Published for SISSA by Springer

Received: November 26, 2025
Accepted: December 9, 2025
Published: February 2, 2026

Reflection positivity in a higher-derivative model with physical bound states of ghosts

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Abstract: The inclusion of higher derivatives is a necessary condition for a renormalizable or superrenormalizable local theory of quantum gravity. On the other hand, higher derivatives lead to classical instabilities and a loss of unitarity at the quantum level. A standard way to detect such issues is by examining the reflection positivity condition and the existence of a Källén-Lehmann spectral representation for the two-point function. We demonstrate that these requirements for a consistent quantum theory are satisfied in a theory we have recently proposed. This theory is based on a six-derivative scalar field action featuring a pair of complex-mass ghost fields that form a bound state. Our results support the interpretation that physical observables can emerge from ghost dynamics in a consistent and unitary framework.

Keywords: Models of Quantum Gravity, Nonperturbative Effects

ArXiv ePrint: 2511.15283

https://doi.org/10.1007/JHEP02(2026)020

JHEP02(2026)020

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1 Introduction

Quantum gravity theories involving higher-derivative fields often suffer from instabilities at the classical level, which are related to the well-known Ostrogradski instabilities [1]. In their quantum counterparts, these theories typically contain one-particle states with negative kinetic energy, known as ghosts. The presence of such particles can produce undesirable features, such as appearing at infinite times in incoming and outgoing states [2]. Unfortunately, it is unavoidable to introduce higher-derivative terms into the action of gravity in quantum theory [4], or even at the semiclassical level, where the gravitational field is a classical background for quantum matter fields [3]. Because of the utmost importance of the subject in quantum gravity, numerous remarkable attempts have been made to solve the problem of ghosts using different approaches. Notably, starting from [5–8], several studies have explored the possibility that ghosts in renormalizable quantum gravity with four derivatives might acquire masses with an imaginary component and, hence, become unstable due to loop corrections. In such scenarios, the ghost mode decays in the distant future, rendering the S-matrix unitary. Alternatively, there was an interesting proposal [10] to construct a theory in which massive ghosts are considered alongside the massless and healthy (i.e., stable) graviton. However, both ideas can hardly be implemented in the *minimal* renormalizable quantum gravity with four derivatives because in this theory there is not enough room for hosting a consistent subsector of composite of ghost fields.

On the other hand, resolving the problem of ghosts may be simpler in superrenormalizable quantum gravity models that incorporate six or more derivatives [9]. The mass spectrum in such models may contain real-mass ghosts accompanied by other massive, healthy (normal) states. Alternatively, there might exist ghost states with complex-conjugate masses. In both scenarios, one can envision a realization of Hawking's idea [10], where he speculated that complex-conjugate ghost states might be accommodated in a consistent quantum theory, provided they remain off-shell and do not contribute to the asymptotic Hilbert space. In the complex-mass scenario, quantum corrections are unnecessary to observe the emergence of the imaginary components of the masses, as described in one of the pioneering papers by Salam and Strathdee [8]. Within the six-derivative (or higher) quantum gravity framework, the complex-mass poles in the propagator are present already at tree level and persist under loop corrections [11].

An appealing feature of superrenormalizable theories with complex-mass ghosts is the possibility that such ghosts do not appear in the asymptotic spectrum but instead form physical bound states, an idea reminiscent of early proposals in the context of QCD [12] and in modern QCD-inspired approaches [13]. However, implementing such a mechanism in quantum gravity is substantially more difficult due to the nonpolynomial nature of the interactions and the presence of higher-derivative couplings, which make the dynamics more intricate than in typical QCD models. In spite of this, we recently introduced a toy model [14] that mimics certain features of higher-derivative quantum gravity and exhibits a bound state formed by complex-conjugate ghost fields. Related approaches exploring similar ghost-confinement mechanisms have also appeared in [26, 27]. In the present work, we address the question of whether the toy model of forming bound states that was constructed as an example in ref. [14] is a consistent quantum field theory.

Consistent quantum field theories are constrained by a set of basic and fundamental principles; see, e.g., [15–20] and also [31] for a recent discussion concerning higher derivative quantum gravity. Among the most relevant principles, one can list the Wightman axioms in Minkowski space, and in the Euclidean formalism, the Osterwalder-Schrader reflection positivity condition and the existence of a Källén-Lehmann spectral representation for the two-point function. Together with certain additional regularity conditions, these requirements ensure the construction of the physical Hilbert space and the quantum fields from the Euclidean Schwinger functions via the Osterwalder-Schrader theorem [15, 16]. The physical significance of these conditions has been thoroughly discussed in the aforementioned references, in particular, in the books [15, 16, 18]. Reflection positivity guarantees the positivity of the inner product in the reconstructed Hilbert space, while the Källén-Lehmann representation, applicable to both Wightman and Schwinger two-point functions, encodes unitarity through the positivity of the spectral density. Analyticity, in turn, embodies causality by enforcing locality and relativistic propagation. Taken together, these conditions guarantee unitarity, causality, and the probabilistic interpretation of the theory in accordance with the principles of quantum mechanics. In the present work, we consider the Källén-Lehmann representation for the bound state and the Osterwalder-Schrader condition of reflection positivity in the corresponding quantum theory.

The paper is organized as follows. The next section, section 2, is a technical part of the introduction, as we discuss the link between reflection positivity and the Källén-Lehmann representation for the Euclidean two-point function. In section 3 we analyze the reflection positivity condition in the model developed in ref. [14]. Section 4 discusses the Källén-Lehmann representation. Finally, in section 5 we draw our conclusions.

2 Reflection positivity from Källén-Lehmann representation

Let us start by analyzing the existing relation between reflection positivity and the positivity of the spectral representation. In particular, we shall show that if a two-point Schwinger correlation function $S_2(x-y)$ satisfies the Källén-Lehmann representation, it automatically verifies reflection positivity. Reflection positivity implies the positivity of the product in the Hilbert space [31]

$$\langle \theta f, S_2 f \rangle = \int d^4x\, d^4y\, (\theta f)(x)\, S_2(x-y)\, f(y) \geq 0\,, \tag{2.1}$$

for any continuous function $f \in C_0(\mathbb{R}^4_+)$ with compact support in the positive time half-space ($y_0 > 0$) of $\mathbb{R}^4$, where θ is the time-reversal transformation defined by $(\theta f)(x_0, \mathbf{x}) = f^*(-x_0, \mathbf{x})$. Assuming a Källén-Lehmann representation for the two-point function

$$S_2(p^2) = \int_0^\infty ds\, \frac{\rho(s)}{p^2 + s}\,, \qquad \rho(s) \geq 0\,, \tag{2.2}$$

which, after a Fourier transform, reads

$$S_2(x-y) = \int_0^\infty ds\; \rho(s)\, \Delta_s(x-y)\,, \qquad \Delta_s(x) = \int \frac{d^4p}{(2\pi)^4}\, \frac{e^{ip\cdot x}}{p^2+s} \tag{2.3}$$

in a position space representation. Thus, the reflection positivity definition (2.1) factorizes as

$$\langle \theta f, S_2 f\rangle = \int_0^\infty ds\; \rho(s)\, I_s \tag{2.4}$$

where

$$I_s = \int d^4x\, d^4y\, (\theta f)(x)\, \Delta_s(x-y)\, f(y)\,. \tag{2.5}$$

Since $f(y) = f(y_0, \mathbf{y})$ has support only at the half space $y_0 > 0$, which means it is only non-vanishing for $y_0 > 0$ (here and in what follows, this means the function is defined to be non-zero in the given region), and $(\theta f)(x_0, \mathbf{x}) = f^*(-x_0, \mathbf{x})$ only for $x_0 < 0$, we set $x_0 = -t'$ and $y_0 = t$ with $t, t' \geq 0$, such that

$$I_s = \int_0^\infty dt \int_0^\infty dt' \int d^3x\, d^3y\; f^*(t', \mathbf{x})\, \Delta_s((-t', \mathbf{x}) - (t, \mathbf{y}))\, f(t, \mathbf{y})\,. \tag{2.6}$$

After a Fourier transformation of the propagator kernel, we have

$$\begin{aligned}\Delta_s((-t', \mathbf{x}) - (t, \mathbf{y})) &= \int \frac{d^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \int \frac{dp_0}{2\pi}\, \frac{e^{-ip_0(t+t')}}{p_0^2 + \omega^2}\,,\\ &= \int \frac{d^3\mathbf{p}}{(2\pi)^3}\, e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}\, \frac{1}{2\omega}\, e^{-\omega(t+t')}\,, \qquad \omega = \sqrt{\mathbf{p}^2 + s}\,,\end{aligned} \tag{2.7}$$

where clearly $\omega > 0$. Inserting (2.7) back to (2.6) leads to

$$\begin{aligned} I_s &= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\omega} \int_0^\infty dt \int_0^\infty dt'\, e^{-\omega(t+t')} \int d^3x\, d^3y\; f^*(t', \mathbf{x}) e^{i\mathbf{p}\cdot\mathbf{x}} f(t, \mathbf{y}) e^{-i\mathbf{p}\cdot\mathbf{y}}\\ &= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\omega} \left(\int_0^\infty dt'\, e^{-\omega t'} \int d^3x\, f^*(t', \mathbf{x}) e^{i\mathbf{p}\cdot\mathbf{x}}\right) \left(\int_0^\infty dt\, e^{-\omega t} \int d^3y\, f(t, \mathbf{y}) e^{-i\mathbf{p}\cdot\mathbf{y}}\right)\\ &= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\omega} \left|\int_0^\infty dt\, e^{-\omega t} \int d^3x\, f(t, \mathbf{x}) e^{-i\mathbf{p}\cdot\mathbf{x}}\right|^2 \geq 0\,. \end{aligned} \tag{2.8}$$

Since we have factorized the dot product into a manifestly positive norm, and the Källén-Lehmann hypothesis requires $\rho(s) \geq 0$, we have

$$\langle \theta f, S_2 f\rangle = \int_0^\infty ds\; \rho(s)\, I_s \;\geq 0. \tag{2.9}$$

This means that the Källén-Lehmann representation with positive spectral density implies Osterwalder-Schrader reflection positivity. It should be remarked that the two-point correlation function satisfies the reflection positivity condition (2.1), even after the removal of possible UV divergences, whenever they are associated with local counterterms. The reason is that subtracting a polynomial term in p^2 from the two-point function contributes as derivatives of a Dirac's delta function in position space. Thus, this operation may add only contact terms that contribute solely at coincident points $x = y$, which are outside the support $\mathbb{R}^4_+$ of the f functions. Therefore, reflection positivity is also preserved by the renormalized two-point correlation functions.

3 Reflection positivity in the bound state model of complex ghosts

Let us now consider the reflection positivity condition in the model with a bound state formed by a pair of complex-mass ghost-like elementary quantum states [14]. The starting point of the model is a six-derivative scalar action with one healthy massless particle and two particles with complex-conjugate masses. By using Gaussian integration in the free theory, one can find an equivalent action with three two-derivative scalar fields. Ignoring the massless part and introducing an interaction term, we arrive at the action

$$S_{\rm gh} = \int d^4x \left\{ \frac{i}{2}\varphi_1 \left(-\partial^2 + m^2\right) \varphi_1 - \frac{i}{2}\varphi_2 \left(-\partial^2 + m^{*2}\right) \varphi_2 - U(\varphi_1, \varphi_2) \right\} , \qquad (3.1)$$

where we choose the interaction of the form

$$U(\varphi_1, \varphi_2) = \frac{1}{4}\, \lambda_{12}\, \varphi_1^2 \varphi_2^2 \,. \qquad (3.2)$$

Here $\lambda_{12} > 0$ is the coupling constant corresponding to the interaction of the two fields. Let us now explore the composite operator made of the product $O_{\varphi_1\varphi_2}(x)$ of two ghosts with complex-conjugate masses

$$O_{\varphi_1\varphi_2}(x) = \varphi_1(x)\varphi_2(x)\,. \qquad (3.3)$$

The respective correlation function $C(x, y)$ defines the nature of a possible bound state,

$$\begin{aligned} C(x,y) &= C(x-y) = \langle O_{\varphi_1\varphi_2}(x)\, O_{\varphi_1\varphi_2}(y)\rangle \\ &= \frac{1}{Z_{\rm gh}} \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2\, O_{\varphi_1\varphi_2}(x)\, O_{\varphi_1\varphi_2}(y)\, e^{-S_{\rm gh}}\,. \end{aligned} \qquad (3.4)$$

Using the first order two-point function $G_B(p)$

$$G_B(p) \;=\; \int \frac{d^4k}{(2\pi)^4}\, D_{\varphi_1}(p-k)\, D_{\varphi_2}(k)\,. \qquad (3.5)$$

with the elementary propagators

$$D_{\varphi_1}(p) = \frac{i}{p^2 + m^2} \qquad \text{and} \qquad D_{\varphi_2}(p) = \frac{-\,i}{p^2 + m^{*2}}\,, \qquad (3.6)$$

we arrive at the momentum representation of the resumed two-point correlation function (3.4) in the form

$$C(p) = G_B(p) \sum_{n=0}^{\infty} \left[\lambda_{12}\, G_B(p)\right]^n = \frac{G_B(p)}{1-\lambda_{12}\, G_B(p)}\,. \tag{3.7}$$

Thus, the fundamental features of (3.4) depend on the expression for the first order two-point function (3.5). This integral can be written in explicit form

$$G_B(p) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left[(p-k)^2+m^2\right]\left[k^2+m^{*2}\right]}\,. \tag{3.8}$$

Our purpose is to explore whether the correlation functions of the composite operator satisfy Osterwalder-Schrader reflection positivity. In the next section, we will derive an explicit, renormalized Källén-Lehmann representation for (3.8), which, by the results of the previous section, establishes its reflection positivity. Here, however, we provide a direct and independent proof that does not rely on the Källén-Lehmann representation. Consider

$$\Delta_m(k) = \frac{1}{k^2+m^2} = \frac{1}{k_0^2+\mathbf{k}^2+m^2}\,, \qquad \Delta_m(x) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\Delta_m(k)\,, \tag{3.9}$$

the free propagator in momentum space, and its Fourier transform to the coordinate space. Since $G_B(p)$ is a convolution of two free propagators in momentum space, its corresponding Fourier transform to position space $G_B(x)$ is the pointwise product and thus we need to apply the reflection positivity test (2.1) to the function

$$G_B(x-y) = \Delta_m(x-y)\Delta_{m^*}(x-y)\,. \tag{3.10}$$

Using the well-known Fourier integral

$$\int \frac{dk_0}{2\pi} \frac{e^{ik_0x_0}}{k_0^2+\mathbf{k}^2+m^2} = \frac{e^{-\omega_{\mathbf{k}}|x_0|}}{2\,\omega_{\mathbf{k}}}\,, \qquad \omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2+m^2}\,, \tag{3.11}$$

the Euclidean free propagator then has the representation

$$\Delta_m(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{k}}}\, e^{i\mathbf{k}\cdot\mathbf{x}}\, e^{-\omega_{\mathbf{k}}|x_0|}\,. \tag{3.12}$$

Reflection positivity requires the positivity of the product in Hilbert space

$$\langle \theta f, G_B f\rangle = \int d^4x\, d^4y\, (\theta f)(x)\, G_B(x-y)\, f(y) \;\geq 0\,. \tag{3.13}$$

Since f has support in $x_0 > 0$, the reflected function θf has support only in $x_0 < 0$. Thus, in the integral (3.13), it is always satisfied that $x_0 < 0$, $y_0 > 0$. Therefore

$$|x_0 - y_0| = -(x_0-y_0) = -x_0 + y_0 = |x_0| + y_0\,,$$

because for $x_0 < 0$ we have $|x_0| = -x_0$. Now, expanding the bilinear form of reflection positivity (3.13) and inserting $G_B(x-y)$ explicitly (3.10), we have

$$\begin{aligned}\langle \theta f, G_B f\rangle = &\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{k}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{2\omega^*_{\mathbf{q}}} \\ &\times \int_{x_0<0} d^4x \int_{y_0>0} d^4y\, f^*(-x_0,\mathbf{x})\, f(y)\, e^{i(\mathbf{k}+\mathbf{q})\cdot(\mathbf{x}-\mathbf{y})}\, e^{-(\omega_{\mathbf{k}}+\omega^*_{\mathbf{q}})(|x_0|+y_0)}\,.\end{aligned} \tag{3.14}$$

Now let us change the variables $u_0 = -x_0 > 0$ and $u = (u_0, \mathbf{x})$ with $\mathbf{u} = \mathbf{x}$. Then clearly $f^*(-x_0, \mathbf{x}) = f^*(u)$ and $|x_0| = u_0$, so the bilinear form (3.14) becomes

$$\langle \theta f, G_B f\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{k}}}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{2\omega^*_{\mathbf{q}}} \\ \times \int\limits_{u_0>0} d^4u \int\limits_{y_0>0} d^4y\, f^*(u)\, f(y)\, e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{u}}\, e^{-i(\mathbf{k}+\mathbf{q})\cdot\mathbf{y}}\, e^{-(\omega_{\mathbf{k}}+\omega^*_{\mathbf{q}})(u_0+y_0)}. \tag{3.15}$$

Let us introduce the auxiliary function

$$F_{\mathbf{k},\mathbf{q}} = \int\limits_{y_0>0} d^4y\; f(y)\; e^{-(\omega_{\mathbf{k}}+\omega^*_{\mathbf{q}})y_0}\; e^{-i(\mathbf{k}+\mathbf{q})\cdot\mathbf{y}} \tag{3.16}$$

and its complex conjugate

$$F^*_{\mathbf{k},\mathbf{q}} = \int_{u_0>0} d^4u\; f^*(u)\; e^{-(\omega^*_{\mathbf{k}}+\omega_{\mathbf{q}})u_0}\; e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{u}}\,. \tag{3.17}$$

Hence, using the modulus $F^*_{\mathbf{k},\mathbf{q}}F_{\mathbf{k},\mathbf{q}} = |F_{\mathbf{k},\mathbf{q}}|^2$, the bilinear form can be expressed as

$$\langle \theta f, G_B f\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{k}}}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{2\omega^*_{\mathbf{q}}}\; \left|F_{\mathbf{k},\mathbf{q}}\right|^2\; \geq 0\,. \tag{3.18}$$

Since we have been able to write the reflection positivity bilinear form as an integral of absolute squares with a positive measure, the first order two-point function $G_B(x-y)$ satisfies the reflection positivity property. Note that in the derivation, it is crucial that the two masses are complex conjugates (to end up with conjugate frequency factors) and have positive real parts so that the Fourier time-integrals yield decaying exponentials. One might be worried about the fact that the first order two-point function integral (3.8) is divergent at large momentum. Since this UV divergence is logarithmic, the simplest way to renormalize it is via a momentum subtraction scheme, as was done in [14].

Let us now define the first order renormalized two-point function as

$$G'_B(p) = G_B(p) - G_B(p_0)\,, \tag{3.19}$$

where the subtraction is done at p_0 and therefore $G_B(p_0)$ is a constant independent of p. In position coordinate space, this renormalized two-point function reads

$$G'_B(x) = G_B(x) - G_B(p_0)\,\delta^{(4)}(x)\,. \tag{3.20}$$

It is easy to see that the subtracted first order two-point function $G'_B(x)$ also satisfies reflection positivity, since by the definition (3.13) we explicitly have

$$\langle \theta f, G'_B f\rangle = \langle \theta f, G_B f\rangle - G_B(p_0)\int d^4x\, d^4y\; (\theta f)(x)\,\delta^{(4)}(x-y)\, f(y)\,. \tag{3.21}$$

The delta function reduces the last integral to

$$\int d^4x\; (\theta f)(x) f(x)\,,$$

but $(\theta f)(x) = f^*(-x_0, \mathbf{x})$ has support only where $x_0 < 0$, while $f(x)$ has support only where $x_0 > 0$. Since the supports are disjoint, the delta integral contribution vanishes; thus,

$$\langle \theta f, G'_B f \rangle = \langle \theta f, G_B f \rangle \,,$$

and so reflection positivity is preserved for the renormalized first order two-point function. The crucial point is that subtracting a constant in momentum space corresponds to adding a contact term proportional to $\delta^{(4)}(x)$ in position space. This contact term does not contribute to the Osterwalder-Schrader condition for test functions supported in the positive-time half-space. Thus, reflection positivity is preserved even after renormalization. More generally, subtracting a polynomial in p corresponds to adding derivatives of delta functions in position space; such contact terms are also invisible to the Osterwalder-Schrader conditions. This is a consequence of the principle that adding local contact terms does not change the physical correlation functions away from coincident points.

4 Renormalized Källén-Lehmann representation

Let us now show that the Euclidean first order two-point function integral (3.8) also admits a Källén-Lehmann spectral representation, we need to establish the following relationship

$$G_B(p^2) = \int_0^\infty ds \, \frac{\rho(s)}{p^2 + s} \,. \tag{4.1}$$

Here, $\rho(s) \geq 0$ must be a positive spectral density function to ensure the unitarity and causality of the theory. Is it clear that if $G_B(p^2)$ diverges, just as (3.8) does, then it is impossible to accomplish such a relation directly. It is usual in such cases to use a suitably subtracted spectral representation to deal with finite quantities. This subtracted/renormalized Källén-Lehmann representation can be derived by taking the derivative of (4.1) with respect to p^2 and then integrating from p_0^2 to p^2, i.e.,

$$G_B^R(p^2) = G_B(p^2) - G_B(p_0^2) = \int_0^\infty ds \, \rho(s) \left(\frac{1}{p^2 + s} - \frac{1}{p_0^2 + s} \right) , \tag{4.2}$$

where now $G_B^R(p^2)$ is finite. There are several ways of showing that $G_B^R(p^2)$ has a spectral representation with positive spectral density. One way starts from the original first order two-point function expression (3.8). After analytic continuation to Minkowski spacetime, we can use Cutkosky cut rules. In this case, the result is almost immediate [21]. The other way, which we detail here, is based on a method that was used in [22] for the case of purely imaginary masses. Here, we will apply it to the more general case of complex masses. We start by considering the derivative of (3.8) with respect to p^2, expressed in terms of Feynman parameters (the overall factor $1/(4\pi)^2$ is omitted for simplicity)

$$\begin{aligned} \frac{dG_B(p^2)}{dp^2} &= -\int_0^1 dx \, \frac{x(1-x)}{x(1-x)p^2 + (1-x)m^2 + xm^{*2}} \\ &= -\int_0^1 dx \left[p^2 + \frac{(1-x)m^2 + xm^{*2}}{x(1-x)} \right]^{-1} = -\int_0^1 dx \left[p^2 - i\frac{(2x-1)m_I^2 + im_R^2}{x(1-x)} \right]^{-1} , \end{aligned} \tag{4.3}$$

where we have defined $m^2 = m_R^2 + i m_I^2$. Next, we perform the following change of variables:

$$\alpha = \frac{(2x-1)m_I^2 + i m_R^2}{x(1-x)} = \begin{cases} x=0 & \rightarrow \quad \alpha_0 = -(1-i\,\theta_m)\infty \\ x=1 & \rightarrow \quad \alpha_1 = +(1+i\,\theta_m)\infty \end{cases} \quad \text{with} \quad \theta_m = \frac{m_R^2}{m_I^2}\,. \tag{4.4}$$

This means that the integration over α is along an infinite line tilted at an angle θ_m with respect to the real-α line. The integral in (4.3) then becomes

$$\begin{aligned} \frac{dG_B(p^2)}{dp^2} &= -\int_{\alpha_0}^{\alpha_1} \left(\frac{dx}{d\alpha}\right) \frac{d\alpha}{p^2 - i\alpha} \\ &= -\,i \int_{\alpha_0}^{\alpha_1} \frac{d\alpha}{2\alpha} \; \frac{\alpha - 2m_I^2 + \sqrt{\alpha^2 + 4(m_I^2)^2 - 4i m_R^2 \alpha}}{(p^2 - i\alpha)^2}\,, \end{aligned} \tag{4.5}$$

where the second term in the integrand is $x = x(\alpha)$. Note that when solving x in terms of α, the root with the minus sign in front of the square root is not the appropriate one, as it does not yield the limiting values $x = 0$ and $x = 1$. We are working in Euclidean space $p^2 \geq 0$; therefore, the integral in (4.5) does not have poles in the upper half-plane but has a square root branch cut starting from a threshold value α_{thr}. This threshold value is the value of α for which the square root in the integrand acquires an imaginary part; in the upper half plane, it is given by

$$\alpha_{\text{thr}} = i \left[2m_R^2 + 2\sqrt{(m_R^2)^2 + (m_I^2)^2} \right]. \tag{4.6}$$

This means that one can transform (4.5) into an integral over a contour Γ in the upper half plane. The integral over the contour at infinity vanishes because the integrand behaves as $1/|\alpha|^2$ for large α. Moreover, the integral over the term $(\alpha - 2m_I^2)/2\alpha$ vanishes because of Cauchy's theorem. Therefore, one is left with two integrals (in opposite directions) along each side of the cut. To perform the integral over the square root more easily, we change the variable again, $\alpha = is$, so that

$$\alpha_{\text{thr}} \;\longrightarrow\; s_0 = 2m_R^2 + 2\sqrt{(m_R^2)^2 + (m_I^2)^2}\,, \tag{4.7}$$

and

$$\begin{aligned} \frac{dG_B(p^2)}{dp^2} &= i \int_\Gamma ds \, \frac{1}{(p^2+s)^2} \frac{\sqrt{-s^2 + 4(m_I^2)^2 + 4m_R^2 s}}{2s} \\ &= i \int_{s_0}^{\infty} ds \, \frac{1}{(p^2+s)^2} \frac{i\sqrt{s^2 - 4(m_I^2)^2 - 4m_R^2 s}}{2s} \\ &\quad + i \int_{\infty}^{s_0} ds \, \frac{1}{(p^2+s)^2} \frac{-i\sqrt{s^2 - 4(m_I^2)^2 - 4m_R^2 s}}{2s} \\ &= -\int_{s_0}^{\infty} \frac{ds}{s\,(p^2+s)^2} \sqrt{s^2 - 4(m_I^2)^2 - 4m_R^2 s}\,. \end{aligned} \tag{4.8}$$

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This last expression confirms the well-known result that the integral of a function with a branch cut is twice the imaginary part of that function. Now, integrating over p^2 and recovering the overall factor, we obtain

$$G_B^R(p^2) = G_B(p^2) - G_B(p_0^2) = \int_{s_0}^{\infty} ds\, \frac{\sqrt{s^2 - 4(m_I^2)^2 - 4m_R^2 s}}{(4\pi)^2 s} \left(\frac{1}{p^2+s} - \frac{1}{p_0^2+s} \right), \tag{4.9}$$

where $G_B(p_0^2)$ is a finite real number corresponding to the subtraction, which is crucial for taming the UV divergence. Is it clear now, comparing (4.9) and (4.2), that the first order two-point function integral (3.8) satisfies a renormalized Källén-Lehmann representation with a manifestly non-negative spectral density given by

$$\rho(s) = \Theta(s - s_0) \frac{\sqrt{s^2 - 4(m_I^2)^2 - 4m_R^2 s}}{(4\pi)^2 s} \geq 0\,. \tag{4.10}$$

In Minkowski space, this corresponds, up to a π factor, to the discontinuity of the first order two-point function $G_B(p^2 = -\tau + i\epsilon)$ along its branch cut, where it develops an imaginary part. An easy proof of this general statement can be seen directly from the Källén-Lehmann representation (4.1) of a correlator $F(p^2)$ and by using the Sokhotski-Plemelj identity:

$$\begin{aligned}
\lim_{\epsilon\to 0^+} \mathrm{Im}[F(p^2 = -\tau - i\epsilon)] &= \lim_{\epsilon\to 0^+} \frac{1}{2i}\Big[F(-\tau - i\epsilon) - F(-\tau + i\epsilon)\Big] \\
&= \lim_{\epsilon\to 0^+} \frac{1}{2i} \int_0^{\infty} ds\, \rho(s) \left(\frac{1}{-\tau - i\epsilon + s} - \frac{1}{-\tau + i\epsilon + s} \right) \\
&= \frac{1}{2i} \int_0^{\infty} ds\, \rho(s)\, (2\pi i \delta(\tau - s)) = \pi\rho(\tau)\,.
\end{aligned} \tag{4.11}$$

Once a renormalized Källén-Lehmann representation is obtained for the renormalized first order two-point function $G_B^R(p^2)$, it is straightforward to obtain such a representation for the resumed two-point correlation function $C^R(p^2)$ defined in (3.7) with the first order renormalized function $G_B(p^2)$. Using the same method that was used to derive the Källén-Lehmann decomposition of $G_B(p^2)$, we can obtain the spectral function for $C^R(p^2)$ by computing its imaginary part in Minkowski space, as in (4.11). From the definition of the correlator, we have

$$\begin{aligned}
\lim_{\epsilon\to 0^+} \mathrm{Im}[C^R(p^2 = -\tau - i\epsilon)] &= \lim_{\epsilon\to 0^+} \mathrm{Im}\left[\frac{G_B^R(-\tau + i\epsilon)}{1 - \lambda_{12} G_B^R(-\tau + i\epsilon)} \right] \\
&= \frac{\mathrm{Im}[G_B^R(-\tau)]}{(1 - \lambda_{12}\mathrm{Re}[G_B^R(-\tau)])^2 + (\lambda_{12}\mathrm{Im}[G_B^R(-\tau)])^2} = \pi\rho_C(\tau)\,,
\end{aligned} \tag{4.12}$$

where $\rho_C(s)$ corresponds to the spectral density of the continuum part (above the branch cut) of the resumed correlator $C^R(p^2)$. An explicit expression for $\rho_C(\tau)$ above (4.12) is obtained from the real and imaginary parts of the renormalized expression in Minkowski space, derived by analytic continuation of the first order correlation function, where we have fixed $m^2 = 1+i$ and $p_0 = 1$ for concreteness:

$$\mathrm{Re}\left[G_B^R(-\tau)\right] = \mathrm{Re}\Bigg\{-\frac{1}{32\pi^2\,\tau}\left(\pi(-\tau-1) - \tau\log 2\right.$$
$$\left.+4\sqrt{(4-\tau)\tau+4}\left[\tan^{-1}\left(\frac{-\tau+2\sqrt{2}+2}{\sqrt{(4-\tau)\tau+4}}\right) - \tan^{-1}\left(\frac{2-\tau}{\sqrt{(4-\tau)\tau+4}}\right)\right]\right)\Bigg\}, \tag{4.13}$$

where

$$\mathrm{Im}[G_B^R(-\tau)] = \pi\Theta\left(s-\left(2+2\sqrt{2}\right)\right)\frac{\sqrt{\tau^2-4-4\tau}}{(4\pi)^2\tau}\,. \tag{4.14}$$

Of course, this is not the whole spectral density for the correlator, since by the definition of $C^R(p^2)$ we also have an isolated contribution from the pole associated with the ghost condensate bound state of mass $\mathscr{M}$. The complete renormalized Källén-Lehmann representation of the resumed two-point correlation function is then

$$C^R(p^2) = \int_0^\infty ds\;\rho(s)\left(\frac{1}{p^2+s} - \frac{1}{p_0^2+s}\right), \tag{4.15}$$

with

$$\rho(s) = R_C\,\delta(s-\mathscr{M}^2) + \rho_C(s)\,\Theta(s-s_0) \geq 0\,, \quad R_C = -\frac{1}{\lambda_{12}^2\mathrm{Re}\left[\frac{d}{ds}G_B(-s)|_{s=\mathscr{M}^2}\right]}\,. \tag{4.16}$$

Here R_C is the residue corresponding to the bound state, and s_0 is the threshold of a physical branch cut. Note that for the noninteracting theory, $\lambda_{12} = 0$, we obtain the spectral function of the first order two-point function, as it should. Also, the value of s_0, given by (4.7), can be rewritten as

$$\begin{aligned} s_0 &= 2m_R^2 + 2\sqrt{(m_R^2)^2 + (m_I^2)^2} = m^2 + m^{*2} + 2\sqrt{m^2 m^{*2}} \\ &= \left(\sqrt{m^2} + \sqrt{m^{*2}}\right)^2 = (m+m^*)^2\,, \end{aligned} \tag{4.17}$$

which generalizes the well-known result of the branch cut for real masses. Using the results of the first section, as we have obtained a renormalized Källén-Lehmann representation for the resumed two-point function correlator, we can claim that it also satisfies reflection positivity, since only a momentum independent subtraction (contact term) is needed to handle the UV divergence.

5 Conclusions

We have investigated the consistency of a six-derivative scalar field theory featuring a bound state formed by a pair of ghost fields with complex-conjugate masses that we have proposed in [14]. We examined whether this model satisfies two fundamental consistency conditions for a quantum field theory: Osterwalder-Schrader reflection positivity and the existence of a renormalized Källén-Lehmann spectral representation with a positive spectral density. We have shown that both conditions are satisfied by the two-point function of the composite (bound state) operator, indicating that physical observables can consistently emerge from the underlying ghost dynamics in such a theory. In fact a quantum field theory is not completely defined until one specifies which are their physical fields. In the present case they are the composite local fields

$$O_{\varphi_1\varphi_2}(x) = \varphi_1(x)\varphi_2(x)\,,$$

Our main goal is to explore the possibility of forming bound states in higher (six or more) derivative models of quantum gravity, where ghost-like states with complex-mass poles naturally appear. For this reason, it would be interesting to extend the simplest toy model ref. [14] endowed with a mechanism of confining ghosts into bound states by including more general types of interactions, e.g., those with derivatives, which are typical for quantum gravity models. One can foresee that the methods of proving the consistency of the theory with bound states, which we used here for the relatively simple scalar model with the single quartic interaction term (3.2), may eventually be used for more sophisticated models of quantum gravity involving ghosts in a controlled way.

Acknowledgments

G.K. acknowledges discussions with David Dudal on spectral representations of bound states of particles with complex masses. This work was supported by Spanish Grants no. PGC2022-126078NB-C21 funded by MCIN/AEI/10.13039/501100011033, Diputación General de Aragón-Fondo Social Europeo (DGA-FSE) Grant no. 2020-E21-17R of the Aragon Government; and the European Union, NextGenerationEU Recovery and Resilience Program on Astrofísica y Física de Altas Energías, CEFCA-CAPA-ITAINNOVA (M.A.); Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) grants nos. 305122/2023-1 (I.Sh.), 313254/2025-7 (G.K.); Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), grant no. 2018/25225-9 (G.K.).

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

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