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Asymptotic freedom and higher derivative gauge theories

M. Asorey,[a] **F. Falceto**[a] **and L. Rachwał**[b]

[a] *Centro de Astropartículas y Física de Altas Energías, Departamento de Física Teórica, Universidad de Zaragoza, C/ Pedro Cerbuna 12, E-50009 Zaragoza, Spain*

[b] *Departamento de Física — ICE, Universidade Federal de Juiz de Fora, Rua José Lourenço Kelmer, Campus Universitário, Juiz de Fora, 36036-900, MG, Brazil*

E-mail: asorey@unizar.es, falceto@unizar.com, grzerach@gmail.com

Abstract: The ultraviolet completion of gauge theories by higher derivative terms can dramatically change their behavior at high energies. The requirement of asymptotic freedom imposes very stringent constraints that are only satisfied by a small family of higher derivative theories. If the number of derivatives is large enough ($n > 4$) the theory is strongly interacting both at extreme infrared and ultraviolet regimes whereas it remains asymptotically free for a low number of extra derivatives ($n \leqslant 4$). In all cases the theory improves its ultraviolet behavior leading in some cases to ultraviolet finite theories with vanishing β-function. The usual consistency problems associated to the presence of extra ghosts in higher derivative theories may not harm asymptotically free theories because in that case the effective masses of such ghosts are running to infinity in the ultraviolet limit.

Keywords: Renormalization Group, Renormalization Regularization and Renormalons

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JHEP05(2021)075

Contents

1 Introduction

Field theories with higher derivatives were first considered as covariant ultraviolet regularizations of gauge theories [1]–[3]. However, in the last years there is a renewed interest in these theories mainly due to the rediscovery that they provide a renormalizable field theoretical framework for the quantization of gravity [4, 5]. Field theories with higher derivatives can be also considered as very efficient effective theories in strongly correlated regimes of standard gauge field theories and extended gravity theories in inflationary scenarios [6, 7]. From a fundamental viewpoint higher derivatives theories were disregarded in the past due to the fact that they face pathological behaviors concerning causality and unitarity principles [8, 9]. However, such a behavior is not a problem in some higher derivative theories such as Lee-Wick theories [10]–[13] and theories with ghost condensation phenomena. We shall show that the small family of consistent gauge theories may be extended to asymptotically free gauge theories.

These theories might provide ultraviolet completions of the Standard Model. Higher derivative theories of spin-0 and spin-1/2 fields with local interactions are finite. However, in general relativity and non-abelian gauge theories one cannot get rid of one-loop ultraviolet (UV) divergences that require renormalization [1]–[3]. We shall show that there are some very special higher derivative theories that do not require renormalization because all UV divergences cancel out. These finite theories do not have UV singularities even classically, which is particularly interesting in gravity theories because they lead to black holes without hidden singularities and evolutions of the Universe without Big Bang singularity [14, 15]. However, this does not exclude that some of those theories can present infrared (IR) divergences.

In this paper, we analyze from this perspective the ultraviolet behavior of higher derivative gauge theories and, in particular, the asymptotic freedom behavior at extremely high energies. We also analyze the emergence of IR divergences and how the renormalization group (RG) can smoothly interpolate between these two asymptotic regimes and solve the consistency problems associated to the appearance of extra ghost fields.

JHEP05(2021)075

2 Higher derivative gauge theories

In the Euclidean formalism a pure gauge theory with higher derivatives can be defined by the following action

$$S = \frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{\mu\nu a} + \frac{1}{4g^2\Lambda^{2n}}\int d^4x\, F^a_{\mu\nu}\,\Delta^n\,F^{\mu\nu a}\,, \tag{2.1}$$

where the operator Δ defined by

$$\Delta^{\nu\, b}_{\mu\, a} = -D^2\delta^\nu_\mu\delta^b_a + 2f^b{}_{ca}\,F_\mu{}^{\nu c} \tag{2.2}$$

is the Hodge-covariant Laplacian operator $\Delta = d^*_A d_A + d_A d^*_A$ acting on the Lie algebra $\mathfrak{g}-$valued 2-forms F of the gauge field strength $F_{\mu\nu}$ and D_μ is the gauge-covariant derivative. The gauge group is denoted by G here and its corresponding algebra by $\mathfrak{g}$. The gauge-covariant exterior differential on forms we denote by d_A and d^*_A is its Hodge-dual.

These new theories are not conformally invariant, in contrast with standard Yang-Mills theory in $d = 4$ spacetime dimensions. But they preserve all solutions of the Yang-Mills vacuum equation of motion $d^*_A F = 0$, including standard instantons ($F = \pm * F$) for Euclidean theory, as exact solutions of their equations of motion.

Although one-loop corrections around such solutions are different than that of ordinary Yang-Mills theories and might have a softer UV behavior, they share the same pathological IR behavior, which usually spoils the instanton physical implications.

For $n \geqslant 2$ the new theories (2.1) are superrenormalizable. They only have one-loop UV divergences [1, 3]. Higher order contributions become finite once the one-loop divergences are renormalized [16, 17].

The calculation of UV divergences can be performed in α-gauge by adding the following gauge-fixing functional to the action (2.1)

$$S_\alpha = \frac{\alpha}{2g^2\Lambda^{2n}}\int d^4x\,\partial^\mu A^a_\mu(-\partial^\sigma\partial_\sigma)^n\partial^\nu A^a_\nu \tag{2.3}$$

and by using dimensional regularization with $\epsilon = 4-d$, where d is the regularized dimension of Euclidean space. Returning to Minkowski spacetime formalism, the result for the UV-divergent contribution of the two-point gluonic function coming from one-loop vacuum polarization diagrams is (see appendix)

$$\Gamma^{ab}_{\mu\nu}(p) = -c_n\frac{C_2(G)}{16\pi^2\epsilon}i\delta^{ab}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right) \tag{2.4}$$

with

$$c_0 = \alpha - \frac{13}{3}, \quad c_1 = -\frac{43}{3} \quad \text{and} \quad c_n = 5n^2 - 23n + \frac{29}{3}, \quad \text{for} \quad n \geqslant 2, \tag{2.5}$$

which agrees for $n \geqslant 2$ with the results of Asorey-Falceto [16, 17] and for $n = 1$ with those of Babelon-Namazie [18]. In the radiative correction to the two-point function (2.4), p is the momentum of the incoming gluon, $\eta_{\mu\nu}$ denotes the Minkowski metric of flat spacetime and $C_2(G)$ is the quadratic Casimir operator of the gauge group G. Notice the independence of c_n on the gauge fixing parameter α for $n \geqslant 1$, because in that case all α-dependent terms

JHEP05(2021)075

are finite at any loop order (see appendix for a detailed explanation). It can be shown that in such a case the divergent contributions to 3-point and 4-point gluonic functions preserve gauge invariance, and thus, all UV one-loop divergences can be removed by a simple counterterm

$$S_{\text{count}} = c_n \frac{C_2(G)}{128\pi^2} \left(\frac{2}{\epsilon} + \log \frac{\Lambda^2_{\text{QCD}}}{\Lambda^2} \right) F^a_{\mu\nu} F^{\mu\nu a}, \tag{2.6}$$

where the Λ_{QCD} scale has been introduced by a renormalization prescription, which adds a finite counterterm to the minimal renormalization scheme to recover in the IR the renormalized two-point function of standard QCD towards our higher derivative theory (2.1) tends to.

In the case $n = 1$, there are still some two-loop divergent contributions which require additional renormalization. In the case $n = 0$, besides the corresponding one-loop counterterm a most careful analysis is required, because the 3-point and 4-point contributions cannot be simply absorbed by a renormalization of the YM coupling constant g, but also need a field renormalization

$$A_{r,\mu} = \left(1 + c_0 \frac{C_2(G)}{32\pi^2 \epsilon} \right) A_\mu, \tag{2.7}$$

which absorbs all α-dependence of c_0. On the other hand the α-independence of one-loop radiative corrections to the coupling constant g follows from the BRST invariance of the theory. Moreover, as it is well known in the last case the renormalization process has to be extended to any loop order.

The effect of higher derivative terms leads to a modified β-function. In the case $n \geqslant 1$, the result is

$$\beta_n = c_n \frac{g^3 C_2(G)}{32\pi^2}, \tag{2.8}$$

whereas in the case $n = 0$, once the wave-functions of gluons and ghosts are renormalized, one gets the standard Yang-Mills β-function of the coupling constant [17].

Summing up all orders of perturbation theory one gets the renormalization group flow of the bare coupling constant

$$g^2_{\text{bare}}(\mu) = \frac{g^2}{1 - \frac{g^2 C_2(G)}{(4\pi)^2} c_n \log \mu/\Lambda}, \tag{2.9}$$

which is running to 0 as the renormalization scale μ goes to infinity. Since the higher derivative terms of the action (2.1) do not get any divergent radiative correction, the coefficient $g^2 \Lambda^{2n} = g^2_{\text{bare}} \Lambda^{2n}_{\text{bare}}$ remains unrenormalized. Thus, the ultraviolet behavior of the bare mass Λ_{bare} parameter is just the oposite of g_{bare}, i.e. $\beta_\Lambda = -\frac{\Lambda}{ng}\beta_n$, which implies that Λ_{bare} is running to infinity as the renormalization scale μ goes to infinity.

This observation allows to extend the family of consistent higher derivative theories beyond the expected tree-level bounds, because the masses of all pathological ghosts breaking causality and unitarity are proportional to Λ_{bare}, and for asymptotically free theories these ghosts become infinitely heavy and decouple from the physical spectrum in the UV regime.

JHEP05(2021)075

3 Asymptotic freedom

The behavior of the renormalized two-point functions is more involved. The one-loop result at leading order in Λ is

$$\Gamma^{ab}_{\mu\nu}(p) = -\frac{C_2(G)}{32\pi^2} i\delta^{ab}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\Pi(p^2), \tag{3.1}$$

with

$$\Pi(p^2) = \left(b_n \log\frac{p^2+\Lambda^2}{\Lambda^2} + c_0 \log\frac{p^2}{\Lambda^2_{\rm QCD}}\right), \tag{3.2}$$

where we identified an arbitrary renormalization scale μ with the higher derivative scale Λ and

$$b_0 = 0, \quad b_1 = -10-\alpha, \quad \text{and} \quad b_n = 14-\alpha-23n+5n^2 \quad \text{for } n \geqslant 2 \tag{3.3}$$

or generally $b_n = c_n - c_0$ for any n.

There are two different asymptotic regimes, the $p \gg \Lambda$ UV regime, where

$$\Gamma^{ab}_{\mu\nu}(p) = -c_n\frac{C_2(G)}{32\pi^2} i\delta^{ab}\log\frac{p^2}{\Lambda^2}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right) \tag{3.4}$$

and the intermediate IR regime $\Lambda_{\rm QCD} < p \ll \Lambda$, where we recover the renormalized two-point function of standard QCD

$$\Gamma^{ab}_{\mu\nu}(p) = -c_0\frac{C_2(G)}{32\pi^2} i\delta^{ab}\log\frac{p^2}{\Lambda^2_{\rm QCD}}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right). \tag{3.5}$$

The renormalization group (RG) flow of the higher derivative Yang-Mills theory interpolates for $n \geqslant 1$ between these two asymptotic regimes with two different beta functions

$$\beta_{\rm UV} = c_n\frac{g^3 C_2(G)}{32\pi^2} \quad \text{and} \quad \beta_{\rm IR} = -\frac{22}{3}\frac{g^3 C_2(G)}{32\pi^2}. \tag{3.6}$$

In the infrared regime one recovers the standard values of the β-function for asymptotically free Yang-Mills scenario. However, in the ultraviolet regime there is a variety of scenarios involving either asymptotic freedom or asymptotic slavery, except in the case $n = 0$ where the β-function remains frozen in the QCD asymptotically free regime for all $p > \Lambda_{\rm QCD}$ (see figure 1).

Assuming that n has to be an integer in order to preserve locality, the number of UV asymptotically free theories is very limited. It reduces to theories with negative coefficients of the β-function, i.e., only to five cases $n = 0, 1, 2, 3, 4$:

$$\widetilde{c}_0 = -\frac{22}{3}, \quad c_1 = -\frac{43}{3}, \quad c_2 = -\frac{49}{3}, \quad c_3 = -\frac{43}{3}, \quad c_4 = -\frac{7}{3}. \tag{3.7}$$

For all other integer values of n, $c_n > 0$. This shows how the limitation to UV asymptotically free theories imposes severe constraints on higher derivative gauge theories.

In order to enlarge the family of theories with UV asymptotically free regimes one can introduce an extra dimensionless coupling λ in the theory, and replace the Hodge Laplacian operator (2.2) in (2.1) by the operator ${}^\lambda\Delta$ defined by

$${}^\lambda\Delta^{\nu\, b}_{\mu\, a} = -\delta^b_a\delta^\nu_\mu D^2 + 2\lambda f^b{}_{ca}\, F_\mu{}^{\nu c}. \tag{3.8}$$

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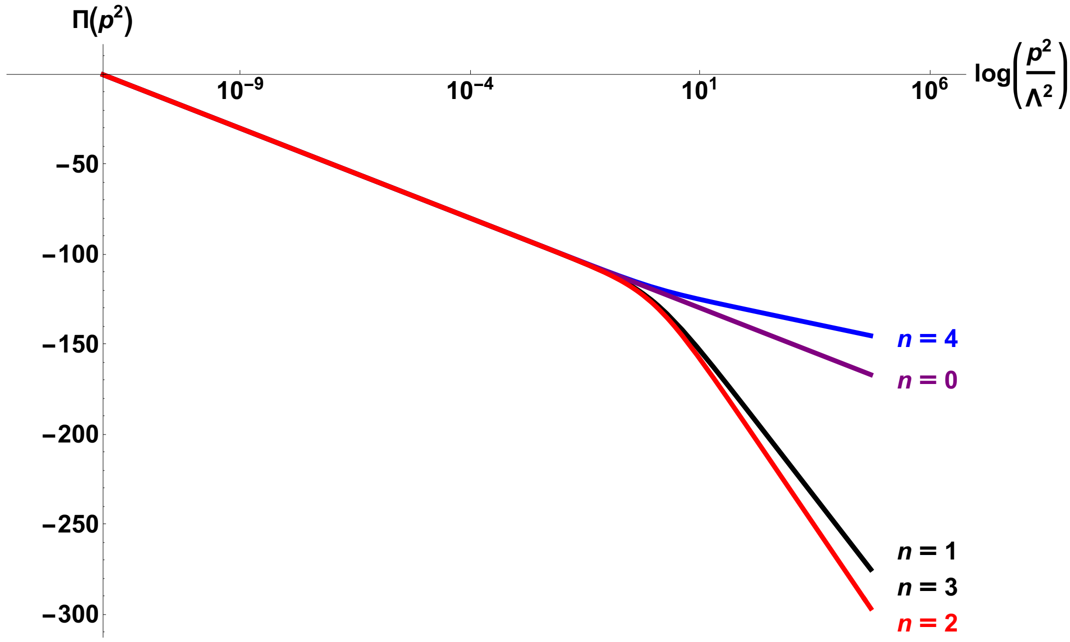

Figure 1. Momentum dependence of one-loop radiative corrections to the 2-point function form factor $\Pi(p^2)$ for different higher derivative theories. The cases $n = 1$ and $n = 3$ give rise to the same form factors. The results for large momenta are independent of the gauge fixing parameter α, whereas in the IR we used a Lorentz gauge with $\alpha = 0$. In the infrared regime $\Lambda_{\rm QCD} < p \ll \Lambda$, the behavior is independent on the number of extra derivatives and agrees with that of standard gauge theories. In the UV regime $p \gg \Lambda$, the behavior depends on the number of extra derivatives, but in all five cases the theory is asymptotically free. We assumed that the higher derivative scale Λ is of the order of 100 TeV, i.e. $\Lambda \approx 10^6 \Lambda_{\rm QCD}$.

For $\lambda \neq 1$ instantons ($d^*_A F = 0$) are not anymore solutions of the Euclidean equations of motion. The Euclidean extension of operator ${}^\lambda\Delta$ is positive for $\lambda = 1$ and $\lambda = 0$, but not for n odd and larger than 1. However, the action of the theory remains positive for any even n, which is a physically relevant requirement. In this case, the structure of one-loop divergences gets modified because now

$$c_n = \begin{cases} -9\lambda^2 - 18\lambda + \frac{38}{3}, & \text{for } n = 1 \\ \\ \left(5n^2 - 18n + 16\right)\lambda^2 - \left(4n^2 + 10n + 4\right)\lambda + 4n^2 + 5n - \frac{7}{3}, & \text{for } n \geqslant 2. \end{cases}$$

Although these coefficients reduce to (2.5) in a continuous way when $\lambda \to 1$.

The coupling λ does not get any divergent radiative contribution and, thus, is kept fixed under RG flow as an extra parameter of the theory. On the other hand, for higher values of $\lambda \gg 1$ the constraint of having a window with asymptotic freedom is even more stringent. In fact, in the limit $\lambda \to +\infty$, only the theory with $n = 2$ is marginally asymptotically free. Actually, for $n = 2$ and $\lambda \to +\infty$ the theory becomes UV-finite. Notice that the minimal

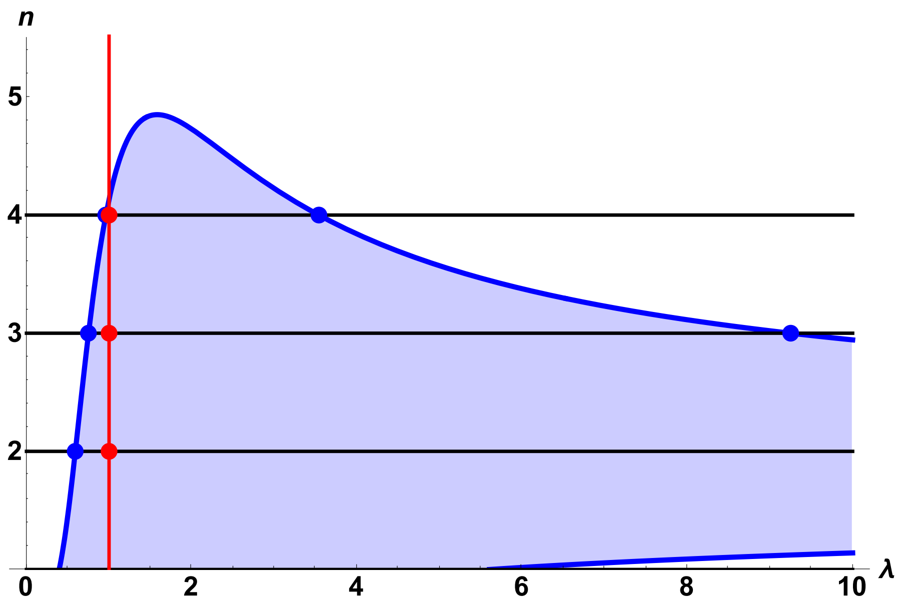

Figure 2. Domain of asymptotically free gauge theories (shadow blue area). Horizontal lines correspond to local theories and λ is the extra parameter of the generalized Laplacian operator (3.8). Red points mark theories with Hodge Laplacians ($\lambda = 1$) and blue points UV-finite theories. In the limit $\lambda \to \infty$ the upper and lower curves converge at $n = 2$.

choice of scalar covariant Laplacian ${}^0\Delta = -D^2$ never gives rise to asymptotically free theories. In fact, the above results imply that the Hodge-covariant Laplacian Δ is the optimal choice to get asymptotic freedom in a larger number of higher derivative gauge theories.

It is also interesting to remark that the theories are UV-finite for special values of the parameter λ:

$$
\begin{array}{lll}
n = 1 & \lambda_1 = -2.55 & \lambda_2 = 0.55 \\
n = 2 & \lambda \;\; = \;\; 0.59 & \\
n = 3 & \lambda_1 = \;\; 0.75 & \lambda_2 = 9.25 \\
n = 4 & \lambda_1 = \;\; 0.96 & \lambda_2 = 3.54
\end{array}
$$

without any need of supersymmetry. Notice that the introduction of operator ${}^\lambda\Delta$ recalls the addition of terms cubic or quartic in field strengths to the action of a higher derivative theory, which is crucial for achieving UV-finiteness [19–21].

For $G = \mathrm{SU}(N)$ the interaction with fundamental quarks generates a lower (absolute) value of the β-function

$$\beta_n = \left(c_n N + \frac{4}{3} N_q\right) \frac{g^3}{32\pi^2} \tag{3.9}$$

for $n \geqslant 1$, and thus, slightly stronger constraints that shrink the window of UV asymptotically free regime. In the case $\lambda = 1$, the window is preserved for small number of quarks

$N_q < \frac{43}{4}N$. For a larger number of quarks the window shrinks till its disappearance in the large N_q limit.

4 Discussion

In summary, gauge theories with higher derivatives can be asymptotically free in the UV regime, if the total number of higher derivatives is not bigger than 8. Otherwise, the theories become UV-slave and exhibit a strong interacting regime both in the UV and in the deep IR. There is an intermediate regime $\Lambda_{\rm QCD}^2 < p^2 \ll \Lambda^2$, where the theory is always asymptotically free with the same scaling properties as in the standard QCD at high energies.

This means that there are UV-completions of gauge theories, where the behavior of the theory for energy scales larger than Λ can dramatically change from asymptotic freedom to asymptotic slavery.

One remarkable feature of higher derivative theories is the possibility of having an UV scale-invariant fixed point with $\beta = 0$, where the theory becomes finite, without any UV divergence [21]. The behavior of these theories in the presence of a θ-term deserves further analysis. In particular, it will be interesting to find the appearance of non-standard effective potentials, which might have implications for axion physics phenomenology.

An interesting remaining open problem is the analysis of the dynamics of the hidden ghost sector and its implications for unitarity, causality and stability of these theories. Ghost fields appear in conjugate pairs of poles of the propagator as in the case of Lee-Wick theories [10]–[13]. However, in the present case of asymptotically free theories the running of the bare mass $\Lambda_{\rm bare}$ towards $+\infty$ in the UV regime can make the pathological effects of ghosts harmless. It is remarkable that this property only holds for asymptotically free theories, which enhances the relevance of such a UV behavior for the consistency of higher derivative theories.

These results are compatible with the behavior of lattice gauge theories. The Wilson formulation of lattice gauge theories satisfies in the Euclidean formalism the reflection positivity property which guarantees unitarity and stability of the corresponding quantum theory. The first two leading terms in the continuum limit of Wilson's action of lattice gauge theories do coincide with those of (2.1) for $n = 1$. On the other hand, non-abelian lattice gauge theories turn out to be also asymptotically free. These two relevant properties can provide an ultimate argument for the consistency of asymptotically free higher derivative gauge theories.

In spite of this fact some physical effects of the pairs of complex poles can still be traced down in the behavior of the trace anomaly on curved backgrounds. Under certain consistency conditions a-theorem establishes that the coefficient of the Gauss-Bonnet term of the anomaly a must evolve in a monotonically decreasing way under the renormalization group flow [22]. However, in the case of the UV asymptotically free theories analyzed in this paper a is positive in standard infrared regime whereas in the UV regime it can reach negative values [23–25]. This breaking of a-theorem raises some questions that require a deeper analysis.

The above results open a new interesting perspective for the analysis of higher derivative theories of quantum gravity, which deserves further study.

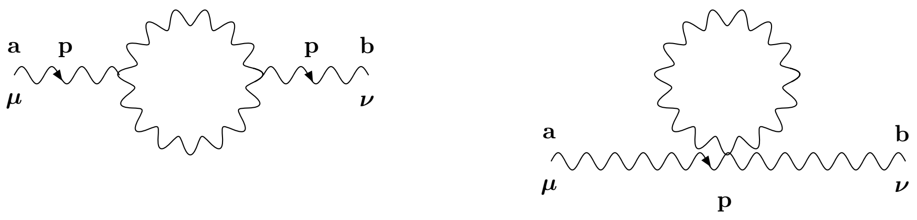

Diagram (1)   Diagram (2)

Figure 3. Gluonic one-loop Feynman diagrams contributing to the 2-point functions of higher derivative theory.

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A Two-point functions: one-loop gluonic UV divergences

One-loop pure gluonic contribution to the two-point gluon function is given by Feynman diagrams of figure 3.

For theories with $n > 1$ the UV-divergent contribution of diagram (1) of figure 3 is given in dimensional regularization by

$$\begin{aligned} {}^{(1)}\Gamma^{ab}_{\mu\nu}(p) = \frac{iC_2(G)\delta^{ab}}{64\pi^2\epsilon} p^2\eta_{\mu\nu}\Bigg[&\left(4n^2+16n+16\right)\lambda^2+\left(8n^2+32n+32\right)\lambda \\ &+\frac{2n^4}{3}+\frac{8n^3}{3}-8n^2+\frac{8n}{3}+\frac{50}{3}+\left(\frac{10n^2}{3}+\frac{32n}{3}+8\right)\alpha\Bigg] \\ -\frac{iC_2(G)\delta^{ab}}{64\pi^2\epsilon} p_\mu p_\nu\Bigg[&\left(4n^2+16n+16\right)\lambda^2+\left(8n^2+32n+32\right)\lambda \\ &-\frac{4n^4}{3}-\frac{16n^3}{3}-16n^2+\frac{8n}{3}+\frac{56}{3}+\left(\frac{4n^2}{3}+\frac{20n}{3}+8\right)\alpha\Bigg], \end{aligned} \tag{A.1}$$

where $\epsilon = 4 - d$.

The corresponding UV-divergent contribution of diagram (2) of figure 3 is

$$\begin{aligned} {}^{(2)}\Gamma^{ab}_{\mu\nu}(p) &= \frac{iC_2(G)\delta^{ab}}{64\pi^2\epsilon}p^2\eta_{\mu\nu}\Big[\Big(-24n^2+56n-80\Big)\lambda^2+\Big(8n^2+8n-16\Big)\lambda \\ &\qquad -\frac{2n^4}{3}-\frac{8n^3}{3}-8n^2-\frac{68n}{3}-8-\left(\frac{10n^2}{3}+\frac{32n}{3}+8\right)\alpha\Big] \\ &\quad -\frac{iC_2(G)\delta^{ab}}{64\pi^2\epsilon}p_\mu p_\nu\Big[\Big(-24n^2+56n-80\Big)\lambda^2+\Big(8n^2+8n-16\Big)\lambda \\ &\qquad +\frac{4n^4}{3}+\frac{16n^3}{3}-\frac{68n}{3}-8-\left(\frac{4n^2}{3}+\frac{20n}{3}+8\right)\alpha\Big]. \end{aligned} \tag{A.2}$$

The sum of these diagrams

$$\begin{aligned} {}^{(1)}\Gamma^{ab}_{\mu\nu}(p)+{}^{(2)}\Gamma^{ab}_{\mu\nu}(p) &= -\frac{iC_2(G)\delta^{ab}}{16\pi^2\epsilon}\Big[\Big(5n^2-18n+16\Big)\lambda^2-\Big(4n^2+10n+4\Big)\lambda \\ &\quad +4n^2+5n-\frac{13}{6}\Big]\Big(p^2\eta_{\mu\nu}-p_\mu p_\nu\Big)-\frac{iC_2(G)\delta^{ab}}{32\pi^2\epsilon}\,p_\mu p_\nu \end{aligned} \tag{A.3}$$

cancels out the n^4 and n^3 terms and the α-dependence. The cancelation of α-dependence can be understood in background gauge formalism [8, 26] as a consequence of the fact that the infinitesimal variation of the effective action under changes of α is given by a functional equation where one of the factors is proportional to the equations of motion of the theory [8]. Since these equations involve higher derivative terms which do no get any divergent contribution from radiative corrections we can conclude that there are no α-dependent UV divergences due to one-loop radiative corrections to any gluonic n-point function, which is in agreement with the above explicit calculation (A.3).

The contribution of Faddeev-Popov ghosts to $\Gamma^{ab}_{\mu\nu}$ is given by diagram (3) of figure 4

$${}^{\rm FP}\Gamma^{ab}_{\mu\nu}(p)=\frac{iC_2(G)\delta^{ab}}{32\pi^2\epsilon}\left[\frac{1}{3}(p^2\eta_{\mu\nu}-p_\mu p_\nu)+p_\mu p_\nu\right]. \tag{A.4}$$

Notice that it is the same as that of the standard Faddeev-Popov ghosts in ordinary Yang-Mills theory. This is a consequence of the factorization of the higher derivative Faddeev-Popov determinant

$$\det[(-\partial^\sigma\partial_\sigma)^n(-\partial^\mu D_\mu)]=\det(-\partial^\sigma\partial_\sigma)^n\det(-\partial^\mu D_\mu) \tag{A.5}$$

and the fact that $-(\partial^\sigma\partial_\sigma)^n$ does not give any contribution to the gluonic n-point functions.

Finally, the sum of all diagrams contributing to the two-point function reads

$$\begin{aligned} \Gamma^{ab}_{\mu\nu}(p) &= -\frac{iC_2(G)\delta^{ab}}{16\pi^2\epsilon}\Big[\Big(5n^2-18n+16\Big)\lambda^2-\Big(4n^2+10n+4\Big)\lambda \\ &\quad +4n^2+5n-\frac{7}{3}\Big]\Big(p^2\eta_{\mu\nu}-p_\mu p_\nu\Big). \end{aligned} \tag{A.6}$$

The explicit transverse structure of the two-point function $\Gamma^{ab}_{\mu\nu}(p)$ is a consequence of BRST invariance of the theory.

JHEP05(2021)075

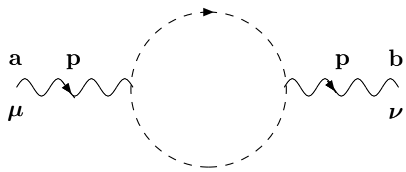

Diagram (3)

Figure 4. One-ghost-loop Feynman diagram contributing to the 2-point functions.

Diagrams involving two external ghosts and any number of external gluons are finite because in the corresponding Feynman graphs there is always one more gluon propagator with higher derivatives than vertices with higher derivatives. This implies that the gluonic wave function does not require renormalization. Thus all divergences can be absorbed by gauge coupling renormalization that by BRST symmetry cannot depend on the gauge fixing parameters. This fact explains why in higher derivatives theories the gluonic 2-point function does not depend on the gauge fixing parameter α as shown by the explicit calculation (A.6). Notice that for this reason the result holds even for gauge fixing conditions with less derivatives whenever the number of extra derivatives of the gauge fixing terms is larger than one. This fact guarantees that both the transverse and longitudinal parts of the wave functions do not acquire any divergent contribution from radiative corrections.

Similar results hold for the UV-divergent contributions in $n = 1$ higher derivative theories. The gluonic one-loop contribution to $\Gamma^{ab}_{\mu\nu}(p)$ is given by the two diagrams of figure 3. The contribution of diagram (1) of figure 3 is

$$ {}^{(1)}\Gamma^{ab}_{\mu\nu}(p) = \frac{iC_2(G)\delta^{ab}}{32\pi^2\epsilon}\left[\left(18\lambda^2 + 36\lambda + \frac{22}{3} + 11\alpha\right)(p^2\eta_{\mu\nu} - p_\mu p_\nu) + (8+3\alpha)\, p_\mu p_\nu\right] $$

and that of diagram (2) is

$$ {}^{(2)}\Gamma^{ab}_{\mu\nu}(p) = -\frac{iC_2(G)\delta^{ab}}{32\pi^2\epsilon}\left[(33+11\alpha)\,(p^2\eta_{\mu\nu} - p_\mu p_\nu) + (9+3\alpha)\, p_\mu p_\nu\right]. \tag{A.7} $$

And in the sum of these two diagrams,

$$ \Gamma^{ab}_{\mu\nu}(p) = \frac{iC_2(G)\delta^{ab}}{32\pi^2\epsilon}\left[\left(18\lambda^2 + 36\lambda - \frac{77}{3}\right)(p^2\eta_{\mu\nu} - p_\mu p_\nu) - p_\mu p_\nu\right] \tag{A.8} $$

the α-dependence cancels out by the same reasons that in the $n \geqslant 2$ case.

When we add the contribution of diagram (3) of figure 4 with one FP ghost loop given in (A.4) we finally obtain

$$ \Gamma^{ab}_{\mu\nu}(p) = \frac{iC_2(G)\delta^{ab}}{16\pi^2\epsilon}\left(9\lambda^2 + 18\lambda - \frac{38}{3}\right)\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right), \tag{A.9} $$

which again has a transverse structure in agreement with BRST symmetry.

JHEP05(2021)075

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JHEP05(2021)075

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