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Bound states of massive complex ghosts in superrenormalizable quantum gravity theories

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Abstract: One of the remarkable differences between renormalizable quantum gravity with four-derivative action and its superrenormalizable polynomial generalizations is that the latter admit a more sophisticated particle mass spectrum. Already in the simplest superrenormalizable case, the theory has a six-derivative Lagrangian, admitting either a real or complex spectrum of masses. In the case of a real spectrum, there are the graviton, massive unphysical ghosts, and normal particles with masses exceeding the ones of the ghosts. It is also possible to have pairs of complex conjugate massive ghost-like particles. We show that in both cases, these theories do not admit a Källén-Lehmann representation and do not satisfy the positivity criterium of consistency in terms of the fields associated to those particles. In the main part of the work, using a relatively simple Euclidean scalar toy model, we show that the theory with complex spectrum forms bound states confining unphysical massive excitations into a normal composite particle. Finally, we discuss the cosmological implications of such a ghost confinement.

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Contents

1 Introduction

The main problem of renormalizable models of quantum gravity is the presence of massive unphysical ghosts and related instabilities. The simplest of such models is based on the fourth-derivative covariant action. Its particle spectrum includes a massless graviton, a tensor ghost, and a normal massive scalar particle [1]. The presence of higher derivatives and ghosts leads to instabilities at the classical [2] and quantum [3] levels and to the loss of quantum unitarity. Numerous attempts to resolve the contradiction between renormalizability and unitarity by taking into account loop corrections [4–7] were not conclusive [8]. The same concerns the low energy effective action of massless bosonic modes of (super)string theory where an infinite amount of fine-tuning is required [9] to eliminate pathological higher derivative terms of the effective action [10, 11]. Although, as remarked by Gross and Sloan [12], such a fine-tuning might not be necessary because the associated pathological ghosts would appear in the ultraviolet regime where string perturbation theory is not reliable, which is not in contradiction with the unitarity of string theory [13].

In any case it is clear that in the field theoretical approach to quantum gravity new ideas are required to formulate a consistent theory.

Extending the action of gravity by introducing additional terms with six derivatives makes the new theory superrenormalizable [14] and opens some new possibilities concerning the ghosts. For instance, in the case of a complex spectrum of ghost-like particles, it is possible to construct a simple realization of the proposal of [6] and prove the unitarity of the S-matrix of gravitational field [15] in the framework of the Lee-Wick quantization [16, 17]. Still, this cannot be a complete solution to the problem of ghosts and stability. There is a shortcoming of an approach based on Lee-Wick applied to gravity, which requires the conservation of energy in the closed (isolated) system of massive particles in the asymptotic states. In gravity, massive particles always create the gravitational field and, in this sense, the mentioned condition is difficult to fulfill, from the physical viewpoint.

It is clear that the main problem is the presence of ghosts and massive normal particles in the asymptotic states. It was suggested by Hawking in [18] that in the four-derivative gravity, the ghost should be considered together with a graviton. In the six-derivative model, this proposal can be simplified by requiring that the massive ghost forms a bound state with a normal particle of a larger mass, or that the two complex conjugate degrees of freedom form a bound state which is a normal (nor ghost neither tachyon) composite particle which is harmless for stability and unitarity. If that mechanism works it could imply a definitive resolution of the fundamental contradiction between renormalizability and unitarity.

Complex-mass particles are a common feature in QCD model studies, where they are thought to signal quark and gluon confinement; one of the first suggestions in this direction was made in [19]. In quantum gravity, this reasoning of fighting massive unphysical ghosts was discussed in recent papers [20–24] employing toy models inspired by those used in QCD. In the present work, we introduce a toy model that has a propagator closely related to one of the six-derivative quantum gravity. In this model, we perform a basic-level quantum calculation showing that the confinement of ghosts is possible for the sufficiently strong coupling. The toy model of our present interest describes scalar field with the Lagrangian possessing six derivatives. However, we shall explore this model keeping referring to the relation with the tensor and scalar ghosts in the six-derivative quantum gravity, as described in the next section. A pertinent point is that since such a theory is superrenormalizable, different from the fourth-derivative theory, there is no gauge-fixing ambiguity in the quantum corrections [14, 25], so there are a lot of similarities in the role of quantum contributions and one can claim the possible solution of the ghosts problem by using the QCD analogy. In particular, in both theories we are free to choose the model with such a spectrum of masses which is appropriate for our purposes.

We would like to remark that *ghost confinement* in the present paper does not mean permanent confinement, like quark and gluon confinement in QCD, rather it is used in the sense of complex-mass particles trapping into normal bound states. We also use the terminology *ghost condensate* to mean a bound state of ghosts.

The remaining of the paper is organized as follows. In section 2 we describe the basic derivation of the propagator of quantum metric in the six-derivative quantum gravity. In section 3 we discuss the action of the toy model with a similar ghost spectrum and reformulate its action in a useful form using auxiliary fields. In section 4 the Källén-Lehmann representation for the six-derivative model is analyzed and shown to give inconsistencies in the UV limit. Section 5 contains the main part of the paper, which is the derivation of the bound states of the ghost-like fields with the complex masses. In section 6 we describe the cosmological implications of ghost confinement, including the role of the Planck-order cut off on the cosmological perturbations and its fundamental importance for the stability of classical solutions of gravity. Also, we critically analyze the possibility of using the gas of the ghost-based bound states as a dark matter candidate. Finally, in section 7, we draw our conclusions. We use Euclidean notations in four-dimensional spacetime.

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2 Ghosts in six-derivative quantum gravity

Let us present a brief survey of the propagator in six-derivative quantum gravity model. More details can be found in [25, 26]. The action of the theory has the form

$$S_{gen} = \int d^4x\sqrt{g}\,\Big\{ -\frac{1}{\kappa^2}\,(R+2\Lambda) + \frac{1}{2}\,C_{\mu\nu\alpha\beta}\,\Phi(\Box)\,C_{\mu\nu\alpha\beta} + \frac{1}{2}\,R\,\Theta(\Box)\,R + \mathcal{O}(R^3_{\dots})\Big\}\,, \tag{2.1}$$

where $\kappa^2 = 16\pi G = 16\pi/M_P^2$, also $C_{\mu\nu\alpha\beta}$ is Weyl tensor, $\Phi(x)$ and $\Theta(x)$ are linear functions of the d'Alembert operator $\Box$ and $\mathcal{O}(R^3_{\dots})$ stands for the terms which are cubic in the curvature tensors (Riemann, Ricci and scalar curvature). Independent on the choice of these terms the theory is superrenormalizable, such that the divergences occur only at the first three loop orders [14]. Furthermore, the six-derivative terms (including the terms in $\mathcal{O}(R^3_{\dots})$ sector) are never renormalized and the fourth-derivative terms get divergences only from the first loop, Einstein's term only from the first and second loop and only the cosmological constant density gets renormalized in the third loop order.

The gauge fixing in six-derivative quantum gravity has to provide the non-degeneracy of the highest-derivative terms in the bilinear part of the action. This means the gauge fixing action has to be

$$S_{gf} \;=\; \frac{1}{2}\int d^4x\sqrt{g}\;\, \chi_\alpha Y^{\alpha\beta}\chi_\beta, \tag{2.2}$$

where

$$\begin{aligned} \chi_\alpha &= \partial_\lambda h^\lambda_\alpha - \beta\partial_\alpha h,\\ Y^{\alpha\beta} &= (\gamma_1\eta^{\alpha\beta}\partial^2 + \gamma_2\partial^\alpha\partial^\beta)\partial^2 + \gamma_3\partial^\alpha\partial^\beta + \gamma_4\eta^{\alpha\beta}\partial^2 + \gamma_5\eta^{\alpha\beta}. \end{aligned} \tag{2.3}$$

For the sake of simplicity, we use here the flat background metric $\eta_{\alpha\tau}$ and the simplest parametrization of quantum metric $g_{\alpha\tau} = \eta_{\alpha\tau} + h_{\alpha\tau}$. There are six arbitrary parameters of gauge fixing β and γ_k. A few more arbitrary parameters are possible for a more general parametrization of the quantum field. In principle, the dependence of all these parameters is an important issue at both the tree level and in the loop corrections. However, in the superrenormalizable gravity models this ambiguity is much less relevant [14, 25, 26]. For any choice of β and γ_k, the Faddeev-Popov gauge ghosts are massless vector fields. Since these fields have nothing to do with the massive unphysical ghosts, we shall not consider gauge ghosts in what follows.

The quantum metric field $h_{\mu\nu}$ can be split into irreducible parts as

$$h_{\mu\nu} \;=\; \bar{h}^{\perp\perp}_{\mu\nu} + \partial_\mu\epsilon^\perp_\nu + \partial_\nu\epsilon^\perp_\mu + \partial_\mu\partial_\nu\epsilon + \frac{1}{4}\,h\,\eta_{\mu\nu}. \tag{2.4}$$

The tensor component (i.e., the spin-2 mode) is traceless and transverse, i.e., $\bar{h}^{\perp\perp}_{\mu\nu}\eta^{\mu\nu} = 0$ and $\partial^\mu\bar{h}^{\perp\perp}_{\mu\nu} = 0$. It can be shown that the propagator of this part of the metric does not depend on the gauge fixing. The vector component (i.e., the spin-1 mode) satisfies $\partial_\mu\epsilon^{\perp\mu} = 0$ and is gauge dependent. There are also two scalar modes ϵ and h, one of them is gauge-dependent while another is invariant.

Finally, there are two physical (gauge-invariant) modes, tensor and scalar. In both cases, after the gauge fixing, the propagators include a massless mode and two massive modes, which can be real or complex. The propagation of the tensor mode is governed by the Weyl-squared term and depends on the form factor Φ. The propagation of the scalar mode depends only on the square of the scalar curvature and the function Θ.

It is customary to analyze the number of degrees of freedom, ghosts and related issues for a zero cosmological constant [1] that can be seen as working in the locally flat reference frame. Assuming zero cosmological constant, the action for both tensor and scalar modes $\psi = (\bar{h}^{\perp\perp}_{\mu\nu},\ h)$, on the flat background, has a general form

$$S_{inv} = \frac{1}{2}\int d^4x\ \psi(\theta_6\,\Box^3 + \theta_4\,\Box^2 + \theta_2\,\Box\,)\psi\,. \tag{2.5}$$

The mass spectrum in both tensor and scalar sectors depends on the corresponding coefficients θ_6, θ_4 and θ_2. In turn, these coefficients depend on the polynomials $\Phi(\Box)$ (tensor sector) and $\Theta(\Box)$ (scalar sector) in the action (2.1). Since the situation with the two fields is similar, we can restrict the analysis to one of them, that can be, e.g., the tensor mode.

The mass spectrum of the classical theory is defined by the poles in the tree-level propagator. The positions of these poles are found in the equations

$$p^2\left(p^4 - \frac{\theta_4}{\theta_6}\,p^2 + \frac{\theta_2}{\theta_6}\right) = 0. \tag{2.6}$$

It is easy to see that there is always a massless mode (e.g., the graviton in the tensor sector) and the two-dimensionful (massive) solutions. For these solutions, the two physically different options are as follows:

i) Two real positive solutions for the poles. In this case, the particle with a larger mass is a normal one and the lighter massive particle is a ghost. This rule of alternating signs is general for any polynomial gravity [14] with the real mass spectrum.

ii) Complex conjugate solutions. This case is known to provide unitary S-matrix in the framework of the Lee-Wick quantization [15, 27]. In what follows we see that there is one more distinguishing feature of this version of the theory — it admits, in principle, a quantum confinement of the complex ghost-like states.

According to the analysis of [11], if all the dimensional parameters of the six-derivative quantum gravity are proportional to the single massive parameter (usually assumed Planck mass), all the masses have Planck order of magnitude. In case of the complex spectrum this concerns the absolute values of the complex masses. Without a special fine tuning, the real and imaginary parts of these masses are of the same order of magnitude.

3 Starting action in equivalent second-order form

Consider the auxiliary fields representation of the six-derivative action of the form

$$\mathcal{S}_{6der} = \int d^4x\,\frac{1}{2}\,\psi\,(-\partial^2)(-\partial^2+m^2)(-\partial^2+m^{*2})\,\psi - U(\psi). \tag{3.1}$$

This action has one massless mode and two complex conjugate modes, in the sense m^* is a complex conjugate of m.

Let's start from the theory with three scalar fields, which has a similar particle contents,

$$\mathcal{S}_3 = \frac{i}{2}\int d^4x\, \varphi_1(-\partial^2+m^2)\varphi_1 - \frac{i}{2}\int d^4x\, \varphi_2(-\partial^2+m^{*2})\varphi_2 + \frac{1}{2}\int d^4x\, \varphi_3(-\partial^2)\varphi_3\,.$$

The propagators of the three fields $\varphi_{1,2,3}$, in the momentum representation are as follows:

$$\begin{aligned}
\langle\varphi_1\varphi_1\rangle &= \Big\langle \varphi_1\Big(-\frac{p}{2}\Big)\varphi_1\Big(\frac{p}{2}\Big)\Big\rangle = \frac{i}{p^2+m^2}\,,\\
\langle\varphi_2\varphi_2\rangle &= \Big\langle \varphi_2\Big(-\frac{p}{2}\Big)\varphi_2\Big(\frac{p}{2}\Big)\Big\rangle = -\frac{i}{p^2+m^{*2}}\,,\\
\langle\varphi_3\varphi_3\rangle &= \Big\langle \varphi_3\Big(-\frac{p}{2}\Big)\varphi_3\Big(\frac{p}{2}\Big)\Big\rangle = \frac{1}{p^2}\,.
\end{aligned} \tag{3.2}$$

On top of this, we assume that the three fields are independent and therefore $\langle\varphi_k\varphi_l\rangle \sim \delta_{kl}$. Now, let us make an assumption that a certain field Ψ and $\varphi_{1,2,3}$ satisfy the linear relation

$$\Psi = \varphi_3 + \alpha_1\varphi_1 + \alpha_2\varphi_2, \tag{3.3}$$

where α_1 and α_2 are unknown coefficients. Replacing (3.3) into the propagator of ψ, we get

$$\begin{aligned}
\langle\Psi\Psi\rangle &= \Big\langle \Psi\Big(-\frac{p}{2}\Big)\Psi\Big(\frac{p}{2}\Big)\Big\rangle = \langle\varphi_3\varphi_3\rangle + \alpha_1^2\langle\varphi_1\varphi_1\rangle + \alpha_2^2\langle\varphi_2\varphi_2\rangle\\
&= \frac{1}{p^2} - \frac{i\alpha_1^2}{p^2+m^2} - \frac{i\alpha_2^2}{p^2+m^{*2}} = \frac{Ap^4+Bp^2+C}{p^2(p^2+m^2)(p^2+m^{*2})}\,.
\end{aligned} \tag{3.4}$$

To have an agreement with the action (3.1), the numerator of the last expression should be a constant. Thus, we arrive at the following equations for α_1 and α_2:

$$\begin{aligned}
A &= -i\alpha_1^2 + i\alpha_2^2 + 1 = 0,\\
B &= -i\alpha_1^2 m^{*2} + i\alpha_2^2 m^2 + m^{*2} + m^2 = 0,
\end{aligned} \tag{3.5}$$

while $C = |m|^4$ independent on the values of $\alpha_{1,2}$. The solutions of (3.5) can be easily found in the form

$$\alpha_1^2 = \frac{-im^{*2}}{m^{*2}-m^2}, \qquad \alpha_2^2 = \frac{im^2}{m^2-m^{*2}} = (\alpha_1^2)^*. \tag{3.6}$$

Using these solutions, we get the propagator of the field Ψ

$$\langle\Psi\Psi\rangle = \frac{|m|^4}{p^2(p^2+m^2)(p^2+m^{*2})}\,. \tag{3.7}$$

This corresponds to the action

$$\mathcal{S}_{6der} = \frac{1}{2|m|^4}\int d^4x\, \Psi\,(-\partial^2)(-\partial^2+m^2)(-\partial^2+m^{*2})\,\Psi\,, \tag{3.8}$$

that boils down to (3.1) after a constant reparametrization of the field $\Psi = |m|^2\psi$. On the other hand, (3.8) can be recognized as a particular version of the six-derivative action of the gauge-invariant modes (2.5) that emerge in the model of superrenormalizable quantum gravity with linear functions Φ and Θ.

In what follows, our purpose is to show that the confinement of the massive modes φ_1 and φ_2 is possible for the case of a complex mass spectrum. To arrive at this result, we will try to simplify things as much as possible. In particular, in the analysis of confinement we omit the massless field φ_3 and also replace the non-polynomial interactions typical for gravity, to the particular quartic interaction of the complex ghost-like fields φ_1 and φ_2. It is clear from the relation (3.3) that quartic interaction ψ^4 contains various quartic interactions of the fields φ_1 and φ_2, but we include only some of them which are relevant for our purposes.

One of the effects of the representation (3.3) and the last reparametrization between Ψ and ψ is that the fields $\varphi_{1,2,3}$ and the six-derivative field ψ have different dimensions. In particular, the fields ϕ_1 and ϕ_1 have usual mass dimension $+1$ and the field ψ has the mass dimension -2. Consequently, the coupling of the quartic interaction between the ghosts can be dimensionless while in the six-derivative theory it is dimensionful. A similar situation holds in the gravitational case, where the reparametrization of the fields may change the dimension of the coupling constants.

Let us also present an equivalent consideration based on the Gaussian integral. As a starting action, consider the action $\mathcal{S}_3$ from (3) and define the common generating functional for the three free fields,

$$\mathcal{Z}(J) = \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 \mathcal{D}\varphi_3 \, \exp\Big\{ - \mathcal{S}_3 + (\alpha_1\varphi_1 + \alpha_2\varphi_2 + \varphi_3) J \Big\}. \tag{3.9}$$

Taking the Gaussian integrals over $\varphi_{1,2,3}$, we get

$$\begin{aligned} \mathcal{Z}(J) = & \Big[\text{Det}\, (-\partial^2)(-\partial^2 + m^2)(-\partial^2 + m^{*2}) \Big]^{-1/2} \\ & \times \, \exp\Big\{ \frac{1}{2} \int d^4x \, J \Big[- i\alpha_1^2(-\partial^2 + m^2)^{-1} + i\alpha_2^2(-\partial^2 + m^{*2})^{-1} + (-\partial^2)^{-1} \Big] \, J \Big\}. \end{aligned} \tag{3.10}$$

In the momentum representation, the expression in the square brackets in the exponential is reproducing (3.4) and, using the coefficients (3.6), we arrive at

$$\mathcal{Z}(J) = \Big[\text{Det}\, (-\partial^2)(-\partial^2 + m^2)(-\partial^2 + m^{*2}) \Big]^{-1/2} \, \exp\Big\{ \frac{1}{2} \int d^4x \, JH^{-1}J \Big\}, \tag{3.11}$$

$$\text{where} \quad H = \frac{1}{m^2 m^{*2}} (-\partial^2 + m^2)(-\partial^2 + m^{*2})(-\partial^2). \tag{3.12}$$

It is easy to recognize that (3.11) is a Gaussian integral

$$\mathcal{Z}(J) = \int \mathcal{D}\Psi \, \exp\Big\{ - \frac{1}{2} \int d^4x \, \Psi H \Psi \, + \, \Psi J \Big\}.$$

The last expression shows that we arrived at the action (3.8), which is equivalent to (3.1). The standard interpretation of this result is that the theory with six derivatives has either a massive ghost and a massive normal particle [14] or a couple of complex conjugate ghost-like states [15]. The question is whether it is possible to avoid fatal instabilities in such a theory at the classical or quantum levels. In the next section, we consider the complex spectrum case in more detail.

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4 Ghosts and Källén-Lehmann representation

As we already discussed in the Introduction, the quantum gravity theories based on higher derivative Lagrangians may be renormalizable [1], superrenormalizable [14, 28, 29] or even finite [27, 30]. On the other hand, these theories may be unstable at the classical level owing to the Ostrogradski instabilities [2]. From the quantum perspective, there are ghost fluctuations that usually lead to instabilities [3] in the form of negative norm states, violations of the fundamental unitary and causal properties [1, 31–34].

Since our present purpose is to describe the possible scheme of confining the ghosts it is worthwhile to stress that there is no another perspective to explain how the problem of higher derivative ghosts can be solved. In this respect, let us add one more argument concerning the inconsistency of the unconfined theory with higher derivatives. There are many ways of pointing out the conflict of higher derivative gravity with fundamental principles of quantum field theory. An efficient approach is showing that these theories do not admit the Källén-Lehmann representation

$$\widehat{S}_2(p) \,=\, \int_0^\infty d\mu \, \frac{\rho(\mu)}{p^2+\mu^2}, \quad \text{with} \quad \rho(\mu) \geqslant 0 \tag{4.1}$$

of the two-point Schwinger function $\widehat{S}_2(p^2)$ whose existence follows immediately from the first principles. A useful necessary condition of functions $\widehat{S}_2$ that have a Källén-Lehmann representation is given by the inequality [35–38]

$$\frac{d}{dp^2}\, p^2 S_2(p) \;\; > 0\,. \tag{4.2}$$

It can be shown that the theory with four- or higher-derivative Lagrangian cannot satisfy this inequality [39]. Let us apply the first of this condition to our six-derivative model. In this case,

$$S_2(p^2) \;=\; \frac{1}{p^2\,(p^2+m^2)\,(p^2+m^{*2})}\;. \tag{4.3}$$

Independent of the magnitude of the mass m, in the UV we have $p^2 \gg |m|^2$. Therefore, at high energies, $S_2(p^2) \;\approx\; 1/p^6$ and we arrive at the estimate

$$\frac{d}{dp^2}\, p^2\, S_2(p^2) \;\approx\; -\,\frac{2}{p^6} \;\leqslant\; 0. \tag{4.4}$$

Thus, the inequality (4.2) is violated in a theory where the propagator behaves as p^{-6} in the UV. This signals the quantum inconsistency of the theories with six derivatives if these theories are treated in a usual perturbative way. The consideration presented above can be extended to the case of real poles and other higher derivative models, where there is a generalization of Källén-Lehmann representation [40]. This extension will be discussed elsewhere, while here we consider only the six-derivative models.

Different from the UV, quantum gravity models with higher derivatives may be consistent in the IR, where the higher derivative terms are regarded as small corrections to GR by definition [41, 42] or if there are imposed restrictions on the frequencies of the quantum perturbations [43, 44] by imposing a Planck-order cut-off. The pertinent question is what can be the origin of such a UV cut-off. In the next section, we argue that the corresponding mechanism could be a confinement of complex massive ghost-like states that emerge in the six or more-derivative quantum gravity with the complex mass spectrum.

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5 Bound states with ghost-like complex massive particles

Consider the following six-derivative Euclidean Lagrangian for the field ψ

$$\mathcal{L} = \frac{1}{2}\,\psi\big[-\partial^2\big(-\partial^2+m^2\big)\big(-\partial^2+m^{*2}\big)\big]\psi - U(\psi), \tag{5.1}$$

where $m^2 = m_R^2 + i m_I^2$ is a complex squared mass and $U(\psi)$ is for now unspecified potential. This potential is the main difference to the free model (3.1) discussed above.

The momentum-space free propagator $D_\psi(p)$ of the ψ-field is given by:

$$D_\psi(p) = \frac{1}{p^2\,(p^2+m^2)\,(p^2+m^{*2})}\,. \tag{5.2}$$

Using the results of section 3, this propagator can be split as a sum of three propagators:

$$D_\psi(p) = \frac{1}{|m|^4}\frac{1}{p^2} - \frac{iA}{p^2+m^2} + \frac{iA^*}{p^2+m^{*2}} \qquad \text{with} \qquad A = \frac{1}{|m|^4}\frac{m^2}{2m_I^2}\,. \tag{5.3}$$

As we know, the Lagrangian in (5.1) is equivalent to the following one:

$$\mathcal{L} = \frac{1}{2}\phi\big(-\partial^2\big)\phi + \mathcal{L}_{\text{gh}}, \tag{5.4}$$

where the first term leads to the first term in (5.3) (after absorbing the $|m|^4$ in the field ϕ) and $\mathcal{L}_{\text{gh}}$ is the Lagrangian of two ghost fields φ_1 and φ_2 (that absorb the A and A^*) that leads to the other two propagators in (5.3):

$$\mathcal{L}_{\text{gh}} = \frac{1}{2}\varphi_1\big[i\,\big(-\partial^2+m^2\big)\big]\varphi_1 + \frac{1}{2}\varphi_2\big[-i\,\big(-\partial^2+m^{*2}\big)\big]\varphi_2 - U(\varphi_1,\varphi_2)\,, \tag{5.5}$$

where $U(\varphi_1,\varphi_2)$ is a Hermitian potential that in principle is related to the $U(\psi)$ potential. The fact that $\mathcal{L}_{\text{gh}}$ leads to a complex Euclidean action[1] should not be of concern since the generating functional

$$Z_{\text{gh}} = \int \mathcal{D}\varphi_1\mathcal{D}\varphi_2\, e^{-S_{\text{gh}}} \qquad \text{where} \qquad S_{\text{gh}} = \int d^4x\, \mathcal{L}_{\text{gh}} \tag{5.6}$$

is real because φ_1 and φ_2 are dummy integration variables.

Let us start with the survey of the dimensions of field and couplings. As we mentioned above, the dimensions of the fields get modified when using auxiliary fields. Consider this in detail. The field ψ in (5.1) has mass-dimension $[\psi] = -1$. Consider a possible interaction term in $U(\psi)$

$$U(\psi) = \frac{\lambda}{4!}\,\psi^4, \qquad \text{hence} \qquad [\lambda] = 8, \tag{5.7}$$

[1]A complex Euclidean action occurs e.g. in QCD, when one needs to consider a baryon chemical potential μ_B to treat high-density nuclear and quark matter. The chemical potential contributes with a complex term to the fermionic part of the QCD action, namely $-i\int d^4x\,\Psi^\dagger\mu_B\Psi$, where Ψ is the quark field. A complex Euclidean action leads to what is known as the *sign problem*; it obstructs the use of the Monte Carlo method, which is the basis of lattice QCD simulations. This obstruction is the main reason for the lack of progress in the knowledge on the properties of the matter in the interior of neutron stars.

wheras the dimensions of the fields φ_i are $[\varphi_i] = 1$ To investigate the possibility that the ghost fields φ_1 and φ_2 form two-particle bound states, we make (perhaps) the simplest possible computation. Let $O_{\varphi_1\varphi_2}(x)$ be the scalar composite operator (correlator)

$$O_{\varphi_1\varphi_2}(x) = \varphi_1(x)\varphi_2(x). \tag{5.8}$$

Then, we consider the correlation function $G(x, y)$

$$\begin{aligned} C(x,y) = C(x-y) &= \langle O_{\varphi_1\varphi_2}(x)\, O_{\varphi_1\varphi_2}(y)\rangle \\ &= \frac{1}{Z_{\text{gh}}} \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2\, O_{\varphi_1\varphi_2}(x)\, O_{\varphi_1\varphi_2}(y)\, e^{-S_{\text{gh}}}. \end{aligned} \tag{5.9}$$

To compute $C(x-y)$, we need to specify $U(\varphi_1, \varphi_2)$. We proceed within an effective field theory (EFT) perspective, i.e., consider all possible interactions with coefficients to be adjusted phenomenologically. The lowest-order terms are the renormalizable ones, namely

$$U(\varphi_1, \varphi_2) \;=\; \frac{1}{4!}\lambda\varphi_1^4 + \frac{1}{4!}\lambda\varphi_2^4 + \frac{1}{4}\lambda_{12}\varphi_1^2\varphi_2^2. \tag{5.10}$$

where $\lambda_{12} = \lambda/3$. We then compute C in perturbation theory; the $\mathcal{O}(\lambda_{12})$ one-loop contribution, $C^{1-\text{loop}}$, is given by

$$\begin{aligned} C^{1-\text{loop}}(x,y) \;=&\; D_{\varphi_1}(x-y)D_{\varphi_2}(x-y) \\ &+ \lambda_{12} \int d^4z\, D_{\varphi_1}(x-z)D_{\varphi_2}(x-z)D_{\varphi_1}(z-y)D_{\varphi_2}(z-y), \end{aligned} \tag{5.11}$$

where D_{φ_1} and D_{φ_1} are the ghost-field free propagators. In momentum space, they are given by

$$D_{\varphi_i}(x-y) \;=\; \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, D_{\varphi_i}(p), \qquad i = 1, 2\,, \tag{5.12}$$

where

$$D_{\varphi_1}(p) = \frac{i}{p^2 + m^2} \qquad \text{and} \qquad D_{\varphi_2}(p) = \frac{-\,i}{p^2 + m^{*2}}\,. \tag{5.13}$$

Note that the factors i and $-i$ in these propagators come from the i and $-i$ multiplying respectively $(\,-\,\partial^2 + m^2)$ and $(\,-\,\partial^2 + m^{*2})$ in (5.5).

Consider the first term in (5.11):

$$\begin{aligned} D_{\varphi_1}(x-y)D_{\varphi_2}(x-y) &= \int \frac{d^4k'}{(2\pi)^4} \frac{d^4k}{(2\pi)^4}\, e^{-ik'\cdot(x-y)-ik\cdot(x-y)}\, D_{\varphi_1}(k')\, D_{\varphi_2}(k) \\ &= \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)} \int \frac{d^4k}{(2\pi)^4}\, D_{\varphi_1}(p-k)\, D_{\varphi_2}(k) \\ &= \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, G_B(p)\,, \end{aligned} \tag{5.14}$$

where $G_B(p)$ is the *bubble integral*

$$G_B(p) \;=\; \int \frac{d^4k}{(2\pi)^4}\, D_{\varphi_1}(p-k)\, D_{\varphi_2}(k). \tag{5.15}$$

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Using the result (5.14) twice and integrating over z, one can show that the second term in (5.11) can be written as:

$$\int d^4z\, D_{\varphi_1}(x-z)D_{\varphi_2}(x-z)D_{\varphi_1}(z-y)D_{\varphi_2}(z-y)$$
$$= \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, G_B(p)\, G_B(p). \tag{5.16}$$

Next, defining the Fourier transform $C(p)$ of $C(x-y)$ through

$$C(x-y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, C(p), \tag{5.17}$$

one can then write (5.11) in momentum space as[2]

$$C(p) = G_B(p) + G_B(p)\big[\lambda_{12}\, G_B(p)\big]. \tag{5.18}$$

One can now iterate this one-loop result to obtain a Dyson's type of equation,

$$\begin{aligned} C(p) &= G_B(p) + G_B(p)\big[\lambda_{12}\, G_B(p)\big] + G_B(p)\big[\lambda_{12}\, G_B(p)\, \lambda_{12}\, G_B(p)\big] \\ &\quad + G_B(p)\big[\lambda_{12}\, G_B(p)\, \lambda_{12}\, G_B(p)\, \lambda_{12}\, G_B(p)\big] + \cdots \\ &= G_B(p) \sum_{n=0}^{\infty} \big[\lambda_{12}\, G_B(p)\big]^n \;=\; \frac{G_B(p)}{1-\lambda_{12}\, G_B(p)}\,. \end{aligned} \tag{5.19}$$

There is a physical bound state if $C(p)$ has a pole at a value $p^2 = -\mathcal{M}^2$ and the residue at the pole is positive. There will be a pole at $p^2 = -\mathcal{M}^2$ when

$$1 - \lambda_{12}\, G_B(p)\Big|_{p^2=-\mathcal{M}^2} = 0. \tag{5.20}$$

If there is a pole, there will exist a critical value of the coupling λ_{12} where the pole first appears. Let us compute the one-loop integral by using spherical coordinates and assuming that we have a pair of complex masses $m^2 = (1+i)\mu^2$ and $m^{*2} = (1-i)\mu^2$. We get

$$\begin{aligned} G_B(p) \;=\; \frac{1}{4(2\pi)^2\, p^2} \int_0^{\infty} dk\, k\, \Big\{(1-i)\mu^2 + k^2\Big\}^{-1} \Big\{(1+i)\mu^2 + k^2 + p^2 \\ - \sqrt{\big[(1+i)\mu^2 + (k+p)^2\big]\,\big[(1+i)\mu^2 + (k-p)^2\big]}\Big\}. \end{aligned} \tag{5.21}$$

The last integral is logarithmically divergent, but this divergence can be easily renormalized by momentum substraction

$$G_B^R(p) \;=\; G_B(p) \;-\; G_B(p_0). \tag{5.22}$$

The result for $1 - \lambda_{12} G_B^R(ip)$ is displayed in figure 1 once the substation point has been fixed at $p_0^2 = 1$ with unit mass scale $\mu = 1$ and coupling constant $\lambda_{12} = \pi^5$. The presence of a pole at $\mathcal{M}^2 = 1.56$ supports the existence of a bound ghost state under these conditions.

[2]To lighten the notation we omit the index $1-$ loop in $C^{1-\text{loop}}(p)$.

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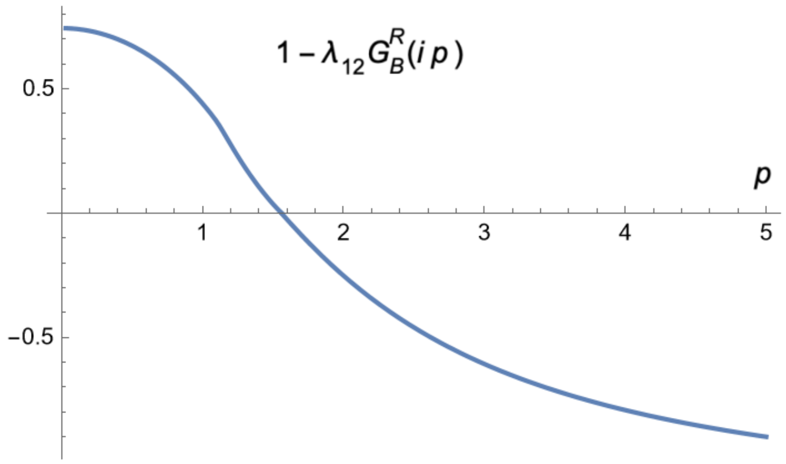

Figure 1. The denominator in the right hand side of eq. (5.19) as a function of $-i\mathcal{M} = p$, where we have chosen $\lambda_{12} = \pi^5$ and $\mu^2 = 1$.

The mass of the bound state $\mathcal{M}$ is the solution of (5.20). As mentioned, to be a physical pole, its residue should be positive. To obtain the residue, we expand $C(p)$ around the point $p^2 = -\mathcal{M}^2$ to get

$$\begin{aligned} C(p)\Big|_{p^2\approx-\mathcal{M}^2} &= \frac{G_B^R(-\mathcal{M}^2)+\cdots}{1-\lambda_{12}\,G_B^R(-\mathcal{M}^2)-(p^2+\mathcal{M}^2)\,\lambda_{12}\,G_B^{R\prime}(-\mathcal{M}^2)+\cdots} \\ &= \frac{R_G^R}{p^2+\mathcal{M}^2}+\cdots\,, \end{aligned} \tag{5.23}$$

with the residue R_G^R given by

$$R_G^R = -\frac{1}{\lambda_{12}^2\,G_B^{R\prime}(-\mathcal{M}^2)}\,. \tag{5.24}$$

For a positive R_G, one must have $G_B^{R\prime}(-\mathcal{M}^2) < 0$.

Now, from the behavior of the two-point function displayed in figure 1 it follows that the residue at the pole $\mathcal{M} = 1.56$ is positive $R_G > 0$. Thus, it corresponds to a physical pole and marks the appearance of a ghosts condensate.

The existence of a physical solution with a bound condensate of ghosts depends on the value of the complex masses of the pair of ghosts $(1 \pm i)\mu$, the coupling constant of the model λ_{12} and the renormalization momentum substraction point p_0. But in our case, we have shown that there is a window of coupling constants where the condensation of ghosts is possible. The critical values of the coupling constant $\lambda_{12}^{\pm}$ enclosing such a window are $\lambda_{12}^- = 0.68\pi^5$ and $\lambda_{12}^+ = 3.91\pi^5$, i.e. only if $\lambda_{12}^- \le \lambda_{12} \le \lambda_{12}^+$ the condensation occurs.

Our analysis indicates that there is a possibility to form bound states from the pair of complex conjugate massive ghosts. Similar results were obtained when using the $\overline{\text{MS}}$ renormalization scheme. We skip the details of these calculations, since the results are equivalent.

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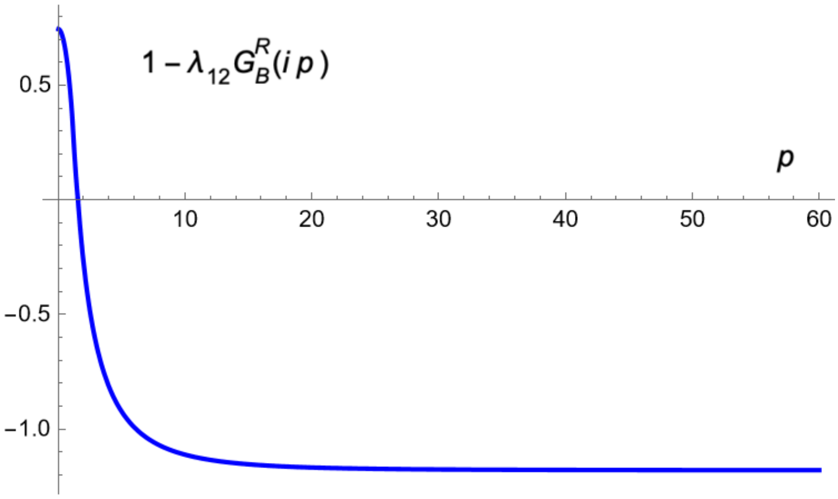

Figure 2. The analytic continuation to imaginary values of p of denominator of the right hand side of eq. (5.19). Here, we have chosen $\lambda_{12} = \pi^5$ and $\mu^2 = 1$.

It is important to note that the possibility of having a second pole with a negative residue, which would correspond to a composite ghost, can be discarded because the function $1 - \lambda_{12} G_B^R(-p^2)$, as can be seen in figures 1 and 2, is a monotonically decreasing function of p, reaching an asymptotic value in the $p^2 \to \infty$ limit:

$$1 - \lambda_{12} G_B^R(-p^2) = -1.18 + \mathcal{O}(1/p^2), \tag{5.25}$$

which is almost saturated at maximum momentum displayed in figure 2.

One can further substantiate the possibility of forming bound states out of a pair of complex conjugate massive ghosts by examining an alternative scalar composite field operator, for example:

$$\widetilde{O}_{\varphi_1\varphi_2}(x) = \varphi_1(x)^2 + \varphi_2(x)^2. \tag{5.26}$$

This operator has the same quantum numbers of $O_{\varphi_1\varphi_2}$ and, as such, is equally suited to study the existence of bound states, as would any other composite operator with the same quantum numbers [45]. An explicit calculation, at the same level of approximation as in the previous calculation, shows that the correlation function

$$\widetilde{C}_B(x,y) = \langle \widetilde{O}_{\varphi_1\varphi_2}(x) \widetilde{O}_{\varphi_1\varphi_2}(y) \rangle, \tag{5.27}$$

also shows the presence of a single real pole in the corresponding renormalized correlator:

$$\widetilde{C}_B^R(p^2) = \frac{\widetilde{G}_B^R(p)}{1 - \frac{\lambda}{2} \widetilde{G}_B^R(p)}, \tag{5.28}$$

where $\widetilde{G}_B^R(p) = \widetilde{G}_B(p) - \widetilde{G}_B(1)$ with $\widetilde{G}_B(p) = \widetilde{G}_B^+(p) + \widetilde{G}_B^-(p)$ and

$$\begin{aligned} \widetilde{G}_B^{\pm}(p) = -\frac{1}{4(2\pi)^2 p^2} \int_0^\infty dk\, k \left\{(1 \pm i)\mu^2 + k^2\right\}^{-1} & \Big\{(1 \pm i)\mu^2 + k^2 + p^2 \\ & - \sqrt{\left[(1 \pm i)\mu^2 + (k+p)^2\right]\left[(1 \pm i)\mu^2 + (k-p)^2\right]}\Big\}. \end{aligned} \tag{5.29}$$

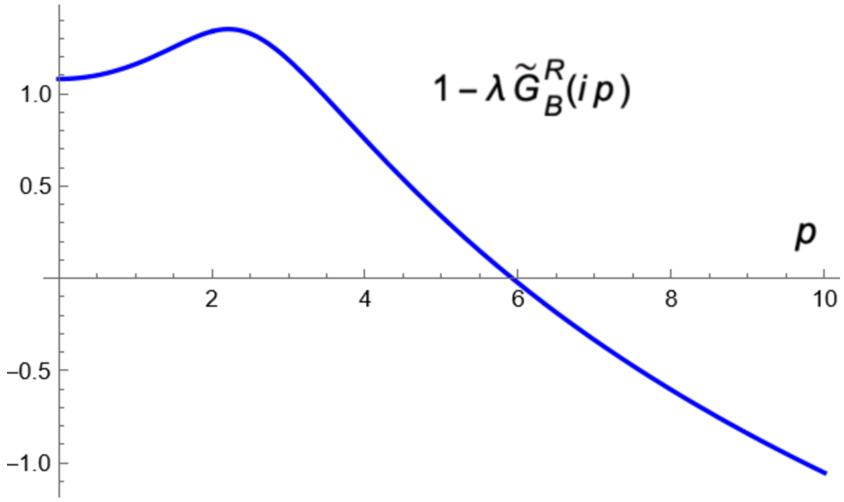

Figure 3. $1-\lambda\widetilde{G}_B^R(ip)$ as a function of p, where we have chosen $\mu^2 = 1$ and $\lambda = \pi^5/4$.

Indeed, the denominator $1-\lambda\widetilde{G}_B^R(ip)$ has a single zero at $\mathcal{M} = 5.92$ with a positive residue $R_{\widetilde{G}} > 0$, as one can see in figure 3. The window of coupling constants λ for the occurrence of the bound state is unbounded, $0 < \lambda < \infty$. This is due to the asymptotic logarithm behavior of $\widetilde{G}_B^R(ip) = 0.031 + (4/\pi^2 + \mathcal{O}(1/p^2))\log p$ in the $p^2 \to \infty$ limit. This different coupling dependence from the previous example is expected, as both the compoiste operators and the coupling are scale dependent (they run with the renormaliation scale), but the pole position should be scale independent. The study of such features is out of the scope of this initial study and is reserved for a future publication.

The above results give a real hope to observe a similar effect in the superrenormalizable quantum gravity model with six derivatives, and probably also in the cases of $2+2n$ derivatives, where $n = 3, 4, \ldots$, because these models admit mass spectrum consisting from complex conjugate pairs of poles plus the graviton.

6 Cosmological implications of ghost confinement

It would be interesting to extend the scheme of confinement of ghosts to the real quantum gravity and arrive at the definite answer concerning the bound states, by using the methods borrowed from nonperturbative QCD. However, this will not be an easy task because even in QCD, definite analytic results concerning confinement of quarks are difficult to obtain. On the other hand, it is interesting to speculate about possible observable manifestations of physical bound states of complex mass ghosts. Since the mass of such a composite particle and the corresponding cutoff on the energy of the original gravitational modes are of the Planck order of magnitude, such observations can belong only to early cosmology, where we meet a growing amount and quality of available data.

Is it possible to find cosmological traces of the confinement of the ghost-like complex modes? Thinking about particle physics and accelerator experiments, the answer is negative because the masses of the bound states are of the Planck order of magnitude, far beyond the scale of the present or future particle physics. An additional feature is that the composite particles do not bring any charges except the mass and, therefore, do not engage in any interactions except gravitational.

The situation is different in the case of early cosmology or black hole perturbations of the metric or (in the cosmological case) density. The first effect of ghost confinement is imposing a Planck cut-off on the energy of the gravitational perturbations. For the sake of definiteness, consider the early cosmology. Looking back in time, as far as a cosmic perturbation becomes trans-Planckian, the pair of complex conjugate ghosts is created and gets confined into a bound state. This situation rules out the observation of the trans-Planckian physics in both cosmology (see e.g., [46–48]) and black hole evaporation (see, e.g., [49]). The Planck-order cut-off in the case of cosmological perturbations may be, in principle, detected by future observational facilities.

It is important to note that the same happens if the gravitons concentrate in a locality with a Planck-order density of energy. Such a concentration is a necessary element of creating a pair of massive complex ghosts from the vacuum forming the bound states rule out the instabilities created by massive ghosts, as discussed in [43, 44]. All in all, the definite answer about forming the bound state of complex ghosts would resolve the contradiction between renormalizability and unitarity in quantum gravity.

Finally, looking forward in time from the Planck-energy epoch in cosmology, what could be the possible effect of the bound states in the later epochs? For example, are these particles, with the masses of the Planck order of magnitude, which interact only gravitationally, realistic candidates to be Dark Matter? To address this question, let us make a numerical estimate. The first thing to note is that the complex ghosts bound states are created at the Planck energy scale, i.e., at energies much higher than the ones typical for inflation.

As in the rest of this paper, we follow the simplest possible approach. At the moment of creating the bound states, we assume that the energy scale is $\mathcal{E}_{in} = M_P$ and the initial energy density of these composite particles is $\rho_{in}^{BS}(M_P) = M_P^4$. Consider that right after this point the inflation starts. This means that we are making a too optimistic estimate because the bound states could be dissolved between the Planck epoch and the beginning of inflation. However, since this point does not change the result, we can use it as a simplification. Now, we evaluate the evolution of the density of bound states at the energy $\mathcal{E} \sim 1/a(t)$, corresponding to the conformal factor $a(t)$ of the cosmological expansion. The described conditions produce the result

$$\rho^{BS}(\mathcal{E}) \;\propto\; \rho_{in}^{BS}\left(\frac{\mathcal{E}}{\mathcal{E}_{in}}\right)^3 \;=\; M_P^4\left(\frac{a_{in}}{a}\right)^3. \tag{6.1}$$

For the density of bound states at the end of N e-folds inflation, we get

$$\rho^{BS}(\mathcal{E}_{\text{end}}) \;\propto\; M_P^4\, e^{-3N}. \tag{6.2}$$

For the critical density at the same epoch, one can use the Friedmann equation

$$\rho_c(\mathcal{E}_{\text{end}}) \;=\; \frac{3}{8\pi G}\, H_{\text{end}}^2 \;=\; \frac{3M_P^2}{8\pi}\, H_{\text{end}}^2. \tag{6.3}$$

Using the standard estimate $H_{\text{end}} \approx 10^{12}\,\text{GeV}$ for the Hubble parameter at the end of inflation, taking the number of e-folds $N = 70$ and $M_p = 10^{19}\,\text{GeV}$, we get

$$\rho^{BS}(\mathcal{E}_{\text{end}}) \;\propto\; \rho_c(\mathcal{E}_{\text{end}}) \,\times\, 10^{-78}\,. \tag{6.4}$$

After the end of inflation, the density of bound states continues dissolving, however in the early Universe, the rest of the matter may show faster decrease of critical density. Then, the ratio with the critical density may loose up to 25 orders of magnitude, but the hierarchy between this density and density of the ghost's bound states still remains very strong.

This result implies that there is no chance to regard the gas of the Planck mass-bound states of complex ghosts as a Dark Matter candidate. The point is that this gas is generated too early and expands too much after the Planck-energy epoch.

7 Conclusions

We have analyzed the toy model with a pair of complex conjugate massive unphysical ghost-like states, i.e., the mass spectrum which is typical for the six-derivative superrenormalizable quantum gravity. It was shown that this theory may describe the confinement of the ghost states if the coupling constant is large enough. This condition can be fulfilled in the models of quantum gravity by either choosing appropriate parameters in the action (2.1) or by describing the force between the two ghosts by exchanging gravitons.

The bound states of the massive complex ghost-like particles form the Planck-density gas in the early Universe. The subsequent expansion between the Planck scale and the end of inflation strongly dissolves this gas, such that it cannot be even a small part of the Dark Matter. Thus, the unique potentially observable consequence of the ghost confinement is the cut-off of the Planck order of magnitude to the energy density of gravitons. The two relevant aspects of this quantum-origin cut-off are the non-existence of the trans-Planckian effects on cosmic perturbations [46–48] and, perhaps the most important, the explanation of why there are no Planck-order frequencies in the initial seed of cosmic perturbations. The last issue is a cornerstone in explaining the stability of classical cosmological solutions in the presence of higher derivatives [43, 44]. We note that there is a significant interest in discussing other physical outputs of a Planck-scale cut-off on the energy density of gravitons (see, e.g., [50, 51]). The confinement of ghosts which we found in the toy model of superrenormalizable quantum gravity, may offer a solid background to such a cut-off.

Looking beyond the scope of the present work, further explanation of the subject requires elaborating models to quantum gravity and, on the other hand, making more complete analyses of the formation of bound states. We hope to advance in these directions in future works.

In particular, there are two aspects of the described bound states that were not discussed in the present paper but will be reported in a separate future work. The first one concerns the existence of the Källén-Lehmann representation for the bound states. This representation is possible and can be demonstrated in ways similar to what was done in QCD [52–54]. Since these proofs are technically elaborated, we leave such an analysis for a separate study. The second issue concerns the six-derivative theory with a real mass spectrum. As mentioned above, such a theory has a massive ghost and a normal particle with a mass exceeding the mass of the ghost. The preliminary answer regarding bound states in this case is negative, but it was obtained with the same model of self-interaction between ghosts that was used in the complex case. We also leave this issue for future work.

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