

Population ageing, health care, and growth: a comment on the effects of capital accumulation

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Abstract

In a recent paper, Hashimoto and Tabata (J Popul Econ, 2010) present a theoretical model in which the increase in the rate of dependence due to the ageing of the population leads to a re-allocation of labor from non-health to health production and, as a consequence, to a decline in economic growth. We argue that these results rely heavily on assumptions of a ‘small economy’ and perfect capital mobility, which tie down the amount of capital. In this paper, we proceed by analyzing the case of an economy in which the availability of capital is endogenously determined by domestic savings. We find that the new ‘capital accumulation effect’ is opposite to the previous ‘dependency rate effect’, leaving the effect on economic growth ambiguous. In particular, if the former prevailed, population ageing would foster economic growth, a result that finds support in recent empirical work.

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1. Introduction

In a recent paper, Hashimoto and Tabata (2010) -H&T in what follows- build a two-sector (health care and non-health care) model that focuses on the role of health care services as ‘consumption goods’. The authors maintain that a rise in the old-age survival probability increases health care demand by older agents, and slows down labor supply growth. Both elements produce a shift in labor from the non-health care sector to the health care sector, lowering the labor productivity growth rate in the non-health care sector (driven by a learning-by-doing mechanism) and, hence, long-run per capita income growth slows down. The authors make clear that they do not consider any positive link between health care services and labor productivity and, consequently, the rise in the old-age dependency ratio caused by the rise in the old-age survival probability inevitably hampers economic growth.

Empirically, there is no consensus about whether an expansion of the life horizon has a positive or a negative effect on growth. For instance, Sala-i-Martin et al. (2004) find that life expectancy is a positive significant determinant of economic growth, in a sample of 98 countries, using a set of 32 variables and the growth rate of per capita GDP between 1960 and 1992. Their results coincide with much of the prior literature surveyed in Bloom et al (2004) and Weil (2007), which observes that countries with better health attain higher rates of economic growth. However, Acemoglu and Johnson (2007) find that higher life expectancy is negatively correlated with economic growth in a cross-country panel analysis. This finding is responded to by Bloom et al. (2009), who claim a strong positive correlation between a country’s initial life expectancy and its subsequent economic growth. Acemoglu and Johnson (2009) reply with new regression results, showing no evidence of positive effects of life expectancy on income per capita at 40- or 60-year horizons. The debate remains open.

The paper by H&T provides support to the negative relationship between longevity and economic growth through population ageing. They point out that population ageing strengthens the competition for resources between sectors, for two parallel reasons: shortages in the labor force, and a higher demand for workers in the health sector. However, this subject also presents inconclusive empirical evidence. First, as the authors mention, Newhouse (1992) found that population ageing is not a main determinant of health demand, since it explains only 4% of the health care expenditure

rise in United States. Second, although Lindh and Malmberg (2009), replicating their OECD study of 1999, do confirm that increasing dependency rates have negative effects on per capita GDP growth, Bloom et al. (2011) estimate that the effect of population ageing on the rate of labor force participation, and the associated effect of changes in labor force participation on economic growth, are of modest size for the majority of countries. Furthermore, Lindh and Malmberg (2009) point to a possible explanation of the negative link between dependency rates of the elderly and economic growth as stemming from a shift of labor towards services, with slower productivity growth, which also involves increasing relative prices for capital due to lower savings rates.

None of the empirical studies concerned with the effects of population dependency rates on aggregate savings present a consensus about the negative, insignificant or positive sign¹. Among these studies, Li et al. (2007) captured our attention by providing new evidence of the effects of both longevity and dependency rates on growth. The key element of their study is the behavior of savings, distinguishing the different roles played by longevity, on one side, and dependency, on the other. Using the World Development Indicators panel data set, they find that the longevity effect on savings is positive, whereas the dependency effect is negative. Coincidentally, the same results apply in investment and growth regressions. In particular, they assess these two opposing forces, finding a positive net effect on economic growth.

In this paper, we argue that the lack of a good match between the results in H&T and the recent evidence provided by Li et al. (2007) lies in the omission of the role of savings as a consequence of the assumption of a *small* economy. Our contribution is to consider the high demand for health care services and the connected high demand for health workers, together with the large savings required to finance a longer retirement. The joint consideration of these elements allows us to go one step further in examining the implications of population ageing for economic growth. In particular, we extend the H&T framework to a general equilibrium context allowing us to analyze how the demand for health care services, as well as savings behavior, are influenced by population ageing. We find that, although greater numbers of the elderly strengthen the demand pressure for health care goods, inducing a shift in labor from the non-health

¹ For example, Edwards (1996) finds a significant negative effect of the old-age dependency rate on the aggregate saving rate, whereas Adams (1971), Gupta (1971), Goldberger (1973), and Ram (1982, 1984) show cases with an insignificant or even a positive effect.

sector to the health sector, and hampering economic growth (ignoring the link between health and productivity), individuals simultaneously modify their savings behavior in order to accommodate the greater needs of retirement. This adaptation favors the accumulation of capital and increases labor productivity in the non-health sector, promoting an opposite shift of labor towards this sector. The net effect of these opposing forces, which we identify as the ‘dependency rate effect’ and the ‘capital accumulation effect’, depends on the degree of substitutability between labor and capital in the production of non-health care goods. When this is high, the dependency rate effect prevails, there is a shift of labor to the health goods sector, and economic growth slows down. However, as capital and labor become less and less substitutable, the capital accumulation effect intensifies and population ageing can be accompanied by an enhancement of economic growth.

The rest of the paper is organized as follows. In Section 2, we extend the H&T framework to a general equilibrium context, determining individual behavior and the market equilibrium. The short-run and long-run effects of population ageing on the economy are analyzed in Section 3. Finally, Section 4 closes the paper with our main conclusions.

2. The model

2.1. *Households*

H&T build the following framework. Individuals can live for three periods, corresponding to childhood, youth and old age. Survival to old age is not assured, with $p \in (0,1]$ being the probability of surviving to the third period of life. An individual born in period $t-1$ is endowed with one unit of time, from which a fraction η_t is supplied to the labor market in t . The remainder is devoted to rearing and educating n_t children (in amounts e and v_t for every child). The effective labor supply is $h_t \eta_t$, with h_t being the level of human capital. Denoting the wage rate w , the labor income amounts to $w_t \eta_t h_t$. Some portion is spent on non-health care goods and the remainder is saved².

² The costs of raising and educating children are exclusively in terms of time.

The surviving elderly agents obtain a capital income in period $t+1$ of $\frac{R_{t+1}}{p}s_t$, where R_{t+1} is the (gross) rate of return on savings. As usual, a perfect annuity market is incorporated, in such a way that the capital income unexpectedly left by individuals who die is shared by the individuals who survive to old age, thus generating a premium $\frac{1-p}{p}$ over the ordinary payoff R on savings. Old-age consumption again involves non-health goods but also health goods in amounts $c_{2,t+1}$ and $z_{2,t+1}$, respectively. The production sector provides both types of product, health care and non-health care goods (denoted by H and N , respectively). The latter are considered as the numeraire good, and q_t denotes the relative price of health care goods.

On the other hand, the time devoted to education of children v contributes to their human capital, according to the education technology $h_{t+1} = \mu v_t^\sigma$.

Every individual derives utility from youth consumption, old-age consumption conditional on survival, and offspring. The expected utility of an individual born in $t-1$ is given by:

$$u^t = \ln c_{1,t} + p \ln \left(c_{2,t+1}^\gamma z_{2,t+1}^{1-\gamma} \right) + \phi \ln \left(n_t h_{t+1} \right), \quad (1)$$

where n_t denotes the number of children and h_{t+1} their level of human capital. Thus, the consumer's problem is given by the maximization of the utility function (1) subject to the budget constraints

$$\begin{aligned} c_{1,t} + s_t &= w_t h_t \eta_t, \\ c_{2,t+1} + q_{t+1} z_{t+1} &= \frac{R_{t+1}}{p} s_t, \end{aligned}$$

the time constraint $\eta_t + (e_t + v_t)n_t = 1$ and the technology of education $h_{t+1} = \mu v_t^\sigma$. From the solution of this problem, the optimal consumption profile is characterized by the following expressions for consumption when young (non-health goods), and old (both non-health and health goods), respectively:

$$c_{1,t} = \frac{1}{(1+p+\phi)} w_t h_t,$$

$$c_{2,t+1} = \frac{\gamma R_{t+1}}{(1+p+\phi)} w_t h_t, \quad (2)$$

$$z_{t+1} = \frac{(1-\gamma) R_{t+1}}{q_{t+1} (1+p+\phi)} w_t h_t, \quad (3)$$

implying an associated individual savings function given by:

$$s_t = \frac{p}{(1+p+\phi)} w_t h_t. \quad (4)$$

At this point, two things are worth highlighting. First, a greater probability of surviving into the third period implies a higher probability of enjoying the consumption derived from the second period savings, which leads to individuals reserving a larger share of income for this last period (Reinhart, 1999). In contrast, the yields on savings fall, since the premium $\frac{1-p}{p}$ obtained through the annuity market from the cut-back set of non-survivors declines. The second force dominates and thus, other things being equal, a higher life expectancy reduces the planned expenditure of both health (3) and non-health goods (2) at the greater age.

Second, given the functional form of the utility function, the share of old age expenditure devoted to health and non-health goods depends exclusively on the preferences parameter γ : $\frac{c_{2,t+1}}{q_{t+1} z_{t+1}} = \frac{\gamma}{1-\gamma}$, implying that the distribution of old-age expenditure is not affected by a change in the probability of surviving.

The solution of the consumer's problem also determines the optimal number of children and the optimal fraction of time devoted to their education as:

$$n_t = n = \frac{\phi(1-\sigma)}{e(1+p+\phi)}, \quad (5)$$

$$v_t = v = \frac{e\sigma}{1-\sigma},$$

respectively, implying that both the human capital of individuals and the fraction of time supplied to the labor market are constant over time:

$$h_t = h = \mu \left(\frac{e\sigma}{1-\sigma} \right)^\sigma,$$

$$\eta_t = \eta = \frac{1+p}{1+p+\phi}.$$

Observe that a rise in the probability of survival does not affect the time devoted to education (and thus, the human capital) by each individual. However, it does induce a reduction in the number of children, which in turn reduces the aggregate time required for their care and education and, as a result, working time expands.

The size of the working population in t is denoted by N_t . This is the population born in $t-1$ from the previous generation, who had n_{t-1} children, thus implying $N_t = n_{t-1}N_{t-1}$. We assume in what follows that $e \leq \frac{\phi(1-\sigma)}{1+p+\phi}$, guaranteeing in (5) a non-negative population growth ($n \geq 1$).

2.2. Firms

The production of health-care goods Y_t^H follows a linear technology:

$$Y_t^H = A_t^H L_t^H,$$

where L_t^H and A_t^H denote the amount of labor hired by firms and the labor productivity index in the health care sector, respectively. This sector does not require capital as an input.

Diverging from H&T, we assume that the production of non-health care goods Y_t^N requires labor (in an amount L_t^N) as well as capital (denoted by K_t) in a constant returns to scale technology represented by the CES production function:

$$Y_t^N = \left[\alpha K_t^{-\theta} + (1-\alpha) (A_t^N L_t^N)^{-\theta} \right]^{-1/\theta},$$

with $0 < \alpha < 1$ and $\theta > -1$. A_t^N denotes the labor productivity index of the non-health care sector. Although our qualitative results would also apply for a generic technology with standard properties, as the one assumed by H&T, we consider the CES case since it makes the problem easier to solve and, above all, because it clearly highlights the key

role played by the elasticity of substitution between inputs in the effects of the population ageing process. The above CES production function presents an elasticity of substitution between capital and labor equal to $1/(1+\theta)$. When $\theta \rightarrow -1$, the inputs are perfect substitutes; as θ increases, the inputs become less and less substitutable. The particular case $\theta = 0$ corresponds to a Cobb-Douglas technology (in which the elasticity of substitution is equal to one).

Denoting by $y^N = Y^N / (A^N L^N)$ and $k^N = K / (A^N L^N)$ the output and capital per unit of effective labor, the technology can be expressed in intensive terms as

$$y_t^N = f(k_t) = [\alpha k_t^{-\theta} + (1-\alpha)]^{-1/\theta}.$$

Again following H&T, the labor productivity indices A_t^N and A_t^H are fostered by the average human capital level of the workers and also, as a consequence of the assumption of a scale external effect at the level of the industry, by the relative hired labor share of the firms of the corresponding sector:

$$A_{t+1}^i = \lambda^i(l_t^i, h) A_t^i, \text{ for } i = H, N. \quad (6)$$

where $l^N = L^N / L$ and $l^H = L^H / L$ denote the fractions of hired labor in the non-health and health sectors, respectively. The function $\lambda^i(\cdot)$ verifies the common conditions that imply the irreversibility of A_t^i : $A_{t+1}^i \geq A_t^i$ for all t . Under this specification, it is clear that the distribution of labor between both sectors plays a key role in determining long-run economic growth.

2.3. *Equilibrium*

In a context of perfect competition, wages (w^H in the health sector, w^N in the non-health sector) and the interest rate will be given by the value of the marginal productivities of capital and labor in the corresponding sector:

$$R_t = \alpha \left(\frac{f(k_t)}{k_t} \right)^{1+\theta}, \quad (7)$$

$$w_t^N = A_t^N (1-\alpha) f(k_t)^{1+\theta}, \quad (8)$$

$$w_t^H = q_t A_t^H . \quad (9)$$

In equilibrium, the wage rate is the same in both sectors $w_t^H = w_t^N = w_t$ and hence, from (8) and (9), the relative price of health care goods is given by:

$$q_t = \frac{w_t^N}{A_t^H} = \frac{A_t^N}{A_t^H} (1 - \alpha) f(k_t)^{1+\theta} . \quad (10)$$

Consequently, an increase in the labor productivity of the non-health sector, relative to that of the health sector, fosters the relative price of health care goods. In parallel, an increase in the intensity of use of capital in non-health firms (pushing up the marginal productivity of labor in this sector) increases such relative price.

With z_t being the individual demand for health goods of any of the pN_{t-1} survivors to old age in t , the market clearing condition for health care goods is given by:

$$A_t^H L_t^H = pN_{t-1} z_t ,$$

which, from (3), can be rewritten as:

$$A_t^H L_t^H = \frac{p}{1+p+\phi} \frac{1-\gamma}{q_t} N_{t-1} R_t w_{t-1} h . \quad (11)$$

An individual supply of labor time η on a population N_t determines the aggregate effective labor L_t as:

$$L_t = \eta N_t h = \frac{1+p}{1+p+\phi} N_t h , \quad (12)$$

which is shared by both health goods and non-health goods sectors in proportions l_t^H and l_t^N . Thus, in labor market equilibrium the following constraint holds:

$$l_t^H + l_t^N = 1 . \quad (13)$$

Making use of (6)-(10) and (12)-(13), expression (11) can be rewritten as:

$$l_t^H = \frac{p(1-\gamma)}{(1+p)n} \frac{w_{t-1} R_t}{w_t} = \frac{p(1-\gamma)}{(1+p)n} \frac{\alpha f(k_{t-1})^{1+\theta}}{\lambda^N (l_{t-1}^N, h) k_t^{1+\theta}} . \quad (14)$$

With respect to capital, used only by the non-health sector, the aggregate supply in period t is determined by the aggregate savings of the previous period: $K_t = N_{t-1} s_{t-1}$. According to (4), this implies

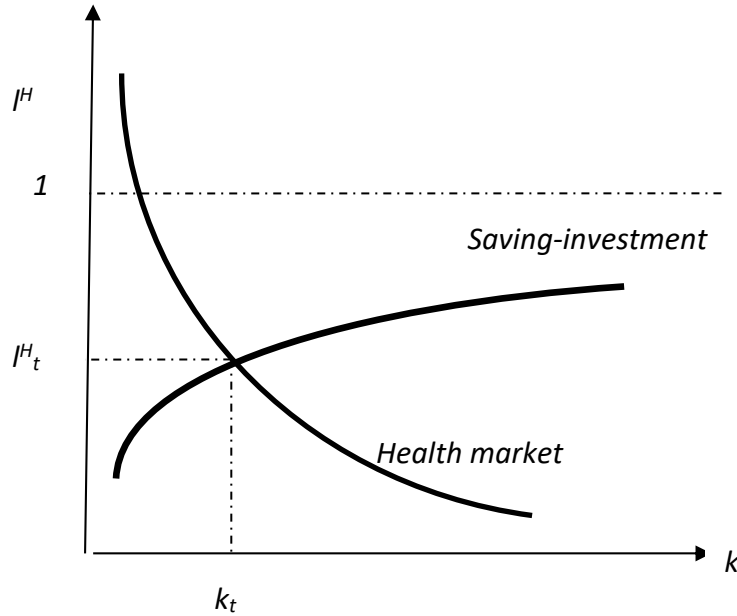
$$K_t = N_{t-1} \frac{p}{1+p+\phi} w_{t-1} h. \quad (15)$$

In intensive terms, taking into account (12) and (13), the capital per unit of effective labor is given by

$$k_t = \frac{p}{n(1+p)} \frac{1}{l_t^N A_t^N} w_{t-1} = \frac{p}{n(1+p)} \frac{1}{\lambda^N (l_{t-1}^N, h)} \frac{(1-\alpha) f(k_{t-1})^{1+\theta}}{(1-l_t^H)}. \quad (16)$$

Given $k_{t-1} > 0$ and $0 < l_{t-1}^N < 1$, the equilibrium in t is determined by equations (14) and (16), providing the new value of capital and the distribution of labor between both productive sectors. Figure 1 depicts both equations³. Expression (14), capturing the equilibrium in the health goods market, defines implicitly one inverse relationship between capital and the share of labor in the health good sector. In turn, the equilibrium savings-investment in expression (16) involves both variables in a positive relationship. The general equilibrium corresponds to the point that fulfills both conditions.

Figure 1. Equilibrium



³ The function in (14) may be convex, concave or linear, depending on the value of θ .

3. The effects of population ageing

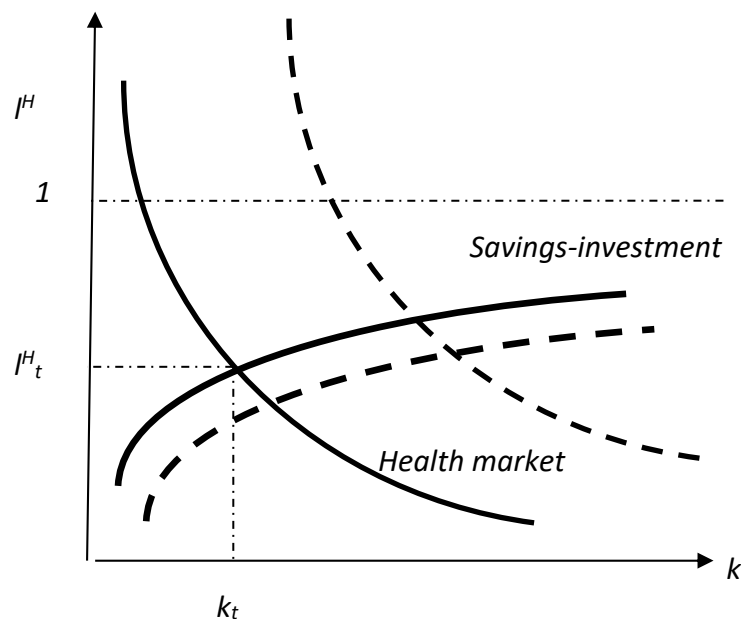
3.1. Short-run effects of population ageing

The effects of an increase in life expectancy on the short-run equilibrium are summarized in the following Proposition:

Proposition 1 *In the short run, a rise in the old-age survival probability p increases the capital per unit of effective labor, but has an indeterminate effect on the distribution of labor between sectors.*

Proof. An increase in p changes the two functions (14) and (16) that determine the equilibrium. On the one hand, the health goods market clearing condition moves upwards because of two effects present in the first fraction on the right-hand side of equation (14): the direct increase in p and the associated fall in n . On the other hand, the same elements cause the savings-investment equation (16) to move to the right. The new equilibrium corresponds to a larger amount of capital. However, the impact on the distribution of labor appears to be indeterminate, as Figure 2 suggests. We will show in proposition 2 that the direction of this impact depends on the elasticity of substitution between inputs in the production of non-health goods.

Figure 2. Short-run effects of an increase in p



What are the reasons for such an ambiguous effect on the distribution of labor between health and non-health goods production? The answer requires us to identify the forces at work behind equation (14), with some of these having to do with the evolution in capital in (16).

Three forces (of a demographic nature) are at work after an increase in p . They appear in the first fraction in (14). First, an increase in individual life-spans increases the number of older individuals, generating an increase in aggregate demand for health care goods. Second, from (5), the increase in p drives a reduction in the fertility rate, increasing the populations of subsequent generations more slowly and, hence, slowing down the working population growth. In parallel, the lower fertility rate also reduces the time devoted to children, increasing the time devoted to work. The net effect of these forces is that the aggregate effective labor force grows more slowly, combined with a greater pressure for the production of health goods, favoring a shift of labor towards the health care sector. This is what we identify as the ‘dependence rate effect’.

The second fraction in (14) includes wages and interest rates. From (7)-(9), these are affected by the evolution of the amount of capital and the labor productivity indices. Focusing on the effect of an increase in p on the evolution of capital, equation (15) shows that, for a given set of values for the variables in $t-1$, the capital available in t is increasing in p . The reason is obvious from (4): as we noticed there, an expectation of longer life increases the rate of savings, allowing for a greater accumulation of capital. Other things being equal, this implies an increase in capital per effective labor in (16), leading to a higher wage as well as to a lower interest rate in t , thus reducing the value of the second fraction in (14). In this way, the greater accumulation of capital acts as a disincentive for labor to move to the health sector, which we can identify as the ‘capital accumulation effect’. Note that, on the one hand, a higher amount of capital boosts wages in the non-health sector, and consequently the relative price of health goods in (10) rises, thus reducing the demand for health goods. On the other hand, the increase in capital reduces the interest rate. Since much of the income in the third period of life comes from the return on savings, that income experiences a cut-back that reinforces the decline in the demand for health goods.

In summary, the extension of life expectancy involves two opposite effects. On one side, the ‘dependency rate effect’ implies a greater pressure for the production of health goods that, in turn, produces a shift of labor towards the health sector. On the

other side, the ‘capital accumulation effect’ boosts wages in the non-health sector and in parallel reduces the income of older individuals and their demand for health, promoting a shift of labor towards the non-health sector. The net effect of these opposite forces, as shown in figure 2, is ambiguous. The analysis of H&T focuses on expression (11) under the assumption of a world with perfect mobility of capital, in which the state plays the role of a ‘small economy’ and thus takes the interest rate in (7) as given; as a consequence, the capital intensity in the production of non-health goods, and the wage in both sectors, become exogenous. In terms of figure 2, the savings-investment function would be vertical. In such circumstances, the ‘capital accumulation effect’ disappears. Then, equation (14) would unambiguously predict a re-allocation of labor towards the production of health goods.

Our incorporation of a new ‘capital accumulation effect’, working in the opposite direction, is what makes the final effect ambiguous. In any case, the assumption of a CES production function of non-health care goods allows us to be more precise about the circumstances in which the extension of life expectancy generates a shift of labor towards one sector or the other, with subsequent consequences for the growth of productivity in each sector and for the whole economy. The results are summarized in Proposition 2.

Proposition 2 *In the short run, an increase in the old-age survival probability p has the following consequences for growth:*

1. *If the elasticity of substitution between inputs is above the unit, the labor productivity growth rate of the health sector increases and that of the non-health care sector decreases, thus decreasing the short-run per capita income growth rate.*
2. *If the elasticity of substitution between inputs is below the unit, these results reverse: the labor productivity growth rate of the health sector decreases and that of the non-health care sector increases, thus increasing the short-run per capita income growth rate.*
3. *If the elasticity of substitution between inputs is the unit, there are no effects on the labor productivity growth rate of any of the sectors, and thus none on the short-run per capita income growth rate.*

Proof. A non-controversial result of the increase in survival probability p is the increase in the amount of capital available, k . From (14) and (16), we have:

$$l_t^H = \frac{1}{1 + \frac{1-\alpha}{(1-\gamma)\alpha} k_t^\rho}. \quad (17)$$

When the elasticity of substitution between capital and labor is above the unit ($\theta < 0$), the expression (17) implies an increase in the fraction of labor hired in the health sector l^H to the detriment of the non-health sector, since, given that in this case the inputs are easily substitutable, the accumulation of capital has a low positive impact on the wage paid by the non-health sector; thus, the ‘dependency rate effect’ prevails, generating a shift in labor from the non-health sector to the health sector. The repercussions for economic growth are obvious. Since the fraction of labor in each sector has a positive scale effect on its productivity growth rate, the increase in the old-age survival probability p increases the labor productivity growth rate of the health care sector, and in parallel reduces that of the non-health sector, determining the per capita income growth rate⁴. Hence, in this case, the increase in p hampers economic growth in the short run.

Conversely, when the elasticity of substitution between capital and labor is below the unit ($\theta > 0$, that is to say, both inputs are relatively necessary to non-health goods production), l^H decreases as k increases in (17). In this case, the ‘capital accumulation effect’ is more intense and leads to a significant rise in the wage paid by the non-health sector. In this situation, labor re-allocation takes place in the reverse direction: towards the non-health sector. Thus, the rise in k will be accompanied by an increase of the labor productivity growth rate of the non-health sector and, hence, by an acceleration of economic growth in the short run.

Finally, in the threshold case of unit elasticity of substitution, corresponding to the Cobb-Douglas function ($\theta = 0$), the availability of capital k has no influence on l^H in (17), indicating that the ‘dependence rate effect’ and the ‘capital accumulation effect’ offset each other in such a way that an extension of longevity provokes no labor

⁴ Note that the wage rate in (8) is related positively with labor productivity in the non-health sector. Although the relationship appears to also be positive with labor productivity in the health sector in (9), the parallel decline in the relative price of health goods (see (10)) cancels the positive effect.

reallocation between sectors. Therefore, the population ageing process has no effect on economic growth.

3.2. Long-run effects of population ageing

After analyzing the short-run equilibrium, this section goes a step further by paying attention to the dynamics of the economy after the shock in longevity and, mainly, on its long-run consequences.

The short-run changes in the allocation of labor and in the accumulation of capital described above are the beginning of a transition dynamics in which the increase in capital modifies the savings decisions as well as the wage in the non-health sector, generating new equilibria over time. Eventually, the economy could reach a new steady state. We will confirm that the short-run effects of the increase in longevity also prevail in the new long-run equilibrium.

The dynamics of capital accumulation can be obtained by substituting (17) into (16), which gives rise to the following expression, relating the value of capital of two consecutive periods:

$$\frac{k_t^{\theta+1}}{(1-\gamma)\alpha + (1-\alpha)k_t^\theta} = \frac{p}{n(1+p)} \frac{f(k_{t-1})^{1+\theta}}{\lambda^N \left(\frac{(1-\alpha)k_{t-1}^\theta}{(1-\gamma)\alpha + (1-\alpha)k_{t-1}^\theta}, h \right)}. \quad (18)$$

The dynamic behavior of the economy, whose motion law is given by (18), is summarized in

Proposition 3 *In the long-run, when $-1 < \theta \leq 0$ the economy always converges asymptotically to a unique interior steady state, k^* . When $\theta > 0$, the economy also converges to a unique steady state provided that θ is close enough to 0.*

Proof. Let us rename as $F(k_t)$ and $G(k_{t-1})$ the left-hand and the right-hand terms of (18), respectively, in such a way that the dynamics are given by $F(k_t) = G(k_{t-1})$. A steady state is a value of capital k^* that verifies $F(k^*) = G(k^*)$. For such equilibrium to be stable, $\left. \frac{dk_t}{dk_{t-1}} \right|_{k=k^*} = \frac{G'(k^*)}{F'(k^*)} < 1$ must hold. By defining $\Phi(k) = G(k)/F(k)$, the steady

states can be identified as the values of capital that make this function equal to the unit $\Phi(k^*) = 1$, being stable when $\Phi'(k^*) < 0$, and unstable otherwise.

From (18), the first derivative of function $\Phi(k)$ is given by:

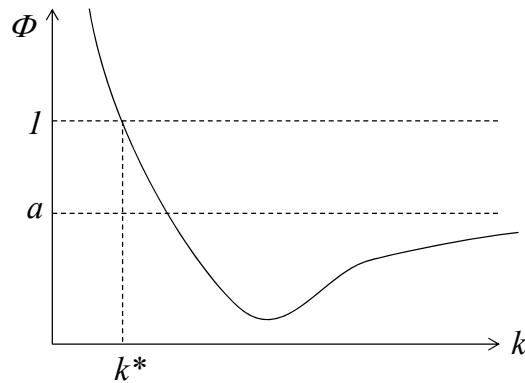
$$\Phi'(k) = \frac{p[\alpha + (1-\alpha)k^\theta]^{-(1+\theta)/\theta}}{n(1+p)\lambda^N k} \left\{ \left[\alpha(1-\gamma) + (1-\alpha)k^\theta \right] \left[\frac{\alpha(1+\theta)}{\alpha + (1-\alpha)k^\theta} - 1 \right] + \alpha(-\theta)(1-\gamma) \left[\frac{\lambda_{t^N}^N}{\lambda^N} \frac{(1-\alpha)k^\theta}{(1-\gamma)\alpha + (1-\alpha)k^\theta} + 1 \right] \right\} \quad (19)$$

Let us begin with the case $-1 < \theta < 0$. The function $\Phi(k)$ verifies $\lim_{k \rightarrow 0} \Phi(k) = \infty$ and

$$\lim_{k \rightarrow \infty} \Phi(k) = \frac{p}{1+p} \frac{(1-\gamma)\alpha^{-1/\theta}}{n\lambda^N(0,h)} = a < 1. \text{ Moreover, the two addends inside brackets in (19)}$$

are of opposite sign: the first is negative and becomes lower as k becomes higher, whereas the second is always positive. We can check that $\Phi'(k) < 0$ for k small and $\Phi'(k) > 0$ for k large enough. That is to say, $\Phi(k)$ decreases with k at the beginning but eventually increases with k , as depicted in Figure 1. The steady state corresponds to the amount of capital that makes $\Phi(k^*) = 1$, which always exists, is unique and stable, as depicted in Figure 3.

Figure 3. Steady state when $-1 < \theta < 0$



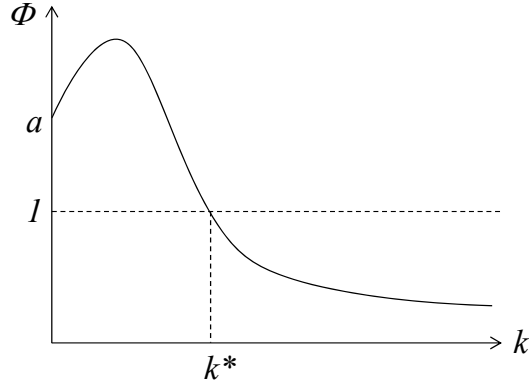
Let us now move to the case $\theta > 0$. The extreme values of the function are now given by $\lim_{k \rightarrow 0} \Phi(k) = a > 0$ and $\lim_{k \rightarrow \infty} \Phi(k) = 0$. Moreover, the second addend inside

brackets in (19) is now negative, with the first also being negative for k high enough. This means that $\Phi(k)$ can increase or decrease with k at the origin, but eventually decreases, tending to zero. Note that with $\theta > 0$ the value of a can be either over or below the unit. The condition

$$\theta < \ln \alpha / \ln \left(\frac{p}{1+p} \frac{1-\gamma}{n \lambda^N(0, h)} \right) \quad (20)$$

does ensure that $a > 1$ and therefore guarantees that a steady state exists, and that it is unique and stable (see Figure 4). Otherwise, there could be no steady state, or multiples of the same.

Figure 4. Steady state when $\theta > 0$ under condition (20)



Finally, when $\theta = 0$, that is to say, the elasticity of substitution between inputs is the unit, the dynamics of capital accumulation given by (18) can be rewritten as

$$k_t = \frac{p}{n(1+p)} \frac{1-\alpha}{\lambda^N((1-\alpha)/(1-\alpha\gamma), h)} k_{t-1}^\alpha, \text{ which is a standard dynamic equation in}$$

which the capital monotonically converges to the steady state.

Taking into account the above proposition, it is possible to analyze the effects of longevity on the capital per effective labor, and the distribution of labor between sectors in the steady-state equilibrium.

Proposition 4 *Provided that the conditions for a steady state equilibrium, unique and stable, hold (proposition 3), the long-run effects of an increase in the old-age survival probability p are the following:*

1. *The long-run capital per effective labor increases.*

2. *If the elasticity of substitution between inputs is above the unit, the labor productivity growth rate of the health sector increases and that of the non-health care sector decreases, and thus the long-run per capita income growth rate decreases.*
3. *If the elasticity of substitution between inputs is below the unit, these results reverse: the labor productivity growth rate of the health sector decreases and that of the non-health care sector increases, and thus the long-run per capita income growth rate increases.*
4. *If the elasticity of substitution between inputs is the unit, there are no effects on the labor productivity growth rate of any of the sectors, and thus none on the long-run per capita income growth rate.*

Proof. An increase in survival probability p positively affects the function $G(k_{t-1})$ and thus moves $\Phi(k)$ upwards. A simple evaluation of Figures 3 and 4 reveals that after such a change the steady-state level of capital per effective labor becomes higher. From expression (17), we can check that this extra accumulation of capital provokes an increase in the fraction of labor hired in the health sector l^H to the detriment of the non-health sector when the elasticity of substitutions between capital and labor is above the unit ($\theta < 0$). On the contrary, the extra accumulation of capital causes an increase in the fraction of labor hired in the non-health sector l^N , to the detriment of the health sector, when the elasticity of substitutions between capital and labor is below the unit ($\theta > 0$). Since the fraction of labor in each sector has a positive scale effect on its productivity growth rate, and the labor productivity growth rate of the non-health care sector governs the per capita income growth rate of the whole economy, the repercussions for economic growth are obvious. The increase in p hampers economic growth in the long run when $\theta < 0$, whereas the increase in p enhances economic growth in the long run when $\theta > 0$. Note that, since the short-run increase in capital is followed by further increases over the transition to the new steady state, the effects of the increase in the old-age survival probability on both the distribution of labor between sectors and the economic growth rate in the steady-state equilibrium are more intense than those in the short-run. Finally, when $\theta = 0$, we have that

$$l^{N*} = \frac{(1-\alpha)}{(1-\gamma)\alpha + (1-\alpha)} \quad \text{and} \quad k^* = \left[\frac{p}{n(1+p)} \frac{1-\alpha}{\lambda^N(l^{N*}, h)} \right]^{\frac{1}{1-\alpha}}, \text{ from where it is immediate}$$

that an increase in survival probability p affects positively the steady-state level of capital per effective labor, but does not affect the allocation of labor between sectors and, thus, neither does it affect the long-run economic growth rate.

Consequently, the results of H&T are maintained in our framework in the long run when $-1 < \theta < 0$, that is to say, when the inputs are easily substitutable. In this case, the accumulation of capital in the steady state has a low positive impact on the wage paid by the non-health sector and the ‘dependency rate effect’ prevails, generating a shift in labor from the non-health sector to the health sector, damaging economic growth.

The above analysis makes clear that the elasticity of substitution between inputs plays a key role in this framework, where individuals modify their savings behavior after an extension of life expectancy, thus determining the behavior of capital. Depending on the degree of input substitution, either the ‘dependency rate effect’ or the ‘capital accumulation effect’ will predominate, in both the short and the long run. This multiplicity of results appears to be in accordance with the empirical evidence on longevity and population ageing presented by Li et al. (2007), and provides support to the open debate about the controversial effects of longevity and population ageing on economic growth.

As a final remark, we want to call attention to a shortcoming of the above analysis that is shared with H&T, namely the assumption of no labor migration between countries (in our case, due to the closed-economy setting). Empirical evidence shows substantial immigration of physicians from developing to developed countries. For instance, Mullan (2005) has estimated that international medical graduates constitute between 23 and 28 percent of physicians in the United States, the United Kingdom, Canada, and Australia, and lower-income countries supply between 40 and 75 percent of these international medical graduates. In the above framework, such migration movements would reduce the shortages of workers derived from the ageing process in the developed countries, thus lowering the relative importance of the ‘dependency rate effect’ and making it more probable that the ‘capital accumulation effect’ would prevail.

4. Conclusions

The relationship between population ageing, demand for health care goods, and economic growth, is complex and controversial. Our contribution to the debate lies in the joint consideration of the demand for health care services and the connected demand for health workers derived from the ageing process, together with the more substantial savings required to finance a longer retirement. We find that, although an increase in the numbers of the elderly strengthens the demand pressure for health care goods, inducing a shift in labor from the non-health sector to the health sector that could damage economic growth (ignoring the link between health and productivity), individuals simultaneously modify their savings behavior in order to accommodate to the greater needs of a longer period of retirement. This latter change favors the accumulation of capital and increases labor productivity in the non-health sector, promoting an opposite shift in labor towards this sector. The net effect of these opposing forces, namely the ‘dependency rate effect’ and the ‘capital accumulation effect’, depends on the degree of substitution between labor and capital in the production of non-health care goods. Specifically, when these inputs are easily substitutable, the ‘dependency rate effect’ prevails and economic growth slows down. However, when the inputs are less substitutable, the ‘capital accumulation effect’ dominates, enhancing economic growth.

The assumption in H&T of a small economy, in a world with perfect mobility of capital, firmly ties the capital-labor ratio of the economy under analysis to the worldwide level, since the interest rate is given by the world rate. Thus, the possible effects through the accumulation of capital disappear when capital is disseminated globally, and its contribution to the worldwide total is negligible. As a result, only the ‘dependency rate effect’ is at work and, thus, population ageing is accompanied by a shift of labor towards the health goods sector and by a slowdown of growth.

Our paper supports the claim that, to the extent that the mobility of capital is not “so perfect” and/or the country is not “so small”, the negative influence of population ageing on growth is reduced, and can even be reversed. From this point of view, the consideration of a given amount of capital can bias conclusions about the consequences of the ageing process. Indeed, Li et al. (2007) show that, on average, the relationship between longevity and growth is positive due to the prevalence of the effects of capital accumulation.

In any case, the overall message is that, even ignoring the connections between health, life expectancy and productivity (healthier people are more productive and can

work for longer), our simple (largely standard) framework gives support to the controversial relationship between population ageing and economic growth reported by the existing empirical evidence.

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