Migration dynamics, growth and convergence*

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Abstract

The aim of the present paper is to analyze the impact of migration dynamics on economic growth and convergence in terms of both the capital/labor ratio and wages. From our results, the following main conclusions can be highlighted: 1) migration positively affects the sending country because of the improvement in the capital/labor ratio and the savings of returning workers; 2) the differences existing between countries do not necessarily disappear in the long term, so the convergence that arises is limited or conditional because it does not necessarily imply an equalization of per capita income, capital/labor ratio, and wages, or the disappearance of migration; and 3) the possibility of migratory flow reversion cannot be excluded in the transitory dynamics.

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1. Introduction

The role played by immigration in the aggregate dynamics of economies has been extensively studied in the 90s, mainly due to the development of endogenous growth models and the growing interest in labor flows. The previously available reference was the exogenous growth model in which convergence in remuneration was formally derived and labor and/or capital flows eventually disappeared when factor mobility was present. However, empirical research using the Summers and Heston database clearly shows, on the basis of data from more than one hundred countries over several decades, that conclusions in favor of absolute convergence cannot be deduced. Only if the scope of study is limited to economies where tastes, technologies and institutions are similar, can absolute convergence be found, as Barro and Sala i Martin (1995) demonstrate in the regional analysis of several European countries, the states of the USA, and the Japanese prefectures. What these results confirm is that, in general, we only have conditional convergence in the growth rate. Nevertheless, if we take this analysis a step further, convergence does not necessarily occur towards the same value of the stationary state, even at regional level, and the persistence of interregional immigration is one of the signs of the absence of absolute convergence in per capita income.

Although the result that poor states, countries or regions, have higher growth rates than rich ones in the transitory dynamics is consistent with the hypothesis that the growth rates should become the same in the long term, the stationary state could be different for each state, country or region. This should produce persistent migration flows and the absence of convergence in per capita income when factor mobility is present.

An increasing interest in solving the contradiction between exogenous growth theory and the evidence has led to the use of endogenous growth models, in which convergence is no longer necessarily derived, at least theoretically. Bertola (1992) and Rauch (1994) concluded that migratory movements are never-ending and always take place in the same direction, because salaries are permanently higher in the host country, which leads to divergence in per capita incomes. In these conditions, the migratory flow is detrimental to the sending country. This result is quite strong because there would be no hope for poor countries, which would be clearly harmed, as the productivity of capital depends on a scale effect in employment.

Other studies have appeared since providing alternative possibilities to these contradictory perspectives. Faini (1996) presented a detailed analysis of the decision to emigrate and concluded that convergence may occur for some combinations of parameters, even with endogenous growth. However, in terms of Faini, convergence means the absence of migration and identical remuneration for the factors, as in the previous perspective of exogenous growth.

Reichlin and Rustichini (1998) use an endogenous growth model with scale effect but introduce the immigrants' skilled-unskilled distinction to overcome the divergent result in some cases in which a reversion in the immigration flows takes place. Our model could be considered an extension of Reichlin and Rustichini's paper in that we pursue the analysis of the same problem, although using an endogenous growth model without scale effect. We find that in our case it is not necessary to introduce the skilled-unskilled distinction in order to achieve a result in which the country that is sending emigrants can benefit[1]. Moreover, we characterize the steady state and the complete dynamics.

In particular, our findings suggest that migration always benefits the sending country in the steady state, that there is convergence towards the same growth rate, but that this convergence does not necessarily imply convergence in salaries or in per capita incomes, being possible the existence of a constant gap with permanent migration flows in the same direction.

These results could be considered the core of our contribution. Nevertheless, we have also tried to overcome some limitations found in the migration literature. The novelty in this respect, related to previous papers, is the simultaneous consideration of all the alternatives to those limitations. In the first place, the emigrant's decision takes into account an element that is not strictly economic but must be introduced as it plays an important role in the decision. This element is the preference of individuals for their country of birth. Secondly, migration should be the result of an endogenous decision in the model, so that individuals decide, according to their own preferences, where they wish to work and retire. Thirdly, one particular case among the proposed alternatives is that immigrants return to their country of birth to spend their retirement once they have emigrated abroad during their active life. In this case the savings of the emigrants could

[1] Including this distinction would add to the paper but make it more complex.

return to their country of origin. Dustman and Kirchkamp (2002) empirically demonstrated that most of the Turkish immigrants to Germany returning to Turkey engage in entrepreneurial activities, investing their savings in their country of origin. However, all the alternatives to this hypothesis are considered in the paper. Fourthly, we use a model of endogenous growth without scale effect in which the production function, used by Kemnitz (2001) in the context of migrations, plays an important role. Fifthly, we try to avoid the limitation of many papers, which only consider the perspective of the residents in one of the countries involved. Immigration into one country always means emigration from another, and the effects on both should be studied. Lastly, we consider migration as a factor that influences and, in turn, is influenced by growth. We theoretically analyze this influence on growth and, consequently, on convergence. The principal conclusion we can draw is that, in spite of using an endogenous growth model, migration has a positive effect on convergence, which corroborates the empirical results of Persson (1994): migration may help to reduce the distance between countries although it does not necessarily eliminate it if there is a technological difference between the countries.

The model is presented in Section 2. The behavior in a closed economy is introduced in Section 3 and in Section 4 we go on to derive what occurs in an open economy with labor mobility and a certain degree of capital mobility. In this section the dynamics and the steady state are determined. Section 5 focuses on all the possible investment alternatives to the hypothesis used in the main part of the paper. Finally, in Section 6 the principal conclusions are presented.

2. The model

The model is developed in a context of overlapping generations with two life periods. The individuals living in the first period are the *workers,* who produce and save. Those in the second period are the *retired*, who receive the returns on the savings carried out in the first period.

Let us suppose that there are only two possible locations for production, A = "country of birth" and B = "foreign country", both with the same native population, $2L$ in total, since in each period L new individuals are born.

2.1 Preference for the country of birth and the possibility of returning home.

The utility of an individual in each period is logarithmic (without altruism and therefore without bequests), with an element of intertemporal discount δ $(0<\delta<1)$ that does not differ between countries:

$$U = \ln(\theta^{p} c_{t}^{1,p}) + \delta \ln(\theta^{q} c_{t+1}^{2,q}); \qquad \text{p, q = A,B} \qquad (1)$$

where c^1 is the consumption of the individual when a worker, c^2 the consumption when she retires and θ is the preference for the country of birth [2]. This parameter allows us to represent heterogeneous individuals in each country according to the intensity of their preference; it is a random variable with a Pareto distribution. For country *A*'s individuals it will have the following properties:

$$\theta^{p}, \theta^{q} \begin{cases} >1 \quad \text{if } p, q = A, \ \theta \in [1,\infty), \ f(\theta) = \dfrac{\varepsilon}{\theta^{\varepsilon+1}}, \ \varepsilon>0, f(.)\text{: density function} \\ 1 \quad \text{if } p,q = B \end{cases}$$

where *p* and *q* indicate the country in which the individual is to be located: p when a worker and q when retired. The parameter shows the migration cost for the worker and the benefits of the return simultaneously. The individual has the following alternatives: to stay in the country of birth all her life (*p=q=A*), to emigrate in order to work and stay in the foreign country (*p=q=B*), to emigrate only on retirement (*p=A, q=B*) or to emigrate in order to work and return to the country of birth (*p=B, q=A*).

2.2 Learning by investing

The economy produces a single good with physical capital and labor, according to a production function of constant returns to scale of the type used by Kemnitz (2001):

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[2] Following Faini (1996).

$$Y = F(K, TL) = F(\frac{K}{TL}, 1)TL\,, \qquad F_1 > 0, F_2 > 0, F_{11}, F_{22} < 0, F_{12} > 0$$

$$y = \frac{Y}{L} = F(\frac{K}{TL}, 1)T \tag{2}$$

where T is a technological labor-increasing factor, exogenous to the firm. This factor reflects the positive diffusion that investment in physical capital has on labor productivity, with the consequent productive learning. We suppose this "learning-by-investing" effect to have the following form:

$$T^p = \frac{1}{\alpha^p}\frac{K^p}{L^p} = \frac{1}{\alpha^p}k^p\,, \qquad \alpha^p > 0$$

where $k^p = \frac{K^p}{L^p}$, K^p and L^p are the capital and the labor in country p (p= A, B), so that the constants α^p (p= A, B) corresponding to the two locations are not necessarily equal, that is, the two countries do not necessarily have the same technology. Thus, we can write:

$$y = F(\alpha^p, 1)\frac{1}{\alpha^p}k^p = \frac{f(\alpha^p)}{\alpha^p}k^p \tag{3}$$

where $f(\alpha^p) = F(\alpha^p, 1)$ and $f^{'}(\cdot) > 0, f''(\cdot) < 0$.

The factor markets are competitive, so that the marginal productivities of labor and capital, given T, correspond to the price of their factors:

$$\begin{aligned} r_t^p &= f^{'}(\alpha^p) \\ w_t^p &= \left[\frac{f(\alpha^p)}{\alpha^p} - f^{'}(\alpha^p)\right]k_t^p \end{aligned} \tag{4}$$

2.3 Different Technologies

We suppose that the production of the good in country A is relatively labor-intensive, that is, the payment for labor in country A for a given capital-labor ratio is less than in country B so that it holds that:

$$\frac{\left[\frac{f(\alpha^A)}{\alpha^A} - f'(\alpha^A)\right]}{\left[\frac{f(\alpha^B)}{\alpha^B} - f'(\alpha^B)\right]} = \xi < 1.$$

It could be questioned why the externality is limited to each specific country and does not lead to a convergence in technology. As it is evident that there can be persistent differences in technology between states, countries or regions, and that the channels through which convergence is achieved are not limited to technology, we are interested in the case in which technological differences are given due to specific features of the countries. The difference in our case means that country B is able to exploit the externality that represents the capital/employment ratio more efficiently than country A. The specific features could be, for example, of an institutional character. In spite of this assumption, we will always consider the results that would be derived if the parameter ξ were equal to one, that is, if the two countries have the same technology. Consequently we obtain the limit result corresponding to technological convergence.

In what follows, we are going to study the resource allocation problem that is posed in these two countries, separating two situations. In the first place, the situation with regard to closed economy, that is, when there is no labor mobility, is discussed. This situation will be used as a reference to evaluate the benefits of any situation with factor mobility. We then show what happens if we allow perfect labor mobility, and a certain degree of capital mobility, that is, if return migratory flows could occur jointly with the capital mobility that they produce. All possible investment alternatives to this hypothesis will also be considered in Section 5.

3. Closed economy

If individuals are not permitted to emigrate from their country of birth, either as workers or when they retire, the only decision they must take in the first period is how much income to save, because $p = q$ and $\theta^p = \theta^q = \theta$. The savings of any individual in country p will be:

$$s_t^p = \frac{\delta}{1+\delta} w_t^p, \qquad p = A, B. \qquad (5)$$

The equilibrium in the goods market allows us to know the accumulation of capital per worker in each period, which is equal to the savings of the individuals in the previous period because a complete depreciation of capital is considered. This relation allows us to say that each country grows individually at the following rate:

$$\frac{k_{t+1}^p - k_t^p}{k_t^p} = \frac{\delta}{1+\delta}\left[\frac{f(\alpha^p)}{\alpha^p} - f'(\alpha^p)\right] - 1 \qquad p = A, B. \qquad (6)$$

As can be seen, there is continuous growth in both countries in the steady state as long as expression (6) is positive.

Therefore, in a closed economy without factor mobility, country *B* would grow at a higher rate than country *A*, causing divergence between the two countries.

If we obtain the dynamic behavior of the relative capital/labor ratio between both countries, $k = \frac{k^A}{k^B}$, we get the following expression:

$$k_{t+1} = \xi k_t \qquad (7)$$

From this expression it can be deduced that there would only be a steady state relative capital/labor ratio if $\xi=1$, in which case all possible values of the relative capital/labor ratio are steady state. As a result, there is no convergence because the differences become permanent, provided that the starting point differs. The two countries grow at the same rate but keep the relative capital/labor ratio and the relative wage constant, so maintaining the existing gap between them.

When $\xi<1$, as the production in country *A* is relatively labor-intensive, what happens is that the relative capital/labor ratio of the two countries will be decreasing, with country *A* poorer and poorer with respect to country *B*. Thus, *a clear divergence will take place if the economies are completely closed* and have different growth rates. Let us see whether this result could be modified by migration.

4. Open economy

4.1 Savings and the decision to emigrate

Having analyzed the behavior of the economies in the case in which there is no labor mobility, let us consider what happens if frontiers are opened for workers in the other country. The first difference that we face is that now the individual has to take several additional decisions, not only in the first period but also in the second. Firstly, she must decide how much income to save, for which the typical problem of utility maximization subject to the restrictions mentioned above arises. From the solution to this problem the same savings function dependent on the return on capital, the intertemporal discount rate and the wage as in expression (5) is obtained.

Secondly, the worker has to take the decision about whether to emigrate or not in the first period of life, and about what to do in the second period, for which she compares the utility that each of the four possible alternatives offers her. Let us call this utility $U^{p,q}$, so that, as before, p reflects the country of residence when the individual is working and q the country in which she retires.

If we analyze the utility of each of the alternatives in detail, we may verify that $U^{AA} > U^{AB}$ and $U^{BA} > U^{BB}$ always hold. This means that, whatever the individual decides during her working life, on retirement she always prefers to return to her country of origin. This conclusion is due to the fact that we have not imposed any return cost and that the income available for the individual's retirement is her own savings. Therefore, she only finds advantages in returning to her country of birth. The parameter of preference for residing in the country of birth and the fact that the income of the second period of life is independent of the country of residence play a key role in making this result possible. This analysis reflects a common fact in migration known as temporary migration. Nevertheless, we still have to answer the question as to how many workers will migrate.

In the decision about whether to emigrate or not, as well as in the decision we have just seen of emigrant's returning, the cost assumptions involved in mobility are decisive. We are considering only one cost of migration: the opportunity cost represented by the parameter θ, in that the preference for the country of origin provides a utility premium that must be compensated in order to decide to leave the home country. This opportunity cost affects in the decisions of the two periods. Depending on the value of this parameter the compensation must be higher or lower, which will influence the number of emigrants: the greater the available compensation, the greater the number of emigrants. By introducing an additional cost of migration and/or an additional cost of returning, we would be increasing the compensation required for each individual and, for a given compensation, the number of emigrants will decrease. Zero emigration would even be possible for a given range of low compensations, which would only reinforce the absence of absolute convergence. Taking into account this circumstance, for the sake of simplicity, we do not introduce any other cost of migration but this opportunity cost, and we will discuss the effects derived from the existence of other possible costs when appropriate. Any additional cost will always mean a lower value of the flow of emigrants.

An individual decides to emigrate as long as the utility obtained after migrating is greater than the one obtained in her country of origin. Thus, as the returning effect is known, a native of country A decides to migrate when young if $U^{BA} > U^{AA}$, which holds if her parameter of preference for the country of birth θ fulfils the condition:

$$\theta < \left(\frac{w_t^B}{w_t^A} \right)^{1+\delta} = \overline{\theta} \tag{8}$$

Once this inequality has been deduced, and knowing that parameter θ follows a Pareto distribution, we may first obtain the number of emigrants M_t as a proportion m_t of the native population, such that:

$$M_t = m_t L. \tag{9}$$

Hence, as the proportion that does not emigrate is $1-m_t$, according to the Pareto distribution of θ, we may write:

$$1 - m_t = \int_{\overline{\theta}}^{\infty} f(\theta) d\theta = \left(\frac{w_t^A}{w_t^B} \right)^{(1+\delta)\varepsilon} = \left(\xi \frac{k_t^A}{k_t^B} \right)^{(1+\delta)\varepsilon} \tag{10}$$

With this result it could be concluded that being able to decide where to spend one's retirement has an influence on migration because the proportion of natives who will decide to emigrate in our model is higher than that derived without permitting the return of the emigrants [see Faini(1996)]. In our case there will be workers who would not have emigrated if they had not considered the possibility of returning, but who will do so because their preference for their country of birth will be compensated by a higher income when they return for their retirement.

In order to guarantee that the proportion of emigrants (*m*) is within the limits *0* and *1*, a technical condition must be fulfilled which is deduced from the fact that θ can only take values higher than *1*. This condition requires that the ratio of wages between the country of birth and the foreign country be inferior to *1*, that is to say, the productivity of labor in the country of birth must be lower than that of the foreign country, which requires the condition $\xi \frac{k^A}{k^B} < 1$, or, to put this in another form:

$$k = \frac{k^A}{k^B} < \frac{1}{\xi}. \tag{11}$$

As we have assumed $\xi<1$, even with $k^A > k^B$ there could be migrations from *A* to *B*. If the condition $k < \frac{1}{\xi}$ holds, it is guaranteed that $0<m<1$. When this condition does not hold, and we have $k>1/\xi$, the migration will be in the opposite direction, that is, from country *B* to country *A*, and this opens up the possibility of a reversal of migration flows that we can derive in our model. The condition in which this reversion takes place is that the technologically laggard country (country A) must initially be relatively much more capital abundant than the technologically advanced country (country B) related to technological backwardness. We will see below the dynamics according to which this migration will not be maintained over time when we tackle the analysis of the dynamics of the relative capital/labor ratio. The point in which the immigration regime change occurs is $k=1/\xi$ from where the immigration from country *A* to country *B* will start. If $\xi = 1$, there will only be migration from *A* to *B* when the capital per worker in the first country is lower than in the second one. The reversion in such a case will exist provided that the capital stock in *A* is greater than the capital stock in *B*.

4.2 Capital stock

The capital of a country in each period is formed by the savings of native workers. We consider that emigrants contribute to capital accumulation in their country of origin with the savings they bring back when they return, although some extensions to this hypothesis will be considered in section 5. Thus, the stock of capital in both countries will be:

$$\begin{aligned} K_{t+1}^{A} &= s_t^A(1-m_t)L + s_t^B m_t L \\ K_{t+1}^{B} &= s_t^B L \end{aligned} \tag{12}$$

In terms of the capital/labor ratio, the resulting expressions are:

$$\begin{aligned} k_{t+1}^{A}\frac{1-m_{t+1}}{1-m_t} &= s_t^A + s_t^B\frac{m_t}{1-m_t} \\ k_{t+1}^{B}\frac{1+m_{t+1}}{1+m_t} &= s_t^B\frac{1}{1+m_t} \end{aligned} \tag{13}$$

In contrast to the situation in the closed economy in which there is no transitory dynamics, in this situation the migration rates corresponding to the contemporary period and the previous one influence the capital/labor ratio. This circumstance allows us to identify the dynamics of the two variables, *k* and *m*, which are the two key variables in the allocation of resources in the two economies. With these two equations and equation (10) we can undertake the study of the steady state.

4.3 Steady state

In order to explain the behavior of the relative capital/labor ratio *k,* we explore one of the conditions that must hold in the steady state: the growth rate of the capital per worker in country *A* must be equal to the growth rate of the capital per worker in country *B*. From expressions (4), (5) and (13) we can deduce that the growth rate in country *A* is:

$$\frac{k_{t+1}^A - k_t^A}{k_t^A} = \frac{1-m_t}{1-m_{t+1}} \frac{\delta}{1+\delta}\left[\frac{f(\alpha^A)}{\alpha^A} - f'(\alpha^A)\right] + \frac{m_t}{1-m_{t+1}} \frac{\delta}{1+\delta}\left[\frac{f(\alpha^B)}{\alpha^B} - f'(\alpha^B)\right]\frac{1}{k_t} - 1, \tag{14}$$

and the growth rate in country *B* is:

$$\frac{k_{t+1}^B - k_t^B}{k_t^B} = \frac{1}{1+m_{t+1}} \frac{\delta}{1+\delta}\left[\frac{f(\alpha^B)}{\alpha^B} - f'(\alpha^B)\right] - 1 \tag{15}$$

A careful inspection of these two expressions is of a particular interest to conclude about the benefits of migration. We know that both rates should be equal in the steady state, but it is worth comparing them to the values they would have when the corresponding economy is closed. It can be concluded that the growth rate of country *A* in the steady state is higher than when the mobility of labor is not permitted. With reference to the growth rate of country *B* in steady state, we observe that it is lower than it would be in a closed economy. As a result, we deduce that the sending economy benefits from migration while the host economy loses out in terms of growth, and therefore in welfare.

The existence and uniqueness of the steady state

From the analysis carried out so far, we have obtained two equations that relate relative capital/labor ratio and migration in the steady state. On the one hand, from equation (10), once we substitute the expression for the wage in (4) for each country, we get:

$$k = \frac{(1-m)^{\frac{1}{(1+\delta)\varepsilon}}}{\xi}, \tag{16}$$

which is a decreasing relationship between the relative capital/labor ratio and the migration rate. It shows the negative relation between the relative wage and the desire of migrating and can be considered as the supply function of emigrants. The higher the relative capital/labor ratio, the lower the relative wage. Consequently, migration will be lower.

On the other hand, applying the condition of an equal growth rate in the two countries in (14) and (15), together with that of the steady state, an expression that relates relative capital/labor ratio and migration in the steady state is obtained:

$$k = \frac{\frac{m}{1-m}}{\frac{1}{1+m} - \xi}. \tag{17}$$

This shows a positive relation between the migration and the relative capital/labor ratio and may be considered as the demand function for emigrants. The growth rate in country *A* depends positively on *m* and negatively on *k*. In order to grow at the same rate as country *B* thanks to migration, a greater demand for emigrants must be offset with an increase in the relative capital/labor ratio, due to the savings brought back by emigrants when they return to the country of origin.

As a consequence, we have two functions defined on the space of two variables, *m* and *k*, where *m* is bounded between *0* and *1* or between *0* and $\frac{1-\xi}{\xi}$ [3] (depending on whether this last value is greater or smaller than one, respectively) and *k* between *0* and $\frac{1}{\xi}$. These two functions show the relationships that should exist between them in the steady state. The first of these equations [equation (16)], convex as long as $(1+\delta)\varepsilon < 1$ and concave otherwise, shows the supply of emigrants from *A* to *B*. The second [equation (17)], increasing until $m=1$ or $m = \frac{1-\xi}{\xi}$ (depending on whether this last value is greater or smaller than one, respectively), shows the demand for emigrants from *A* to *B*, so that it grows as the capital/labor ratio between countries *A* and *B* grows. Hence, we are able to represent the equilibrium of this two-equation system graphically in Figure 1. We do this both for $(1+\delta)\varepsilon < 1$ and for $(1+\delta)\varepsilon > 1$, although the conclusions obtained later would be valid for both cases. The equilibrium exists, is unique and interior given the conditions imposed.

Figure 1. *Determination of the steady state*

[Insert figure 1]

The behavior of these two functions implies that, although it is not possible to obtain explicitly the concrete steady state value of k, we know that there is a capital/labor ratio in the steady state in which a constant and continuous migration of workers from country A to country B takes place if $\xi<1$. When $\xi=1$ there will be no migration and we shall have k=1 in steady state, that is, there will be absolute convergence.

Sensitivity analysis in the steady state

In what follows we carry out an analysis of the sensitivity of this steady state. First we shall see what happens if the technological distance between the two countries is reduced, that is to say, if ξ increases. On the one hand we have a shift of the supply function of emigrants (*S*) towards the origin because the migration is smaller for each capital/labor ratio due to the increase of the relative salary. On the other, there is another shift of the demand function for emigrants (*D*) towards the left, since for each value of migration the relative capital/labor ratio must increase in order to maintain the same growth rate as in country B. The equilibrium moves from E_0 to E_1 as can be seen in Figure 2.

Figure 2. *Effect of an increase in ξ*

[Insert figure 2]

Thus, in the new steady state, migration decreases without any doubt. However the effect on the relative capital/labor ratio is not determined: it may increase, decrease

[3] When $\xi=1$ the asymptote shifts towards the left until it coincides with the vertical axis, so that the equilibrium is situated at $m=0$ and $k=1$.

or stay the same. If the effect on income of the fall in migration is not offset internally, the ratio may decrease. This will occur for low values of ξ and high values of m. As ξ approaches one, the effect tends to be positive. This result means, first, that an improvement in technology does not always have a linear effect on the standard of living of the less developed country, and secondly that, in spite of this, an improvement in technology eventually terminates in conditional or absolute convergence. In fact, when $\xi=1$, the relative capital/labor ratio in the steady state is one, that is, absolute convergence takes place.

If the preference for the country of birth, ε, increases, the supply function of emigrants is modified in such a way that, if the function is convex, it becomes more convex but maintaining the same initial and final values. The reason is that each value of the relative capital/labor ratio (except the extremes) generates a lower value of migration. If, on the contrary, the function is concave, its concavity is smoothed. Therefore, in the new equilibrium, moving from E_0 to E_1, both migration and relative capital/labor ratio decrease. Fewer workers choose to work abroad because the country of birth is more preferred and so a given volume of savings on returning home are not accumulated in the country of origin. This variation causes a greater gap between the countries.

Both results indicate that convergence could only be achieved through technological improvement or through lowering the preference for the country of birth. This latter option provides a conditional convergence and does not eliminate the gap, while technological improvement may mean transitory recessions in convergence but could eventually provide even absolute convergence.

Figure 3. *Effect of an increase in ε.*

[Insert figure 3]

4.4 Dynamics of the system

Up to now, we have seen that there is a long-term equilibrium and that it is unique and interior. In this section we are going to study its stability. Analyzing the dynamic system is not a trivial question because the dynamic path of the relative capital/labor ratio takes the form of the following implicit function:

$$k_{t+1}\frac{(\xi k_{t+1})^{(1+\delta)\varepsilon}}{2-(\xi k_{t+1})^{(1+\delta)\varepsilon}}=(\xi k_t)^{(1+\delta)\varepsilon}\left[\xi k_t-1\right]+1 \tag{18}$$

The range of values of k_t for which migration from *A* to *B* exists coincides with the interval $[0,\frac{1}{\xi}]$. From equation (18) it can be deduced that the values of k_{t+1} corresponding to the two extreme points of this interval coincide, while the values of k_{t+1} for any of the intermediate points are lower than them. The initial point tells us that although the sending country, *A*, does not have capital available, in the following period, due to the savings with which emigrants return, the value will be positive.

Equalizing to zero the first derivative of equation (18) the minimum value of k_t is obtained:

$$k_t^{\min}=\frac{(1+\delta)\varepsilon}{\left[(1+\delta)\varepsilon+1\right]\xi} \tag{19}$$

Therefore, from the analysis carried out so far, it is known that we are dealing with a function in which there is a unique extreme point that corresponds to a minimum, which can be found above, on, or below the bisector (the line $k_{t+1}=k_t$), giving rise to a well-posed casuistry.

Furthermore, for the final point with migration from *A* to *B* ($k_t=\frac{1}{\xi}$) the function is always below the bisector if $\xi<1$. It has already been demonstrated that a unique steady state exists, so the dynamic path must cut the bisector at only one point. The points situated to the right of the steady state must, then, be below the bisector.

Otherwise there would be more than one steady state[4]. This indicates that, on the right of the steady state the gap between k_t and k_{t+1} widens out with the value of k_t, in such a way that from the final point of the dynamic path to the right, this condition requires migration from country *B* to country *A*.

It has already been mentioned that, depending on whether the minimum is above or below the bisector, we will be faced with different possibilities related to the dynamic stability. Figure 4 sums up these situations.

Figure 4. *Possibilities of the dynamic function*

[Insert figure 4]

When the minimum is above the bisector (in which the particular situation where $\xi = 1$ is included), it is clear that the equilibrium is stable. Alternatively, when the minimum is below the bisector the steady state is reached through convergent oscillations and, therefore, the migratory flows will show a stable non-monotonous behavior in the short term[5]. In other words, the sending country will experience a cyclical pattern in migration flows, as well as in the relative capital/labor ratio, around a value where it will eventually stabilize. This dynamic function allows the existence of migration reversion if k_t is initially greater than $1/\xi$. Then migration will flow from country *B* to country *A*, but with k_t falling from the right to the left. When this variable reaches the value $1/\xi$ the migration flow will reverse its direction, will start from country *A* to country *B* and will continue until the steady state with $k_t = k_{t+1}$.

[4] This property can be analytically demonstrated for the final point of the dynamic function when $\xi = 1$, the point $(k_t, k_{t+1}) = (1,1)$. At this point, a marginal fall in ξ makes the value of the final point $k_t = \frac{1}{\xi}$ increase more than its corresponding k_{t+1}, because the condition $\left.\frac{dk_{t+1}}{d\xi}\right|_{\xi=1} < \left.\frac{dk_t}{d\xi}\right|_{\xi=1} = 1$ holds.

[5] The proof can be obtained from the authors upon request.

5. Alternative hypothesis for the investment of the immigrant's savings

The basic model described in the previous sections considers the assumption that immigrants invest all their savings in the home country. It might be asked to what extent the results obtained till now depend on this assumption. To solve the question we shall consider, first, the opposite case in which all the savings of the immigrants is invested in the country where they have been working, although they have decided to live in their country of birth. Then we shall consider any intermediate situation.

Immigrants' savings invested totally in the host country

When all of the immigrant's savings are invested in the host country, the capital stock in the two countries will be the following:

$$\begin{aligned} K_{t+1}^A &= s_t^A (1-m_t)L \\ K_{t+1}^B &= s_t^B (1+m_t)L \end{aligned} \tag{20}$$

Taking into account the expression of the savings function we can obtain the growth rate of the two countries:

$$\begin{aligned} \frac{k_{t+1}^A - k_t^A}{k_t^A} &= \frac{1-m_t}{1-m_{t+1}} \frac{\delta}{1+\delta} \left[\frac{f(\alpha^A)}{\alpha^A} - f'(\alpha^A) \right] - 1 \\ \frac{k_{t+1}^B - k_t^B}{k_t^B} &= \frac{1+m_t}{1+m_{t+1}} \frac{\delta}{1+\delta} \left[\frac{f(\alpha^B)}{\alpha^B} - f'(\alpha^B) \right] - 1 \end{aligned} \tag{21}$$

From these expressions we may assert that, if the proportion of immigrants arriving at country *B* increases over time, the growth rate in *A* will be greater than in the closed economy, and the opposite happens in country *B*. But we cannot say anything about what happens in the steady state without the proof of its existence. In order to conclude about this point, we consider the dynamic of the relative capital/labor ratio, which is the following:

$$\frac{(k_{t+1})^{(1+\delta)\varepsilon+1}}{2-(\xi k_{t+1})^{(1+\delta)\varepsilon}} = \xi \frac{k_t^{(1+\delta)\varepsilon+1}}{2-(\xi k_t)^{(1+\delta)\varepsilon}} \tag{22}$$

that we can write as $x_{t+1} = \xi x_t$, where $x_t = \dfrac{k_t^{(1+\delta)\varepsilon+1}}{2-(\xi k_t)^{(1+\delta)\varepsilon}}$.

From this last expression we can conclude that a steady state relative capital/labor ratio only exists if *ξ=1*, in which case all possible starting values of the relative capital/labor ratio will be steady state. As a result, there is no convergence because the differences between the two countries become permanent, provided that the starting point differs. The two countries grow at the same rate, keeping the relative capital/labor ratio and the relative wage constant, so maintaining the existing gap between them.

As the production in country *A* is relatively labor-intensive, when *ξ<1* growth is not the same for both countries. According to equation (22) the relative capital/labor ratio continuously decreases, a clear divergence will take place in which country *B* grows more than country *A* up to the limit at which the point $(m_t, k_t)=(1, 0)$ is reached[6]. This is an extreme case in which we find the divergent result typical in the endogenous growth models with scale effect. At this limit point the country *B* grows at the same rate as in the closed economy, but in the transition the growth rate will be lower. On the other hand, country *A*'s individuals will be benefited. In this case, the conclusion related to the welfare is very different from models with scale effect.

Immigrants' savings invested partially in the host country

Let us now assume that immigrants can invest a part of their savings in the country of origin *(σ)* and the rest in the host country *(1-σ)*. This hypothesis implies the following equations for capital accumulation:

$$\begin{aligned} K_{t+1}^A &= s_t^A (1-m_t)L + \sigma\, s_t^B m_t L \\ K_{t+1}^B &= s_t^B L + (1-\sigma)\, s_t^B m_t L \end{aligned} \qquad (23)$$

[6] This would be a situation in which production will not exist in country *A* and the only residents in this country will be the old individuals that worked in country *B* during the previous period, from where they receive their income and acquire the goods they consume in their second period of life.

Following the same procedure as in the previous section, we can establish the following increasing relationship between the relative capital/labor ratio and migration in the steady state[7]:

$$k = \frac{\frac{m}{1-m}\sigma}{\frac{1+(1-\sigma)m}{1+m} - \xi} \tag{24}$$

The characteristics of this function are similar to equation (17). It shows the positive relationship between migration and the relative capital/labor ratio[8] that we have previously denominated as the demand function for emigrants. An increase in the demand for emigrants from country A must be compensated in the steady state with an increase in the relative capital/labor ratio in order to maintain the same growth rates in the two countries. The only difference is the presence of parameter σ. When the value of this parameter decreases, the function shifts downwards.

The other equation that allows the proof of the existence of the steady state is equation (16), which explains the migration as a decreasing function of the relative capital/labor ratio. This relation is independent of parameter σ. As equation (24) displays a behavior similar to equation (17) we can say that for a positive value of the parameter σ, even for values close to zero, a steady state with finite and positive values of *m* and *k* will always exist. Figure 5 shows the result.

Figure 5. Steady state determination with $0<\sigma<1$

[Insert Figure 5]

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[7] When σ =1 (all the returning migrants' savings are invested in the home country) we have the case analyzed in the previous sections of the paper. This equation becomes equation (17).

[8] This relationship holds for $m<1$ or for $m \leq \frac{1-\xi}{\xi-(1-\sigma)}$, depending on whether the last term is greater or smaller than one, respectively, provided it is positive ($\xi > 1-\sigma$). When it is negative the relationship holds for $m<1$.

Related to the growth rate, we can see that the result for the gains from migration with positive values of the parameter σ is the same as in the case in which all savings are invested in the country of origin. The growth rates for the two countries are the following:

$$\frac{k_{t+1}^A - k_t^A}{k_t^A} = \frac{\delta}{1+\delta}\frac{1-m_t}{1-m_{t+1}}\left[\frac{f(\alpha^A)}{\alpha^A} - f'(\alpha^A)\right] + \\ + \sigma\frac{\delta}{1+\delta}\frac{m_t}{1-m_{t+1}}\left[\frac{f(\alpha^B)}{\alpha^B} - f'(\alpha^B)\right]\frac{1}{k_t} - 1$$

$$\frac{k_{t+1}^B - k_t^B}{k_t^B} = \frac{\delta}{1+\delta}\frac{1}{1+m_{t+1}}\left[\frac{f(\alpha^B)}{\alpha^B} - f'(\alpha^B)\right] + \\ + (1-\sigma)\frac{\delta}{1+\delta}\frac{m_t}{1+m_{t+1}}\left[\frac{f(\alpha^B)}{\alpha^B} - f'(\alpha^B)\right] - 1 \tag{25}$$

where it can be see that in the steady state the growth rate in the country *A* is greater than in the closed economy and the growth rate in country *B* is lower than in the closed economy. The country that sends emigrants improves and the country that receives emigrants worsens compared to the closed economy situation.

Related to the dynamics, we can obtain the law of motion of the relative capital/labor ratio:

$$k_{t+1}\frac{(k_{t+1})^{(1+\delta)\varepsilon}}{2-(\xi k_{t+1})^{(1+\delta)\varepsilon}} = \frac{(\xi k_t)^{(1+\delta)\varepsilon}\left[\xi k_t - \sigma\right] + \sigma}{1+(1-\sigma)\left[1-(\xi k_t)^{(1+\delta)\varepsilon}\right]} \tag{26}$$

It can be easily verified that this is a function that rotates towards the left over the point $k_t=1/\xi$ when the parameter σ decreases, in such a way that the new steady state have a smaller *k* and a smaller *m*. At the limit, when σ is equal to zero the solution is the point $(k_t,k_{t+1})=(0,0)$. This parameter plays an important role. Decreases in its value produce lower accumulation of capital per worker in country *A* with respect to country *B*. However it does not affect the decreasing function (16), which shows the desire to emigrate for every capital/labor ratio between the countries. Therefore, in the steady state, the relative capital/labor ratio decreases in relation to the basic model analyzed in the previous sections of the paper, and migration increases. Our results strongly suggest that in this case, with $\sigma<1$, the permanent gap between the countries would be greater than in the basic model and also constant in the long run, which produces a permanent and more intense migration flow. The limit case with $\sigma=0$ is the only situation we have found with the divergent result that was typical in the first wave of endogenous growth models with migrations.

6. Conclusions

This paper presents a model that describes an economy characterized by the heterogeneity of individuals and countries, where the movement of workers is allowed and where it is considered that migration, apart from the personal benefits it brings through improvements in per capita income, also brings a cost in terms of welfare derived from the preference for the country of birth. Furthermore, the possibility of the return of emigrants is considered, taking into account the possibility of the contribution of savings coming from the income obtained in the host country to the total savings of the country of birth.

In this framework, the steady state of the dynamic system is unique and stable, with migration from one country to the other in the long run whereas there are technological differences. Although each country converges towards its own steady state, in which both of them grow at the same rate, neither their capital/labor ratios nor their wages necessarily become the same, which causes permanent migration. The gap between the countries neither disappears in the long run (as the models of exogenous growth would indicate), nor increases permanently (as many models of endogenous growth show) except in the case in which all the immigrants' savings are invested in the host country. Our general result is consistent with the evidence from empirical studies, according to which there is no absolute convergence, understood as a tendency towards homogeneity in the remuneration of the productive factors between countries. However, there is, indeed, convergence in the growth rates towards the same value, which maintains the gaps in the capital/labor ratio and in salaries in the long run.

Another remarkable conclusion that is derived from the model is that individuals decide to return to their country of birth as a consequence of utility maximization among the alternatives they face. Nevertheless, only some individuals decide to emigrate. This decision depends on their preference for the country of birth and on the wage differential between the countries. These workers could contribute to capital accumulation in their country of birth and the sending economy could benefit in the steady state if it permits the movement of labor, growing more than it would in a closed economy. On the contrary, the growth of the host country will be lower.

Lastly, it has been shown that changes in the exogenous parameters of the countries produce interesting effects. In particular, a narrowing of the technological gap lowers the pressure to migrate, as was to be expected. But this does not always lead to an improvement in the steady state's relative capital/labor ratio of the less developed country, although finally, when the gap tends to disappear, improvement is guaranteed. Decreases in the preference for one's own country may also mean a boost to convergence.

A further research it would be worth to consider is the introduction of the immigrant's skilled-unskilled distinction in both countries in order to explore the consequences on growth and convergence. Mainly it would be of interest to know the dynamics that the relative gains derived from migration might generate in the two countries involved and the conditions in which the host country would be benefited.

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Figure 1

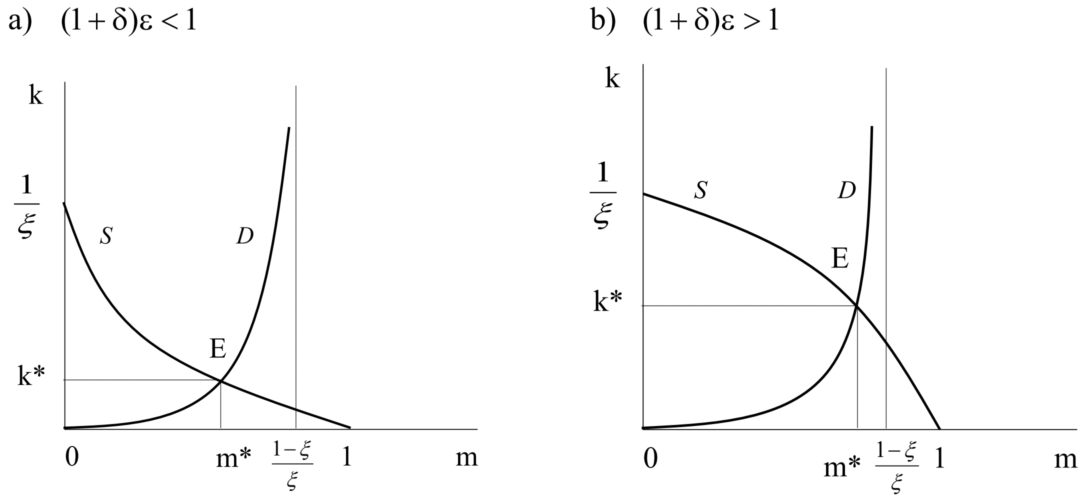


Figure 2

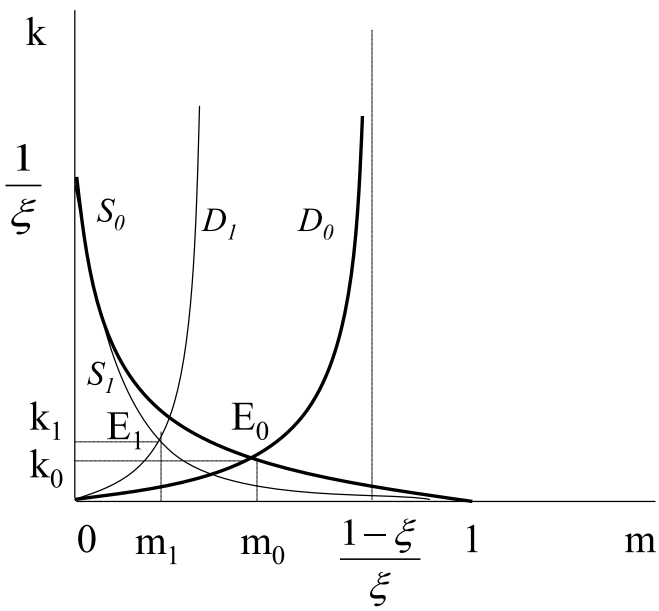


Figure 3

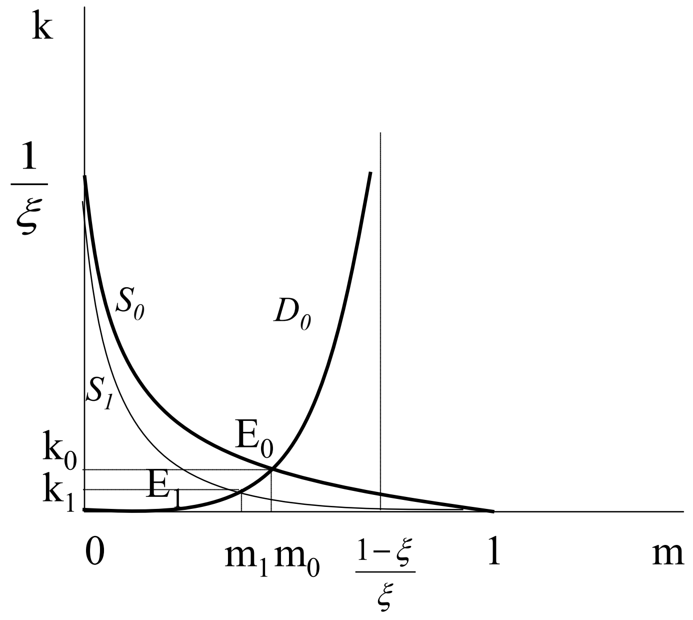


Figure 4

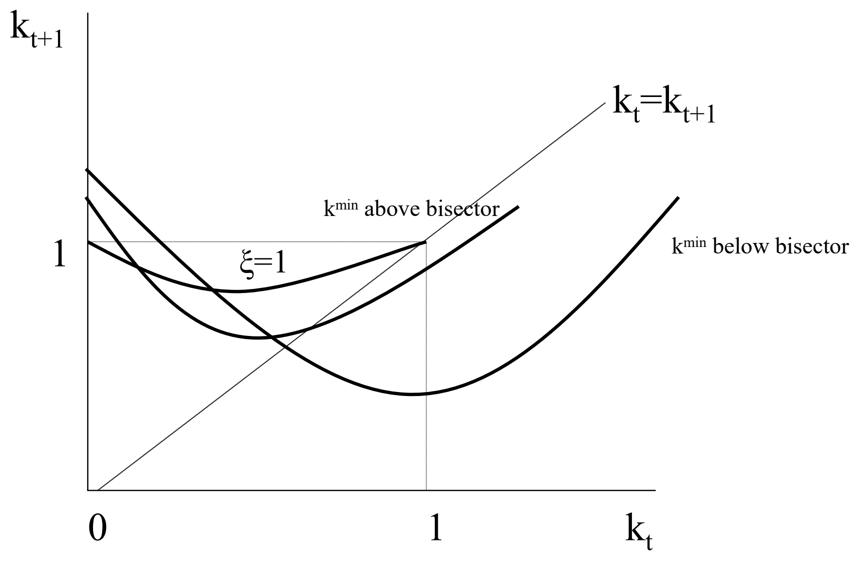
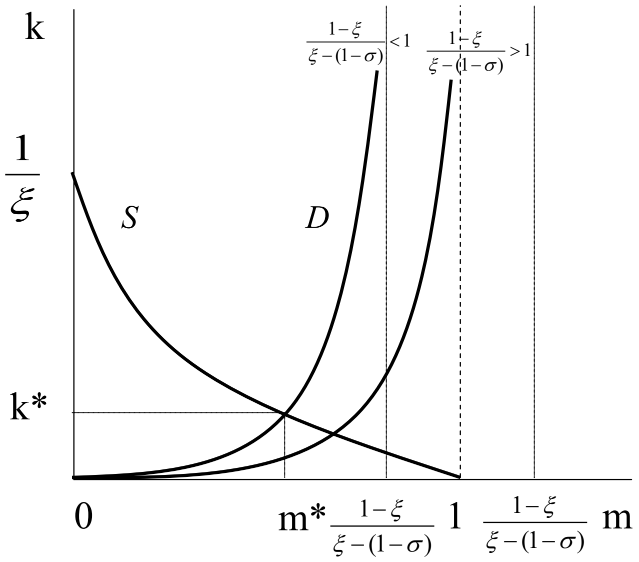


Figure 5.


k
$\frac{1-\xi}{\xi-(1-\sigma)}<1$
$\frac{1-\xi}{\xi-(1-\sigma)}>1$
$\frac{1}{\xi}$
S
D
k*
0
m*
$\frac{1-\xi}{\xi-(1-\sigma)}$
1
$\frac{1-\xi}{\xi-(1-\sigma)}$
m