| Journal Code: | | | | Article ID | | | | Dispatch: 16.06.13 | CE: Mary Edylize F. Denosta |
|---|---|---|---|---|---|---|---|---|---|
| O | C | A | | 2 | 0 | 7 | 8 | No. of Pages: 11 | ME: |

OPTIMAL CONTROL APPLICATIONS AND METHODS
*Optim. Control Appl. Meth.* (2013)
Published online in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/oca.2078

# Education and age at migration

Rosa Aisa, Josefina Cabeza and Gemma Larramona*,†

*Universidad de Zaragoza, Gran Vía, 2, 50005 Zaragoza, Spain*



SUMMARY

The population of the developed world is growing older, leading to shortages in the labor market, and the age and level of education of migrants will be important factors in the labor market in the coming decades. This paper presents a theoretical framework in which the timing of migration and the educational attainments of immigrants are determined endogenously. The combination of social and economic factors related to migration allows us to detect different patterns in these variables. The main finding is that the optimal age for migration, in the case of individuals who attain a higher educational level, is lower than those who have a lower level of education. Copyright © 2013 John Wiley & Sons, Ltd.

Received 2 August 2012; Revised 27 May 2013; Accepted 29 May 2013

KEY WORDS: age at migration; three-stage optimal control; human capital

## 1. INTRODUCTION

The bulk of the literature on immigration from an economic perspective has concentrated on the effects on welfare, with a view to formulating immigration policies aimed at facilitating assimilation. The optimum approach to policy design may be to ascertain certain key characteristics of migrants, in such a way that it would then be possible to assess the effects of migration on the host country's economy, and implement appropriate policies to ease the assimilation of migratory flows. The variable considered in the majority of papers is that of education (see, for instance, [1–4]). In contrast, only a small number of papers have focused on another important characteristic of migrants, their age. The general conclusion is that the best decision is to migrate at the youngest possible age, or never [5]. However, this conclusion seems to be inconsistent with data. For instance, focusing on the two European countries that received the highest flows of immigrants in 2008 [6], Spain and Italy, most decisions to migrate are taken between the ages of 25 years and 29 years (Table I), whereas an individual can be considered a potential migrant as soon as he or she enters the labor market (around 18 years old).

One possible explanation of the divergence between the data in Table I and the developed theoretical models is that these frameworks only consider the stream of income associated with migration, without taking into account other important elements, such as social factors or the human capital of the migrant. Our objective in this paper is to contribute to a better understanding of the mechanisms through which migrants decide both at what age to migrate and their qualification level at the time of departure. It is clear that migration provides economic gains, a factor that we introduce in our model by assuming a market made up of two countries – home country and host country – where there is economic asymmetry, in the sense that wages and salaries in the former are lower than in the latter. The immediate gain of migration is the difference between salaries in the host and home

*Correspondence to: Gemma Larramona, Universidad de Zaragoza, Gran Vía, 2, 50005 Zaragoza, Spain.
†E-mail: gemmalar@unizar.es

Table I. Number of migrants received by age, 2008.

| | Spain | % | Italy | % |
|---|---|---|---|---|
| 15–19 | 58,283 | 10 | 39,340 | 9 |
| 20–24 | 111,304 | 19 | 80,180 | 18 |
| 25–29 | 129,009 | 22 | 89,660 | 20 |
| 30–34 | 97,790 | 16 | 69,886 | 16 |
| 35–39 | 68,602 | 11 | 52,436 | 12 |
| 40–44 | 47,902 | 8 | 38,471 | 9 |
| 45–49 | 32,942 | 5 | 30,627 | 7 |
| 50–54 | 22,655 | 4 | 22,416 | 5 |
| 55–59 | 16,557 | 3 | 14,136 | 3 |
| 60–64 | 14,101 | 2 | 8511 | 2 |

countries [7]. Additionally, our framework integrates the human capital of the migrant as well as social factors, in both the home and the host countries, to avoid any discrepancy between theory and data.

From a social point of view, the individual who emigrates suffers a loss of relationships in his homeland and must make a great deal of effort to adapt to the new culture. The contribution [8] takes this consideration into account, introducing a parameter in the utility function that represents migrant preferences for the country of birth, in such a way that migration to another country implies a decline in utility. We follow the example of Faini's work, but rather than assuming a constant preference parameter, we incorporate the effect of time within this parameter. The paper [9] also includes this, considering that individuals with less preference for their country of origin migrate earlier, and those who have deeper home roots prefer to wait until earlier emigrants establish a better network. Although this assumption is plausible, the specification of migrants' preference for the country of birth has the shortcoming that the later the departure time of the emigrant, the lower the preference for the country of origin. The present paper establishes a more adequate specification. First, we consider that the longer a migrant stays at home (that is to say, the older he or she is), the greater the preference for the country of birth. Second, we take into account that the longer an immigrant stays in the destination country, the greater the level of integration; the migrant assimilates cultural aspects and establishes new relationships. Finally, although migration always produces a loss of relationships, it is also true that the longer an emigrant waits for departure, the more developed are the networks to be found in the host country, and the more able he or she is to settle, thus producing a higher level of welfare for the immigrant. All these considerations imply that the individual age at migration establishes a mechanism that makes the social costs of migration endogenous.

The education level of migrants is also taken into account. The literature of migration points to a close connection between the final educational level of migrants and age at migration. The majority of papers study the effect of age at migration on the education level attained by migrants in the host country. The contribution [10] reports that immigrants from Mexico who arrive in the US before the age of 6 years, complete as much education as the second generation of Mexican-Americans. However, those who arrive after the age of 15 years complete less schooling. The paper [11] investigates the effects of arrival age on immigrants to the Netherlands, showing that migration at an early age appears to have a disadvantageous effect on immigrant educational achievement, depending on their country of origin. However, to our knowledge, there is no recent work examining the relationship between the education level of migrants attained in the country of origin and the age when the migration takes place. The lack of studies of this aspect may be due to data limitations. Fortunately, we have a microdata set of a representative sample of Spanish immigrants (for 2007), which allows us to test the relationship between the prior educational level of migrants and their age at migration. Spain can be considered a good example, because it has received a substantial number of immigrants in recent years. Figure 1 shows a clear negative relation between the prior human capital of the migrant and the age at migration.

The incorporation of education level in our framework allows us to test the main implications of the model with Spanish data. The optimal age at migration will be lower for those who choose more





DOI: 10.1002/oca

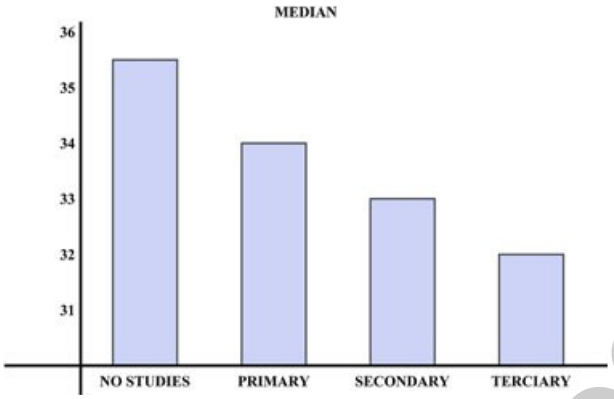



Figure 1. Age at migration, median by educational level, and Spanish data 2007.



education than for the less educated, in spite of more time spent obtaining qualifications in the country of origin. In particular, we find that the link between the optimal age at migration and education level operates in two directions: the age at migration affects education level, and education, in turn, has an influence on the optimal departure time.

On the one hand, the age at migration negatively affects the level of education, because it shortens the period in which the emigrant earns a higher wage: the later the departure time, the lower the stream of income obtained from migrating, and, hence, the lower the incentives to invest more in education. This economic effect is similar to that highlighted by studies focusing on migration incentives in the accumulation of human capital [2, 3]. These studies establish that, due to the wage differential between the origin and destination countries, non-migrants have a lower stream of income than potential migrants and, thus, fewer incentives for greater educational attainment. Therefore, early migration extends the period during which income is higher, a fact that encourages investment in education. On the other hand, age at migration is also affected by education via the salary gap. The higher the level of education of the migrant, the greater the expected wage benefits associated with migration and, hence, the greater the incentives for an earlier migration. This effect is the only one that appears in the model of paper [5], who finds that the best decision is to emigrate at the earliest possible age. What, then, influences migrants to delay the age of migration? The answer is that social forces delay migration; by postponing the moment, the emigrant can reduce the social costs associated with migration because of the more consolidated networks he or she may find in the destination country. All these considerations lead to lesser educated migrants having more incentive to delay the age at migration than those with higher education and vice versa. This finding supports the idea that educational requirements could be incorporated in immigration policy, not only to attract educated workers but also younger workers. This is an important concern in advanced economies, which have a high proportion of older workers.

The remainder of this paper is organized as follows: the nature of the problem faced by migrating individuals, the key economic factors, and the optimal levels of the age at migration and the period of education, are presented in Section 2. The calibration of the model, and comparative statics are analyzed in Section 3. Section 4 closes with a review and discussion of our main conclusions.

## 2. THE MODEL

The model is developed in continuous time. Individual welfare comes from the stream of consumption c.t / over the individual's lifetime. Assuming that the instantaneous utility derived from consumption is logarithmic, the utility of an individual born in t D 0 can be represented as follows:

$$\int_0^T 8.t / \ln c.t / e^{-pt} dt \quad , \tag{1}$$





DOI: 10.1002/oca

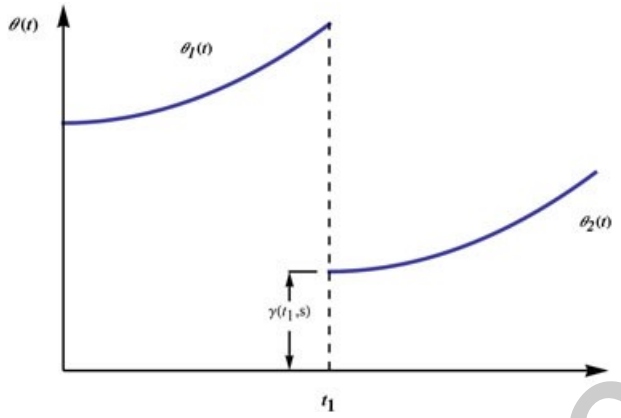



Figure 2. Illustration of the function 8.

where T is the life expectancy of this individual, p is the inter-temporal discount rate of the utility, and 8 *(t)* is a parameter that reflects his or her preference for the place of residence. In a similar way to the paper [8], we introduce the fact that the emigrant experiences a reduction of utility associated with departure from the country of birth, revealing the existence of psychological and social moving costs. Faini [8] postulates that individuals have a preference for living in their country of birth, because migration produces a *loss of social relationships and the need to adapt to a new environment*. In this model, as in paper [9], the loss of utility associated with migration depends on the departure time of the emigrant, because the delay in the timing of migration facilitates the consolidation of networks. However, we use a very different specification, and the parameter $8.t/$ adopts the following form:

$$8_1.t/ \mathrm{D}\ e^{\mathbf{t}_1 t}, \quad t \ 2\ \mathbb{0}, t_1],$$

$$8_2.t/ \mathrm{D}\ y.t_1, s/\ e^{\mathbf{t}_2.t-t_1/}, \quad t \ 2\ .t_1, T],$$

where $t_1$ is the age at migration and s is the period of education. We assume that the emigrant completes his entire education in his country of birth $t_1 > s$, because our purpose is to analyze the relationship between the prior educational level of migrants and their age at migration. We also assume that $\mathbf{t}_1 > 0$, $\mathbf{t}_2 > 0$ and $e^{\mathbf{t}_1 t_1} > y.t_1, s/$. In this way, three facts are established (Figure 2):

1. The longer an individual stays in the home country, the greater is the preference for this country $.\mathbf{t}_1 > 0/$.
2. The longer an individual lives in the host country, the greater is the preference for this country, through better adaptation to the new environment $.\mathbf{t}_2 > 0/$.
3. Migration produces a decline in utility through the loss of relationships and the necessity to adapt $\left(e^{\mathbf{t}_1 t_1} > y\ .t_1, s/\right)$. However, the longer an individual delays migration, the more consolidated networks will be found in the host country, and the lower will be the jump in utility. Moreover, working before migration also may reduce the drop in utility, because previous experience facilitates integration in the host labor market. Therefore, we assume that $y\ .t_1, s/$ depends positively on the departure time of the emigrant. In any case, whatever the timing of migration, the utility in the home country is always higher than in the host country, that is to say, $8\ 0 \leq t_1 \leq T$, the condition $e^{\mathbf{t}_1 t_1} > y\ .t_1, s/$ fulfills.

We consider that individuals emigrate to obtain a better standard of living. As in other studies of migration, we establish the wage difference as the main reason for migration. In this framework,





DOI: 10.1002/oca

earnings are the product of a rate $w.t/$, that is the wage for one unit of human capital, multiplied by each individual's human capital. This human capital will be greater for the more skilled workers. We assume that individuals are heterogeneous with respect to their human capital level h, which depends exclusively on the period of education, s :

$$h = f(s),$$

with $f'(s) > 0$. Following contribution [12], we assume that the only education cost is the opportunity cost. The rate of return of human capital is given by the economic conditions of the country of residence. We assume that the rate of return of human capital in the country of birth $w_1$ is constant, reflecting the economic stagnation of this economy, assuming growth of $w_1$ does not substantially change the results. The host country rate of return of one unit of human capital $w_2.t/$ is higher and grows at a rate $\theta_2$, reflecting the better economic conditions in the destination country, or a greater productivity of human capital in the host country. For simplicity, we assume that this growth rate is positive and constant, indicating that the differential in wage levels between more-educated and less-educated emigrants increases over time. Taking all these considerations into account, the problem for an emigrant born at $t = 0$ takes the following form:

$$\text{Max} \int_0^s e^{\theta_1 t} \ln c.t/ e^{-\rho t} dt + \int_s^{t_1} e^{\theta_1 t} \ln c.t/ e^{-\rho t} dt + \int_{t_1}^{T} y.t_1, s/e^{\theta_2.t-t_1/} \ln c.t/ e^{-\rho t} dt, \quad (2)$$

$$\text{s.t} \quad \dot{k}.t/ = r_1 k.t/ - c.t/ \quad t \in [0, s/, \quad (3)$$

$$\dot{k}.t/ = r_1 k.t/ + w_1 f(s) - c.t/ \quad t \in [s, t_1/ \quad (4)$$

$$\dot{k}.t/ = r_2 k.t/ + w_2.t_1/e^{\theta_2.t-t_1/} f(s) - c.t/ \quad t \in [t_1, T], \quad (5)$$

$$k.0/ = 0, \quad k.T/ > 0, \quad (6)$$

with $t_1 > s$. The parameters $r_1$ and $r_2$ are the interest rates in the country of birth and the host country, respectively, and $k.t/$ is the amount of assets that an individual owns at time t. A dot over the variable indicates its variation in time. Equation (26) is the restriction for the individual while obtaining skills, Equation (17) is the restriction for the individual while working in the country of origin, and (9) represents this same restriction for the worker in the destination country. We have not considered the explicit economic costs of moving, although these can easily be included, assuming that the expected wage is net, after these costs.

The endogenous variables are the age at migration $t_1$ and the period of education s. To solve this three-stage optimal control problem, we extend the conditions developed by contribution [13] for a two-stage model, where the switch point is a choice variable and appears as an argument of the integrands. This technique has also been applied to solve certain adoption problems under embodied technical change [14] and other problems addressed by the environmental literature [15]. The formulated problem is nontrivial, but it can be made tractable by decomposing it into a sequence of three auxiliary problems, corresponding to the period in which the emigrant has left the country of birth $t_1 \leq t \leq T$, the period in which the emigrant works in the home country $s \leq t < t_1$, and the period in which the emigrant is at school $0 \leq t < s$. For a brief introduction of multistage optimal control models [16].

We begin with an auxiliary problem corresponding to the third stage of the multistage problem, once the individual has migrated to the host country.





DOI: 10.1002/oca

*Auxiliary Problem I*

In the host country, the emigrant faces the following problem:

$$\text{Max } J_3 = \int_{t_1}^{T} y(t_1, s) e^{\theta_2 (t - t_1)} \ln c(t)\, e^{-\rho t} dt,$$

$$\text{s. t.} \quad \dot{k}(t) = r_2 k(t) + w_2 e^{\theta_2 (t - t_1)} f(s) - c(t), \quad t_1 \leq t \leq T,$$

$$k(t_1) = k_1, \quad k(T) > 0.$$

In this auxiliary problem, the emigrant's consumption $c(t)$ is the endogenous variable, whereas the starting time $t_1$, the level of education $s$, and the starting level of individual assets *in the host country* $k(t_1)$ are exogenous variables, in such a way that the term $t_1$ in the integrand of the performance index $J_3$ does not imply any conflict when Pontryagin's maximum principle is used. The variable $c(t)$ is called the control variable and the variable $k(t)$ *is* called the state variable. The corresponding Hamiltonian is defined as $H_3(k, c, \lambda_3, t, t_1, s) = y(t_1, s) e^{\theta_2 (t - t_1)} \ln c(t)\, e^{-\rho t} + \lambda_3(t)\left[r_2 k(t) + w_2 e^{\theta_2 (t - t_1)} f(s) - c(t)\right]$, where $\lambda_3(t)$ is called the costate variable. The solution meets the following necessary conditions, detailed in [17]:

$$\frac{y(t_1, s) e^{\theta_2 (t - t_1)} e^{-\rho t}}{c(t)} = \lambda_3(t), \tag{7}$$

$$-r_2 \lambda_3(t) = \dot{\lambda}_3(t), \tag{8}$$

$$\dot{k}(t) = r_2 k(t) + w_2 e^{\theta_2 (t - t_1)} f(s) - c(t), \tag{9}$$

$$\lambda_3(t) > 0, \tag{10}$$

$$k(t_1) = k_1, \quad k(T) > 0. \tag{11}$$

From expressions (7) and (8), we obtain the optimal path of consumption, given by the following:

$$\dot{c}(t) = (r_2 - \rho + \theta_2) c(t). \tag{12}$$

By using (9), (11), and (12), and assuming that $r_2 \neq \theta_2$ and $\rho \neq \theta_2$, it can be derived that

$$k(t_1) = c(t_1) \frac{1 - e^{(\theta_2 - \rho)(T - t_1)}}{\rho - \theta_2} - w_2 f(s) \frac{e^{(\theta_2 - r_2)(T - t_1)} - 1}{\theta_2 - r_2}, \tag{13}$$

for $t_1 \leq t \leq T$, where $c(t_1)$ is the level of consumption at moment $t_1$ in the host country. Introducing (12) and (13) into the objective function $J_3$, we now define the optimal performance index $J_3^*(k(t_1), t_1, s)$ as follows:

$$J_3^*(k(t_1), t_1, s) = \max_{c} J_3 = y(t_1, s) e^{-\theta_2 t_1} \left[ \ln\left( k_1 + w_2 f(s) \frac{e^{(\theta_2 - r_2)(T - t_1)} - 1}{\theta_2 - r_2} \right) \frac{\rho - \theta_2}{1 - e^{(\rho - \theta_2)(T - t_1)}} + \frac{r_2 - \rho + \theta_2}{\rho - \theta_2} \right.$$
$$\left. \frac{e^{(\rho - \theta_2) t_1} - e^{(\rho - \theta_2) T}}{\rho - \theta_2} - \frac{r_2 - \rho + \theta_2}{\rho - \theta_2} (T - t_1) e^{(\rho - \theta_2) T} \right], \tag{14}$$

allowing us to formulate the following auxiliary problem.





DOI: 10.1002/oca

*Auxiliary Problem II*

$$\text{Max } J_2 = \int_s^{t_1} e^{\theta_1 t} \ln c(t) e^{-pt} dt + J_3(k(t_1), t_1, s),$$

$$\text{s. t} \quad \dot{k}(t) = r_1 k(t) + w_1 f(s) - c(t), \qquad s \le t < t_1,$$

$$\text{and} \quad k(t_1) \text{free}, \; k(s) = k_s.$$

In this auxiliary problem, the emigrant's consumption $c(t)$ is the control variable, whereas we assume that the starting capital after education, $k(s)$, and the period of education, $s$, are given. The Hamiltonian is defined as $H_2(k, c, \lambda_2, t, t_1, s) = e^{\theta_1 t} \ln c(t) e^{-pt} + \lambda_2(t)[r_1 k(t) + w_1 f(s) - c(t)]$, where $\lambda_2(t)$ is the costate variable. The solution meets the following necessary conditions:

$$\frac{e^{\theta_1 t} e^{-pt}}{c(t)} = \lambda_2(t), \tag{15}$$

$$-r_1 \lambda_2(t) = \dot{\lambda}_2(t), \tag{16}$$

$$\dot{k}(t) = r_1 k(t) + w_1 f(s) - c(t), \tag{17}$$

$$\lambda_2(t_1 -) = \lambda_3(t_1 +), \tag{18}$$

$$k(t_1 -) = k(t_1 +), \tag{19}$$

with $t_1^- = \lim_{\varepsilon \to 0} t_1 - \varepsilon$ and $t_1^+ = \lim_{\varepsilon \to 0} t_1 + \varepsilon$ being $\varepsilon > 0$. Equations (18) and (19) establish the continuity of the costate variable path and the state variable path, respectively. From (15) and (16), we deduce that the optimal path of consumption at this stage takes the following expression:

$$\dot{c}(t) = (r_1 - p + \theta_1) c(t). \tag{20}$$

By using (17) and (20), and assuming $p > \theta_1$, it is possible to characterize the path of assets in the source country once the emigrant has entered the labor market. At $t_1$, we have

$$k(t_1) = k_s e^{r_1 (t_1 - s)} + w_1 f(s) \frac{e^{r_1 (t_1 - s)} - 1}{r_1} - c(t_1) \frac{e^{(p - \theta_1)(t_1 - s)} - 1}{p - \theta_1}, \tag{21}$$

for $s \le t < t_1$, where $c(t_1)$ is the level of consumption at moment $t_1$ *in the country of birth.*

Evaluating the expressions (7) and (15) in $t_1$ and applying the condition (18), we detect that the break in the consumption path of the emigrant at moment $t_1$ is $y(t_1, s) e^{-\theta_1 t_1}$ due to the sudden drop in the level of utility derived from adaptation to the new environment. Therefore, postponing the time of migration reduces the utility discontinuity. Note that the utility function is strictly concave, so the individual tends to smooth consumption whenever possible. Consumption discontinuities are allowed if there are incentives to wait: the later the timing of migration, the more consolidated networks in the host country and the lower the social costs of migration.

From expressions (13), (19), and (21), we derive the following expression:

$$c(t_1) = \frac{k_s e^{r_1 (t_1 - s)} + w_1 f(s) \frac{e^{r_1 (t_1 - s)} - 1}{r_1} + w_2 f(s) \frac{e^{(r_2 - \theta_2)(T - t_1)} - 1}{r_2 - \theta_2}}{y(t_1, s) e^{-\theta_1 t_1} \frac{1 - e^{-(p - \theta_2)(T - t_1)}}{p - \theta_2} + \frac{e^{(p - \theta_1)(t_1 - s)} - 1}{p - \theta_1}}, \tag{22}$$





DOI: 10.1002/oca

where $c(t_1)$ is the level of consumption at moment $t_1$ in the country of birth. Expression (22) allows us to define the optimal performance index $J_2(k(s), t_1, s)$ as

$$J_2(k(s), t_1, s) = \operatorname{Max}_{\{e^{-(\rho-\gamma_1)s} - e^{-(\rho-\gamma_1)t_1}\}} \ln \frac{k_s e^{(r_1-\gamma_1)t_1 - s)} w_1 f(s)^{(r_1-\gamma_1)t_1-s)/r} + w_2 f(s)^{(e^{(r_2-\gamma_2)(T-t_1)}-1)/r}}{e^{\frac{1}{1-\gamma_1}} - 1} \Bigg\}^{\frac{1-\gamma_1}{2-\gamma_2}}$$

$$= \frac{1}{\rho-\gamma_1} \ln y(t_1, s) e^{-\gamma_1 t_1} \left[1 - e^{-(\rho-\gamma_2)(T-t_1)}\right] + \frac{1}{\rho-\gamma_2} \ln y(t_1, s) e^{-\gamma_2 t_1} e^{(\rho-\gamma_2)(T-t_1)} + \frac{1}{\rho-\gamma_1} \ln e^{-\frac{r_1-\rho+\gamma_1 t_1}{\rho-\gamma_1}(t_1 - s)} e^{-(\rho-\gamma_1)s}$$

$$+ \frac{1}{\rho-\gamma_2} \ln \left\{ \frac{k_s e^{(r_1-\gamma_1)t_1-s)} w_1 f(s)^{\frac{e^{(r_1-\gamma_1)t_1-s)}-1}{r_1}} + w_2 f(s)^{\frac{e^{(r_2-\gamma_2)(T-t_1)}-1}{2-r_2}}}{y(t_1, s) e^{-\gamma_1 t_1} \frac{1-e^{-(\rho-\gamma_2)(T-t_1)}}{\rho-\gamma_2} + e^{(\rho-\gamma_1)(t_1-s)} - 1} \right\}^{\frac{2-r_2}{\rho-\gamma_1}} y(t_1, s) e^{-\gamma_1 t_1} + \frac{r_2-\rho+\gamma_2 t_2}{\rho-\gamma_2}$$

$$y(t_1, s) e^{-\gamma_2 t_1} \frac{e^{-(\rho-\gamma_2)t_1} - e^{-(\rho-\gamma_2)T}}{\rho-\gamma_2} - \frac{r_2-\rho+\gamma_2 t_2}{\rho-\gamma_2}(T - t_1) e^{-(\rho-\gamma_2)T} y(t_1, s) e^{-\gamma_2 t_1}, \tag{23}$$

which allows us to reduce the initial three-stage problem to a tractable, single-stage problem, characterized by the following auxiliary problem:

*Auxiliary Problem III*

$$\operatorname{Max} J_1 = \int_0^s e^{\gamma_1 t} \ln c(t) e^{-\rho t} dt + J_2(k(s), t_1, s) = J_1(t_1, s),$$

$$\text{s. t} \quad \dot{k}(t) = r_1 k(t) - c(t), \qquad 0 \le t < s,$$

$$k(s) \text{ free}, \quad k(0) = 0.$$

The Hamiltonian is $H_1(k, c, \lambda_1, t, t_1, s) = e^{\gamma_1 t} \ln c(t) e^{-\rho t} + \lambda_1(t)[r_1 k(t) - c(t)]$, where $\lambda_1(t)$ is the costate variable. The solution meets the following necessary conditions:

$$\frac{e^{\gamma_1 t} e^{-\rho t}}{c(t)} = \lambda_1(t), \tag{24}$$

$$-r_1 \lambda_1(t) = \dot{\lambda}_1(t), \tag{25}$$

$$\dot{k}(t) = r_1 k(t) - c(t), \tag{26}$$

$$\lambda_1(s^-) = \lambda_2(s^+), \tag{27}$$

$$k(s^-) = k(s^+), \tag{28}$$

$$\int_0^s \frac{\partial H_1}{\partial t_1} dt + \int_s^{t_1} \frac{\partial H_2}{\partial t_1} dt + \int_{t_1}^T \frac{\partial H_3}{\partial t_1} dt = H_3(t_1^+) - H_2(t_1^-), \tag{29}$$

$$\int_0^s \frac{\partial H_1}{\partial s} dt + \int_s^{t_1} \frac{\partial H_2}{\partial s} dt + \int_{t_1}^T \frac{\partial H_3}{\partial s} dt = H_2(s^+) - H_1(s^-), \tag{30}$$

with $s^- = \lim_{\varepsilon \to 0} s - \varepsilon$, $s^+ = \lim_{\varepsilon \to 0} s + \varepsilon$, $H_3(t_1^+) = \lim_{\varepsilon \to 0} H_3(t_1^+ \varepsilon)$, $H_2(t_1^-) = \lim_{\varepsilon \to 0} H_2(t_1^- - \varepsilon)$, $H_2(s^+) = \lim H_2(s^+ \varepsilon)$ and $H_1(s^-) = \lim H_1(s^- - \varepsilon)$, being $\varepsilon > 0$. Equations (27) and (28)





DOI: 10.1002/oca

54 establish the continuity of the costate and the state variable paths, respectively. Equations (29) and



DOI: 10.1002/oca

(30) are the matching conditions for a three-stage optimal control with two explicit switch points dependence. From (24) and (25), we deduce that the optimal path of consumption in this stage is equal to that in the previous stage:

$$\dot{c}(t) = (r_1 - \rho)\, c(t), \tag{31}$$

which, together with expression (26), leads to:

$$k(s) = -c(s)\frac{e^{(\rho - r_1)s} - 1}{\rho - r_1}. \tag{32}$$

From the expressions (22), (27), (28), (31) and (32), we obtain:

$$c(t_1) = \frac{\left[w_1 \frac{e^{r_1 t_1} - s - 1}{r_1} + w_2 \frac{e^{(\rho - r_2)T - t_1 - 1}}{\rho - r_2}\right]}{y(t_1, s)\, e^{(\rho - r_2)t_2 - r_1 t_1} \frac{1 - e^{-(\rho - r_2)T - t_1}}{\rho - r_2} + \frac{e^{(\rho - r_1)t_1 - 1}}{\rho - r_1}}. \tag{33}$$

where $c(t_1)$ is the level of consumption at moment $t_1$ in the country of birth. Finally, applying the matching conditions (29) and (30), we obtain two equations that provide a solution for $t_1$ and s*:

$$\begin{aligned}
&\frac{\frac{\partial c(t_1)}{\partial t_1}}{c(t_1)}\frac{1 - e^{-(\rho - r_1)t_1}}{\rho - r_1} + \left[\frac{\frac{\partial c(t_1)}{\partial t_1}}{c(t_1)} + r_2 - r_1\right]\frac{e^{-(\rho - r_2)t_1} - e^{-(\rho - r_2)T}}{\rho - r_2} y(t_1, s) + \\
&\left[\ln c(t_1) + \frac{r_1 - \rho}{\rho - r_1} e^{-(\rho - r_1)t_1} + \ln c(t_1) - t_2 (T - t_1) - r_1 t_1 + \frac{r_2 - \rho}{\rho - r_2}\right] \\
&\frac{e^{-(\rho - r_2)t_1} - e^{-(\rho - r_2)T}}{\rho - r_2}\frac{\partial y(t_1, s)}{\partial t_1} + \frac{r_2 - \rho}{\rho - r_2} e^{-(\rho - r_2)T} y(t_1, s) = \\
&= \left[\ln c(t_1) + t_2 (T - t_1) - r_1 t_1 + \frac{r_2 - \rho}{\rho - r_2}\right] e^{-(\rho - r_2)t_1} y(t_1, s) + \\
&+ \left[\frac{r_1 - \rho}{\rho - r_1} + \frac{r_2 - \rho}{\rho - r_2} e^{-(\rho - r_2)T}\right](T - t_1)\frac{\partial y(t_1, s)}{\partial t_1},
\end{aligned} \tag{34}$$

$$\begin{aligned}
&\frac{\frac{\partial c(t_1)}{\partial s}}{c(t_1)}\frac{1 - e^{-(\rho - r_1)t_1}}{\rho - r_1} + \left[\frac{\frac{\partial c(t_1)}{\partial s}}{c(t_1)} + r_2 - r_1\right]\frac{e^{-(\rho - r_2)t_1} - e^{-(\rho - r_2)T}}{\rho - r_2} y(t_1, s) - \\
&- \frac{r_2 - \rho}{\rho - r_2} e^{-(\rho - r_2)T}(T - t_1)\frac{\partial y(t_1, s)}{\partial s} = \\
&= -\left[\ln c(t_1) + t_2 (T - t_1) - r_1 t_1 + \frac{r_2 - \rho}{\rho - r_2}\right]\frac{e^{-(\rho - r_2)t_1} - e^{-(\rho - r_2)T}}{\rho - r_2}\frac{\partial y(t_1, s)}{\partial s},
\end{aligned} \tag{35}$$

with $c(t_1)$ given by (33).

The character of the equations, as they stand, makes it impossible to determine the solution explicitly for $t_1$ and s*, and so we must solve numerically, which is carried out in the following section.

## 3. NUMERICAL SOLUTION

The auxiliary problem is solved numerically making use of the program Wolfram MATHEMATICA 7. To obtain an optimal point close to the Spanish average timing of migration, we set the following values for the parameters that can be considered as biennial: $\rho = 0.066$ corresponds to an annual inter-temporal discount rate around 3% (commonly used in the models [12]); $w_1(t_1) = 0.27$ and $w_2(t_1) = 0.4$ match the annual earnings $w_1(t_1) = 0.13$ and $w_2(t_1) = 0.20$ in hundreds of thousands of Euros, in 2006, for Spain, and the rest of the world, respectively; $\varepsilon_2 = 0.00001$ establishes a nearly null wage growth in the destination country; $T = 38$ approaches a life expectancy





DOI: 10.1002/oca

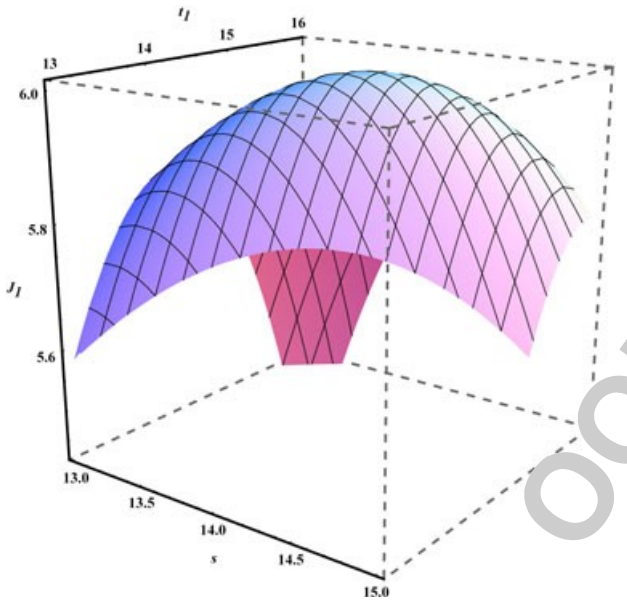



Figure 3. The performance index $J_1$ for the variation of $t_1$ and s.



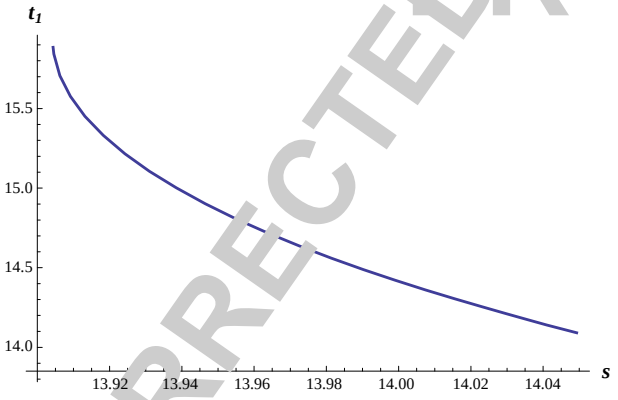



Figure 4. Optimal age at migration and period of education for different values of $w_1(t_1)$.

of 76 years; $r_1 = 0.001$ and $r_2 = 0.1$ match an almost null return to capital in origin and an annual return to capital of 4.9% in destination, respectively and, following contribution [18], we have considered the following form for the function $f(s)$:

$$f(s) = e^{\frac{\gamma}{1-\alpha}s^{1-\alpha}}$$

and utilized their estimates for the parameters, $\alpha$=0.58 and $\gamma$=0.32. Finally, we have chosen $t_1 = 0.003$, $t_2 = 0.001$, and $y(t_1, s) = e^{0.003 t_1 - s}$ to show a pair of optimal values of the age at migration and the period of education ($t_1^*, s^*$) within the range of plausibility. The performance index $J_1$ is plotted in the vicinity of $t_1^*$ and $s^*$ to show the optimality of both (Figure 3). Observe that the functional form selected for $y(t_1, s)$ leads to a jump down at $t_1$ when $s > 1$, which is critical in this model.

To test the relationship between the age of the individual at migration and the level of education, we assign different possible values to the parameter $w_1(t_1)$. Figure 4 shows the set of optimal pairs ($t_1^*, s^*$) obtained. We can observe that age at migration and level of education move in opposite directions. Intuitively, the effect of the age at migration on the educational level is negative, because later migration shortens the period in which the emigrant earns a higher wage: the later the departure time, the lower the stream of income obtained, and, hence, the lower the incentive to invest





DOI: 10.1002/oca

more in education. Additionally, education negatively affects the age at migration due to the earnings gap. The higher the level of education of the migrant, the higher the expected wage benefits associated with migration and, hence, the greater the incentive to earlier migration. Thus, the longer the period of education, the younger will be the age at migration, which is a result supported by the Spanish data.

## 4. CONCLUSION

We have designed a three-stage optimal control problem that endogenously determines age at migration and level of education. The combination of social and economic factors related to migration allows us to detect different patterns in the age at migration and the level of schooling. Our main finding is the negative relationship between these two variables. We can say that host countries may be more interested in higher skilled workers, not only because of their human capital but also because the migrants are younger. Immigration policies based on human capital selection provide younger and more qualified workers who, apart from being more productive, contribute longer to the economy of the host country, facilitating assimilation and alleviating the aging phenomenon commonly found in advanced host economies.

ACKNOWLEDGEMENTS

Our thanks go to the anonymous referees. All remaining errors are ours alone. Q5

REFERENCES

1. Borjas GJ. Self-selection and the earnings of immigrants. *American Economic Review* 1987; **77**(4):531–53.
2. Beine M, Docquier F, Rapoport H. Brain drain and economic growth theory and evidence. *Journal of Development Economics* 2001; **64**:275–289.
3. Beine M, Docquier F, Rapoport H. Brain drain and human capital formation in developing countries winners and losers. *Economic Journal* 2008; **118**:631–652.
4. Chiquiar D, Hanson G. International migration, self-selection, and the distribution of wages evidence from Mexico and the United States. *Journal of Political Economy* 2005; **113**(2):239–281.
5. Hartog J, Winkelmann R. Comparing migrants to non-migrants: the case of Dutch migration to New Zealand. *Journal of Population Economics* 2003; **16**:683–705.
6. OECD International Migration Outlook 2010. SOPEMI.
7. Harris J, Todaro M. Migration, unemployment and development: a two sector analysis. *American Economic Review* 1970; **60**:126–142.
8. Faini R. Increasing returns, migration and convergence. *Journal of Development Economics* 1996; **49**:121–136.
9. Aisa R, Cabeza J, Larramona G. Timing of migration. *Economics Bulletin* 2007; **6**:1–10.
10. Grogger J, Trejo SJ. Falling behind or moving up? The intergenerational progress of Mexican Americans 2002, Public Policy Institute of California. Available from: http://www.ppic.org/content/pubs/report/r{_}502JGR.pdf. Q6
11. Van Ours JC, Veenman J. Age at immigration and educational attainment of immigrants. *Economics Letters* 2006; **90**:310–316.
12. Kalemli-Ozcan S, Ryder HE, Weil DN. Mortality decline, human capital investment and economic growth. *Journal of Development Economics* 2000; **62**:1–23.
13. Tomiyama K, Rossana RJ. Two-stage optimal control problems with an explicit switch point dependence. *Journal of Economic Dynamics and Control* 1989; **13**(3):319–337.
14. Boucekkine R, Saglam C, Vallée T. Technology adoption under embodiment a two-stage optimal control approach. *Macroeconomic Dynamics* 2004; **8**:250–271.
15. Amit R. Petroleum reservoir exploitation switching from primary to secondary recovery. *Operations Research* 1986; **34**:534–539.
16. Grass D, Caulkins JP, Feichtinger G, Tragler G, Behrens D. *Optimal Control of Nonlinear Processes*. Springer: Heidelberg, 2008.
17. Takayama A. *Analytical Methods in Economics*. MI University of Michigan Press: Ann Arbor, 1993.
18. Bils M, Klenov P. Does schooling cause growth or the other way round? *American Economic Review* 2000; **90**(5):1160–83.





DOI: 10.1002/oca

# Author Query Form

## Journal: Optimal Control Applications and Methods

## Article: oca_2078

Dear Author,

During the copyediting of your paper, the following queries arose. Please respond to these by annotating your proofs with the necessary changes/additions.

- If you intend to annotate your proof electronically, please refer to the E-annotation guidelines.
- If you intend to annotate your proof by means of hard-copy mark-up, please refer to the proof mark-up symbols guidelines. If manually writing corrections on your proof and returning it by fax, do not write too close to the edge of the paper. Please remember that illegible mark-ups may delay publication.

Whether you opt for hard-copy or electronic annotation of your proofs, we recommend that you provide additional clarification of answers to queries by entering your answers on the query sheet, in addition to the text mark-up.

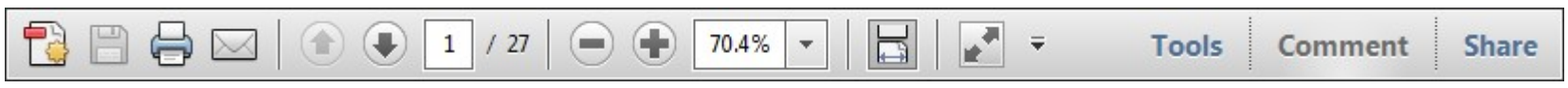
| Query No. | Query | Remark |
|---|---|---|
| Q1 | AUTHOR: Please check if captured author's affiliations are correct. | |
| Q2 | AUTHOR: Please check the suitability of the suggested short title. | |
| Q3 | AUTHOR: Equation number have been renumbered, please check. | |
| Q4 | AUTHOR: Please give address information for this manufacturer: town, state (if applicable), and country. | |
| Q5 | AUTHOR: Please check captured Acknowledgement if correct. | |
| Q6 | AUTHOR: Please provide accessed date for Reference 10. | |

WILEY-BLACKWELL

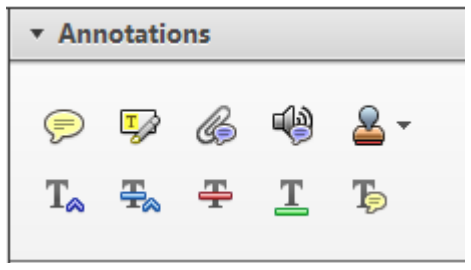
# USING e-ANNOTATION TOOLS FOR ELECTRONIC PROOF CORRECTION

**Required software to e-Annotate PDFs: Adobe Acrobat Professional or Adobe Reader (version 7.0 or above). (Note that this document uses screenshots from Adobe Reader X)**
**The latest version of Acrobat Reader can be downloaded for free at: http://get.adobe.com/uk/reader/**

Once you have Acrobat Reader open on your computer, click on the Comment tab at the right of the toolbar:




This will open up a panel down the right side of the document. The majority of tools you will use for annotating your proof will be in the Annotations section, pictured opposite. We've picked out some of these tools below:

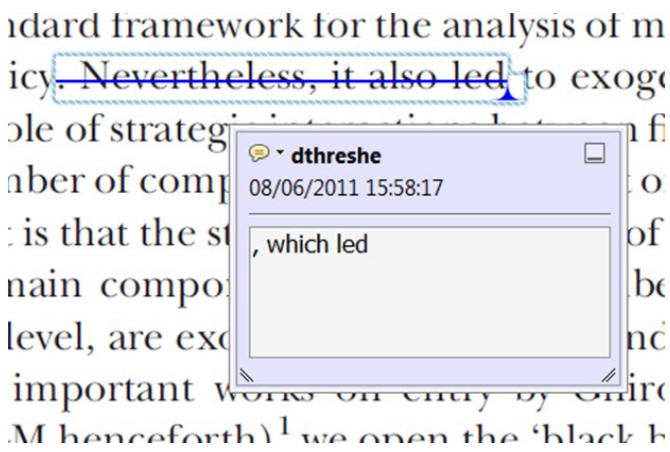



## 1. Replace (Ins) Tool – for replacing text.

Strikes a line through text and opens up a text box where replacement text can be entered.

**How to use it**

- Highlight a word or sentence.
- Click on the Replace (Ins) icon in the Annotations section.
- Type the replacement text into the blue box that appears.

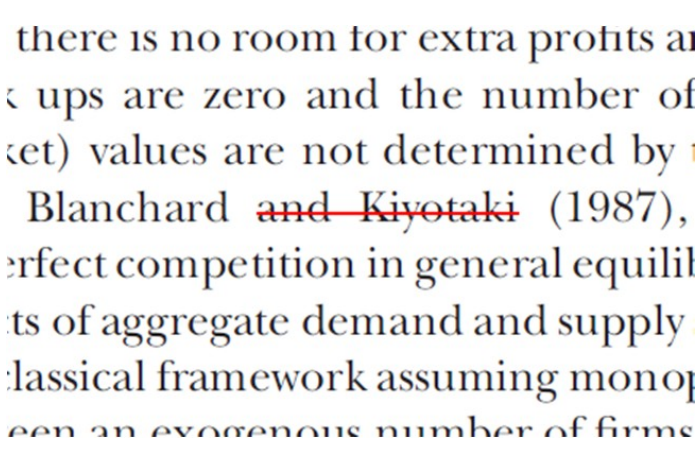



## 2. Strikethrough (Del) Tool – for deleting text.

Strikes a red line through text that is to be deleted.

**How to use it**

- Highlight a word or sentence.
- Click on the Strikethrough (Del) icon in the Annotations section.

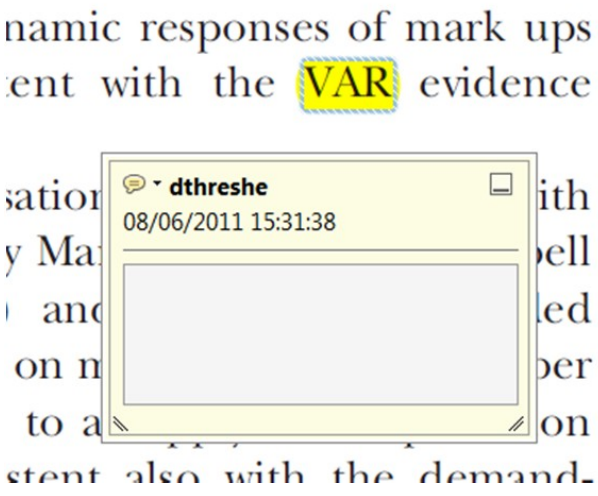


## 3. Add note to text Tool – for highlighting a section to be changed to bold or italic.

Highlights text in yellow and opens up a text box where comments can be entered.

**How to use it**

- Highlight the relevant section of text.
- Click on the Add note to text icon in the Annotations section.
- Type instruction on what should be changed regarding the text into the yellow box that appears.

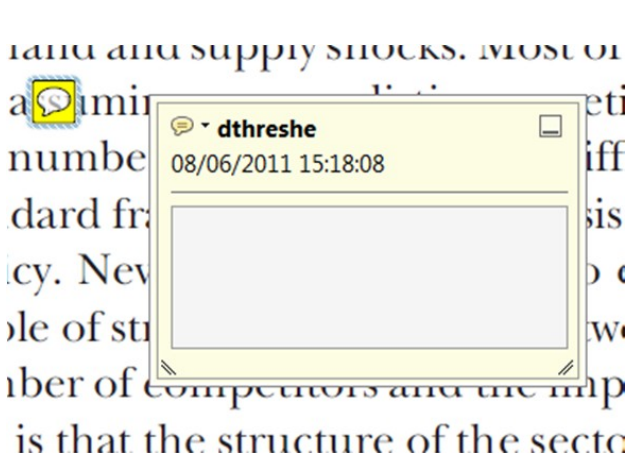



## 4. Add sticky note Tool – for making notes at specific points in the text.

Marks a point in the proof where a comment needs to be highlighted.

**How to use it**

- Click on the Add sticky note icon in the Annotations section.
- Click at the point in the proof where the comment should be inserted.
- Type the comment into the yellow box that appears.



WILEY-BLACKWELL

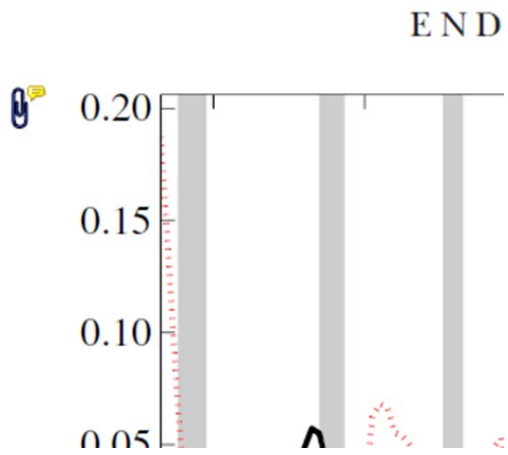
# USING e-ANNOTATION TOOLS FOR ELECTRONIC PROOF CORRECTION

### 5. Attach File Tool – for inserting large amounts of text or replacement figures.

Inserts an icon linking to the attached file in the appropriate pace in the text.

**How to use it**

- Click on the Attach File icon in the Annotations section.
- Click on the proof to where you'd like the attached file to be linked.
- Select the file to be attached from your computer or network.
- Select the colour and type of icon that will appear in the proof. Click OK.

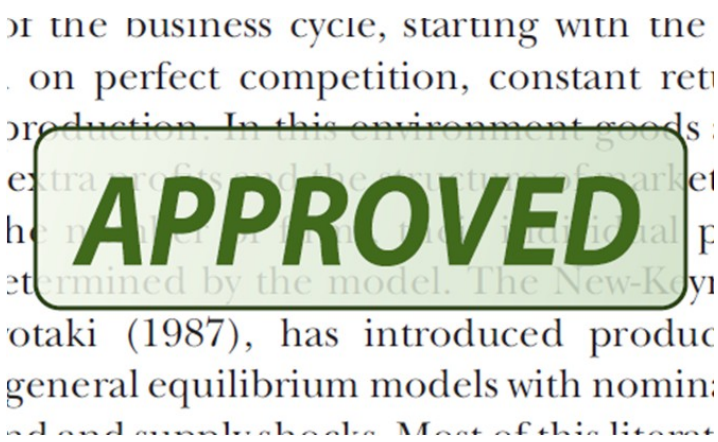


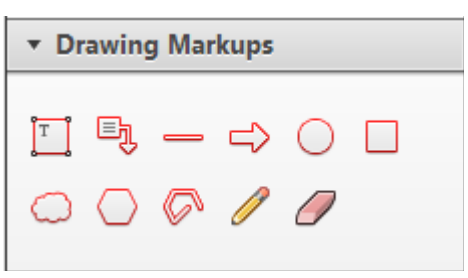

### 6. Add stamp Tool – for approving a proof if no corrections are required.

Inserts a selected stamp onto an appropriate place in the proof.

**How to use it**

- Click on the Add stamp icon in the Annotations section.
- Select the stamp you want to use. (The Approved stamp is usually available directly in the menu that appears).
- Click on the proof where you'd like the stamp to appear. (Where a proof is to be approved as it is, this would normally be on the first page).



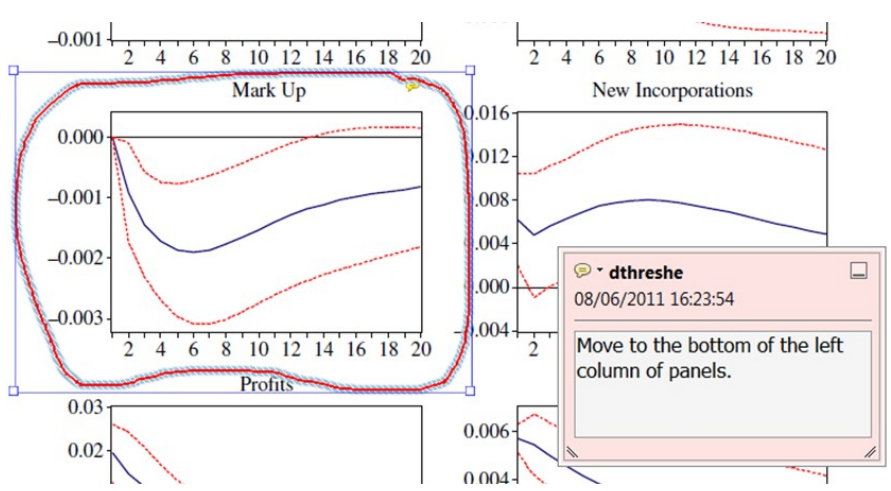




### 7. Drawing Markups Tools – for drawing shapes, lines and freeform annotations on proofs and commenting on these marks.

Allows shapes, lines and freeform annotations to be drawn on proofs and for comment to be made on these marks..

**How to use it**

- Click on one of the shapes in the Drawing Markups section.
- Click on the proof at the relevant point and draw the selected shape with the cursor.
- To add a comment to the drawn shape, move the cursor over the shape until an arrowhead appears.
- Double click on the shape and type any text in the red box that appears.




**For further information on how to annotate proofs, click on the Help menu to reveal a list of further options:**