*Energy Science & Engineering*

SHORT COMMUNICATION OPEN ACCESS

# Assessing Computational Complexity in Selecting Periods for LMDI Techniques in Energy-Related Carbon Dioxide Emissions: An Alternative Approach

Juan David Rivera-Niquepa[1,2] | Jose M. Yusta[2] | Paulo M. De Oliveira-De Jesus[1]

[1]Department of Electrical and Electronic Engineering, School of Engineering, Universidad de los Andes, Bogota, Colombia | [2]Department of Electrical Engineering, Universidad de Zaragoza, Zaragoza, Spain

**Correspondence:** Paulo M. De Oliveira-De Jesus (pm.deoliveiradejes@uniandes.edu.co)

**Received:** 2 April 2025 | **Revised:** 29 May 2025 | **Accepted:** 6 June 2025

**Funding:** This study was funded by the Ministerio de Ciencia Tecnología e Innovación, Colombia, Convocatoria 1 de 2018.

**Keywords:** computational time | LMDI | mean square error | periods selection | time series

## ABSTRACT

The Logarithmic Mean Divisia Index (LMDI) decomposition analysis is widely employed to examine the drivers behind changes in carbon dioxide emissions related to energy consumption. This analysis has been applied using single-period, year-by-year, and multi-period time frames worldwide. However, these time frames often overlook trend changes in carbon emission time series, which may lead to inaccurate and biased identification of driving factors. This study replicates previous findings and proposes a novel multi-period methodology for defining time frames in decomposition analysis. The proposed approach addresses the limitations of traditional methods by accounting for trend changes in the time series and performing an exhaustive search to optimally identify the most suitable time frames for LMDI-based decomposition. The methodology comprises two stages: the first generates an exhaustive list of possible time series partitions, and the second determines the optimal partition by minimizing the total mean square error (TMSE) using sequential linear models. The results, supported by computational performance tests, demonstrate that the proposed method effectively identifies optimal time frame definitions, making it particularly suitable for annualized case studies on carbon dioxide emissions decomposition in the context of the energy transition.

## 1 | Introduction

The purpose of decomposition analysis is to identify the factors driving changes in a variable over a given time frame. This technique has been applied in fields such as economics, engineering, energy, and the environment, primarily to identify the drivers of changes in carbon dioxide emissions [1]. Various methods for decomposition analysis have been reported. According to [1], the Laspeyres Parametric Partitioning Method (LASP-PDM), Paasche Parametric Partitioning Method (PAA-PDM), Arithmetic Mean Divisia Method (AMDI), and the Logarithmic Mean Divisia Index (LMDI) are particularly noteworthy.

The LMDI method, the most widely used, was developed and presented by Ang in [2] and was updated in [3]. Two variations of the method, known as LMDI-I and LMDI-II, emerged over time. However, due to its properties of consistency in aggregation and perfect decomposition without residues, the LMDI-I formulation is the most popular in the literature [3]. Detailed state-of-the-art review studies can be found in [4, 5]. A comprehensive review of the classification of these mathematical models, specifically in the application of decomposition analysis to carbon dioxide emission studies, is presented in [6, 7].

Regarding the time frame for decomposition analysis, which must be defined before implementing the technique, three approaches have been identified in the literature. The first is the year-by-year (yy) analysis, which considers a general time range and performs cumulative decomposition for all years within that range. Although exhaustive in terms of the time period, it does not facilitate deep and disaggregated analysis, making it suitable only for studies from a very broad perspective. The second approach is the single-period (sp) time frame, which considers only the initial and final years of the time period. This approach allows for disaggregated analysis in the decomposition but does not account for interannual variations within the analyzed period, potentially leading to the loss of relevant information. Finally, the multi-period (mp) approach involves dividing the time series into constant sub-periods, usually 4 or 5 years each. This approach provides fewer periods for analysis, enabling disaggregated analysis while capturing some of the interannual information within the time series [8].

Some recent case studies on carbon dioxide emissions using different time frame approaches are highlighted in the literature. Country-level studies with sp time frames are reported in [9–11]. These case studies correspond to China and the Netherlands, respectively, and focus on the energy, manufacturing, and power sectors. Other continental-level studies using the sp approach in the power sector are presented in [12] for Latin America, [13] for Asia, and [14] for the European Union. Global-level studies for the energy and power sectors are found in [15–17], all using the sp time frame.

The yy time frame approach has been applied mainly to country-level case studies. Some recent studies in the energy sector include China [18–21], Spain [22, 23], Saudi Arabia [24], Colombia [25], Cameroon [26], the United States [27], Iran [28], and Tunisia [29]. In the power sector, notable studies with a year-to-year focus include those on Ghana [30], China [31], New York [32], the United States [33], and the Philippines [34]. In the transportation sector, applications include studies on China [35, 36] and India [37].

Finally, with the mp approach, studies have been reported at both the continental and country levels. At the continental level, notable studies in the energy sector include those on the global scale [38], Organization for Economic Cooperation and Development (OECD) countries [39], the European Union [40, 41], and Africa [42]. In the power sector, studies with a mp time frame have been reported for Latin America [43], the European Union [44], and OECD countries [45]. Country-level studies with mp decomposition in the energy sector include China [46–49] and Malaysia [50]. In the power sector, studies with the mp approach include Colombia [51] and China [52]. A case study on the iron and steel industry sector in Mexico is reported in [53].

The literature review indicates that different time frame approaches are used for implementing decomposition analysis. The lack of a defined specific time frame introduces an element of arbitrariness, as it depends on the criteria of the authors of each study. To eliminate this arbitrary factor [8], a methodology was presented for defining a time frame (mp) before decomposition analysis. This methodology, based on the minimization of the total mean square error (TMSE), was designed to identify decomposition periods before applying the LMDI decomposition technique. It was executed on both an illustrative case and an aggregated case of carbon dioxide emissions related to energy consumption in OECD countries in Europe. The methodology resulted in the identification of a time series partition that accounts for changes in the trend of carbon dioxide emissions. However, no rigorous validation of the proposed methodology was performed, nor was a performance test implemented to establish its scope and limitations. The methodology presented in [8] was subsequently implemented and extended in case studies for Spain [54] and Portugal [55]. In the case of Spain, energy-related carbon dioxide emissions were evaluated, and the analysis was extended to a disaggregated approach considering seven sectors of the economy. In the case of Portugal, energy-related carbon dioxide emissions were analyzed again, incorporating both decomposition and decoupling approaches. These studies conducted a comparison between the mp, trend-based time frame definition approach and the fixed time frame definition approach. All of them concur that accounting for trend changes in the time series is essential for the accurate identification of the drivers behind changes in energy-related carbon dioxide emissions.

According to [8, 54, 55], the implementation of decomposition analysis using the LMDI method must incorporate a rigorous prior definition of the time frame. Considering trend changes contributes to a less biased and more accurate identification of the underlying drivers. Although these studies introduced and extended an improved approach to defining time frames for decomposition analysis, there remains a lack of validation and performance testing in previous methodologies. Validation is essential to ensure that the selected time frame represents the optimal partition of the time series with respect to trend changes. This can be achieved through a rigorous evaluation of the TMSE between the input data and the sequential linear models. Furthermore, performance testing is necessary to assess the applicability and limitations of the proposed approach. Given that the identification of time frames involves combinations of time series partitions, it is important to determine the data length and resolution that the methodology can effectively handle.

*To address this research gap, this paper proposes a comprehensive methodology for identifying the optimal time frame in the context of decomposition analysis*. The methodology is developed within the mp time frame approach, taking into account significant trend changes in the analyzed time series. It consists of two main stages: the first involves generating an exhaustive list of possible time series partition combinations, and the second applies a TMSE minimization procedure to identify the global minimum, thereby determining the optimal partition. In addition, a rigorous computational performance test is conducted to evaluate the scope and limitations of the proposed approach.

The remainder of this document is organized as follows: Section 2 presents and describes the development of the proposed methodology. Section 3 shows the results of

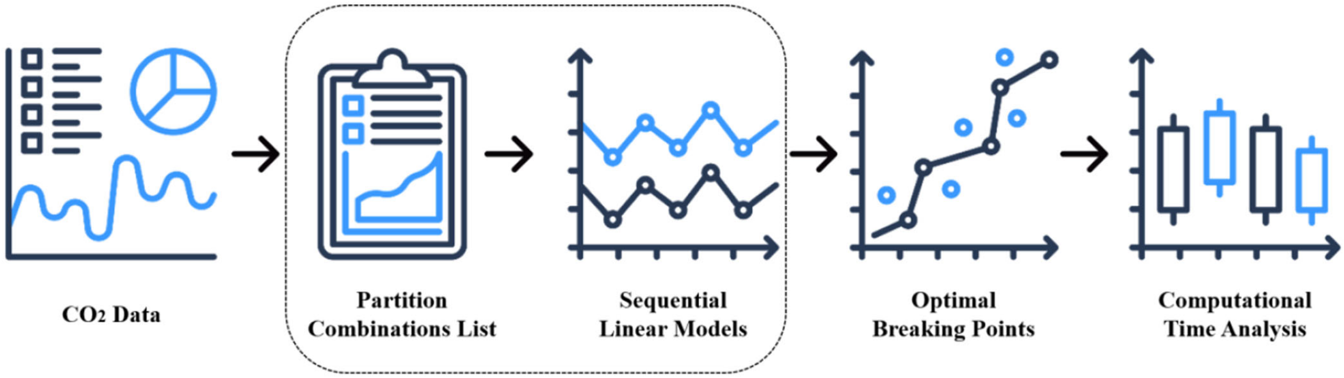


**FIGURE 1** | Methodological framework.

the methodology and compares them with previously reported methods. Additionally, results from four different application cases are reported. Section 4 discusses the results, and Section 5 presents the conclusions and outlines future work.

## 2 | Methodology

To identify the time frame using the mp approach before the application of the LMDI technique, we propose an exhaustive two-stage search methodology. Figure 1 illustrates the proposed methodology. It begins with the input data, which consists of the time series of carbon dioxide emissions. Based on this information, the two-stage method is executed. The first stage involves generating all possible combinations of partitions of a time series. In the second stage, linear models are sequentially obtained for each partition combination. For each partition, the TMSE is calculated, and the partition with the lowest TMSE is identified. This partition corresponds to the combination of linear models that best fits the temporal behavior of the time series. The boundaries of these partitions provide the trend change points that define the decomposition periods for the LMDI technique analysis (optimal breaking points). To complement the analysis, a performance test is conducted to evaluate the computational time required by the proposed two-stage algorithm, depending on the characteristics of the input time series.

A more detailed explanation of the proposed two-stage approach is presented below.

### 2.1 | Stage 1: Partition Combinations List Generation

The first stage of the methodology is summarized in the exhaustive combination generation function, as presented in Algorithm 1.

Given a time series $y_t = (y_1, y_2, ..., y_N)$, a minimum number of partitions $\min_p$, and a maximum number of partitions $\max_p$, the sum of the partition extents must correspond to the target $T_N = \text{length}(y_t)$, and the numbers to be combined will be $N_s =$ range ($\min_p$, $T_N$). These are the inputs to the combination generation algorithm.

As shown in Algorithm 1, the main function (GEN_COMBINATIONS) uses input parameters that include the length of the time series, the range of partition lengths to be combined, and constraints on the minimum and maximum number of partitions. The secondary function (EXTEND_SEQUENCES) takes a list of sequences and attempts to append elements within the specified range of partition lengths until the total length of the time series is reached. The algorithm considers a sequence of partitions valid if the sum of the partition lengths equals the total length of the time series and excludes any sequence that contains partitions shorter than the specified minimum, including the final partition. Once a valid partition is generated, the algorithm continues extending the sequence to obtain all possible combinations of partitions within the time series. The output of this stage is an exhaustive list ($V_{sqs}$) of all valid partition combinations that could define potential time frames for decomposition analysis.

**ALGORITHM 1** | Exhaustive combination generation function.

**Input:** $y_t$, $\min_p$, $\max_p$, $N_s$, $T_N$
**Output:** $V_{sqs}$
**function** GEN_COMBINATIONS ($N_s$, $T_N$, $\min_p$, $\max_p$)
    **function** EXTEND_SEQUENCES ($S_{qs}$, $N_s$, $T_N$)
        Initialize $n_{sqs}$
        **for** each $S_q$ in $S_{qs}$ **do**
            **for** each $n$ in $N_s$ **do**
                $n_{sq} = S_q +$ [n]
                **if** sum ($n_{sq}$) $\le T_N$ **then**
                    Append $n_{sq}$ to $n_{sqs}$
                **end if**
            **end for**
        **end for**
        **return** $n_{sqs}$
    **end function**

(Continues)

Initialize $S_{qs}$ with [[n] for $n$ in $N_s$ where $n \le T_N$]
Initialize $V_{sqs}$ with $S_{qs}$ where:
  sum (seq) $= T_N$
  len (seq) $\ge \min_p$
  seq[-1] $\neq \min_p - 1$
**for** each iteration from 1 to $(\max_p - 1)$ **do**
  $S_{qs} =$ EXTEND_SEQUENCES$(S_{qs}, N_s, T_N)$
  Append sequences to $V_{sqs}$ where:
    sum (seq) $= T_N$
    len (seq) $\ge \min_p$
    seq[-1] $\neq \min_p - 1$
**end for**
**return** $V_{sqs}$
**end function**

## 2.2 | Stage 2: Sequential Linear Models and TMSE Calculation

After obtaining the list $V_{\text{sqs}}$ of exhaustive combinations of partitions, we proceed to the second stage of the methodology, which is summarized in Algorithm 2. This stage involves the sequential calculation of linear regressions and TMSE. The algorithm is summarized below.

The inputs to Algorithm 2 are the list of $V_{\text{sqs}}$ partitions, the time series $y_t$, and the time scale $x_t$. The algorithm takes each sequence of $V_{\text{sqs}}$ partitions, obtains the linear models, and calculates the mean square error (MSE) for each partition. After evaluating each partition, the TMSE for the sequence is computed and stored. This process is repeated until all partition sequences are exhaustively evaluated. At the end, the minimum TMSE is identified, and the best partition in the $V_{\text{sqs}}$ list is selected. This partition corresponds to the best fit of the time series and provides the breakpoints indicating trend changes in the time series (optimal breaking points). These results constitute the optimal time frame definition. Using the identified breakpoints and sub-periods, it is possible to perform an improved decomposition analysis based on the LMDI method. However, this paper focuses solely on the optimal definition of the time frame and its corresponding validation; the decomposition analysis itself is beyond the scope of this study.

To limit the generation of combinations in Algorithm 1, a maximum number of partitions, as given by Equation (1), is established.

The calculation of the TMSE for a sequence of partitions in Algorithm 2 is illustrated in Equation (2).

$$\max_p = \frac{1}{4}\left(2T_N + (-1)^{(T_N+1)} - 3\right) \tag{1}$$

**ALGORITHM 2 |** Sequential linear regression and TMSE calculation.

**Input:** $V_{sqs}$, $x_t$, $y_t$
**Output:** $TMSE$
Initialize $TMSE$
**for** each $V_{sq}$ in $V_{sqs}$ **do**
  Initialize $MSE_{sq}$ as 0
  Initialize $i_s$ as 0
  **for** each index $i$ and size $\Delta_i$ in $V_{sq}$ **do**
    **if** $\Delta_i = 0$ **then**
      **continue**
    **end if**
    **if** $i <$ len$(V_{sq})$-1 and $V_{sq}$[i+1] = 0 **then**
      $Sub_y = y[i_s\!: i_s + \Delta_i]$
      $Sub_x = x[i_s\!: i_s + \Delta_i]$
    **else**
      $Sub_y = y[i_s\!: i_s + \Delta_i + 1]$
      $Sub_x = x[i_s\!: i_s + \Delta_i + 1]$
    **end if**
    Initialize lin_mod as a linear regression model
    Fit lin_mod with $Sub_x$ and $Sub_y$
    $\text{Pred}_y =$ lin_mod.predict$(Sub_x)$
    Compute mse = mean_squared_error $(Sub_y, Pred_y)$
  Add mse to $MSE_{sq}$
  Update $i_s = i_s + \Delta_i$
  **end for**
  Append $MSE_{sq}$ to $TMSE$
**end for**
$\min_e =$ argmin $(TMSE)$
$V_{\text{sqs}}\ [min_e]$

$$\text{TMSE} = \sum \text{MSE} = \frac{1}{n}\sum_{t=1}^{n}(\hat{y}_t - y_t)^2 \tag{2}$$

## 2.3 | Data Source and Assumptions

To assess the effectiveness of the proposed methodology, four case studies were analyzed. The first two are from [8]: the first being an illustrative case, and the second examining aggregated carbon dioxide emissions for OECD countries in Europe during the period 2000–2019, as reported in [56]. Additionally, the energy-related carbon dioxide emissions for Spain [54] and Portugal [55] during the period 1995–2020 were evaluated. The carbon dioxide emissions data were obtained from the International Energy Agency (IEA) database, accessible at https://www.iea.org/data-and-statistics.

## 3 | Results

The proposed methodology was applied to four case studies, and the results are summarized in Figure 2. For each case, the behavior of the TMSE is depicted for all exhaustively evaluated combinations, with the partition corresponding to the lowest TMSE specifically highlighted. This partition represents the selection of periods that capture significant trend changes in the time series. Thus, an appropriate time frame is established for conducting the decomposition analysis using the mp approach.

Additional details regarding the solution for each case study are provided in Table 1.

Table 1 presents the number of combinations evaluated for each case study, along with the properties of the optimal partitioning solution for the time series. These properties include the minimum TMSE, the number and extent of the identified sub-periods, the years of trend changes within the analyzed period, and the computational time required for the exhaustive evaluation of combinations. The results were obtained using a personal computer equipped with an Intel Core i7-5500U processor, 2.9 GHz CPU, and 8.00 GB of RAM.

The results presented in Figure 2 are consistent with those reported in previous studies. The proposed approach successfully identified the same time frames reported in [8] for both the illustrative case and the OECD-Europe case. The four and five respective sub-periods, along with their optimal breakpoints, were accurately detected. Furthermore, the proposed approach is capable of providing the behavior of the TMSE across all possible partitions of the time series. This serves as a validation

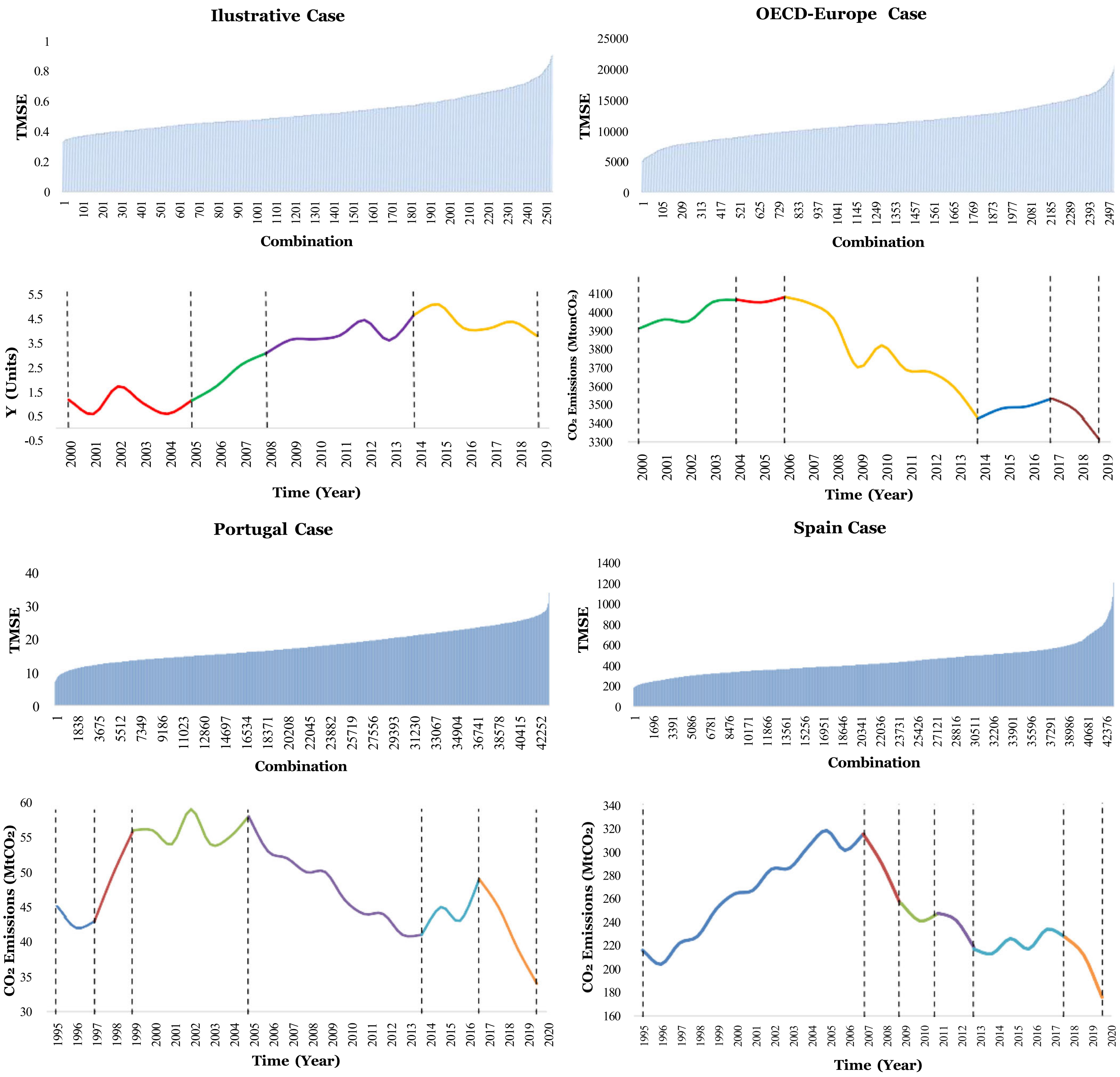


**FIGURE 2** | TMSE behavior and time series partition results.

**TABLE 1** | Characteristics of the solutions in the case studies.

| Case | Span | Combinations | Min (TMSE) | Periods | Breaking points | CPU time (s) |
|---|---|---|---|---|---|---|
| Illustrative | 2000–2019 | 2583 | 0.32902 | 5, 3, 6, 6 | 00-05-08-14-19 | 7.0176 |
| OECD-Europe | 2000–2019 | 2583 | 4950.67 Mt $CO_2$ | 4, 2, 8, 3, 3 | 00-04-06-14-17-19 | 7.052 |
| Portugal | 1995–2020 | 44067 | 6.381 Mt $CO_2$ | 2, 2, 6, 9, 3, 4 | 95-97-99-05-14-17-20 | 147.667 |
| Spain | 1995–2020 | 44067 | 153.70 Mt $CO_2$ | 12, 2, 3, 2, 4, 3 | 95-07-09-12-14-18-20 | 152.367 |

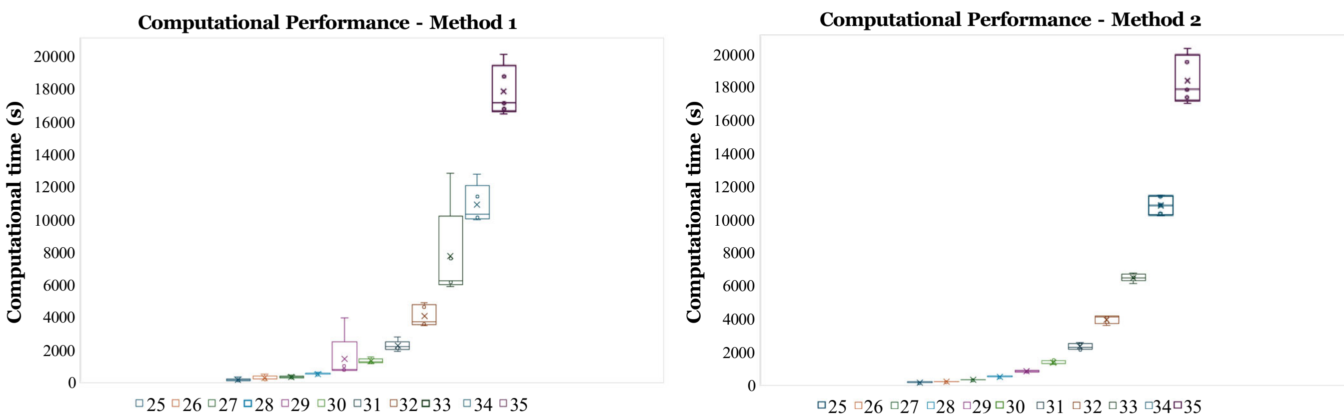


**FIGURE 3** | Computational complexity test results: Method 1-proposed approach [8] and Method 2-proposed approach.

mechanism, effectively confirming that the identified time frame corresponds to the optimal partition of the time series based on the most significant trend changes. Similarly, as shown in Figure 2, the results for Spain and Portugal are fully validated under the proposed approach, as the identified time frames again coincide with those reported in [54, 55], respectively. Another key aspect illustrated in Figure 2 is the comparison between the maximum and minimum levels of the TMSE. For example, the smallest difference was observed in the illustrative case, where the TMSE corresponding to the optimal partition was approximately half of the maximum TMSE obtained. In contrast, the largest difference was found in the Portugal case, where the TMSE at the optimal partition was approximately five times lower than the maximum TMSE. These findings provide strong evidence that the partitioning of a time series should not be determined using arbitrary criteria.

As shown in Table 1, other notable aspect observed in the results is the increase in the number of combinations evaluated and, consequently, the computational time required. This is particularly evident in the cases of Portugal and Spain, where the longer time series necessitated greater computational complexity. To assess the increase in computational complexity of the period selection methodology, time series of varying lengths were tested. Figure 3 presents the results for the method described in [8] (Method 1) and the approach proposed in this study (Method 2).

Figure 3 presents box plots illustrating the dispersion of computational time for time series ranging from 25 to 35 years, comparing the methodology reported in [8] with the approach introduced in this study. The results indicate that both methodologies exhibit similar overall computational performance. However, the proposed methodology demonstrates greater stability, with lower dispersion across different input time series. In terms of computational cost, the results show that CPU time increases significantly as the length of the time series grows. For time series longer than 35 years, more than 6 h of computational time is required to identify the global optimum of the time series partition.

## 4 | Discussion

The proposed methodology demonstrated the capability to identify the optimal partitioning of a time series by accounting for the most significant trend changes. Furthermore, the exhaustive approach ensures that the identified solution represents the global optimum among all possible partition combinations for the given input parameters.

The results presented in Figure 2 are consistent with those reported in previous studies. In all four cases evaluated, the number of sub-periods and corresponding breakpoints were accurately identified. This confirms that the proposed algorithm is suitable for establishing an optimal time frame for decomposition analysis using the LMDI method. An additional contribution of this work is the complete validation of the identified solutions. This was achieved through the calculation, presentation, and evaluation of the TMSE across the exhaustive set of time series partition combinations. The identification of the minimum TMSE demonstrates that the selected time frame effectively captures the most significant trend changes in the time series.

The results presented in Figure 2 and Table 1 clearly confirm that the selection of the time frame is a critical component of decomposition analysis and should not be based on arbitrary criteria. In none of the cases evaluated was the optimal time frame composed of fixed periods of 4 or 5 years, as is commonly assumed in many studies that apply the mp approach. The integration of the proposed methodology with mp LMDI decomposition analysis provides a more robust framework for identifying the drivers of carbon dioxide emissions. Defining the time frame before conducting the decomposition offers several advantages over widely used methods such as cumulative yy decomposition, sp analysis, and mp analysis with fixed sub-periods (mp). As noted in [8], this approach can yield more accurate and less biased estimates of the underlying drivers of emissions changes. Additional benefits include reduced data complexity, the capacity to account for year-to-year trend variations, and improved feasibility for conducting more detailed disaggregated analyses.

The results presented in this study represent a significant contribution to the field of carbon emissions research based on LMDI decomposition. One of the main implications is that many previously published studies—particularly those employing mp time frames with fixed sub-periods—could be reconsidered using the approach proposed here. This has the potential to generate new conclusions and valuable insights into the underlying drivers of carbon dioxide emissions.

The results presented in Table 1 and Figure 3 indicate that the proposed comprehensive approach becomes substantially more complex as the length of the time series increases. The number of partition combinations grows significantly with the addition of only a few years to the input data. As a result, for time series with very high temporal resolution, computational times may become prohibitive. Nevertheless, the methodology remains highly suitable for application in a wide range of studies involving time series with annual resolution. This includes decomposition analyses of the drivers of changes in carbon dioxide emissions, which are extensively addressed in the existing literature.

## 5 | Conclusions

This paper addressed the challenge of computational complexity in identifying an appropriate time frame for conducting decomposition analysis using the LMDI technique. A novel exhaustive methodology was introduced that accounts for significant trend changes and identifies the optimal partitioning of a carbon dioxide emissions time series. The proposed approach consists of two stages: the first involves the exhaustive generation of combinations of partitions, whereas the second evaluates these combinations using sequential linear models and calculates the TMSE. This methodology effectively identifies the optimal time frame based on relevant trend changes in the time series, thereby enabling a more accurate identification of the drivers behind changes in carbon dioxide emissions. The application of this approach may contribute to a better understanding of the global energy transition. Furthermore, the unbiased and precise identification of these drivers can assist decision-makers in the design of public policies that benefit both the environment and society.

The proposed methodology was applied to four case studies: an illustrative example and three applications focusing on energy-related carbon dioxide emissions in OECD-Europe, Portugal, and Spain. The results demonstrated that the approach successfully identifies trend changes within a time series and generates a partition that specifies the number of sub-periods, their durations, and the breakpoints. This partitioning yields an mp time frame without fixed sub-periods. The time frame defined by this methodology addresses and overcomes limitations found in existing approaches reported in the literature. This represents a significant contribution to the state of the art in the fields of carbon emissions analysis and energy transition. The findings reported in this study may encourage other researchers to revisit their results and inspire the continued development of data-driven techniques to inform policymakers, researchers, and the academic community on issues of sustainability and energy transition. The computational complexity of the proposed methodology was also assessed. Due to its exhaustive nature, computational time increases substantially as longer time series are evaluated. Although the approach may become computationally prohibitive for very extensive time series, it remains feasible for annualized data spanning up to 50 years. This renders the methodology applicable to a wide range of case studies, including decomposition analyses of the drivers behind changes in carbon dioxide emissions—an issue of critical importance in the context of climate change mitigation. This opens new avenues for research across various disciplines related to climate change, energy, economics, and environmental studies.

For future work, it is recommended to apply the proposed approach, in conjunction with LMDI decomposition analysis, to additional case studies and to compare the results with those obtained from existing methods. Furthermore, there is a need to develop alternative methodologies for defining time frames in LMDI analyses that reduce computational burden while maintaining efficiency in identifying decomposition periods. Another promising application of the proposed methodology is its use with higher temporal resolution data, such as in evaluating the drivers behind changes in hourly carbon intensity in the power sector.

**Acknowledgments**

The authors would like to thank the Vice Presidency of Research & Creation's Publication Fund at Universidad de los Andes for its financial support.

**References**

1. P. F. González, M. Landajo, and M. J. Presno, *The Driving Forces of Change in Environmental Indicators: An Analysis Based on Divisia Index Decomposition Techniques* (Springer, 2014).

2. B. Ang, F. Zhang, and K. Choi, "Factorizing Changes in Energy and Environmental Indicators Through Decomposition," *Energy* 23, no. 6 (1998): 489–495.

3. B. W. Ang, "LMDI Decomposition Approach: A Guide for Implementation," *Energy Policy* 86 (2015): 233–238.

4. B. W. Ang and F. Q. Zhang, "A Survey of Index Decomposition Analysis in Energy and Environmental Studies," *Energy* 25, no. 12 (2000): 1149–1176.

5. X. Y. Xu and B. W. Ang, "Index Decomposition Analysis Applied to $CO_2$ Emission Studies," *Ecological Economics* 93 (2013): 313–329.

6. T. Goh, B. W. Ang, and X. Y. Xu, "Quantifying Drivers of $CO_2$ Emissions From Electricity Generation–Current Practices and Future Extensions," *Applied Energy* 231 (2018): 1191–1204.

7. L. Ma, C. Chong, X. Zhang, et al., "LMDI Decomposition of Energy-Related $CO_2$ Emissions Based on Energy and $CO_2$ Allocation Sankey Diagrams: The Method and an Application to China," *Sustainability* 10, no. 2 (2018): 344.

8. J. D. Rivera-Niquepa, D. Rojas-Lozano, P. M. De Oliveira-De Jesus, and J. M. Yusta, "Methodology for Selecting Assessment Periods of Logarithmic Mean Divisia Index Decomposition Techniques," *Energy Strategy Reviews* 50 (2023): 101241.

9. W. Li, Z. Ji, and F. Dong, "Spatio-Temporal Analysis of Decoupling and Spatial Clustering Decomposition of $CO_2$ Emissions in 335 Chinese Cities," *Sustainable Cities and Society* 86 (2022): 104156.

10. Y. An, D. Zhou, and Q. Wang, "Carbon Emission Reduction Potential and Its Influencing Factors in China's Coal-Fired Power Industry: A Cost Optimization and Decomposition Analysis," *Environment, Development and Sustainability* 24, no. 3 (2022): 3619–3639.

11. A. Faridzad and M. G. Ghadim, "$CO_2$ Intensity Decomposition Analysis in the Netherlands' Manufacturing Industry: An Application of Monetary and Physical Indicators," *Environment, Development and Sustainability* 25, no. 8 (2023): 8799–8817.

12. P. M. De Oliveira-De Jesus, "Effect of Generation Capacity Factors on Carbon Emission Intensity of Electricity of Latin America & the Caribbean, a Temporal IDA-LMDI Analysis," *Renewable and Sustainable Energy Reviews* 101 (2019): 516–526.

13. B. W. Ang and T. Goh, "Carbon Intensity of Electricity in ASEAN: Drivers, Performance and Outlook," *Energy Policy* 98 (2016): 170–179.

14. P. Fernández González, M. Landajo, and M. J. Presno, "Tracking European Union $CO_2$ Emissions Through LMDI (Logarithmic-Mean Divisia Index) Decomposition. The Activity Revaluation Approach," *Energy* 73 (2014): 741–750.

15. P. Sadorsky, "Energy Related $CO_2$ Emissions Before and After the Financial Crisis," *Sustainability* 12, no. 9 (2020): 3867.

16. L. Mundaca and A. Markandya, "Assessing Regional Progress Towards a 'Green Energy Economy'," *Applied Energy* 179 (2016): 1372–1394.

17. B. W. Ang and B. Su, "Carbon Emission Intensity in Electricity Production: A Global Analysis," *Energy Policy* 94 (2016): 56–63.

18. F. Dong, J. Li, J. Huang, et al., "A Reverse Distribution Between Synergistic Effect and Economic Development: An Analysis From Industrial $SO_2$ Decoupling and $CO_2$ Decoupling," *Environmental Impact Assessment Review* 99 (2023): 107037.

19. T. Shen, R. Hu, P. Hu, and Z. Tao, "Decoupling Between Economic Growth and Carbon Emissions: Based on Four Major Regions in China," *International Journal of Environmental Research and Public Health* 20, no. 2 (2023): 1496.

20. W. Chen, Q. Zhang, Z. Gao, Y. Geng, Y. Cheng, and X. Tian, "Exploring the Drivers of Energy-Related $CO_2$ Emissions in Western China: A Case Study of Haixi," *Environment, Development and Sustainability* 25, no. 10 (2023): 11957–11971.

21. X. Wang and L. Yan, "Driving Factors and Decoupling Analysis of Fossil Fuel Related-Carbon Dioxide Emissions in China," *Fuel* 314 (2022): 122869.

22. P. F. González, M. J. Presno, and M. Landajo, "Tracking the Change in Spanish Greenhouse Gas Emissions Through an LMDI Decomposition Model: A Global and Sectoral Approach," *Journal of Environmental Sciences* 139 (2024): 114–122.

23. J. M. Cansino, A. Sánchez-Braza, and M. L. Rodríguez-Arévalo, "Driving Forces of Spain's $CO_2$ Emissions: A LMDI Decomposition Approach," *Renewable and Sustainable Energy Reviews* 48 (2015): 749–759.

24. R. G. Alajmi, "Factors That Impact Greenhouse Gas Emissions in Saudi Arabia: Decomposition Analysis Using LMDI," *Energy Policy* 156 (2021): 112454.

25. L. I. Patiño, V. Alcántara, and E. Padilla, "Driving Forces of $CO_2$ Emissions and Energy Intensity in Colombia," *Energy Policy* 151 (2021): 112130.

26. M. A. Hossain, J. Engo, and S. Chen, "The Main Factors Behind Cameroon's $CO_2$ Emissions Before, During and After the Economic Crisis of the 1980s," *Environment, Development and Sustainability* 23 (2021): 4500–4520.

27. Q. Wang and S. Wang, "Decoupling Economic Growth From Carbon Emissions Growth in the United States: The Role of Research and Development," *Journal of Cleaner Production* 234 (2019): 702–713.

28. B. Mousavi, N. S. A. Lopez, J. B. M. Biona, A. S. F. Chiu, and M. Blesl, "Driving Forces of Iran's $CO_2$ Emissions From Energy Consumption: An LMDI Decomposition Approach," *Applied Energy* 206 (2017): 804–814.

29. H. Achour and M. Belloumi, "Decomposing the Influencing Factors of Energy Consumption in Tunisian Transportation Sector Using the LMDI Method," *Transport Policy* 52 (2016): 64–71.

30. E. F. Oteng-Abayie, F. A. Asaki, E. Duodu, S. Mahawiya, and B. A. Gyamfi, "Decomposition Analysis of Electricity Generation on Carbon Dioxide Emissions in Ghana," *Heliyon* 10, no. 7 (2024): e28212.

31. R. Li, Z. Chen, and J. Xiang, "A Region-Scale Decoupling Effort Analysis of Carbon Dioxide Emissions From the Perspective of Electric Power Industry: A Case Study of China," *Environment, Development and Sustainability* 25, no. 5 (2023): 4007–4032.

32. M. Isik and P. O. Kaplan, "Understanding Technology, Fuel, Market and Policy Drivers for New York State's Power Sector Transformation," *Sustainability* 13, no. 1 (2020): 265.

33. X. T. Jiang and R. Li, "Decoupling and Decomposition Analysis of Carbon Emissions From Electric Output in the United States," *Sustainability* 9, no. 6 (2017): 886.

34. A. K. Sumabat, N. S. Lopez, K. D. Yu, et al., "Decomposition Analysis of Philippine $CO_2$ Emissions From Fuel Combustion and Electricity Generation," *Applied Energy* 164 (2016): 795–804.

35. H. Ma, J. Liu, and J. Xi, "Decoupling and Decomposition Analysis of Carbon Emissions in Beijing's Tourism Traffic," *Environment, Development and Sustainability* 24 (2022): 5258–5274.

36. M. Liu, X. Zhang, M. Zhang, et al., "Influencing Factors of Carbon Emissions in Transportation Industry Based on CD Function and LMDI Decomposition Model: China as an Example," *Environmental Impact Assessment Review* 90 (2021): 106623.

37. S. Jain and S. Rankavat, "Analysing Driving Factors of India's Transportation Sector $CO_2$ Emissions: Based on LMDI Decomposition Method," *Heliyon* 9, no. 9 (2023): e19871.

38. C. H. Simbi, F. Yao, J. Zhang, et al., "Decoupling for a Greener Future: A Spatio-Temporal Analysis of $CO_2$ Emissions and Economic Growth," *Environmental Science and Pollution Research* 31, no. 46 (2024): 56886–56900.

39. C. Magazzino, P. Pakrooh, and M. Z. Abedin, "A Decomposition and Decoupling Analysis for Carbon Dioxide Emissions: Evidence From OECD Countries," *Environment, Development and Sustainability* 26, no. 11 (2024): 28539–228566.

40. M. Papież, S. Śmiech, and K. Frodyma, "The Role of Energy Policy on the Decoupling Processes in the European Union Countries," *Journal of Cleaner Production* 318 (2021): 128484.

41. V. Moutinho, A. C. Moreira, and P. M. Silva, “The Driving Forces of Change in Energy-Related $CO_2$ Emissions in Eastern, Western, Northern and Southern Europe: The LMDI Approach to Decomposition Analysis,” *Renewable and Sustainable Energy Reviews* 50 (2015): 1485–1499.

42. C. Habimana Simbi, J. Lin, D. Yang, et al., “Decomposition and Decoupling Analysis of Carbon Dioxide Emissions in African Countries During 1984–2014,” *Journal of Environmental Sciences* 102 (2021): 85–98.

43. P. M. De Oliveira-De Jesus, J. J. Galvis, D. Rojas-Lozano, and J. M. Yusta, “Multitemporal LMDI Index Decomposition Analysis to Explain the Changes of ACI by the Power Sector in Latin America and the Caribbean Between 1990–2017,” *Energies* 13, no. 9 (2020): 2328.

44. M. Karmellos, D. Kopidou, and D. Diakoulaki, “A Decomposition Analysis of the Driving Factors of $CO_2$ (Carbon Dioxide) Emissions From the Power Sector in the European Union Countries,” *Energy* 94 (2016): 680–692.

45. H. Kim, M. Kim, H. Kim, and S. Park, “Decomposition Analysis of $CO_2$ Emission From Electricity Generation: Comparison of OECD Countries Before and After the Financial Crisis,” *Energies* 13, no. 14 (2020): 3522.

46. T. Chun, S. Wang, X. Xue, et al., “Decomposition and Decoupling Analysis of Multi-Sector $CO_2$ Emissions Based on LMDI and Tapio Models: Case Study of Henan Province, China,” *Environmental Science and Pollution Research* 30 (2023): 88508–88523.

47. Y. Huang, Y. Wang, J. Peng, et al., “Can China Achieve Its 2030 and 2060 $CO_2$ Commitments? Scenario Analysis Based on the Integration of LEAP Model With LMDI Decomposition,” *Science of the Total Environment* 888 (2023): 164151.

48. Y. Wang, S. Jeong, Y. Hang, and Q. Wang, “Determinants of Net Energy-Related $CO_2$ Emissions in China: A Source-to-Sink Decomposition Analysis,” *Environmental Impact Assessment Review* 98 (2023): 106979.

49. J. Yang, W. Cai, M. Ma, et al., “Driving Forces of China's $CO_2$ Emissions From Energy Consumption Based on Kaya-LMDI Methods,” *Science of the Total Environment* 711 (2020): 134569.

50. C. H. Chong, W. X. Tan, Z. J. Ting, et al., “The Driving Factors of Energy-Related $CO_2$ Emission Growth in Malaysia: The LMDI Decomposition Method Based on Energy Allocation Analysis,” *Renewable and Sustainable Energy Reviews* 115 (2019): 109356.

51. J. D. Rivera-Niquepa, D. Rojas-Lozano, P. M. De Oliveira-De Jesus, and J. M. Yusta, “Decomposition Analysis of the Aggregate Carbon Intensity (ACI) of the Power Sector in Colombia—A Multi-Temporal Analysis,” *Sustainability* 14, no. 20 (2022): 13634.

52. Y. He, Y. Xing, X. Zeng, et al., “Factors Influencing Carbon Emissions From China's Electricity Industry: Analysis Using the Combination of LMDI and K-Means Clustering,” *Environmental Impact Assessment Review* 93 (2022): 106724.

53. C. Sheinbaum, L. Ozawa, and D. Castillo, “Using Logarithmic Mean Divisia Index to Analyze Changes in Energy Use and Carbon Dioxide Emissions in Mexico's Iron and Steel Industry,” *Energy Economics* 32, no. 6 (2010): 1337–1344.

54. J. D. Rivera-Niquepa, J. M. Yusta, and P. M. De Oliveira-De Jesus, “Kaya Factor Decomposition Assessment of Energy-Related Carbon Dioxide Emissions in Spain: A Multi-Period and Multi-Sector Approach,” *Sustainable Energy Technologies and Assessments* 74 (2025): 104156.

55. J. D. Rivera-Niquepa, P. M. De Oliveira-De Jesus, and J. M. Yusta, “Trend-Based Multi-Period Decomposition and Decoupling Methodology for Energy-Related Carbon Dioxide Emissions: A Case Study of Portugal,” *Utilities Policy* 93 (2025): 101863.

56. I. E. Agency, *$CO_2$ Emissions From Fuel Combustion 2019* (International Energy Agency, 2019).