})^i - b^0(\mathbf{a})^i \equiv u\hat{e}_1^i, \tag{A.9}$$

$$(\mathbf{v})^i \equiv \frac{1}{2}\varepsilon^{ijk}T_{jk} = (\mathbf{a} \times \mathbf{b})^i \equiv v\hat{e}_2^i, \tag{A.10}$$

and thus

$$(\mathbf{w})^i \equiv (\mathbf{u} \times \mathbf{v})^i \equiv uv\hat{e}_3^i, \tag{A.11}$$

where  $\hat{e}_1, \hat{e}_2$  and  $\hat{e}_3$  are three orthonormal space vectors on the direction of  $\mathbf{u}, \mathbf{v}$  and  $\mathbf{w}$ , respectively. The norm of these vectors are,

$$u = \sqrt{(a^0)^2(\mathbf{b})^2 + (b^0)^2(\mathbf{a})^2 - 2(a^0b^0)(\mathbf{a} \cdot \mathbf{b})}, \tag{A.12}$$

$$v = \sqrt{(\mathbf{a})^2(\mathbf{b})^2 - (\mathbf{a} \cdot \mathbf{b})^2}.$$

Note that

$$\begin{aligned} T^2 &= T_{\mu\nu}T^{\mu\nu} = 2(v^2 - u^2), \\ &= 2(a^2b^2 - (a \cdot b)^2) = -\frac{1}{2}\hat{h}^2. \end{aligned} \tag{A.13}$$

The negative values of  $T^2$  correspond to real eigenvalues for  $\hat{h}$  and the positive ones correspond to purely imaginary eigenvalues. By making use of the analogy with the electromagnetic tensor  $F$  we will call the  $T^2 < 0$  ‘‘electric’’ case and  $T^2 > 0$  the ‘‘magnetic’’ case.

Now, we define the rotation and boost generators in the spinor representation,

$$\mathcal{J}_i = \frac{1}{2}\varepsilon_{ijk}\mathcal{S}^{jk}, \quad \mathcal{K}_i = \mathcal{S}_{0i}, \tag{A.14}$$

where the spatial indices are referring to the  $e$  basis defined above. Then, the  $\hat{h}$  operator turns out to be

$$\hat{h} = -4(u\mathcal{J}_1 + v\mathcal{K}_2). \tag{A.15}$$

Performing a boost transformation on the eigenspinor in the  $\hat{e}_3$  direction

$$\psi_h = e^{-i\eta\mathcal{K}_3}\psi'_h, \tag{A.16}$$

the  $\hat{h}$  operator transforms as

$$\begin{aligned} \hat{h}' &\equiv e^{i\eta\mathcal{K}_3}\hat{h}e^{-i\eta\mathcal{K}_3} = -4[(u \cosh \eta - v \sinh \eta)\mathcal{J}_1 \\ &+ (v \cosh \eta - u \sinh \eta)\mathcal{K}_2]. \end{aligned} \tag{A.17}$$

Because  $-1 < \tanh \eta < 1$ , we can distinguish two cases. For  $u > v$  we set  $\tanh \eta = \frac{v}{u}$  so that

$$\hat{h}' = -4\sqrt{u^2 - v^2}\mathcal{J}_1. \tag{A.18}$$

However, for  $v > u$ , we can set  $\tanh \eta = \frac{u}{v}$  such that

$$\hat{h}' = -4\sqrt{v^2 - u^2}\mathcal{K}_2. \tag{A.19}$$

Since the eigenvalues of  $\mathcal{J}$  and  $\mathcal{K}$  are  $\pm\frac{1}{2}$  and  $\pm\frac{i}{2}$ , respectively, we have  $h_i = 2\varepsilon_i\sqrt{u^2 - v^2}$  for  $u > v$ , and  $h_i = 2i\varepsilon_i\sqrt{v^2 - u^2}$  for  $v > u$  as we expected. The convention here is  $\varepsilon_1 = +1$  and  $\varepsilon_2 = -1$ .

The eigenspinors in the chiral representation for  $u > v$  can be written as

$$\psi'_i = \begin{pmatrix} \alpha_i \xi_i \\ \beta_i \xi_i \end{pmatrix}, \tag{A.20}$$

with  $(\mathbf{u} \cdot \boldsymbol{\sigma})\xi_i = -\varepsilon_i u \xi_i$ .

Notice that these eigenvectors have the property, in the  $e$  basis,

$$\gamma^1\psi_i = \varepsilon_i\gamma^0\gamma^5\psi_i. \tag{A.21}$$

However, for  $v > u$ , the eigenspinors have the form,

$$\psi'_i = \begin{pmatrix} \gamma_i \chi_i \\ \delta_i \sigma^1 \chi_i \end{pmatrix}, \tag{A.22}$$

with  $(\mathbf{v} \cdot \boldsymbol{\sigma})\chi_i = \varepsilon_i v \chi_i$ . Similarly, the eigenspinors have the property, in the  $e$  basis,

$$\gamma^3\psi'_i = i\varepsilon_i\gamma^1\gamma^5\psi'_i. \tag{A.23}$$

The constants  $\alpha_i, \beta_i, \delta_i, \gamma_i$  reflect the fact that the eigenspinors are twofold degenerate.

Now, we are ready to find the solutions of the equations of motion in terms of the spinors  $\psi_i$ . Performing the same transformation on  $\hat{M}$  we obtain after some algebra:

In the electric case ( $u > v$ ), we can set, by choosing appropriately the parameter  $\eta$ ,  $a'_3 = b'_3 = 0$  and find

$$a'_0 = a_0\sqrt{1 - \frac{v^2}{u^2}} = a_0\frac{|h|}{2u}, \tag{A.24}$$

$$b'_0 = b_0\sqrt{1 - \frac{v^2}{u^2}} = b_0\frac{|h|}{2u}.$$

In the other hand, in the magnetic case ( $v > u$ ), we can set  $a'_0 = b'_0 = 0$  and find

$$\begin{aligned} a'_3 &= -a_0 \sqrt{\frac{v^2}{u^2} - 1} = a_0 \frac{|h|}{2u}, \\ b'_3 &= -b_0 \sqrt{\frac{v^2}{u^2} - 1} = b_0 \frac{|h|}{2u}. \end{aligned} \quad (\text{A.25})$$

where  $|h| \equiv \sqrt{|u^2 - v^2|}$  and where we have considered that the 2-direction is perpendicular to  $\mathbf{a}$  and  $\mathbf{b}$ . Hence, in the electric case the equations of motion,  $\hat{M}'\psi_i = 0$ , are

$$[(a'_0 - \varepsilon_i b_1)\gamma^0 - (b'_0 - \varepsilon_i a_1)\gamma^0\gamma^5 - m]\psi_i = 0, \quad (\text{A.26})$$

where we have used (A.21). This equation fixes the constants in Eq. (A.20)

$$\begin{aligned} \alpha_i &= \mathcal{N} m, \\ \beta_i &= \mathcal{N} [(a'_0 + \varepsilon_i b_1) - (b'_0 + \varepsilon_i a_1)], \end{aligned} \quad (\text{A.27})$$

where  $\mathcal{N}$  is a normalization constant. The equations of motion,  $M'\psi_i = 0$  in the magnetic case are

$$[(a_1 - i\varepsilon_i b'_3)\gamma^1 - (b_1 - i\varepsilon_i a'_3)\gamma^1\gamma^5 - m]\psi'_i = 0, \quad (\text{A.28})$$

where we have used property (A.23). This implies that the constants in Eq. (A.22) are

$$\begin{aligned} \gamma_i &= \mathcal{N}' m, \\ \delta_i &= \mathcal{N}' [(b_1 - i\varepsilon_i a'_3) - (a_1 - i\varepsilon_i b'_3)], \end{aligned} \quad (\text{A.29})$$

where  $\mathcal{N}'$  is another normalization constant.

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