Mutual fund tournaments: A network DEA model using interim rankings to forecast risk-taking and fund inflows

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Abstract

Investors are attracted to well-performing funds, with the top-performing receiving a disproportionately high share of money inflows. This behaviour, combined with incentives like status and monetary rewards, drives the tournament effect, whereby fund managers adjust their portfolio risk to either catch up with competitors or lock their position. Data Envelopment Analysis (DEA) is useful for studying this complex behaviour because it does not assume a predetermined relationship between variables. In this study, we use the Network DEA approach to assess and forecast how efficiently mutual funds compete in this tournament.

We propose and test a model with three stages: reacting to mid-year rankings, improving year-end rankings, and receiving inflows in the subsequent quarter. This study is the first to use DEA to examine dynamic behaviour in mutual fund tournaments. Our findings show that managers who improve their year-end ranks compared to their mid-year ranks are more likely to attract inflows efficiently. However, changes in portfolio beta, concentration, and equity exposure are not directly linked to the rewards at the end of the tournament. Our results remain consistent across different time frames and variable specifications.

Keywords: Behavioural OR, Data Envelopment Analysis, Finance, Forecasting.

Conflict of interest The authors declare that they have no conflict of interest.

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1 Introduction

The performance-chasing behaviour of mutual fund investors is well-documented. Research shows that investors allocate money based on past performance (Sirri and Tufano 1998, Berk and Green 2004, Ferreira et al. 2012, Ben-David et al. 2022). Over the past two decades, this has intensified competition among mutual fund companies for inflows and asset-based fees (Investment Company Institute 2024).

The link between mutual fund performance and managers' subsequent risk-taking has been widely studied. Managers adjust portfolio risk based on past performance (Brown et al. 1996, Chevalier and Ellison 1997, Busse 2001, Taylor 2003, Huang et al. 2011). Performance-based bonuses may intensify this behaviour (Ma et al. 2019) and affect market efficiency (Sato, 2016).

In their seminal research, Brown et al. (1996) found that mid-term losing managers gamble and increase portfolio volatility, while mid-year winners lock in their position. This tournament behaviour is reinforced by an asymmetric relationship between past performance and money flows (Gruber 1996, Chevalier and Ellison 1997, Sirri and Tufano 1998, Huang et al. 2007). Tournaments may also be driven by job security, higher salaries, or reputation (Khorana 1996, Qiu, 2003, Kempf et al. 2009).

However, some studies suggest winners may also gamble (Chevalier and Ellison 1997, Busse 2001, Qiu 2003, Taylor 2003, Sheng et al. 2019). Gärling et al. (2020) propose the above-average bias to explain these rank-based risk strategies, while Eriksen and Kvaløy (2017) and Gärling et al. (2020) suggest that skilled managers see risk-taking as a winning strategy. Recently, Ma and Tang (2019) and Li et al. (2022) have tried to assess the complexity of tournaments by focusing on alternative competition indicators, but reconciling contradictory findings in the literature remains challenging.

Motivated by this challenge, we propose an innovative application of a Network Data Envelopment Analysis (DEA) model to evaluate tournament behaviour in the mutual fund industry. This model captures the dynamic nature of managers' risk-taking decisions and their consequences on performance and inflows. Unlike previous studies, our approach integrates the entire process of tournaments, offering deeper insights into the complex tournament interactions.

We divide tournament behaviour into three stages: how managers react to past performance in terms of portfolio risk, how these risk changes impact subsequent performance, and how these performance changes attract money inflows. Network DEA models, which do not require predefined functional forms for the variables included in the model, are well-suited for modelling this complexity. This network approach allows us to break down and evaluate tournaments into individual processes. As Kao (2014) notes, ignoring these individual processes could provide misleading results, and an overall system could be considered as efficient even if these individual processes are not.

Our empirical findings support that an intense reaction to mid-year ranking does not directly influence subsequent inflows if this reaction has not efficiently improved the year-end ranking. Our network captures this complex structure from the risk-taking reaction to money inflows.

Our study contributes to behavioural operational research, a new sub-discipline advocated by Hämäläinen et al. (2013), which analyses behavioural aspects using operational research methods. Following Becker (2016), our study is part of the innovative application of operational research to behavioural finance.

The paper is organised as follows: Section 2 presents incentives for tournaments and reviews DEA applications in mutual funds, Section 3 introduces the network model and variables, Section 4 presents the data, Section 5 reports the empirical results, and Section 6 concludes.

2 Literature review

2.1 Incentives for Tournament Behaviour in the Mutual Fund Industry

The literature on mutual fund tournaments mainly analyses managers' risk-shifting behaviour in relation to mid-year performance. Brown et al. (1996) suggest that mid-year losers will gamble and increase their volatility, while winners will try to lock in their position and play it safe by reducing their volatility. Other studies provide similar evidence (Goriaev et al. 2005, Acker and Duck 2006, Basak et al. 2008, Schwarz 2012). Agarwal et al. (2022) find that interim losers increase holdings in 'lottery stocks' during the last two quarters, while Li et al. (2022) noted an increase in active share.

Huang et al. (2011) suggest two main reasons for risk changes. First, risk-taking strategies might be caused by perverse-motivated trades of unskilled or agency-prone managers who trade to increase their performance ranking-based compensations. The second reason might be time-sensitive investment opportunities. If risk shifting responds to superior active management abilities, one would expect funds that shift risk to perform subsequently better. However, funds that increase risk often perform worse.

Gärling et al. (2020) review behavioural explanations for the risk shifting evidence when companies use performance-based incentives. Competition, social comparison and skilled strategies explain this behaviour induced by performance rankings. However, without incentives, social comparison does not lead to competition (Gärling et al. 2021)

Three key factors explain tournaments in the mutual fund industry. First and foremost, consistent evidence presents past performance rankings as one of the topmost decisive factors in investors' choice of a mutual fund (Sirri and Tufano, 1998). Mutual fund companies typically charge a fee that is calculated as a fixed percentage of the assets under management (Gil-Bazo and Ruiz-Verdú 2009). Therefore, best performing funds subsequently attracts the highest money inflows increasing their revenue (Gruber 1996, Chevalier and Ellison 1997, Sirri and Tufano 1998). Ha and Ko (2017) found that a change in fund risk is associated with a positive and convex relationship with the subsequent net flows. Ben-David et al. (2022) concluded that investors rely more on simple performance indicators that are readily available such as fund rankings and ratings than on complex performance measures. This suggests that prior-period rankings could a better predictor of subsequent inflows into funds than more complex measures.

Secondly, Kirchler et al. (2018) emphasized non-monetary incentives, such as self-image and status. Relatedly, O'Connell and Teo (2009) supported the view that professional investors' self-worth is linked to their investment abilities. Sheng et al. (2019) noticed that managers typically have two main monetary compensation incentives: an explicit incentive, which is a fixed percentage of the assets under management, and an implicit incentive, which is derived from the future inflow attracted by their performance. Thus, alongside non-monetary interests, managers have reasonable monetary interests in performing better than their peers. Performing better attracts more inflows and improves self-image and job security (Khorana 1996, Qiu 2003). Kempf et al. (2009) suggested that when there is some prospect of employment risk, interim losers tend to decrease their risk compared with winning managers, in an attempt to secure their jobs. It also supports the idea that in the presence of low employment risk, compensation incentives become the main drivers of managers' behaviour. Hu et al. (2011) also examine the role of employment risk in tournaments, documenting the presence of a U-shaped relation between prior performance and subsequent risk.

Finally, empirical evidence shows an asymmetry in the performance-flow relationship (Gruber 1996, Chevalier and Ellison 1997, Sirri and Tufano 1998, Huang et al. 2011). The penalization for interim losers is disproportionately lower than the gain in inflows derived by interim winners. This asymmetry then generates a stronger incentive for underperforming managers (interim losers) to gamble by increasing their volatility.

Nevertheless, there is no consensus on the performance level that drives risk increases. Chevalier and Ellison (1997) provided evidence of increase in tracking error of portfolios consistent with tournament theory, specifically in the last quarter of the year. Sheng et al. (2019) concluded that mid-year performers above the median hold riskier assets to maintain the lead. Busse (2001), using the same data as Brown et al. (1996) but with a higher frequency, did not find evidence of interim losers changing their risk level. Taylor (2003), based on a theoretical model, suggested that when managers compete against a publicly available benchmark, the winners are more likely to gamble. Qiu (2003) showed that managers near the top are more likely to gamble due to the "winner takes all" phenomenon, while employment concerns curb risk-taking. The dysfunctional consequences of the use of league tables (rankings) in performance evaluation have been documented in other literature too. In the context of investment decision making by organisational sub-units, Keasey et al. (2000) report an increase in risk seeking behaviour when there is a possibility of a sub-unit moving from second to first in the league table. Elsewhere, Kuziemko et al. (2014) show that individuals accept gambles with the potential to move them out of last place that they would reject otherwise.

Rather than viewing the mixed findings as contradictory, we believe there are more nuanced insights to uncover in the tournament theory than the simplistic view that losers will gamble, and winners will index. For instance, Anderson (2012) provides an analytical framework for understanding decision-making in ranking games and gambling scenarios, concluding that it is sometimes better to quit while ahead rather than risk losing what has already been gained. Gaba et al. (2004) offer a detailed examination of how altering performance variability and correlations can influence the results of winner-take-all contests, where only the top performer receives the reward. To sum up, competition for recognition in rankings of top funds, which can lead to large inflows of money from investors and large rewards for fund managers, shows complex and difficult-to-model interactions in this real-world ranking game. Our network DEA model does not assume any functional relationship between the tournament variables and allows us to build a network approach that captures the actual tournament interactions without any pre-established functional forms.

2.2 DEA Applications to Mutual Funds

DEA is a non-parametric frontier model introduced by Charnes et al. (1978) to evaluate the efficiency of decision-making units (DMUs). Reviews of real-world DEA applications confirm its wide application in the financial industry (Cook and Seiford 2009, Emrouznejad and Yang 2018), particularly in banking, mutual funds, and insurance (Kaffash and Marra 2017).

In the mutual fund industry, DEA is primarily used to evaluate mutual fund performance, offering an alternative to traditional performance measures like risk-adjusted returns and alphas (Sharpe 1966, Jensen 1968). Murthi et al. (1997) presented a pioneer DEA model to assess mutual fund performance. Murthi et al. (1997) stressed three main advantages of DEA over parametric performance models. First, DEA is flexible, enabling simultaneous use of multiple inputs and outputs. Second, it compares DMUs with similar inputs and outputs, identifying inefficient units and measuring inefficiency magnitude. Applied to mutual funds, DEA positions each fund relative to a frontier of the most efficient ones, revealing lagging funds and factors driving inefficiency. Finally, DEA requires no functional form for the input-output relationship, eliminating the need for parametric connections.

Based on these three advantages, early papers used DEA to evaluate mutual fund performance as a single-stage production process (McMullen and Strong 1998, Basso and Funari 2001, Choi and Murthi 2001, Galagedera and Silvapulle 2002). From this early stage, literature on single-stage DEA for mutual fund performance has expanded, with notable examples including Joro and Na (2006), Lozano and Gutiérrez (2008), Glawischnig and Sommersguter-Reichmann (2010), Kerstens et al. (2011), Lamb and Tee (2012a, 2012b), Branda (2015), Brandouy et al. (2015), and Liu et al. (2015).[1]

Growing studies use single-stage DEA to analyse other mutual fund aspects, including socially responsible investments (Basso and Funari 2003, 2014, Pérez-Gladish et al. 2013, Ayadi et al. 2015), managers' skills (Banker et al. 2016, Andreu et al. 2019), fund size and performance (Basso and Funari 2017), and sustainability information disclosure (Zhou et al. 2018).

More recent studies assess mutual fund performance using a multi-stage DEA process, providing detailed insights into various management aspects beyond a single performance score. Premachandra et al. (2012) employ a two-stage DEA model to break down overall efficiency into operational management and portfolio management components. Galagedera et al. (2016)

[1] Kaffash and Marra (2017) include these papers in the main path of DEA developments of the mutual fund industry.

extend this model with leakage variables. Sánchez-Gónzalez et al. (2017) propose a three-stage network DEA model to evaluate the interactions between the operational management efficiency of mutual fund companies and the efficiency of their core competencies, dividing the core into portfolio management and marketing stages. Galagedera et al. (2018) further model mutual fund management as a serially linked three-stage process, including operational management, resource management, and portfolio management.

These advancements in network DEA methodology applied to mutual fund performance offer a valuable approach to addressing the scarcity of DEA research on behavioural dynamics within the mutual fund industry. Given the nature of behavioural finance, defining an appropriate DEA model can be challenging. Conceptualising these behavioural dynamics as a multi-stage process could open new research avenues for modelling complex behavioural patterns in finance. Our paper seeks to contribute to this emerging field.

3 Model and Variables

3.1 Proposed Network Structure

To the best of our knowledge, our approach is the first application of a network DEA structure to analyse dynamic behaviour in the mutual fund industry, extending earlier studies of three-stage network DEA models such as Sánchez-Gónzalez et al. (2017) and Galagedera et al. (2018).

Our network approach (Figure 1) comprehensively evaluates the tournament interactions in a mutual fund j through three stages: the Reaction Stage, which assesses the relevance of the tournament response in terms of risk management; the Recompense Stage, which examines the impact of this response on subsequent performance rankings; and the Reward Stage, which evaluates the visibility of this impact in terms of monetary inflows. Without this holistic approach, a mutual fund's significant tournament response, even if it positively affects its relative performance, may hold little value for the fund company if this behaviour is not rewarded and translated into increased money flows. Our three-stage approach allows for the evaluation of each individual stage within our network structure and overcomes the problems of overall systems deemed efficient, even when their individual processes are not.

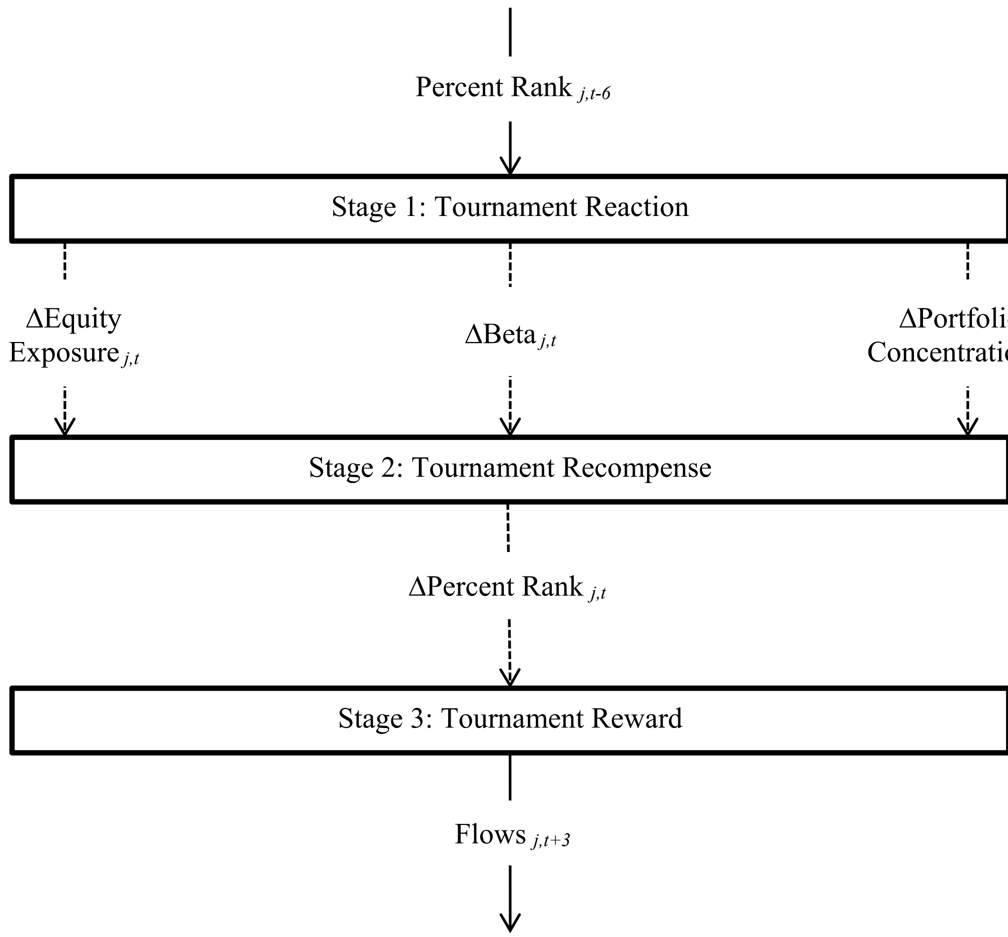

Fig. 1 Three-Stage Network of Mutual Fund Tournaments.

This figure shows the tournament interactions in a mutual fund *j* as a three-stage network structure. *Percent Rank*$_{j,t-6}$ is the input variable at Stage 1: Tournament Reaction, and *Flows* $_{j,t+3}$ is the output variable at Stage 3: Tournament Reward. There are also four intermediate variables (dotted lines). *ΔEquity Exposure* $_{j,t}$, *ΔBeta*$_{j,t}$, and *ΔPortfolio Concentration*$_{j,t}$ are outputs at Stage 1: Tournament Reaction and are used as inputs at Stage 2: Tournament Recompense; *ΔPercent Rank*$_{j,t}$ is an output at Stage 2: Tournament Recompense that is considered as an input at Stage 3: Tournament Reward.

At the Reaction Stage, our network approach captures the intensity of the reaction of managers as a change in the risk level of their portfolios in the second half on the year (from month *t-6* to year-end *t*), as a consequence of their relative performance in the first half of the year (measured at month *t-6*). At the Recompense Stage, our model evaluates the impact of this risk management on their year-end relative performance, assessing whether managers' tournament reactions result in changes in relative performance in the second half. Finally, at the Reward Stage, our model evaluates the success of this behaviour based on the money inflows into funds in the first trimester of the subsequent year (from year-end *t* to month *t+3*). Thus, our three-stage approach differentiates the timing of the response of the manager to the prior relative performance of fund *j* and the potential consequences in terms of money flows.

According to Kao (2014), Figure 1 corresponds to an extension of a basic two-stage into a basic three-stage network structure.[2] Our network includes a dynamic component with different model variables corresponding to sequential points to reflect the dynamic behaviour of tournaments. However, our network approach does not explore the interacting dynamics of inputs and outputs between different years. Similar to a multi-period network production system (Esmaeilzadeh and Matin 2019), each fund in our network model generates time-dependent inputs, outputs, and intermediate variables over a defined time period. Using this information, our network approach evaluates how managers react to their ranking position in the first half of the year, and the consequences of this reaction in terms of subsequent rankings and flows. This evaluation will be reiterated annually throughout our sample period.

Using four intermediate variables might raise concerns about the curse of dimensionality. Therefore, we adhere to the DEA convention that the minimum number of decision-making units should exceed three times the number of variables (Coelli et al. 2005). This condition is well satisfied in our analyses.

Huang et al. (2011) identified three mechanisms through which mutual funds can shift risk: by modifying their liquidity ratio, by altering their exposure to systematic risk or by changing their exposure to idiosyncratic risk. First, to increase the level of risk in search of a ranking improvement, mutual fund managers can reduce their cash holdings and/or increase their equity holdings, all other things being equal. The documented significance of asset allocation as a

[2] Halkos et al. (2014) survey and classify the two-stage DEA models and present the applications of this network approach across the literature.

relevant factor to explain mutual fund returns justifies the choice of the change in equity allocation to capture the managers' reaction to mid-year rankings. Second, managers can also replace low-beta stocks with high-beta stocks, thus increasing their exposure to systematic risk. Literature has extensively shown that market exposure is one of the significant pricing factors of mutual funds. The variation of beta aims to capture managers' reaction in terms of this market exposure. Finally, fund managers can also react by concentrating their holdings on fewer stocks or fewer industries, thus increasing their exposure to idiosyncratic risk. Portfolio concentration has shown to be a significant measure of active management, and how this management bets on different idiosyncratic characteristics of their portfolio holdings. The choice of the change in the portfolio concentration constitutes an effective and direct measure to compute fund managers' reaction in terms of idiosyncratic risk.

Mid-year winners and losers has been used in several tournament studies such as Brown et al. (1996), Busse (2001), Taylor (2003), and Goriaev et al. (2005) to explain the reaction of fund managers. The straightforward identification and significance of mid-year rankings in the previous tournament literature justifies the choice of this variable in our model. Following this previous rationale, at the Reaction Stage, we model that mutual fund *j* reacts to its mid-year ranking by changing its risk level from month *t-6* to year-end *t* through three different mechanisms: 1) the percentage of the portfolio allocated to equity assets as representative of the riskiest asset,[3] 2) the portfolio beta as representative of the systematic risk, and 3) the portfolio concentration as representative of the idiosyncratic risk.

This Reaction Stage aligns with the seminal paper of Brown et al. (1996) who concluded that mid-year losing managers, not having much more to lose, will tend to gamble and increase their levels of portfolio risk, while mid-year winners will try to lock in their positions by playing it much safer than those managers at the bottom of performance ranking. Thus, a fund with a poor relative ranking in month *t-6* with significant increases in equity portfolio allocation, portfolio beta and portfolio concentration will lead to high DEA scores at the Reaction Stage, providing evidence of an important tournament response. On the other hand, a fund with a poor relative ranking in month *t-6* with small changes in its portfolio risk will provide low DEA scores and thus a weak tournament response. Our model could even lead to lower DEA scores of mid-year winning funds compared to mid-year losing funds when the former tries to lock their previous

[3] See Ibbotson and Harrington (2021) for an analysis of historical returns of the major asset types of the US market.

performance positions with decisions on equity portfolio allocation, portfolio beta and portfolio concentration contrary to risk-shifting strategies. Further, the Reaction Stage also covers the scenario of mid-year winners gambling more than mid-year losers do (Busse 2001, Sheng et al. 2019). In this case, the values of the prior rankings (considered as inputs at this stage), and the level of the subsequent changes in the portfolio risk (considered as outputs at this stage) will rank the intensity of the reaction of both winners and losers and the DEA score at this stage.[4,5]

At the Recompense Stage, our model evaluates how efficient the risk management is. This efficiency is evaluated in terms of the impact of the tournament response on the subsequent performance rankings. The first aim of managers with an important tournament reaction is to improve their previous performance ranks. The Recompense Stage evaluates whether the efforts of managers after mid-year have brought about any changes in their performance ranks. Thus, the outputs at the Reaction Stage are now the inputs of the Recompense Stage, building the first linking node in our network structure. Significant improvements in the subsequent performance ranking will lead to higher DEA scores with smaller increases of portfolio risk rather than with larger increases of portfolio risk. On the other hand, negative consequences in the subsequent performance ranking will be represented by lower DEA scores with larger risk-shifting strategies rather than with less important risk changes. While the changes in the equity allocation, portfolio beta and portfolio concentration showed the intensity of fund managers' reaction at first stage, this reaction is now evaluated in terms of improvement of the year-end ranking. This is the rationale of the first linking node of our network structure.

Finally, at the Reward Stage our model evaluates how efficient the impact of tournament behaviour has been in terms of money flows. Previous literature has extensively provided evidence for the "winner takes all phenomenon" in which winning funds capture a disproportionate share of total inflows (Gruber 1996, Chevalier and Ellison 1997, Sirri and

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[4] Although DEA scores are based on the rationale of efficiency, it is necessary to underscore that Tournament Reaction scores evaluate the intensity of managers' reaction to mid-year rankings rather than how efficient this reaction has been. "Losers" and "Winners" at this first stage should be then associated with lower and higher levels of reaction relative to mid-year rankings, respectively. The efficiency of this reaction will be evaluated by the Recompense stage.

[5] An alternative to our tournament model could consider mid-year ranking as an exogenous variable instead of a discretionary one. While this approach could lead to easier interpretations of managers' reaction as this DEA model would not aim at reducing this input in the first stage, tournament literature establishes that mid-year rankings are not merely exogenous outcomes but results of previous management decisions that directly influence subsequent tournaments. Treating mid-year ranking as discretionary reflects the endogenous nature of this variable as result of the decision-making process during the first half of the year and its connection with the behavioural dimension of managers' reaction. Further models could discuss the exogenous or endogenous nature of the Reaction stage.

Tufano 1998, Qiu 2003, Huang et al. 2007). The tournament response to performance ranking aims to improve the future performance ranking. Indeed this improvement is not only motivated by the need for good reputation, which could be important for managers' own career plan, but it is also extremely important for mutual funds in terms of money flows, as these flows can play a relevant role in the fee structure of the fund company.[6] The Reward stage evaluates if the successful/unsuccessful results of risk shifting strategies at year-end are translated into money flows. Thus, the input of the Reward Stage is the output of the Recompense Stage, building the second linking node in our network structure. Funds that obtain significantly higher flows from minor ranking improvements will achieve the highest DEA scores at the Reward Stage, while significantly lower flows after substantial ranking improvements result in the lowest DEA scores, indicating that investors have not noticed the positive impact of the tournament.

3.2 Proposed Network Structure

In this section, we describe the suitable procedure to model the network structure presented in Figure 1. We work with n funds (j = 1, …, n) consisting of 3 stages (k = 1,2,3). Let m_k and r_k be the numbers of inputs and outputs at stage k, respectively. The link from stage k to stage h is denoted by (k,h) and the set of links by L. The inputs of fund j at stage k are $\left\{x_j^k \in R_+^{m_k}\right\}$ and the outputs of fund j at stage k are $\left\{y_j^k \in R_+^{r_k}\right\}$, where ($j$=1,…, n; k= 1, 2, 3). The link variables from stage k to stage h are $\left\{z_j^{(k,h)} \in R_+^{t(k,h)}\right\}$ (j=1,…, n; $(k,h) \in L$), where $t_{(k,h)}$ is the number of items in link (k,h). $\lambda^k \in R_+^n$ is the intensity vector of stage k. Our network structure (Figure 1) includes one input to stage 1, one output from stage 3, three link variables from stage 1 to 2, and one link variable from stage 2 to 3. Thus, the production possibility set $\{(x^k, y^k, z^{(k,h)})\}$ for our model is:

$$
\begin{array}{lll}
x^1 \geq X^1\lambda^1 & & \\
y^3 \leq Y^3\lambda^3 & & \\
z^{(1,2)} = Z^{(1,2)}\lambda^1 & \text{(as intermediate outputs from 1),} & \\
z^{(2,3)} = Z^{(2,3)}\lambda^2 & \text{(as intermediate output from } 2\text{),} & (1) \\
z^{(1,2)} = Z^{(1,2)}\lambda^2 & \text{(as intermediate inputs to } 2\text{),} & \\
z^{(2,3)} = Z^{(2,3)}\lambda^3 & \text{(as intermediate input to } 3\text{),} & \\
e\lambda^k = 1,\ \lambda^k \geq 0 & (k = 1, 2, 3) &
\end{array}
$$

The restriction $e\lambda^k=1$ in Equation 1 includes the different scales at which funds could operate at the tournament stages, thereby assuming the variable returns-to-scale (VRS) hypothesis.

[6] This effect might be particularly relevant in mutual fund industries in which management fees are largely based on assets under management instead of performance-based fees. This is the case of Spain (Díaz-Mendoza et al. 2014).

We follow the widely used slacks-based measure (SBM) proposed by Tone (2001) to solve the network proposed in Figure 1. SBM is a non-radial DEA model for measuring efficiency when inputs and outputs may change non-proportionally. SBM works with excess inputs and output shortfalls simultaneously and can be applied under CRS and VRS assumptions.

According to the non-oriented VRS approach of the SBM model, a target fund $\{x_o^k, y_o^k\}$ will be efficient at each stage k in terms of Pareto-Koopmans when it has no input excesses and no output shortfalls for any optimal solution. The general SBM formulation for each stage is:[7]

$$\rho_o^{SBM_k} = \min_{\lambda^k, s^{k-}, s^{k+}} \frac{1 - \frac{1}{m_k}\left(\sum_{i=1}^{m_k} \frac{s_i^{k-}}{x_{io}^k}\right)}{1 + \frac{1}{r_k}\left(\sum_{r=1}^{r_k} \frac{s_r^{k+}}{y_{ro}^k}\right)}$$

subject to: (2)

$$X^k \lambda^k + s^{k-} = x_o^k$$
$$Y^k \lambda^k - s^{k+} = y_o^k$$
$$e\lambda^k = 1,\ \lambda^k, s^{k-}, s^{k+} \geq 0,$$

where $X^k = (x_1^k, \cdots, x_n^k) \in R^{m_k \times n}$; $Y^k = (y_1^k, \cdots, y_n^k) \in R^{r_k \times n}$, and s^{k+} and s^{k-} are the slacks, i.e., the non-negative vectors of input excesses and output shortfalls of stage k, respectively. In this approach, the link variables $Z^{(k,h)}$ and their slacks must be included in the sets of ordinary inputs X^k or outputs Y^k previously defined in our network structure (Figure 1). Let $(\lambda_o^{k*}, s_o^{k-*}, s_o^{k+*})$ and $\rho_o^{SBM_k} = 1$ be an optimal solution of the model presented in Equation 2.

Following Tone and Tsutsui (2009), there are two major SBM alternatives for evaluating tournaments as represented in Figure 1: an SBM-based separation model and a Network SBM model (NSBM). In the separation approach, we could evaluate each of the three stages individually using intermediate variables as ordinary inputs or outputs as in the aforementioned SBM model (Equation 2), thereby omitting any continuity between the three stages. While the SBM-based separation model does not capture the interaction between the stages proposed in our network dynamics of mutual fund tournaments, we include it initially in the empirical analyses to follow by way of a comparator against which we are able to highlight the nuanced behavioural insights offered by the NSBM model to which we now turn.

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[7] Appendix A details the SBM formulation (Equation 2) and the NSBM formulation (Equation 3) considering the specific structure and variables of our tournament network (Figure 1).

In the NSBM approach, Tone and Tsutsui (2009) proposed the weighted SBM model (Cooper et al. 2007, Tsutsui and Goto 2009) to decompose the overall score of the network structure into a weighted score of partial efficiencies where the weights are set exogenously. We follow one of the NSBM extensions proposed by Tone and Tsutsui (2009) to integrate the slacks of the link variables individually and independently into the NSBM objective function.

After setting exogenously the relative importance w^k of each stage k in the overall efficiency measure, this NSBM approach evaluates the non-oriented overall efficiency of a target fund $\{x_o^k, y_o^k, z_o^{(k,h)}\}$ under VRS assumption, including the slacks $s^{(f,k)-}$ of the intermediate input from stage f to stage k at link (f,k), and the slacks $s^{(k,h)+}$ of the intermediate output from stage k to stage h at link (k,h). The model does not allow neither intermediate outputs from stage k to f, nor intermediate inputs from stage h to k. The numbers of non-intermediate inputs and non-intermediate outputs at stage k are m_k and r_k, respectively. P_k is the set of stages having the link $(f,k)\in L$ (predecessor of stage k), and $t_{(f,k)}$ is the number of intermediate variables in that link; and F_k is the set of stages having the link $(k,h)\in L$ (successor of stage k), and $t_{(k,h)}$ is the number of intermediate variables in that link. Our network structure (Figure 1) works with one input to stage 1, one output from stage 3, three intermediate variables from stage 1 to 2, and one intermediate variable from stage 2 to 3. Thus, our NSBM approach is represented as follows:

$$\rho_o^{NSBM} = \min_{\substack{\lambda^1,\lambda^2,\lambda^3,\\ s^{1-},s^{3+},s^{(1,2)-},s^{(2,3)-},s^{(1,2)+},s^{(2,3)+}}} \frac{\sum_{k=1}^{3} w^k \left[1 - \frac{1}{m_k + \sum_{f\in P_k} t_{(f,k)}} \left(\sum_{i=1}^{m_k} \frac{s_i^{k-}}{x_{io}^k} + \sum_{f\in P_k} \frac{s_f^{(f,k)-}}{z_{fo}^{(f,k)}} \right)\right]}{\sum_{k=1}^{3} w^k \left[1 + \frac{1}{r_k + \sum_{h\in F_k} t_{(k,h)}} \left(\sum_{r=1}^{r_k} \frac{s_r^{k+}}{y_{ro}^k} + \sum_{h\in F_k} \frac{s_h^{(k,h)+}}{z_{ho}^{(k,h)}} \right)\right]}$$

subject to: (3)

$$\begin{array}{ll}
X^1\lambda^1+s^{1-}=x_o^1 & e\lambda^1=1 \\
Y^3\lambda^3-s^{3+}=y_o^3 & e\lambda^3=1 \\
Z^{(1,2)}\lambda^2+s^{(1,2)-}=z_o^{(1,2)} & Z^{(1,2)}\lambda^1=Z^{(1,2)}\lambda^2 \\
Z^{(2,3)}\lambda^3+s^{(2,3)-}=z_o^{(2,3)} & Z^{(2,3)}\lambda^2=Z^{(2,3)}\lambda^3 \\
Z^{(1,2)}\lambda^1-s^{(1,2)+}=z_o^{(1,2)} & Z^{(1,2)}\lambda^2=Z^{(1,2)}\lambda^1 \\
Z^{(2,3)}\lambda^2-s^{(2,3)+}=z_o^{(2,3)} & Z^{(2,3)}\lambda^3=Z^{(2,3)}\lambda^2 \\
\multicolumn{2}{l}{\lambda^1,\lambda^2,\lambda^3,s^{1-},s^{3+},s^{(1,2)-},s^{(2,3)-},s^{(1,2)+},s^{(2,3)+}\geq 0}
\end{array}$$

A tournament will be overall efficient under the NSBM model in Equation 3 when the optimal input and output slacks (s^{1-*}, s^{3+*}) together with optimal intermediate input and output slacks ($s^{(1,2)-*}$,$s^{(2,3)-*}$, $s^{(1,2)+*}$, $s^{(2,3)+*}$) result in $\rho_o^{NSBM}=1$.

Our NSBM model is aligned with the cooperative approach proposed by Liang et al. (2008) to evaluate two-stage processes. According to the rationale of the network structure of mutual fund tournaments in Figure 1, we cannot assume leader or follower stages for the evaluation of the whole tournament dynamics because all of them are part of the success or failure of this mutual fund behaviour. For instance, could we consider as a tournament success a significant increase in the performance ranking after a risk-shifting strategy? The answer would depend on the improvement in subsequent money flows. On the contrary, a significant improvement of these money flows after a decrease in the performance ranking could respond to other mechanisms different from tournaments. This alignment with the cooperative rationale will result in a more centralized result of the intermediate variables as assumed by our NSBM approach. Further, the importance w^k of each stage k in the NSBM model presented in Equation 3 is exogenously defined to provide different weights compared with the equal weight assumption that assigns the same importance to all. This exogenous definition should be justified in terms of importance for the model. In case of a neutral approach to tournament dynamics where the response, recompense a reward could have equal importance, an equal weight w^k for each stage k seems the most appropriate exogenous definition.

As noted above, our empirical application will compare the results of the SBM-based separation model (Equation 2) and the NSBM (Equation 3) to assess the new insights of our network approach that the conventional separation model does not offer.

3.3 Inputs, Intermediate Variables and Outputs

Table 1 lists and defines the inputs, outputs and intermediate variables used in our three-stage network representation of mutual fund tournaments in Figure 1. These variables are obtained for three different sequential periods depending on when the tournament stage occurs each year, i.e., months t-6, t and t+3, thereby capturing the time dynamics of the model. We set the mid-year term (t-6=30th June) as the onset of managers comparison of their performance rankings against that of the rest of their peers and the starting point for the Reaction Stage. With reference to this, the subsequent period refers to the end of the year when the tournaments have taken place (t=31st December). This two-period model is based on the suggestion of Brown et al. (1996) and is similar to the one used in Karoui and Meier (2015) and Schwarz (2012). Finally, t+3 refers to the end of the subsequent quarter (31st March) once the tournaments are over and when the short-term flows response to previous performance results is expected to have materialised. According to tournament (Brown et al. 1996) and flows literature (Chevalier and Ellison 1997, Sirri and Tufano 1998, Berk and Green 2004) this baseline (June-December-March) covers the most accurate time sequence for tournaments within the year and short-term flows response. However, the empirical application of our model also allows for alternative specifications of this baseline time frame to get robust evaluations of the tournament behaviour for different time dynamics.

The variables included in our network structure are mainly a consequence of the discretional decisions of the fund managers. According to the classification of non-discretionary variables proposed by Thanassoulis et al. (2007), we exclude those purely external-driven factors and uncontrollable internal decisions. Thus, there is no need to engage with an enhanced DEA model that handles non-discretionary inputs and outputs differently depending on their classification as internal or external to the tournament process.

Table 1 Inputs, Outputs and Intermediate Variables

Stage	Inputs	Outputs
Tournament Reaction	*Percent Rank*$_{j,t-6}$ is the percentile rank of the cumulative gross return of fund j from 1st January to 30th June.	***ΔEquity Exposure***$_{j,t}$ is the normalised variation of fund j in its portfolio allocation to Equity from 30th June to 31st December. ***ΔBeta***$_{j,t}$ is the normalised variation in the CAPM beta of fund j from 30th June to 31st December. ***ΔPortfolio Concentration***$_{j,t}$ is the normalised variation in the Herfindahl-Hirschman index of portfolio of fund j from 30th June to 31st December.
Tournament Recompense	***ΔEquity Exposure***$_{j,t}$ is the normalised variation of fund j in its portfolio allocation to Equity from 30th June to 31st December. ***ΔBeta***$_{j,t}$ is the normalised variation in the CAPM beta of fund j from 30th June to 31st December. ***ΔPortfolio Concentration***$_{j,t}$ is the normalised variation in the Herfindahl-Hirschman index of portfolio of fund j from 30th June to 31st December.	***ΔPercent Rank***$_{j,t}$ is the normalised variation in the percentile rank of the cumulative gross return of fund j between 30th June and 31st December.
Tournament Reward	***ΔPercent Rank***$_{j,t}$ is the normalised variation in the percentile of the cumulative gross return rank of fund j between 30th June and 31st December.	*Flows*$_{j,t+3}$ is the normalised value of the implied net money flows for fund j from 31st December to 31st March of the subsequent year.

This table shows the inputs, outputs and intermediate variables used in our three-stage network model of mutual fund tournaments and how they are computed. Intermediate variables are printed in bold.

Where necessary and appropriate, variables listed in Table 1 are normalised in the range [0,1], in the same line as Sánchez-González et al. (2017) and Andreu et al. (2019).[8] This scaling down removes the potential time effects and overcomes the problem of negative values of these variables in the NSBM model represented by Equation 3. Further, by employing this normalisation approach, we avoid mixing volume variables with indices, ratios or percentages, thereby averting a potential and important pitfall in DEA applications (Dyson et al. 2001). Specifically, all the variables of our input/output set undergo normalisation, except for the variable *Percent Rank*, given that it is expressed as a percentage rather than a volume. It should be noted that Papaioannou and Podinovski (2023) show that percentage variables are generally inconsistent with the assumption of convexity, which leads to scalability and incorrect efficiency targets in DEA models. This problem arises when the denominators of the ratios for the individual DMUs are different, but this is not the case for the variable *Percent Rank*, which is always related to the same denominator 100. The normalisation of percent rankings ensures that there is no convexity problem in the linear combinations of the funds in our sample.

Although our normalisation approach is aimed at handling negative data and avoiding the use of volume and ratio variables at the same time, this process does not solve the lack of translation invariance condition of the SBM-type models. In pursuit of both robustness and accuracy within our approach, we will also run our network tournament structure (Figure 1) following the Base Point (BP) transformation proposed by Tone et al. (2020). Although this BP-SBM model is not translation invariant either, it enables SBM-type models to handle negative data and offers important advantages[9] over the Modified SBM (MSBM) model proposed by Sharp et al. (2007), which is the only SBM-type model fulfilling the translation invariance property. This analysis will be shown later in the robustness tests of Appendix C.

Our network structure aims to include a whole set of variables that have the potential to significantly elucidate tournament behaviour. Omitting correlated variables, as pointed out by Dyson et al. (2001), could cause DEA results to vary greatly and they therefore recommend their inclusion. The findings of López et al. (2016) indicate that the results of DEA models with only positive correlations among the variables could be significantly affected by the degree of those correlations. The low levels of significantly positive correlations between the variables listed in

[8] This rescaling subtracts the minimum value recorded for the variable from each fund's value in the variable and divides the result by the difference between the maximum and minimum value of the variable in the period.
[9] Details of these advantages can be found in section 5 of Tone et al. (2020).

our model supports that our approach is evaluating different tournament attributes.[10] The isotonicity assumption is upheld in our model and variables, as we expect efficient changes in our measures of risk management, namely equity exposure, portfolio beta, and portfolio concentration, to lead to better relative performance, which would result in increased inflows.

4 Data

We choose the Spanish mutual fund market for the application of our tournament model. Spain is one of the most relevant Euro mutual fund industries and is characterised by an important concentration in terms of management: small and mostly independent mutual funds coexist with large and mostly bank-owned mutual funds.[11] Previous evidence highlights the significant networking effects at the marketing stage and questions the primary role of portfolio management skills in the overall efficiency of Spanish fund companies (Sánchez-González et al. 2017). The incentives for tournaments of funds sold through major bank distribution channels should differ from those of small and independent funds with lower levels of marketing and distribution power. This heterogeneity and relevance in the Eurozone make Spain a suitable market for our network approach to capture the different tournament dynamics potentially present in this highly concentrated competition map.

The primary data used in this study are obtained from the Spanish supervisor (CNMV). Our initial database includes open-end Spanish domiciled funds that were in operation during the period under study (January 2010 - December 2015). The choice of this sample period covers the years with the largest money outflows from the Spanish industry in the last two decades, alongside a sharp recovery of money inflows in 2014-2015 (Inverco 2016). Thus our sample offers extremely different management contexts within which to identify tournaments through our proposed model, providing assurances about the robustness of findings. While updating of the research period might be expected to enhance the descriptive power of the sample, we would not expect it to significantly impact the conclusions of our model for the simple reason that risk-shifting behaviour observed in tournaments is pervasive and persistent in the literature. Nevertheless, we encourage future applications to other data sets for a general model validation.

[10] Detailed correlations are shown in Appendix B.

[11] As of December 2016, Spain was the fifth largest Euro mutual fund industry in terms of the number of funds (Investment Company Institute 2020). Top 5 and Top 10 of the 83 Spanish bank-owned fund companies shared 58% and 78% of the total assets of the industry, respectively. The median size of the Euro and Domestic Equity funds in Spain is 22 million €, whereas the median size of the largest 25% funds is 151 million € (Inverco 2016).

The initial database comprises 551 funds. In total, 42 index funds are dropped given that they are not actively managed. Our analysis is focused on the two main investment categories in the Spanish fund industry: Euro and Domestic Equity Funds, which represent a total of 184 funds. We hand collected data on daily returns, monthly total net assets (TNA) and quarterly holdings.

Finally, we also exclude a total of 35 funds because the reported information does not entirely fulfil the data availability required by our model (for instance, funds terminated before 31st December or funds not reporting subsequent money flows for the first quarter because they were terminated before 31st March). In order to obtain reliable results, we require funds included in any given year in the study to exist in January and survive at least until March of the subsequent year, when flows are computed. Our final sample consists of a total of 149 distinct equity funds across the sample period and a cumulative total of 624 fund year observations.

Table 2 reports the summary statistics by year for the sample of mutual funds analysed in this paper. The yearly fund observations steadily decrease from 135 at the beginning to 84 at the end of the sample period. This decreasing number of funds in our sample period is consistent with both the merger and acquisition process of some relevant Spanish bank-owned fund companies and the termination of small funds managed by small independent companies. Meanwhile, the cumulative TNA under management steadily increases during the period of study up to approximately the triple of the initial amount by the end of the period of study.

Table 2 Descriptive Statistics

	2010	2011	2012	2013	2014	2015
Total Number of Funds	135	119	107	88	91	84
Total Net Assets (in thousands)	34,868	36,110	44,104	66,637	86,585	96,975
Mid-year Equity Exposure (%)	84.70	83.85	88.80	86.12	85.49	88.75
Year-end Equity Exposure (%)	85.26	82.86	87.61	85.85	87.26	89.77
Mid-year Portfolio Beta	0.90	0.87	0.88	0.84	0.85	0.83
Year-end Portfolio Beta	0.88	0.92	0.83	0.84	0.87	0.90
Mid-year Portfolio Concentration	531	505	550	535	547	544
Year-end Portfolio Concentration	517	535	551	540	538	550
Mid-year Portfolio Return (%)	5.78	2.59	7.67	10.19	0.78	7.49
Year-end Portfolio Return (%)	6.28	1.90	1.93	2.32	0.53	5.99
Net Implied Flows (%)	1.29	-4.08	6.62	18.69	-1.76	-4.01

This table reports the descriptive statistics by year for the sample of mutual funds employed in this study. Total Number of Funds and Total Net Assets refer to year-end data. Equity Exposure, Portfolio Beta, Portfolio Concentration and Portfolio Return refer to the variables described by Table 1 and are reported at mid-year (30th June) and at year-end (31st December). Net Implied Flows refer to the data in the first quarter of the next year.

At aggregate level, the variables reported are relatively stable and no important annual disparities can be observed. The equity exposure is consistent with the legal requirement of the Spanish supervisor (CNMV) which states that Spanish equity funds should maintain a minimum of 75% in equity. Nonetheless, this average annual data is always maintained below 90%. Table 2 also reports defensive portfolio betas and low levels of portfolio concentration. These conservative strategies make the evaluation of tournaments in Spain even more intriguing, as the potential tournament reaction and recompense may be greater than in other mutual fund industries with higher levels of risk exposure thereby leaving little room to detect heightened risk taking in response to interim performance rankings.

5 Empirical Analysis

5.1 Empirical Results

Figure 2 outlines our empirical analysis. First, we obtain baseline results from the NSBM model in Equation 3. Then, we compare these with SBM-based separation models from Equation 2 for each tournament stage displayed in Figure 1. We use nonparametric tests for rank-based performance persistence across time and funds. Our results remain robust with alternative specifications.

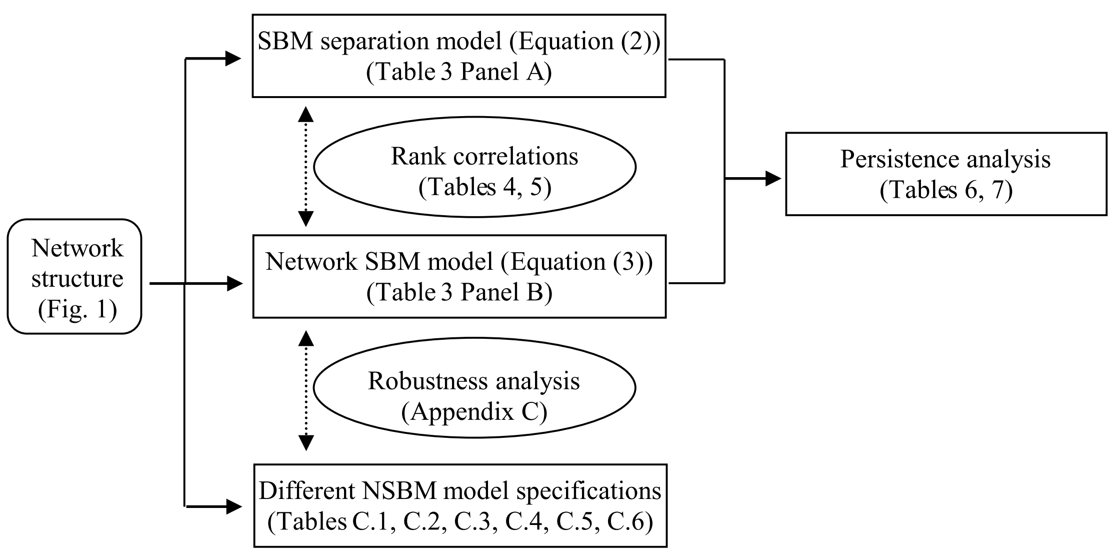

Fig. 2 Graphical Outline of Empirical Analyses.

This figure shows the design of the empirical application of the three-stage network structure of tournaments.

Table 3 compares the annual results of the SBM separation model and the NSBM model under VRS. Both models assign equal importance to all three stages. Panel A of Table 3 shows stage scores for the SBM separation model with overall scores provided by the arithmetic mean of the scores. Panel B reports the scores for the NSBM model, which links the three stages.

The number of funds deemed efficient varies between the SBM separation model and the NSBM model. Neither model follows similar patterns in individual stages or overall tournament analyses. In the SBM model, the Reaction Stage has the highest number of funds with the maximum score. This suggests that some fund managers, though few, sharply adjust their equity exposure, beta, and portfolio concentration based on their mid-year rankings. However fewer managers translate efficiently these adjustments into year-end rankings. Finally, changes at the year-end ranking are weakly linked to inflows in the first quarter of the next year. In contrast, for the NSBM model, the equally-weighted scores in the Reward stage exceed those observed in the SBM model. These findings support that changes in the year-end ranking are more efficiently linked to inflows in the first quarter of the next year for the NSBM model than the SBM model.

The NSBM findings show that the number of efficient managers is consistently low across all three stages. It is evident that the network structure makes it difficult for managers to achieve tournament efficiency because the network stages are linked to each other. However, the consistent distribution of efficient managers across stages, despite their limited numbers, illustrates that our network successfully captures the complex and interconnected challenge of the decision-making process in mutual fund tournaments.[12]

Annual standard deviations indicate that the Reaction Stage consistently has the highest variability in both SBM and NSBM models, showing the greatest disparity of tournament scores occurs at this initial stage. This suggests managers employ diverse strategies in response to their mid-year rankings, likely influenced by different performance ranking-based incentives, reputational issues and job security perceptions.

No fund is Overall Tournament efficient in the SBM model. Conversely, except for 2012, a few funds are considered efficient each year when using the NSBM model, which incorporates links between stages. Our network findings support that while many managers can strongly react to mid-year percent ranks, executing a completely successful tournament remains complex.

[12] Our design is based on the documented relationship between flows and past performance (Gruber 1996, Chevalier and Ellison 1997, Sirri and Tufano 1998). This design allows for further incorporation of additional inputs at the Reward Stage and for distinct weightings of the three stages.

Table 3 Tournament Scores: SBM Separation and Network SBM Models under VRS

Panel A SBM separation model	2010	2011	2012	2013	2014	2015
Total number of mutual funds	135	119	107	88	91	84
Tournament Reaction						
Number of efficient mutual funds	15	10	10	11	11	9
Equally weighted score	0.245	0.172	0.147	0.217	0.168	0.204
(Standard deviation)	(0.313)	(0.293)	(0.289)	(0.328)	(0.290)	(0.290)
Tournament Recompense						
Number of efficient mutual funds	8	11	8	6	9	6
Equally weighted score	0.420	0.409	0.437	0.347	0.500	0.461
(Standard deviation)	(0.220)	(0.265)	(0.234)	(0.226)	(0.276)	(0.224)
Tournament Reward						
Number of efficient mutual funds	3	3	2	2	3	6
Equally weighted score	0.085	0.160	0.076	0.070	0.120	0.278
(Standard deviation)	(0.187)	(0.203)	(0.176)	(0.159)	(0.215)	(0.273)
Overall Tournament						
Number of efficient mutual funds	0	0	0	0	0	0
Equally weighted score	0.250	0.247	0.220	0.211	0.263	0.314
(Standard deviation)	(0.148)	(0.128)	(0.112)	(0.139)	(0.140)	(0.144)
Panel B Network SBM model	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
Tournament Reaction						
Number of efficient mutual funds	3	2	0	2	2	2
Equally weighted score	0.156	0.330	0.217	0.250	0.268	0.318
(Standard deviation)	(0.204)	(0.339)	(0.243)	(0.274)	(0.279)	(0.331)
Tournament Recompense						
Number of efficient mutual funds	4	4	4	4	4	8
Equally weighted score	0.459	0.887	0.565	0.640	0.670	0.915
(Standard deviation)	(0.190)	(0.095)	(0.193)	(0.184)	(0.187)	(0.116)
Tournament Reward						
Number of efficient mutual funds	4	4	4	4	4	8
Equally weighted score	0.459	0.887	0.562	0.640	0.670	0.914
(Standard deviation)	(0.190)	(0.095)	(0.193)	(0.184)	(0.187)	(0.116)
Overall Tournament						
Number of efficient mutual funds	3	2	0	2	2	2
Equally weighted score	0.358	0.702	0.448	0.510	0.536	0.716
(Standard deviation)	(0.166)	(0.129)	(0.167)	(0.171)	(0.177)	(0.141)

This table shows the total number of mutual funds and the number of tournament-efficient funds per stage and year. It also provides the equally weighted average of the scores obtained by the SBM separation model (Panel A) and the Network SBM model (Panel B). The standard deviations are in brackets.

Table 4 Rank Correlation across Tournament Stages

Panel A: SBM separation model

	2010			2011			2012		
	TRc	TRw	OT	TRc	TRw	OT	TRc	TRw	OT
Tournam. Reaction (TR)	0.004	-0.107	0.602**	-0.092	-0.149	0.420**	0.003	-0.375**	0.337**
Tournam. Recompense (TRc)	1	-0.520**	0.592**	1	-0.149	0.510**	1	-0.462**	0.521**
Tournam. Reward (TRw)		1	-0.183**		1	0.227*		1	-0.175
Overall Tournament (OT)			1			1			1
	2013			2014			2015		
	TRc	TRw	OT	TRc	TRw	OT	TRc	TRw	OT
Tournam. Reaction (TR)	0.070	-0.218*	0.712**	-0.133	0.042	0.331**	0.114	-0.0556	0.461**
Tournam. Recompense (TRc)	1	-0.605**	0.520**	1	-0.349**	0.604**	1	-0.3283	0.331**
Tournam. Reward (TRw)		1	-0.313**		1	0.088		1	0.513**
Overall Tournament (OT)			1			1			1

Panel B: Network SBM model

	2010			2011			2012		
	TRc	TRw	OT	TRc	TRw	OT	TRc	TRw	OT
Tournam. Reaction (TR)	0.079	0.079	0.409**	-0.089	-0.089	0.881**	0.196*	0.201*	0.573**
Tournam. Recompense (TRc)	1	1**	0.906**	1	1**	0.254**	1	1**	0.858**
Tournam. Reward (TRw)		1	0.906**		1	0.254**		1	0.861**
Overall Tournament (OT)			1			1			1
	2013			2014			2015		
	TRc	TRw	OT	TRc	TRw	OT	TRc	TRw	OT
Tournam. Reaction (TR)	0.135	0.135	0.592**	0.205	0.205	0.621**	0.193	0.193	0.878**
Tournam. Recompense (TRc)	1	1**	0.813**	1	1**	0.834**	1	1**	0.551**
Tournam. Reward (TRw)		1	0.813**		1	0.834**		1	0.551**
Overall Tournament (OT)			1			1			1

Panel A of this table shows the Spearman rank correlations across the tournament rankings obtained by the SBM separation model under VRS for the different stages. Panel B provides similar information for the Network SBM model under VRS. * 5% significance level; ** 1% significance level.

Table 5 Rank Correlation: SBM Separation vs Network SBM Models under VRS

	2010	2011	2012	2013	2014	2015
Tournament Reaction	0.729**	0.516**	0.470**	0.581**	0.387**	0.613**
Tournament Recompense	-0.481**	0.026	0.105	-0.055	-0.253*	-0.172
Tournament Reward	0.954**	0.839**	0.618**	0.622**	0.941**	0.883**
Overall Tournament	0.133	0.749**	0.629**	0.470**	0.477**	0.836**

This table shows Spearman rank correlations for each stage and year between the tournament scores obtained by the SBM separation and the Network SBM models under VRS. * 5% significance level; ** 1% significance level.

Tone and Tsutsui (2009) advised against directly comparing the efficiency scores of different stages due to varying number of outputs and inputs. Instead, comparing efficiency rankings provides a clearer picture. Panel A of Table 4 shows rank correlations across stages when links are omitted. These results indicate that stages are either negatively correlated or independent, except for the Overall Tournament results, likely because the SBM score is an equally weighted average of the individual scores of the three stages. The NSBM model in Panel B of Table 4 offers a more accurate picture by incorporating links between stages. It shows that while strong responses at the Reaction stage do not affect later stages (evidenced by low and predominantly insignificant correlations with the Reward and Recompense stages), efficiency at the Recompense stage drives efficiency at the Reward stage (evidenced by significant correlations between the two stages tending towards unity). Both results are not contradictory and are captured by our NSBM model. A strong mid-year reaction does not directly impact subsequent inflows, but efficiently improving year-end ranking has a significant impact on future flows.

To analyse the differences between the SBM separation model and the NSBM model, we applied Spearman correlations to the efficiency ranks from both models. The results, shown in Table 5, indicate a high correlation between the models for the overall tournament, except for the year 2010. For individual stages, Table 5 shows significant correlations between the Reaction Stage and the Reward Stage, the first and last stages of each model. However, the Recompense Stages of both models are negatively or not significantly correlated. These results support the hypothesis that the Recompense Stage is an important link to capture tournaments within the mutual fund industry, as illustrated in Figure 1.[13]

Our findings underscore tournament behaviour as a sophisticated phenomenon with various stages and interactions, establishing the NSBM model as a suitable tool for its evaluation. Appendix C includes additional specifications to enhance the robustness of our baseline findings.

5.2 Tournament persistence

Do efficient tournament responses persist over time, or are they limited to specific years? To answer this question, we use a k-means clustering technique to categorise funds into four groups: Top Winners, Winners, Losers, and Bottom Losers, ordered by tournament efficiency.

Table 6 displays the average NSBM scores and the number of funds for each cluster. Fund managers generally do not efficiently react to their mid-year performance ranks, as evidenced by the consistently large size of the Bottom Losers cluster at the Reaction Stage. However, fund managers are more efficient improving their percent ranks because of their reaction, and investors usually reward these changes in percent ranks. Overall tournament data indicate that, apart from the final year of the study, most fund managers fail to execute a tournament strategy efficiently. These findings highlight the complexity of leading successful tournament strategies.

[13] Kendall rank correlations provide results consistent with Table 4 and Table 5.

We utilize contingency tables to assess tournament persistence. We test mobility across clusters within each stage. Panel A of Table 7 displays the results, including the Immobility Ratio (IR), the percentage of funds with improved performance (MU), and the percentage with worsened performance (MD). Overall, this table does not reveal clear patterns. The Immobility Ratios at the Reaction Stage tend to be high, indicating that funds that inefficiently react to their percent rank often remain inefficient a year later. However, these ratios are not significant.

Panel B of Table 7 shows the transition probability matrices by comparing each year to the next across stages and overall. Significant values appear sporadically, indicating a general lack of persistence. At the Reaction Stage, the Bottom Losers cluster exhibits the highest transition probability, suggesting a tendency for these funds to remain in the same cluster, though this result is seldom significant. There is also limited evidence that Top Winners are likely to become Losers in the Reaction Stage the following year, questioning the existence of persistent skills.

Corroborating Carhart's (1997) seminal work on persistence in mutual fund performance, efficient tournaments are not persistent over time. Our findings are of special importance for investors as successful tournaments in the past should not be considered as an accurate predictor of future success in these strategies. The practical implication for investors is clear; they should not try to chase past tournaments.

Table 6 Summary Statistics of the Tournament-Efficiency Clusters

	2010	2011	2012	2013	2014	2015
Tournament Reaction						
Top Winners	0.95 (05)	0.87 (29)	0.68 (14)	0.91 (06)	0.81 (14)	0.89 (19)
Winners	0.38 (22)	0.54 (08)	0.46 (20)	0.63 (10)	0.51 (10)	0.38 (10)
Losers	0.15 (29)	0.24 (23)	0.15 (16)	0.47 (10)	0.25 (17)	0.18 (17)
Bottom Losers	0.04 (79)	0.07 (59)	0.04 (57)	0.09 (62)	0.07 (50)	0.07 (38)
Tournament Recompense						
Top Winners	0.84 (18)	0.97 (22)	0.95 (12)	0.96 (10)	0.89 (23)	0.98 (26)
Winners	0.52 (44)	0.90 (72)	0.65 (38)	0.76 (22)	0.67 (48)	0.92 (47)
Losers	0.36 (58)	0.82 (24)	0.47 (51)	0.60 (41)	0.48 (16)	0.79 (10)
Bottom Losers	0.21 (15)	0.03 (01)	0.14 (06)	0.36 (15)	0.13 (04)	0.03 (01)
Tournament Reward						
Top Winners	0.84 (18)	0.97 (22)	0.94 (12)	0.88 (20)	0.94 (14)	0.98 (26)
Winners	0.54 (32)	0.90 (72)	0.65 (36)	0.67 (35)	0.75 (27)	0.92 (47)
Losers	0.40 (46)	0.82 (24)	0.47 (52)	0.52 (25)	0.61 (39)	0.79 (10)
Bottom Losers	0.28 (39)	0.03 (01)	0.17 (07)	0.28 (08)	0.32 (11)	0.03 (01)
Overall Tournament						
Top Winners	0.90 (07)	0.90 (26)	0.76 (11)	0.85 (09)	0.87 (11)	0.94 (16)
Winners	0.48 (30)	0.75 (21)	0.61 (16)	0.59 (30)	0.60 (33)	0.73 (27)
Losers	0.33 (59)	0.63 (71)	0.44 (48)	0.46 (31)	0.45 (38)	0.64 (40)
Bottom Losers	0.21 (39)	0.03 (01)	0.27 (32)	0.29 (18)	0.23 (09)	0.02 (01)

This table shows the average NSBM scores per cluster and year for each stage. The number of funds is in brackets.

Table 7 Summary Statistics of the Tournament-Efficiency Clusters

Panel A: Mobility ratios and chi-square tests

	Tournament Reaction				*Tournament Recompense*				*Tournament Reward*				*Overall Tournament*			
	IR	MU	MD	χ2	IR	MU	MD	χ2	IR	MU	MD	χ2	IR	MU	MD	χ2
2010-2011	0.41	0.41	0.18	14.5	0.27	0.51	0.22	10.2	0.20	0.60	0.20	6.0	0.28	0.53	0.18	8.5
2011-2012	0.42	0.26	0.32	15.1	0.35	0.16	0.49	2.6	0.34	0.16	0.50	2.8	0.35	0.20	0.45	18.6^{*}
2012-2013	0.47	0.19	0.34	7.1	0.37	0.19	0.44	15.7	0.40	0.36	0.24	14.5	0.30	0.47	0.23	15.8
2013-2014	0.31	0.27	0.42	11.4	0.23	0.62	0.15	14.6	0.50	0.30	0.20	7.1	0.33	0.38	0.28	3.9
2014-2015	0.40	0.37	0.23	7.3	0.49	0.28	0.23	6.8	0.40	0.37	0.23	8.1	0.39	0.33	0.28	4.9

Panel B: Transition probability matrices

	Tournament Reaction				*Tournament Recompense*				*Tournament Reward*				*Overall Tournament*			
2011 2010	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL
TW	0.00	0.00	0.75^{**}	0.25	0.13	0.67	0.20	0.00	0.13	0.67	0.20	0.00	0.17	0.17	0.67	0.00
W	0.31	0.06	0.13	0.50	0.19	0.56	0.26	0.00	0.19	0.53	0.28	0.00	0.30	0.11	0.56	0.04
L	0.33	0.14	0.19	0.33	0.11	0.73^{*}	0.14	0.02	0.19	0.68	0.11	0.03	0.25	0.15	0.60	0.00
BL	0.21	0.05	0.16	0.57	0.36	0.36^{*}	0.29	0.00	0.16	0.59	0.25	0.00	0.11	0.26	0.63	0.00
2012 2011	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL
TW	0.18	0.23	0.23	0.36	0.15	0.30	0.50	0.05	0.15	0.30	0.50	0.05	0.16	0.11	0.26	0.47
W	0.14	0.29	0.43^{*}	0.14^{*}	0.11	0.37	0.46	0.06	0.11	0.35	0.46	0.08	0.25^{*}	0.05	0.40	0.30
L	0.05	0.27	0.14	0.55	0.10	0.33	0.52	0.05	0.10	0.33	0.52	0.05	0.05^{*}	0.18	0.51	0.26
BL	0.15	0.11	0.09	0.65^{*}	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00^{*}	0.00	0.00
2013 2012	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL
TW	0.11	0.22	0.11	0.56	0.25	0.58^{**}	0.08^{**}	0.08	0.58^{**}	0.25	0.08	0.08	0.00	0.33	0.44	0.22
W	0.17	0.11	0.11	0.61	0.11	0.19	0.52	0.19	0.19	0.46	0.31	0.04	0.33^{**}	0.33	0.25	0.08
L	0.11	0.00	0.11	0.78	0.08	0.18	0.55	0.20	0.15^{*}	0.37	0.34	0.15	0.13	0.41	0.31	0.15
BL	0.02^{*}	0.11	0.13	0.74	0.25	0.25	0.25	0.25	0.50	0.25	0.25	0.00	0.00^{*}	0.26	0.35	0.39^{*}
2014 2013	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL
TW	0.17	0.17	0.33	0.33	0.20	0.30	0.40^{*}	0.10	0.25	0.20	0.35	0.20	0.11	0.56	0.22	0.11
W	0.11	0.33^{*}	0.22	0.33	0.27	0.59	0.09	0.05	0.20	0.31	0.43	0.06	0.14	0.36	0.39	0.11
L	0.00	0.00	0.22	0.78	0.36^{*}	0.46	0.13	0.05	0.09	0.30	0.43	0.17	0.13	0.32	0.48	0.06
BL	0.18	0.08	0.15	0.60	0.00^{*}	0.73	0.27	0.00	0.00	0.38	0.50	0.13	0.11	0.28	0.44	0.17
2015 2014	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL	TW	W	L	BL
TW	0.29	0.14	0.21	0.36	0.36	0.55	0.09	0.00	0.38	0.62	0.00	0.00	0.27	0.36	0.36	0.00
W	0.44	0.11	0.22	0.22	0.27	0.62	0.09	0.02	0.35	0.50	0.12	0.04	0.16	0.35	0.45	0.03
L	0.21	0.21	0.21	0.36	0.23	0.46	0.31^{*}	0.00	0.20	0.66	0.14	0.00	0.21	0.26	0.53	0.00
BL	0.16	0.09	0.20	0.56^{*}	0.50	0.50	0.00	0.00	0.38	0.38	0.25	0.00	0.00	0.33	0.67	0.00

Panel A provides the following data based on the probability matrices: *Immobility Ratio* (IR), *the percentage of funds with improved performance* (MU) and *the percentage of funds with worsened performance* (MD). A chi-square test χ2 is applied to check for the persistence hypothesis. Panel B reports the transition probability matrices for each sub-sample period and tournament stage. Each element p_{ij} of each transition matrix corresponds to the probability of transiting from cluster *i* to cluster *j,* i.e., the number of funds in cluster *i* of year *t-1* that are now part of cluster *j* of year *t* in relation to the total of funds included in the transition matrix for that sub-sample period. The significance is based on Haberman adjusted residues. * 5% significance level; ** 1% significance level.

6 Conclusions and Practical Implications

This study introduces a nuanced tournament model for the mutual fund industry, examining how managers react to interim performance rankings, adjust portfolio risk to enhance year-end rankings, and how these changes are rewarded by investors through subsequent fund inflows.

Applying our model to a real mutual fund market supports our initial assumptions. Our findings highlight the complexity fund managers face in responding to interim performance rankings and improving their year-end performance relative to peers. The results indicate that the Reaction Stage is not necessarily correlated with the Reward Stage. Changes in portfolio risk in response to interim performance rankings do not always lead to increased inflows.

Consistent with existing literature, the key to attracting flows in the following quarter lies in managers' ability to enhance performance rankings through portfolio adjustments. Success at the Recompense Stage, involving improved year-end performance, is critical in the final tournament results. These findings validate our proposed model, which remains robust under alternative variable specifications. Moreover, the lack of persistence in tournaments at individual stages or overall suggests that maintaining a consistently efficient tournament strategy is challenging.

Our findings have significant managerial implications for stakeholders, including mutual fund companies, managers, investors, and market supervisors. Performance-based bonuses can greatly influence manager behaviour in response to interim rankings, particularly concerning portfolio risk, with market performance implications. Monitoring these incentives is important to understand their impact on relative performance and fund inflows. If interim loser managers fail in their risk-taking strategies, it could adversely affect investor returns and mutual fund companies' profitability. Investors should be aware that successful tournaments in the past do not act as a significant predictor of future successful tournament strategies.

Market supervisors should be vigilant about performance-based compensation policies, as they can encourage excessive risk-taking among fund managers. The intra-year frequency of these remunerations could lead managers to engage in multiple tournaments within a year. Supervisors should oversee remuneration practices to prevent unsuccessful tournaments and mitigate the increased downside risk that could harm investors.

Our empirical analysis is limited to the Spanish market. Further applications to other data sets would help to calibrate the model for each industry with specific time frames aligned with managers' compensation calendars to deepen understanding of interim tournament dynamics.

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Appendix A. Detailed Formulation of the SBM and NSBM models

SBM Model at Reaction Stage

$$\rho_o^{SBM_{Reaction}} = \min_{\lambda^{Reaction},\, s^{Reaction-}_{Percent\ Rank},\, s^{Reaction+}_{\Delta Equity\ Exposure},\, s^{Reaction+}_{\Delta Beta},\, s^{Reaction+}_{\Delta Portfolio\ Concentration}} \frac{1-\frac{1}{1}\left(\frac{s^{Reaction-}_{Percent\ Rank}}{x^{Reaction}_{Percent\ Rank_o}}\right)}{1+\frac{1}{3}\left(\frac{s^{Reaction+}_{\Delta Equity\ Exposure}}{y^{Reaction}_{\Delta Equity\ Exposure_o}}+\frac{s^{Reaction+}_{\Delta Beta}}{y^{Reaction}_{\Delta Beta_o}}+\frac{s^{Reaction+}_{\Delta Portfolio\ Concentration}}{y^{Reaction}_{\Delta Portfolio\ Concentration_o}}\right)}$$

subject to:

$$X^{Reaction}\lambda^{Reaction}+s^{Reaction-}_{Percent\ Rank}=x_o^{Reaction}$$
$$Y^{Reaction}\lambda^{Reaction}-s^{Reaction+}_{\Delta Equity\ Exposure}-s^{Reaction+}_{\Delta Beta}-s^{Reaction+}_{\Delta Portfolio\ Concentration}=y_o^{Reaction}$$
$$e\lambda^{Reaction}=1$$
$$\lambda^{Reaction}, s^{Reaction-}_{Percent\ Rank}, s^{Reaction+}_{\Delta Equity\ Exposure}, s^{Reaction+}_{\Delta Beta}, s^{Reaction+}_{\Delta Portfolio\ Concentration}\geq 0$$

SBM Model at Recompense Stage

$$\rho_o^{SBM_{Recompense}} = \min_{\lambda^{Recompense},\, s^{Recompense-}_{\Delta Equity\ Exposure},\, s^{Recompense-}_{\Delta Beta},\, s^{Recompense-}_{\Delta Portfolio\ Concentration},\, s^{Recompense+}_{\Delta Percent\ Rank}} \frac{1-\frac{1}{3}\left(\frac{s^{Recompense-}_{\Delta Equity\ Exposure}}{x^{Recompense}_{\Delta Equity\ Exposure_o}}+\frac{s^{Recompense-}_{\Delta Beta}}{x^{Recompense}_{\Delta Beta_o}}+\frac{s^{Recompense-}_{\Delta Portfolio\ Concentration}}{x^{Recompense}_{\Delta Portfolio\ Concentration_o}}\right)}{1+\frac{1}{1}\left(\frac{s^{Recompense+}_{\Delta Percent\ Rank}}{y^{Recompense}_{\Delta Percent\ Rank_o}}\right)}$$

subject to:

$$X^{Recompense}\lambda^{Recompense}+s^{Recompense-}_{\Delta Equity\ Exposure}+s^{Recompense-}_{\Delta Beta}+s^{Recompense-}_{\Delta Portfolio\ Concentration}=x_o^{Recompense}$$
$$Y^{Recompense}\lambda^{Recompense}-s^{Recompense+}_{\Delta Percent\ Rank}=y_o^{Recompense}$$
$$e\lambda^{Recompense}=1$$
$$\lambda^{Recompense}, s^{Recompense-}_{\Delta Equity\ Exposure}, s^{Recompense-}_{\Delta Beta}, s^{Recompense-}_{\Delta Portfolio\ Concentration}, s^{Recompense+}_{\Delta Percent\ Rank}\geq 0$$

SBM Model at Reward Stage

$$\rho_o^{SBM_{Reward}} = \min_{\lambda^{Reward},\, s^{Reward-}_{\Delta Percent\ Rank},\, s^{Reward+}_{Flows}} \frac{1-\frac{1}{1}\left(\frac{s^{Reward-}_{\Delta Percent\ Rank}}{x^{Reward}_{\Delta Percent\ Rank_o}}\right)}{1+\frac{1}{1}\left(\frac{s^{Reward+}_{Flows}}{y^{Reward}_{Flows_o}}\right)}$$

subject to:

$$X^{Reward}\lambda^{Reward}+s^{Reward-}_{\Delta Percent\ Rank}=x_o^{Reward}$$
$$Y^{Reward}\lambda^{Reward}-s^{Reward+}_{Flows}=y_o^{Reward}$$
$$e\lambda^{Reward}=1$$
$$\lambda^{Reward}, s^{Reward-}_{\Delta Percent\ Rank}, s^{Reward+}_{Flows}\geq 0$$

NSBM Model:

$$\rho_o^{NSBM} = \underset{\substack{\lambda^{Reaction},\lambda^{Recompense},\lambda^{Reward},\ s^{Reaction-}_{Percent\ Rank},\ s^{Reward+}_{Flows} \\ s^{(Reaction,\ Recompense)-}_{\Delta Equity\ Exposure},\ s^{(Reaction,\ Recompense)-}_{\Delta Beta},\ s^{(Reaction,\ Recompense)-}_{\Delta Portfolio\ Concentration},\ s^{(Recompense,\ Reward)-}_{\Delta Percent\ Rank} \\ s^{(Reaction,\ Recompense)+}_{\Delta Equity\ Exposure},\ s^{(Reaction,\ Recompense)+}_{\Delta Beta},\ s^{(Reaction,\ Recompense)+}_{\Delta Portfolio\ Concentration},\ s^{(Recompense,\ Reward)+}_{\Delta Percent\ Rank}}}{min}$$

Reaction Stage

$$\frac{\frac{1}{3}\left[1-\frac{1}{1+0}\left(\frac{s^{Reaction-}_{Percent\ Rank}}{x^{Reaction}_{Percent\ Rank_o}}+0\right)\right]}{\frac{1}{3}\left[1+\frac{1}{0+3}\left(0+\frac{s^{(Reaction,\ Recompense)+}_{\Delta Equity\ Exposure}}{y^{(Reaction,\ Recompense)}_{\Delta Equity\ Exposure_o}}+\frac{s^{(Reaction,\ Recompense)+}_{\Delta Beta}}{y^{(Reaction,\ Recompense)}_{\Delta Beta_o}}+\frac{s^{(Reaction,\ Recompense)+}_{\Delta Portfolio\ Concentration}}{y^{(Reaction,\ Recompense)}_{\Delta Portfolio\ Concentration_o}}\right)\right]}$$

$$+$$

Recompense Stage

$$\frac{\frac{1}{3}\left[1-\frac{1}{0+3}\left(0+\frac{s^{(Reaction,\ Recompense)-}_{\Delta Equity\ Exposure}}{x^{(Reaction,\ Recompense)}_{\Delta Equity\ Exposure_o}}+\frac{s^{(Reaction,\ Recompense)-}_{\Delta Beta}}{x^{(Reaction,\ Recompense)}_{\Delta Beta_o}}+\frac{s^{(Reaction,\ Recompense)-}_{\Delta Portfolio\ Concentration}}{x^{(Reaction,\ Recompense)}_{\Delta Portfolio\ Concentration_o}}\right)\right]}{\frac{1}{3}\left[1+\frac{1}{0+1}\left(0+\frac{s^{(Recompense,\ Reward)+}_{\Delta Percent\ Rank}}{y^{(Recompense,\ Reward)}_{\Delta Percent\ Rank_o}}\right)\right]}$$

$$+$$

Reward Stage

$$\frac{\frac{1}{3}\left[1-\frac{1}{0+1}\left(0+\frac{s^{(Recompense,\ Reward)-}_{\Delta Percent\ Rank}}{x^{(Recompense,\ Reward)}_{\Delta Percent\ Rank_o}}\right)\right]}{\frac{1}{3}\left[1+\frac{1}{1+0}\left(\frac{s^{Reward+}_{Flows}}{y^{Reward}_{Flows_o}}+0\right)\right]}$$

subject to:

$$X^{Reaction}\lambda^{Reaction}+s_{Percent\ Rank}^{Reaction\ -}=x_o^{Reaction} \qquad e\lambda^{Reaction}=1$$

$$Y^{Reward}\lambda^{Reward}-s_{Flows}^{Reward\ +}=y_o^{Reward} \qquad e\lambda^{Reward}=1$$

$$Z^{(Reaction,\ Recompense)}\lambda^{Recompense}+s_{\Delta Equity\ Exposure}^{(Reaction,\ Recompense)\ -}+s_{\Delta Beta}^{(Reaction,\ Recompense)\ -}+s_{\Delta Portfolio\ Concentration}^{(Reaction,\ Recompense)\ -}= \\ =z_o^{(Reaction,\ Recompense)}$$

$$Z^{(Reaction,\ Recompense)}\lambda^{Reaction}-s_{\Delta Equity\ Exposure}^{(Reaction,\ Recompense)\ +}-s_{\Delta Beta}^{(Reaction,\ Recompense)\ +}-s_{\Delta Portfolio\ Concentration}^{(Reaction,\ Recompense)\ +}= \\ =z_o^{(Reaction,\ Recompense)}$$

$$Z^{(Reaction,\ Recompense)}\lambda^{Reaction}=Z^{(Reaction,\ Recompense)}\lambda^{Recompense}$$

$$Z^{(Recompense,\ Reward)}\lambda^{Reward}+s_{\Delta Percent\ Rank}^{(Recompense,\ Reward)\ -}=z_o^{(Recompense,\ Reward)}$$

$$Z^{(Recompense,\ Reward)}\lambda^{Recompense}-s_{\Delta Percent\ Rank}^{(Recompense,\ Reward)\ +}=z_o^{(Recompense,\ Reward)}$$

$$Z^{(Recompense,\ Reward)}\lambda^{Recompense}=Z^{(Recompense,\ Reward)}\lambda^{Reward}$$

$$\lambda^{Reaction},\lambda^{Recompense},\lambda^{Reward},s_{Percent\ Rank}^{Reaction\ -},s_{Flows}^{Reward\ +}\geq 0$$

$$s_{\Delta Equity\ Exposure}^{(Reaction,\ Recompense)\ -},s_{\Delta Beta}^{(Reaction,\ Recompense)\ -},s_{\Delta Portfolio\ Concentration}^{(Reaction,\ Recompense)\ -},s_{\Delta Percent\ Rank}^{(Recompense,\ Reward)\ -}\geq 0$$

$$s_{\Delta Equity\ Exposure}^{(Reaction,\ Recompense)\ +},s_{\Delta Beta}^{(Reaction,\ Recompense)\ +},s_{\Delta Portfolio\ Concentration}^{(Reaction,\ Recompense)\ +},s_{\Delta Percent\ Rank}^{(Recompense,\ Reward)\ +}\geq 0$$

Appendix B. Correlations across Variables

Table B.1 Pearson Correlations across Variables (2010-2015)

2010	*ΔEquity*	*ΔBeta*	*ΔPortfolio Concentration*	*ΔPercent Rank*	*Flows*	*Percent Rank*
ΔEquity	1	0.047	0.051	-0.179*	0.112	-0.012
ΔBeta		1	0.060	-0.223**	-0.109	0.222**
ΔPortfolio Concentration			1	0.053	0.081	-0.031
ΔPercent Rank				1	0.135	-0.535**
Flows					1	0.012
Percent Rank						1
2011	*ΔEquity*	*ΔBeta*	*ΔPortfolio Concentration*	*ΔPercent Rank*	*Flows*	*Percent Rank*
ΔEquity	1	0.001	0.087	-0.128	-0.052	0.104
ΔBeta		1	-0.054	-0.088	0.016	-0.159
ΔPortfolio Concentration			1	0.121	-0.025	0.021
ΔPercent Rank				1	0.170	-0.599**
Flows					1	0.022
Percent Rank						1
2012	*ΔEquity*	*ΔBeta*	*ΔPortfolio Concentration*	*ΔPercent Rank*	*Flows*	*Percent Rank*
ΔEquity	1	0.032	0.044	-0.035	-0.080	-0.018
ΔBeta		1	-0.052	-0.065	-0.028	0.239*
ΔPortfolio Concentration			1	-0.020	-0.181	0.027
ΔPercent Rank				1	-0.120	-0.528**
Flows					1	0.312**
Percent Rank						1
2013	*ΔEquity*	*ΔBeta*	*ΔPortfolio Concentration*	*ΔPercent Rank*	*Flows*	*Percent Rank*
ΔEquity	1	-0.099	0.089	-0.033	0.018	0.046
ΔBeta		1	-0.061	-0.088	0.205	0.281**
ΔPortfolio Concentration			1	0.175	0.005	-0.038
ΔPercent Rank				1	-0.094	-0.738**
Flows					1	0.220*
Percent Rank						1
2014	*ΔEquity*	*ΔBeta*	*ΔPortfolio Concentration*	*ΔPercent Rank*	*Flows*	*Percent Rank*
ΔEquity	1	0.041	0.011	-0.049	0.084	0.108
ΔBeta		1	0.026	-0.165	-0.188	0.028
ΔPortfolio Concentration			1	-0.082	0.074	0.077
ΔPercent Rank				1	0.117	-0.726**
Flows					1	-0.170
Percent Rank						1
2015	*ΔEquity*	*ΔBeta*	*ΔPortfolio Concentration*	*ΔPercent Rank*	*Flows*	*Percent Rank*
ΔEquity	1	-0.109	-0.005	0.294**	-0.013	-0.233*
ΔBeta		1	0.174	-0.165	-0.007	0.226*
ΔPortfolio Concentration			1	-0.030	0.010	0.058
ΔPercent Rank				1	-0.001	-0.530**
Flows					1	-0.067
Percent Rank						1

This table shows the Pearson correlations across the variables listed in Table 1.
* 5% significance level; ** 1% significance level.

Appendix C. Robustness Analysis

To test for robustness, we apply the NSBM model and the SBM separation model to alternative variable specifications: for the time splitting (*t-3, t, t+3*) in line with Chevalier and Ellison (1997) and for the use of the percentile rank of the implied net money flows from 31st December to 31st March instead of variable $Flows_{j,t+3}$. We further test for the use of the normalised value of the cumulative gross return of fund *j* from 1st January to 30th June instead of variable $Percent\ Rank_{j,t-6}$ and the normalised variation in the cumulative gross return of fund *j* between 30th June and 31st December instead of $\Delta Percent\ Rank_{j,t}$. Table C.1, Table C.2 and Table C.3 show the detailed results of these alternative analyses.

In Table C.4, we assess the correlation between the results obtained in the main NSBM models and the ones obtained using alternative variable specifications. By and large, they reveal that our findings on tournament behaviours are consistent when modelling for tournament reaction covering the period year-start to September instead of year-start to mid-year (Panel A of Table C.4), when employing percent ranks of the implied net money flows from 31st December to 31st March instead of the variable $Flows_{j,t+3}$ (Panel B of Table C.4) and when using the normalised value of the cumulative gross return of fund *j* from 1st January to 30th June instead of $Percent\ Rank_{j,t-6}$ (Panel C of Table C.4).

Further, we apply the Base Point (BP) transformation proposed by Tone et al. (2020) to the variables included in our network structure. Details of this Base Point transformation can be found in section 2 of Tone et al. (2020). This robustness analysis aims to know whether the application of our original NSBM model using normalised variables (Table 1) is consistent with the BP transformation. Although this BP-SBM model is not translation invariant, Tone et al. (2020) justify the advantages of this approach over the MSBM proposed by Sharp et al. (2007), which is currently the only translation invariant SBM-type model in the literature. The results of the efficiency scores using the BP-transformed variables are presented in Table C.5.

Both normalisation and BP transformation handle negative data. However, if convexity was a problem in the linear combinations of our normalised variables, the comparison between the scores obtained from both assessments would reveal a lack of consistency. Table C.6 shows the correlation between the scores of our original NSBM model (Equation 3) using normalised variables and the ones obtained using the BP transformation assessed by Tone et al. (2020). An average rank correlation higher than 0.99 significantly supports the use of normalised variables.

Table C.1 SBM Separation and Network SBM Models under VRS
(Alternative Time Splitting: *t-3, t, t+3*)

Panel A SBM separation model	2010	2011	2012	2013	2014	2015
Total number of mutual funds	135	119	107	88	91	84
Tournament Reaction						
Number of efficient mutual funds	15	15	15	7	10	6
Equally weighted score	0.184	0.281	0.265	0.193	0.193	0.140
(Standard deviation)	(0.314)	(0.330)	(0.326)	(0.299)	(0.321)	(0.280)
Tournament Recompense						
Number of efficient mutual funds	5	4	8	11	11	6
Equally weighted score	0.474	0.417	0.367	0.581	0.547	0.400
(Standard deviation)	(0.196)	(0.185)	(0.235)	(0.222)	(0.258)	(0.205)
Tournament Reward						
Number of efficient mutual funds	3	2	3	2	5	3
Equally weighted score	0.0253	0.058	0.092	0.0736	0.216	0.149
(Standard deviation)	(0.170)	(0.142)	(0.180)	(0.169)	(0.241)	(0.187)
Overall Tournament						
Number of efficient mutual funds	0	0	0	0	0	0
Equally weighted score	0.237	0.252	0.241	0.283	0.319	0.230
(Standard deviation)	(0.142)	(0.131)	(0.124)	(0.108)	(0.153)	(0.107)

Panel B Network SBM model	2010	2011	2012	2013	2014	2015
Tournament Reaction						
Number of efficient mutual funds	0	2	2	1	4	2
Equally weighted score	0.092	0.332	0.233	0.254	0.289	0.341
(Standard deviation)	(0.148)	(0.330)	(0.218)	(0.217)	(0.296)	(0.361)
Tournament Recompense						
Number of efficient mutual funds	4	6	5	3	6	6
Equally weighted score	0.429	0.903	0.547	0.557	0.686	0.917
(Standard deviation)	(0.176)	(0.095)	(0.198)	(0.173)	(0.186)	(0.116)
Tournament Reward						
Number of efficient mutual funds	4	6	5	3	6	6
Equally weighted score	0.429	0.903	0.547	0.557	0.686	0.917
(Standard deviation)	(0.176)	(0.095)	(0.198)	(0.173)	(0.186)	(0.116)
Overall Tournament						
Number of efficient mutual funds	0	2	2	1	4	2
Equally weighted score	0.316	0.712	0.442	0.456	0.554	0.725
(Standard deviation)	(0.135)	(0.137)	(0.169)	(0.144)	(0.186)	(0.157)

This table is similar to Table 3 but for the use of the alternative time framework (*t-3, t, t+3*). It shows the total number of mutual funds and the number of tournament-efficient funds per stage and year. It also provides the equally weighted average of the tournament scores obtained by the SBM separation model (Panel A) and the Network SBM model (Panel B). The standard deviations of the scores are in brackets.

Table C.2 SBM Separation and Network SBM Models under VRS
(Alternative Variable: $Flows_{j,t+3}$)

Panel A SBM separation model	2010	2011	2012	2013	2014	2015
Total number of mutual funds	135	119	107	88	91	84
Tournament Reaction						
Number of efficient mutual funds	15	10	10	11	9	9
Equally weighted score	0.245	0.172	0.147	0.217	0.168	0.204
(Standard deviation)	(0.313)	(0.293)	(0.289)	(0.328)	(0.289)	(0.290)
Tournament Recompense						
Number of efficient mutual funds	8	11	8	6	11	6
Equally weighted score	0.420	0.409	0.437	0.347	0.500	0.461
(Standard deviation)	(0.220)	(0.265)	(0.234)	(0.226)	(0.276)	(0.224)
Tournament Reward						
Number of efficient mutual funds	3	4	4	3	6	6
Equally weighted score	0.135	0.176	0.105	0.143	0.165	0.269
(Standard deviation)	(0.256)	(0.230)	(0.196)	(0.260)	(0.276)	(0.277)
Overall Tournament						
Number of efficient mutual funds	0	0	0	0	0	0
Equally weighted score	0.267	0.252	0.229	0.236	0.278	0.311
(Standard deviation)	(0.154)	(0.131)	(0.113)	(0.142)	(0.153)	(0.145)

Panel B Network SBM model	2010	2011	2012	2013	2014	2015
Tournament Reaction						
Number of efficient mutual funds	3	2	0	2	3	2
Equally weighted score	0.231	0.241	0.262	0.260	0.272	0.251
(Standard deviation)	(0.276)	(0.277)	(0.310)	(0.302)	(0.308)	(0.304)
Tournament Recompense						
Number of efficient mutual funds	5	6	8	7	7	8
Equally weighted score	0.684	0.686	0.691	0.702	0.690	0.709
(Standard deviation)	(0.274)	(0.274)	(0.275)	(0.281)	(0.278)	(0.270)
Tournament Reward						
Number of efficient mutual funds	5	6	6	7	7	8
Equally weighted score	0.684	0.686	0.688	0.702	0.690	0.709
(Standard deviation)	(0.274)	(0.274)	(0.274)	(0.281)	(0.278)	(0.270)
Overall Tournament						
Number of efficient mutual funds	3	2	0	2	3	2
Equally weighted score	0.533	0.537	0.547	0.555	0.550	0.557
(Standard deviation)	(0.226)	(0.225)	(0.233)	(0.236)	(0.246)	(0.241)

This table is similar to Table 3 but for the use of the percentile rank of the implied net money flows from 31st December to 31st March instead of the variable $Flows_{j,t+3}$ (see Table 1). It shows the total number of mutual funds and the number of tournament-efficient funds per stage and year. It also provides the equally weighted average of the tournament scores obtained by the SBM separation model (Panel A) and the Network SBM model (Panel B). The standard deviations of the scores are in brackets

Table C.3 SBM Separation and Network SBM Models under VRS
(Alternative Variables: *Percent Rank*$_{j,t-6}$, *ΔPercent Rank*$_{j,t}$)

Panel A SBM separation model	2010	2011	2012	2013	2014	2015
Total number of mutual funds	135	119	107	88	91	84
Tournament Reaction						
Number of efficient mutual funds	13	8	10	6	7	4
Equally weighted score	0.342	0.197	0.152	0.207	0.217	0.273
(Standard deviation)	(0.292)	(0.286)	(0.289)	(0.285)	(0.266)	(0.241)
Tournament Recompense						
Number of efficient mutual funds	7	9	7	6	10	6
Equally weighted score	0.398	0.387	0.441	0.411	0.558	0.540
(Standard deviation)	(0.181)	(0.235)	(0.226)	(0.210)	(0.242)	(0.203)
Tournament Reward						
Number of efficient mutual funds	3	2	2	2	3	4
Equally weighted score	0.030	0.035	0.072	0.079	0.114	0.384
(Standard deviation)	(0.148)	(0.131)	(0.157)	(0.165)	(0.206)	(0.282)
Overall Tournament						
Number of efficient mutual funds	0	0	0	0	0	0
Equally weighted score	0.257	0.206	0.221	0.233	0.296	0.399
(Standard deviation)	(0.123)	(0.109)	(0.110)	(0.124)	(0.115)	(0.135)

Panel B Network SBM model	2010	2011	2012	2013	2014	2015
Tournament Reaction						
Number of efficient mutual funds	2	0	0	2	2	35
Equally weighted score	0.179	0.387	0.264	0.279	0.413	0.404
(Standard deviation)	(0.164)	(0.291)	(0.208)	(0.200)	(0.217)	(0.287)
Tournament Recompense						
Number of efficient mutual funds	4	4	3	3	5	5
Equally weighted score	0.396	0.875	0.551	0.559	0.654	0.915
(Standard deviation)	(0.161)	(0.094)	(0.178)	(0.175)	(0.179)	(0.115)
Tournament Reward						
Number of efficient mutual funds	4	4	3	3	5	5
Equally weighted score	0.396	0.875	0.551	0.559	0.654	0.915
(Standard deviation)	(0.161)	(0.094)	(0.178)	(0.175)	(0.179)	(0.115)
Overall Tournament						
Number of efficient mutual funds	2	0	0	2	2	3
Equally weighted score	0.323	0.713	0.455	0.466	0.574	0.745
(Standard deviation)	(0.143)	(0.119)	(0.158)	(0.154)	(0.172)	(0.133)

This table is similar to Table 3 but for the use of the normalised value of the cumulative gross return of fund *j* from 1st January to 30th June instead of *Percent Rank*$_{j,t-6}$ (see Table 1), and the normalised variation in the cumulative gross return of fund *j* between 30th June and 31st December instead of *ΔPercent Rank*$_{j,t}$ (see Table 1). It shows the total number of mutual funds and the number of tournament-efficient funds per stage and year. It also provides the equally weighted average of the tournament scores obtained by the SBM separation model (Panel A) and the Network SBM model (Panel B). The standard deviations of the scores are in brackets.

Table C.4 Rank Correlation across Different NSBM Models and Variable Specifications

Panel A: NSBM (*t-6, t, t+3*) vs NSBM (*t-3, t, t+3*)

	2010	2011	2012	2013	2014	2015
Tournament Reaction (TR)	0.5875**	0.3636**	0.4989**	0.5102**	0.5442**	0.4079**
Tournament Recompense (TRc)	0.8023**	0.6575**	0.8491**	0.7831**	0.8429**	0.7922**
Tournament Reward (TRw)	0.8023**	0.6575**	0.8491**	0.7831**	0.8429**	0.7922**
Overall Tournament (OT)	0.7297**	0.3928**	0.7109**	0.5673**	0.8375**	0.5543**

Panel B: NSBM (*t-6, t, t+3*) vs NSBM (*t-6, t, t+3*) RankFlows

	2010	2011	2012	2013	2014	2015
Tournament Reaction (TR)	0.9293**	0.9161**	0.9703**	0.9591**	0.9606**	0.9041**
Tournament Recompense (TRc)	0.8137**	0.8361**	0.9059**	0.9083**	0.9508**	0.9441**
Tournament Reward (TRw)	0.8137**	0.8361**	0.9083**	0.9083**	0.9508**	0.9441**
Overall Tournament (OT)	0.7737**	0.5606**	0.8832**	0.8632**	0.9102**	0.7784**

Panel C: NSBM (*t-6, t, t+3*) vs NSBM (*t-6, t, t+3*) Returns

	2010	2011	2012	2013	2014	2015
Tournament Reaction (TR)	0.8946**	0.9673**	0.9292**	0.9289**	0.8829**	0.9574**
Tournament Recompense (TRc)	0.8848**	0.8969**	0.9474**	0.8957**	0.9310**	0.9420**
Tournament Reward (TRw)	0.8848**	0.8969**	0.9461**	0.8957**	0.9310**	0.9420**
Overall Tournament (OT)	0.8867**	0.9388**	0.9358**	0.9004**	0.8975**	0.9503**

Panel A of this table shows the Spearman rank correlations of the tournament rankings obtained by the NSBM model under VRS for the temporal specification (*t-6, t, t+3*) against the tournament rankings obtained by the NSBM model under VRS for the temporal specification (*t-3, t, t+3*). Panel B of this table shows the Spearman rank correlations of the tournament rankings obtained by the NSBM model under VRS (*t-6, t, t+3*) against the tournament rankings obtained by the same NSBM model specification using the percentile rank of the implied net money flows from 31st December to 31st March instead of variable $Flows_{j,t+3}$ (see Table 1). Panel C of this table shows the Spearman rank correlations of the tournament rankings obtained by the NSBM model under VRS (*t-6, t, t+3*) against the tournament rankings obtained by the same NSBM model specification using the normalised value of the cumulative gross return of fund *j* from 1st January to 30th June instead of $Percent\ Rank_{j,t-6}$ (see Table 1), and the normalised variation in the cumulative gross return of fund *j* between 30th June and 31st December instead of $\Delta Percent\ Rank_{j,t}$ (see Table 1).* 5% significance level; ** 1% significance level.

Table C.5 Network SBM Model under VRS using BP-Transformed Variables

NSBM using BP-transformed variables	2010	2011	2012	2013	2014	2015
Total number of mutual funds	135	119	107	88	91	84
Tournament Reaction						
Number of efficient mutual funds	3	1	0	2	2	2
Equally weighted score	0.161	0.339	0.218	0.257	0.277	0.327
(Standard deviation)	(0.201)	(0.334)	(0.234)	(0.271)	(0.276)	(0.327)
Tournament Recompense						
Number of efficient mutual funds	4	3	3	4	4	7
Equally weighted score	0.459	0.888	0.566	0.639	0.671	0.916
(Standard deviation)	(0.188)	(0.095)	(0.189)	(0.184)	(0.187)	(0.116)
Tournament Reward						
Number of efficient mutual funds	4	3	3	4	4	7
Equally weighted score	0.459	0.888	0.566	0.639	0.671	0.916
(Standard deviation)	(0.188)	(0.095)	(0.189)	(0.184)	(0.187)	(0.116)
Overall Tournament						
Number of efficient mutual funds	3	1	0	2	2	2
Equally weighted score	0.360	0.705	0.450	0.512	0.540	0.719
(Standard deviation)	(0.165)	(0.128)	(0.160)	(0.171)	(0.176)	(0.140)

This table is similar to Panel B of Table 3 but for the use of BP-transformed variables instead of the normalisation approach for the variables included in our original network model. It shows the total number of mutual funds and the number of tournament-efficient funds per stage and year. It also provides the equally weighted average of the tournament scores obtained by the Network SBM model. The standard deviations of the scores are in brackets.

Table C.6 Rank Correlation across NSBM Models: Normalised vs BP-Transformed Variables

NSBM using normalised variables (*t-6, t, t+3*) vs NSBM using BP-transformed variables (*t-6, t, t+3*)	2010	2011	2012	2013	2014	2015
Tournament Reaction (TR)	0.9988**	0.9984**	0.9923**	0.9962**	0.9974**	0.9979**
Tournament Recompense (TRc)	0.9998**	0.9999**	0.9995**	0.9994**	0.9999**	1.0000**
Tournament Reward (TRw)	0.9998**	0.9999**	0.9992**	0.9994**	0.9999**	1.0000**
Overall Tournament (OT)	0.9996**	0.9994**	0.9985**	0.9994**	0.9993**	0.9986**

This table shows the Spearman rank correlations of the tournament rankings obtained by the original NSBM model specification (*t-6, t, t+3*) under VRS using the normalised variables listed in Table 1 against the tournament rankings obtained by the original NSBM model specification (*t-6, t, t+3*) under VRS using the BP-transformed variables proposed by Tone et al. (2020). * 5% significance level; ** 1% significance level.