Gibrat's Law for CO2 emissions revisited

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Abstract:

This study revises the results of Ahundjanov and Akhundjanov (2019), who concluded that Gibrat's Law could not be rejected for CO_2 emissions and per capita CO_2 emissions. We revisit this topic using a more recent and distinct statistical methodology. Our main result is that Gibrat's Law for CO_2 emissions, both in absolute and in per capita terms, does not hold in a majority of cases. The fulfillment of Gibrat's Law in this case seems to be a short-term phenomenon, in contrast to the patterns observed in cities within countries such as the USA.

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Conflict of Interest

None.

Ethical Approval

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1. Introduction

The need to manage CO_2 emissions has emerged in the global policy agenda (Kim and Kim,2012, Peña et al.,2022). The reason is that currently greenhouse effect gases are probably growing, and the climate of Earth is becoming warmer (Schmalensee et al.,1998). These authors claim that the anthropogenic emissions are the most important source of potential warming. One way of studying the distribution of these emissions is by means of testing the fulfillment of the so called Gibrat's Law of proportional effect, a concept originally defined for firms (Gibrat,1931) that is also frequently applied to city population (González-Val et al.,2014).

If Gibrat's Law applies to CO_2 emissions, then the initial level of emissions wouldn't necessarily affect how fast these emissions grow later. This means that a country with high initial CO_2 emissions wouldn't necessarily have faster or slower growth in emissions compared to a country that starts with a lower initial value. In this context, Gibrat's Law gives us a fresh way to look at CO_2 emissions, suggesting that successful plans to cut emissions don't always need to depend on how big emissions were at the start. Against this background, if Gibrat's Law does not hold, the evolution can be divergent (positive correlation between growth rate and initial value) or convergent (negative correlation), which has important policy implications.

In a recent and interesting paper, Ahundjanov and Akhundjanov (2019) stated that Gibrat's Law cannot be rejected with CO_2 emissions and CO_2 emissions per capita (CO_2pc), using a variety of parametric and nonparametric tools, mainly based on Eeckhout (2004). We wondered if the results would be essentially different with other methodologies, alternative to the one proposed by the previous authors. We followed recent standard statistical tests to assess whether the log-growth rates of CO_2 emissions are statistically independent of their initial log-values. To give robustness to our results we employ seven different statistical tests. Specifically, we use the test for DEP R package (Miecznikowski et al., 2018), where Vexler (VEXLER), Kallenberg TS2 and V, Maximal Information Coefficient (MIC), Hoeffding (HOEFFD), Empirical Likelihood (EL) and Kendall (KENDALL) tests are employed. We use both CO_2 (absolute) and CO_2pc (per capita) emissions, because, according to Akhundjanov et al. (2017), we must take into account the impact of population. In addition, we enlarge the sample initially used by Ahundjanov and Akhundjanov (2019) to further increase the robustness of the results.

Our main outcome and contribution is that Gibrat's Law does not hold always for CO_2 emissions, rather the opposite, especially for CO_2pc emissions.

In this paper the periods are separated according to different lengths (short run for one year, med-term for 5 years, long-run for 10 years). To end this Section, we briefly explain the implications of Gibrat's Law or Law of proportionate growth. We say this Law is satisfied when the growth rate of a quantitative variable is independent of its initial size.

2. Brief Literature Review

The study of the statistical evolution of CO_2 emissions in the literature is not new. Apart from Ahundjanov and Akhundjanov (2019), who stated that Gibrat's Law cannot be rejected with CO_2 emissions measured both in levels and per capita, most of the literature highlight possible reductions of CO_2 emissions in some cases by shifting to renewable energies (Kang et al.,2019), but with peaks when fossil fuel burning is used (Raupach et al.,2007), suggesting a presence of non-independence as they find. Other studies have obtained a relative decoupling between CO_2 emissions and GDP (Mikayilov et al.,2018). This suggests a growth rate of CO_2 emissions, in opposition to Gibrat's Law, or that in specific moments the CO_2 emissions in China had increased (Lv et al.,2020). More recent works show examples of absolute and conditional β convergence of CO_2 emissions (Zeng et al.,2023) or they expose the relevant nexus between CO_2 emissions, growth and development (Lv and Li,2021; Bolea et al.,2023).

3. Data

We have taken the datasets from the source mentioned in Ahundjanov and Akhundjanov (2019), obtained from Oak Ridge National Laboratory (ORNL) for 207 countries and territories, although we have enlarged the considered years to 1994, 1995, 1999, 2000, 2004, 2005, 2009 and 2010 (in the original paper data from 1995, 2000, 2005 and 2010 are taken). As stated before, we have analyzed both the CO_2 and CO_2pc emissions (in kilotons (kt) for CO_2 and metric tons per capita (mt per capita) for CO_2pc). We have also considered the log-growth rates g of these quantities (dimensionless), based on the well-known formula

$$g = \ln(x_{final}) - \ln(x_{initial}) \quad , \qquad (1)$$

for each relevant pair of periods. For the log-growth rates we have studied the four samples one year apart (short-term), the six samples five years apart (mid-term) and the four samples ten years apart (long-term).

4. Results

We have performed the above mentioned statistical tests for independence, for which the null hypothesis is:

$$H_0\text{: } g \text{ and } \ln(x_{initial}) \text{ are statistically independent.} \qquad (2)$$

The non-rejection of H_0 directly implies that Gibrat's Law is satisfied. Note that the employed tests allow for non-linear dependencies of the variables used, in this case g and $\ln(x_{initial})$. In Table 1 (CO_2 emissions), we observe that for each of the tests there are many cases of rejection of the null hypothesis, a 61.22% of the instances.

Regarding CO_2pc, there is a strong pattern of rejection of the null hypothesis for any test, the cases of non-rejection being scarce (87.76% of rejections). So, the conclusion is that for CO_2pc, the Gibrat's Law does not hold universally. In spite of the fact that Gibrat's Law does not hold for CO_2 emissions in general, it is more accepted in the short than in the mid and long term. The reason could be that in the long-term the short-term deviations are accumulated and then there is a higher dependence on the growth rate. Thus, it seems, according to our results, that countries “correct” their pattern of raw CO_2 emissions in the medium and long run, so the log-growth rates become statistically dependent on the initial log-emissions. This seems to be contrary to what happens to cities in a country like the USA, where the long-term nature of Gibrat’s Law is documented (see, e.g., Desmet and Rappaport, 2017 and references therein).

Table 1: *Outcome of the tests of independence of g and ln($x_{initial}$).*

Var.	*Term*	*Period*	VEXLER	TS2	V	MIC	HOEFFD	EL	KENDALL
CO_2	Sh.	1994-1995	**122.111 (0.053)**	13.942 (0.002)	13.942 (0.002)	**0.242 (0.117)**	**0.002 (0.141)**	**1.743 (0.063)**	**-1.135 (0.255)**
		1999-2000	**120.802 (0.210)**	6.196 (0.035)	**6.196 (0.066)**	0.263 (0.022)	**0.002 (0.128)**	**1.580 (0.093)**	**-0.684 (0.502)**
		2004-2005	**120.700 (0.232)**	**0.016 (0.896)**	**6.620 (0.054)**	0.300 (0.001)	**0.001 (0.204)**	**1.494 (0.115)**	**-0.231 (0.805)**
		2009-2010	**121.324 (0.126)**	**0.850 (0.374)**	**0.850 (0.404)**	**0.250 (0.067)**	**0.001 (0.241)**	**1.465 (0.124)**	**-0.87 (0.378)**
	Mid	1994-1999	122.579 (0.034)	7.051 (0.024)	7.051 (0.043)	**0.244 (0.094)**	0.010 (0.007)	3.118 (0.002)	-4.219(0)
		1995-2000	**121.498 (0.104)**	6.378 (0.027)	11.963 (0.005)	0.326 (0)	0.010 (0.007)	3.019 (0.003)	-2.522 (0.011)
		1999-2004	122.981 (0.019)	5.741 (0.037)	**5.741 (0.068)**	0.271 (0.014)	0.010 (0.008)	2.644 (0.009)	-2.316 (0.023)
		2000-2005	123.521 (0.009)	7.903 (0.017)	7.903 (0.030)	**0.247 (0.078)**	0.015 (0.001)	3.190 (0.002)	-2.95 (0.005)
		2004-2009	**119.301 (0.614)**	5.235 (0.043)	**5.235 (0.084)**	**0.223 (0.317)**	0.007 (0.021)	2.212 (0.018)	-2.333 (0.02)
		2005-2010	122.770 (0.027)	5.914 (0.034)	**5.914 (0.069)**	**0.240 (0.129)**	0.007 (0.018)	2.565 (0.009)	-2.393 (0.017)
	Long	1994-2004	**121.047 (0.172)**	9.228 (0.010)	9.228 (0.017)	**0.224 (0.310)**	0.011 (0.003)	3.297 (0.001)	-3.017 (0.003)
		1995-2005	**120.709 (0.229)**	9.048 (0.010)	9.048 (0.018)	0.268 (0.016)	0.014 (0.001)	3.373 (0.001)	-3.088 (0.002)
		1999-2009	**121.465 (0.116)**	8.851 (0.012)	8.851 (0.019)	**0.238 (0.146)**	0.013 (0.003)	2.902 (0.005)	-3.018 (0.003)
		2000-2010	123.556 (0.008)	11.943 (0.004)	11.943 (0.005)	**0.231 (0.208)**	0.018 (0.000)	3.821 (0.001)	-3.525 (0.001)
CO_2 *pc*	Sh.	1994-1995	135.997(0)	8.552 (0.012)	8.552 (0.022)	0.452 (0)	0.019 (0.000)	4.948 (0)	-3.157 (0.002)
		1999-2000	131.389(0)	**0.495 (0.501)**	**0.495 (0.516)**	0.440 (0)	**0.001 (0.231)**	2.714 (0.005)	**0.574 (0.554)**
		2004-2005	139.496(0)	**1.646 (0.219)**	10.691 (0.008)	0.474 (0)	0.009 (0.010)	4.132 (0.000)	**-1.485 (0.138)**
		2009-2010	129.888(0)	**1.846 (0.194)**	**1.846 (0.215)**	0.381 (0)	**0.005 (0.052)**	2.975 (0.004)	**-1.328 (0.176)**
	Mid	1994-1999	126.996(0)	14.736 (0.002)	22.171 (0)	0.394 (0)	0.032(0)	6.343 (0)	-4.219 (0)
		1995-2000	122.906 (0.006)	**4.455 (0.054)**	9.882 (0.012)	0.421 (0)	0.011 (0.006)	3.341 (0.001)	-2.236 (0.024)
		1999-2004	125.212(0.001)	**4.045 (0.068)**	11.092 (0.008)	0.318 (0.000)	0.009 (0.008)	2.812 (0.004)	-2.051 (0.041)
		2000-2005	128.735(0)	7.476 (0.017)	7.476 (0.033)	0.397 (0)	0.017 (0.000)	4.273 (0)	-3.066 (0.001)
		2004-2009	133.906(0)	11.493 (0.005)	11.493 (0.006)	0.384 (0)	0.026(0)	5.141 (0)	-3.563(0)
		2005-2010	132.105(0)	27.357 (0)	14.503 (0.001)	0.407 (0)	0.021(0)	5.655 (0)	-3.767(0)
	Long	1994-2004	130.804(0)	20.961 (0)	36.065(0)	0.330 (0)	0.043(0)	7.873(0)	-4.733(0)
		1995-2005	130.160(0)	15.916 (0.001)	27.681(0)	0.354 (0)	0.041(0)	6.654 (0)	-4.303(0)
		1999-2009	130.456(0)	18.699 (0)	18.699 (0.000)	0.379 (0)	0.036(0)	6.125 (0)	-4.459(0)
		2000-2010	130.161(0)	43.643 (0)	35.181(0)	0.375 (0)	0.050(0)	8.746 (0)	-5.652(0)

Note: The format is test statistic(p-value). Non-rejections at the 5% level of the null hypothesis of independence are marked in bold and, hence, correspond to the cases where Gibrat’s Law holds. Sh.: Short

The fulfillment of the Gibrat's Law for CO_2 emissions achieves 82.14% for the short-term and the 21.43% for the mid and long run. It is verified the Gibrat's Law for CO_2pc emissions the 35.71% of the cases in the short run, the 4.67% for mid-term and the 0% in the long run. The practical significance of Gibrat's Law validity concerns the fact that its compliance means that, on average, the growth rates of emissions are the same for all countries, independently of the initial volume of those emissions.

This relevant discrepancy between Ahundjanov and Akhundjanov (2019) and our results can be due to the kind of employed statistical methodology. Our case deals with non-parametric methods, while Ahundjanov and Akhundjanov (2019) combine parametric and non-parametric approaches. Additionally, the samples of both papers are not exactly the same.

An easy way to complement what is done until here consists on performing the classical Gibrat regressions ($g = \alpha + \beta \ln(x_{initial})$) for all the possible cases. The estimate of the β parameter is always negative and significant in most cases, in particular, for CO_2pc and for the long-term, as expected. This fact defines a convergent evolution: the initially highest pollutants are those with the lowest growth rates of CO_2 and, so, the differences between the highest and lowest pollutants are attenuated.

4. Conclusions

We have examined the fulfilling of Gibrat's Law claimed by Ahundjanov and Akhundjanov (2019) by analyzing the data from the same source (although with a bit enlarged number of samples for the sake of robustness of the results) but using seven more tests of independence of the log-growth rate variable *g* and the initial log-sizes $\ln(x_{initial})$. The main result, and also the main contribution, is the rejection of the null hypothesis of statistical independence in most cases, so Gibrat's Law does not seem to hold as a whole, contrary to what the previous authors claim. However, CO_2 emissions, especially in absolute terms, do not reject the Law in the short-run, so results are, at least to a certain extent, mixed. It is interesting to note that for CO_2 emissions the fulfillment of Gibrat's Law is more a short term feature, contrary to what happens with cities in the USA, where Gibrat's Law is more a long-term one.

For the cases, in the mid and, mainly, in the long-term, where the Gibrat's Law is not verified, the predominant behavior is a negative correlation (convergent evolution) between the growth rates and the initial size. This negative correlation implies that, seeking for controlling the CO_2 emissions, the behavior of the leader countries in the ranking regarding the highest pollutants of CO_2 seems to have achieved the top value and, consequently, the main effort has to be focused on reducing the growth rate emissions of the lowest pollutant countries.

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