Geotech Geol Eng (2026) 44:133
https://doi.org/10.1007/s10706-025-03562-9

ORIGINAL PAPER

Strength Parameter Estimation from Triaxial Data Using the Úcar Failure Criterion

Norly Belandria · Luis E. Arlegui · Francisco Javier Torrijo · Roberto Úcar

Received: 28 July 2025 / Accepted: 1 December 2025 / Published online: 16 February 2026

Abstract This study presents a methodology for fitting the uniaxial compressive strength (UCS) and tensile strength (TS) of rocks and concrete material, using the nonlinear failure criterion proposed by Úcar. A selected of triaxial test data was compiled from the literature and systematically filtered to retain only data consistent with the brittle failure behaviour. For each lithology, the relationship between the principal stresses was modelled through nonlinear regression, allowing estimation of the UCS and TS parameters. These were used to calculate and adjust $\xi = TS/UCS$, parameter of the Úcar failure criterion, quantifying the variation using the Taylor series expansion. Failure envelopes models are proposed for the six rock types analyzed as well as for the concrete. A comparative case study based on Coburg limestone, using the proposed regression methodology of the Úcar criterion versus the Hoek and Brown criterion against Bayesian and heuristic alternatives, yielded successful results. A key advantage of the Úcar criterion is that it requires only UCS and TS as input parameters. The proposed methodology is especially valuable during the early stages of geotechnical and rock engineering projects, or under budget limitations, where experimental datasets are often sparse or unavailable. In addition, the general rock-type models developed herein enable strength prediction for intact rock in contexts where only lithology is known or available data are limited.

Keywords Data triaxial · Uniaxial compressive strength (UCS) · Tensile strength (TS) · Nonlinear regression · Úcar criterion · Hoek and Brown criterion

Abbreviations

σ_1	Major principal stress at failure
σ_3	Minor principal stress at failure
$\sigma_c = UCS$	Uniaxial compressive strength
$\sigma_t = TS$	Tensile Strength
ξ	Úcar criterion: parameter TS/UCS
k_1, k_2	Úcar criterion: parameters dependent on ξ

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s10706-025-03562-9.

N. Belandria (✉) · R. Úcar
Departamento de Geomecánica, Universidad de Los Andes, Mérida, Venezuela
e-mail: norlyb@gmail.com

R. Úcar
e-mail: robertoucar@gmail.com

N. Belandria · L. E. Arlegui (✉)
Departamento de Ciencias de la Tierra, Universidad de Zaragoza, Zaragoza, Spain
e-mail: 875639@unizar.es

F. J. Torrijo
Centro de Investigación en Arquitectura, Patrimonio y Gestión para el Desarrollo Sostenible (PEGASO), Universitat Politècnica de València, Valencia, Spain
e-mail: fratorec@trr.upv.es

m_i, s	Hoek-Brown criterion: parameters
σ_{ci}	Unconfined compressive strength of intact rock
σ_{x1}	Standard deviation or standard error mean of TS
σ_{x2}	Standard deviation or standard error mean of UCS
ρ_{x1x2}	Pearson correlation coefficient between UCS and TS
SD	Standard deviation
sem	Standard error mean
$\sigma_{c\text{-adjus}}$	Unconfined compressive strength adjusted
$\sigma_{t\text{-adjus}}$	Tensile strength adjusted
$\xi_{\text{-adjus}}$	$\sigma_{t\text{-adjus}}/\sigma_{c\text{-adjus}}$
$\Delta\xi$	Variation of ξ

1 Introduction

In rock engineering, it is widely recognized that the ground's response to stress conditions induced by engineering activities is governed primarily by the strength characteristics of the rock mass. Although these characteristics differ from those of the intact rock, they are intrinsically linked, as previously noted by Hansagi (1965). Consequently, determining the strength at failure of intact rock represents a fundamental step in the characterization of any rock mass, regardless of the nature of the engineering application.

The mechanical behaviour of rock under varying confining stress conditions can be assessed through the application of failure criteria. These criteria not only predict the onset of failure but also serve as a mathematical framework—typically expressed in the form of constitutive equations—for use in numerical simulations and modelling.

A wide range of failure criteria have been proposed in the literature, several of which have seen widespread application, including those by Coulomb (1776), Griffith (1921, 1924), Murrell (1965), and Hoek and Brown (1980, 2019). Other, less commonly used criteria have also been compiled by Edelbro (2003). Most of these approaches are empirical in nature, typically derived from regression analyses of experimental datasets.

Many were originally formulated on the basis of triaxial laboratory testing, which remains a fundamental method for characterizing rock strength and soil. Researchers have also performed triaxial tests for stability assessment in underground rock engineering (Fan et al., 2025), tunnels (Gui et al., 2025), experimental triaxial tests (Haque and Ansary, 2025; Du W et al., 2025) anisotropic failure models (Ganesan and Mishra, 2024).

However, triaxial testing is both time-consuming and costly, prompting the search for alternative methodologies. In contrast, uniaxial compressive strength (UCS) and tensile strength (TS) can be obtained more efficiently under laboratory conditions. Although these tests are less comprehensive than triaxial ones, they yield parameters of critical importance in rock mechanics. In this study, we propose a novel approach to fit values UCS and TS from triaxial test data using nonlinear regression analysis. Here, tensile stresses are considered negative, following standard geological convention.

In contrast, the estimation of tensile strength remains a more challenging task due to the inherent complexity of its measurement. Pérez-Rey et al. (2024) conducted and compared direct tensile strength (DTS) tests in four laboratories across different countries, using diverse testing equipment and methodologies on two rock types—granite and sandstone—to highlight the variability in results.

Experimental triaxial data collected from investigations are currently available and can be used to predict and adjust geotechnical parameters. In this context, the present study applies nonlinear regression analysis to laboratory-derived triaxial test data, leveraging the advantage that the functional form of the relationship between the principal stresses (σ_1, σ_3) becomes explicit once a failure criterion is adopted.

The objective of this research is to evaluate the accuracy of fit uniaxial compressive strength (UCS) and tensile strength (TS) through the incorporation the nonlinear formulation of Úcar's failure criterion (Úcar 2021) and the Python programming language. Once trained, the predictive model may serve as a valuable tool in preliminary stages of geotechnical projects—particularly during exploration phases or under budgetary and time constraints—where reliable numerical modelling is often precluded due to the lack of experimentally determined parameters.

Úcar's failure criterion is especially well-suited for such applications, as it provides reliable estimates of rock strength based solely on UCS and TS.

Furthermore, incorporating triaxial data for a specific lithology enhances the predictive performance and allows for an estimation of the UCS/TS ratio. This includes more accurate determination of the parameter $\xi = TS/UCS$, which governs Úcar's criterion, as well as an analytical comparison between the Hoek–Brown's failure criterion (Cordón et al. 2024; Hoek and Brown 2019). The expected range of this ratio can be further constrained using Taylor's analytical formulation. Finally, graphical representations of predicted UCS and TS values, along with their associated variability, provide useful visual tools for selecting appropriate failure models to each rock type studied.

2 Methodology

This section describes the methodology adopted to fit the uniaxial compressive strength (UCS) and the tensile strength (TS) of rocks using Triaxial laboratory data, then fitted using nonlinear regression techniques from formulation of Úcar's failure criterion.

2.1 Input data

A substantial dataset that encompasses various types of rock was compiled from multiple sources. The core dataset originates from Sheorey (1997), which includes triaxial test results previously reported by Everling (1960); Gnirk (1963); Brace (1964); Hobbs (1964); Gnirk and Cheatham Jr (1965); Dunikowski et al. (1969); Franklin and Hoek (1970); Hobbs (1970); Horino and Ellickson (1970); Glushko and Kirnichanskiy (1974); Heard et al. (1974); Chan (1977); Gowd and Rummel (1980); Dlugosz et al. (1981); Borecki et al. (1982); Dayre and Giraud (1986); Betournay et al. (1991); Hareland et al. (1993). Additional triaxial datasets were obtained from Schwartz (1963), Torres (1992), and Guo et al. (2020), as well as from the commercial software RocData™v4.2. Most of these datasets report values of triaxial compressive strength (TCS).

Given the diversity of sources, the dataset can exhibit heterogeneity in terms of experimental procedures, equipment, and sample characteristics. As poor-quality data can considerably affect the reliability of the analysis, rigorous quality control measures were applied. These were informed by standard practices in rock mechanics to identify and correct potential inconsistencies, ensuring the robustness and integrity of the final dataset.

2.2 Selected Data

The experimental data were classified by lithology, including granite, quartzite, limestone, sandstone, marble, coal, and concrete. The latter was incorporated due to its brittle failure behaviour, which is relevant in many rock engineering contexts. Given the heterogeneity of the compiled data included in as a supplementary material to this study (appendix A), a systematic methodology was implemented to minimize errors arising from the inclusion of inappropriate samples and to improve overall data quality. This process was guided by established recommendations from the rock mechanics literature.

The criteria used to filter out inconsistent or noise prone data during the exploratory analysis are as follows:

- **Exclusion of ductile behaviour:** Samples exhibiting ductile responses were removed based on the criterion proposed by Mogi (1973), whereby brittle–ductile transition occurs when $\sigma_1 = 4.4\ \sigma_3$. Several studies (Mogi 1966; Byerlee 1968; Scholz 1968; Hu et al. 2018; Hoek and Brown 2019) have examined this transition in intact rocks, generally agreeing that brittle behaviour is expected within the range $3 < \sigma_1/\sigma_3 < 5$. Accordingly, a conservative threshold of $\sigma_1/\sigma_3 = 4$ was adopted in this study.
- **Minimum data density:** Only datasets containing at least three valid triaxial test points were retained. As noted in standard guidelines, a robust triaxial testing programme should ideally include a minimum of three to five tests, supplemented by uniaxial compression tests, and preferably multiple observations at each level of confining stress σ_3.
- **Triaxial test data:** Fig. 1a shows all the data collected for this research. The dataset is shared in the supplementary material. This data comprises a total of 1098 (σ_3, σ_1) pairs. Of these, 108 samples correspond to Marble, 125 to Granite, 387 to Sandstone, 213 to Coal, 129 to Limestone, 62 to Quartzite, and 74 to Concrete. On the other hand, Fig. 1b illustrates the selected data considering

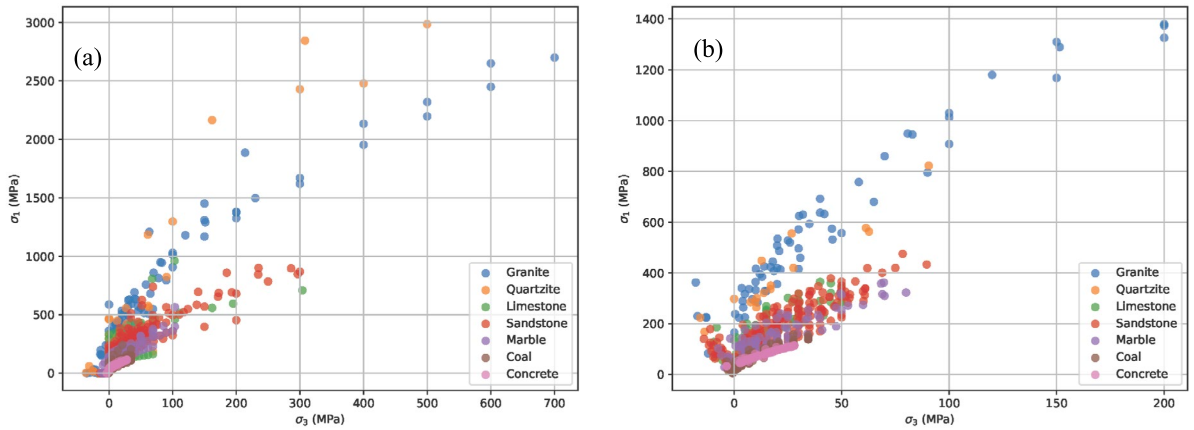


Fig. 1 Distribution of σ_3 and σ_1 values by lithology (**a**) Dataset collected. **b** Dataset selected

the exclusion of ductile behaviour and minimum data density. The studied and analyzed data is distributed as follows: 89 samples for Marble, 74 for Granite, 232 for Sandstone, 145 for Coal, 73 for Limestone, and 25 for Quartzite, for a total of 709 samples, representing 64.57% of the total samples.

- **Uniaxial compressive and tensile strength test data:** The study of the simple compressive and tensile strength of each sample is shown in Figs. 2 and 3, respectively. These pairs of (σ_t, σ_c) values will represent the input data for the nonlinear regression analysis carried out using the Python program. The collected data corresponds to 149 (σ_t, σ_c) pairs, representing 13.57% of the total samples. These can be seen in Appendix C, is as follows: 15 for granite, 8 for quartzite, 19 for limestone, 54 for sandstone, 17 for marble, 31 for coal, and 6 for concrete.

It is important to note that not all samples include experimentally measured (σ_t, σ_c) values. Among the 149 samples, 75 lack TS measurements (σ_t) and 3 lack UCS measurements (σ_c). To address these missing data, a preliminary least-squares analysis was performed in an Excel spreadsheet to estimate the individual (σ_t, σ_c) values for each rock type and sample. These estimated values are reported in the last two columns of Appendix C. Through this approach, 50.33% of the tensile strength values and 2.01% of the compressive strength values were inferred using least squares.

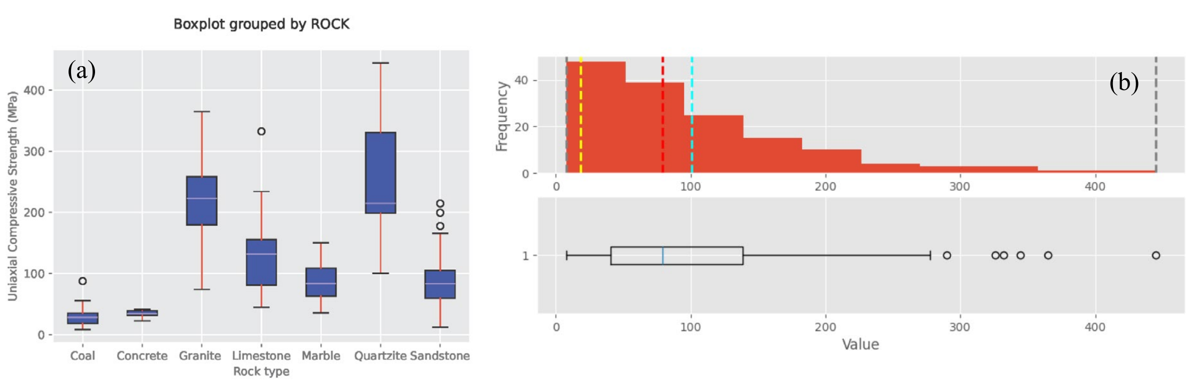


Fig. 2 Uniaxial compressive strength (UCS), grouped by rock type (**a**) and general (**b**) showing the frequency distribution and points outliers

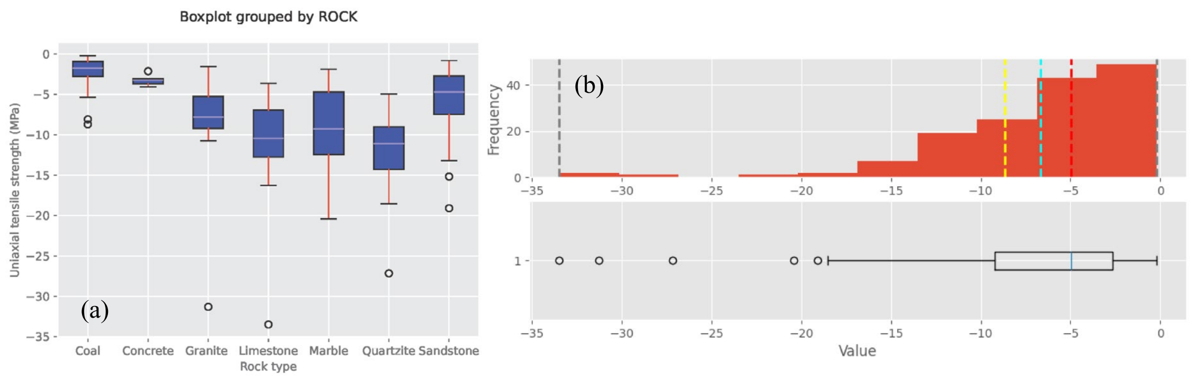


Fig. 3 Tensile strength (TS), grouped by rock type (**a**) and general (**b**) showing the frequency distribution and points outliers

The UCS and TS values obtained through least-squares estimation were subsequently incorporated into the dataset for each lithology, following a data-cleaning procedure that retained values between the 10 and 90 percentiles. These cleaned datasets were then used to compute the adjusted UCS and TS parameters corresponding to Úcar's failure criterion for each rock type in a generalized manner. After filtering, a total of 113 samples remained: 11 granite, 5 quartzite, 13 limestone, 40 sandstone, 12 marble, 26 coal, and 6 concrete, representing 75.84% of the processed dataset.

It is important to emphasize that the failure modes observed in uniaxial compressive strength tests—including single-plane shear, axial splitting, double shear, and more complex fracture patterns—can introduce significant variability into the measured values (Chakraborty et al. 2019). Similarly, Perras and Diederichs (2014) noted that tensile strength tests are frequently affected by experimental artefacts, reflecting the inherent difficulty of obtaining consistent and valid TS measurements. These authors also highlighted that direct tensile testing remains the most reliable method when properly executed, with Hobbs (1964) and Brace (1964) recommending dog-bone specimen geometries to ensure optimal performance. In this study, the available laboratory-derived UCS and TS values—whether experimental or least-squares-adjusted—were used as input for the nonlinear regression analysis.

2.3 Failure Criteria

The entire procedure is based on the nonlinear failure criterion proposed by Úcar (2021), which defines an explicit relationship between the major and minor principal stresses (σ_1 and σ_3). This criterion has demonstrated notable predictive performance when applied in a data-driven context (Cordón et al. 2024; Úcar et al. 2024), and its formulation also allows the derivation of parameters used in other widely accepted failure models, such as the Hoek–Brown criterion (Hoek and Brown 1980, 2019).

The principal advantage of Úcar's criterion lies in its analytical structure, which enables the estimation of uniaxial compressive strength (UCS) and tensile strength (TS) directly from the fitted curve, as both parameters are explicitly embedded in the formulation.

The general form of the criterion is expressed as follows (Úcar 2021):

$$\overline{\sigma}_1 = k_1\left(\overline{\sigma}_3 - \xi\right) + k_2\left(\overline{\sigma}_3 - \xi\right)^{1/2} \tag{1}$$

$$\mathrm{k}_1 = \frac{(1+|\xi|)+\sqrt{(1+7|\xi|)(1-|\xi|)}}{2|\xi|} \tag{2}$$

$$k_2 = \frac{1-k_1(-\xi)}{\sqrt{-\xi}} \tag{3}$$

where, $\overline{\sigma}_1 = \sigma_1/\mathrm{UCS}$ is the principal major stress, $\overline{\sigma}_3 = \sigma_3/\mathrm{UCS}$ is the principal minor stress, $\xi = \mathrm{TS}/$

UCS is the ratio between tensile and uniaxial compressive strength, and the coefficients k_1 and k_2 are functions of ξ. These expressions form the basis for the nonlinear regression analysis employed in this study to estimate UCS and TS from triaxial data.

For comparison, the Hoek–Brown failure criterion, originally proposed by Hoek and Brown (1980), is given by:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\sqrt{m_i\frac{\sigma_3}{\sigma_{ci}} + 1} \tag{4}$$

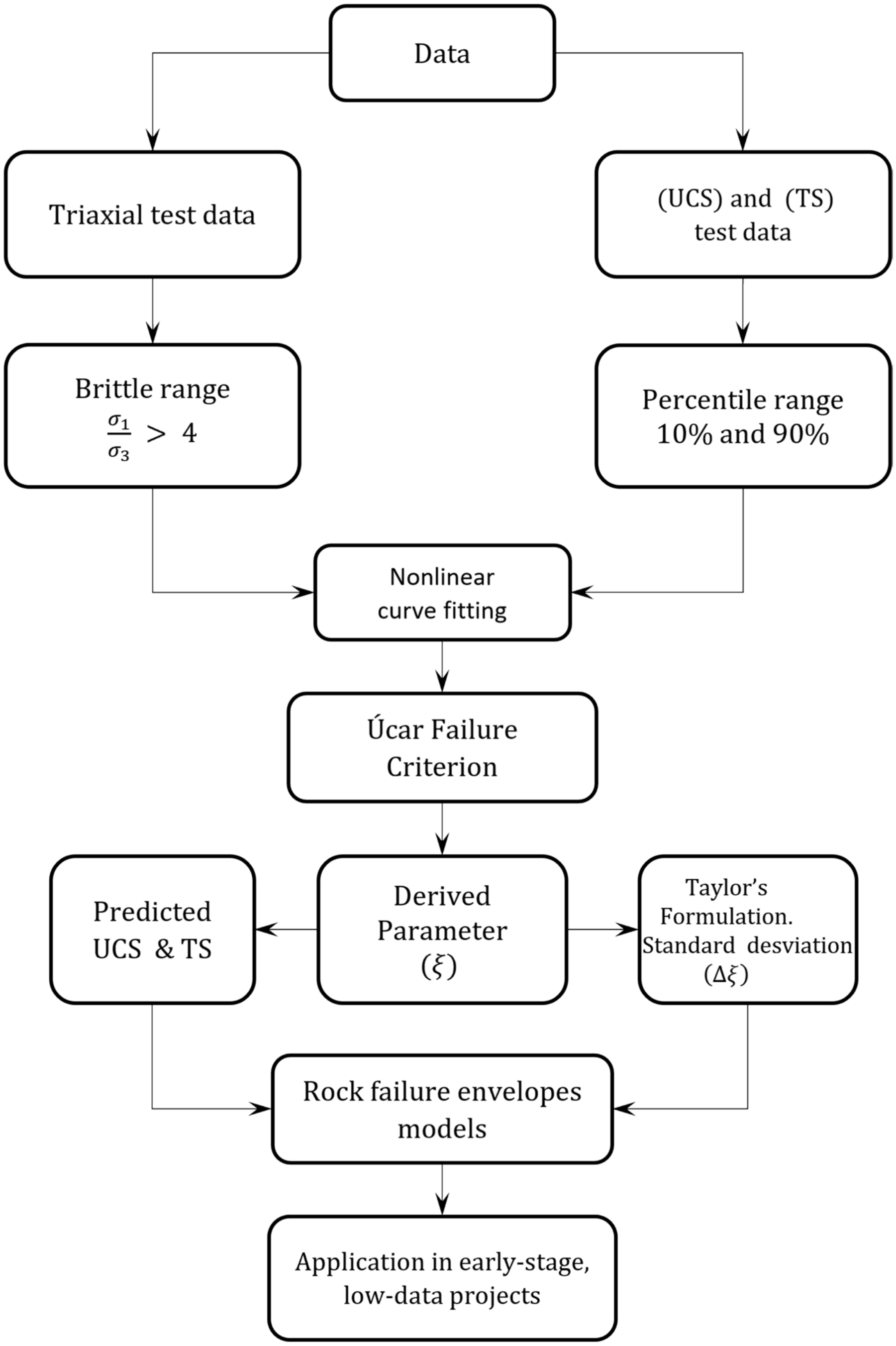


Fig. 4 Flowchart of the nonlinear regression workflow using the Úcar failure criterion

where σ_{ci} is the unconfined compressive strength of the intact rock, and m_i is a material constant that characterizes the brittle behaviour of the rock.

Although the Hoek–Brown criterion is widely used, it does not allow direct computation of UCS and TS from the curve itself. However, previous research has identified empirical relationships between the ratio $\sigma_{ci}/\ \sigma_t$ and the Hoek–Brown parameter m_i. For example, Ramsey and Chester (2004) and Bobich (2005) proposed the following expression, later cited by Hoek and Brown (2019):

$$\frac{\sigma_{ci}}{|\sigma_t|} = 0.81 \cdot m_i + 7 \tag{5}$$

This equation is based on a regression analysis of triaxial data. Since mi cannot be directly measured and remains a subject of ongoing research, this relationship is a generalized proposal for different rock types that relates σ_c/σ_t to m_i. On the other hand, Hoek and Brown (2019) have indicated that other methods exist for fitting curves to triaxial data, such as the modified Cuckoo search (Walton et al. 2011), which is included in Rocscience's RocData program, and the method proposed by Bozorgzadeh et al. (2018) and Contreras et al. (2018), which uses Bayesian statistics to quantify the strength of intact rock. These methods are used to compare the failure criterion model proposed by Úcar with that of Hoek and Brown.

Table 1 initial data on the types of rocks studied

Rock	UCS (MPa)	TS (MPa)	CC Lineal
Granite	220.09	−7.48	0.982
Quartzite	245.95	−14.96	0.806
Limestone	123.59	−9.95	0.970
Sandstone	85.37	−5.46	0.930
Marble	86.51	−8.55	0.979
Coal	27.94	−2.09	0.968
Concrete	34.07	−3.58	0.979

2.4 Nonlinear Regression Analysis Based on the Úcar Failure Criterion

To improve predictive accuracy, the nonlinear regression analysis was applied using the Úcar failure model defined by Eq. 1. The implementation was carried out in Python, using established scientific computing libraries.

The dataset was randomly partitioned into training and validation. the 80% of the data allocated for training and 20% for validation. In this framework, the minor principal stress σ_3, was used as the input feature (X), UCS and TS was entered as initial guess data, while the major principal stress σ_1 was the target variable (y).

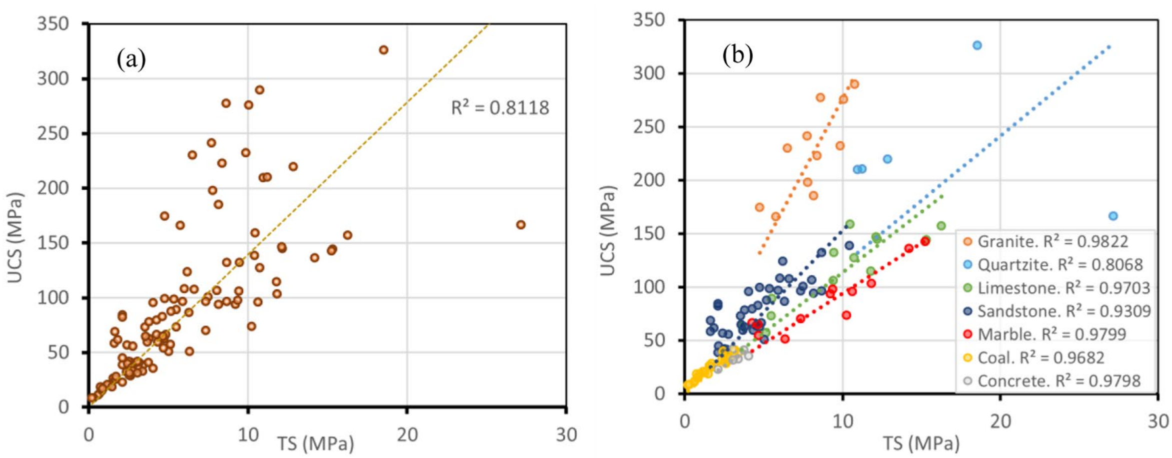
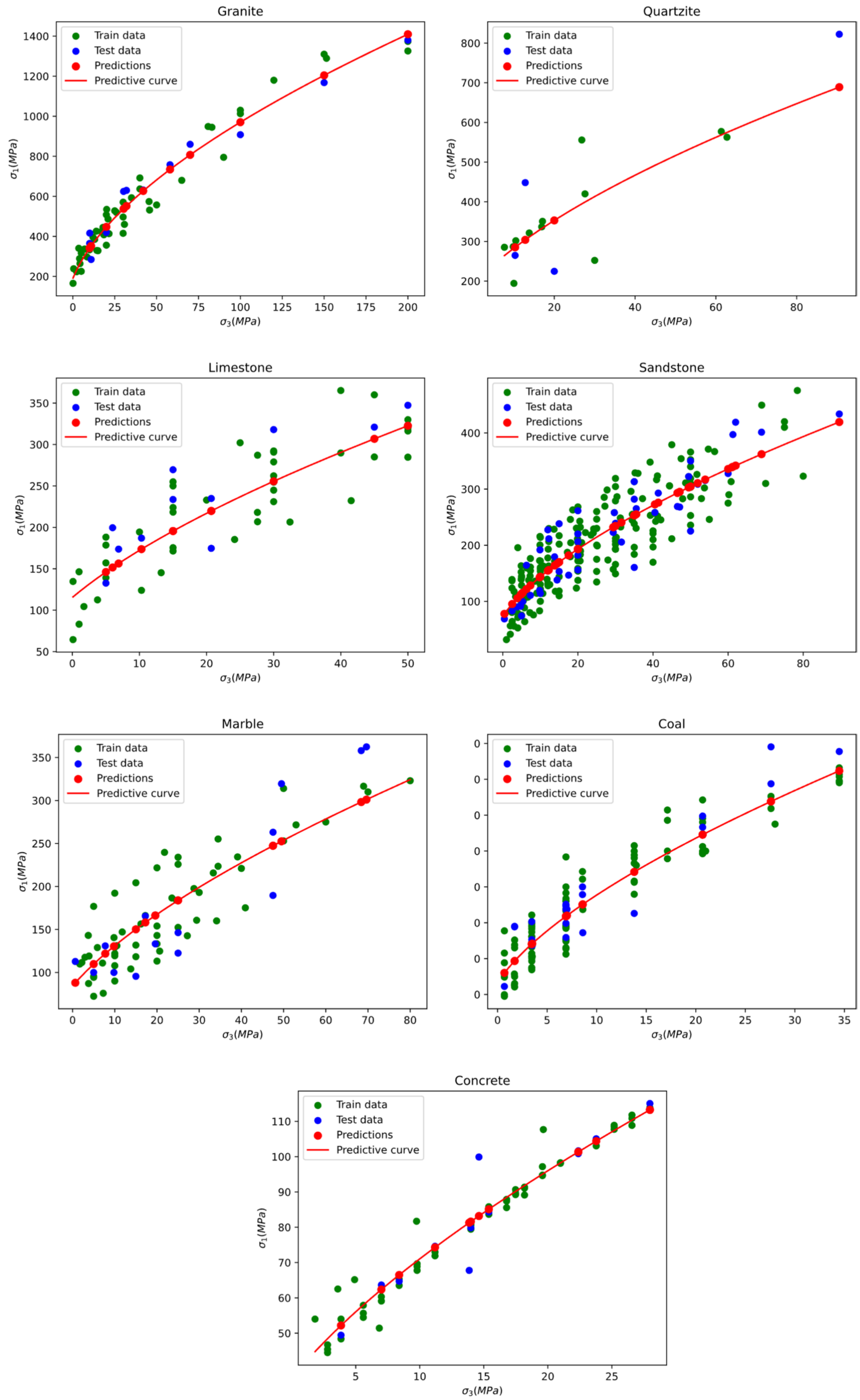


Fig. 5 Shows the relationship between uniaxial compressive strength (UCS) and tensile strength (TS) obtained from experimental data. (**a**) General linear correlation coefficient between uniaxial compressive strength and tensile strength. (**b**) Linear correlation coefficient between uniaxial compressive strength and tensile strength for each type of rock


Granite
Quartzite
Limestone
Sandstone
Marble
Coal
Concrete
Train data
Test data
Predictions
Predictive curve
$\sigma_1(MPa)$
$\sigma_3(MPa)$

◂**Fig. 6** Graphs of the nonlinear regression analysis model for each rock type

To calibrate the model, a nonlinear least-squares fitting was performed. This procedure aimed to minimize the discrepancy between the predicted values of σ_1 (as given by Eq. 1) and the actual experimental observations. The optimization yielded fitted values of UCS and TS for each rock type in the training set.

The resulting model was then used to generate predictions for the validation subset, and the performance of the fit was evaluated using the standard deviations (SD) and standard error mean (sem). Visualizations of the nonlinear regression curves were produced, and the results were further used to make predictions for additional user-specified input values of σ_3.

This methodology facilitates the fit of UCS and TS values from test data. The entire modelling workflow is summarized in the flowchart shown in Fig. 4.

2.5 Taylor's Formulation

To assess the uncertainty associated with the parameter $\xi = TS/UCS$, Taylor's series expansion was applied following the propagation of uncertainty framework. This analysis was conducted after obtaining the standard deviation and standard error mean of UCS and TS through the nonlinear regression analysis.

The general form of the Taylor series–based variance propagation is expressed as:

$$V\left[g\left(x_i\right)\right] = \sigma_{x_1}^2\left(\frac{\partial g}{\partial x_1}\right)^2 + \sigma_{x_2}^2\left(\frac{\partial g}{\partial x_2}\right)^2 + 2\rho_{x_1x_2}\sigma_{x_1}\sigma_{x_2}\left(\frac{\partial g}{\partial x_1}\right)\left(\frac{\partial g}{\partial x_2}\right) \quad (6)$$

In this context:

- $g\left(x_i\right) = \xi = \frac{x_1}{x_2} = \frac{TS}{UCS}$
- σ_{x_1}: standard deviation or standard error mean of TS.
- σ_{x_2}: standard deviation or standard error mean of UCS.
- $\rho_{x_1x_2}$: Pearson correlation coefficient between UCS and TS.

This formulation enables the quantification of variability in the ratio ξ, incorporating both individual variances and the correlation between UCS and TS. The complete mathematical development of Eq. (6) is provided in Appendix B. An example application for determining the variation of ξ of Granite rock is expanded in appendix B.

In summary, the proposed methodology integrates a data selection with a nonlinear regression analysis grounded in the formulation of Úcar's failure criterion. Input data was carefully filtered to ensure representativeness and quality, and then used to input the failure model for each type of rock via nonlinear regression. This approach enables the fit of the UCS and TS parameters with improved reliability. Furthermore, the uncertainty associated with the predicted TS/UCS ratio (ξ) was assessed using Taylor's variance propagation. The complete workflow offers a practical and replicable way of fitting the intact rock strength in new studies.

3 Results

3.1 Experimental Data Uniaxial Compressive Strength and Uniaxial Tensile Strength

Table 1 initial guess shows the results of experimental data collected from numerous researchers on the average strength of the six rock types between the 10 and 90% percentiles studied. These experimental data (UCS and TS) correspond to the initial input data in the regression analysis.

On the other hand, the general linear correlation coefficient between uniaxial compressive strength and tensile strength is 0.8118 shown in Fig. 5a, and relationship for each type of rock in Fig. 5b and Table 1.

3.2 Fitted Data Uniaxial Compressive (UCS) and Tensile (TS) Strength

Figure 6 illustrates the results for each rock type, showing the training and test datasets, the fitted values, and the nonlinear regression curves model for each rock type derived from Úcar's failure formulation.

The descriptive statistics for each material, including UCS and TS means, standard deviations (SD),

Table 2 Descriptive statistical values obtained from nonlinear regression analysis for each rock type

Rock	UCS (MPa)	TS (MPa)	SD (UCS)	SD (UTS)	sem (UCS)	IC 95% (UCS)	sem (UTS)	IC 95% (UTS)
Granite	189.69	−4.7	23.21	1.23	3.39	6.60	0.179	0.350
Quartzite	193.47	−9.38	61.89	7.92	17.87	34.84	2.286	4.458
Limestone	115.4	−8.97	12.58	2.5	1.92	3.74	0.381	0.743
Sandstone	73.79	−3.95	8.92	1.95	0.64	1.25	0.140	0.272
Marble	84.4	−7.92	9.84	2.27	1.29	2.52	0.298	0.581
Coal	26.87	−1.74	2.75	0.38	0.29	0.56	0.040	0.078
Concrete	37.19	−4.46	1.7	0.48	0.24	0.46	0.067	0.130

Table 3 Variation of ξ obtained using Taylor's formulation.

Rock	UCS (mean)	UTS (mean)	UCS (IC 95%)	UTS (IC 95%)	C.C. Lineal	Δξ
Granite	189.69	−4.70	6.60	−0.35	0.982	±0.001
Quartzite	193.47	−9.38	34.84	−4.46	0.806	±0.017
Limestone	115.40	−8.97	3.74	−0.74	0.970	±0.004
Sandstone	73.79	−3.95	1.25	−0.27	0.930	±0.003
Marble	84.40	−7.92	2.52	−0.58	0.979	±0.004
Coal	26.87	−1.74	0.56	−0.08	0.968	±0.002
Concrete	37.19	−4.46	0.46	−0.13	0.979	±0.002

Values the UCS and TS in MPa

Table 4 Summary of statistical values, mean and variations of the rock strength fitted

Rock	UCS (MPa)	TS (MPa)	ξ
Granite	189.69 ± 6.60	−4.7 ± 0.350	−0.025 ± 0.001
Quartzite	193.47 ± 34.84	−9.38 ± 4.458	−0.048 ± 0.017
Limestone	115.40 ± 3.74	−8.97 ± 0.743	−0.078 ± 0.004
Sandstone	73.79 ± 1.25	−3.95 ± 0.272	−0.054 ± 0.003
Marble	84.40 ± 2.52	−7.92 ± 0.581	−0.094 ± 0.004
Coal	26.87 ± 0.56	−1.74 ± 0.078	−0.065 ± 0.002
Concrete	37.19 ± 0.46	−4.46 ± 0.130	−0.120 ± 0.002

Variation of the parameters ξ, derived from Taylor's formulation

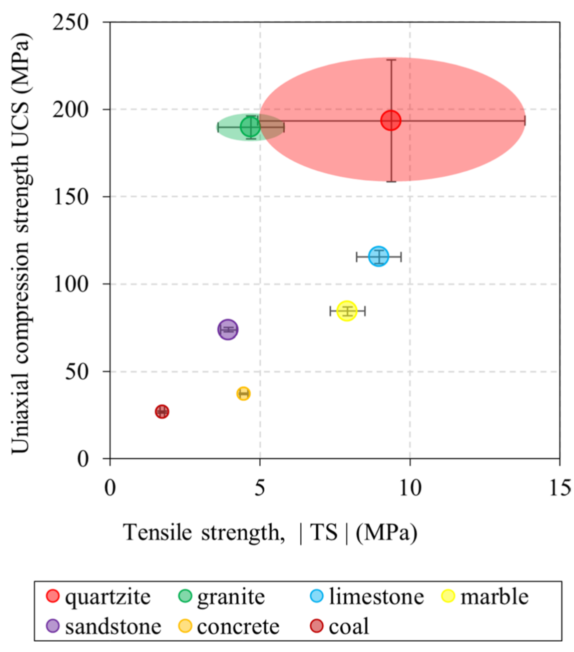


Fig. 7 Graphical representation of UCS and TS values and their 95% confidence intervals

standard error mean (sem), and 95% confidence intervals, are provided in Table 2. These data allow for the computation of the parameter $\xi = TS/UCS$. The variability of ξ was assessed using Taylor's series expansion (Eq. (6) and Table 3). The correlation coefficient using between UCS and TS is shown in the Fig. 5.

A consolidated summary of the fitted UCS and TS values, including their 95% confidence intervals and Variation of the parameters ξ, derived from Taylor's formulation is provided in Table 4. These results are also depicted in Fig. 7, which shows the UCS–TS relationship for each rock type, incorporating their respective uncertainty ranges.

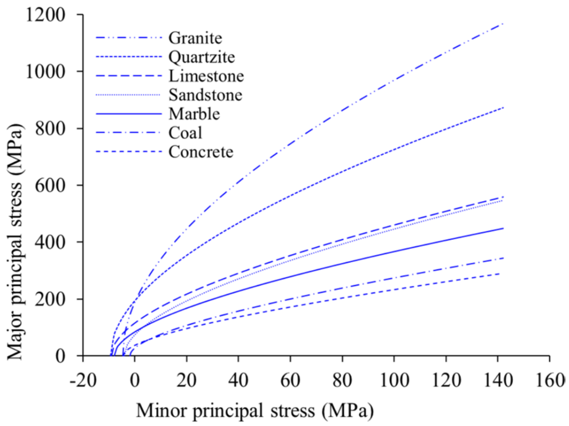


Fig. 8 Failure envelopes for each rock type based on Úcar's criterion using fitted mean values

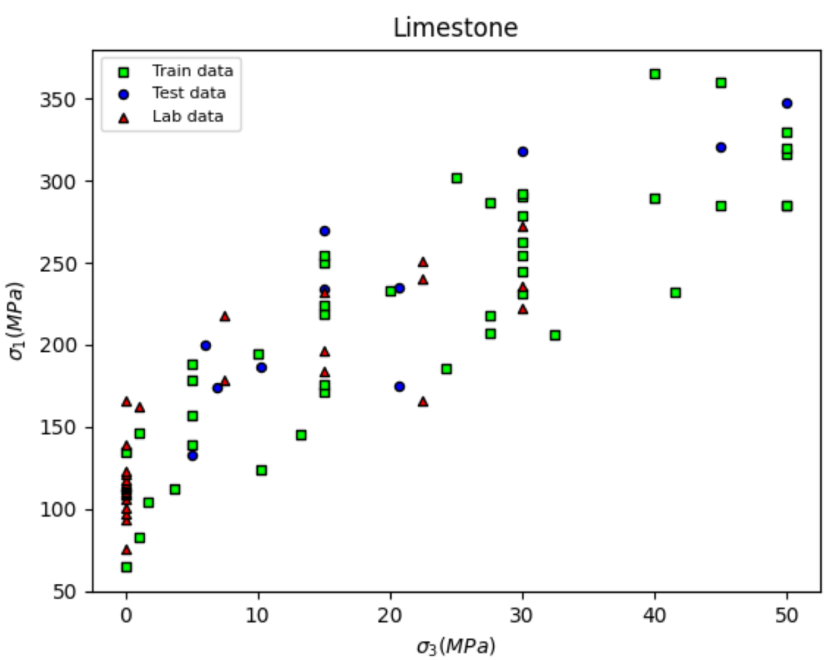
3.3 Úcar's Failure Criterion Model

The predictive failure envelopes obtained for each rock type using Úcar's criterion are presented in Fig. 8. These envelopes were constructed using the fitted mean values UCS and TS derived from the nonlinear regression analysis (Table 4).

According to the results, granite exhibits the highest strength among the materials studied, characterized by a high UCS and a relatively low TS, resulting in a small value of the parameter ξ. In contrast, concrete displays the lowest mechanical resistance, as reflected in its lower UCS and higher value ξ.

4 Discussion

This study analyzed six lithologies (granite, quartzite, limestone, sandstone, marble, and coal) alongside concrete. The fit value of UCS and TS for each material was performed using laboratory data. The application of nonlinear regression analysis enabled fitting of the UCS and TS parameters (see Table 4). Among the materials studied, quartzite showed the highest variability, mainly due to the limited quantity of available data. This suggests that expanding the dataset would likely improve the stability and precision of parameter estimation.



Lab test data

σ_3 MPa	σ_1 MPa
0	75.2
0	93.1
0	97.1
0	100.3
0	106.2
0	109.1
0	109.6
0	110.6
0	111.9
0	113.5
0	114
0	114
0	114.4
0	115.3
0	116.3
0	117.8
0	121.1
0	123.2
0	139.5
0	165.6
1	162.3
7.5	178.5
7.5	218
15	183.6
15	196.8
15	232.1
22.5	165.8
22.5	240.7
22.5	250.9
30	222.3
30	235.7
30	272.4

Fig. 9 Dataset used for comparison of the Úcar criterion models of the limestone based on nonlinear regression analysis (Table 4 and supplementary material), and Hoek–Brown failure for Coburg Limestone (Lab. test data by Hoek and Brown 2019)

Table 5 Comparison of UCS, m_i and TS values for Coburg limestone obtained by different analyses

Analysis	UCS (MPa)	mi	TS (MPa)
Úcar Model	115.4*	–	– 8.96*
Bayesian	114.5	11.5	–
Modified Cuckoo Regression	116.5	10.6	–

* Data taken from Table 4

The predictive values of ξ, defined as the ratio between mean value the TS and UCS, revealed systematic differences among the materials. The lowest value ξ was observed in granite, while the highest occurred in concrete. This corresponds to a TS-to-UCS ratio of 3.4% for granite and 10.5% for concrete. Notably, lower values of ξ are associated with higher values of k_1 and k_2 in Úcar's criterion, whereas higher ξ values lead to lower k_1 and k_2. Across the full range of materials analyzed, it was observed that $k_1 < 1$ and $k_2 < 10$ in all cases.

The resulting failure envelopes derived from Úcar's model displayed clear distinctions among rock types. Granites exhibited the steepest curve and highest overall strength, while coal and concrete showed the gentlest slopes and the lowest strength. These variations reflect the intrinsic mechanical differences across lithologies and validate the sensitivity of Úcar's formulation to material-specific parameters.

The validation of the methodology of the adjusted values of the resistance of the rocks (UCS and TS), of the failure criteria of Úcar, as well as the parameter ξ was compared with a case study based on Coburg limestone data from Hoek and Brown (2019).

Figure 9 illustrates the training and test data used for limestone in the Úcar failure criterion model. Table 5 show the predicted mean values—UCS = 115.4 MPa and TS = – 8.96 MPa—agree with the measurements made by Hoek and Brown using laboratory test data from Coburg limestone.

For comparison, Hoek and Brown (2019) reported values UCS and m_i based on Bayesian analysis (Student's t distribution) and a modified Cuckoo optimization approach implemented in RocData™. Table 5 summarizes the resulting UCS and m_i values from each method. The results show a close agreement between the nonlinear regression- based prediction and those obtained via more traditional statistical and heuristic approaches.

Figure 10 presents the Úcar failure envelope model generated from the limestone rock model proposed in this research using nonlinear regression the training

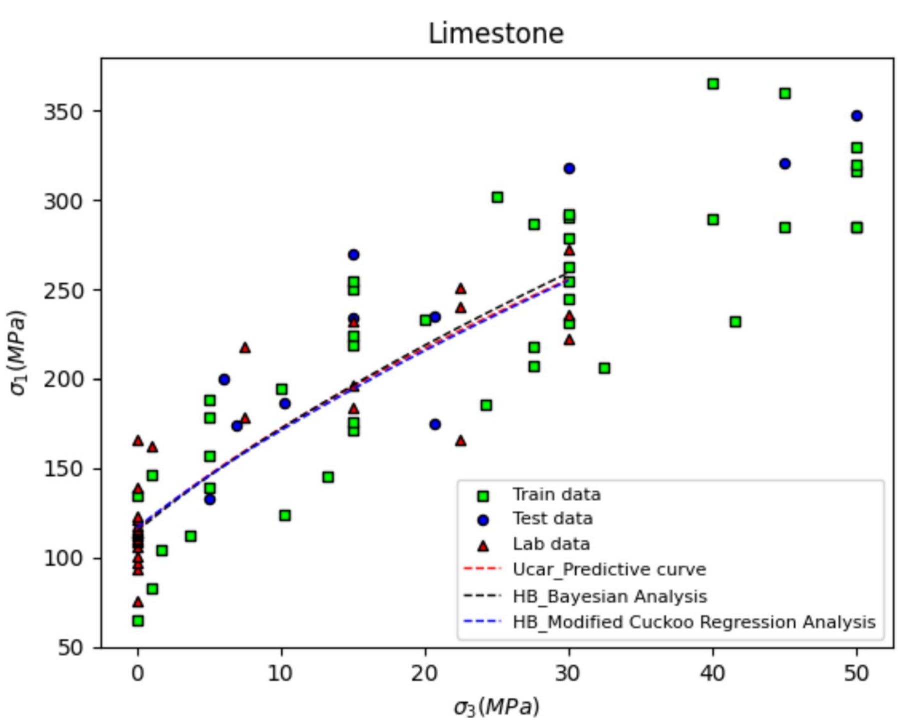


Fig. 10 Comparison of the Úcar criterion models of the limestone (parameter obtained in this investigation) and of the Hoek and Brown criterion obtained for the Lab. test data on Coburg limestone (Hoek and Brown 2019) applying Bayesian analysis (Student'sdistribution) and RocData modified Cuckoo regression analysis

and test data (see Limestone Rock Fig. 8). The similarity between the curves proposed by the two methodologies (Bayesian analysis and Cuckoo regression analysis) proposed by Hoek and Brown (2019) using laboratory test data (Figs. 9 and 10) confirms that the nonlinear regression proposed for limestone in this research provides a valid and competitive alternative for estimating strength parameters from limited or no input data.

The advantage of using the strength parameters of any of the six rock types analyzed (Table 5 and Fig. 8) is that no testing is required; simply knowing the rock type in a project allows for predicting the rock's strength behaviour and using the Úcar criterion as the initial failure criterion.

5 Conclusions

A nonlinear regression analysis was developed to obtain adjusted values of uniaxial compressive strength (UCS) and tensile strength (TS) by applying the Úcar failure criterion, resulting in calibrated Úcar-based strength models for six rock types and one concrete material.

The methodology was based on an extensive compilation and rigorous filtering of experimental data. Using the Python, the statistical parameters were provided and outlier data were visualized, allowing the use of the standard deviation and standard error mean as the range of variability of the strength parameters (uniaxial compressive strength and tensile strength).

A further contribution of this research is the estimation of parameter variation. The uncertainty of the parameters (UCS, TS, and ξ) required the variability of the mean values of UCS and TS, which were quantified by nonlinear regression of the selected triaxial data using the standard error of the mean with a 95% confidence level, while the uncertainty or variation of the relationship $\xi = TS/UCS$ was evaluated using Taylor's formulation.

The main advantage of the proposed methodology lies in its ability to predict key parameters and the uncertainty of UCS, TS, and ξ, which can be easily used in rock mechanics through the Úcar failure criterion model.

The predictive models developed in this study offer a practical and scalable tool, requiring only the identification of the rock type. This makes the approach particularly valuable in early project stages or in situations where laboratory testing is limited or unavailable due to constraints in time, budget, or sample quality.

Finally, it is important to highlight the advantage of the Úcar failure criterion compared with other criteria commonly used in rock mechanics. A distinctive feature of the Úcar failure criterion is that the quadratic relationship between the principal stresses σ_1 and σ_3 defines a conic section —specifically, a parabola (Úcar 2021)—which accurately represents the general behavior of the experimental strength data obtained for the studied materials.

Moreover, the coefficients k_1 and k_2 governing this relationship are derived analytically as functions of the parameter $\xi = TS/UCS$. Taken together of the geometric properties of the parabola and the concept of latus rectum, these elements confirm the strong analytical foundation and practical relevance of the Úcar failure criterion.

Acknowledgements This work is a contribution of the Geotransfer Research Group (E32 23R, funded by Gobierno de Aragón, Spain), Grupo de Investigación en Geología Aplicada (GIGA, University of Los Andes, Venezuela) and Research Centre PEGASO (Universitat Politècnica de València, Spain).

Funding Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature.

Data availability All data supporting the findings of this study are available within the paper and its Supplementary Information.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Betournay, M, Gorski B, Labrie D, Jackson R, Gyenge M (1991) New Considerations in the determination of Hoek and Brown material constants. Paper presented at the 7th ISRM Congress, Aachen, Germany, pp ISRM–7CONGRESS

Bobich J (2005) Experimental analysis of the extension to shear fracture transition in Berea sandstone. PhD thesis, Texas A&M University

Borecki M, Kwasniewski M, Pacha J, Oleksy S, Berszakiewicz Z, Guzik J (1982) Triaxial compressive strength of two mineralogic/diagenetic varieties of coal measure. fine-medium grained Pniowek and Anna sandstones tested under confining pressure up to 60 MPa. Proc Instiytutu PBKiOP politechnikislaskiej 119(2)

Bozorgzadeh N, Escobar M, Harrison J (2018) Comprehensive statistical analysis of intact rock strength for reliability-based design. Int J Rock Mech Min Sci 106:374–387. https://doi.org/10.1016/j.ijrmms.2018.03.005

Brace W (1964) Brittle fracture of rocks. State of stress in the earth's crust, pp 111–174

Byerlee J (1968) Brittle-ductile transition in rocks. J Geophys Res 73(14):4741–4750. https://doi.org/10.1029/JB073i014p04741

Chakraborty S, Bisai R, Palaniappan S, Pal S (2019) Failure modes of rocks under uniaxial compression tests: an experimental approach. J Adv Geotech Eng 2(3):1–8. https://doi.org/10.5281/zenodo.3461773

Chan S (1977) Engineering properties of rocks and rock masses in the deep mines of the Coeur D'Alene Mining District

Contreras L, Brown E, Ruest M (2018) Bayesian data analysis to quantify the uncertainty of intact rock strength. J Rock Mech Geotech Eng 10(1):11–31. https://doi.org/10.1016/j.jrmge.2017.07.008

Cordón V, Arlegui L, Simón J, Toro R, Úcar R, Belandria N, Torrijo F (2024) Úcar's nonlinear rock failure criterion as a predictive tool and as a curve fitting equation. In: New challenges in rock mechanics and rock engineering. CRC Press, p 607–612. https://doi.org/10.1201/9781003429234

Coulomb C (1776) Essai sur une application des règles de maximis & minimis a quelques problèmes de statique, relatifs a l'architecture. Mem Roy des Sciences, Paris 3.

Dayre M, Giraud A (1986) Mechanical properties of granodiorite from laboratory tests. Eng Geol 23(2):109–124. https://doi.org/10.1016/0013-7952(86)90033-5

Dlugosz M, Gustkiewicz J, Wysocki A (1981) Apparatus for investigation of rocks in a triaxial state of stress. II: Some investigation results concerning certain rocks. Arch Gorn 26:29-41 26:29-41

Du W, Lehane B, Doherty J (2025) Wang Z (2025) Numerical Analysis of Triaxial Tests for Interpreting Sample Stiffness with Existing Measurement Systems. Geotech Geol Eng 43:280. https://doi.org/10.1007/s10706-025-03261-5

Dunikowski A, Korman S, K¨ohsling J (1969) Laboratory tests on indices of physicomechanical properties of rocks in three-axial state of stress. Przegl Gorn 25(11):523–528

Edelbro C (2003) Rock mass strength: a review. Lulea Tekniska Universitet, Tech rep, p 16

Evans B, Fredrich J, Wong T (1990) The brittle-ductile transition in rocks: recent experimental and theoretical progress. Brittle-Ductile Transit Rocks 56:1–20

Everling G (1960) Rock mechanical investigations and basis for determination of rock pressure according to deformation of drill holes. Gluckauf 96:390–409

Fan H, Wang Z, Liu J, Wang Y (2025) Failure mechanism of sandstone under true triaxial cyclic loading: insights from energy evolution. Geotech Geol Eng 43(7):1–16. https://doi.org/10.1007/s10706-025-03301-0

Franklin J, Hoek E (1970) Developments in triaxial testing technique. Rock Mech 2:223–228

Ganesan G, Mishra A (2024) A modified anisotropic Hoek and Brown failure criterion for transversely isotropic rocks. Geotech Geol Eng 42:4869–4889. https://doi.org/10.1007/s10706-024-02818-0

Glushko V, Kirnichanskiy G (1974) Engineering geological prognosticating of stability of the openings in deep coal mines. Nedra, Moscow

Gnirk P (1963) Indentation experiments on dry rocks under pressure. J Pet Technol 15(09):1031–1039. https://doi.org/10.2118/498-PA

Gnirk P, Cheatham Jr (1965) An experimental study of single bit-tooth penetration into dry rock at confining pressures 0 to 5,000 psi. Soc Pet Eng J 5(02):117–130. https://doi.org/10.2118/1051-PA

Gowd T, Rummel F (1980) Effect of confining pressure on the fracture behaviour of a porous rock. Int J Rock Mech Min Sci Geomech Abstr 17(4):225–229. https://doi.org/10.1016/0148-9062(80)91089-X

Griffith A (1921) The phenomena of rupture and flow in solids. Philos Trans R Soc Lond A Math Phys Eng Sci 221:163–198

Griffith A (1924) The theory of rupture. In: International congress for applied mechanics. 55–63

Gui Z, Chen J, Xu Y, Wu W, Rong X (2025) Hu J (2025) Development of a variable-angle triaxial test device and its application in excavation-induced rock damage. Geotech Geol Eng 43:303. https://doi.org/10.1007/s10706-025-03268-y

Guo B, Wang L, Li Y, Chen Y (2020) Triaxial strength criteria in Mohr stress space for intact rocks. Adv Civ Eng 2020(1):8858363. https://doi.org/10.1155/2020/8858363

Hansagi I (1965) Numerical determination of mechanical properties of rock and of rock masses. Int J Rock Mech Min Sci Geomech Abstr 2(2):219–223. https://doi.org/10.1016/0148-9062(93)90012-3

Haque M, Ansary M (2025) Cyclic triaxial test-based user-specified constitutive model of sand: application to tunnel–sand–pile interaction. Geotech Geol Eng 43:515. https://doi.org/10.1007/s10706-025-03492-6

Hareland G, Polston C, White W (1993) Normalized rock failure envelope as a function of rock grain size. Int J Rock Mech Min Sci Geomech Abstr 30(7):715–717. https://doi.org/10.1016/0148-9062(93)90012-3

Harr M (1987) Reliability-Based Design in Civil Engineering. McGraw-Hill, New York

Heard H, Abey A, Bonner B, Schock R (1974) Mechanical behavior of dry westerly granite at high pressure. Tech rep, California Univ, Livermore (USA), Lawrence Livermore Lab

Hobbs D (1964) The strength and the stress-strain characteristics of coal in triaxial compression. J Geol 72(2):214–231. https://doi.org/10.1086/626977
Hobbs D (1970) The behavior oof broken rock under triaxial compression. Int J Rock Mech Min Sci Geomech Abstr 7(2):125–148. https://doi.org/10.1016/0148-9062(70)90008-2
Hoek E, Brown E (1980) Empirical strength criterion for rock masses. J Geotech Eng 106(9):1013–1035. https://doi.org/10.1061/AJGEB6.0001029
Hoek E, Brown E (2019) The Hoek-Brown failure criterion and GSI–2018 edition. J Rock Mech Geotech Eng 11(3):445–463. https://doi.org/10.1016/j.jrmge.2018.08.001
Horino F, Ellickson M (1970) A method for estimating strength of rock containing planes of weakness, vol 7449. US Department of Interior, Bureau of Mines
Hu K, Zhu Qz, Chen L, Shao J, Liu J (2018) A micromechanics-based elastoplastic damage model for rocks with a brittle–ductile transition in mechanical response. Rock Mech Rock Eng 51:1729–1737. https://doi.org/10.1007/s00603-018-1427-z
Mogi K (1966) Pressure dependence of rock strength and transition from brittle fracture to ductile flow. Bull Earthq Res Inst 44:215–232
Mogi K (1973) Rock fracture. Annu Rev Earth Planet Sci 1:1–63
Murrell S (1965) The effect of triaxial stress systems on the strength of rocks at atmospheric temperatures. Geophys J Int 10(3):231–281. https://doi.org/10.1111/j.1365-246X.1965.tb03155.x
Pérez-Rey I, Muñiz-Menéndez M, Frühwirt T, Konietzky H, Jacobsson L, Perras M, Alejano L (2024) Assessment of direct tensile strength tests in rock through a multi-laboratory benchmark experiment. Rock Mech Rock Eng 57(5):3617–3634. https://doi.org/10.1007/s00603-023-03751-z
Perras MA, Diederichs MS (2014) A review of the tensile strength of rock: concepts and testing. Geotech Geol Eng 32(2):525–546. https://doi.org/10.1007/s10706-014-9732-0
Ramsey JM, Chester FM (2004) Hybrid fracture and the transition from extension fracture to shear fracture. Nature 428(6978):63–66. https://doi.org/10.1038/nature02333
Scholz C (1968) Microfracturing and the inelastic deformation of rock in compression. J Geophys Res 73(4):1417–1432. https://doi.org/10.1029/JB073i004p01417
Schwartz A (1963) An investigation of the strength of rock. PhD thesis, Georgia Institute of Technology. Directed by George M. Sowers
Sheorey P (1997) Empirical rock failure criteria. (No Title)
Torres R (1992) Nuevos criterios sobre la resistencia del concreto. PhD thesis, Tesis de Maestría, Facultad de Ingeniería, Universidad de Los Andes, 160
Úcar R (2021) Determination of a new failure criterion for rock mass and concrete. Geotech Geol Eng 39(5):3795–3813. https://doi.org/10.1007/s10706-021-01728-9
Úcar R, Arlegui L, Belandria N, Torrijo F (2024) Estimating rock strength parameters across varied failure criteria: application of spreadsheet and R-based orthogonal regression to triaxial test data. J Rock Mech Geotech Eng 17(8):4685–4699. https://doi.org/10.1016/j.jrmge.2024.11.024
Walton S, Hassan O, Morgan K, Brown MR (2011) Modified cuckoo search: a new gradient free optimisation algorithm. Chaos, Solitons Fractals 44(9):710–718. https://doi.org/10.1016/j.chaos.2011.06.004