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RESEARCH

The evolutionary political economy of dichotomized societies

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Abstract

Referendum practices and other forms of antagonistic political participation have become more commonplace in the last two decades. These practices have significant social and economic consequences and, as such, economic theory must study their development. In this work, we study the cases of binary political choices, encompassed in what we call the evolutionary political economy of dichotomized societies. We see the origins of these phenomena in certain streams of socio-political thought and analyze the conditions of their evolution. We also link our study with contributions to polarization research in socio-physics and mathematical sociology. Drawing on these fields, we present a new model that allows us to analyze these processes and obtain scenarios with different implications. Drawing on the model, we ask questions such as: can we determine specific conditions under which a referendum may end up truly reflecting the structural trend of public opinion? Are there situations in which dichotomized political processes may lead to surprising results? Can we characterize polarization as an emergent property of evolving political economies? Depending on specific parametric regimes, very different answers to these questions emerge.

Keywords Polarization · Political economy · Referendum · Evolutionary modeling · Dynamics

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1 Introduction

The first two decades of the 21st century have been exceptionally turbulent ones. The Great Recession, which began in 2008 and the subsequent citizen movements (Occupy), the demand and acceptance of referendum practices for issues with strong political and economic effects (BREXIT), political polarization in many areas of the world even regarding presidential elections, the pandemic lockdowns with posterior supply-side bottlenecks and inflationary consequences, and the current geopolitical confrontations, have all pulled apart The End of History anticipated by Francis Fukuyama (1992), and have exacerbated political tensions. Faced with these turbulent challenges and their exasperation effects, certain ideologies (as carriers of beliefs regarding how society should be organized, but often ignoring the complexity of problems and creating oversimplified analyses) have spread far and wide. They have led to social debates in which the issues were simply posed in a *for-vs.-against* manner. This has been accompanied by the increasing prominence of citizens taking a direct stance, often antagonistically, with immediate connectivity to digital and social networks and revealing a like–dislike propensity to post emotional opinions on complex themes.

This situation reminds us of the image of an ongoing (often informal) "referendum-society", understood not in its formal legalized sense but referring to the original etymology of the word referendum: namely, the increasing prevalence of antagonistically framed participatory processes in democracy seeking to restore debate and even effective power back to its origins, that is, to "the people". With the added difficulty that, due to the circumstances of our century, and influenced by certain streams of contemporary thought, those conforming "the people" have become highly polarized.

In this work, we shall refer to any process of dynamic positioning by boundedly rational citizens through action, political learning, and direct debate around opposing stances as an evolutionary political process within a dichotomized framework. We aim to analyze large-scale processes of this type (exacerbated by the upheavals and technical means of our century) from the perspective of institutional evolutionary economics (Dopfer and Potts 2008; Nelson 2018; Hodgson 2019; Dosi et al. 2020; Dosi 2023). In these processes, citizens freely take a stance and may reconsider their views in uncertain learning conditions regarding topics such as redistribution issues, remaining in or leaving (as a nation or region) a specific political and economic placement, being in favor of – or against – a controversial political candidate; and other more atomized questions which, in our open, emotionally charged, digital societies are constantly posed in antagonistic terms.

These debates all occur in our societies (through different networks and digital platforms) continuously, and develop independently of whether the process eventually leads to a legally regulated vote with governmental effects or not, and/or whether they actually produce legislative or administrative changes or not. The debates develop massively across the population within dichotomized realms (face-to-face talks, traditional or digital discussions, short posts), even in cases

where experts present well-supported arguments. Furthermore, the dichotomized processes we see in reality and shall study often surpass the institutions established by states; they often call for a significant re-allocation of resources and try to shape decision-making; changes in attitudes develop in conditions of imperfect information and uncertain learning; and majorities revise their claims within contexts which they do not understand well.

Obviously, economic theory must take part in the analysis of these processes, given their allocative and deliberative components which shape reality and produce economic consequences. In fact, problems related to collective action have been a classic issue in economics since, at least, the first liberals and utilitarian thinkers. At present, social choice theory (SCT), which sets out from the works of Arrow (1951, 1963) and Sen (1970), has established in mainstream economics that the aim of this theory is the analysis of individual preference relations regarding socio-political themes, the best ways to "aggregate" these preference relations fulfilling reasonable conditions, and the study of whether these "social preferences" produce consistent prescriptions by rational decision-makers. Matters concerning SCT include the implementation of redistributive measures associated with the Second Fundamental Theorem of Welfare Economics, or the application of policies to correct market failures (Debreu 1959; Arrow and Hahn 1971). However, although it has been argued that SCT fits well with the classical vision of democracy (as a method of social decision focused on reaching the common good via the integration of free individual rational actions taken in static contexts; Downs 1957 and Arrow et al. 2011), in our opinion, the SCT approach is excessively rigid when dealing with the processes described above (Almudi and Fatas-Villafranca 2021).

We recognize that the development of SCT has undoubtedly influenced the neo-contractualist philosophers (Rawls 1971; Nozick 1974), in public choice theory (Buchanan and Tullock 1962; Mueller 1993), and – of course – in mainstream Neoclassical-Neokeynesian thinking (Drazen 2000; Grossman and Helpman 2001; Benassy 2011). However, these approaches face their limits when trying to deal with dynamic processes of antagonistic participation, involving boundedly rational citizens with endogenous preferences, interacting in complex contexts, and with ongoing access to advanced communication technologies.

An interesting framework within which the dynamics of socio-political issues and public opinion formation processes have been successfully addressed is the so-called socio-dynamic quantitative approach (see, for example, Weidlich 2006; Chakrabarti et al. 2006; Galam 2008, 2012). In these works, general systems theory, nonlinear dynamics, and statistical physics have been applied to tackle fundamental questions such as voting dynamics, complex group decision-making, and the conditions for spontaneous self-organization in collective action. In these works, generally, authors apply computational methods to social systems and/or infer master equations that resemble the probability density function of the macro-variables used to describe the state of the system (e.g., hierarchical levels in voting systems, different agents' possible actions in collective problems, distributions of opinions in public debates). At the micro level, some kind of heuristic rules (single, collective, systemic ones) are defined to explain the changing behavior and learning of the internal entities composing the system (agents, groups, hierarchical structures underlying organizations)

which, in turn, explain the transition dynamics operating at the macro level. Specific stochastic forces also operate at both the micro and macro levels, leading to unpredictable emergences in the original nonlinear systems.

As we explain in the corresponding sections, the model we develop in this paper is inspired in contemporary streams of socio-political thought and also has some formal similarities with respect to the general structure of socio-dynamic quantitative frames. However, and very much in line with the scope of evolutionary economics, while the aforementioned works in socio-physics establish some kind of direct analogy between physical and social systems (agents as particles, waving dynamics in massive social systems), in our model, analogies are closer to evolutionary biology and replicator dynamics in social studies. Our approach seeks to build new bridges between the political economy of opinion formation studies and the evolutionary strand of economic thought. Thus, we address the political economy of polarized societies from a formal neo-Schumpeterian perspective.

We shall organize this work as follows: we review the aforementioned ideas in Section 2. After revising the literature, we epitomize features of the antagonistic processes under consideration and incorporate them into a theoretical model in Section 3. We present an evolutionary model designed to study political deliberation processes in what we call "for-vs.-against" situations. Then, we proceed to the analysis of the model. As we shall see in Section 4, there are multiple equilibria and alternative parametric ranges that delimit situations where direct participation and referendum practices are more recommendable than in other cases. Likewise, we characterize ranges for which political polarization may emerge, or not. In Section 5, we extend our model to the case of a replicator-cum-mutation evolutionary setting. In Section 6, drawing upon our results, we critically discuss some arguments in the literature. Finally, in Section 7, we offer our conclusion.

2 Literature review and features for the model

2.1 A brief reflection on contemporary political philosophy

The turbulent events that have shaken the first two decades of our century show us that political polarization appears in periods of crisis in the context of active, unsettled citizens with emotionally overloaded views (Chirot 2020). Polarizing processes seem to emerge from populations of citizens who take a stance on specific issues but are also hesitant in critical situations and are potentially open to changing their views if they are "properly" persuaded to do so. It has been observed that twitched polarization emerges from overwhelmed citizens operating in complex environments, who are not sure which ideas to stand by, are open to receiving "help, orientation and advice", but really need quick and "easy" solutions (Novak 2018).

During the last two decades, many issues have ended up being looked at in these terms (BREXIT, presidential elections, or the rise of the nationalism vs. globalism debates are some examples). These debates often deal with the possibility of achieving great changes through antagonistic strategies, an assessment of which would need expert knowledge beyond the reach of most citizens in their

everyday lives. However, citizens (at least provisionally) seem to end up taking a clear stance anyway (even if it is just via clicks on social media) either in favor of or against "the" relevant issue. Sometimes, citizens show intense commitment in supporting their views, even to the extreme of shaking the socio-political mood (or even the status quo).

In the case of contemporary societies, an essential aspect that has changed concerning similar situations in the past is that late-capitalism critical citizens, despite being in an emotionally committed mood facing (at least in principle) critical situations, live this commitment as part of hybrid experiences alongside overall lifestyles (social networking, cultural habits, symbolic demonstrations). They are not, in the majority, intrepid revolutionaries, and seem to be significantly influenced by peers (through positive or negative externalities) in their "social (including cyber-social)" proximities, being aware of the opportunity costs of moving too far. More recently, for these citizens, engaging in political arguments, social critique, and even being open to revising their positions on trending topics have become parts of their digital habits. Some authors (Zizek 2012; Durand 2020) point out that these aspects of current political life could be a "light" late-capitalism development of debates that took place during the second half of the 20th century. Let us revise these debates.

To begin with, there is a set of 20th-century controversies that confront classical concepts of democracy, with an incipient rise of evolutionary thinking in open-ended societies. Authors such as Popper (1945) and Schumpeter (1942) stand out. Also, in line with this progress, Austrian economics took a certain hold in society (liberal positions regarding market questions, with a more reluctant stance on politics; Mises 1949, and Hayek 1960). In some cases, these positions were stretched to extremes, as we see with Murray Rothbard (1978) who, in his libertarian manifesto, opts for new, more daring and more participative ways to conceive liberty.

On the other side, it is interesting to observe that, on the opposite pole of the political spectrum, and in an attempt to overcome the structural doctrines of the mid-20th century (Claude Levi-Strauss), in Western societies (during the 1960s), a mobilizing critical theory appeared with effects on May 68. It influenced cultural studies and mass media and advocated for freedom and direct action (Marcuse 1964). The moderate version of this line defends a reflective ethical ideal seeking to reach a pragmatic consensus – Habermas (1986). Critical theory aims to place important political decisions in the hands of citizens by appealing to their participation.

As a rare distillation of the aforementioned visions – the liberal one and the critical theory (opposing views, but with points in common regarding the consideration of freedom – see Klatch 1999), the late 1960s and the 1970s show, on the one hand, the slow crystallization of (anarcho-) libertarian pro-market developments; in parallel (on the other hand) they saw the post-structuralist lefts (Lacan 1960; Foucault 1970, also Derrida or Deleuze). Here, we see from both sides a destabilization of the neat positions characteristic of "Modern" thinking, a force that led to the proliferation of arguments conforming to the ideologies of the (so-called) "de-centered subjects" (Althusser 1970). We witness, therefore, the rise of claims for social antagonisms with traces of psychological tensions, often materialized in linguistic "games" representing social power relations. Instead of looking for essential truths, authors

such as Rorty (1979) claimed instructive local "narratives" for each specific issue, thus fracturing the political arena and paving the way for the next movement.

The next step occupied the final decades of the 20th century. There, we see mixtures of former streams of thought giving rise to postmodern views – Lyotard (1984), Baudrillard (1983), and Vattimo (1997). In these authors, one can perceive a degree of pro/anti schizophrenia regarding industrial societies, technology, markets, the "emancipatory" role of consumption, pop culture, and slogans. Driven by their representation of a fluid world (Bauman 2000) postmodern authors "were ready" to declare the end of large modern narratives (Marxism, the scientific rationales of 18th-century enlightenment, national institutions). In their place, they cite the irreconcilable differences between fragmented opinions and tales, the competitive "market" society of self-centered individuals, and the prevalence of subjective perspectives that often hold sway over facts. They advocated for sub-institutionalized adaptable versions of democracy to reach micro-consensus. This is the essence of the micro-politics of specific solutions (Sennet 1998; Lipovetsky 2006). These solutions were thought to be implementable by generalizing digital hyper-connected devices and social networking (Durand 2020). The move towards a like/dislike "tweet" atmosphere is straightforward.

Finally, the most recent step in this outline of social thinking and action relevant to our work comes with the onset of the unsettled decades of the new millennium. They have dismantled the post-modern comfort, and have engendered new political demands. These real demands have been faced during the last 20 years in a paradoxical manner. On the one hand, the demands were packed as a global pre-revolutionary stream to challenge the state, the status quo, – in general a global oppressing enemy which varied depending on the extreme pole of the political spectrum flagging the "revolution". On the other hand, the demands were divided in a very particularized way by propelling "for-vs.-against" debates; this induced single citizens to behave as pseudo-revolutionary agents motivated to allocate time, energy, and resources to fight the threatening "others". These "others" supported (at least rethorically) anachronic institutions that were oriented to destroy vital components of the citizens' (self-centered) personal identity. This has been seen from both sides of the political range.

An influential vision that embodies this new reality is the so-called populist reason (Laclau 2005). This vision has taken the place (taking it to extremes and revealing a polyvalent applicability) of the post-Marxist arguments (Althusser 1976), has tried to remove post-modern politics (Derrida 1997), and has led to the support of pre-revolutionary actions in dichotomized terms. A symbolic color, a polarizing presidential candidate, a new party, artistic images, and aesthetic re-constructions of history were all seen as potential attractors for differentiated citizens. These citizens need accurate, quick solutions to meet their demands, which are real in our turbulent century. However, at the same time, they are persuaded to fight against "the" global oppressing enemy (which is domain-specific depending on the issue). For the time being, a paradoxical outcome of the populist reason has been that the eradicated and busy (always connected) contemporary individuals have become polarized in twitched micro-debates, and the alignment towards "social emancipation" has not occurred.

Regarding our theoretical proposal, it is worth noting that the currents we have just outlined point to the role played by:

(i) Factors of institutional estrangement seen in education, media, and cultural trends that have been exacerbated by economic uncertainty and hyper-connection.
(ii) A degree of uninformed wavering by citizens bewildered by the acceleration of technical and social changes, perceiving local externalities in conditions of atomistic social life, and suffering real economic shocks; and
(iii) The sentimentalized polarization of populist reason (Laclau 2005) which deepens and links all kinds of simplified antagonisms in "packed" issues. This combination leads to the reappearance of nostalgic ideals of the past, distant and micro-related idealized fables, and different versions (embodied in leaders promising security) of a return to the political Eden – in opposition to the 'malevolent' antagonistic option offered for each case (Chirot 2020). In economic terms, we would say that we are witnessing populations of domain-specific unsettled and emotionally overloaded boundedly rational citizens (Nelson 2018; Dosi et al. 2020; Dosi 2023) permeable to populist antagonistic ideologies, looking for easy explanations and fixes, but often hesitant on what they are said; citizens that provide resources and action for the social "battle", pursue quick solutions that they truly need and – at the same time – are shaped by local externalities and global interactions.

2.2 Sociophysics, mathematical sociology, and public opinion

The analysis of phenomena related with the material we have described in the previous section has led to the production of a great deal of formal and quantitative work, in fields like sociophysics and mathematical sociology. Regarding the aims of our work, we would like to mention how sociophysics and quantitative sociology have devoted an increasing number of studies – during the last five or six decades – to explain public opinion dynamics and polarization (see e.g., Abelson 1964; Chatterjee and Seneta 1977; DeGroot 1974; Friedkin and Johnsen 1990; Galam 2008, 2012, 2017, 2022; Banisch and Olbrich 2019). The aim of these models is usually to explain how confrontation or conformity of positions emerge from local and global interactions and initial conditions, in settings that very much resemble the fluid and de-centered antagonistic political micro-debates described in the previous section. We see in socio-physical models and quantitative sociology the fruitful use of multi-agent dynamic stochastic models from which different typologies of changes (involving slow vs. fast, or gradual vs. sudden and radical changing paths) are characterized and explored. The models incorporate a large number of plausible psychological and sociological assumptions and interactive elements from which the dynamics of public opinion emerge. Broadly speaking, in the case of polarized opinion formation models, some key cognitive and social features are considered the main driving forces to explain changes in the opinion of agents. For example, emotional reinforcement and learning mechanisms, in terms of approval/disapproval

by peers, can operate (Banisch and Olbrich 2019); the lines of argumentation and persuasion among individuals observed in data from social networks take place in the models (Mäs and Flache 2013), and/or open (informal) discussion settings versus more ruled-constrained situations are often pointed as the leading explanation for position change and opinion formation (Galam 2008, 2022). Once the previous social mechanisms are defined in each case, and in order to obtain the final state of the social systems, some updating rules (for voting, social distance) drive the dynamics of change underlying the shifting relative importance of competing opinions. In this manner, the final outcomes in terms of the prevalence, disappearance, conformity of the competing opinions can be obtained as emergent properties in the simulations (or as analytically obtained limit-distributional states) of the system. As we will see in Section 3, our model can be close to the general spirit of Weidlich (2006); Galam (2008, 2023), but with analogies closer to biological mathematics than to statistical physics.

2.3 Some features of our model

In essence, it is clear from the preceding subsections that the importance of participative antagonistic dynamics and tools have grown this century (Glencross 2016; Altman 2019), favoring the appearance of deliberative mini-publics or local tools (Matsusaka 2020, Cronin 1999) which enable boundedly rational citizens to debate public issues in real time (Gerber and Lupia 1995; Setala 2014). The reasons for this can be found in economic reality, the evolution of political thought, and new technical tools. However, serious doubts and controversy still exist regarding the outcomes of these antagonistic practices (Broder 2000). Bearing in mind all we have talked about, we shall now gather the debates and evidence seen with the aim of integrating some relevant aspects (in a stylized manner) in our model in Section 3. We shall try to deal with some of the issues by analyzing whether the dynamics of dichotomized societies can be formalized as a specific type of population dynamics model (Sandholm 2010). More precisely, we have to represent large-scale processes of citizen involvement in complex political debates (posed in a for-vs.-against manner), and explore whether the modeled dynamics show robust properties and a limited number of possible equilibria conveying the trend of the (boundedly rational) public (Page and Shapiro 1992); or, on the contrary, whether it reveals complex dynamics (driven by local externalities and global learning) which may be distorted in short periods of time, thus engendering unpredictable opinion states.

To start out building this framework, we want to extract certain characteristics of population dynamics models that bring out what we have seen above (in the descriptive section and regarding the quantitative literature) and we will connect these traits with recent contributions to evolutionary economics (Friedman (1991), Sandholm (2010), Nelson (2018), Dosi (2023)). Thus, we can single out the following features from the revision above:

1. We should build up an evolutionary model considering the bounded rationality of citizens, imperfect understanding of the dynamic situation that is to be played,

learning with limited attention, and endogenous change in shares of heterogeneous citizens. The changing shares of citizens behaving in certain ways and holding specific opinions (for vs. against an issue) influence other citizen's payoffs via externalities and institutional change.

2. In reality, as we have seen in the preceding subsections, problems and debates around political decisions are complicated. Given the limited capabilities of citizen attention, the complexity of problems, and the fact that citizens' involvement in the political arena means that they gradually allocate private resources (energy, time, income) to foster their preferred option (or to collectively make an influence on institutions, or to persuade others) then, our heterogeneous boundedly rational citizens update their positions by considering both, the satisfaction obtained from political commitment, but also the opportunity costs derived from this activity.
3. The processes considered in the preceding sections resemble interactions in large populations of small agents (citizens). The antagonistic processes discussed above seem to fit well with a two-populations of citizens setting, in which citizens support "for-vs.-against" positions on an issue; we can consider that they do it with a higher or lower degree of intensity (commitment of resources in support of their preferred political option). One population includes citizens "for" a specific issue, and the other population includes citizens "against" the issue. Inside both populations, citizens behave in diverse ways (in terms of their degree of commitment to persuade others).
4. According to the processes we revised in the preceding subsections, we can consider that both populations (for vs. against) may evolve as the agents update their behaviors and degrees of commitment (intra-population dynamics). Then, intra-population dynamics in global interaction could influence – through different emerging commitment levels – the selection among said populations at a higher level (inter-population "for-vs.-against" competition). Citizens may change their opinion in the debates (influenced by highly committed others) and are conditioned by the power distribution that arises from political confrontation. In turn, depending on how inter-population competition develops, the higher or lower social and institutional prevalence of (for vs. against) opinions should change, and, through this channel, inter-population global competition could affect intra-population dynamics. More precisely, succeeding in fostering their preferred option, so that the share of the population that supports "my option" increases globally, can be a source of satisfaction that shapes the revision of behavioral patterns at the intra-population level. On the contrary, failing to disseminate and establish the supported opinion may erode satisfaction.
5. The number of citizens in dichotomized political economies is always very large (e.g., in pre-referendum processes in a nation where the final overall distribution gradually emerges).
6. The number of roles is finite. Notice that we are assuming that citizens, apart from making up part of one out of two populations, also show a specific behavioral pattern regarding commitment when defending their position (persuading others, being open to be persuaded, and trying to shape institutions). For the sake of clarity, we will consider that we have a finite number of roles: people "for" the issue

divided between those supporting their position strongly or weakly and citizens "against" the issue (also with high vs. low degree of commitment).

7. We can assume that citizens are small in society, so that their individual behaviors have little (but not zero either) effect on other citizens' payoffs. More precisely, we assume that citizens' payoffs depend on the behavior of others through the distribution of the other citizens' choices and through externalities produced by how masses of citizens behave and opining different things interact in several directions. These masses and distributions self-organize as citizens engage in debates, maintain or change behavior and opinions, and try to shape institutions. The widespread use of digital communication devices allows citizens to be continuously interconnected.
8. Technically, and to sum up, the discussion above leads us to model "masses" of citizens who behave (some of them) in a similar way and other groups who behave in a very different manner within both the "for" and "against" populations. The payoff functions in both populations depend on citizens' own behavior, peer's behavior and discourses, institutional shaping, and on the co-evolving distributions of the behavior of opponents and peers. We represent the process of social debate with institutional effects as an evolving process of interaction in which local externalities of different signs and cooperative and competitive behaviors co-exist. Citizens may update their commitment as society changes, and these intra-population processes (both in the "for" and the "against" population) drive inter-population flows (opinion change). In turn, as the dichotomized structure of public opinion evolves, this produces effects at the intra-population level (by shaping payoffs and citizen learning in the competing populations). In the next section, we will lay out the specific revision protocols and learning/selection processes underlying behavioral and opinion changes. As we shall explain, the analysis of the co-evolving populations and emergent flows will allow us to characterize different trajectories of public opinion in dichotomized settings. We formalize all these elements in Section 3.

3 The model

3.1 Citizen commitment

The society that we propose in this section is composed of two (sub-)populations of boundedly rational economic agents (citizens) who hold an opinion in favor of (for), or against (against), a specific issue. The position that achieves a higher share of total society support (we can leave aside abstention) wins in, what is, a problem of collective opinion formation. The society can be de-composed in said two (sub-) populations of citizens (those "against" and those "for" something), and we can also distinguish internally, within each population, between those that defend their option with a high degree of intensity, and others defending the option with lower intensity. Thus, there is intra-population diversity in behavior (high commitment vs. low commitment) both, in the "for" and the "against" populations. Citizens may revise their level of support (revising their strategy of high vs. low commitment of resources

in support of their ideas –intra-population dynamics), and/or they may change their "for" vs. "against" option (opinion shift regarding the issue under consideration; in this way, we consider inter-population flows when changing opinion). We represent each citizen's degree of commitment at t by the *proportion* of private resources devoted to fostering the desired option (for vs. against) at t. Let us formalize our preceding considerations:

Assumption I The mass of citizens in society is distributed among two large populations of opinion $\pi = \{F, A\}$ at t (the "for" population F, and the "against" population, A). We also assume that citizens may position themselves within each population with a low level of commitment in support of their option –low strategy (x_1), or, on the contrary, with a strategy of high commitment (x_2),$0 < x_1 < x_2 < 1$. We assume similar parameters (x_1), and (x_2) in both populations, and we consider $x_2 = x_1 + a, a > 0$.

Let s_{jt}^{π} be the share of citizens within population π whose level of commitment at time t is $x_j, j = 1,2$. Therefore, $0 \leq s_{jt}^{\pi} \leq 1$, and $\sum_j s_{jt}^{\pi} = 1$. That is to say, for each of the two populations of opinion in society $\pi = \{F, A\}$ at time t, we have a share of citizens s_{jt}^{π} within population π whose level of commitment (share of private resources devoted to support their option) is $x_j, j = 1,2$.

3.2 Citizen payoff

Assumption II Formally, we can represent the payoff functions for citizens in each population $\pi \in \{F, A\}$:

$$\begin{aligned} u_{1t}^{\pi} &= \left(\gamma_t^{\pi}(1-\varphi) + \varphi s_{2t}^{\pi}\right)x_1 \\ u_{2t}^{\pi} &= \left(\gamma_t^{\pi}(1-\varphi) - \varphi s_{1t}^{\pi}\right)x_2 \end{aligned} \tag{1}$$

with γ_t^{π} being the share of population $\pi \in \{F, A\}$ in society $0 \leq \gamma_t^{\pi} \leq 1$, $\sum_{\pi} \gamma_t^{\pi} = 1$. Parameter $\varphi, 0 < \varphi < 1$, captures the intensity of an externality effect (permeability to peers' contribution), vs. strictly partisan motivations.

We are assuming in Eq. (1) that devoting a share of resources x_j in pursuit of and to support a preferred option when a dichotomized political process is developing, is in principle a "good" (something that gives satisfaction) for citizens. Of course, the level of commitment and political participation through resource allocation also bears opportunity costs. This is the reason why, as we will see, citizens are involved in an interdependent strategic situation with their peers and, eventually, they may revise their commitment and even the option they support. For the time being, let us note that the –let us say- *unit payoff* of x_j in Eq. (1) is captured by: $\left(\gamma_t^{\pi}(1-\varphi) + \varphi s_{2t}^{\pi}\right)$ in the case of a low-commitment strategy x_1 -in support of option $\pi \in \{F, A\}$; and it is $\left(\gamma_t^{\pi}(1-\varphi) - \varphi s_{1t}^{\pi}\right)$ in the case of the high-contribution strategy (x_2).

Note that, in both cases, we assume that succeeding in fostering their own preferred option so that the share of the population that supports "my option" $\gamma_t^{\pi}, \pi \in \{F, A\}$ is large and even increases, is a source of satisfaction for citizens (the

perception of being effective in socio-political action as γ_t^π becomes as large as possible is a source of citizen payoff from socio-political participation).

On the other hand, note in Eq. (1) and in the "unit payoffs" $\left(\gamma_t^\pi(1-\varphi)+\varphi s_{2t}^\pi\right)$ and $\left(\gamma_t^\pi(1-\varphi)-\varphi s_{1t}^\pi\right)$ that, in the case of the low-commitment strategy (x_1), we assume that the low-committed mass of citizens receive a positive externality s_{2t}^π from more-committed peers; this externality is higher, the higher the share of citizens within "my" population (with a similar political position to mine) that, fighting in support of my option, devote more efforts than I do in support of a common ideal. Thus, the higher the mass of peers playing x_2, the more the mass of less-committed peers can relax their commitment and devote more resources to alternative (non-political) activities. The relative bias between *permeability to externalities* vs. *strictly partisan motivations* in Eq. (1) is given by the value of parameter φ. In fact, the higher the value of this parameter, the more permeable to externalities are citizens involved in the socio-political debate.

Likewise, in the case of highly committed citizens (those contributing a share of their resources x_2), we assume that they perceive a negative externality s_{1t}^π, because they interpret the low commitment of peers as a type of free-rider behavior. This is the reason why we incorporate the factor $\left(\gamma_t^\pi(1-\varphi)-\varphi s_{1t}^\pi\right)$ in Eq. (1). The higher the share of low-committed peers, the stronger the negative externality; again, parameter φ captures the externality vs. partisan motivations in the payoff function.

Finally, let us mention that unit payoffs in (1) include endogenously changing components, which will change as the revision protocols and persuasion mechanisms that we present below operate. Of course, Eq. (1) holds for both $\pi \in \{F, A\}$. They hold in both populations. Note that we are denoting by s_{jt}^π the share of citizens within population π whose level of commitment at time t is $x_j, j = 1,2$. Therefore, $0 \le s_{jt}^\pi \le 1$, and $\sum_j s_{jt}^\pi = 1$, for $\pi \in \{F, A\}$.

From all the above-mentioned, it is clear that we can define the average level of contribution (in support of the corresponding option) within each sub-population as $x_t^\pi = \sum_j s_{jt}^\pi x_j$. The average payoff within each subsystem at t is $u_t^\pi = \sum_j s_{jt}^\pi u_{jt}^\pi$. Finally, it is also straightforward that the average level of citizen contribution/commitment in society at time t will be $x_t = \sum_\pi \gamma_t^\pi x_t^\pi$. These aggregates will play a key role in the revision protocols and evolutionary mechanisms that follow both at the intra-population level and in the inter-population dynamics.

3.3 Intra-populations revision protocol

Assumption III We assume that citizens within both populations $\pi \in \{F, A\}$ may meet randomly (intra-population random matching), and revise their behaviors by, eventually, switching strategy (within the same population) in a better-valuation direction. More precisely, for $\pi \in \{F, A\}$, if we denote by f_{ij}^π the rate at which citizens with strategy j switch to strategy i within π, we assume that this switching rate is given by:

$$f_{ij}^\pi = \gamma[u_i^\pi - u_j^\pi]_+ = \gamma max\left(u_i^\pi - u_j^\pi; 0\right), \gamma > 0.$$

Now, assuming that $\delta s_j^\pi s_i^\pi$ gives the probability for a random interaction between citizens with strategy j and those with strategy i in $\pi \in \{F, A\}$, in a small time interval Δt, the flow of citizens from strategy j to strategy i is given by $\delta s_j^\pi s_i^\pi f_{ij}^\pi \Delta t$. Then, the change in the share of citizens playing strategy i ($i = 1,2$), in $\pi \in \{F, A\}$ is:

$$\Delta s_i^\pi = \sum_{j=1}^{2} \delta s_i^\pi s_j^\pi (f_{ij}^\pi - f_{ji}^\pi)\Delta t,$$

where $f_{ij}^\pi - f_{ji}^\pi = \gamma\left(u_i^\pi - u_j^\pi\right)$. Thus, the continuous evolution of the share of citizens with behavioral pattern i within π is described by the differential equation:

$$\dot{s_i^\pi} = \sum_{j=1}^{2} \delta s_i^\pi s_j^\pi \left(f_{ij}^\pi - f_{ji}^\pi\right) = \delta s_i^\pi \sum_{j=1}^{2} s_j^\pi \gamma\left(u_i^\pi - u_j^\pi\right)$$

$$= \gamma \delta s_i^\pi \left(u_i^\pi - \sum_{j=1}^{2} s_j^\pi u_j^\pi\right), i = 1,2; \pi \in \{F, A\}$$

Or equivalently, just by changing velocity and taking $s_i^\pi(t) = s_i^\pi(\delta\gamma\tau)$ we have the system of two differential equations for each population intra-revision dynamics:

$$\dot{s_i^\pi} = s_i^\pi\left(u_i^\pi - u^\pi\right),\ u^\pi = \textstyle\sum_{j=1}^{2} s_j^\pi u_j^\pi,\ i = 1,2; \pi \in \{F, A\} \tag{2}$$

Of course, the payoff functions that appear in System (2) are those given by Eq. (1) in Assumption II. If we observe together the expressions (1) and (2), what we see are two similar revision protocols, taking place at the intra-population level (among citizens supporting the alternative options $\pi \in \{F, A\}$), according to which boundedly rational citizens may revise their level of commitment – from low to high commitment, or vice versa. Notice that citizens have an incentive to contribute intensely in support of their political option (it is a source of payoff for them) but, since they may benefit from peer efforts or, on the other hand, may feel that less committed peers are free-riding them, there is a possibility of updating behaviors in a "better valuation" direction. This possibility arises as citizens randomly meet, debate, and compare behaviors. This social interaction underlies the intra-population process.

Furthermore, as we explain in the following sub-section, we shall consider that, since the intra-population dynamics (above) will generate trajectories for the corresponding average commitment levels, $x_t^\pi = \sum_j s_{jt}^\pi x_j$ for $\pi \in \{F, A\}$, the differential implication levels for supporters of the two alternative options, will have an influence on the evolution of the share of supporters for each option in the whole society. That is to say, it is going to drive the dynamics of γ_t^π, for $\pi \in \{F, A\}$. As can be seen in Eqs. (1) and (2), what happens to the dynamics of γ_t^π at the global (society) level is not only driven by but also drives the overall model dynamics (through the endogenous payoffs (1)). Thus, there is a process of co-evolution linking the two populations. We deal with these mechanisms to close the model in Assumption IV.

3.4 Inter-populations opinion change

Now, we assume that the population $\pi \in \{F, A\}$(from those two opinion populations which conform to society) which engenders stronger levels of average commitment in support of its "for-vs.-against" option will end up (gradually) gaining a relative presence in society (a higher share of supporters in society). This may also allow the growing group to increase its influence on prevailing institutions. Formally:

Assumption IV We take into consideration that citizens within both populations $\pi \in \{F, A\}$ engender trajectories for their corresponding average levels of population-commitment $x_t^{\pi} = \sum_j s_{jt}^{\pi} x_j$; and we recall also that $x_t = \sum_{\pi} \gamma_t^{\pi} x_t^{\pi}$ is the average level of political commitment in overall society. Then, we assume that the population (political option) which engenders a higher level of commitment grows in support, whereas the other position loses supporters. In terms of inflows and outflows of citizen masses reflected in a replicator dynamic for the share of supporters in society, we can represent the dynamics of opinion shifts between the two options $\pi \in \{F, A\}$ as follows:

$$\dot{\gamma_t^{\pi}} = \gamma_t^{\pi} \left(x_t^{\pi} - x_t \right), \pi \in \{F, A\}. \tag{3}$$

Expression (3) means that the population (political option) with a higher than average commitment gains in share of supporters $\dot{\gamma_t^{\pi}} > 0$. Likewise, the population with lower-than-average support loses social presence ($\dot{\gamma_t^{\pi}} < 0$). If we consider the average level of population-level commitment/citizen contribution $x_t^{\pi} = \sum_j s_{jt}^{\pi} x_j$, $\pi \in \{F, A\}$ as a joint effort/input in creating a common (population (opinion)-specific) deposit of arguments and initiatives in support of the corresponding option $\pi \in \{F, A\}$, it is easy to see how Eq. (3) may come from a process of random meetings and mutual attempts to persuade (on the basis of the proper deposit of arguments produce at the intra-population level from x_t^{π}). Thus, (e.g.,) for the case of the "against" population, let us denote by $(\delta \gamma^A \gamma^{\pi})$, $\pi \in \{F, A\}$ the probability of random meetings among "against" citizens and other citizens in society in a political debate and persuasion process (the case for the "for" citizens mirrors this one). Then, in continuous time (as we have seen in Assumption III) if we assume that average population commitment "proxies" (or feeds in a directly proportional way) the common deposit of arguments and initiatives, and citizens resort to this deposit in global political action, we can state that the process of opinion change (inflow-outflow) for citizens "against" in society may be represented by:

$$\dot{\gamma_t^{A}} = \sum\nolimits_{\pi} \delta \gamma^A \gamma^{\pi} \left(x^A - x^{\pi} \right) = \delta \gamma^A (x^A - x)$$

If we consider $\delta = 1$, which is just a change in velocity, and we set the same process for $\dot{\gamma_t^{F}}$, we obtain Expression (3). Notice that Assumptions (I) to (IV) and Expressions (1), (2), and (3) work together, driving the model dynamics. In a dichotomized process of opinion formation, we are interested in exploring the global dynamics of $\gamma_t^{\pi}, \pi \in \{F, A\}$, and also how the intra-population distribution of shares $s_{jt}^{\pi}, j = 1,2, \pi \in \{F, A\}$ evolves as time goes by. Note that we are modeling something

like a linked-two-layer process: two population fields in which citizens revise their commitment strategies; and a second level in which opinion change takes place. There are influences "up-down" and "bottom-up", as well as intra-population and inter-population effects and, in this way, both political levels co-evolve.

Then, the analysis of the model dynamics may allow us to deal with such questions as: does it make a difference to announce a debate (or even a formal/informal "referendum" in our sense) early or late in the process preceding the final action (if we assume that the majority option wins)? Can we distinguish parametric conditions for which an evolving situation of pro vs. against evolves in one specific way or another? How many equilibria (if any) exist in this process, and what are their properties (stability)? Can we characterize the conditions for polarization to emerge (that is, situations in which society evolves towards a division, in similar proportions, around shocking antagonistic political options)? In our model, there are both non-cooperative elements (the political fight between the two opinion populations, as well as the non-cooperative intra-population externalities that instill strategic interactions among peers), and also a cooperative component, since in the payoff of peers (citizens in the same population, no matter the strategy they play) γ_t^{π} appears as a source of satisfaction (see (1)) in favor of which both low- and high-committedpeers contribute. Now, considering Assumptions (I) to (IV) and Eqs. (1–3), since we are dealing with (linked) two-dimensional dynamics in each of the two revision protocols of intra-population behaviors, and in the process of political action at the level of global opinion change – (six dimensions in total), we can convey the model dynamics in a system of three nonlinear differential equations obtained from the linked projections of the three simplexes (Hofbauer and Sigmund 1998; Sandholm 2010).

4 Dynamic analysis: Equilibria and stability

The analysis of the model can be carried out by focusing on the linked simplex projections for the replicator dynamics that follow from Eqs. (1), (2), and (3). For simplicity, we are going to study the model focusing on the evolution of $v(t) = \gamma_t^A$, $y(t) = s_{1t}^F$, $z(t) = s_{1t}^A$. That is to say, we are going to analyze the model by focusing on the dynamics of: the share of citizens with a low commitment in each opinion-population ("for" and "against" an issue (s_{1t}^F, s_{1t}^A)), and the dynamics of the share of society "against" the issue γ_t^A. The complementary dynamics are straightforward once we understand the dynamics of $v(t) = \gamma_t^A$, $y(t) = s_{1t}^F$, $z(t) = s_{1t}^A$.

4.1 Existence of equilibria

If we denote $v(t) = \gamma_t^A$, $y(t) = s_{1t}^F$, $z(t) = s_{1t}^A$, and we work out the interlinked simplex-projections from (1), (2), and (3) considering all we have said, we arrive at the following system of differential equations:

$$\dot{v} = f(v, y, z) = av(1 - v)(y - z) \tag{4}$$

$$\dot{y} = g(v, y, z) = (a\varphi)y(1 - y)[\frac{x_1}{a} + y - \frac{(1 - \varphi)}{\varphi}(1 - v)] \tag{5}$$

$$\dot{z} = h(v, y, z) = (a\varphi)z(1 - z)(\frac{x_1}{a} + z - \frac{(1 - \varphi)}{\varphi}v) \tag{6}$$

with $a > 0, x_1 > 0, x_2 = a + x_1 < 1$ and $\varphi \in (0,1)$.

The system determines the time evolution of the orbits in the unit-cube $[0,1] \times [0,1] \times [0,1]$, which is an invariant space-set for the dynamics. We can see straight away that each face of the cube is invariant and thus the edges of the cube (where two faces meet) are also invariant. The vertices of the cube (where edges join) are invariant too. These vertices are equilibrium points of the system. Apart from the vertices, the model shows two edges as equilibria of the system: the edge for the equations $y = 0$, $z = 0$ (linking vertices (0,0, 0) and (1,0, 0)) and the edge for $y = 1$, $z = 1$ (linking vertices (1,1, 1) and (0,1, 1)). The condition $\dot{v} = \dot{y} = \dot{z} = 0$ is fulfilled for all these points.

As well as the vertices and the edges mentioned, the system can have (depending on the values of the parameters) four additional equilibria in edges, and one more inside the cube. Specifically, the points $\left(0, \frac{1-\varphi}{\varphi} - \frac{x_1}{a}, 0\right)$, $\left(0, \frac{1-\varphi}{\varphi} - \frac{x_1}{a}, 1\right)$, $\left(1,0, \frac{1-\varphi}{\varphi} - \frac{x_1}{a}\right)$ and $\left(1,1, \frac{1-\varphi}{\varphi} - \frac{x_1}{a}\right)$ are equilibria of the system if $0 < \frac{1-\varphi}{\varphi} - \frac{x_1}{a} < 1$, or, equivalently, if $1 < \frac{1}{\varphi} - \frac{x_1}{a} < 2$. Likewise, $\left(\frac{1}{2}, \frac{1}{2}\frac{1-\varphi}{\varphi} - \frac{x_1}{a}, \frac{1}{2}\frac{1-\varphi}{\varphi} - \frac{x_1}{a}\right)$ is also an equilibrium point (in this case, inside the cube) if $0 < \frac{1}{2}\frac{1-\varphi}{\varphi} - \frac{x_1}{a} < 1$.

In none of these cases does the interior equilibrium play a relevant role as it can be seen that the dynamics of the system are volume-expanding inside the cube. To see this, we calculate the divergence of the system $\dot{\boldsymbol{u}} = \boldsymbol{F}(\boldsymbol{u})$, where $\boldsymbol{F}$ is the vectorial function $\boldsymbol{F}(v, y, z) = (f(v, y, z), g(v, y, z), h(v, y, z))$ and we can see that it is always positive. This can also be done by multiplying the system by a scalar function $k(v, y, z)$ which is always positive, as the initial dynamics are equivalent (only the velocity of the process changes) to that of the system $\dot{\boldsymbol{u}} = k(\boldsymbol{u}) \bullet \boldsymbol{F}(\boldsymbol{u})$. In our case, if we take $k(v, y, z) = \frac{1}{v(1-v)y(1-y)z(1-z)}$, which is defined, and always positive inside the cube: $(0,1) \times (0,1) \times (0,1)$, we obtain:

$$div(k\boldsymbol{F}) = \frac{\partial(kf)}{\partial v} + \frac{\partial(kg)}{\partial y} + \frac{\partial(kh)}{\partial z} = \frac{a\varphi[y(1 - y) + z(1 - z)]}{v(1 - v)y(1 - y)z(1 - z)} > 0$$

Thus, the system is volume-expanding, and the orbits of the cube's interior always tend towards the cube's boundary (faces, edges, or vertices). The asymptotic behavior of the system can be deduced from the aforementioned and, considering the forces operating in the model through the faces and edges of the cube. Although we will consider as initial conditions only orbits which originate inside the cube, let us visualize the dynamics on the boundary, in the three relevant parametric cases, to fully understand the dynamics.

Fig. 1 Case 1: $\frac{1-\varphi}{\varphi} - \frac{x_1}{a} \leq 0$

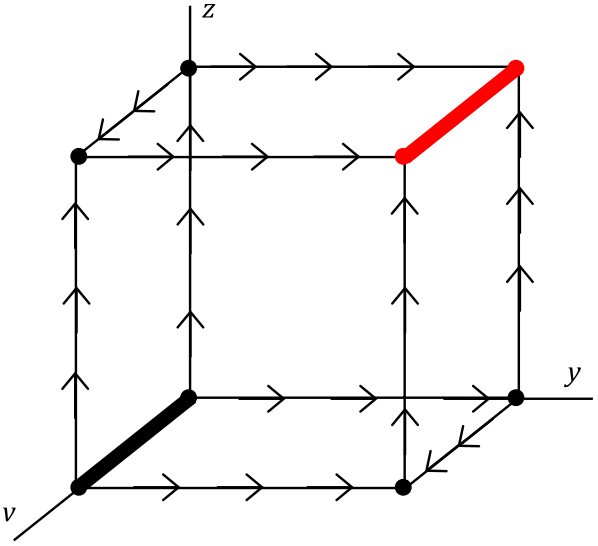
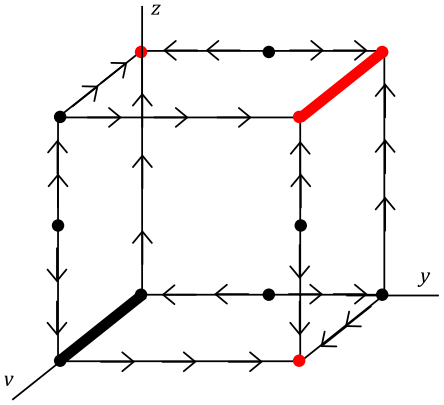


Fig. 2 Case 2: $0 < \frac{1-\varphi}{\varphi} - \frac{x_1}{a} < 1$



We begin with Fig. 1 (parametric regime in which $\frac{1-\varphi}{\varphi} \leq \frac{x_1}{a}$) where the attractor is the edge linking vertices (1,1, 1) and (0,1, 1)) including said vertices. The long-term evolution of the orbits always approach the edge $(\widehat{v}, 1,1)$, with $\widehat{v} \in [0,1]$.

On the other side, in Fig. 2 (case 2 for which $0 < \frac{1-\varphi}{\varphi} - \frac{x_1}{a} < 1$) the attractor is the edge linking vertices (1,1, 1) and (0,1, 1) together with said vertices, and also two more vertices (1,1, 0) and (0,0, 1). As we will see below, depending on the initial conditions within the cube, orbits can approach (1,1, 0), (0,0, 1), or $\left(\widehat{v}, 1,1\right)$ with the basin of attraction of the edge separating one area from another.

Finally, in Fig. 3 ($\frac{1-\varphi}{\varphi} - \frac{x_1}{a} \geq 1$), the attractor of the system is formed by two vertices, (1,1, 0) and (0,0, 1), and by two edges: the edge which links vertices (1,0, 0) and (0,0, 0), and the edge linking vertices (1,1, 1) and (0,1, 1). Depending on the initial conditions (as we will see below), the orbits can approach (1,1, 0), (0,0, 1), or

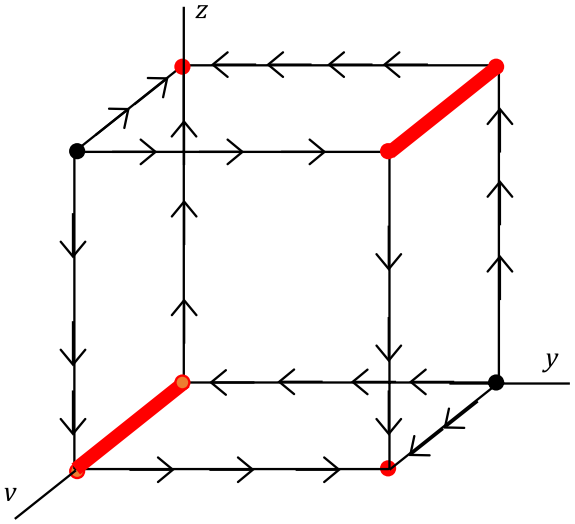


Fig. 3 Case 3: $\frac{1-\varphi}{\varphi} - \frac{x_1}{a} \geq 1$

equilibria on the edges $(\hat{v}, 0,0)$ or $\left(\hat{v}, 1,1\right)$, with $\hat{v} \in [0,1]$. The basins of attraction of the edges will separate the areas.

4.2 Stability analysis

The stability analysis of the model has been carried out (to a large extent) already; we have detected and represented the equilibrium situations in the alternative parametric regimes, we have proved that the dynamics are always volume expanding within the cube, and we have seen the dynamics on the boundary, with the asymptotic behavior of the system being driven (largely) by the behavior in the boundary (visualized in Figs. 1, 2 and 3). Now, we are going to see that once the dynamics of the model are characterized for the alternative regimes, we find an interesting degree of sensitivity to initial conditions that also influences the specific limit state of the system. The sensitivity to initial conditions together with the specific parametric regime we are in, will allow us to interpret what the model reveals regarding the evolutionary dynamics of opinion in dichotomized modern societies. Thus, let us now illustrate, visualize, and interpret the dynamics within the cube with the aid of Figs. 4, 5 and 6, which draws upon what we have already obtained in the preceding analysis (above). Let us begin with Fig. 4.

In Fig. 4, we see the system dynamics for the parametric case 1 (or regime 1) in which $\frac{1-\varphi}{\varphi} - \frac{x_1}{a} \leq 0$. In this case, we observe in Fig. 4 how the dynamics tend to the attracting edge, but with a significant dependence on the initial conditions (along the edge, we have infinite alternative equilibria, and the attracting one –for different orbits- depends remarkably on the initial condition within the cube). In order to understand what the dynamics in Fig. 4 mean regarding the complicated dynamics of our dichotomized societies, let us note, firstly, that the parametric condition can

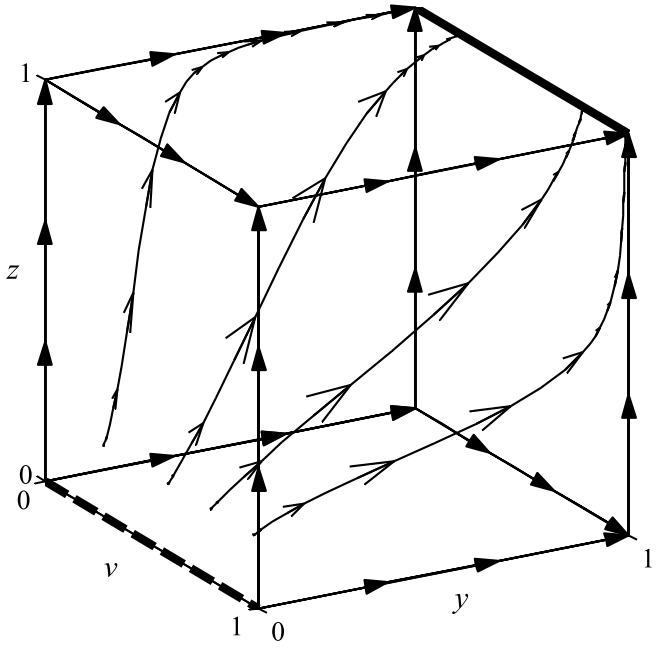


Fig. 4 Case 1: $\frac{1-\varphi}{\varphi} - \frac{x_1}{a} \leq 0$

be rewritten as $\frac{1-\varphi}{\varphi} \leq \frac{x_1}{a}$. Thus, if $\frac{1-\varphi}{\varphi}$ is sufficiently small, then the specific edge $(\widehat{v}, 1,1)$, with $\widehat{v} \in [0,1]$ is the attractor, and the initial condition is rather determinant in explaining how society will move to a specific configuration of behaviors and opinions (pro vs. against) on an issue. Since we know that $v(t) = \gamma_t^A$, $y(t) = s_{1t}^F$, $z(t) = s_{1t}^A$, Fig. 4 shows that when $\frac{1-\varphi}{\varphi}$ is small (the strictly partisan component of the payoffs in (1) is relatively small in comparison with the externality component) then everything can happen in terms of the final predominance of for-vs.-against opinions on an issue $\widehat{v} \in [0,1]$, but the final result does not differ radically from the initial opinion distribution. It is important that, as we can see if we look at Eqs. (4) to (7) and Fig. 4, political competition in this regime flows as follows: imagine that we depart from conditions within the cube from which $v(t) = \gamma_t^A$ begins to grow. This implies (from Eq. 4) that the initial condition is such that $y(t) = s_{1t}^F > z(t) = s_{1t}^A$. A growing $v(t) = \gamma_t^A$ generates in Eq. (6) that $z(t) = s_{1t}^A$ is going to grow but very slowly in the parametric regime under consideration (case 1 where $\frac{1-\varphi}{\varphi} \leq \frac{x_1}{a}$). Likewise, the growing $v(t) = \gamma_t^A$ leads (Eq. 5) to an increase in $y(t) = s_{1t}^F$. In both the "for" and "against" population the citizens are moving towards low commitment levels, but since $y(t) = s_{1t}^F > z(t) = s_{1t}^A$ in the initial conditions under consideration, the decrease in commitment is faster in the "for" population than in the "against" population (Eqs. 4–6), and this effect reinforces the trend towards the predominance of people "against" the issue. We could obtain exactly the reversal interpretation if (Fig. 4) we departed from a situation with decreasing $v(t) = \gamma_t^A$, and $y(t) = s_{1t}^F < z(t) = s_{1t}^A$. Although everything can happen (in Fig. 4) in terms of the final

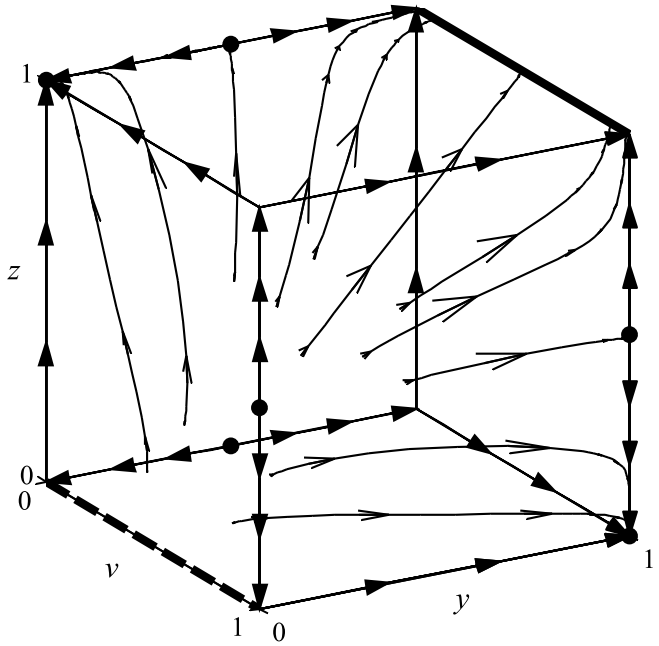


Fig. 5 Case 2: $0 < \frac{1-\varphi}{\varphi} - \frac{x_1}{a} < 1$

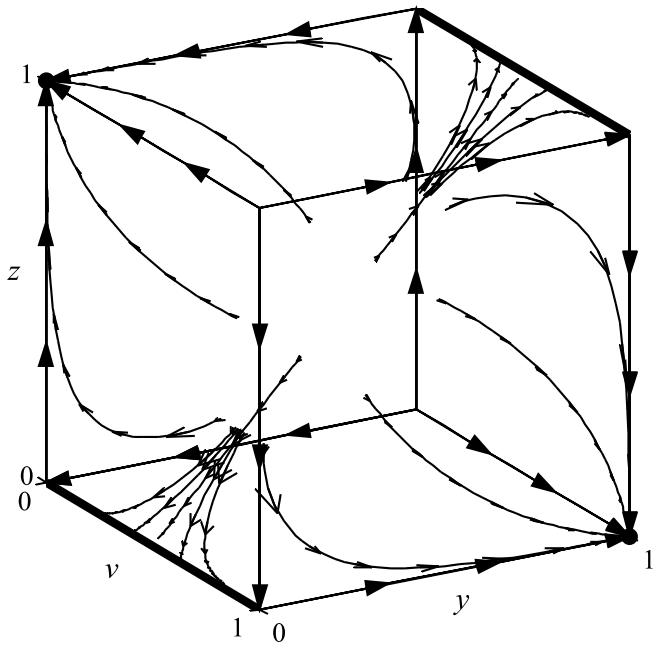


Fig. 6 Case 3: $\frac{1-\varphi}{\varphi} - \frac{x_1}{a} \geq 1$

prevalence of "for" vs. "against" populations, we see that the initial conditions clearly determine the final outcome. The process respects the structural trend of opinion in this regime. Why?

Notice that if we rewrite the parametric condition for the regime in Fig. 4 as $\varphi \geq \frac{1}{1+\frac{x_1}{a}}$, we see that when the parameter φ surpasses the specific threshold, then the peer externality component in the payoff functions (1) is sufficiently strong as to allow for a continuous transfer of citizens from high-commitment to low-commitment inside both subpopulations (pro vs. against). In this regime, citizens are highly sensitive to the opportunity costs of committing too many resources to the political action and, if they don't perceive quick prevalence, they tend to perceive political competition in mild terms. Furthermore, in this parametric regime (and Fig. 4), the subpopulation that retains highly committed people longer is the one that finally prevails; and retaining highly committed citizens depends on how the initial prevalence of for-vs.-against opinions appears in the initial conditions and develops in the early phase of the transition process (prevalence of pro vs. against in the payoff functions (1) in Section 3, represented relative visibility, power for institutional shaping and self-satisfaction for those that prevail). In any case, what is sure is that competition will end up (sooner or later) developing among citizens with low levels of commitment (because of the φ effect).

During the early phases of the transient time, and with an uncertain result on what position may prevail (as the process may self-organize blindly in real conditions), there is a highly uncertain interior point from which one of the options slightly begins to dominate. This effect together with the externality influence on payoffs in this regime, determine that, while citizens in both options reduce their commitment level ($\dot{y} > 0, \dot{z} > 0$), the one that prevails in terms of $v(t) = \gamma_t^A$ loses highly committed individuals more slowly. The final result emerges in favor of the opinion whose citizens keep a sufficient level of commitment. The final state has a lot to do with the initial conditions.

The high sensitivity to free-rider effects in (1) and, consequently, the low levels of commitment mean that once the power or the institutions are sufficiently (not necessarily much) shaped by the prevailing opinion (once a slightly clear direction emerges for $v(t) = \gamma_t^A$) then this effect self-reinforces through Eq. (4) and discourages even more the commitment levels (see Eqs. 5 and 6). Of course, once a specific dominant direction emerges, the group that perceives a decline in opinion quickly moves towards a low-commitment strategy (to avoid free-riding of peers). This effect reinforces the dominant emergent direction that had begun to appear slightly at first.

This is absolutely relevant, and it seems to represent those dichotomized real situations in which society is formed by citizens that gradually become detached of socio-political and public issues, and the relative final social presence of any of both options is much dependent on initial conditions. In this circumstances, there could be those that silently begin to reach slightly superior quotas of power the ones that silently end up gaining the political battle. Of course this result has large implications for people seeking to defend certain ideas but who are not true political defenders of their opinions. If they do not do the job, the other will do their job

(in their own direction) for them. The result can also explain situations with no big failures in polling results since the direction of opinion appears clear (never certain) at a certain point. To sum up, in this regime the dynamics reveal a certain tendency to maintain the *status quo*; if initially the social context is very fractured (two similar shares of opposing opinions), then retaining highly committed citizens for a longer period is essential to keep visibility and institutional control. It could be the case that intense actions and narratives may emerge in reality from both sides of the spectrum trying to avoid citizen detachment during the early phases of the process. However, the high value of φ plays a crucial role for citizens who are very conscious of the opportunity costs of political involvement.

Besides this, in those situations in which the initial conditions present uneven masses of citizens supporting for-vs.-against positions, since – in this regime – the relative prevalence of both opinions becomes essential in determining the direction of the public, the largely asymmetric initial distribution in favor of one option leads citizens in the minority option to detach (fast) from the socio-political process (fast convergence to low commitment levels because of the φ effect). In terms of proposing dichotomic choice processes, we can say that the final outcomes would represent well the structural trajectory of opinion (no sharp changes). Let us now move to the next regime (case 2 in Fig. 5).

In case 2 (in Fig. 5) the parameters verify the condition $0 < \frac{1-\varphi}{\varphi} - \frac{x_1}{a} < 1$. In this case, we can rewrite the expression as $\frac{x_1}{a} < \frac{1-\varphi}{\varphi} < 1 + \frac{x_1}{a}$. Here, we have an intermediate parametric regime. Thus, now, the ratio $\frac{1-\varphi}{\varphi}$ is higher than in the case 1 (Fig. 4) but has both upper and lower bounds. It is higher than the threshold $\frac{x_1}{a}$, though it also has an upper bound in $1 + \frac{x_1}{a}$. This situation leads the system to engender the dynamics depicted in Fig. 5. The situation in this second regime is more complex, an insight which is clearly conditioned by the lower value of φ as compared with the situation in case 1 (Fig. 4). Note that in this new regime (Fig. 5), both motivations in (1) play a significant role and, because of the lower value of φ, the strictly partisan motivation is going to be fundamental in many cases.

For this regime (case 2), we have two different types of relevant attractive equilibria: the edge $(\hat{v}, 1,1)$ with $\hat{v} \in [0,1]$, whose basin of attraction very much resembles the dynamics specified in case 1 (Fig. 4), but now with a trend that in general represents a tendency towards conformity situations (both around "for" or "against" options depending on initial conditions). On the other side, we now have two (opposite) vertices that also represent conformity situations where uniformity of opinion emerges ((1,1, 0), (0,0, 1)), but through a slightly different mechanism.

Thus, as we can see in Fig. 5, regarding the orbits tending to the attracting edge, they are characterized by initial conditions in which the masses of citizens with low commitment in both options are high and rather similar (no highly uneven). In this cases what is essential is the initial proportion of people defending the "for" vs. "against" options. The one that initially prevails is the one that ends up prevailing and with a level of citizen commitment that becomes low in both populations. It resembles Fig. 4.

What is really new and surprising in Fig. 5 (case 2 with a lower φ) is the case for orbits tending to the opposite attractive points. In these cases, which are

characterized by high initial levels of commitment in both populations, although the strictly partisan motivation (as given by the initial value of $v(t) = \gamma_t^A$ is also crucial), the option that ends up winning needs highly committed citizens, whereas the option that finishes almost disappearing is characterized by citizens highly detached from politics. In fact, and this is a very important difference when comparing case 2 with case 1 (Fig. 5 with Fig. 4), in the case 2 regime the option that prevails tends to do it through convincing citizens to the extent of reaching almost conformity. The partisan component and political change of opinion in this regime is much more relevant.

Furthermore, as we see in Fig. 5, there is a highly uncertain interior zone of the cube (around the central part) from which, in principle, for orbits departing from its proximities, it is almost impossible to know the final result. The direction and sense of the dynamics become clearer, once an uncertain point in the transient is reached from which one of the options slightly begins to dominate, and this determines the final result in favor of the opinion (pro vs. against) with a higher average level of commitment.

For initial conditions with relatively low initial (and similar) commitment levels near the center of the cube, there is a point from which once a specific dominant direction emerges, the group that perceives a decline in opinion quickly moves towards a low commitment strategy (to avoid free riding of peers). This effect reinforces the dominant emergent direction that had begun to appear slightly at first. Note that the turning point in the interior of the cube is not predictable in real processes. Of course, this result has large implications for people seeking to launch a dichotomized debate, or to pose a referendum, and it can explain failures in polling results depending on the moment of time in which the polls were carried out. Once citizens see that their option declines in support and peers begin to retire, they quickly decrease commitment thus amplifying the defeat of own option. Also, most of them move (changing opinion) towards the prevailing option. It is a situation that somehow resembles what is called "the fury of the recent convert" type of behavior. We refer to situations in the basins of attraction for the (opposite) vertices ((1,1, 0), (0,0, 1)). These situations may explain phenomena of "late-winner new fanatic proselitors in battles that has already begun to be won"; the largest increase in winners' commitment is observed when the for-vs.-against debate has already taken a clear direction.

Finally, we can analyze Fig. 6, where the dynamics emerging from the parametric regime in which $\frac{1-\varphi}{\varphi} \geq 1 + \frac{x_1}{a}$ are depicted. This is a regime (case 3) in which partisan motivations are very strong and play their role in (1), and in the dynamics driven by Eqs. (4–6). The ratio $\frac{1-\varphi}{\varphi}$ is high and very significant now (in case 3 regime). As we see in Fig. 6, this is the most complex regime. If we look at the cube's center in Fig. 6, we see now that the sensitivity to initial conditions is really high. More precisely, small differences in the initial configuration of public opinion, can determine large final differences in our evolving dichotomized society. Now we have two opposite edges as attractors (in which everything can happen regarding for-vs.-against final configurations, since $\widehat{v} \in [0,1]$), but we also see two opposite vertices (far from the attractive edges) as point attractors (see Fig. 6). In this regime – characterized by strong partisan preferences in Eq. (1) – , the type of process is highly influenced by whether we depart from

high or low commitment levels in both populations. In a society with high commitment since the beginning, we see that both populations are going to be highly committed until the end; there is fierce (dangerous) political competition at high levels with high uncertainty until the late stages of the process. There is high uncertain, a very active and confronted political climate, and polarization emerges easily (Fig. 6). On the contrary, when the initial levels of political involvement are low, although everything can eventually happen (regarding for-vs.-against competition), the competition deploys until the end with mild levels of commitment.

For the case of the two attracting isolated vertices, here we find that it is precisely the population (for or against) that engenders and maintains high commitment levels, the one that finally prevails, with the other becoming detached, and in both resting points we find conformity (no polarization at all). This is extremely clear if we look at the opposite attracting vertices out of the edges in Fig. 6. Finally, we would like to emphasize that in this case 3 (Fig. 6), characterized by high levels of partisan motivations, and a not very significant effect of externalities, the probability of ending up in highly polarized societies with similar shares of the population supporting shocking opposite opinions on an issue is high. The bundles of orbits in the cube, which tend towards centric points within the attracting edge in terms of pro vs. against are very significant. The situation is much more dangerous when the dynamics tend towards the lower edge. In these cases, we should be very careful in launching dichotomized processes of political competition.

On the contrary, when we looked at Fig. 4, we saw that when externalities to peer's behavior are highly influential in citizen preferences (this is what happens in case 1, Fig. 4) then the dynamics seem to be much more predictable since early in the process, and the probability of polarization is lower or, at least, predictable. In these cases, promoting evolving processes of opinion formation around antagonistic positions, or even carrying out referendum processes on controversial issues is less dangerous, as we should expect much clearer results since the very beginning of the process.

In the following section, we are going to see what happens in our model when, apart from considering the systematic determining factors parametrized in Eqs. (1–3) and that lead us to the system driven by Eqs. (4–6), we incorporate a source of novelty. More precisely, in the case of replicator dynamics with mutation (Section 5), we are going to explore the extent to which random components (factors not considered in Section 4) change the model dynamics when incorporated to processes of political competition in dichotomized settings. New results will come about.

5 Replicators with mutation

In order to incorporate random motivations in the model as a source of novelty (apart from the systematic factors seen in Section 4), we could consider now that, while we can maintain Assumptions I, II (Eq. 1), we can incorporate new suppositions -Assumption III' and Assumption IV' as follows (instead of Assumptions III and IV in Sections 3 and 4):

Assumption III′ We assume that citizens within both populations $\pi \in \{F, A\}$ may meet randomly (intra-population random matching), and revise their strategies by, eventually, switching strategy in a better-valuation direction. More precisely, for $\pi \in \{F, A\}$, if we denote by f_{ij}^{π} the rate at which citizens with strategy j switch to strategy i within π, we assume that this switching rate is given by:

$$f_{ij}^{\pi} = \gamma \left[u_i^{\pi} - u_j^{\pi}\right]_+ = \gamma max\left(u_i^{\pi} - u_j^{\pi}; 0\right), \gamma > 0.$$

Now, assuming that $\delta s_j^{\pi} s_i^{\pi}$ gives the probability for a random interaction between citizens with strategy j and those with strategy i in $\pi \in \{F, A\}$, in a small time interval Δt, the flow of citizens from strategy j to strategy i is given by $\delta s_j^{\pi} s_i^{\pi} f_{ij}^{\pi} \Delta t$. This is what we now call the systematic component of strategy update.

We now incorporate a mutation component according to which each citizen may also change their strategy (intra-population) randomly with a constant probability. If we consider that now, the change in the share of citizens playing strategy i ($i = 1,2$), in $\pi \in \{F, A\}$ is a linear combination of the systematic (payoff-increasing) mechanism and the random component (with a parameter $\mu \in (0,1)$ for the relative weight of the random factor in the linear combination), we have :

$$\Delta s_i^{\pi} = (1-\mu)\left(\sum_{j=1}^{2} \delta s_i^{\pi} s_j^{\pi}\left(f_{ij}^{\pi} - f_{ji}^{\pi}\right)\Delta t\right) + \mu\left(\frac{1}{2} - s_i^{\pi}\right)\Delta t,$$

where $f_{ij}^{\pi} - f_{ji}^{\pi} = \gamma\left(u_i^{\pi} - u_j^{\pi}\right)$. Thus, the continuous evolution of the share of citizens playing strategy i within π is described by a system of two differential equations for each population intra-revision dynamics:

$$\dot{s_i^{\pi}} = (1-\mu)s_i^{\pi}\left(u_i^{\pi} - u^{\pi}\right) + \mu\left(\frac{1}{2} - s_i^{\pi}\right),\ u^{\pi} = \sum_{j=1}^{2} s_j^{\pi} u_j^{\pi}\ \ i = 1,2; \pi \in \{F, A\} \tag{2'}$$

Of course, the payoff functions which appear in system (2'), are those given by Eq. (1) in Assumption II.

Assumption IV′ We take into consideration that citizens within both populations $\pi \in \{F, A\}$ engender trajectories for their corresponding average levels of population-commitment $x_t^{\pi} = \sum_j s_{jt}^{\pi} x_j$; recall also that $x_t = \sum_{\pi} \gamma_t^{\pi} x_t^{\pi}$ is the average level of political commitment in the overall society. Then, we assume that the population (political option pro vs. against an issue), which engenders a higher level of commitment, grows in support, whereas the other position loses supporters. In terms of inflows and outflows of citizen masses reflected in a replicator dynamics for the share of supporters in society, we can represent the dynamics of opinion shifts between the two options $\pi \in \{F, A\}$ as follows (now incorporating a random mutation component, with $\mu \in (0,1)$ denoting the relative weight of the random factor in the dynamics):

$$\dot{\gamma_t^{\pi}} = (1-\mu)\gamma_t^{\pi}\left(x_t^{\pi} - x_t\right) + \mu\left(\frac{1}{2} - \gamma_t^{\pi}\right),\ \pi \in \{F, A\}. \tag{3'}$$

5.1 Existence of equilibria and stability

Let us see now what new outcomes we can obtain from this extended version of the model. If we denote, again, for simplicity $v(t) = \gamma_t^A$, $y(t) = s_{1t}^F$, $z(t) = s_{1t}^A$, and we work out from (1), (2'), and (2') the interlinked simplex-projections of the model, we arrive at the following new system of three differential equations driving the model dynamics in the version of replication with mutation:

$$\dot{v} = f(v,y,z) = (1-\mu)av(1-v)(y-z) + \mu\left(\frac{1}{2} - v\right) \tag{7}$$

$$\dot{y} = g(v,y,z) = (1-\mu)(a\varphi)y(1-y)\left[\frac{x_1}{a} + y - \frac{(1-\varphi)}{\varphi}(1-v)\right] + \mu\left(\frac{1}{2} - y\right) \tag{8}$$

$$\dot{z} = h(v,y,z) = (1-\mu)(a\varphi)z(1-z)\left(\frac{x_1}{a} + z - \frac{(1-\varphi)}{\varphi}v\right) + \mu\left(\frac{1}{2} - z\right) \tag{9}$$

with $a > 0, x_1 > 0, x_2 = a + x_1 < 1$ and $\varphi, \mu \in (0,1)$.

The system determines the time evolution of the orbits within the unit-cube $[0,1] \times [0,1] \times [0,1]$, which is an invariant space set for the dynamics.

The system represented by Eqs. (7), (8), and (9) is a convex combination of the following two subsystems (or components):

(i) The systematic part of the *evolutionary replicator dynamics* obtained if we consider $\mu = 0$. As we have seen in the previous section, the dynamics for this subsystem are volume-expanding, and the orbits within the cube always tend towards the cube's boundary.

(ii) The *purely random component*, that we obtain when $\mu = 1$. It is straightforward to show that, in this case, we have a linear system with a unique (interior) globally stable equilibrium $v = y = z = \frac{1}{2}$. Thus, all the orbits within the cube tend towards this resting point in the center of the unit cube.

For the general case with $\mu \in (0,1)$ it is easy to show that the dynamics within the unit cube $\mathbb{U}$ are volume-contracting. More precisely, in the cube's face $v = 0$ we have that $\dot{v} = \frac{\mu}{2} > 0$, whereas in the opposite face $v = 1$ we have $\dot{v} = -\frac{\mu}{2} < 0$; likewise, in the cube's face $y = 0$ we have $\dot{y} = \frac{\mu}{2} > 0$, whereas in the face $y = 1$ we obtain $\dot{y} = -\frac{\mu}{2} < 0$; finally, in the cube's face $z = 0$ we have $\dot{z} = \frac{\mu}{2} > 0$ and in the opposite face $z = 1$ we have that $\dot{z} = -\frac{\mu}{2} < 0$. As a consequence, for all $\mu > 0$ the orbits within the unit cube remain always within the cube.

Furthermore, for $\mu \in (0,1)$ the cube's faces are no longer invariant and the system's attractor is in the cube's interior, although now we obtain a new invariant set: the segment Γ connecting the points $\left(\frac{1}{2}, 0, 0\right)$ and $\left(\frac{1}{2}, 1, 1\right)$. The points in Γ, which can be denoted by $\left(\frac{1}{2}, \beta, \beta\right)$ with $\beta \in [0,1]$, verify that $\dot{v} = 0$, $\dot{y} = \dot{z}$; there-

fore, any orbit with an initial condition such as $\left(\frac{1}{2}, \beta_0, \beta_0\right) \in \Gamma$ satisfies that $v(t) = \frac{1}{2}$, $y(t) = z(t)$, that is to say it verifies that, $(v(t), y(t), z(t)) \in \Gamma$ for all $t \geq 0$.

The one-dimensional dynamics of the system on Γ is given by the following differential equation in the interval [0,1]:

$$\dot{y} = G(y) = a\varphi(1-\mu)y(1-y)\left(\frac{x_1}{a} + y - \frac{1-\varphi}{2\varphi}\right) + \mu\left(\frac{1}{2} - y\right)$$

Since we have $G(0) = \frac{\mu}{2} > 0$ and $G(1) = -\frac{\mu}{2} < 0$, it is clear that the system has at least one interior equilibrium point which is asymptotically stable: $G(\hat{y}) = 0$, with $\hat{y} \in (0,1)$. Depending on the parametric values, the system could have up to three resting points in Γ (in this case, the intermediate equilibrium would be unstable and the other two resting points would be attracting ones). We illustrate in Fig. 7 both options.

Besides this, within the unit cube $\mathbb{U}$, but out of the segment Γ, we could have even more equilibrium points. These new equilibria (in those cases in which they exist) appear in a pairwise manner: thus, it is clear that if $(\hat{v}, \hat{y}, \hat{z}) \in \mathbb{U}$ is an equilibrium point for the system (7), (8), (9), within the unit cube (it satisfies $\dot{v} = \dot{y} = \dot{z} = 0$), then the point $(1 - \hat{v}, \hat{z}, \hat{y}) \in \mathbb{U}$ is also another interior equilibrium for the system.

This symmetry property with respect to the curve Γ is valid for all the orbits of the system. Thus, if $(v(t), y(t), z(t)) \in \mathbb{U}$ is a solution of the system given by (7), (8), (9), then $(v_s(t), y_s(t), z_s(t)) = (1 - v(t), z(t), y(t)) \in \mathbb{U}$ is another solution of the system

$$\begin{aligned}
\dot{v}_s &= -\dot{v} = -f(1-v, z, y) = f(v_s, y_s, z_s) \\
\dot{y}_s &= \dot{z} = h(v, y, z) = g(1-v, z, y) = g(v_s, y_s, z_s) \\
\dot{z}_s &= \dot{y} = g(v, y, z) = h(1-v, z, y) = h(v_s, y_s, z_s)
\end{aligned}$$

As a consequence, depending on the parametric values, the attracting set of the system (which represents its long-run asymptotic behavior) can be formed by only one equilibrium point or, in other cases, by several resting points.

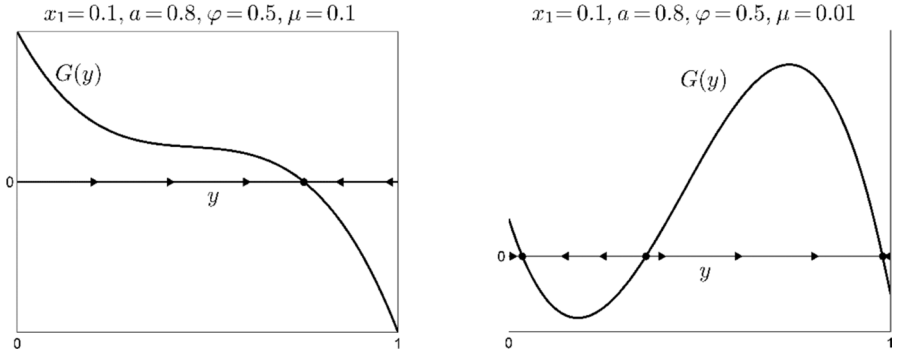


Fig. 7 Alternative options for equilibria in the one-dimensional system on Γ

In Figs. 8 and 9, we illustrate with simulations (for different parametric values) both options. In Fig. 8 we represent the case in which the attracting set of the system is formed by one equilibrium. On the contrary, in Fig. 9 we see the case with several equilibrium points. The interpretation of the formal study is clear in this case of a replicator with mutation system (Eqs. 7, 8, 9). As we have seen in the analysis and can be visualized in Figs. 8 and 9, the relative importance of the random component (as determined by the value of parameter μ) is crucial for the overall dynamics of this generalized version of the model. When this parameter is very close to zero (Fig. 9) the dynamics very much resemble those explained in Section 4. These would be situations in which the systematic part of the dynamics, as given by Eqs. (4–6) in Section 4, are the key drivers of the evolving political economy within our dichotomized society. Almost no randomness means a rather stable environment in which the operational motivations, depending on the parametric values, are those explained in Section 4. The crucial novelty of this section (Section 5) appears when μ is sufficiently high in Eqs. (7), (8), (9); that is to say, when socio-economic shocks, sudden political irruptions, corruption scandals, social turmoil, geopolitical confrontations, surprises, radical uncertainty open the way to random elements that may alter citizen's behavior in unexpected ways. This is the case in Fig. 8. As the dynamics in these cases tend to the center of the unit cube, this means that we have *maximum political polarization* as an emergent result. Similar masses of shocking antagonistic opinions (for vs. against important issues), and a significant (and similar) share of the population in both sides of the political spectrum highly committed and intensely involved, is what characterizes these situations. Also, as we have seen in the analysis, this dangerous, highly polarized social attractor (within sufficiently uncertain societies) is globally stable, there is no way out of this dangerous outcome (see Fig. 8). Obviously, the combined implications of the surprising results

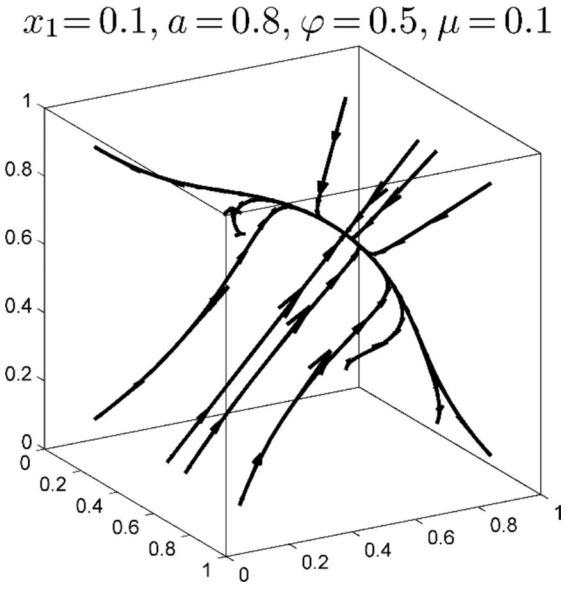


Fig. 8 The dynamics with only one equilibrium

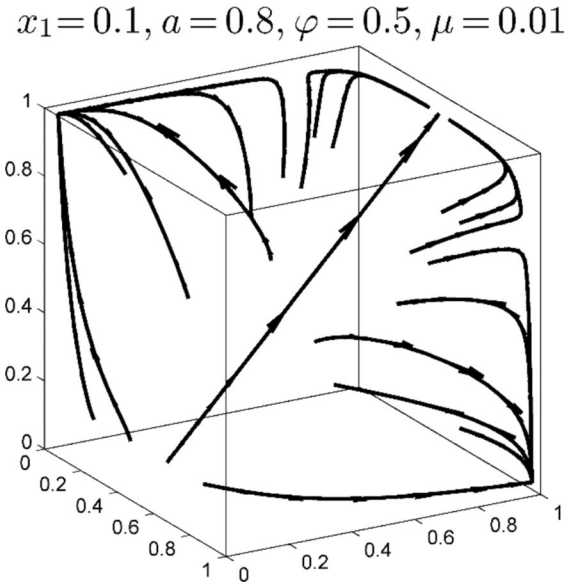


Fig. 9 The dynamics with multiple equilibria

in Sections 4 and 5 lead us, in Section 6, to an overall (but brief) discussion of open issues on the dynamics of dichotomized evolving societies. This is a theme of utter importance in our turbulent times.

6 Discussion

We started out by stating that various schools of social thought from the last third of the 20th century placed an emphasis on the plurality of perspectives, individual emancipation with regards to functional structures of society, language and communication as basic elements providing the meaning and formation of political and esthetic ideas, and the continuous local and direct public participation in politics as forming ideals for the new millennium. Some of these facts and features have been represented in previous sociophysics models and studies on quantitative sociology.

In our model, we have taken the possibility of these visions seriously and we have found scenarios where the debate, in conditions of freedom and symmetry, can reach complete conformity (defended by Habermas (1986) as the objective of communicative reason and the ethics of discourse). These situations are not problematic and have appeared in alternative parametric regimes in Section 4, and in the case of low randomness in Section 5. However, the possibilities of conflict increase when opinion-forming evolves in situations of "for" vs. "against" heading towards a final split which is more or less equal in support and with highly committed partisans. Our model generated this possibility under certain specific conditions in Section 4, but it was a globally stable social attractor, a very dangerous attracting and highly plausible situation indeed, in the case with relatively high μ in Section 5. For this kind of situations, which appear in for certain parametric regimes in the two versions of the model, we

might find corresponding real dynamics leading to what Daniel Chirot (2020) called the tragic consequences of radical-idealism (utopic) participatory antagonism. The proposals from the different (in our model, two) parts of the debate for easy, emotionalized and simple solutions to complex economic and political problems (the dissolution of political States and institutions, the incorporation in large and promising technological-emancipatory new social structures, the introduction of nostalgic renewed values – barely understood but colored with attractive and favorable feelings, or futuristic utopias which promise more than they can deliver) may dangerously lead to situations of social rupture and economic instability (Durand 2020). In our model, we have detected alternative scenarios in which these dangerous dynamics can emerge.

Finally, we would like to bring out in this brief final discussion a warning or prudent advice for contemporary times. Although it is undeniable that human hyper-creativity, a limited role of controlling institutions and governments in participatory technologically advanced societies, the self-organizing properties of capitalistic-democratic societies, and trusting in human freedom within well-educated innovative societies are core principles for good socio-economic and political governance in modern times, we should be (in our reforms) respectful (or at least aware of) the best practices, traditions, and institutions of mutual respect and co-existence that history has brought about. Gradual change, compromise instead of antagonism, and expert analysis followed by moderate reforms are the best ways to avoid extreme polarization and chaos. If these conditions do not apply, we can see worrying circumstances as those currently observed in the European Union, the US, and perhaps in an increasing number of societies worldwide.

7 Conclusion

Recent historical events contest the idea that we have reached the end of history in the sense of Fukuyama (1992). Similarly, it does not look like post-modern thinking and the deconstruction of social structures had led us to a definitive digital Arcadia. An indication that many currents of thought from the end of the twentieth century have questionable foundations is seen in the fact that we are now experiencing the appearance of new schools of social thought based on neo-realist and neo-materialist ideas. Hence, the concept of New Realism (Harman 2011; Quentin Meillassoux) returns to a realistic ontology based on firm realities and objects and an epistemology capable of generating solid knowledge. We can see the same through other authors within this current such as Ian Hamilton Grant (2014) and Maurizio Ferraris. Furthermore, new currents of philosophical materialism (Romero et al. 2022) have also returned to a solid debate on political concepts, an interest for the role of science, and a capacity to seek firm truths, and a material ontology.

From a quantitative point of view, we observe the productive field of sociophysics, in which new public opinion models continuously explain emerging regularities in multi-agent computational settings and allow us to better understand the socio-political changes of the present. Models in line with Weidlich (2006) and the Galam (2008, 2012) families of models are very significant in relation to our work, and open lines for future research that may bring socio-physical insights closer to the evolutionary economics conventional readership. Both the sociological and

quantitative debates on public opinion formation during (at least) the last fifty years have cleared up many of the details regarding collective choice in dynamic settings. Their social and economic influence has been immense. This is seen in their incorporation of heterogeneity of standpoints, social shaping, social conflicts, and in the questioning of economic essentialisms, as well as in the defense of a complex evolving systems perspective to analyze political processes (Hodgson 2015; Dosi 2023).

As potential research lines which very much follow from the limitations of our model, we suggest, firstly, trying to link pre-existing evolutionary studies on technological and economic change with socio-political aspects that socio-physics, differential equations models like ours, and current political studies are uncovering in complementary ways. Evolutionary economists increasingly recognize the need to complete their studies on technical change with institutional and socio-political insights. The aforementioned research fields and potential extensions of our model may open future paths to carry out the advances. Secondly, there may exist fruitful ways of mutual fertilization among neat differential equation models like ours and the whole universe of models in socio-physics, which deal with public opinion dynamics from stochastic multi-agent settings and/or models from statistical physics and explore the emergence of regularities such as polarization, endogenous fluctuations of opinion, or conformity results. Of course, the development of both research paths goes beyond this paper's limits and scope but are attractive lines for future work.

Having said that, we conclude by warning that in a digitalized environment, technically hyper-connected, deconstructed by post-modern thinking and affected by the successive crises since the start of the new millennium, we must be conscious of the risks we take on when we overuse or misuse direct consultation with citizens overwhelmed by an excess of information (Dahl 1989). In our model, we have incorporated feasible features, realistic in the present-day society, and strived to define conditions in which choosing the option of a referendum or deliberative democracy may be recommended and work well, compared with ranges of parameters for conditions which could unchain uncontrollable conflicts. The final decision will always fall within the real dynamics each society develops, but given our results and some current lines of thought, we believe we must warn of the effects of taking up an unquestionable stance in favor of one side or another of the theoretical debate.

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Author contribution I. A. has contributed to the conceptualization of the main ideas, formal design of the model and formal analysis. She has also contibuted to surpervised and analysed the results and to write the final draft of the article. F.F. has contributed to the conceptualization of the main ideas, formal design of the model and formal analysis. He has also contibuted to surpervised and analysed the results and to write the final draft of the article. F.J. V. has contributed to the conceptualization of the main ideas, formal design of the model and formal analysis. He has also contibuted to surpervised and analysed the results and to write the final draft of the article.

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Data availability We do not analyze or generate any data sets in this work, because our study proceeds within a theoretical and mathematical approach.

Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

Competing interests The authors declare no competing interests.

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