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Research Paper

Optimal operation and marginal costs in a complex polygeneration system including thermal energy storage and DHCN pipelines☆

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ABSTRACT

This paper proposes a thermoeconomic analysis to determine the hourly marginal costs of the optimal operation of a complex polygeneration system when the energy demand for a specific energy service increases at any time step, without modifying the operation mode of the system. The work analyses a case study which considers a mixed integer linear programming (MILP) model of an energy community (EC), comprising nine tertiary sector buildings and a central unit supplied by natural gas, solar energy, and electricity. The buildings exchange electricity through a local electric grid as well as heating and cooling through a district heating and cooling network (DHCN). The paper focuses on the marginal cost (MC) analysis of the electricity and heating demands regarding two of the EC buildings by evaluating representative time steps of a typical winter day. Additionally, a detailed analysis of the cost formation process for polygeneration heat production is conducted, clarifying the influence of thermal energy storage (TES) and DHCN on the marginal cost of heat production. The proposed marginal cost analysis reveals strategies for managing increased heat or electricity demands with minimal impact on the objective function. While the applied methodology offers robustness and transparency, it should be noted that the model under analysis does not include dynamic inefficiencies such as start-up/shut-down of technologies, and renewable variability is represented through deterministic time series. Thus, the mentioned optimal operation refers to the most cost-effective response to marginal demand changes within fixed operational modes. Obtained results indicate that optimal marginal paths have the potential to reduce operation costs by 26% compared to non-optimal ones.

1. Introduction

In the context of energy systems and thermoeconomic analysis, the marginal costs of internal flows and final products indicate the cost of producing one additional unit of a given energy flow and, as shown in this paper, are essential to reveal the optimal operating strategy to obtain such a production cost. Thermoeconomics represents, essentially, a merge between thermodynamic principles and economic analysis. Its primary objective is to demonstrate opportunities for energy and cost savings in the design, assessment, diagnosis, and optimization of energy conversion systems.

From the late 20th century, several works laid the foundations of thermoeconomics through different approaches and applications. El-Sayed and Evans [1], linked thermodynamics and economics in order to evaluate what they called as "internal economy" of complex systems to guarantee a rational use of non-renewable resources. Tribus and El-Sayed [2] used a similar methodology to highlight economic impacts of design decisions, later extended by Frangopoulos [3], but with an optimization approach called "thermo-economic functional analysis". Still in the period before the 1990's, Tsatsaronis and Winhold [4] combined exergetic and economic analyses to assign costs to exergy losses, identifying inefficiencies in energy-conversion plants. Gaggioli [5] also contributed with valuable insights into thermoeconomic analysis of energy systems and process plants. In the early 1990's, two major contributions shaped modern thermoeconomics. Lozano and Valero [6] developed the theory of exergetic cost, enabling systematic analysis and optimization of energy systems by coupling exergy with systems theory. Shortly after, Tsatsaronis [7] introduced exergoeconomics as an integrated framework combining exergy with traditional engineering economics to improve design and performance of energy systems.

As stressed by Lozano et al. [8], many thermoeconomic

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Nomenclature

TES	Thermal Energy Storage
EC	Energy Community
CU	Central unit
DHCN	District Heating and Cooling Network
DHN	District Heating Network
DCN	District Cooling Network
MILP	Mixed-Integer Linear Programming
DS	Distribution Substation
BOI	Boiler
ICE	Internal Combustion Engine
HST	Hot water Storage
ST	Solar Thermal panels
PV	Photovoltaic Panels
ABS	Absorption chiller
CC	Compression Chiller
HP	Heat Pump
CST	Chilled water Storage
MGT	Micro Gas Turbine
MC	Marginal Cost
LP	Linear Programming
COP	Coefficient of Performance
B	Building number (according to Table 1)
c	Installed component in a given building
k	Building number (according to Table 1)
m	Month
d	Day
h	Hour
x	Hour related to delayed or advanced energy production
t	Representing m (month), d (day), and h (hour)
λ	Lagrange multiplier or marginal cost

Subscript

h	Heat
c	Cold
HST_in_out	Heating in (positive) or out (negative) of HST
CST_in_out	Cooling in (positive) or out (negative) of HST
Dem	Demand
cen.unit	Central unit
waste	Waste energy
ICEc	Internal Combustion Engine in the central unit
BOIc	Boiler in the central unit
STc	Solar Thermal Panels in the central unit
bgt	Bought or purchased
sold	Sold

methodologies, including those previously presented, are based on determining the unit costs of internal flows and final products in energy supply systems. Such unit costs can be of two different types: average costs and marginal costs. They play a crucial role in various analyses, allowing for a comprehensive understanding of the energy system economics [9–11]. For the purpose of clarification, in the present work, unit marginal cost will be denoted only as marginal costs, while unit average costs can be named also as unit costs.

It is crucial in energy system economics the distinction between unit costs and marginal costs [12]. When changing external conditions (such as variations in energy demand), unit costs are generally not able to properly explain the optimal plant operation and system behaviour, since they are only indicative of the average production cost of a given flow (i.e., how much of the resources have been consumed for its production, divided by how much was produced). In contrast, a marginal cost is a derivative regarding the cost of producing one additional unit of a given energy flow.

As emphasized in the literature, marginal costs provide a clear path to understanding and managing cost behaviour throughout an energy supply system [13]. For instance, in the study developed by Lozano et al. [8], they analysed a grid-connected simple trigeneration system under various operational scenarios. Employing a linear programming model, they identified the most cost-efficient operational mode based on a marginal cost analysis. Based on this study, the paper published by Pina et al. [14] also analysed the optimization of simple trigeneration systems, emphasizing the role of thermal energy storage (TES) in improving efficiency by connecting production and consumption phases. Through a thermoeconomic approach, the paper assessed the marginal costs of internal flows and final products, elucidating the system's optimal operation and the pivotal contribution of TES. The analysis delineated the formation of marginal costs, tracing a clear path from final products back to resource consumption.

Several works have advanced both thermoeconomics and the application of marginal cost analysis. In the late 1980′s, Rossiter and Ranade [15] examined techniques to calculate marginal costs of the products of a steam-power plant, identifying potential energy-saving pathways. In the early 2000′s, Hui [16] presented a mathematical model approach for calculating marginal values of intermediate flow materials and utilities, supporting the identification of bottlenecks in energy production systems and providing decision-makers with insights into the true economic impacts. In the same period, Sjodin and Henning [17] compared three methods for calculating marginal costs of district heating in a Swedish utility: a manual spreadsheet method, a linear-programming model, and a least-cost dispatch simulation model. Their results emphasized the benefits of using marginal cost pricing to improve resource allocation and encourage efficient customer behaviour. Expanding mathematical modelling approaches, Quelhas et al. [18] presented a multiperiod network flow model for analysing the economic interdependencies between electric and fuel supply systems. According to the authors, the model offered faster solutions when compared to standard linear programming, using marginal costs to evaluate inter-network dynamics. More recently, Sun et al. [19] and Martinez-Sanchez et al. [20] applied marginal cost analysis to specific types of projects and facilities. The former has addressed the complexity of steam costing for energy conservation projects, showing that simplified methods are inadequate and that optimization-based marginal and cumulative cost profiles better evaluate steam savings. Martinez-Sanchez et al. [20] developed a bottom-up methodology for assessing the economic impacts of solid waste management strategies at existing facilities by calculating marginal costs. A case study on two waste-to-energy plants demonstrated how marginal costs of waste diversion, which can be significantly higher than average costs, depend on plant responses and thermal load adjustments.

Over the past five years, marginal cost analysis has gained prominence in energy system transition research, particularly in the context of decarbonisation, resilience, and multi-energy integration strategies. Cole et al. [21] applied it to evaluate emissions abatement in achieving 100 % renewable electricity in the United States, with costs rising steeply from ~\$170/ton CO_2 to ~\$930/ton CO_2 as decarbonisation range between 90 and 100 %. Binsted et al. [22] applied a marginal cost analysis to investigate the United States economy-wide pathways to net-zero CO_2 emissions by 2050, showing that costs escalate sharply if carbon capture technologies are unavailable. Similarly, Zhang et al. [23] assessed coupled energy sectors (electricity, transport, and industry), defining marginal cost of emissions abatement as the incremental system cost per unit of emissions reduction and identifying cost-effective technologies mixes.

In the context of energy systems with multiple technology options, Terlouw et al. [24] present a multi-objective optimization framework integrating residential, industrial, and mobility sectors, where marginal cost analysis highlights trade-offs between economic and environmental objectives. They later extended the approach to grid-connected and off-grid energy systems in Crete [25], highlighting the economic feasibility of energy autonomy. Zhang et al. [26] introduced marginal cost-based pricing into a decentralized peer-to-peer energy sharing framework, incorporating carbon costs and electricity consumption rights, enabling fair and efficient energy transactions across distributed energy systems.

Resilience-oriented studies also rely on marginal cost analysis. Ren et al. [27] introduce the concept of marginal cost of resilience in district integrated energy systems, showing that costs rise exponentially as resilience levels increases, quantifying trade-offs between economic efficiency and system robustness under extreme natural disasters. McPherson and Stoll [28] analysed the impact of demand response to what they called thermal (e.g., coal and diesel) and renewable (e.g., wind and solar) generators in a region of India. Marginal costs played a central role in their analysis demonstrating that shifting from high-marginal-cost thermal plants to near-zero-marginal-cost renewables reduce system costs. Finally, Sigurjonsson and Clausen [29] applied a particular marginal cost analysis to a flexible polygeneration plant (producing bio synthetic natural gas, electricity, and heating), determining optimal operating modes under varying market conditions, thereby improving system utilization and economic viability.

Although these studies provide valuable insights into the application and analysis of marginal costs in energy supply systems, none evaluate the marginal cost formation process in a systematic way, by optimally connecting the available energy resources to the energy demand. Besides, it is hard to find in literature studies dealing with the optimal operation of complex polygeneration systems with distributed production, different demand profiles, multiple production technologies, district heating and cooling networks, and thermal storage integration from a marginal cost perspective.

Although the topic of joint production in complex energy systems has been extensively addressed in thermoeconomics, important transcendental questions have not been tackled in literature:

Assumption of a single operating state: in most studies, the analysed systems are assumed to operate in a single design or operating state, with costs assigned to internal and final products based on that state – specifically, unit costs or average costs. Although such calculations can (and should) satisfy cost balances at both the equipment and plant levels, in cases of joint production the resulting costs are inherently arbitrary, as they depend on the chosen allocation rules.

Cost Formation Process: cost calculation should primarily help to clarify how costs are formed for internal flows and plant products. This corresponds to the economic-price aspect, the dual of the physical-magnitude aspect of flows. These two aspects are complementary: together they define a productive trajectory that explains how products are obtained through successive transformations. At the same time, by consistently accounting for both internal and external resource use, they also elucidate how product costs are formed.

Disregarding variations of key parameters: in general, other studies do not consider changes in (i) environmental conditions, (ii) market prices of resources and products, and (iii) modes of operation of production systems. However, polygeneration systems must respond to these changes.

The present work contributes to these questions. It demonstrates that, unlike unit costs (which are arbitrary for co-products and by-products, with some exceptions) marginal costs play a crucial role in the economic analysis of the system's operation. In the case, for instance, of varying energy demands (as studied in the present paper), or the degradation and hence the increase of energy systems' internal losses (irreversibilities), the optimization model not only provides the related marginal costs but also allows to establish the linked production trajectory, which is achievable through the performance of an analysis and interpretation of such marginal costs. In this way, it is possible to achieve a profound understanding of how the system adapts to changes.

In order to evaluate the benefits of a marginal cost analysis and interpretation for a complex polygeneration system, a case study of an energy community (EC) was investigated. The case study is the project of an EC comprising nine tertiary sector buildings and a central unit in the city of Pordenone, northeast of Italy, demanding electricity, heating, and cooling. Moreover, the buildings are set to (i) share heat and cooling through a set of pipelines network (district heating and cooling network – DHCN), (ii) share electricity among them through a local electric grid, which is connected to the main national electric grid, and (iii) consume natural gas and solar energy as local resources. The EC is modelled through a mixed-integer linear programming (MILP) model, which allows to perform the optimization of the whole polygeneration system. The results obtained from such optimization[1] provide valuable and detailed insights about the optimal energy supply system installed in each building and in the central unit, the optimal set of installed DHCN pipelines, the costs and environmental impacts related to the entire system, and the trade-off solutions between total annual costs and CO_2 emissions. However, the mentioned insights do not provide information to determine the optimal operation of the entire system when the energy demand in a given building and in a given time-step increase.

Therefore, the aim of this paper is to analyse and interpret the hourly marginal costs related to the optimal economic solution (which is the starting point of the analysis) of the polygeneration system of the abovementioned EC. The mentioned analysis and interpretation of marginal costs is developed for a typical winter day in January (the analysis and interpretation procedure would be similar for any other typical day).

The primary contributions and novelties of this paper are twofold: (i) the analysis and interpretation of the hourly marginal costs of a complex, highly integrated polygeneration system featuring distributed production, different demand profiles, multiple production technologies supported by TES and DHCN, as well as the sharing of purchased and self-produced electricity among the EC buildings; and (ii) the outline of the system's optimal operation path when the energy demand of a given energy service is marginally increased.

The structure of the paper is organized as follows: section 2 provides the main details about the case study under analysis; section 3 presents the main aspects regarding the mathematical model, the obtained marginal cost values, and the explanation about the energy service production types; the results are presented through section 4, where representative marginal cost values are analysed and interpreted; finally, section 5 presents the main conclusions of the work.

2. Case study of a complex polygeneration system

The case study under analysis is a polygeneration system designed for an EC comprising nine tertiary sector buildings plus a central unit in a northeast city of Italy, demanding electricity, heating, and cooling. Further information about the buildings is available in the Appendix A. Furthermore, the buildings are set to (i) share heating and cooling through a set of pipelines network (DHCN), (ii) share electricity among them through a local electric grid (connected also to the main national electric grid), and (iii) consume natural gas and solar energy as local resources.

Before dive into the details about the polygeneration system under analysis, it is important to understand the concept of superstructure applied to the field of energy supply systems. Essentially, the superstructure of a given energy system comprises the possible technologies and their interconnections customized to meet the energy demand of the

[1] Developed in a previous work [31]

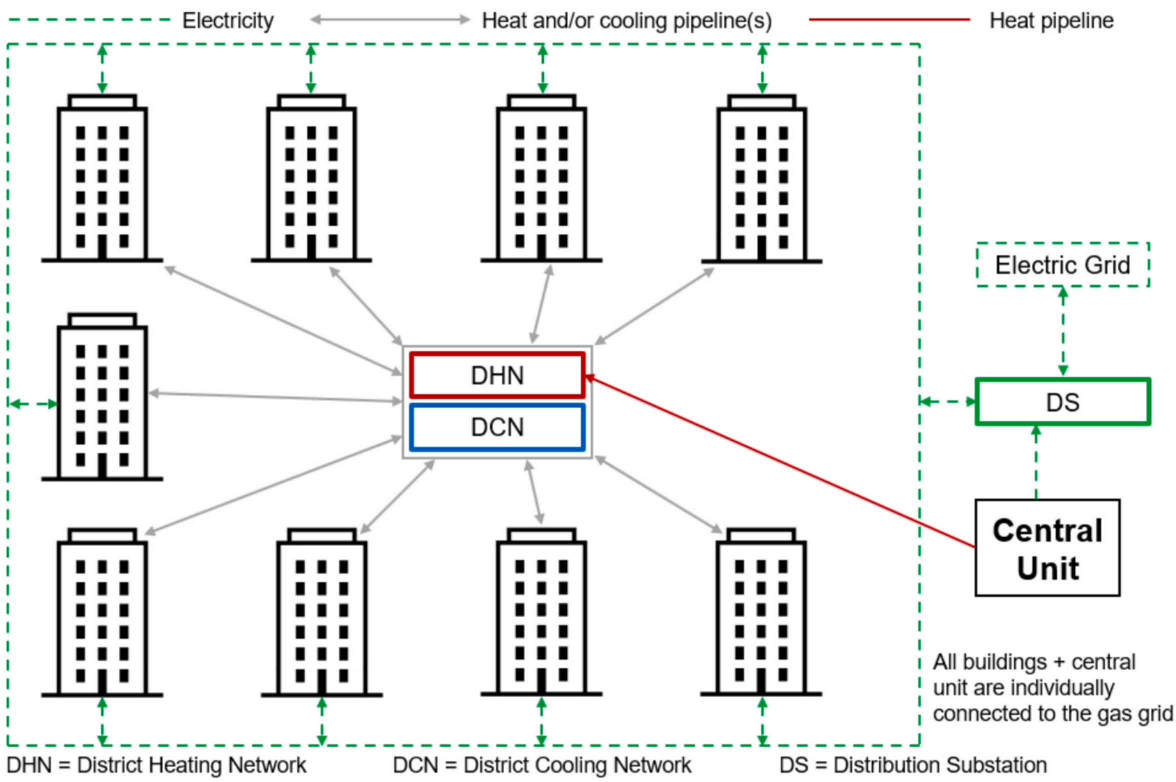


Fig. 1. Energy community superstructure.

consumer centre along the time. In other words, the technologies comprising the superstructure can be selected considering characteristics such as type of fuel/product, operation modes, temperature levels, and how they can cooperate to make better use of the available resources to reliably cover the energy demands [30]. After defining the superstructure, the algorithm of the mathematical optimization model can be written accordingly. In this way, the optimization process can present a solution according to specifications including input data, constraints, and objective function.

2.1. *Energy community overall superstructure*

Since the present work is focused on the analysis and interpretation of the marginal costs associated with the internal energy flows and final products of the polygeneration system designed for the mentioned EC, the main objectives of this section are (i) to briefly present the EC superstructure and the generic superstructure adopted for all the buildings, and (ii) present the optimal structure of the selected buildings that will be used as representative cases for the marginal cost analysis.

As already mentioned, for the purposes of this work, the main objective of presenting the case study is to show and explain the starting point of the marginal cost analysis and interpretation procedure. The reader may find in our previous work [31] an in-depth explanation of the optimization process applied for the same case study. For this reason, this section will briefly explain the overall superstructure of the EC, which is divided into the following parts (discussed on sections 2.1.1 and 2.1.2):

- EC superstructure, which specifies the total number of buildings, how they are connected to the electric grid, how the central unit can be connected with the buildings and electric grid, and the possible types of connections among the buildings,
- superstructure of a given building, which shows all the technologies that can be adopted as well as the possible interactions among them.

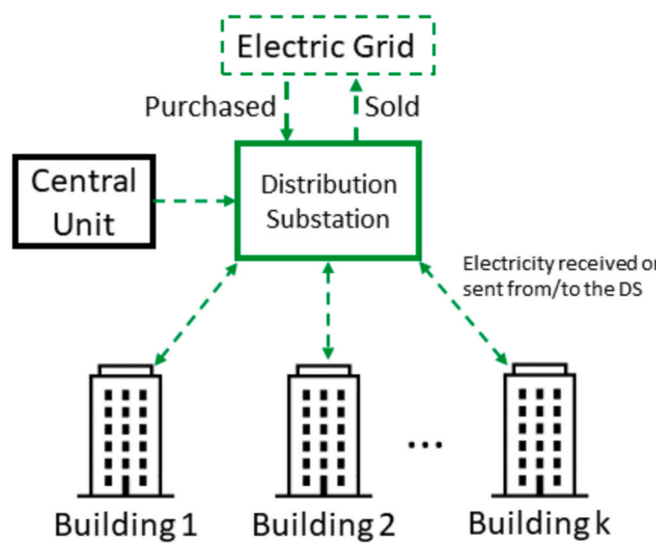


Fig. 2. Electricity balance management performed by the distribution substation (DS).

2.1.1. *EC superstructure*

The superstructure of the EC is presented in Fig. 1. Such illustration is intended to show all possible connections between the buildings in terms of heating, cooling, and electricity. Regarding heating and cooling, the buildings can connect to each other through the DHN and/or DCN. As observed, the buildings are not directly connected to the electric grid. Instead, they can exchange electricity with the distribution substation (DS), which will be explained next. The central unit can send electricity to the DS and heating to the EC through a heating pipeline. There is no cooling production in the central unit.

The electricity balance management of the distribution substation

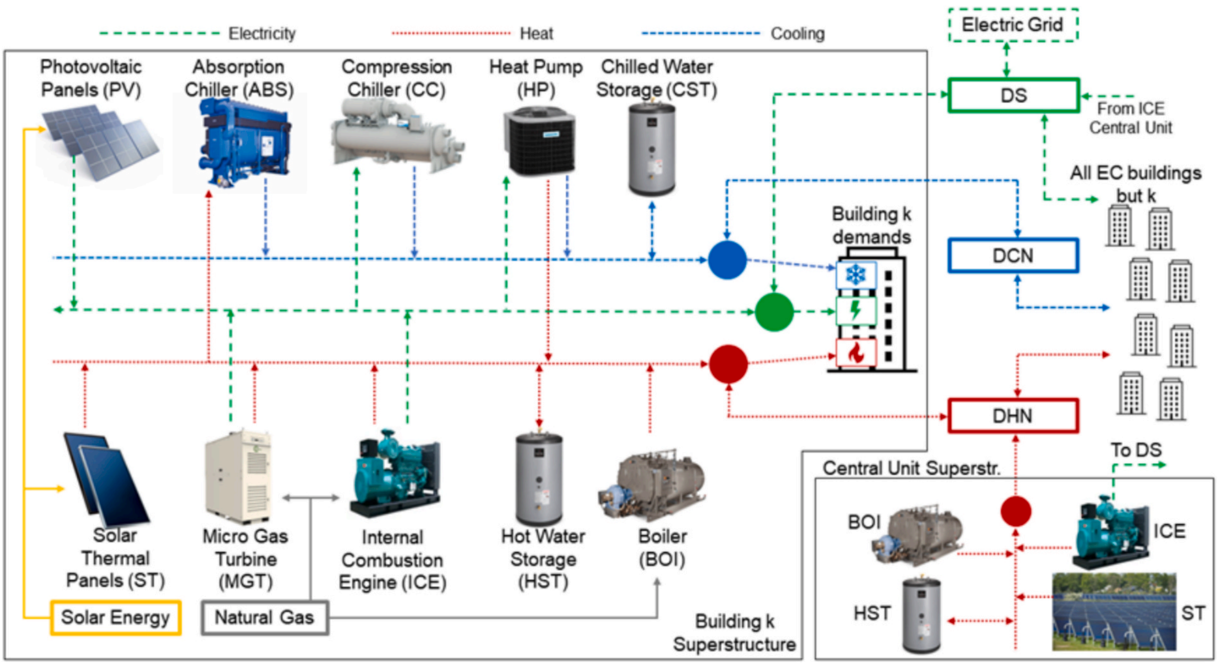


Fig. 3. Superstructure of a given building plus the central unit.

(DS) is one of the main details of the EC superstructure [32]. The buildings are connected exclusively with the DS (low voltage) which, at its turn, is connected to the regional (medium voltage) and national electric grid high voltage – Fig. 2). Such connection is set to function in both directions (not at the same time), i.e., depending on the obtained solution, a given building will be: (i) sending electricity to the DS at some hours of the year (self-production surplus), (ii) receiving electricity from the DS at some other hours of the year (self-production deficit), or (iii) neither receiving nor sending electricity from/to the DS (self-production is equal to the electricity demand at these hours). The central unit does not have an electricity demand. For this reason, all produced electricity would be sent to the DS. Then, in agreement with the objective function and in an hourly basis, the DS will (i) purchase the necessary amount of electricity from the national electric grid, or (ii) sell the total surplus of self-produced electricity by the entire EC, after having satisfied the electricity demand of each building.

2.1.2. Building superstructure

The superstructure of a given building is presented in Fig. 3. This figure can be thought of as a "zoom-in" in a given building in Fig. 1. For the better understanding of the reader, the analysis of Fig. 3 can be focused on the five main details presented in the figure itself: (i) the Building k Superstructure, i.e., all the technologies that are possible to be installed in a given building k; (ii) the candidate technologies considered for the Central Unit Superstructure, i.e., boiler (BOI), internal combustion engine (ICE), hot water storage (HST), and solar thermal panels (ST); (iii) the three possible ways of connecting the buildings in terms of electricity, heating, and cooling, that is, DS, DHN, and DCN; (iv) all the other EC buildings but k, i.e., the representation of the other EC buildings; and (v) the available energy resources (solar energy, natural gas and electricity from the national grid).

2.2. Energy community optimal structure

This is the starting point of the work developed within this study. The proposed MC analysis is performed once the structure and operation of the polygeneration system has been defined and established through a concrete optimal solution, which in this case was the optimal economic solution obtained in a previous work [31]. This solution was obtained by performing a single-objective economic optimization of the EC model, which result provided the technologies to be installed, together with their capacities, and operation. Table 1 presents the optimal structures for each building, according to the mentioned solution and, by analysing Table 1 and Fig. 3, it is possible to identify not only the technologies to be installed in each building, but also their optimal installed capacities.

Since the MC analysis in this study focuses solely on two of the nine buildings within the EC, Fig. 4 and Fig. 5 illustrate the optimal system configurations for these buildings—namely, building 2 (Theatre) and building 7 (Hospital). These figures detail the optimal installed capacities, annual energy flows, heating and cooling connections with other buildings, and electricity connections to the DS. Additionally, Fig. 6 highlights that the optimal configuration for the central unit includes

Table 1
Optimal structure and installed capacities for each building.

Building	ICE (kW)	MGT (kW)	BOI (kW)	ABS (kW)	HP (kW)	CC (kW)	PV (m2)	ST (m2)	HST (kWh)	CST (kWh)
1 – Town Hall	–	–	63	–	160	49	126	74	252	413
2 – Theatre	280	–	12	–	400	–	54	146	838	93
3 – Library	–	–	–	–	35	3	200	–	–	147
4 – Primary School	–	–	3	–	–	–	200	–	–	–
5 – Retirement Home	–	–	16	–	35	21	197	3	9	146
6 – Archive	–	–	11	–	–	70	192	8	23	197
7 – Hospital	800	–	810	–	630	49	62	138	4000	461
8 – Secondary School	–	–	–	–	480	–	200	–	–	–
9 – Swimming Pool	–	–	–	–	500	–	200	–	–	–

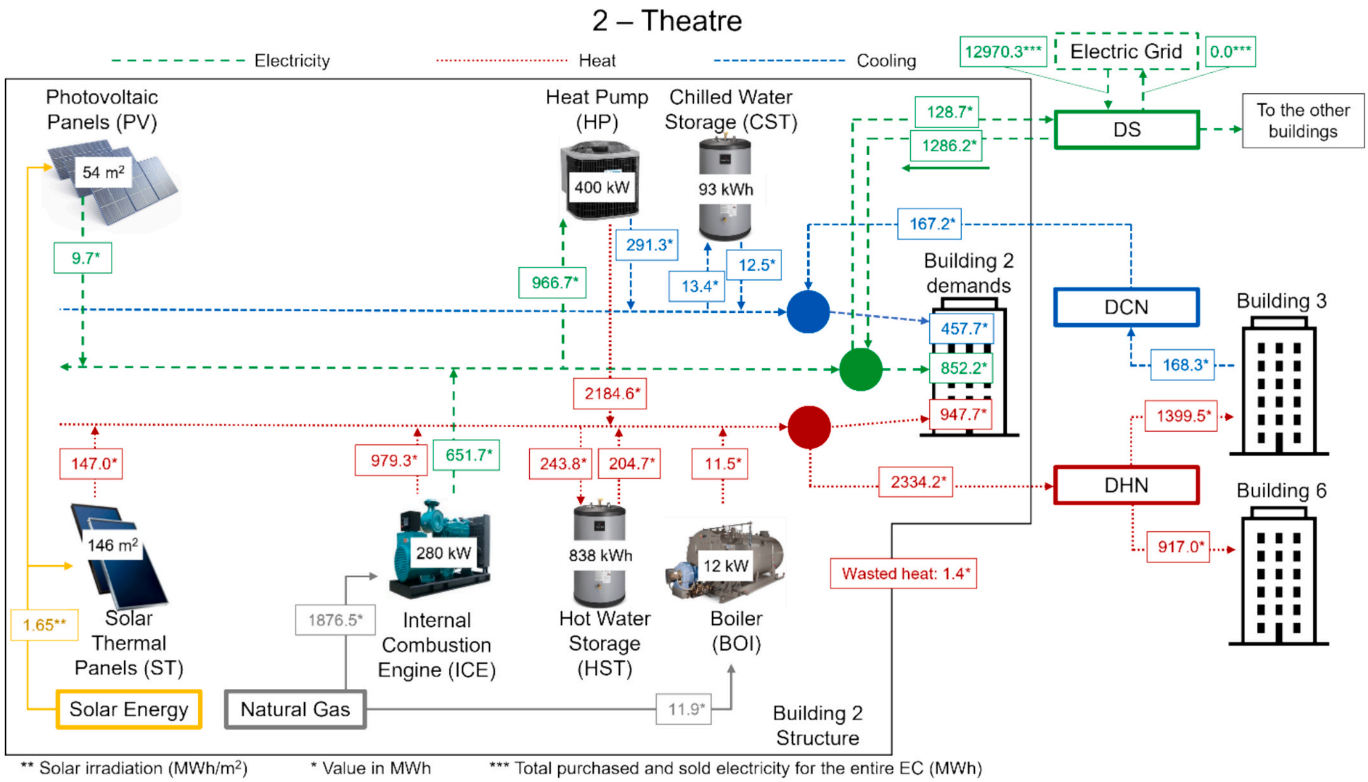


Fig. 4. Optimal structure for building 2 (Theatre). Installed capacities, annual energy flows, and DHCN connections.

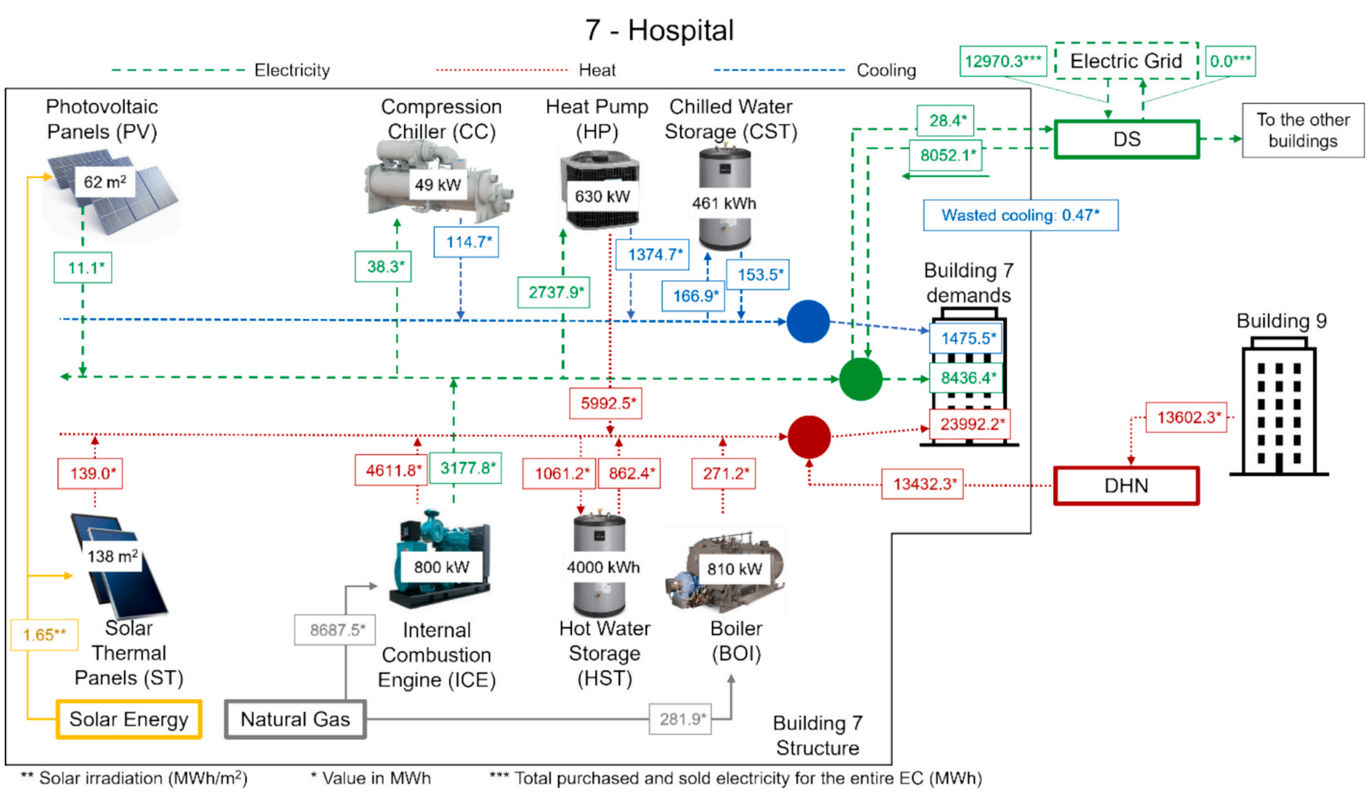


Fig. 5. Optimal structure for building 7 (Hospital). Installed capacities, annual energy flows, and DHCN connections.

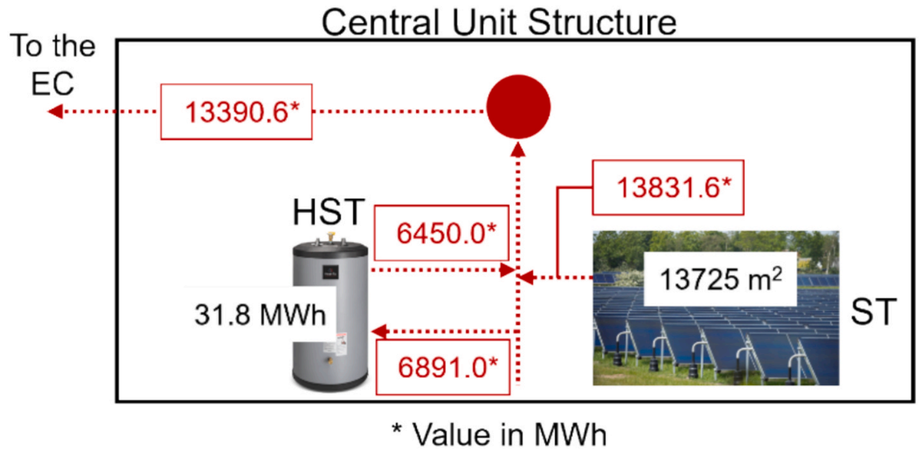


Fig. 6. Optimal structure for the central unit. Installed capacities, annual energy flows, and DHN connection (no cooling production).

only hot water storage and solar thermal panels. In these figures the installed capacity of the different plant components considers also the peak load capacity. The solar irradiation is presented the annual solar energy received per square meter on a tilted surface.

3. Mathematical model

The optimal solution described in the previous section has been obtained through a mixed integer linear programming (MILP) model (with hourly resolution), which defines the annual optimal synthesis, design, and operation of the optimal polygeneration system for the EC under analysis (the reader may refer to our previous work [31] to have a more complete description of both the case study and the mathematical model). Then, the model was configured to a linear programming (LP) model by fixing all binary variables (structure and operation were fixed). This procedure builds on the work of Williams H.P. [33], who suggests that to extract economic insights from a model containing binary or integer variables, these variables should be fixed at their optimal values. This allows for analysing the impact of marginal changes on the continuous variables effectively.

The results from the LP optimization process include also the dual values (or the marginal costs) associated with the internal energy flows and final products, which can be used to analyse and interpret the optimal operation of the system for the case of marginal variations in the previously stablished conditions. As the system under analysis is a polygeneration system supported by TES and DHCN, three pieces of information are essential for the interpretation of the marginal cost values unveiling the marginal path (optimal operation strategy that should be applied when there is a marginal variation of the production conditions): (i) the hourly energy balances, (ii) the hourly dual values associated to the constraints under analysis, and (iii) the TES and pipelines thermal losses. Regarding the constraints, this work will focus only on the energy balances, i.e., electricity and heating balances of the polygeneration system installed in each building plus central unit.

While the developed model offers a robust LP formulation for the EC described in Section 2, the limitations of the present approach should be carefully considered.

Therefore, the aim of this section is to explain each one of these essential points, providing the necessary tools to fully understand not only the analysis and interpretation of the marginal costs related to the system described in section 2, but also the limitations regarding the applied methodology in order to ensure that the findings are understood within the appropriate context.

3.1. Model limitations

The proposed model has been originally developed as a MILP model specifically tailored to represent the EC configuration described in Section 2. Then, the model was configured to a LP formulation in order to develop the marginal cost analysis developed within the present work. While this approach offers robustness and transparency, the inherent methodology limitations must be acknowledged. First, the model does not incorporate non-linear operation dynamics such as start-up and shut-down of the considered technologies, nor the associated inefficiencies of such dynamic behaviours. Second, variability in renewable energy generation is addressed using deterministic hourly time-series, without use the stochastic or probabilistic methods to represent uncertainty. Therefore, the "optimal operation" in this work refers to the most cost-effective response to marginal changes within the fixed operational modes of the pre-defined polygeneration system, rather than a dynamic optimum accounting for a range of real-world variabilities. These limitations, while consistent with the intended scope of this study, should be carefully considered when interpreting the results and their applicability to practical scenarios.

3.2. Objective function

The objective function is not directly involved in the analysis and interpretation of marginal costs presented in this work. Instead, the objective function was applied in an upstream optimization process, and the marginal cost values analysed here are outputs from that prior step. Therefore, for a comprehensive description not only of the objective function, but also of the case study and the mathematical model, the reader may refer to our previous work [31].

3.3. Energy balances for the superstructure

This section presents the general equations that were used to define the energy balances for the superstructure introduced in section 2.1. The corresponding energy balance equations for the optimal structure (section 2.2), as determined by the optimal economic solution, are provided in section 3.2.1.

The equations connecting energy supply resources with the energy demands of each building are the energy balances. The marginal cost analysis will be concentrated in such constraints through the dual values associated to them. It has the aim to evaluate the effect in the objective function when the hourly demand of any energy product (heating, cooling, or electricity) increases by one unit without changing neither the optimal structure nor the operation mode of the whole system. The hourly dependency of the variables, represented by m (month), d (day), and h (hour), is replaced by t for simplicity.

Equation (1) provides the heat balance for building 8 ($B = 8$). For buildings 1 to 7 ($1 \leq B \leq 7$) and 9 ($B = 9$), the heat balance follows the same equation; however the term $Heat_{cen.unit}(t, B) = 0$ since the central unit is set to be connected only to building 8. Building 8, in turn, is able to eventually redistribute heat from the central unit to the other buildings, if it is defined by the optimal solution. As observed, the equation comprises the variables regarding (i) heat producing technologies plus heat storage (within the building superstructure), (ii) heat coming from central unit, (iii) net amount of heat received through DHN pipelines, (iv) wasted heat, and (v) heat demand of a given building.

$$\left[\sum_{c=1}^{6}\left(Heat_{MGT}(t,c,B)+Heat_{ICE}(t,c,B)+Heat_{HP}(t,c,B)-Heat_{ABS}(t,c,B)\right)\right]+\left[\sum_{k=1}^{9}\left(Q_h(t,k,B)\bullet(1-p_h(B,k))-Q_h(t,B,k)\right)\right]+Heat_{BOI}(t,B)+Heat_{ST}(t,B)-Heat_{HST\_in\_out}(t,B)-Heat_{Dem}(t,B)+Heat_{cen.unit}(t,B)-Heat_{waste}(t,B)\geq 0 \tag{1}$$

The term regarding the heat received from the central unit (Eq. (1)) is obtained from another heat balance applied to the technologies comprising the central unit (Eq. (2)).

$$Heat_{ICEc}(t)+Heat_{BOIc}(t)+Heat_{STc}(t)-Heat_{HSTc\_in\_out}(t)-Heat_{cen.unit}(t)\geq 0 \tag{2}$$

Equation (3) provides the cooling balance for a given building of the EC. As seen, the equation comprises the variables regarding (i) cooling producing technologies plus cooling storage (within the building superstructure), (ii) net amount of cooling received through DCN pipelines, (iii) wasted cooling, and (iv) cooling demand of a given building.

$$\left[\sum_{c=1}^{6}\left(Cool_{ABS}(t,c,B)+Cool_{HP}(t,c,B)\right)\right]+\left[\sum_{k=1}^{9}\left(Q_c(t,k,B)\bullet(1-p_c(B,k))-Q_c(t,B,k)\right)\right]+Cool_{CC}(t,B)-Cool_{CST\_in\_out}(t,B)-Cool_{Dem}(t,B)-Cool_{waste}(t,B)\geq 0 \tag{3}$$

Equation (4) provides the electricity balance of the distribution substation (DS). The first term regards the summation of the electricity sent or received to/from the buildings, whereas the second term refers to the electricity received from the engine installed in the central unit. The last two terms refer to the purchased and sold electricity, respectively. Equations (5) and (6) guarantee that the purchased or sold electricity will not be a negative number.

$$\left[\sum_{B=1}^{9}Elec_{DS}(t,B)\right]+Elec_{ICEc}(t)+E_{bgt}(t)-E_{sold}(t)=0 \tag{4}$$

$$E_{bgt}(t)\geq 0 \tag{5}$$

$$E_{sold}(t)\geq 0 \tag{6}$$

3.3.1. Energy balances for the optimal structure

As previously mentioned, the starting point for the proposed MC analysis and interpretation is a specific optimal solution derived from the optimization process discussed in the previous sections. The final energy balance equations for each building, which determine the optimal energy flows, are based on the optimal configuration of each building. Given that this study focuses on conducting a representative MC analysis and interpretation for only two of the nine buildings in the EC and for two of the three utilities consumed, only the optimal energy balance equations for Buildings 2 and 7, for the utilities of heating and electricity, will be presented.

Equation (7) presents the energy balance equation shown in Eq. (1) but applied specifically to the optimal structure of building 2 (Theatre). The first two terms ($Heat_{ICE}(t,c,2)$ and $Heat_{HP}(t,c,2)$) represent the hourly heat production by the internal combustion engine and heat pump, respectively. The parameter t represents the time dependency of the variable in terms of m (month), d (day), and h (hour), whereas the parameter c regards the number of units of a given technology that should be installed in the building, i.e., for the analysed solution, it should be installed $c=2$ ICE units (140 kW each) and $c=4$ HP units (100 kW each) in the building 2. The variables $Q_h(t,2,3)$ and $Q_h(t,2,6)$ concern the hourly amount of heat that building 2 should send to buildings 3 and 6, respectively, while the terms $Heat_{BOI}(t,2)$, $Heat_{ST}(t,2)$, $Heat_{Dem}(t,2)$, and $Heat_{waste}(t,2)$ regard, respectively, the hourly amount of heat that should be produced by the boiler, the solar thermal panels, that is demanded by building 2, and that is wasted by building 2. Finaly, the term $Heat_{HST}(t,2)$ represents the hourly discharging (when the variable acquires a negative value) or the hourly charging (when the variable acquires a positive value) of the hot water storage device in building 2. The variables regarding MGT and ABS do not appear in Eq. (7) since these technologies should not be installed in building 2, as indicated in Table 1.

$$\left[\sum_{c=1}^{2}\left(Heat_{ICE}(t,c,2)\right)+\sum_{c=1}^{4}\left(Heat_{HP}(t,c,2)\right)\right]+\left[-Q_h(t,2,3)-Q_h(t,2,6)\right]+Heat_{BOI}(t,2)+Heat_{ST}(t,2)-Heat_{HST}(t,2)-Heat_{Dem}(t,2)-Heat_{waste}(t,2)\geq 0 \tag{7}$$

The analysis is analogous for Eq. (8), i.e., the energy balance equation shown in Eq. (1) but applied specifically to the optimal structure of building 7 (Hospital). The main differences between Eq. (8) and Eq. (7) are (i) the Hospital would have to install four ICE units (200 kW each) and six HP units (105 kW each), and (ii) the Hospital does not send heat to any building; instead, it receives an hourly amount of heat from building 9 $Q_h(t,9,7)$, taking into account the pipeline heat losses ($p_h(7,9)$) between both buildings.

$$\left[\sum_{c=1}^{4}\left(Heat_{ICE}(t,c,7)\right)+\sum_{c=1}^{6}\left(Heat_{HP}(t,c,7)\right)\right]+\left[Q_h(t,9,7)\bullet(1-p_h(7,9))\right]+Heat_{BOI}(t,7)+Heat_{ST}(t,7)-Heat_{HST}(t,7)-Heat_{Dem}(t,7)-Heat_{waste}(t,7)\geq 0 \tag{8}$$

The DS hourly electricity balance (see section 2.1.1) regarding the optimal structure of the EC is presented through Eq. (10). As observed, based on the optimal economic solution, the central unit does not have the ICE. Moreover, the first term of the same equation depends on the hourly electricity balance of each building. For that reason, and in accordance with the optimal economic solution, Eq. (10) and Eq. (11) provide the individual electricity balances for buildings 2 and 7, respectively, to keep focusing on the same examples of buildings as for the heating balances.

By analysing Eq. (10), it is possible to observe that building 2 has two types of electricity producing technologies, ICE (two units) and PV panels, while there is one type of electricity consuming technology: HP (four units). If the hourly electricity demand of building 2 ($Elec_{Dem}(t,2)$)

Table 2
Hourly marginal costs (in €/kWh) associated with an additional unit on the electricity demanded by the DS from the electric grid. Values for a January working day. Abbreviations: MC (marginal cost), DS (distribution substation).

Hour	MC (DS)	Hour	MC (DS)	Hour	MC (DS)
1	0.1169	**9**	0.1450	**17**	0.1533
2	0.1169	**10**	0.1533	**18**	0.1533
3	0.1169	**11**	0.1533	**19**	0.1533
4	0.1169	**12**	0.1533	**20**	0.1533
5	0.1169	**13**	0.1533	**21**	0.1450
6	0.1169	**14**	0.1533	**22**	0.1450
7	0.1169	**15**	0.1533	**23**	0.1450
8	0.1169	**16**	0.1533	**24**	0.1450

is not covered by the internal net produced electricity, the term $Elec_{DS}(t, 2)$ is in charge of requiring more electricity from the DS. On the other hand, the same term can be used to send the surplus of net produced electricity to the DS, if it is the case. This explanation is analogous to the electricity balance of building 7, presented through Eq. (11). There are only two differences: (i) building 7 has more ICE and HP units, and (ii) there is another electricity consuming technology, i.e., the compression chiller (CC).

$$\left[\sum_{B=1}^{9} Elec_{DS}(t,B)\right] + E_{bgt}(t) - E_{sold}(t) = 0 \tag{9}$$

$$\left[\left(\sum_{c=1}^{2} Elec_{ICE}(t,c,2)\right) + Elec_{PV}(t,2) - \left(\sum_{c=1}^{4} Elec_{HP}(t,c,2)\right) - Elec_{Dem}(t,2)\right] = Elec_{DS}(t,2) \tag{10}$$

$$\left[\left(\sum_{c=1}^{4} Elec_{ICE}(t,c,7)\right) + Elec_{PV}(t,7) - Elec_{CC}(t,7) - \left(\sum_{c=1}^{6} Elec_{HP}(t,c,7)\right) - Elec_{Dem}(t,7)\right] = Elec_{DS}(t,7) \tag{11}$$

3.4. Dual values

As observed in the state-of-the-art presented in the introduction, marginal costs can offer valuable insights within different contexts. However, their calculation poses challenges, particularly in systems characterized by high levels of energy integration. In this sense, computational tools have emerged as essential aids in overcoming these challenges, facilitating the calculation of marginal costs and the analysis of the influences of changes in input data. In combination with the optimal system operation, a linear programming optimization model,

Table 3
Hourly marginal costs (in €/kWh) associated with the hourly heating demand increase of one unit of the $\boldsymbol{Heat_{Dem}(t,B)}$ for the nine buildings. Values for a January working day. Abbreviation: CU (central unit).

Hour	Buildings									CU
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
1	–	0.0559	–	–	0.0567	0.0567	0.0799	0.0774	0.0789	0.0759
2	–	0.0571	–	–	0.0579	0.0578	0.0852	0.0522	0.0532	0.0763
3	–	0.0582	–	–	0.0591	0.0590	0.0869	0.0522	0.0532	0.0766
4	–	0.0594	–	–	0.0603	0.0602	0.0887	0.0522	0.0532	0.0770
5	–	0.0606	–	–	0.0615	0.0614	0.0905	0.0522	0.0532	0.0774
6	0.0905	0.0619	0.0621	0.0627	0.0628	0.0627	0.0923	0.0793	0.0809	0.0778
7	0.0905	0.0631	0.0634	0.0640	0.0787	0.0785	0.0942	0.0797	0.0813	0.0782
8	0.0905	0.0644	0.0647	0.0653	0.0654	0.0653	0.0961	0.0801	0.0817	0.0786
9	0.0905	0.0657	0.0660	0.0667	0.0673	0.0672	0.0832	0.0805	0.0822	0.0790
10	0.0683	0.0671	0.0673	0.0680	0.0711	0.0710	0.0705	0.0683	0.0697	0.0683
11	0.0683	0.0684	0.0687	0.0694	0.1655	0.1652	0.0691	0.0669	0.0683	0.0669
12	0.0683	0.0698	0.0701	0.0708	0.0833	0.0724	0.0679	0.0672	0.0671	0.0672
13	0.0683	0.0713	0.0716	0.0723	0.0723	0.0722	0.0679	0.0676	0.0671	0.0676
14	0.0683	0.0727	0.0730	0.0738	0.0738	0.0737	0.0679	0.0679	0.0671	0.0679
15	0.0683	0.0742	0.0745	0.0753	0.0753	0.0752	0.0674	0.0683	0.0697	0.0683
16	0.0683	0.0757	0.0760	0.0768	0.0768	0.0767	0.0688	0.0686	0.0679	0.0686
17	0.0683	0.0773	0.0776	0.0784	0.0784	0.0783	0.0702	0.0690	0.0693	0.0690
18	0.0683	0.0926	0.0930	0.0939	0.0940	0.0938	0.0716	0.0693	0.0707	0.0693
19	0.0683	0.0945	0.0949	0.0959	0.0959	0.0958	0.0709	0.0686	0.0700	0.0696
20	–	0.0964	0.0968	0.0978	0.0979	0.0977	0.0723	0.0700	0.0714	0.0700
21	–	0.0663	0.0666	0.0672	0.0673	0.0672	0.0667	0.0646	0.0659	0.0704
22	–	0.0676	0.0679	0.0686	0.0686	0.0685	0.0753	0.0646	0.0659	0.0707
23	–	0.0690	–	–	0.0700	0.0699	0.0769	0.0646	0.0659	0.0711
24	–	0.0704	–	–	0.0715	0.0714	0.0784	0.0646	0.0659	0.0714

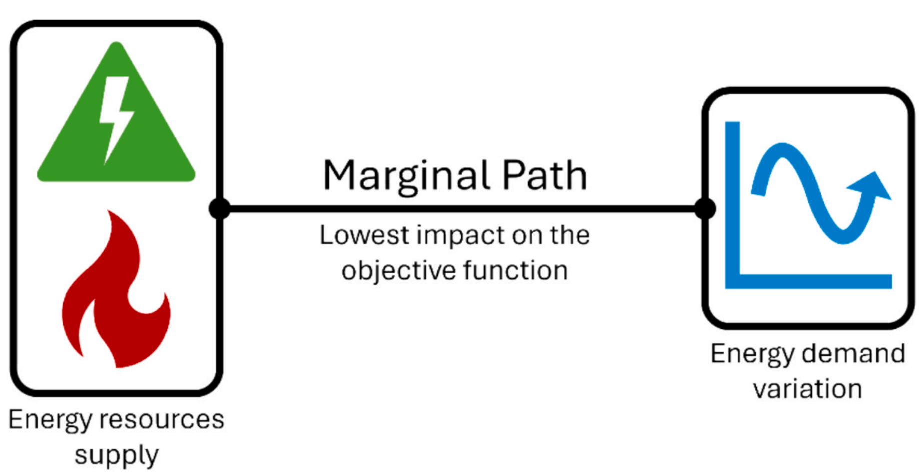


Fig. 7. Marginal path illustration. Energy resources supply, for the case presented in this work, on the left (electricity and natural gas) and energy demand variations on the right.

Table 4
Marginal cost value associated with 1 kWh of heat production from the BOI.

[A] Marginal amount of heat to be produced	$Heat_{BOI} = 1kWh_h$
[B] Efficiency equation	$Fuel_{BOI} = \frac{Heat_{BOI}}{\eta_{BOI}}$
BOI efficiency (η_{BOI})	95 %
[C] Natural gas price (€/kWh$_{NG}$)	0.085
[D] Maintenance cost (€/kWh$_h$)	0.001
[C • B] +[D • A] Marginal cost of heat from BOI (€/kWh$_h$)	0.0905

Table 5
Marginal cost values associated with 1 kWh of heat production from HP.

Marginal amount of heat to be produced	$Heat_{HP} = 1kWh_h$	
[A] Efficiency equation	$Elec_{HP} = \frac{Heat_{HP}}{COP}$	
[B] Maintenance cost (€/kWh$_{el}$)	0.001	
COP	**[C] Elect. price (€/kWh)**	**[A • B] +[A • C] Marginal cost from HP (€/kWh$_h$)**
2.26	0.1169	0.0522
	0.1450	0.0646
	0.1533	0.0683
2.30	0.1169	0.0513
	0.1450	0.0635
	0.1533	0.0671
2.17	0.1169	0.0543
	0.1450	0.0673
	0.1533	0.0711

such as those built in FICO Xpress software [34], provides the dual values for each constraint and at any stablished time-step. Such dual values indicate the amount by which the objective function will vary when the constant term of a constraint is increased by one unit [35]. For instance, for an energy balance constraint, the dual values corresponding to the restrictions of the energy demand represent the marginal costs (λ) related to the energy demand, i.e., they clearly indicate what would be the cost if the energy demand is increased by one unit of energy. A proper interpretation of the marginal cost, provides detailed information about how the energy supply system would react in the case of energy demand increase.

Following the same reasoning of Pina (2019) [12], the hourly dual values associated to the constraints presented from Eq. (7) to Eq. (11) are presented in Table 2 and Table 3. Such dual values can be interpreted as the Lagrange multipliers or the marginal costs λ associated with an additional unit of energy demand [35,36].

For the present analysis, such demand increase can happen on the (i) hourly heat demand of each building (λ of the $Heat_{Dem}(t,B)$), (ii) hourly heat demanded from central unit (λ of the $Heat_{cen.unit}(t)$), and (iii) hourly electricity demand resulted from the summation $\sum_{B=1}^{9} Elec_{DS}(t,B)$.

This section presents a definition and illustration for the marginal path, the marginal cost values associated with the heating and electricity demands, and the marginal costs associated with the obtention of an energy service from a specific technology. The marginal cost values associated with cooling demand will not be evaluated in this study, as the typical day analysed represents a winter day, although the procedure for their calculation and interpretation is similar to the marginal cost values analysed in this work.

3.4.1. Marginal cost values

Dual values represent the marginal costs associated with fulfilling a marginal increase in the constant term of each constraint. In the case of the present work, such constant term represents the hourly energy demand of the buildings in each energy balance. Thus, in this case, the marginal path is the connection between the point where the primary energy resource enters the system's boundaries, and the point where the marginal variations on the energy demand take place, as illustrated in Fig. 7. For instance, when there is a marginal increase in heating demand for a particular building, the optimal marginal cost can reveal which technology (or combination of technologies) should be employed to meet this additional demand with the lowest cost, i.e. the lowest impact on the objective function being evaluated. This could involve using the BOI, HP, ICE, HST, or other heating supply systems, depending on the optimal structure of the polygeneration system designed for the building. In the following, it will be presented the hourly marginal costs for a typical winter day.

Table 2 presents the hourly marginal costs associated with the increase of one kWh in the electricity demanded by the DS from the electric grid, whereas Table 3 presents the hourly marginal costs regarding the heating demands from all nine buildings plus the ones regarding the heating demanded from the central unit. The hourly marginal costs presented on Table 2 are equal to the hourly electricity price since, for the typical day under analysis, an increase on the hourly electricity demand would be covered by purchasing electricity from the grid. The different marginal costs presented on Table 3 are linked to the optimal hourly operation of the system and are in accordance with the marginal path illustrated through Fig. 7. It should be noted two aspects regarding this table: (i) the presented hourly marginal costs correspond to one typical day (working day) of January, and (ii) the hourly marginal costs of the electricity demand are linked to the electricity connection between the DS and the electric grid (since each building is connected to the DS, which in turn is connected to the electrical grid). The DS determines the necessity or not of buying an additional unit of electricity from the grid based on the summation $\sum_{B=1}^{9} Elec_{DS}(t,B)$ (Eq. (9)), which represents the hourly net amount of electricity exchanged between the buildings and the DS.

3.4.2. Marginal costs associated with technologies

With the aim to support the marginal cost analysis, this section is intended to calculate and present the marginal cost values associated with heat and cooling production from key technologies within the optimal economic solution. The goal is to know how much it would cost to produce one extra kWh of heat by using BOI or HP. The calculations and marginal cost values for ICE will not be included since they were not necessary for the marginal cost analysis presented within this paper.

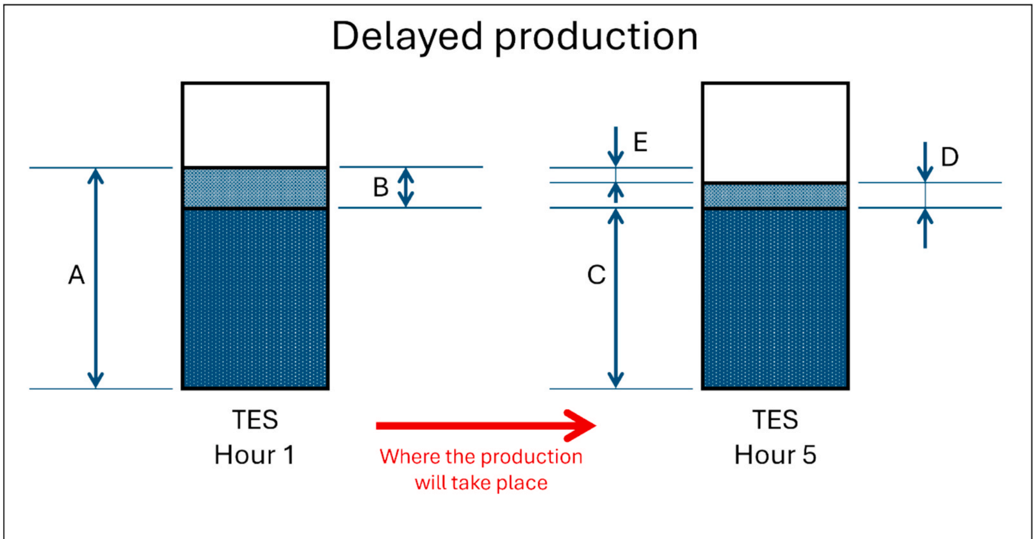
Heat production

Table 4 presents the equations and input data necessary to calculate the marginal cost value associated with the production of 1 kWh of heat by using the BOI. The result from such calculation can be used for a BOI installed in any building since the efficiency and associated costs are the same.

Table 5 shows the equations and the necessary input data to calculate the marginal cost value associated with the production of 1 kWh of heat by using the HP. In this case, there is not only one result; instead, the associated marginal cost will depend on the three different HP nominal capacities with different COP levels. Only one type of such HP technologies is allowed to be installed in a given building. The COP values shown in Table 5 refer to the HP working during the winter, according to the manufacturer [37], when the ΔT between the cold and hot reservoirs is higher compared to the machine working on summer. For additional information about the nominal capacities installed in each building, maintenance costs, and hourly electricity prices, the reader may refer to our previous work [31].

3.5. TES and DHCN pipelines thermal losses

Thermal energy storage (TES) devices, both for heating and cooling, as well as the district heating and cooling network (DHCN) pipelines own the benefit of supporting the energy supply system of the EC. Nevertheless, an inherent characteristic is the presence of heat losses.



A	Amount of stored energy at hour 1, which is a result from the optimization process.
B	Amount of energy discharged from the TES, which was a consequence of a marginal unit increment in the energy demand at hour 1.
C	Amount of stored energy that should be in the TES (for the case of no charging or discharging between hours 1 and 5), which is a result from the optimization process and takes into account the energy losses between hours 1 and 5.
D	Amount of energy that should be compensated at hour 5 IF "B" has happened at hour 1. It is calculated through Eq. (8).
E	TES energy losses between hours 1 and 5.

Note that $B > D$ and $B = D + E$

LESS energy needs to be produced at hour 5 and, for this reason, the production cost will be **lower**. This will be the **marginal cost at hour 1** (since hour 1 should "pay" for a "service provided" by hour 5).

Fig. 8. Illustration of delayed energy production type.

The heat loss factor regarding TES is 2 % per hour that the heat has been stored, whereas the loss factors for pipelines is 5 % per km that the heat travels from source to destination. Therefore, the purpose of this section is to present the equations governing heat losses in the thermal energy storage (TES) and district heating and cooling network (DHCN) pipelines, and to explain how these losses impact the marginal cost values. Understanding these effects is essential for analysing and interpreting the marginal costs associated with the optimal operation of the EC's polygeneration system.

3.5.1. TES: simultaneous, advanced, and delayed production of energy services

Before delving into the detailed definition of these types of energy service production, it is important for the reader to understand their relationship with the TES and the necessity of these different operational modes. This relationship is tied to the real-time availability of generating additional energy demanded by the technologies at the consumer facility. When immediate production is insufficient, energy generation may need to be shifted over time with the support of TES. The following paragraphs will explain this concept in detail. As will be observed, the three types of energy service production are essential to understand the contribution of the TES to the marginal cost associated with the energy supply system at a given hour h. In agreement with Pina (2019) [12], the three mentioned energy service production types will be detailed below.

Simultaneous

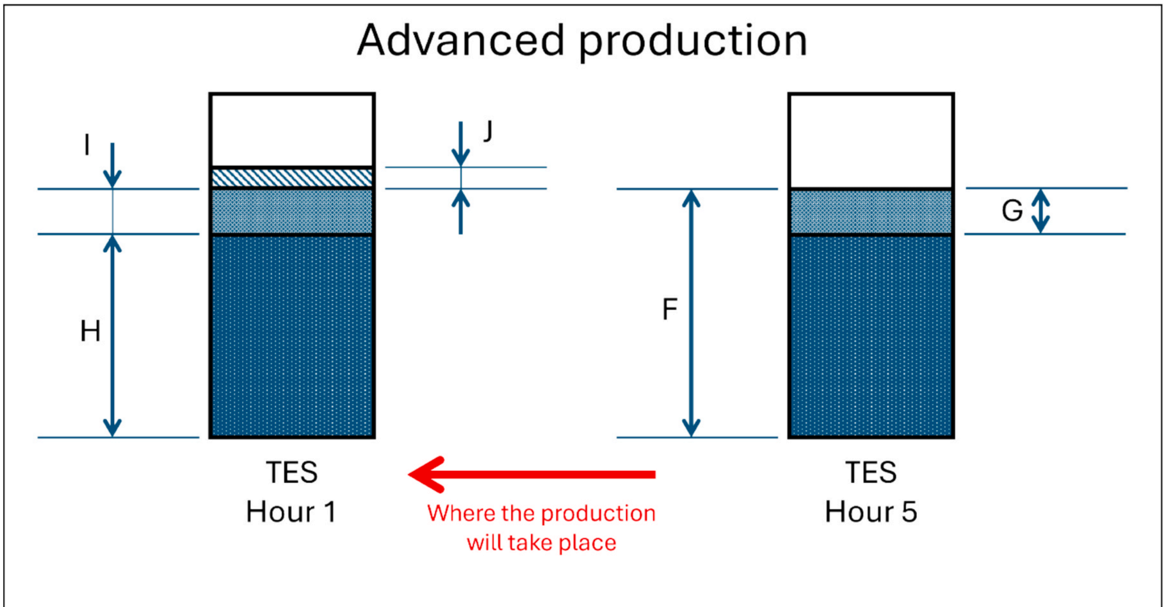
Simultaneous production means that it is more convenient to produce the energy service at the same hour h it is demanded, i.e., if an additional unit of energy is demanded at hour h, such energy service can be produced at that very same hour since (i) there is available capacity to do so, and (ii) using the support of the TES would be more costly.

Delayed

This production type means that the energy service cannot be produced (or it is not convenient) at the same hour h it is demanded and must be shifted to a later hour x. The reason for this can vary according to the energy supply system structure and its optimal operation. Equation (12) can be used to calculate how much energy should be produced at hour x in order to compensate for the energy withdraw at hour h, bearing in mind that the exponent $(h - x)$ can be negative. It means that, when it comes to delayed production, less energy should be produced at hour x since the additional unit of energy demanded at hour h will not be in the TES device from hour h to x and, therefore, energy losses will not take place. Fig. 8 illustrates this idea, where $h = 1$ and $x = 5$.

Advanced

Advanced production means that the energy service cannot be produced (or it is not convenient) at the same hour h it is demanded and must be shifted to a previous hour x. The reason for this depends on the optimal structure and operation of the energy supply system. For instance, one reason could be that all heat-producing technologies are at full capacity at hour h. If an additional unit of heat is demanded at that hour, the only left option would be the TES. Assuming that this addi-



F	Amount of stored energy at hour 5, which is a result from the optimization process.
G	Amount of energy discharged from the TES, which was a consequence of a marginal unit increment in the energy demand at hour 5.
H	Amount of stored energy at hour 1, which is a result from the optimization process.
I	Amount of energy that should be produced at hour 1 in order to compensate the lack of "G" at hour 5.
J	Amount of energy that should be produced at hour 1 in order to compensate the TES energy losses between hour 1 and 5. Therefore, at hour 1, ***I + J*** should be produced in order to compensate "G" at hour 5.
Note that $I = G$	
MORE energy needs to be produced **at hour 1** and, for this reason, the production cost will be **higher**. This will be the **marginal cost at hour 5** (since hour 5 should "pay" for a "service provided" by hour 1).	

Fig. 9. Illustration advanced energy production type.

tional unit of heat taken from the TES (at hour h) will be needed in the following hours,[2] it means that this additional heat must be compensated for in an earlier hour x. Considering the required amount of heat at hour h ($Heat_{out}(h)$), the amount of heat that must be produced and stored at hour x ($Heat\_prod_{in}(x)$), can be obtained through Eq. (12). It means that, when it comes to advanced production, more energy should be produced to compensate for the heat losses inherent to the TES device. Fig. 9 illustrates this idea, where $h = 5$ and $x = 1$. As observed, J will need to be produced at hour 1 in order to compensate for the TES energy losses.

$$Heat_{out}(h) = Heat\_prod_{in}(x) \bullet (1 - heat_{loss\_HST})^{(h-x)} \tag{12}$$

3.5.2. DHCN: Remote production of energy services

The previous section explained how the energy production can be shifted in time through the TES. The present section explains how the energy production can be shifted spatially, i.e., how production can take place in a different location with respect to where the additional energy demand occurs. In the context of the case study presented in section 2, the remote energy production (the fourth energy service production type) refers to the production of an energy service (heat or cooling) in a different building and sending it to the energy demanding building through pipelines of the DHCN. For this reason, the pipeline heat losses that will occur when transferring the energy from one building to another must be taken into account. Equation (13) provides the heat loss equation for the DHCN pipelines. If buildings X and Y are connected through a DHN pipeline and building X is sending heat to building Y, it means that, if building Y needs an additional unit of heat ($Heat_{building\_Y}$) from building X, the latter should send the amount of heat ($Heat_{building\_X}$) expressed in Eq. (13). The same logic applies to cooling pipelines, although they are not analysed in this work from the marginal cost analysis viewpoint. All loss factors regarding the heating pipelines considered in the DHN are provided through Table 6.

$$Heat_{building\_X} = \frac{Heat_{building\_Y}}{(1 - loss_{pipe})} \tag{13}$$

[2] It is important to note that the starting point of the marginal cost analysis is the optimized energy supply system, i.e., all energy flows are already optimal, including the TES charging/discharging quantities.

Table 6
Loss factors ($loss_{pipe}$) regarding heating pipelines.

To / From	Buildings 1	2	3	4	5	6	7	8	9
1	0	0.0225	–	–	0.0115	0.01	–	–	–
2	0.0225	0	0.004	–	0.0125	0.013	–	–	–
3	–	0.004	0	0.01	–	–	–	–	–
4	–	–	0.01	0	–	–	0.07	0.07	–
5	0.0115	0.0125	–	–	0	0.0015	–	–	–
6	0.01	0.013	–	–	0.0015	0	–	–	–
7	–	–	–	0.07	–	–	0	–	0.0125
8	–	–	–	0.09	–	–	–	0	0.02
9	–	–	–	–	–	–	0.0125	0.02	0

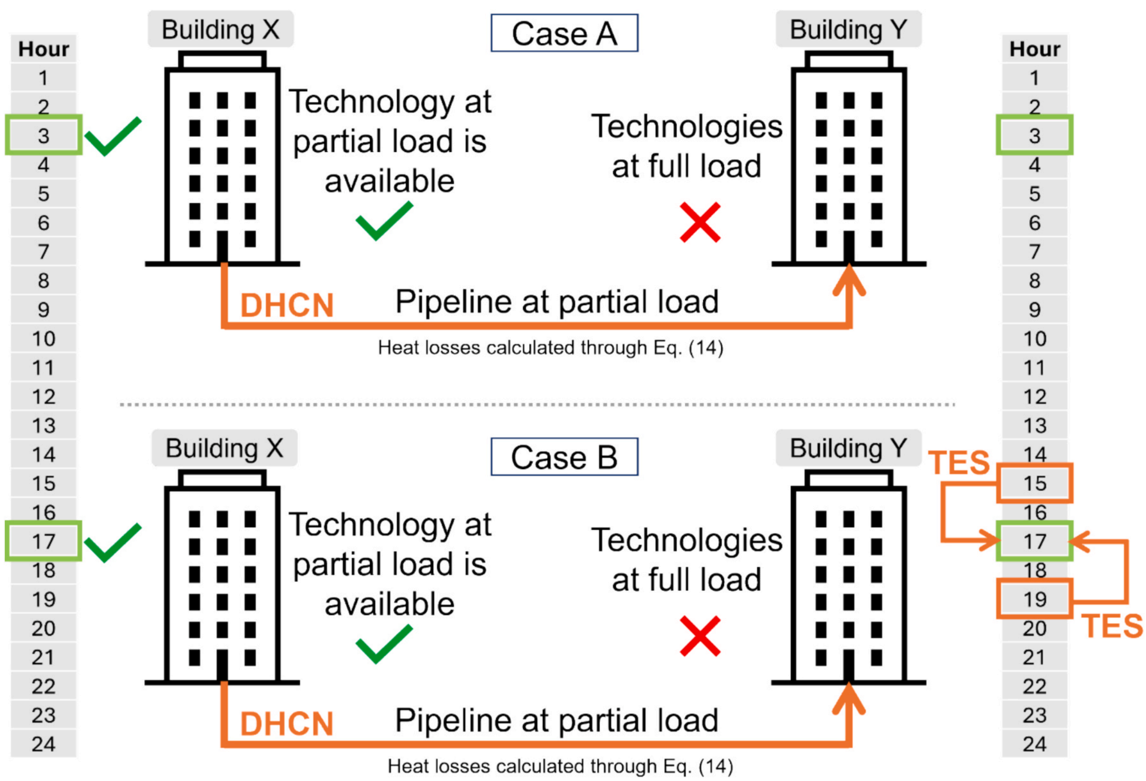


Fig. 10. Illustration regarding the combination of the simultaneous and remote energy production types (Case A) and the combination of remote with advanced or delayed energy production types (Case B).

It is also possible to combine the remote energy service production type with the production types discussed in section 3.4.1. Fig. 10 illustrates two possible scenarios. In Case A, an example of simultaneous and remote production types is shown. Here, Building X, which has a technology operating at partial load, sends heat to Building Y, where all technologies are running at full capacity. If Building Y requires an additional unit of energy at hour 3, it must request that energy from Building X, which is capable of producing it at that same time. Case B demonstrates the combination of remote production with delayed and advanced production modes. Suppose Building X has a technology operating at partial load at hour 17 and is sending energy through the pipeline to Building Y at that time. If Building Y, running all technologies at full capacity, requires an additional unit of energy at hour 15, it can delay its energy production to hour 17 through the TES and request the extra energy from Building X. Similarly, in the case of combining remote and advanced production, if Building Y's energy demand increases at hour 19, for instance, it can advance production to hour 17 through the TES to meet the additional energy demand by requiring more energy from Building X.

Table 6 presents the loss factors for heat pipelines. All non-zero values represent possible connections between the buildings.

4. Marginal costs analysis and interpretation

Once the energy supply system of the Energy Community (EC) has been optimised, i.e. the optimal structure of the system as well as the size of each piece of equipment, and the optimal annual operation have been established [31], the thermoeconomic analysis of the obtained hourly dual values – meaning the hourly marginal costs related to the economic optimum – provide insights to unveil the marginal path when there is a slight variation of the surrounding conditions. In the context of this work, for instance, the method reveals the optimal operational path corresponding to a slight variation of the energy demands of the analysed EC. In other words, among the various possible ways to respond to a rising demand for energy services, the analysis of the obtained hourly marginal costs allows to determine the way of operating the complex energy system to increase its production with a minimum cost.

Thus, this section aims to analyse in detail and interpret the hourly

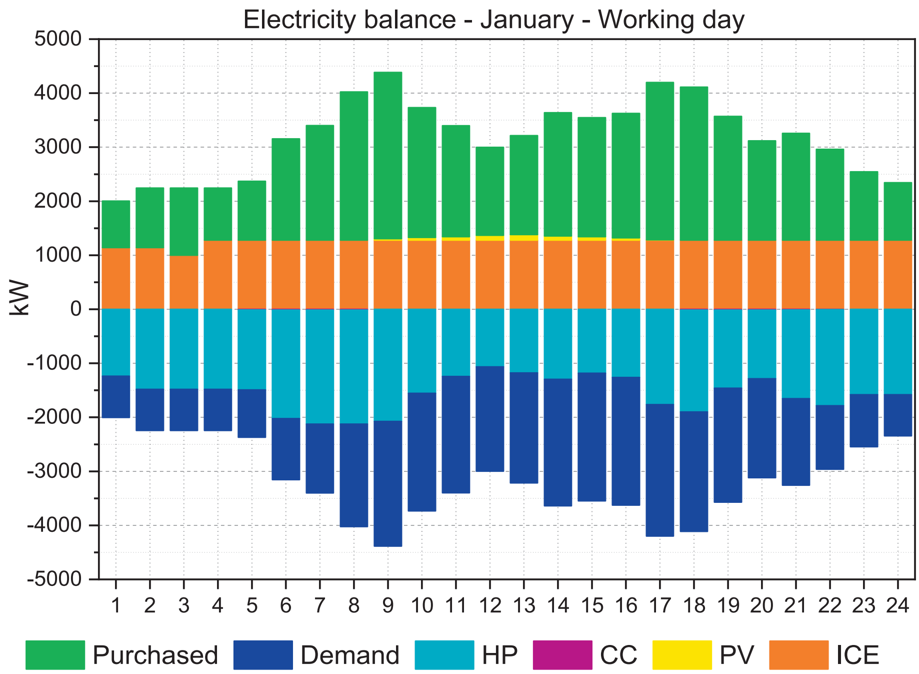


Fig. 11. Electricity balance for the entire EC (all buildings together) and for a January working day.

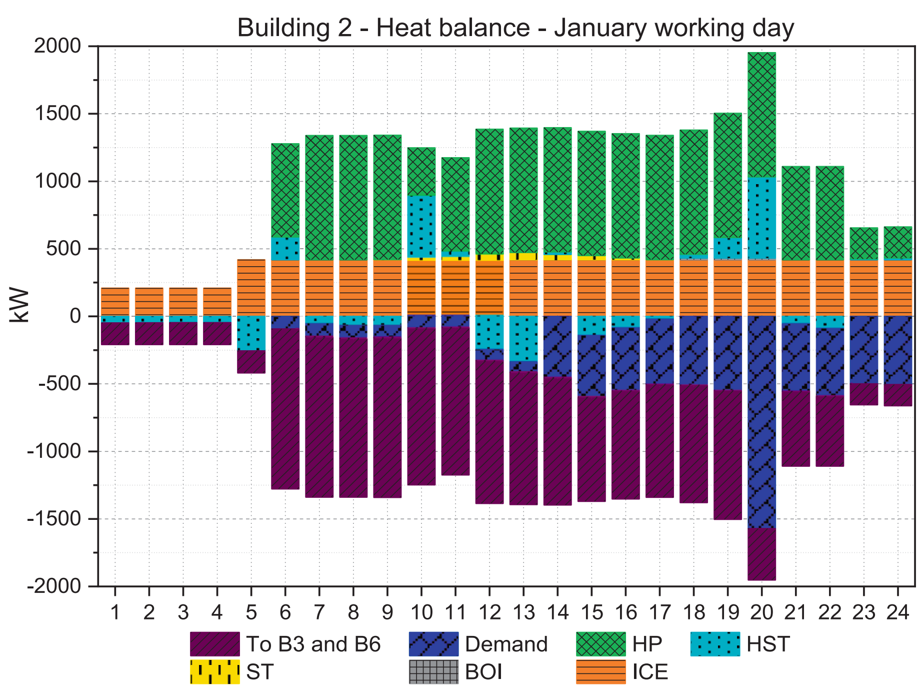


Fig. 12. Building 2 (theatre) heat balance in a winter day.

marginal costs provided by the optimal solution obtained from the LP model of the polygeneration system integrated into the EC. As mentioned in section 3, the model has an hourly resolution and is defined for two typical days per month. To ensure clarity, in the present marginal cost analysis, only one specific day has been selected which was a January working day. The marginal cost values regarding the electricity and heating demands for the nine EC buildings plus central unit, have been provided in section 3.3.1. According to Pina (2019) [12], marginal costs should be evaluated individually for each final product. Therefore, sections 4.1 and 4.2 will individually evaluate the marginal costs related to electricity and heating, respectively.

A complete analysis of the marginal costs regarding the cooling demand can be found in the reference [30].

4.1. *Electricity marginal costs*

The hourly marginal costs associated with the electricity demand (Table 2), are equal to the electricity purchase price[3] for each time band.

[3] The reader may refer to our previous work [31] in order to see how the electricity purchase price was defined.

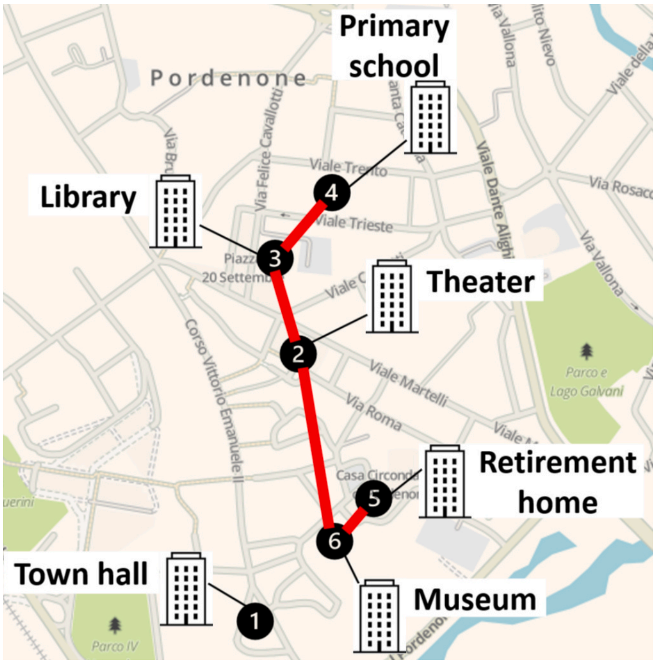


Fig. 13. Optimal economic solution: DHN pipeline connections between buildings 2, 3, and 4 (to the north) and 2, 6, and 5 (to the south), besides building 1 with no pipeline connections.

As the marginal cost analysis carried out within this paper is dealing with the optimal economic solution, it is important for the reader to bear in mind that, for this solution, the EC does not sell electricity at any time throughout the year. Instead, it buys electricity most of the time (in the other times the EC is self-covering its electricity demand). In fact, for the January working day, DS is buying electricity from the grid for all 24 h (Fig. 11). It means that, if any building of the EC would need an additional unit of electricity, the DS would have to buy it from the electric grid at the price of the corresponding hour. For example, if a given building needs an additional unit of electricity at hour 10, the DS would have to purchase it from the grid at the cost of 0.1533 €/kWh.

At this point, a plausible question would be: wouldn't it be possible (and cheaper) for any EC building to self-produce this additional amount of demanded electricity instead of buying it from the grid? To obtain the answer, it is important to note three main aspects: (i) based on the optimal energy supply system structure of each building, the only two ways of self-producing electricity are through PV or ICE, (ii) for the case of PV panels, little solar radiation means little electricity production, even if the installed capacity is greater than the production, and (iii) the ICE units are at full capacity at the January working day under analysis. Therefore, the only left way to obtain one additional unit of electricity is by purchasing it from the grid.

4.2. Heating marginal costs

The hourly marginal costs, associated with the buildings heat demand (for the nine buildings together) and with the heat demanded from central unit are provided through Table 3.

Buildings 2 (theatre) and 7 (hospital) are the ones with the most complex optimal structure among all buildings of the analysed EC. For this reason, and in the interest of conciseness, this section will present the analysis and interpretation of representative marginal cost values from these buildings, which will help to illustrate and explain the four energy service production types (simultaneous, advanced, delayed, and remote) introduced in section 3.4. It is worth remembering that all marginal cost values analysed within this paper regard the optimal economic solution of the polygeneration system integrated to the EC,

Table 7
Marginal costs calculation for building 2, regarding hours 1 to 17. A heat pump is responsible for producing heat at hour 10 and supply the other hours through the TES. Delayed (D) cases from hour 1 to 9, Advanced (A) cases from hour 11 to 17, and Simultaneous (S) case at hour 10. Abbreviations: "[D]" column represents the average cost reduction when comparing to a scenario in which the marginal increase in heat demand is met by the BOI ($Heat_{BOI}$) running at hour 10, instead of the HP; (MC) marginal cost in €/kWh.

Marginal amount of heat to be produced at hour 10 (kWh_h)				$Heat_{HP} = Heat_{out}(h) = 1$ kWh_h	
[A] Efficiency equation				$Elec_{HP} = \frac{Heat_{HP}}{COP}$	
[B] Maintenance cost (€/kWh_{el})				0.001	
[C] Electricity price at hour 10 (€/kWh_{el})				0.1533	
Hour	**MC from LP model (Table 3)**	$Heat_{out}(h)$ **(Eq. (12) for** $x = 10$**)**	**Prod. type**	**How MC was calculated [A·B] + [A·C]**	**[D] (%)**
1	0.0559	0.8337	D	0.0559	-26
2	0.0571	0.8508	D	0.0571	-26
3	0.0582	0.8681	D	0.0582	-26
4	0.0594	0.8858	D	0.0594	-26
5	0.0606	0.9039	D	0.0606	-26
6	0.0619	0.9224	D	0.0619	-26
7	0.0631	0.9412	D	0.0631	-26
8	0.0644	0.9604	D	0.0644	-26
9	0.0657	0.9800	D	0.0657	-26
10	**0.0671**	**1.0000**	**S**	**0.0671**	-26
11	0.0684	1.0204	A	0.0684	-26
12	0.0698	1.0412	A	0.0698	-26
13	0.0713	1.0625	A	0.0713	-26
14	0.0727	1.0842	A	0.0727	-26
15	0.0742	1.1063	A	0.0742	-26
16	0.0757	1.1289	A	0.0757	-26
17	0.0773	1.1519	A	0.0773	-26

and one specific typical winter day, in this case a January working day, with average ambient temperature in the range between 1 °C and 7 °C (refer to Appendix B for more details).

4.2.1. Building 2 (Theatre)

Comparing to buildings 1, and 3 to 6, the optimal solution resulted in a more robust structure of the polygeneration system to be installed in building 2. In fact, building 2 performs the four types of energy service production. Although the heat demand of the theatre takes place only from hour 6 to 24 (Fig. 12), this building presents marginal cost values for all 24 h under analysis. This is because building 2 should send heat to buildings 3 and 6 through the DHN pipelines, and thus, produce heat for 24 h. Then, buildings 3 and 6 can send heat to buildings 4 and 5, respectively (the reader may refer to Fig. 13 and/or to our previous work [31] in order to better understand the connections between the buildings).

According to the optimal economic solution, there are four ways to produce heat in building 2: through heat pump (HP), boiler (BOI), internal combustion engine (ICE), and solar thermal panels (ST). From hour 1 to 4, one of the ICE units is at full load (the other one is turned off). Then, from hour 5 to 24, both ICE units are at full load (Fig. 12). The ST plant cannot deliver more heat due to limitations in its size and solar irradiance. Thus, the only two left options are the HP and BOI.

According to Table 4 and Table 5, the lower marginal cost value would be obtained from the HP. In fact, looking at Table 3, it is possible to identify that, at hour 10, the marginal cost (0.0671 €/kWh_h) is equal to the one regarding the HP (Table 5) with $COP_h = 2.3$[4] and electricity price = 0.1533 €/kWh. Therefore, hour 10 is a case of simultaneous energy service production, i.e., if building 2 needs a marginal amount of

[4] The reader may refer to our previous work [31] in order to better understand the reason for this COP.

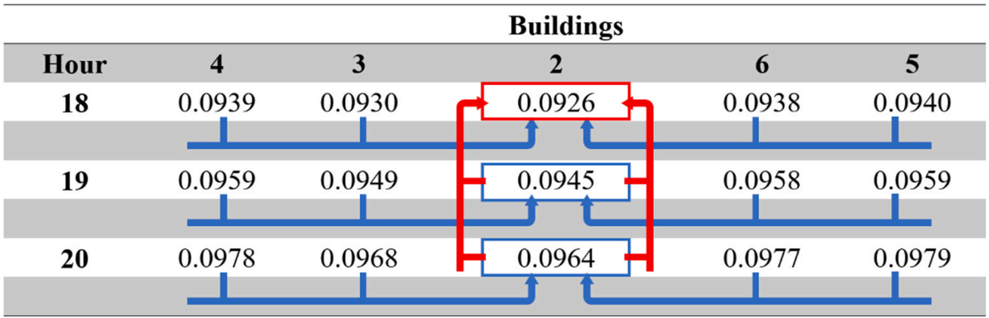


Fig. 14. Marginal costs for building 2, regarding hours 18 to 20. Cases of delayed and remote production.

heat at that time, it can be obtained from the HP since, from the two HP units in operation at that time, one of them is at partial load. Another important aspect from hour 10 is that building 2 is sending an electricity surplus to the DS, which means that, if the heat demand increases at that time, the theatre would send less electricity to the DS in order to power the HP. Such lack of electricity would obligate the DS to purchase more electricity from the grid at the price in force at hour 10 (0.1533 €/kWh).

From hour 1 to 9 and 11 to 17, the energy service production types are delayed and advanced, respectively, as observed on Table 7. At these hours, both ICE and HP units are at full load. Instead of turning the BOI on (which would be more costly), the energy supply system can rely on the hot water storage (HST) to link each one of the mentioned hours to hour 10. The difference is that, for a marginal increase in the heat demand at hours 1 to 9, the HP will have to produce less heat at hour 10. On the contrary, for a marginal increase in the heat demand at hours 11 to 17, the HP will have to produce more heat at hour 10 in order to compensate for HST heat losses. The reasons regarding both cases are illustrated in Fig. 8 and Fig. 9 and explained in section 3.4.1.

To offer a quantitative comparison, two scenarios can be analysed as alluded to earlier: (i) the marginal increase in heat demand from hours 1 to 9 and 11 to 17 met by the HP working at hour 10, and (ii) an alternative scenario where the same marginal increase in heat demand (for the same hours) is met by the BOI working at hour 10. Column [D] of Table 7 shows the average cost reduction when comparing these scenarios, i.e., running the HP at hour 10 results in an average cost reduction of 26 % for each kWh of heat produced when compared with running the BOI at the same hour 10. In both cases, the TES is utilized to shift production in time.

Hours 18 to 20 are supported by the BOI installed in building 2 for two main reasons: (i) both HP and ICE units installed in that building are at full load at these hours, and (ii) at these hours, all heat-producing technologies installed in buildings 3, 4, 5, and 6 (buildings with which building 2 is connected through heat pipelines) are also at full load. It is interesting to note that, for these reasons, buildings 3, 4, 5, and 6 also need support to cover their possible marginal increase in heat demand from hour 18 to 20. The only way to receive such support is through the DHN pipelines. Therefore, for the solution under analysis, the BOI installed in building 2 covers the possible marginal increase in its heat demand and support the possible marginal increase in heat demand of buildings 3, 4, 5, and 6 by means of the DHN pipelines.

Fig. 14 presents the marginal costs related to the hours 18 to 20 for the buildings 2, 3, 4, 5, and 6. It should be noted that the order of the buildings in the table was altered for convenience (according to Fig. 13); this is how the buildings are connected. To the north, building 2 is connected to building 3, which is connected to building 4. To the south, building 2 is connected to building 6, which is connected to building 5. Therefore, besides the delayed production type, this group of buildings (within the hours 18 to 20) also presents the remote production type (section 3.4.2). The following three examples will better explain the relationship between the marginal costs presented in Fig. 14.

Focusing on building 2, if a marginal amount of heat is demanded at hour 18, it will be provided by the BOI installed in the same building at the cost of 0.0926 €/kWh$_h$, which is a case of simultaneous production.

Still on building 2, if a marginal amount of heat is demanded at hour 20, the heat production is advanced to hour 18. The amount of heat that must be produced is obtained through Eq. (12) considering the loss factor related to the HST and by making $h = 20$ and $x = 18$. If the marginal demanded heat is $Heat_{out}(20) = 1kWh_h$, then the heat that must be produced by the BOI at hour 18 is $Heat\_prod_{in}(18) = 1.0412kWh_h$.

In order to exemplify the remote connections, the interpretation of a marginal increase in the heat demand of buildings 3 and 4 will be provided. If building 4 needs an additional amount of heat at hour 19, it must require more heat from building 3 (remote production) since the BOI installed in building 4 is at full load. Knowing that the heat loss factor in the pipeline between these two buildings is 0.01 (Table 6), the marginal amount of heat that building 3 must send to building 4 is $Heat_{building\_3} = 1.0101kWh_h$ (Eq. (13)). At hour 19, the HP of building 3 is also at full load. Then, the only left option for building 3 is to ask $1.0101kWh_h$ more from building 2. Then, at hour 19, the amount of heat that building 2 must send to building 3 is $Heat_{building\_2} = 1.0142kWh_h$ (Eq. (13)) since the heat loss factor in the pipeline between these two buildings is 0.004. Following the same logic of the example explained in the previous paragraph, the marginal demanded heat at the hour 19 of building 2 is $Heat_{building\_2} = Heat_{out}(19) = 1.0142kWh_h$. Then, the heat that must be produced by the BOI at hour 18 (building 2) is $Heat\_prod_{in}(18) = 1.0349kWh_h$ (Eq. (12)). Such marginal path (from hour 19 of building 4 to hour 18 of building 2) is the reason why the marginal cost at the hour 19 of building 4 is 0.0959 €/kWh$_h$.

Another relevant case occurs when a building covers a marginal increase in its heat demand by reducing the amount of heat supplied to another building. If building 2 needs a marginal amount of heat at hour 21 it will obtain support from the HP installed in building 5 through the remote production type. At hour 21, HP and ICE of building 2 are at full load, and the BOI is turned off. Then, a cheaper option to compensate for an increase of one unit of heat demand at hour 21 of building 2 is to send one unit less of heat to buildings 6. However, at hour 21, the only heat source for building 6 is the heat from building 2. For this reason, building 6 should send one unit less of heat to building 5. Since, at hour 21, building 5 has one HP at partial load, the lack of one unit of heat from building 6 will be compensated with one more unit of heat generated by the HP of building 5 (COP = 2.17 and electricity price = 0.1450 €/kWh) at the marginal cost of 0.0673 €/kWh$_h$ (Table 3). It is interesting noting that, in this case, there is no heat transferred from building 5 to building 2. Instead, as mentioned before, the marginal amount of demanded heat in building 2 is covered by sending less heat to building 6 (which will send less heat to building 5). For this reason, there will be no pipeline heat losses and the HP in building 5 will need to produce less heat to compensate for the lack of heat in buildings 2 and 6. Therefore, based on Eq. (13), if building 2 needs $Heat_{building\_2} = 1kWh_h$,

Hour	Building 7	Building 9	Building 8	Central unit
11	0.0691	0.0683	0.0669	0.0669
12	0.0679	0.0671	0.0672	0.0672
13	0.0679	0.0671	0.0676	0.0676
14	0.0679	0.0671	0.0679	0.0679
15	0.0674	0.0697	0.0683	0.0683

Fig. 15. Marginal costs for buildings 7, 9, 8, and central unit (in this order – see Fig. 14). Green arrows: marginal path passing through the HSTc of the central unit; red box: case of simultaneous heat production; red dashed arrows represent a specific case of marginal path explained throughout the text.

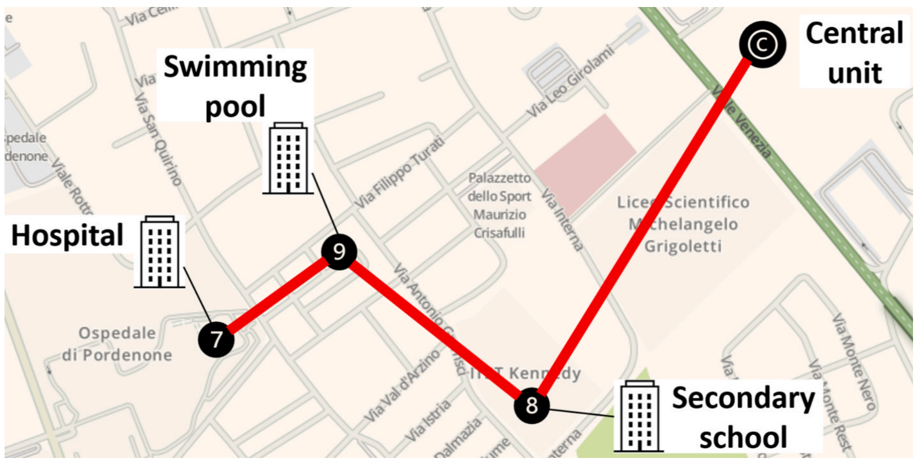


Fig. 16. Optimal economic solution: DHN pipeline connections between buildings 7, 9, 8, and central unit.

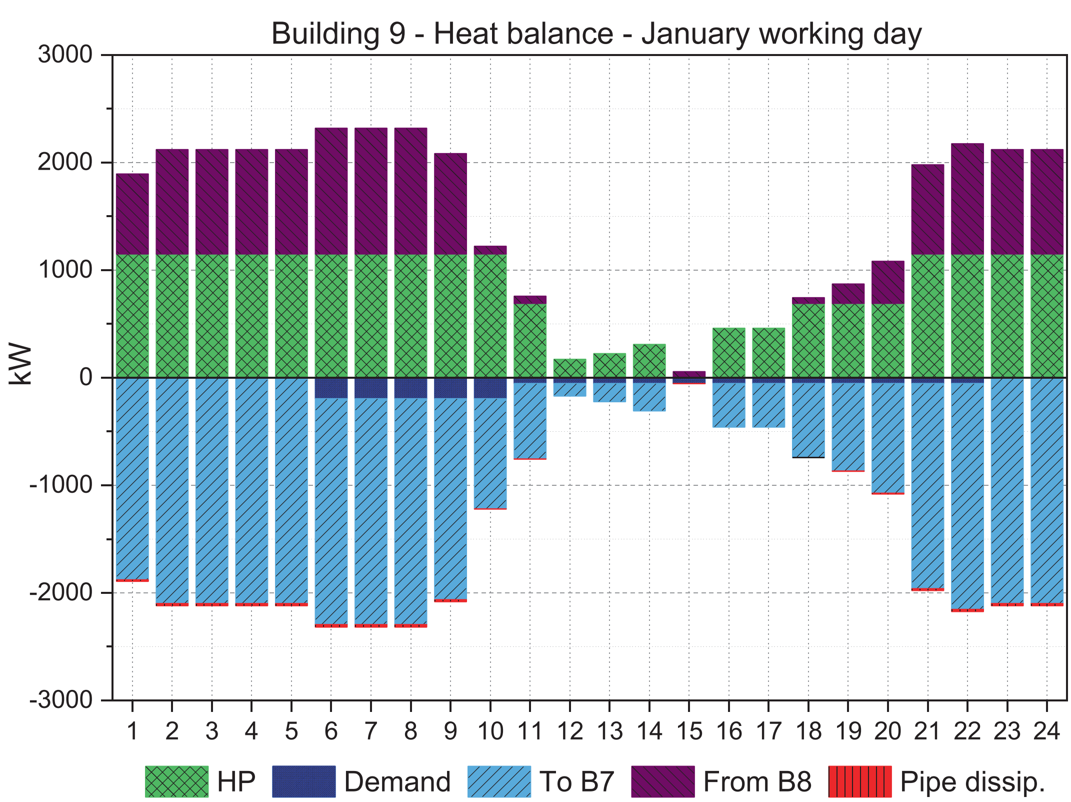


Fig. 17. Heat balance for building 9 (Swimming pool) – January working day.

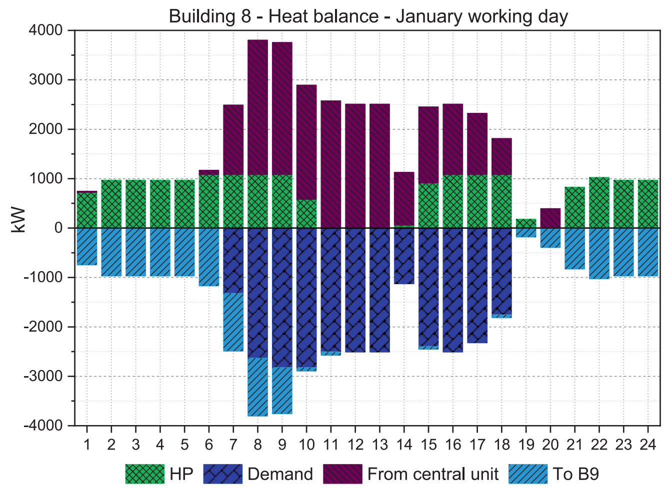


Fig. 18. Heat balance for building 8 (Secondary school) – January working day.

the amount of heat that building 6 will have to produce is $Heat_{building\_6} = 1kWh_h \bullet (1 - 0.013) = 0.987kWh_h$. Then, the amount of heat that building 5 will have to produce is $Heat_{building\_5} = 0.987kWh_h \bullet (1 - 0.0015) = 0.9855kWh_h$. This is the reason why the marginal cost at hour 21 of building 2 is cheaper (0.0663 €/kWh$_h$).

Finaly, if building 2 needs a marginal unit of heat at hour 22, it delays the production to hour 21. Then, the marginal path is the same for hour 21 (building 2) up to building 5. The difference is that the marginal cost of hour 22 (building 2) takes into account the heat loss of the HST from hour 21 to hour 22. Hours 23 and 24 follow the same logic of hour 22.

4.2.2. Building 7 (Hospital)

Another interesting and representative marginal path is the one related to hour 11 of building 7. As indicated in Fig. 15 (dashed red arrows), the mentioned marginal path indicates a first step of remote production by passing through buildings 9, 8, and central unit (at the same hour 11). Then, in a second step, the marginal path indicates delayed production by continuing from hour 11 to hour 15 through the heat storage of the central unit (HSTc) and then (third step), the marginal path indicates again remote production by returning to building 8 at hour 15 (where it has a HP at partial load). Therefore, if building 7 requires a marginal amount of heat at hour 11, it would require such amount of heat from building 9 (see Fig. 16). Then, building 9 would have to provide building 7 with $Heat_{building\_9} = 1kWh_h/(1 - 0.0125) = 1.0127kWh_h$ (Eq. (13)). As building 9 cannot generate any amount of heat at hour 11 (three HPs at full load and two turned off, Fig. 17), it would request the heat from building 8. Then, building 8 would have to provide building 9 with $Heat_{building\_8} = 1.0127kWh_h/(1 - 0.02) = 1.0333kWh_h$. However, building 8 is not able to self-generate that extra amount of heat at hour 11 (HP is turned off, Fig. 18). Then, building 8 requests $1.0333kWh_h$ to the central unit (at hour 11) which is charging its heat storage – HSTc (Fig. 19) at this time. Thus, the central unit can charge $1.0333kWh_h$ less of heat to its storage device and send it to building 7 (through buildings 8 and 9, in this order). However, such missing amount of heat in the HSTc must be offset. The optimal marginal path found by the optimal solution is by sending less heat to building 8 at hour 15 (delayed production) since its HP is at partial load. Using Eq. (12) and knowing that HSTc heat loss is 0.005, it is possible to calculate the exact amount of heat that the central unit will not send to building 8 anymore at hour 15: $Heat\_prod_{in}(15) = 1.0333kWh_h/(1 - 0.005)^{(11-15)} = 1.0128kWh_h$. Therefore, taking into account that it was not considered any heat losses between central unit and building 8, the later must produce $1.0128kWh_h$ at hour 15, by using a 2.26 COP$_h$ HP, at the electricity price of 0.1533 €/kWh. Hence, the marginal cost value (hour 11, building 7) must be equal to $0.0691€/kWh_h$. The reader may check this number by using Table 5 and making $Heat_{HP} = 1.0128kWh_h$.

5. Conclusions

The analysis and interpretation of some relevant hourly marginal costs related to the optimal operation of the polygeneration system

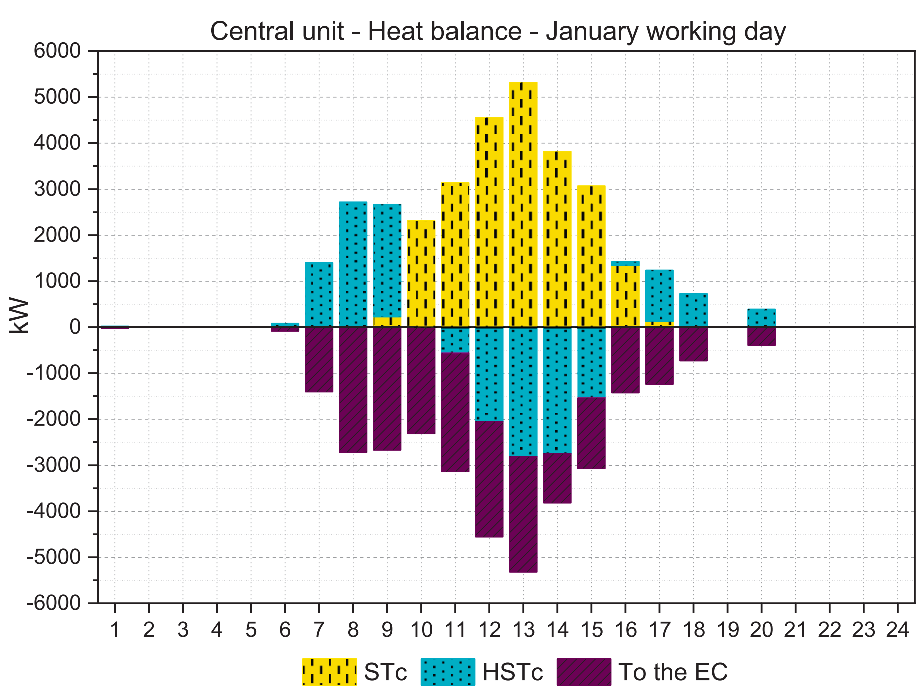


Fig. 19. Central unit heat balance – January working day.

designed for the buildings in the energy community (EC) have been presented in this paper. This analysis and interpretation provided insights that the MILP optimization itself could not provide. The mentioned insights regard mainly the determination of the optimal operation of the system when the hourly energy demand of the building is increased (without changing the operation mode or configuration of the analysed system). In other words, the insights provide the optimal marginal paths connecting energy resource supply and energy demand, which is the cheapest way to provide the marginal amount of demanded energy.

The analysis and interpretation of some representative marginal cost values regarding the polygeneration system installed in buildings 2 and 7 revealed complex operational dynamics across different hours and different buildings. Four types of marginal heat production have been evaluated: simultaneous, advanced, delayed, and remote. The second and third types of heat production are possible thanks to the presence of thermal energy storage (TES) in the polygeneration system. In this way, the heat production can be shifted in time to a previous hour (advanced) or to a later hour (delayed). The fourth type of heat production is possible thanks to the DHN of pipelines between the buildings, which allows a shift in space for the heat production.

From a quantitative perspective, the potential benefit of choosing the optimal marginal path will depend on the available alternatives for covering the same energy service demand. As an example, the comparison discussed on section 4.2.1 indicated an average reduction of 26 % in the marginal cost when choosing the marginal path regarding the heat pump instead of that regarding the boiler. Therefore, the results from the marginal cost analysis provide a pathway of how to optimally operate the polygeneration system in the eventuality of energy demand fluctuations.

It is noteworthy to stress that although the specific findings derived from the analysis of this particular EC are context-dependent and not directly transferable to other cases, the broader goal of this work is to demonstrate a general methodology: given a complex energy system with defined energy resources, installed technologies, demand profiles, and operational strategy, it is possible to compute the marginal costs of the energy services produced and uncover the optimal marginal operational paths in the event of demand fluctuations.

Moreover, the proposed model is sensitive to the variability in external factors such as climatic conditions (namely meteorological data) or energy demands as well as to other factors, such as energy prices. Consequently, the performance of the individual components within the energy system is not assumed to be constant, but rather responsive to these variations. The model proposed in this work does not capture certain key operational non-linear dynamics, such as the start-up or shut-down of devices, nor the associated inefficiencies of such dynamic behaviours. Moreover, the variability in renewable energy generation is addressed using deterministic hourly time-series instead of stochastic or probabilistic methods that consider uncertainty. Thus, the “optimal marginal path” represents the most cost-effective response to marginal changes within the considered fixed operational modes of the pre-defined polygeneration system.

Therefore, this study contributes with valuable insights about several possible marginal paths and the reasons why some paths are less convenient than others. More specifically, it offers the analysis and interpretation of the hourly marginal costs of a complex and highly integrated polygeneration system and the outline of different marginal paths through advanced, delayed, simultaneous, and remote energy services production types, thus contributing to the efficiency improvement of energy supply systems.

CRediT authorship contribution statement

Ronelly José De Souza: Writing – review & editing, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Luis M. Serra:** Writing – review & editing, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Miguel A. Lozano:** Writing – review & editing, Supervision, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Mauro Reini:** Writing – review & editing, Supervision, Project administration, Methodology, Investigation, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Luis M. Serra reports financial support was provided by Spanish State Research Agency. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. – Details of the buildings

This section provides the reader with essential information about the energy community (EC), such as type of the buildings, energy demand profile, location of the buildings, and peak energy demand.

Although the EC has not been implemented, the buildings considered in this study do exist and are located in the city of Pordenone, in northeastern Italy. The nine buildings consist of: the town hall, theatre, library, primary school, retirement home, museum, hospital, secondary school, and swimming pool, in addition to a central unit (the location of each building is shown in Fig. A1). Unlike residential buildings, the facilities under analysis are characterized by highly diverse energy demand profiles. The energy demand of the buildings consists of electricity, heating, and cooling.

The heating demand comprises sanitary hot water (SHW) and spacing heating (SH), which are assumed to be summed in one hourly heating demand for each building. Moreover, the mentioned heating demand does not include the absorption chiller heating demand, which depends on the optimal solution of the MILP model.

The electricity demand of each building is composed mainly by consumption of electric-driven equipment, lighting, etc., while the electricity demand of electric chillers and heat pumps is calculated by the optimal MILP model solution.

Since marginal costs related to cooling demand are not analysed in this work, the corresponding demand profile is not presented here. However, the reader may refer to reference [31] for further details.

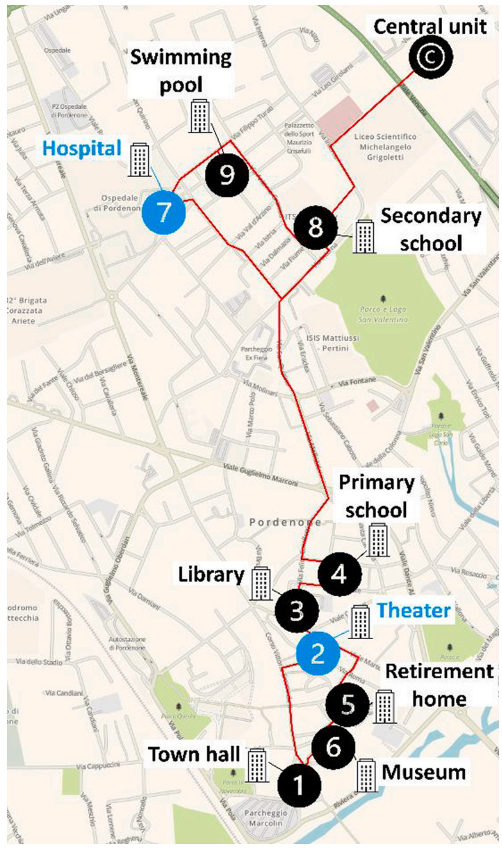

Fig. A1. Description and location of each building. The buildings highlighted in blue are the ones analysed in this work.

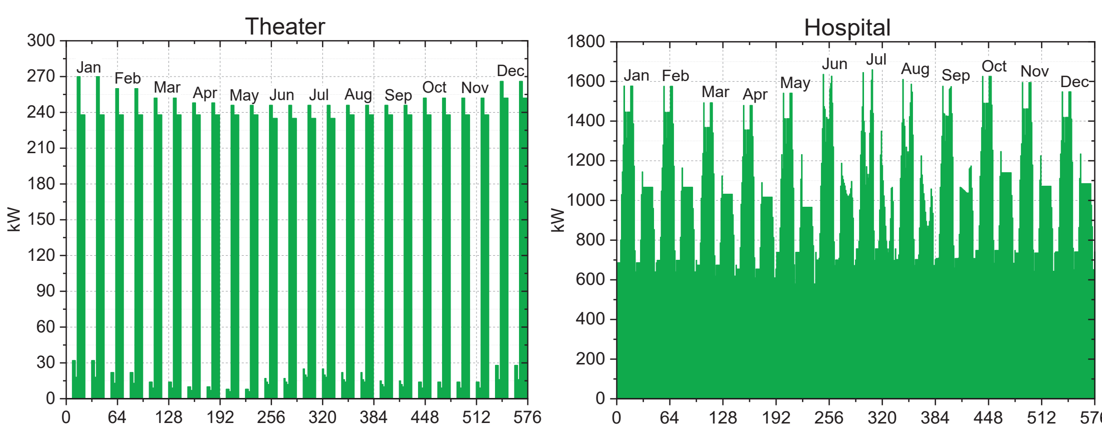

Fig. A2. Hourly electricity demand for the theatre and hospital (two typical days per month).

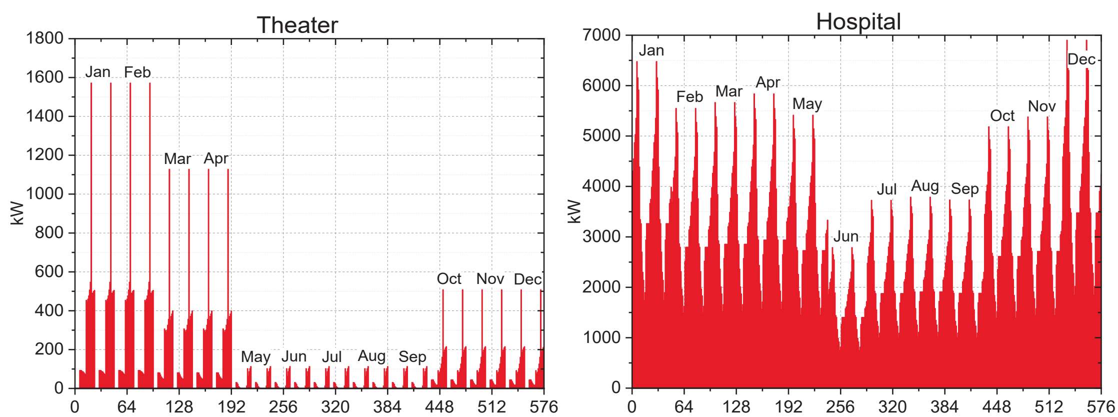

Fig. A3. Hourly heating demand for the theatre and hospital (two typical days per month).

Table A1 presents the annual electricity and heating demands as well as the peak demands for each building. The energy demand for each building was defined based on scientific literature, technical reports, and direct contact with the building administration.

Fig. A2 and Fig. A3 show, respectively, the hourly electricity and heating demand profiles for the theatre and the hospital, across each typical day. The horizontal axis represents time, showing two 24-hours representative days per month. As observed, electricity is demanded throughout the whole year, for both buildings, whereas heating demand is substantially lower during summer due to the absence of SH demand. In other buildings not analysed in this work (such as the library, schools, and swimming pool [30]) electricity and heating demands follows the occupancy levels, with higher consumption during the school year (from September to June). In contrast, the theatre and hospital have an approximately constant electricity demand throughout the year since their occupancy is not influenced by vacation periods.

Table A1
Total annual energy demands and peak demands per building.

Building	Electricity Annual MWh/y	Peak kW_{el}	Heating Annual MWh/y	Peak kW_{th}
1. Town hall	346.6	189	618.9	397
2. Theater	852.2	270	947.7	1572
3. Library	492.2	110	523.8	287
4. Primary school	73.8	54	926.9	572
5. Retirement home	489.0	101	637.4	238
6. Museum	82.5	36	387.3	231
7. Hospital	8840.2	1659.4	23,992.2	6902.9
8. Secondary school	410.3	200	3603.9	2822.6
9. Swimming pool	126.2	23.7	360.8	241.6
TOTAL	11,713.0	2643.1	31,998.9	13,264.1

Appendix B – Climatic data

Fig. B1 provides the hourly solar irradiation at the city of Pordenone, Northeast of Italy, over the period of one year (latitude 45.955 and longitude 12.659). The data was obtained from the online application PVGIS [38] made available by the European Commission. The data is based on the PVGIS-SARAH2 radiation database, considering an elevation of 20 m, slope of 30°, and azimuth of 0°. The hourly air temperature, for the same coordinates, is shown through Fig. B2.

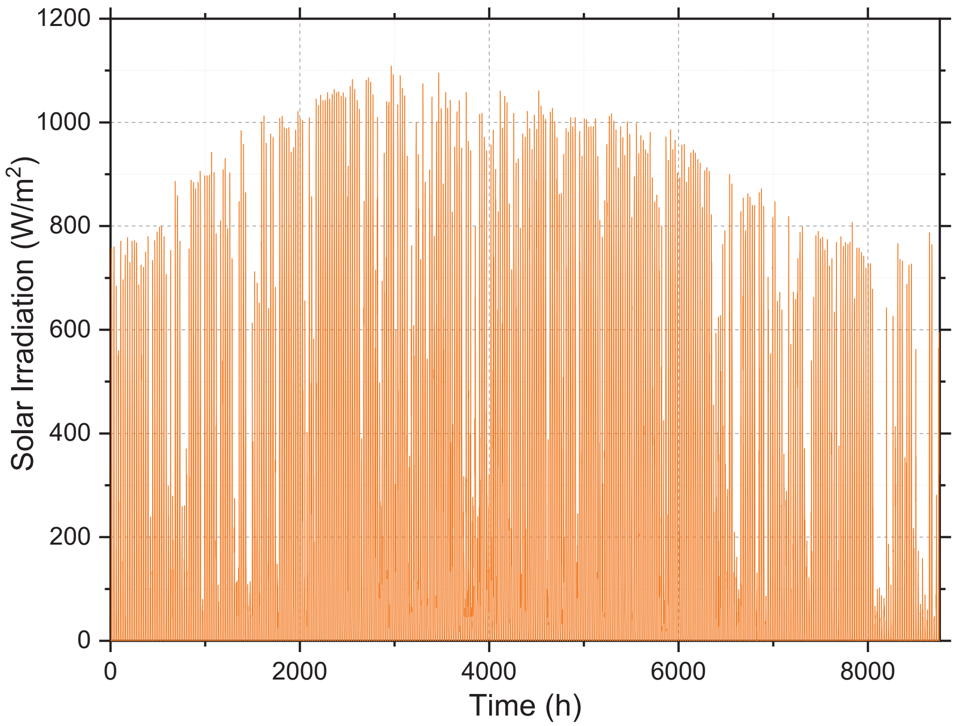

Fig. B1. Hourly solar irradiation at the city of Pordenone, Northeast of Italy [38].

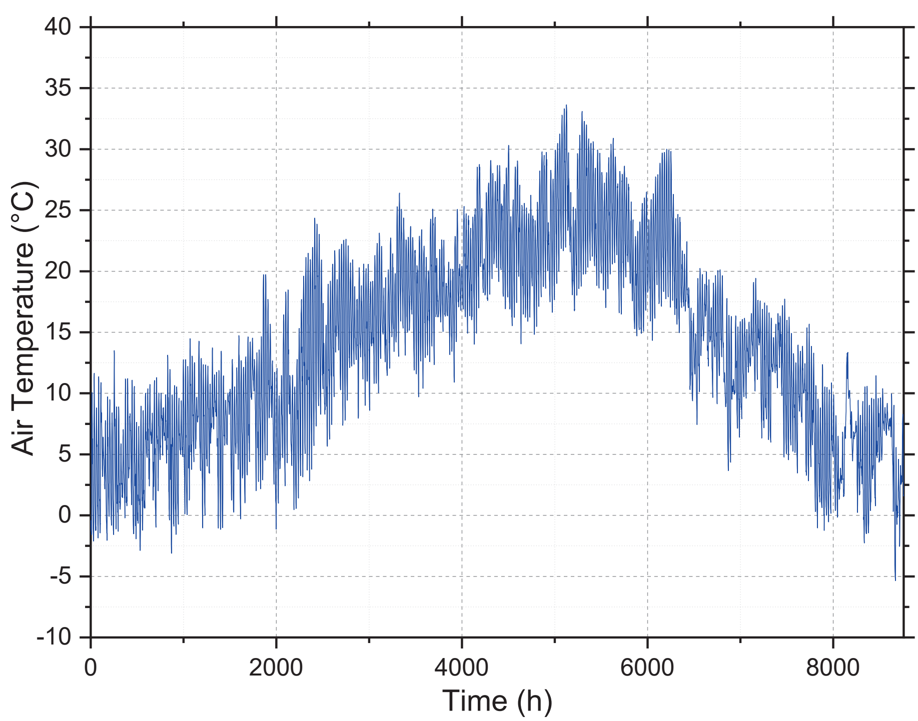

Fig. B2. Hourly air temperature at the city of Pordenone, Northeast of Italy [38].

Data availability

Data will be made available on request.

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