



Expanding the horizons of thermoeconomics: Insights from the School of Zaragoza

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ABSTRACT

The article discusses the future developments of Thermoeconomics, focusing on the current works of the School of Zaragoza, which integrates the Second Law of Thermodynamics with Economics to assess the physical origin of costs, focusing on process irreversibility. Traditionally used in energy systems, the authors argue that Thermoeconomics should expand to chemical, geological, and economic sectors. Key concepts from the Zaragoza School include Thanatia, a hypothetical state representing Earth's mineral depletion, used to measure resource depletion costs; Thermodynamic Rarity, highlighting the loss of Earth's mineral capital; Pristinia, for evaluating deviations from an ideal fertile crust; and Circular Thermoeconomics, based on the idea of waste as external irreversibility, promoting a circular economy to minimize waste. They also introduce Relative Free Energy, a new thermodynamic function that improves thermal system diagnostics. The authors suggest that Thermoeconomics could align economic activities with environmental realities by using physical metrics like exergy to evaluate resource consumption, leading to a sustainable economic model incorporating environmental costs. The paper advocates for broader application and further research in Thermoeconomics to manage the true cost of human activities.

1. Thermoeconomics: Towards a physical metric of economics

Thermoeconomics¹ originated with the approach of allocating co-product costs in proportion to exergy. Notable contributions from scholars in the 1960s, 1970s, and 1980s are those from [1–9]. Since then, the second law studies have developed concepts like true efficiency, exergy costs, system diagnosis, optimization, and simulation of energy structures, all centered around designing cost-effective energy structures [10–27]. Beyond these results, certain foundational concepts at the intersection of Thermodynamics and Economics, required further reflection through the lens of Second Law Analysis. In our opinion, these were efficiency, consumption, and cost.

Exergy efficiency is a more precise metric than the one based on the First Law. Thermodynamics, as well as Economics, use indiscriminately the word consumption. We speak of consumption of raw materials, energy consumption of some equipment, or consumption per unit of product. Specific consumption is a common indicator used in industry that relates the input of a machine to its output.

Cost, in turn, is a more general concept that accounts for all inputs to a system, translating each into monetary units. While consumption is measured in physical units (e.g., kg, kWh, or kWh/kg) and is closely

related to efficiency, unitary or absolute cost is expressed in monetary units.

But what if efficiency, consumption, and cost are expressed in exergy terms? Second Law efficiency, in fact, becomes the inverse of the unit exergy consumption for any machine or process. Meanwhile, exergy cost represents the cumulative exergy consumption of energy and material inputs of any industrial facility composed of different processes [12]. More importantly, we demonstrated that the definition of the exergy efficiency of each process determines the exergy cost of the products of the entire plant. Thus, efficiency and Cost become unified concepts under the Second Law analysis [11,16].

This theoretical result is so powerful that it supports the argument that exergy might be an alternative metric to money. It also offers a new perspective in Thermoeconomics: the goal is no longer solely to convert exergy into monetary costs, but rather to explore the physical cost of any natural or human process. Exergy can quantify all natural and/or synthetic resources as a measure of any material or energetic manifestation

Remarkably, the structure of Thermoeconomics remains isomorphic regardless of the chosen metric — whether money, exergy, or any other

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¹ This paper provides a broad perspective on Thermoeconomics as developed by the Zaragoza school and is not intended as a comprehensive review of the field's state of the art.

conceivable metric. In fact, it is a structure derived from the input-output analysis proposed by Leontief [28]. Exergy costs, however, offer the advantage of being measured in kWh, bypassing the vagaries and fluctuations of monetary costs. Moreover, we have obtained a very solid result: exergy cost represents the cumulative, prorated irreversibilities involved in producing any product or service.

The exergy cost of an energy stream (B^*) equals the remaining exergy (B) plus the accumulated irreversibilities during the process (I), all expressed in energy units (kWh).

$$B^* = B + \sum I$$

The Second Law of Thermodynamics is conclusive: degradation is a physical and unavoidable process in any real activity, and this degradation can be quantified through entropy generation, which is linked to degradation via the Gouy–Stodola Law: $I = T_0 S_g$. So why not use Thermodynamics to account for the depletion of the planet's natural resources?

In its physical expression, cost accounts for the loss of natural resources. This is the central paradigm of thermoeconomics. This loss can be identified with the accumulated irreversibility from cradle to commodity. To achieve this, we simply need to extend the concept of dead state to assess exergies. In other words, we have expanded Thermodynamics to evaluate the planet's resources, opening up immense possibilities for future research and application.

To clarify, let us consider the *production factors* like Capital, K , Labor, L , Materials, M , Energy, E and Environmental Costs, O , as defined in Economics [29]. How do these relate to Thermoeconomics? Firstly, any exchange of K , L , M , E , and O will generate entropy and, as a result, lead to irreversibilities. Second, as early as the 1970s, these production factors were proposed to be reduced to exergy destruction.. Researchers such as Fratzscher [30,31], Szargut [32], Wall [33], and Grubbstrom [34,35] demonstrated that it is possible to develop a physical costing paradigm in which “production factors” can be directly or indirectly related to exergy consumption, which in turn is connected to the consumption of natural resources. In this way:

- Capital: Producing capital requires human labor, natural resource extraction, and the environmental damage caused by pollution and depletion. Therefore, capital carries with it a “backpack” of labor, resource extraction, environmental impact, and natural depletion. Borrowing capital to start a new initiative will influence future labor, drive resource extraction and depletion, and generate environmental impact. Besides, any transfer of capital requires energy to move it from one point to another, and materials are integral components of technology. Humanity created money and sustained itself with people's confidence. Without confidence, there is no value. Energy is eternal; money is not. The true origin of capital lies in nature. However, the relationships between capital and natural resources are not linear. Humans often take many valuable natural elements for granted, such as life, the biosphere, or the geosphere, and in other cases, they do not. At the Zaragoza School of Thermoeconomics, we are not concerned with money itself (as we are not economists), but rather with understanding the natural backpack that accompanies any human activity. Irreversibility terms, i.e., kWh, measure such an activity. We are not alone in this pursuit; the work of Sciubba, [22], in assigning an exergy value to the capital of a given ecosystem is particularly noteworthy.
- Human labor: People need to eat, live, defend themselves, reproduce, excrete, build companies, use, destroy, and create artifacts, among other activities. Every form of labor carries a “backpack” of natural resource extraction, environmental impact, and the depletion of nature.
- Energy, Materials and Environmental Costs: These are precisely the focus of our accounting. How much is the physical cost of any human activity when measured in terms of exergy costs?

In conclusion, production factors such as K , L , M , E , and O can be reduced to Exergy loss/Irreversibility terms (kWh).

In our view, Thermoeconomics is not the “economics of physics” but rather the “physical shadow of economy”. At least in theory, any action can be estimated in terms of irreversibility through the input-output model and using exergy as a metric of physical value. Perhaps in the near future, by using AI, we will be able to associate an irreversibility cost to all human activities.

Money is, in fact, a proxy for use value, but this is not the only one. How do we measure the destruction of ecosystems and planetary resources in general? Which currency should we use? Dollars? Euros? Yuans? Yens? Pesos? Denarius? Drachms? Gold coins? Money is a measure of the present value of our civilization. However, humanity's impact on the planet cannot be assessed using the thousands of currency types that have existed throughout history. Instead, the irreversibility metric can track this impact from the beginning of our civilization to the present and into the future. The kWh measure is immutable.

Whereas the Second Law is universal and applies to any economic or physical activity, it provides a physical structure parallel to economics itself. This allows us to view Economics from the perspective of the physical world. For example, while economic activity pursues profit, irreversibilities are produced every time we carry out any process. Irreversibilities can be seen as the opposite of profit. Furthermore, the economy views nature as a supplier of raw materials or solace. It is designed to value the exchange of services between humans. However, we do not pay nature for its resources; we only pay the extractor. We do not compensate for nature's regeneration or even its depreciation. To overcome this shortcoming, the economy includes the unavoidable reparations of the natural environment as externalities. In short, money was created to measure exchanges between individuals or organizations. It is a human construct that nature does not recognize, as it follows only the principle of action and reaction.

The idea of linking the economy to nature is an ancient one, recognized by the early physiocrats, who considered nature the source of all wealth because they believed it regenerated itself. Interestingly, when the currency's value is questioned, the gold carat is used as a safe haven.

Many authors, especially economists, have built bridges between Thermodynamics and economics; among them, Nicholas Georgescu-Roegen was a precursor with his book “The Law of Entropy and the Economic Process” [36]. Although he called it *bioeconomics*, this book laid the foundation of *ecological economics*. Interpreting Georgescu-Roegen's message, Bonaiuti says [37]: “The economic cycle is radically entropic. This cycle consists of demand-stimulating production that provides the necessary income to feed the new demand in a seemingly indefinite process. In any case, this cycle is not circular but irreversible and dissipative”. In fact, new exergy is destroyed at every turn. Economic exchange takes place at the expense of natural depletion. Economic optimism turns into thermodynamic pessimism.

Moreover, if the biosphere is essentially a closed system in materials, the unlimited growth of production (and income) clashes with the Second Law [38]. In 1971, Thermodynamics had not yet fully incorporated the advances in Second Law Analysis, particularly exergy; however, Georgescu-Roegen's message was qualitative and even metaphorical. Today, with the alternative perspective described above, it is possible to convert these ideas into quantitative values. These ideas are now well established within the thermodynamic framework of sustainability, though they remain far from mainstream economic thinking. For example, Hall [39] developed the concept of energy return on energy investment (EROI) within his biophysical economics. Additionally, the contributions of H. Odum [40] and collaborators on the concept of exergy, Sousa and Domingos [41] defining irreversibility in economic behavior, and Kjelstrup et al. [42] proposing exergy to supplement existing scales for assessing the use of natural resources, are notable.

In this paper, we focus on a perspective of Thermoeconomics through the concept of exergy and the vast room for future development by integrating the extraction of geological resources with Economics. This new approach enables us to assess the planet's natural resources and provides a framework for evaluating the damage our civilization is inflicting on it. It helps clarify the costs involved and supports informed action.

The purpose of this paper is to describe the conceptual advances of the Zaragoza school of thermoeconomics and the need for future development based on these advances. We have divided the paper into four sections: The first section addresses Thanatia, exergy costs, and the rarity of minerals. The second section introduces Pristinia, a planetary model for assessing the degradation of fertile soils. The third section presents the principles of a new Circular Thermoeconomics, based on the thermodynamic concept of waste. Finally, the paper concludes with an introduction to Relative Free Energy. This new thermodynamic function that depends on both the fluid and the machine in operation, offering a fresh perspective for diagnosing thermal systems.

2. Thanatia: A planetary reference state for exergy calculation

One of humanity's most pressing problems in the coming decades, if not centuries, is the rapid destruction of the planet's natural resources, particularly non-renewable minerals. These resources are extracted, processed, and used, and ultimately, in the final stages, they are dispersed into the earth's crust and oceans or contribute to the pollution of the atmosphere and freshwater. Therefore, a significant contribution of Thermoeconomics to planetary sustainability would be the development of a physical theory to assess the periodic cost of this destruction, aiming to minimize and manage it more effectively.

The entropic generation, and with it, the irreversibility of the processes, depends on the reference state one chooses. In conventional process exergy analysis, the choice of a reference state is linked to the type of problem one wants to solve, i.e., we explicitly or implicitly delimit the relevant costs/irreversibilities. Consequently, if one wants to evaluate the destruction of the planet's resources, one needs to define a reference environment, also called *dead state*, or depletion state, *DS*, in mining terms. This reference environment enables the assessment of both resource depletion and waste generation costs at various levels.

The most important characteristic of a reference environment is that it serves as an attractor for the system's intensive properties. Spontaneous evolution occurs due to the differences in temperature, pressure, concentration, or composition between a system and its environment. Therefore, defining the temperature, pressure, composition, and concentration of all chemical substances in the reference state that may interact with the system is crucial. The exergy loss, both in quantity and quality, will then enable the assessment of the degradation of the planet's mineral resources.

Thanatia would represent a hypothetical end state of the current exponential trajectory of human-induced degradation: The Earth's crust, hydrosphere, and atmosphere would have reached the maximum dissipation of all spent materials [43]. Although this scenario is hypothetical and indicates a final depletion, it is purely instrumental and devoid of social or economic connotations, which would obviously take place long before it is reached. This model was developed by Alicia Valero in her PhD thesis in 2008, [44], with the current geochemical and geological information on the atmosphere, hydrosphere, and crust described below.

The so-called crepuscular or twilight atmosphere in Thanatia would form after the depletion of all conventional fossil fuel reserves, with an atmospheric injection of about 2000 Gigatonnes of carbon. According to previously published simulations, this crepuscular atmosphere would have a carbon dioxide content of 683 ppm, an average surface temperature of 17 °C (maximum carbon dioxide-induced warming of 3.7 °C above pre-industrial temperatures), a pressure of 1.021 bar and a composition by volume, of 78.8% N₂, 20.92% O₂, 0.93% Ar and

0.0015% trace gases. (Note that, even if the atmosphere were filled with anthropogenic CO₂, it would be "only" 0.07% [45]).

We assume that the crepuscular hydrosphere would maintain the current chemical composition of the oceans at an average temperature of about 17 °C. This implies that all ice sheets would have melted and dissolved into saline waters. Saline water makes up about 97.5% of the entire hydrosphere, while the remaining 2.5% is freshwater, most of which is stored as ice in Antarctica and Greenland. The rest, though it may seem vast, is marginal in comparison and exists as groundwater, in lakes, rivers, soil, and atmospheric moisture. Only 0.3% of the total freshwater is concentrated in lakes, reservoirs, and river systems. This portion of the hydrosphere is essential for human life but is also the most vulnerable to pollution. Additionally, due to climate change, a significant portion of the glaciers and other ice in the cryosphere is melting.

Without any significant error, we can state that the crepuscular continental crust can approximate the average mineralogical composition of the present-day Earth's upper crust [46]. According to the US Geological Survey [47], the mass of non-fuel industrial mineral resources worldwide is of the order of 10¹⁵ kg. Furthermore, the World Energy Council [48] estimates the amount of available conventional and unconventional fossil fuels to be about 10¹⁶ kg. This means that all concentrated mineral resources of fuel and non-fuel origin represent only 0.001% of the total mass of the Earth's upper continental crust (about 10²² kg, according to Yoder [49]).

The problem we faced was identifying what type and how many significant and distinct (in a thermodynamic sense) minerals are present in the upper continental crust. We proposed a model based on the mineralogical composition studied by Grigoriev [50], ensuring consistency between species and chemical elements. The resulting crustal model comprises at least the 292 most common minerals.

In short, the current state of the biosphere is incredibly close to Thanatia. In our hypothetical reference state, the atmosphere would have reached the state predicted by long-term climate change models. Even if the amount of CO₂ doubled, there would be less carbon dioxide than argon, and this noble gas is considered a trace element. Even if all mineral deposits were increased tenfold in the Earth's crust, the crust would contain no more than 0.01% of them (less than the crustal mass estimation error). Thus, if all commercially exploitable mineral deposits disappeared, the composition of the crust would be virtually the same. Accordingly, the dispersion and degradation of all already used materials mean nothing compared to the amount of common rocks (primarily silicates) mostly found in the crust.

As for the hydrosphere, the pollution of all freshwaters and the melting of the Arctic, Greenland, and Antarctic ice would have a quantitatively more significant effect, but not more than 2.5% of the total waters of the blue planet. The hydrological cycle driven by solar radiation would still exist. But, it must be accountable that water in the atmosphere is only 0.00093% of the total water while rivers account for 0.00015% of all the water on the planet [44]. Therefore, we hold that the hydrosphere of Thanatia would be composed almost entirely of standard salt water.

The present Earth is not in the equilibrium state; its continental masses evolve slowly over geological timescales but more rapidly and spontaneously in chemical terms towards Thanatia. This progression follows the logic of the second law, and there is no catastrophism in this idea. The speed at which we reach Thanatia depends on human actions. However, before we reach that point, it is not difficult to imagine that economic and perhaps civilizational collapse could occur sooner, marking a possible end of the Anthropocene period [51].

2.1. The exponential behavior of mining demand and the exergetic "U"

Our society does not comprehend exponential behavior. The extraction of minerals on the planet is growing so that by 2050, more than twice (or perhaps three times) as much will be consumed as is

extracted today. By 2050, more minerals will have been extracted than in the entire history of human civilization. Phosphate mines limit the world's agriculture and food supply. A mobile phone requires more than 35 chemical elements in the PCBs [52]; a conventional or electric vehicle requires more than 50; and renewable energies, electrochemical storage, and green hydrogen production require huge quantities of rare metals [53]. Therefore, it is necessary to review the message of entropy: its exponential character will help us to understand and assess those behaviors.

It is relevant to analyze the phenomenon of mixtures from a thermodynamic point of view. As is well known, the irreversibility of a mixing process takes the form:

$$I = -RT_0 \sum_{i=1}^n x_i \ln x_i = -RT_0 \ln 10 \sum_{i=1}^n x_i \log_{10} x_i,$$

where x_i are the mole fractions of the n substances to be mixed.

In the case of a mixture of two substances, A and B

$$I = -RT_0 [x_A \ln x_A + (1 - x_A) \ln(1 - x_A)] \quad (\text{kWh})$$

$$\frac{dI}{dx_A} = -RT_0 \left[\ln \frac{x_A}{1 - x_A} \right]$$

This equation is symmetric around $x = 0.5$, where it has a minimum, and is asymptotic with $x_i = 0$. This means that the irreversibility increases exponentially when wanting to decontaminate A from B or B from A. The separation of the last contaminating particles requires an exponentially increasing exergy. The graphical representation of the irreversibility versus the logarithm of the mole fraction resembles a U-shaped curve. We call this behavior the exergetic “U”.

This exergetic “U” is essential from a practical point of view. Ore grades of many mineral deposits are often found at very low concentrations, typically in the range of 10^{-1} , 10^{-2} , ..., 10^{-n} type. These low ranges imply that when depleting a mine with an initial concentration of 10^{-1} and reducing it to one of 10^{-2} , the exergy required for the same amount of extraction increases tenfold. This trend continues with factors of 100, 1000, and so on, while what remains are increasingly small, declining residual traces. For example, this trend is historically proven with gold, where its ore grades have been reduced by several orders of magnitude over the last 120 years [54]. In addition, as the actual processes are far from reversible, extraction and beneficiation of minerals require an amount of energy several orders of magnitude higher than the thermodynamic minimum.

It is not our technological ignorance that imposes such behavior but Thermodynamics, which describes the ideal case of separation. In other words, even if today's mining operations operate gigantic equipment, the physical decline of global ore grades will require more and more energy per tonne of ore extracted.

Unfortunately, metallurgical processes are also governed by the law of entropy. The energy costs associated with reducing, purifying, and refining metals increase significantly and always remain far from their thermodynamic minimum, especially as the desired purification degree becomes higher.

In other words, the law of entropy is evident in the earth's crust. It is not an orderly reservoir of minerals, like in a “department store”. This phenomenon had to be added to the Thanatia theory. Many techno-optimistic messages say there are enough minerals to sustain global development based on new technologies because abundant minerals are in the earth's crust [55]. Apart from the fact that there are unexplored areas with some constraints, they forget that mineral concentrations are not homogeneous but declining in general. The best ore grades are typically exploited first through the “low-hanging fruit” method, and as these resources are depleted, the quality of the remaining ore declines, eventually making further extraction unfeasible. As a result, the energy, water, tailings movement, and the environmental, social, and economic impacts of mining will increase exponentially, perhaps within years, decades, or—though less likely—centuries.

2.2. Exergy, extraction and replacement costs and thermodynamic rarity

The chemical reactivity of non-fossil minerals is generally weak and indistinguishable between different minerals. As a result, their chemical exergy cannot be used to create a theory for valuing them. Their demand is instead driven by the physicochemical properties of the elements or compounds they contain. These minerals must be extracted, processed, and refined to become useful, all of which require energy, often from chemical or electrical sources. Therefore, rather than valuing minerals based on their exergy content, such as through Szargut's well-known Reference Environment model [12], we must assess their value based on the amount of exergy needed to process them from cradle to market. In other words, we value them by their exergy cost of extraction, which can be measured in kWh.

In addition to the tonnage of a mineral deposit, a mine's value might be closely linked to its mineral concentration or ore grade. As the mine is exploited, this value decreases over time. Therefore, the variation in extraction costs is an important indicator, as it reflects the depletion of the mine and, by extension, the loss of the planet's mineral wealth. This trend highlights the growing difficulty and increasing costs associated with extracting lower-grade ores as mining progresses.

However, extracting cobalt is not the same as extracting aluminum. The latter is abundant in the earth's crust, whereas cobalt is much scarcer. Therefore, scarcity is also an indication of value. While one relates the value of scarcity to the physical cost of forming it, there is a problem because the cost of exergy depends on the chosen analysis limits. The resources used to measure it have a cost that expresses the amount of resources used to produce it, and so on, down to the cradle. This accounting must start from the earth's crust, which can only be measured in exergy terms once a reference state describing the dead state of the crust itself has been defined.

It is important to recognize the connection between the search for natural costs and the need to identify the terrestrial biosphere as the cradle or reservoir, as well as the origin, of all chemical elements. To value these costs, a theory is needed to define the “physical effort” in extracting the minerals. Exergy is the key, but quantifying it requires a reference environment, which we call *Thanatia*, as previously explained. An economic theory of costing mineral scarcity without defining a physical reference environment would indeed be highly controversial.

In addition, mineral deposits often contain a variety of metallic compounds that are extracted together (metal companions). In the beneficiation or downstream processes, allocating costs between products and waste is necessary. This brings us back to the exergy analysis.

Let us clarify the concepts. We define the *exergy cost of extracting and processing a mineral* as the amount of exergy required to extract the mineral and process it until it reaches the market as a given commodity. This cost is typically measured in units such as kWh. Ideally, if standardized energy consumption data were available for all mines globally, it would be possible to systematically classify and catalog these exergy costs. Unfortunately, the absence of standardized data from mines around the world is a significant limitation, as it hinders efforts to compare and assess the true exergy costs of mineral extraction across different regions and types of mines.

However, these costs do not consider the scarcity of metals in the upper crust. Indeed, when a metal is very scarce, its composition in *Thanatia* would be on the order of 10^{-n} , with a very high value for n . For example, for cobalt, n is 9, while for aluminum, n is 3. In other words, aluminum is a million times more abundant in the crust than cobalt. Extracting 1 kg of cobalt from *Thanatia* with the same technology would require a million times more waste rock removal than 1 kg of aluminum. However, the price of cobalt is not a million times higher than that of aluminum, which shows that the market may be more or less sensitive to extraction costs but not at all to geological scarcity. Therefore, the loss of mineral capital from the earth must be sensitive to genuine geological scarcity to consider the limits of depletion.

To properly assess geological scarcity, it is necessary to resort to the exergetic “U”, which formulates an exponential behavior for exergy consumption in the systematic replenishment from Thanatia to the mine. Continuing with the example of cobalt, its concentration in present-day mines is of the order of 10^{-3} , so the cost of replenishing that mine from Thanatia would require one million times the energy of current cobalt extraction. No current or future economy could bear such costs, which makes cobalt management a global problem.

We raise three considerations to clarify our concept of geological scarcity:

- Companies legally depreciate the deterioration of their equipment. But who amortizes nature for the deterioration of its mineral endowment? Our civilization, for the most part, cannot renew nature at the same rate at which it depletes its resources. However, there is a crucial difference between the biosphere and the geosphere. While solar energy helps regenerate the biosphere, it would take millions of years to regenerate the geosphere. How could future generations or countries extracting raw materials be compensated for this loss, particularly for scarcer resources?
- The physical cost of recovering non-renewable resources is often overlooked. It is generally assumed that once a resource is extracted, it is irrecoverable and does not have a direct way of being “rewarded” back to nature. However, this assumption deserves scrutiny. What, then, is the true cost of recovering a non-renewable resource? For bio-based fossil fuels, the recovery process can often be mitigated by replacing the resource with an energetically equivalent amount of biomass, essentially maintaining a renewable loop. However, for metals, the recovery process is far more complex. A metal already used in equipment that has reached the end of its useful life could potentially be recovered through chemical extraction, albeit with significant energy and material costs. Alternatively, it could be concentrated from a lower-grade ore body. In extreme cases, it could be extracted from common rock or from Thanatia, where the concentration of chemical elements in the earth’s crust is immense. These latter forms, whether drawn from Thanatia or technological limitations, represent the most accurate and consistent models of resource recovery. For this reason, we will adopt them as the framework for evaluating the true cost of non-renewable resource extraction.
- The avoided cost of extractable mineral deposits is a crucial concept in understanding the true value of mineral resources. The earth’s crust is not a neatly organized store of minerals but rather a complex system of geological formations. When we refer to mineral deposits, we are specifically talking about those sites where minerals are relatively easy to extract compared to the widespread presence of minerals in common rocks, such as Thanatia. These mineral deposits are far scarcer than the rocks that make up the bulk of the earth’s crust. Nature essentially provides these deposits “for free”, bypassing the enormous energy and material costs associated with extracting metals from common rocks. From a practical perspective, geological scarcity is determined by the availability of mineral deposits that are as valuable as their tonnage, concentration, and composition. Therefore, the true mineral endowment of the planet resides in its extractable deposits, and we speak of depletion when the ability to extract these minerals comes to an end. The law of entropy is also relevant to the earth’s crust.

These considerations are the basis of the Thanatia theory, which, using the exergetic “U”, formulates an exponential behavior for the consumption of exergy in the systematic replacement from Thanatia to the concentration of the ore in the mine.

Consequently, we define *replacement cost of an ore* (at zero level) as the exergy that would be necessary to replace, with current technology, the concentration of an ore found in its mine from its concentration in Thanatia [56]. In essence, this cost is an avoided cost or bonus that

nature offers us for free (see Fig. 1). An imaginary cost can be assessed by measuring the energy it would take to concentrate a chemical from a common rock to a concentration equal to that of the mine. Examples of these avoided costs are presented in Thanatia’s books [43,57].

2.3. The exergy replacement cost: the technological barrier and the depreciation of mines

The *exergy replacement cost*, ERC, is a concept to value the wealth of a mineral deposit, as it shows the technical and thermodynamic distance of the deposit with respect to the depletion state.

The exergetic cost of extraction increases as ore grades and/or accessibility decrease. Sooner or later, this progression is a natural entropic phenomenon affecting rarer elements and with higher demand, [58].

As has been explained in Section 2.1, if ore grades are reduced by n orders of magnitude, the extraction cost will be multiplied by at least 10^n times the current mining cost. Unfortunately, mining will hardly reach 10 to 100 times the current mining costs due to physical, environmental, economic, and social impacts. This is regardless of technological improvements that could shift the technological barrier to lower ore grades.

So, in practical terms, the depletion of a mine does not coincide with Thanatia, but when its ore grade reaches the technological/mineralogical barrier [59,60], it is no longer profitable to continue mining it. As explained before, the environmental, social, and economic impacts will have grown exponentially.

Therefore, in Thanatia’s theory, absolute geological scarcity is the most important concept. Still, it is only one of the many depletion states particular to each mine. Relative scarcity is then obtained, limited by the technological barrier, i.e., by in-extractability.

Consequently, the exergy replacement cost is not unique for each chemical element, as it can refer to Thanatia as an absolute depletion state, which we will denote by ERC_0 , or refer to the practical mine depletion state, which we will denote ERC_{MB} . The former depends solely on the composition of the mineral in the crust. In contrast, the latter depends on mining technology and socio-economic reasons that are difficult to specify but very important from a practical point of view.

The technological barrier ultimately depends on what the market can pay. This barrier is mainly related to each orebody. However, without reliable data, a single ore depletion grade value can be assigned for all ore bodies of the same chemical material.

Similar to a mortgage system, when a mining company begins extraction, it would guarantee the actual cost of extraction along with the full Exergy Replacement Cost (ERC), effectively serving as the first payment on the loan. As extraction progresses, the energy required for further extraction increases, while the ERC gradually decreases in proportion to the depletion of the ore. Once the mine is exhausted, the replenishment of the ore (ERC) reaches zero, and the loan is considered fully repaid. Note that the borrowing company theoretically faces a constant cost over the life of the mine. This cost is the sum of the actual cost of extraction plus the replacement cost. The former increases as the ore grade decreases, and the latter decreases to zero. In this way, the depreciation of the ore deposit, converted into cash, always remains in the hands of the lending country.

The ERC is, in fact, a tax on the loss of mineral wealth, which should be invested to return resources to nature, in some way or another, regenerating the geosphere. Note that different authors, mainly [61–63], propose to tax in proportion to entropy generation or exergy destruction. They ignored the scarcity of critical materials. In our theory, we contemplate all real or apparent exergy losses.

The physical costs considered in traditional mining account for the consumption of energy, water, and reagents from cradle to market, equivalent to the exergy costs of extraction. Other things are added to these, such as permits, installation, maintenance, transportation, direct labour, royalties, and closing the mine at the end of its use.

All of them, in one way or another, are conventional and adopted by international law. However, the exergy cost of replacement is systematically ignored, i.e., the value of the scarcity of the mineral is ignored. Mining countries and territories only receive income if the mine is open and resources are extracted. Once the mine is exhausted, the primary concern becomes land restoration, with little recognition of the wealth that was once extracted. The economic value of the mineral resources is not compensated once extraction ends, leaving behind the environmental and social consequences of depletion.

Our proposal has several implications: First, the prices of raw materials would increase significantly. Evidently, the world economy should assume that mineral resources are not renewable and should be rewarded more fairly than is currently the case. At the very least, future generations could be compensated for the depletion of natural wealth. While exchanging mineral wealth for money would be a limited form of compensation, it would still offer some form of restitution. Second, if technological barriers determine actual depletion, it is never truly limited by the Thanatia threshold. Therefore, the age of plenty is shorter than might be expected. The availability of raw materials is limited to “picking to low-hanging fruits”.

2.4. The thermodynamic rarity and physical value of a mineral

The Thanatia theory proposes that the physical value of a mineral should be the sum of the cost of extraction and the cost of replacement, and we call it *rarity*. The word “rarity” was chosen because it relates to the difficulty of obtaining something valuable. Therefore, it encompasses not only the effort involved in extraction and refining but also the irreversible loss of mineral wealth [58,64]

Moreover, when evaluating the replacement costs of metals that are not as scarce as cobalt or gold, the extraction and refining costs of certain metals are close to their replacement costs. In fact, some raw materials are expensive to isolate and refine, regardless of their scarcity. This is the case for rare earth elements or platinum group metals, which require significant effort to separate from each other, as well as for obtaining extremely purified silicon for semiconductors. These materials are situated on the right side of the “exergetic U”.

Rarity serves as an indicator of a mineral’s thermodynamic value, assigning a higher value to substances that are more valuable from a physical perspective. This conveys an important message to policymakers involved in mineral governance: the rarity of a mineral matters, not just its quantitative scarcity.

The greater the rarity of any material component, the more critical its recovery becomes. Accordingly, rarity can be used to allocate costs among components of any physical–chemically mixed object. Indeed, the exergy of metals is lower than that of fossil fuels and does not reflect their value. To clarify, what is more valuable: a kilogram of natural gas or its exergy equivalent in gold? Similarly, the cost of extraction does not account for the value of scarcity. For example, nickel and cobalt can have nearly the same extraction cost if they are both sourced from the same magmatic sulfides, despite their very different levels of scarcity and overall economic value.

Note that when a material is wasted, all the energies used to form it lose their *raison d’être*, and the raw materials to replace it will be rarer and more expensive in terms of kWh; consequently, the Mineral Endowment of the Planet decreases as a consequence of dispersion. In the same way, the rarity of a composite material is the sum of the rarities of each component. The exergy used in separating its components should be distributed according to their rarity as a metric of their physical value.

Like the exergy cost of replacement, the rarity also depends on the composition of the mineral at depletion. As we have already discussed, the replacement concept from Thanatia is extremely radical because mining would stop as technically infeasible long before reaching the concentration found in Thanatia. Depletion occurs when the ore grade reaches its technological barrier or cut-off grade. Therefore, we scale

the concept of rarity to a R_1 , R_2 , R_3 , corresponding to a replacement cost starting from a concentration 10, 100, or 1000 times higher than that of the mineral in Thanatia. Logically, the rarity calculated from the concentration in Thanatia would be R_0 . In general, the subscript can be obtained from the following expression:

$$i = \log_{10} \frac{\text{Oregrade of the technological barrier}}{\text{Composition of the element in Thanatia}}$$

For the Thanatia theory, the technological barrier or cut-off grade is related to the maximum energy cost of extracting, beneficiating, and refining. This, ultimately, depends on what the market can pay and what society can accept. This barrier is mainly related to each ore deposit and its refining process to the market. Tentatively and according to the PhD thesis of our colleague [65], we have set it at 106 GJ/ton of the metal. That is equivalent to 278 kWh/g, and in terms of concentration, would represent about 100 times lower ore grades than those currently being exploited.

The accompanying figure explains the exergetic path of metal from Thanatia to the mine and then to the market. X represents the concentration of the metal. $X = 1$ would be the pure metal. X_B is the concentration after the beneficiation process. X_m is the metal concentration in current mines; X_L is the concentration in landfills, while X_C is the concentration in the crust (in Thanatia). The mineralogical barrier for each metal is somewhere in the red zone.

Unfortunately, the lack of statistics on mining and metallurgical energy consumption means that many concepts described here have yet to be developed. We leave it to young researchers to assess the losses of mineral wealth on a planetary scale, as well as to outline and evaluate the mineral wealth losses of countries or mines over time, investigate market failures, and, perhaps, propose a new way to quantify nature’s value.

3. Pristinia: An exergy approach for assessing the regeneration costs of soil fertility

When we studied the use of exergy to assess the loss of the planet’s mineral endowment, we realized that fertile soils were excluded from our analysis. The problem of soil degradation is progressing even more rapidly than mineral depletion. Audrey Azoulay, Director-General of UNESCO, at an international conference in Agadir (Morocco) in 2024 [66], warned that 90% of the planet’s land surface could be degraded by 2050, with significant risks to biodiversity and human life. This message builds on the European Union’s Joint Research Centre’s World Atlas of Desertification [67], which estimated that, globally, 75% of land was already degraded, directly impacting 3.2 billion people. A 49 percent increase in agricultural production is expected to be required by 2050 [68]. Furthermore, in coordination with its international partners, UNESCO wants to support a *world soil health index*. This index will be a standardized measure for assessing and comparing soil quality across different regions and ecosystems.

We refer to the depletion of fertile soils as the fourth dimension of Thanatia (in addition to the atmosphere, hydrosphere, and crust), [69]. If physical degradation implies irreversibility, why not study soils under the second law? Under the technical and financial support of fertilizer companies, we did it. We realized that this problem is even more complex than that of mineral depletion. Fertile soils are neither purely abiotic nor biotic but rather an extraordinary mixture of both. According to the 20th International Congress of Soil Science [70]: “soil health encompasses a variety of attributes, including its physical, chemical, and biological properties, all of which contribute to the soil’s ability to support plant growth, support diverse ecosystems, and provide essential nutrients to the crops on which we depend for our livelihood”.

In the last decades, crop production yield has been raised by intensive agriculture based on high inputs of inorganic fertilizers and pesticides, resulting in severe environmental impacts, erosion, and soil quality loss. The agricultural sector causes approximately 25 per cent

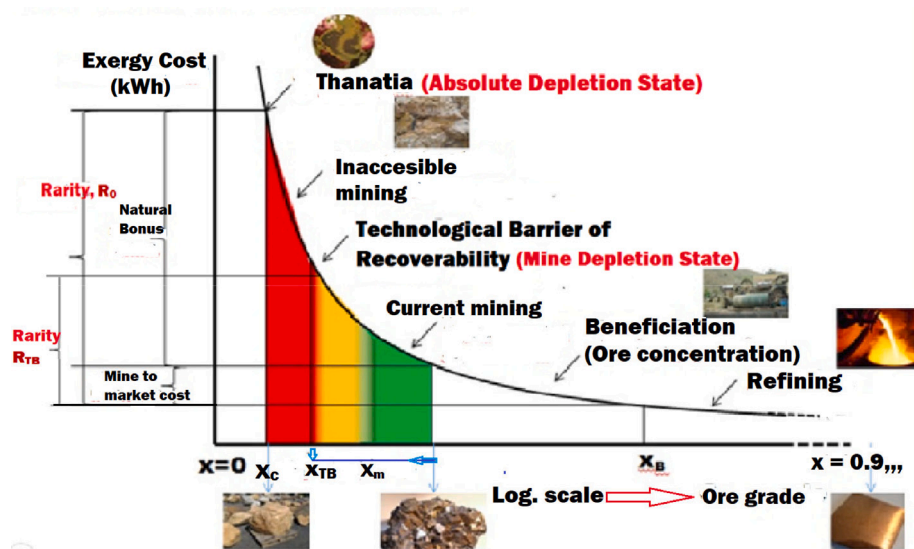


Fig. 1. The concept of Thermodynamic Rarity.

of the global greenhouse gases. In Europe, it is estimated that there are 12 million hectares affected by erosion [71].

More in detail, compared with the 15 km of upper crust, fertile soils only occupy, on average, 15 cm of topsoil. According to edaphology studies [72–76], we discerned three main structures in soil: physical, chemical, and biological. First, their physical texture comprises a variable composition of clay, sand, and silt. Second, its chemical composition, with an inorganic part of at least 19 nutrients and a rich and varied organic part, is essential to improve its physical characteristics and to serve as food for soil biota. The 19 inorganic nutrients are divided into macronutrients like N, P, K, Mg, Ca, and S, then micronutrients like Fe, Mn, Cu, Ni, Mo, B, Zn, and Cl, and then other beneficial elements essential for certain species or under specific conditions like Na, Si, Co, Se, and Al. Third, their biological content includes macro and microbiota with bacteria, fungi, and viruses. Besides that, water, air, and a specific range of temperatures are vital needs.

While mines are getting depleted without physical replacement, degraded soils can and must be regenerated. Therefore, the thermodynamic distance between the current degraded state of the soil and its healthy, fertile state is of greater importance than the distance between its current state and its maximum possible degradation. This marks a conceptual shift where Thanatia, the concept of ultimate depletion, is no longer applicable. Instead, the focus must shift to the regeneration cost, bringing the soil back to an ideal model of fertility. Rather than considering a Depletion State, we must now conceive of a Plentiful State, one where soils are restored to their optimal condition, capable of supporting sustainable ecosystems and agriculture.

Moreover, each area of the globe has different local textures, and fertility also depends on climate. Therefore, it is necessary to establish a suitable methodology to serve as a starting point for assessing soil degradation. We adopt a new strategy, proposing an optimum soil that allows us to analyze all real soils as deviations from an optimum towards depletion. Accordingly, we introduce a perfect fertile planetary crust, OPTSOIL [77], composed of nutrients, OPTNUT, organic matter, OPTSOM, and optimum biota, OPTMIC, which has to be suited to local climatic conditions, is invariable and independent of different textures, but depends on its water content and aeration. We call this imaginary, copiously fertile crust *Pristinia* instead of Thanatia. Thus, any real agricultural soil will be in an intermediate state between *Pristinia* and Thanatia.

The departure of each agricultural soil from the optimum soil can be associated with a sum of physical, chemical, and biological causes that can be assessed exergetically. The idea is to evaluate a spectrum

of actual degradations of agroecosystems, including those occurring in the soil, and to obtain the physical cost of their regeneration. As shown in Fig. 2, the exergy cost of mechanical processes, fertilizers, pesticides, plant protection products, water, and diffuse emissions generated in the agricultural process is quantified. In addition, the restoration processes necessary to return the soil from the final state to the initial state have been defined and evaluated. In this sense, losses due to erosion, nutrient amendments, organic matter, salinity, and acidification are quantified through their exergy costs.

The following steps should focus on the importance of soil microbiota, OPTMIC, using Jørgensen's concept of eco-energy [78]. Likewise, the evaluation of organic matter in terms of CO₂ sequestration achieved, which is controversial because of the difficulty of demonstrating its long-term stabilization.

Even though this active research is in progress, our methodology has been awarded the recognition of a reference document from the European Committee for Standardization (CEN), with the title: "Methodology to quantify the global agricultural crop footprint including soil impacts" [79].

This European directive, CWA 17898, specifies a methodology for identifying, characterizing, and implementing a single indicator to assess the quality and degradation of agricultural soils and the overall impact of the agriculture processes. The agricultural impacts are assessed through the associated mechanical, fertilization, and irrigation activities. Furthermore, soil impacts are evaluated by accounting for soil erosion and parameters such as nutrients, texture, and organic matter. The developed methodology allows a simple but robust assessment of soil biogeochemical processes and the loss of fertility and degradation.

The CWA 17898 has recognized the usefulness of exergy and its derived costs, and it is the first time that exergy analysis has left from academia to become a European standard for valuing agricultural land degradation in physical terms (kWh) instead of fluctuating monetary valuations. Just as each mine must find its particular depletion state, the *Pristinia* model needs to define the optimal and healthy state for each soil.

From 2019 to the present [80], we have been working on developing the idea of *Pristinia* as an example of an analysis of exergy costs of regeneration, as opposed to the classical one of replacement. The Second Law, which quantifies the degradations of any physico-chemical process, is more suitable than a purely economic analysis, as it is possible to disaggregate and objectively quantify the various deviations from the optimal soil. We also believe that the European standard we

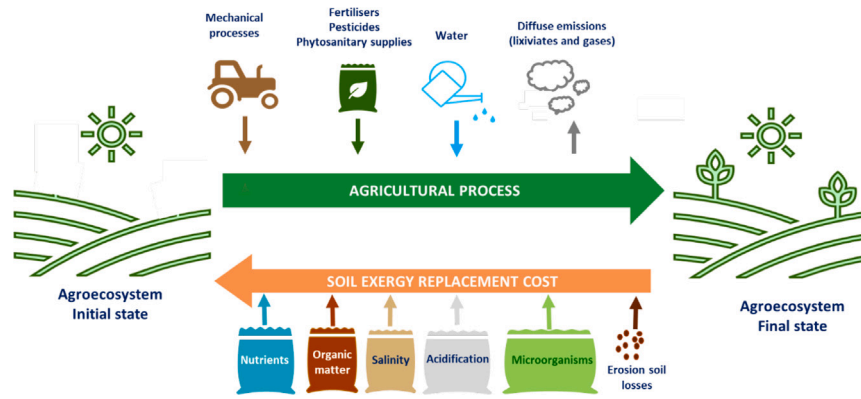


Fig. 2. Graphical summary of exergy-based methodology for the evaluation of agroecosystems.

developed aligns with UNESCO's demand to create a "world soil health index".

Further development will aim to assess soil by considering not only its agricultural production and carbon sequestration functions but also all the different ecosystem services it provides, such as habitat provision, biodiversity, water regulation and purification, and environmental quality, among others.

4. Circular thermoeconomics: A resource and waste cost accounting theory

The concept of waste has not been sufficiently analyzed in Thermoeconomics. By waste, we mean any unwanted material or energy flow, solid, liquid, gaseous, heat, noise, or radiative flow. From a thermodynamic point of view, it is simply the external irreversibility that generates entropy outside the system under analysis. Waste is harmful because it still has exergy, and we must consume it to eliminate it.

Every time we produce, we generate waste. And every product, sooner or later, also becomes waste. This consideration is essential because it allows us to distinguish between waste from production processes and the end-of-use waste of material goods. The former corresponds to the remains of the resources used in the processes; let us call them primary waste because they are produced simultaneously as the products are obtained. The latter corresponds to the degradation or elimination of material goods of an inorganic nature or the discarding of parts of organic substances.

Municipalities usually collect this type of waste, which we call secondary waste here. Their treatment, reduction, and disposal require more energy, water, and raw materials, which constitute the input for their processing. At the same time, new products and tertiary waste are produced, which constitute their output. Thermoeconomic analysis can, therefore, be applied to them.

The purpose of thermoeconomics is to obtain a coherent and significant set of costs in a given energy structure [81]. Costing allocation looks for the resources needed to produce both intermediate and final products.

The irreversibility cost formula [82] allows us to calculate the (direct) exergy cost as the exergy content of a flow plus the sum of the irreversibilities generated to produce that flow.

$$B_i^* = B_i + \sum_j^n \phi_{ij} I_j \quad (1)$$

The coefficients ϕ_{ij} represent the apportioned part of the irreversibility of the equipment j - that has been generated to produce one unit of the flow i -. These coefficients depend exclusively on the definition of the production structure of the plant and the definition of the efficiency of each of its components. Therefore, this formula allows

us to analyze the cost formation process and is also the connection between Thermodynamics (exergy and irreversibility) and Economics (costs) derived from the definition of the productive purpose of the plant components.

As waste is an integral part of production, just as one assesses the process of product cost formation, the process of waste cost formation must also be evaluated. Considering that waste has exergy, it is rational to think it could still be used for production. But there is a lack of technology and a surplus of ignorance of nature. Externalization is the endorsement of the harm we cause outside the production system. At best, we pay money, in the form of taxes or fines, for society to mitigate them, or we send them to those countries that accept them in return for payment, thus creating an increasingly unequal world. Otherwise, society must adapt to environmental degradation, leaving the burden on future generations. Otherwise, society must adapt to environmental degradation, leaving the burden on future generations since nature is incapable of getting rid of it, as nature is unable to get rid of it because of the speed and the amount of waste we are producing.

Before they generate an unmitigable impact, a responsible society must "internalize" waste and its costs. This requires developing techniques that rigorously assess, for example, both the CO₂ emitted and the CO₂ avoided in each waste treatment process.

Many thermoeconomic analysts [83–99] have tried to assign a proper exergy cost to environmental effects. Some have posed the typical environmental economics problem of the cost of polluting versus the cost of reducing or remediating the impact of the waste in the process plant. Their authors named the allocation procedures differently, including Cumulative Exergy Consumption, Thermo-ecological Analysis, Extended Exergy Accounting, Environomics, Exergo-environmental Analysis, etc.

Historically, Thermoeconomics has focused on the second law analysis of power plants. The problem was, therefore, mostly confined to emissions associated with combustion, whether of biomass or fossil fuels. At most, the analysis was extended to waste caused by polygeneration. [100,101].

However, whereas Thermoeconomics can and should be extended to any production process, be it energetic, chemical, mining, metallurgical, etc., or to industrial symbiosis and circular economy, waste is no longer only gaseous but also liquid, solid, noise, unwanted heat transfer and radiation. Therefore, a broad view of what we will call circular thermoeconomics is needed. The point of view changes. Waste is concomitant with production. As mentioned above, just as there is a process of product cost formation, there is also a process of waste cost formation. We no longer look at the environmental impact of waste but at the cost of producing it, as well as the cost of reducing it and the negative cost of recovering its energy [95].

The result is clear: the cost of the products increases. And the more complex the waste, the higher the costs added to production and, thus,

the higher the prices of all goods. This is something that today's society does not easily accept.

Since waste has exergy, it can be used to improve the plants' efficiency in a cyclical production chain, thus reducing production costs and creating the positive effect of reducing resource consumption and waste emissions.

In many cases, this use of waste energy requires new industrial processes and/or new companies that make up the links in the industrial chains. Innovation can come from the thermoeconomic analysts themselves, leaving behind more or less irrelevant procedural discussions on achieving a better cost allocation.

The process of waste cost formation must be a priority objective in this new Circular Thermoeconomics developed by Valero and Torres in [102]. Waste costs must be rigorously evaluated because they are the compass for decisions to improve production processes, both within production plants and in increasingly complex industrial chains. The keyword "recycling" must be intrinsically linked to efficiency and cost. It can be used interchangeably by Thermoeconomics, Circular Economy, and Industrial Symbiosis.

Although it may appear to be self-interestedly oriented, using exergy makes it possible to quantify any material or energy waste in kWh. This makes it possible to create a general theory based on exergy. Furthermore, exergy can be evaluated with any other energy indicator, e.g., in kg or natural gas equivalent. This allows us to convert it into CO₂ equivalent emitted in the production or, alternatively, to evaluate the CO₂ equivalent savings between two production structures. This is important because it gives rigor to the huge simplifications of GHG emission calculations in increasingly complex systems today.

When applying Thermoeconomics to the exergy recovery process of waste, a pinch point of recoverability always appears where total recycling is unattainable because a further increase in recycling does not correspond to the same efficiency increase. The *pinch points* of waste must be identified and systematically diminished. Reality imposes the fact that it is not possible to recycle everything. Hence, we should not speak of Circular Economy but of Spiral Economy [64]. The latter makes it possible to assess the degree of spiraling of a material cycle, understood as the percentage of exergy loss with respect to the exergy brought into play in the whole cycle. In this way, spirality can be related to the concept of pinch point, as the economically viable exergy percentage of recovery of a residual flow, measured in the potential for narrowing of its intensive properties of the type ΔT , ΔP , etc.

It is remarkable that the search for better energy efficiency is often linked to reduced material efficiency. Nonetheless, this fact has been systematically forgotten. For example, cars, which are nothing more than mobility production plants, consume less and less fuel per kilometer. But, we need to remember that the material complexity of their digitization has radically reduced their material efficiency. A vehicle from 50 years ago contained only steel, aluminum, copper, rubber, glass, wood, and some plastic. Today's vehicles contain over 50 chemical elements, with many plastics, alloys, and micro-components full of critical raw materials [53]. Their recyclability is low, and they are not designed to recover their spare parts. Circular Thermoeconomics can provide compelling answers to this problem because, since the exergy costs of extracting and replacing materials are involved in the analysis, and it is possible to evaluate the ratio between the exergy savings and the exergy invested in obtaining them, ExROI [39]. The results yield important conclusions on how our society is squandering the earth's mineral capital in favor of a pseudo-energy efficiency of all processes. This is a solid reason to recycle materials, especially the most critical ones.

The message we can give is that goods are cheap because we internalize the price of products, neither the cost of the scarcity of natural resources nor the impact our waste has on planetary habitability.

Nature closes its material cycles because the waste generated by one living becomes the resources of others, making the biosphere-geosphere complex a sustainable system driven by solar energy and

the planet's internal heat. Nature has its rhythms, and the closing of these cycles can even take geological time, as is the case with fossil resources. On the contrary, our society is in a hurry and lets its waste go underground (landfills), turn into gases (incineration), or be diluted in the atmosphere and the hydrosphere.

This is no small task. The production of waste and the depredation of natural resources are closely related to the nine planetary boundaries that Rockstrom et al. [103] points out: climate change, ocean acidification, stratospheric ozone, nitrogen, and phosphorous cycle, global freshwater use, deforestation and topsoil degradation, biodiversity loss, chemical pollution, and atmospheric aerosol loading. These must be added, among others, to the more recently reported phenomena, such as oceanic pollution by microplastics or the massive extraction of critical raw materials to sustain new digital and renewable technologies.

In light of this scenario, thermoeconomists cannot ignore the importance of optimizing the use of waste most efficiently. Beyond the processes of remediation or abatement of waste, it is necessary to consider the use and reuse of industrial and individual waste. This is the origin of Circular Thermoeconomics. We must learn from nature. The waste of one production plant should be the resource of another. We must design to make the use of waste materially and energetically viable. Cyclical industrial chains must be created to give new life to waste, promoting "liver industries" [104].

The concepts of industrial and urban-industrial symbiosis must be extended to the entire productive system, resulting in a social change of attitudes. In [104], we present a municipal-industrial symbiosis as a conceptual solution to face the future environmental and raw material scarcity problems that industrial society and its cities would face, transforming the problem of Municipal Solid Waste into an opportunity for society.

An introduction to Circular Thermoeconomics can be found in [102, 105]. We invite new researchers to develop this theory and the multiple applications proposed by these reflections.

5. Relative free energy: A new thermodynamic property

In our experience, it often creates controversy among radical exergy followers when we describe Relative Free Energy, but not for those who consider that the Second Law Analysis does not end at exergy. Therefore, we refrain from comparative comments. With mathematics alone, it is easier to discern truth or false. We leave its applications for future young researchers. Here, simply physics and mathematics.

Any machine that uses a working fluid is designed to change its intensive properties, such as its pressure, temperature, chemical potential, or electromagnetic potential, making its energy transfer to another part of the machine.

Suppose now that the working fluid malfunctions due to intrinsic causes in the machine. If the fuel quality, f , does not change, it will be necessary to increase the amount of fuel to get the same product, P . Then, this excess fuel becomes dissipated in the form of heat.

The enthalpy and entropy balances of the working fluid process, under the conditions (h_1, s_1) and P constant, will be:

$$\begin{aligned}(h_1 - h_2) dm &= m dh_2 \\ (s_2 - s_1) dm + m ds_2 &= dS_g\end{aligned}$$

As the machine's deterioration process is compensated by higher fuel consumption, both dm and the entropy generated are positive. So is the heat dissipated, $m dh_2$, and the entropy increase of the fluid, $m ds_2$. If one defines the deterioration temperature of flow in state #2 as the quotient:

$$T_{d_2} \equiv dh_2/ds_2$$

the result of the combined balances is as follows:

$$\left(\frac{dm}{dS_g}\right)_r = \frac{T_{d_2}}{(h_1 - h_2) - T_{d_2}(s_1 - s_2)}, \quad (2)$$

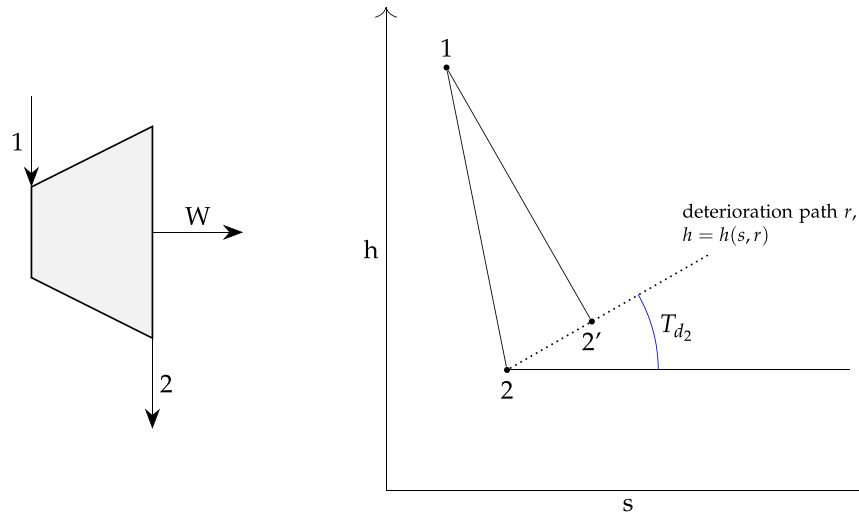


Fig. 3. The deterioration path of a turbine.

where r is the deterioration path in the (h, s) plane of the outflow, as show in Fig. 3 for the case of a turbine. Eq. (2) states that any deterioration process in an energy component has an associated T_d in the exit stream of supplying fuel. This parameter possesses temperature dimensions even if it is not a gauging temperature; it can be calculated by measuring the quotient dh_2/ds_2 experienced by the output stream of the component's fuel.

This equation is the mathematical expression of the relationship between the additional consumption of the resource needed to compensate for an internal degradation of the machine when the output of the machine is kept constant. This equation is mathematically exact, and we do not know that it has been obtained before by any thermodynamic researcher.

Eq. (2) and the function ℓ , whose variation is defined as:

$$\ell_1 - \ell_2 \equiv h_1 - h_2 - T_d(s_1 - s_2)$$

where presented in [106], and was called the *relative free energy*, RFE. Surprisingly, it is similar to the exergy variation of the working fluid, which may explain the attachment and intuition of process analysts to exergy. However, RFE is mathematics, while exergy comes from consensus.

However, the deterioration temperature no longer depends on the chosen reference temperature for exergy calculations. Although T_d has temperature dimensions, it is not a gauging temperature; it can be calculated by measuring the ratio dh_2/ds_2 experienced by the machine's fuel at its output.

For a given deterioration cause r of a machine, a geometric path in the (h, s) plane of the possible exit flow states can be identified. Let $h_2 = h_2(s_2, r)$ be the function describing this outflow deterioration path, and it can be considered as the mathematical description of the effects on the exiting flow caused by such deterioration.

As it is shown in Fig. 4, the intersection of the tangent line of the deterioration path r with the h -axis is, and its slope is T_d :

$$\ell = h - T_d s \quad (3)$$

It means ℓ is a function of T_d , with the caveat that both ℓ , h , and s refer to the same zero state.

In Ref. [107], it is shown that (3) is the Legendre transform of the deterioration r .

According to the definition of relative free energy (3), the expression (2) could be written as:

$$\left(\frac{dm}{dS_g} \right)_r = \frac{T_d}{\ell_1 - \ell_2} \quad (4)$$

Note that this equation is always positive under the specified conditions. Moreover, since $T_d > 0$, then $\ell_1 > \ell_2$, i.e., the working flow has lost part of its relative free energy. In fact, for the same deterioration, $dS_g = \text{const}$, the higher the T_d , the greater the resource required to yield the same product. This trade-off is not linear because whereas the numerator increases, the denominator also decreases. On the contrary, when T_d tends to T_0 , the impact of this compensation will decrease, only in the case where $T_d = T_0$, the decrease of relative free energy will coincide with the exergy decrease.

As the environment is an attractor, the T_d of the components of an installation will decrease until it reaches T_0 , just as the relative free energy will tend to exergy.

To evaluate T_d , it is necessary to know the cause(s) of machine deterioration and its behavior, which is additional information to be incorporated into the plant diagnosis. In other words, the cost of a malfunction is directly related to the physical behavior of the machine.

The relative free energy function, ℓ , is the amount of specific energy of the exiting stream of a fuel, which is not affected by the deterioration of the machine. When the exit flow changes its state due to deterioration, h and s vary by following the path $h_r = h_r(s_r)$. Consequently, the following expression is obtained:

$$d\ell_2 = dh_r - \left(\frac{dh}{ds} \right)_r ds_r = 0 \quad \text{or} \quad \ell_2 = \text{const.} \quad (5)$$

Besides that, there is a new definition of efficiency (or its inverse specific consumption). For example, in a turbine, it is:

$$k_L = \frac{F_L}{P_L} = \frac{\mathcal{L}_1 - \mathcal{L}_2}{W} = \frac{H_1 - H_2 - T_d(S_1 - S_2)}{W} \quad (6)$$

Where $\mathcal{L}$ is the corresponding extensive property of ℓ , i.e. $\mathcal{L} = m\ell$. This "relative" efficiency, k_L , has an astounding property that exergy lacks.

When a malfunction in the turbine follows a trajectory r at W constant, it follows:

$$\ell_1 dm = \ell_2 dm + W dk_L \quad (7)$$

because $\ell_1 - \ell_2$ is constant along trajectory r . Therefore:

$$\frac{dk_L}{k_L} = \frac{dm}{m} \quad (8)$$

In other words, any additional percentage of malfunction following a trajectory r at W constant results in an extra percentage of fuel mass entering the turbine. This property is auspicious for simplifying the mathematical apparatus of plant diagnosis. Note instead that exergy does not fulfill this property since:

$$ex_1 dm = ex_2 dm + m dex_2 + W dk \quad dex_2 \neq 0$$

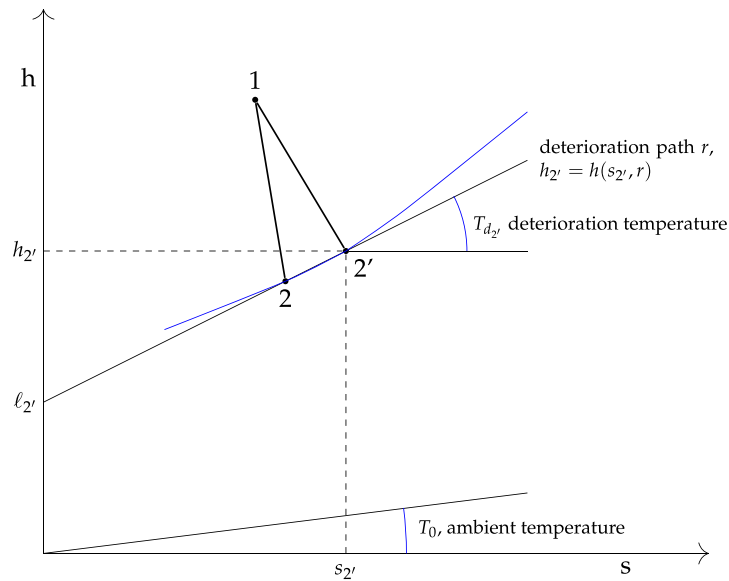


Fig. 4. Geometric representation of relative free energy and deterioration temperature.

which may induce false dysfunctions. Furthermore, the definition of k_L incorporates the information on the physical behavior of the machine through T_d .

A noteworthy observation is that companies usually account for the resource consumption per unit of installation output. This accounting at the facility level is the one we propose in this section at the component machine level. Therefore, whereas we have for each cause of deterioration, r , the derivative $(\partial m_{local}/\partial S_g)$ and whereas we could evaluate how the amount of local equipment resources is related to the facility resources (of the same quality), i.e., $(\partial m_{plant}/\partial m_{local})$, we could obtain a new theory of energy system diagnostics. In theory our proposal would obtain the derivative: $(\partial m_{plant}/\partial S_g)$, that is $(\partial F_T/\partial S_g)$, where F_T are the plant resources. Note that what says “plant” here could also mean any disaggregated part of the overall plant structure without resorting to the reference environment. This is what companies need for good energy consumption management, i.e., the genuine costs of the deterioration of each installation component.

Regrettably, actual installations may have several simultaneous causes of degradation, and their impact on overall efficiency would not necessarily be the sum of each cause separately.

Using the Legendre transform, we have shown that the RFE and the deterioration temperature are as accurate as the actual path of degradation of the working fluid in a machine. They have a physical basis and are not merely theoretical inventions. Unlike other thermodynamic functions that only depend on the equilibrium states of the system, the pair (RFE, T_d) only exists, whereas the machine exists. It is like transferring the deterioration message from the machine to Thermodynamics.

Throughout history, new fundamental properties have opened up new avenues and applications for Thermodynamics. We have seen that the causes of a machine’s deterioration affect the working fluid’s thermodynamic properties at the machine’s output. Each cause “ r ” generates a deterioration path related to increasing the resources that must be expended to compensate for it.

Unbelievably, this function was first presented in 1992 at the ECOS conference [106], with the earliest work by Alefeld at the University of Munich [108] that went almost unnoticed, perhaps because of the excessive cult of exergy. Despite this, the use of this function in diagnostic applications and cost assessment of malfunctions in industrial energy systems is still to be developed. We leave it to future Thermoeconomics practitioners to develop such ideas.

6. Conclusions

For science to be positive, it must be quantifiable and predictive. In general, applying the Second Law of Thermodynamics to the social sciences and economics tends to be more philosophical than operational. However, by redefining cost as the sum of irreversibilities and considering it from “cradle to market”, we can connect Thermodynamics with Economics in a comprehensive and operational way. This represents a union of Physics and Economics in a quantifiable, predictive manner.

The first ideas of Thermoeconomics, along with its very name, are attributed to Myron Tribus, Robert Evans, and Yehia El-Sayed in the 1960s [2,4]. These ideas predate Georgescu-Roegen’s publication of *The Law of Entropy and the Economic Process* [36], in which he argued that Economics had overlooked the concept of irreversibility.

Unfortunately, Thermoeconomics has primarily focused on analyzing industrial thermal systems, often overlooking the broader implications suggested by its name. Beyond merely converting irreversibility into monetary terms and comparing it to investments in production equipment, a more comprehensive approach is needed. Specifically, it is essential to measure economic activities in terms of irreversibility to value the services the planet provides properly. This requires a shift in perspective, one that goes beyond industrial applications and considers the broader environmental costs.

By doing so, we can balance the nature we lose against the benefits gained from any activity. This approach will help us assign value to the regeneration, repair, replacement, and restoration of materials that are crucial for sustaining humanity for future generations. It allows us to shift from a simple yes/no dichotomy to a more nuanced, quantifiable understanding of the losses we are inflicting on nature.

The supply of planetary wealth is not driven by the economic equilibrium of supply and demand, but by planetary limits. In light of this, agreements between individuals or countries are secondary; the assumption of irreversibility will condition the future of humanity itself. Mineral phosphates are limited in their ability to sustain future generations. Electric mobility is constrained by the large number of chemical elements—over 50—that are required for high-tech vehicles. The digital economy depends on non-recyclable microchips and printed circuit boards, which have extremely short lifespans and contain rare metals. Renewable energy is similarly constrained, and the energy transition must account for its material impact.

In this context, the concept of a planetary absolute dead state, Thanatia, offers a powerful framework for assessing the loss of mineral

wealth in terms of both quality and quantity. Thanatia allows for this assessment through a timeless methodology, where exergy analysis remains valid regardless of the chosen reference state. In other words, while a mine can be depleted technologically and economically, it may still contain ore grades that are out of reach with current technology. This highlights the importance of understanding the relative depletion state of all operating mines, which may be far from Thanatia itself. This well-known phenomenon leads to two theoretical consequences: First, the exergy required to extract the ore from the mine's current condition to its depletion state can be treated as the depreciation cost of the deposit. This imagined-to-be-real cost should theoretically be paid to the country to preserve the resource for future generations, offering scientific reasoning for countries to value their mineral deposits beyond the cost of exploitation and royalties. Second, by discarding planetary concentrations at the depletion state that are higher than those of Thanatia, we are realistically reducing the mineral endowment of the planet, thus bringing the End of Plenty closer to a more plausible scenario than Thanatia itself. In other words, choosing specific economic-based depletion states brings the possibility of social collapse closer. However, the theory of Thanatia remains valid because what is rare is valuable and must be conserved regardless of the reference state.

Similarly, current agricultural practices are degrading fertile soils, which are the most limited natural resource of all. Once again, the second law allows us to objectively assess all deviations from a model of a copiously fertile crust, *Pristina*, as compared to Thanatia. Through exergy regeneration methodology, we have a single indicator to evaluate all the factors that contribute to soil formation. Recognition of this methodology, which has been awarded reference status by the European Committee for Standardization, paves the way for refining and expanding its applications. Remarkably, the *Pristina* concept can be extended to regenerate all ecosystems, and we are currently developing its methodology.

Another pressing global issue is waste production—not just solid, but also liquid, gaseous, noise, heat, and radiation of all kinds. These waste streams are produced simultaneously as goods are manufactured and consumed, disturbing and negatively affecting the planet through climate change, pollution, and ecosystem degradation. These wastes remain harmful because they still possess exergy. Harnessing them minimizes remediation and abatement costs. Society must learn to internalize waste, giving it new life and mitigating its negative impacts. To achieve this, industrial symbiosis must be widely adopted, where the waste or byproduct of one company becomes the resource of another, closing the material cycles (or spirals). This approach requires the development of a new industrial sector focused on assessing and optimizing losses and savings. The Circular Thermoeconomics described here will play an essential role in the future of Second Law analysis.

Finally, the concept of relative free energy and its conjugate, the deterioration temperature, can take exergy beyond diagnosing energy systems. It can also explain why such diagnoses should not depend on the analyst's choice of reference state. Throughout history, the introduction of new basic functions has been a landmark scientific event, opening new avenues for Thermodynamics. The same can be said for the relative free energy concept, which deserves to be embraced by the thermoeconomic community.

In conclusion, at the Zaragoza School of Thermoeconomics, we believe that the “analysis of the second law” should be extended to address significant problems facing our civilization, such as the conservation of the planet's energy, water [109], and material resources; the diagnosis and intelligent management of fertile soils to feed current and future populations; and the minimization of waste by closing material supply chains, promoting new industrial sectors focused on waste valorization, still in its early stages.

Lastly, we propose that this concept be referred to as the “Thermodynamics of Sustainability”. However, by introducing the concept of cost in all its forms, the term “Thermoeconomics” remains more fitting. Not only does it honour the term coined in the 1960s, but it also aligns with the broader application of Thermodynamics in Economics, quantifying the value of nature.

CRediT authorship contribution statement

Antonio Valero Capilla: Supervision, Methodology, Formal analysis, Conceptualization. **Alicia Valero Delgado:** Writing – original draft, Project administration, Methodology, Funding acquisition, Conceptualization. **César Torres Cuadra:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Alicia Valero Delgado reports financial support was provided by Spanish Ministry of Science and Technology under project RESET PID-116851RB-100. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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