Toy metallophone: harmony or beauty? An opportunity to learn physics

Francisco José Torcal Milla

Applied Physics Department, Instituto de Investigación en Ingeniería de Aragón (i3A), Universidad de Zaragoza, Zaragoza (Spain)

Email: fjtorcal@unizar.es

Music and physics are two areas that are closely related[1]. Many works have been published regarding this relationship and using it to teach physics concepts[2] or find out the parameters of the instrument or the propagation medium[3]. It is a common practice to purchase toy musical instruments for children. Typically, though not always, the musical quality of a toy instrument is influenced by its price and intended purpose. Some musical toys exhibit impressive sound quality, even resulting in finely tuned. However, in certain instances, visual aesthetics or overall appeal may outweigh musical harmony. In this manuscript, we conducted an analysis of the sound produced by a toy metallophone, as shown in Fig. 1a. When struck, the green, blue, yellow, and red bars produce distinct tones. Let us begin by measuring the fundamental frequency of each metallic bar. To do it, we use a free android app called *Spectroid*, developed by Carl Reinke[4]. There are also alternative programs for iOS[5]. Fig. 2 shows a snapshot of the android app in which the frequency of the more powerful peak that corresponds to the fundamental frequency of the bar is read. In addition, we have measured the dimensions of the bars since the frequencies and overtones depend on their dimensions and material, Fig. 1b. Table 1 shows the measured frequencies and the lengths of each bar. The width, w, and thickness, t, are the same for all of them: $w = 2.83\ cm$ and $t = 2.16\ mm$.

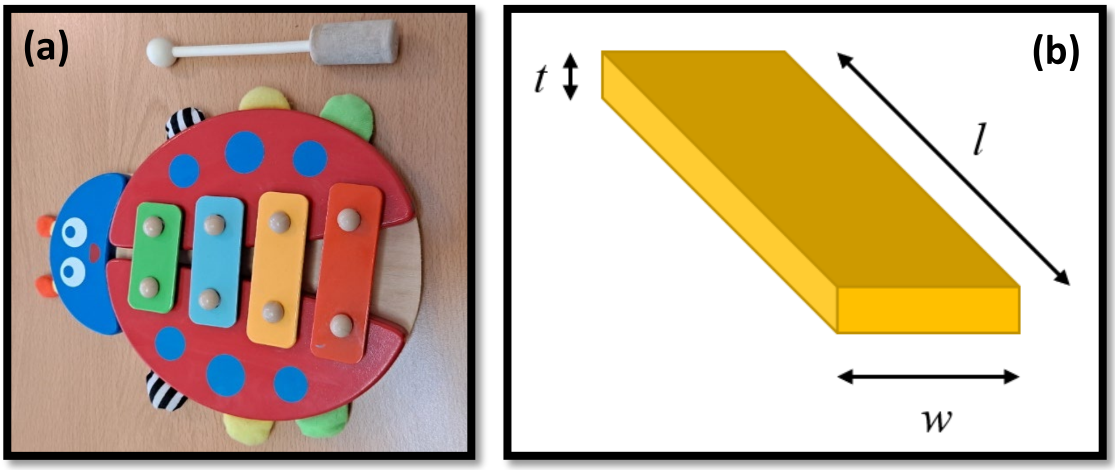

Figure 1.- (a) Toy metallophone, (b) dimensions of the bars.

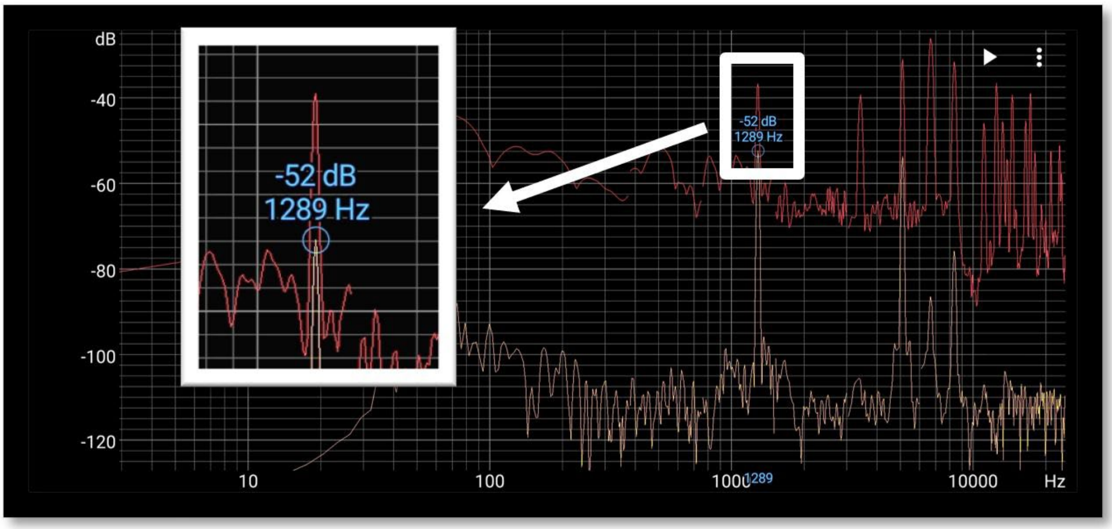

Figure 2.- Snapshot of the Spectroid app for Android when one of the metallic bars is knocked. A zoom of the more powerful peak is shown.

Table 1.- Frequency of the fundamental tone and length of each metallic bar.

Color of the bar	Red (f_R)	Yellow (f_Y)	Blue (f_B)	Green (f_G)
Measured frequency (Hz)	1289	1617	2109	2859
Length, l (cm)	9	8	7	6

The measured lengths suggest that aesthetics has been the goal of the instrument, since the length of the bars increases every centimeter, but let us analyze the fundamental frequencies emitted by each bar. To evaluate the harmony of the instrument, we need to find out the relationship between all fundamental frequencies. Once we have measured them, we may compare their ratios with those of the successive tones of any conventional scale such as the diatonic[6,7] or the equally tempered major scales, Table 2. Let us take the values shown in Table 1 and obtain the ratio between all pairs of frequencies, Table 3. As can be observed, there are some ratios that result near to some relationships shown in Table 2 but we think that they are a mere coincidence. Usually, to construct a harmonic instrument, some degrees or notes of a certain scale should be present and it does not seem the case.

Table 2.- Ratio of frequencies in the diatonic and equally tempered major scales.

Degree of the scale	First	Major second	Major third	Fourth
Frequency ratio (diatonic)	1:1	9:8 = 1.125	5:4 = 1.25	4:3 = 1.333
Frequency ratio (Equally tempered)	$2^{0/12}$	$2^{2/12}$	$2^{4/12}$	$2^{5/12}$

Degree of the scale	Fifth	Major sixth	Major seventh	Octave
Frequency ratio (diatonic)	3:2 = 1.5	5:3 = 1.666	15:8 = 1.875	2:1
Frequency ratio (Equally tempered)	$2^{7/12}$	$2^{9/12}$	$2^{11/12}$	$2^{12/12}$

Table 3.- Ratio of frequencies given by the bars of the toy metallophone.

$f_i : f_j$	$f_G : f_B$	$f_G : f_Y$	$f_G : f_R$
Ratio	1.355	1.768	2.218

Once we have measured the frequencies emitted by the toy metallophone, let us take advantage of the instrument to learn some physics. First of all, let us relate the fundamental mode of vibration emitted by each bar with its physical dimensions and material properties. When the vibrating object is a tensed string or the air inside a pipe, the tones emitted are related to the length of the string and the tension or the length of the pipe and the medium. For example, the harmonic frequencies for a string fixed at both ends, or a pipe with both sides closed or both sides opened, are

$$f_n = \frac{n v}{2 L}, \tag{1}$$

with n integer, v the speed of the wave in the string or medium, and a the length of the string or pipe. In the case of solid rectangular bars, they cannot be considered as one-dimensional[8,9]. The fundamental mode of vibration in this case can be expressed from Eq. (19) of Ref. 8 in the form

$$f = \frac{9\pi}{8 l^2}\sqrt{\frac{Y t^2}{12\rho}}, \tag{2}$$

where Y is the Young's modulus, ρ is the density of the bar, and the other parameters are shown in Fig. 1. By using the measured frequencies, Table 1, and calculating the approximate density from the mass and dimensions of each bar, we may obtain the Young's modulus of the material which the bars are made of. The calculated density results around ρ=7850 kg/m^3. Eq. (2) may be rewritten as

$$f_l = a / l^2, \tag{3}$$

with $a = (9\pi t)\sqrt{Y}\big/\left(8\sqrt{12\rho}\right)$. By substituting all numbers, the slope of Eq. (3) is reduced to $a = 2.486 \cdot 10^{-5}\sqrt{Y}$. Then, from the slope of the linear fitting, Fig. 3, we may obtain the Young's modulus of the material. Besides, as can be observed, the R-square of the fitting is 1, which reveals the validity of Eq. (2). The Young's modulus results $Y = \left(10.188 / 2.486 \cdot 10^{-5}\right)^2 = 1.679 \cdot 10^{11}$ Pa. Looking at the Young's modulus and density of different metals[10], we may conclude that the bars of the toy metallophone could be made of a kind of cast iron or stainless steel.

Finally, taking the diatonic scale ratios and the larger bar length as starting point, the lengths of the other bars should be those shown in Table 4 to produce the mentioned degrees of the scale. They have been calculated by taking the ratios of the diatonic scale and using Eq. (2).

Table 4.- "Correct" lengths of the bars to produce the diatonic scale.

Degree of the scale	First	Major third	Fifth	Major seventh
Frequency (Hz)	1289	1611.25	1933.5	2416.875
Length, l (cm)	9	7.95	7.26	6.49

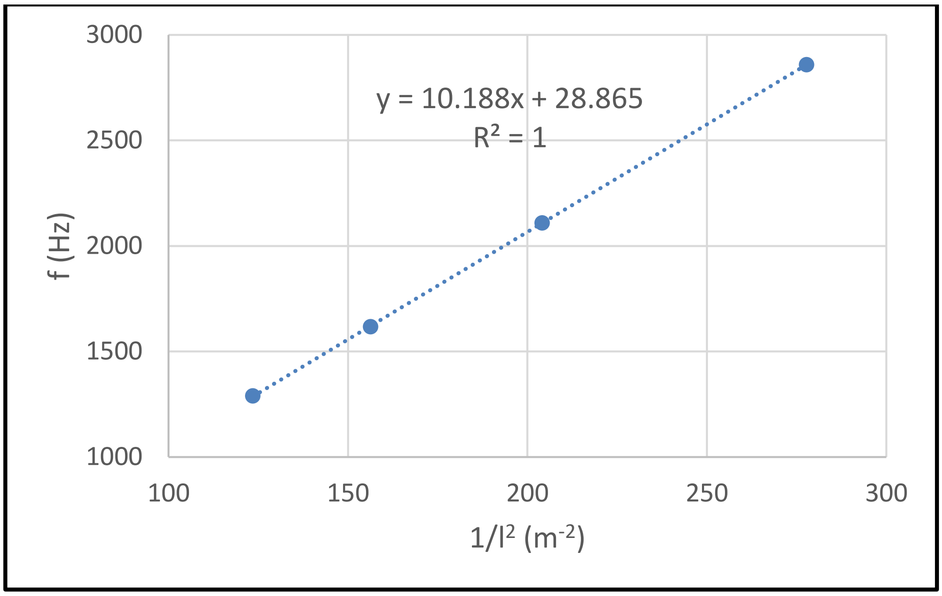

Figure 3.- Measured frequency vs $1/l^2$ for the four bars of the toy metallophone.

In this work, we examine the harmony of a toy metallophone by analyzing the ratios between the frequencies emitted by its metallic bars and comparing them to the ratios provided by the diatonic and equally tempered musical scales. Based on our findings, we reach the conclusion that the primary objective of this toy musical instrument is aesthetic appeal rather than achieving harmonic balance. Despite the absence of harmony, it can still serve as a valuable tool for learning basic principles of physics. By considering the frequencies and dimensions of the metallic bars, we can deduce the material from which they are made, potentially identifying a type of cast iron or stainless steel.

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Francisco Jose Torcal-Milla is an Associate Professor of Physics in the Department of Applied Physics at the University of Zaragoza (Spain). His areas of interest include applied optics and applied physics. He is also passionate about teaching and classroom experimentation.