



Original Research

A Comparative Study of Number Sense in Japanese and Spanish First-Grade Mathematics Textbooks

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Abstract: Japan and Spain represent different cultural contexts and educational practices that could influence the way students approach mathematical concepts. To understand the differences in approaches to number sense in mathematics learning opportunities within the textbook between Japan and Spain, we compare the arithmetic section of the first year of primary school in the official Japanese textbook and in the textbook published by the top-selling publisher in Spain. Through qualitative analysis, the meaning of numbers, general aspects reviewed in the literature, and representation registers were analyzed. Among the results obtained, we found a key distinction between the two countries in the emphasis they place on the different mathematical operations. In addition, the introduction of number operations also differs. For example, the Spanish textbook usually presents systematic counting tasks together with number writing and then gradually integrates the basic operations, while the Japanese textbook emphasizes the understanding of mathematical concepts by breaking numbers into smaller parts and introducing the ideas step by step. Finally, the similarities are also remarkable. Both textbooks predominantly use symbolic representation, demonstrating a common reliance on numerical symbols in the early teaching of arithmetic.

Keywords: Textbooks, Number Sense, Japan, Spain, First Grade

Introduction

The way mathematical ideas are presented in textbooks plays a pivotal role in how students engage with and understand mathematical concepts. Moreover, textbooks largely determine a country's curriculum (Monterrubio and Ortega 2009). Their use in the classroom determines teaching practices, and they can be more influential than ministerial orders (Conejo and Ortega 2014; Stein and Kim 2009; Tarr et al. 2006). In fact, according to a study conducted in Spain (Braga Blanco and Belver Domínguez 2016), 81.3% of teachers acknowledge that they tend to use textbooks "quite a lot or a lot" in their classes, and 71.9% of parents also consider them an essential resource. Furthermore, in Japanese schools, textbooks are positioned as the "primary instructional material," and classroom instruction is organized through them to operationalize the national curriculum guidelines; thus, many teachers rely heavily on textbooks when constructing their lessons (Textbook Research Center 2024). Although research has been conducted on textbooks from Japan, Iran, Singapore, and

Australia, there is limited comparative analysis between countries with distinct cultural and pedagogical traditions. Spain and Japan represent very different cultural contexts and educational practices, and it is through analyzing the proposals in their textbooks that we can make sense of the differences in how their students approach mathematical concepts.

Comparative studies of mathematics textbooks from various countries have revealed significant differences in how representations are used. Vincent and Stacey (2008), for example, analyzed Australian eighth-grade mathematics textbooks and found that these textbooks often lacked depth in representing complex mathematical ideas. In contrast, Fan and Zhu (2000) examined Chinese mathematics textbooks and noted that they frequently utilized multiple representations to explain mathematical concepts, resulting in a more comprehensive understanding among students. In another study, Pepin and Haggarty (2001) compared mathematics textbooks from England, France, and Germany and found that English textbooks mainly used symbolic representations, while French textbooks presented a more balanced combination of visual, contextual, and symbolic representations, which promoted reasoning and problem-solving skills. Similarly, O’Keeffe and O’Donoghue (2015) analyzed mathematics textbooks from Ireland and other countries, focusing on how content is represented and developed. Their findings revealed that textbooks that incorporate both conceptual and procedural understanding tend to be more effective in improving students’ mathematical competence. This suggests that textbooks should balance the use of different representations to support not only the development of procedural skills but also a deeper conceptual understanding. A recent study by Incikabi et al. (2023) analyzed the mathematical literacy demands in fifth-grade mathematics textbooks from Turkey, Singapore, and Australia using the PISA (Programme for International Student Assessment) framework. The study highlighted that while all textbooks offered opportunities for students to formulate and employ mathematical concepts, there was a lack of tasks requiring higher-order skills for interpreting and evaluating solutions.

Regarding number sense in particular, several studies have examined textbooks for the early years of primary education in different countries (Arnal-Palacián et al. 2023; Cheng and Wang 2012; Sood and Jitendra 2007). Among them, we find the work of Sayers et al. (2021), who, using the eight categories of foundational number sense (FoNS) (Andrews and Sayers 2015), conducted a frequency analysis of three Swedish textbooks, analyzing how each of them could offer students opportunities to consolidate their learning. Also using a quantitative approach, Sood and Jitendra (2007) analyzed three traditional primary school textbooks in the United States to examine the number sense content within them. They concluded that the teaching of number sense concepts should enable students to make sense of numbers by creating a learning environment that considers modeling, questioning, and scaffolded instruction during this initial learning. Similarly, an international comparison of Chinese and US first-grade textbooks (Cheng and Wang 2012) found that Chinese textbooks

focused more on number meaning and representation, place value, base ten concepts, and connecting number sense to number operations, while US textbooks focused more on counting numbers, patterns, and connecting number sense to data analysis.

In summary, the existing body of research highlights the importance of diverse representations in mathematics textbooks to enhance students' conceptual understanding and problem-solving skills. The findings also emphasize the need for a developmentally appropriate sequence of content to align with students' cognitive stages.

The aim of this study is to identify differences in approaches to number sense in mathematics learning opportunities within the textbook between Japan and Spain. Given that a high percentage of teachers make extensive use of textbooks in their classes (Braga Blanco and Belver Domínguez 2016), activities related to number sense that appear in their textbooks will be analyzed in order to explore the reasons behind these differences. This comparison will offer insights into the evaluation of the written curriculum through its textbooks of each country and provide ideas on how they might improve their mathematical learning experiences in relation to number sense based on the activities in their textbooks.

The reason for considering both countries is that Japan consistently ranks at the top of international studies related to mathematics learning at primary and secondary levels, such as PISA (Organisation for Economic Co-Operation and Development [OECD] 2023), while Spain ranks below the OECD average. This study, therefore, compares two countries whose mathematics results in international tests are far apart.

Literature Review

Curriculum: Primary Education in Japan and Spain

We define curricula as learning guides designed to address the educational needs of learners, facilitate learning, and establish relationships between teachers and learners (Campbell-Phillips 2020). This serves as our starting point to identify similarities and differences that will guide primary school teachers in both countries. Beyond the curriculum, the main purpose of education is to develop citizens who are able to function appropriately in the society in which they live. Education is widely recognized as the foundation of a successful career, the development of critical thinking and reasoning that provides us with the tools for informed decision-making (Campbell-Phillips 2020). Thus, contemporary societies take into account that individuals need a wide range of competences to face the complex challenges of today's world (OECD 2005). The OECD, which includes the two countries participating in this study, Spain and Japan, has long emphasized training the individual in key competences.

Japan is known for its emphasis on problem-solving and critical thinking through diverse representations. Globally, a similar emphasis on activities involving grasping and processing social phenomena mathematically and engaging in proactive problem-solving is observed in Japan (Ministry of Education, Culture, Sports, Science and Technology [MEXT] 2018). The

Course of Study, which serves as the framework for mathematics education in Japan, states the following as one of the abilities to be developed: the capacity to perceive events mathematically, identify mathematical problems, and independently and collaboratively solve problems (MEXT 2018). In Spain, it is stated as a fundamental aspect of mathematics education that “mathematical competence enables the development and application of a mathematical perspective and reasoning to solve various problems in different contexts” (Ministerio de Educación y Formación Profesional [MEFP] 2022, 24406). From these points, it can be inferred that mathematics education in Spain emphasizes the ability to mathematically process and solve various problems in everyday life and society.

The curriculum for primary education in Japan is revised approximately every ten years. In the most recent revision in 2017, qualities and abilities aimed at fostering development through primary, middle, and high school education were clarified along three pillars: “knowledge and skills,” “ability to think, make judgments, and express themselves,” and “ability to learn and humanity.” Furthermore, it was stipulated that mathematical activities mindful of real-world connections should be implemented throughout each school stage. The “learning content” for each grade is also presented in a way that corresponds to these three pillars. Furthermore, the preamble to the objectives states that learning should be conducted by employing a “mathematical perspective and thinking.” Moreover, for each grade, specific mathematical activities are outlined, including solving arithmetic problems identified from “everyday events” and “arithmetic learning situations,” as well as engaging in activities to express and communicate the process and results of problem-solving.

The Spanish Primary Education curriculum must facilitate the educational development of students, ensuring their integral education through the development of competences (MEFP 2022). The curriculum, created to guide teaching and learning in the schools that teach at this stage, establishes different areas of knowledge and for each of them develops specific competences, assessment criteria associated with the specific competences and basic knowledge that, through the learning situations designed by the teaching staff, will enable the educational objectives and the development of the key competences of the stage to be achieved in an integrated manner. Mathematics, among other subjects, is taught in all years. Through these areas, the so-called key competences will be developed, which are considered to be transversal to each of the learning areas and students must acquire them throughout the stage. In direct relation to these key competences, a series of operational descriptors are defined which, together with the objectives of the stage, will help to define the specific competences of each area, field or subject (MEFP 2022).

The specific competences in the area of mathematics are organized into five fundamental axes: problem solving, reasoning and proof, connections, communication and representation, and socio-affective skills (MEFP 2022). In order to assess the acquisition of these specific competences, the basic knowledge corresponding to each cycle is defined, so

that teachers are clear about the organization of the contents in each cycle and within the Primary Education stage. If we look again at the area of mathematics, we see that the basic knowledge is structured around the concept of mathematical sense: number sense, sense of measurement, spatial sense, algebraic sense, stochastic sense, and socio-affective sense (MEFP 2022). The curriculum itself defines the term number sense as the development of skills and ways of thinking based on the understanding, representation and flexible use of numbers and operations to, for example, guide decision-making (MEFP 2022).

Number Sense in Primary Education

Despite the importance of developing number sense in primary education students (Anghileri 2006; Fernández-Ahumada et al. 2023), this is a construct that is currently complex to define (Aragón-Mendizábal et al. 2017) and for which there is still no consensus (Gersten et al. 2005), despite it being considered an important factor in the learning and application of mathematics by the UK Committee of Inquiry into the Teaching of Mathematics in Schools almost four decades ago (Cockcroft 1982).

Given its holistic nature, it is important to deconstruct it in order to address it in a curricular way (Dunphy 2007). Some authors, such as Sowder (1992), have already pointed out that number sense is the following: (1) decomposing numbers naturally, (2) using relationships between operations flexibly and creatively in problem solving, (3) understanding the base ten positional number system, (4) estimating, and (5) giving meaning to number and recognizing its magnitude. In the early ages, number sense involves the ability to discriminate small quantities, subitization, cardinality, comparison of numerosity in different sets, i.e., approaching numbers from three elements: counting, knowledge of numbers, and numerical operations (Jordan et al. 2008). It is precisely from counting that the possibility of carrying out the first additions and subtractions arises, using counting techniques to solve them. These initial techniques used repeatedly provide an evolution toward other more efficient processes (Cid et al. 2003). Researchers further emphasize that effective number sense also involves composing numbers by reassembling parts into strategically chosen wholes. For instance, when solving $8 + 6$, a child may think of $(8 + 2) + 4$ to make ten first and then reach fourteen. This example shows that meaningful part-whole reasoning relies on both decomposition and composition (Carpenter et al. 1999).

The types of numbers used in school mathematics are natural, integers, rational and real. The first of these, the natural numbers, are those that should be acquired in Primary Education, while the rest will be understood during Secondary Education (Rico et al. 2008). In relation to their meaning, some authors (Cid et al. 2003; Alcalde et al. 2014) point out up to six: numerical sequence, cardinal, ordinal, symbolic, operational and measurement. In order to understand the base ten positional number system, two stages must be addressed: (1) reading and writing digits and (2) reading and writing numbers with two or more digits. These two stages present

very different difficulties for pupils (Cid et al. 2003) and, in both cases, must be addressed after oral arithmetic. Among the difficulties that may be encountered are: the place value of zero and the relationships between the different orders. These difficulties could be accentuated if the task children are exposed to involves a whole ten or not using decimal grouping at all (Salinas 2007). Focusing on the category of recognizing numbers, it is important to design appropriate activities that develop skills in recognizing the concept of number from a very early age and enhance future understanding of mathematical concepts (Aisyah et al. 2024). This category includes identifying, naming, and writing the symbol for any number. This category is found within number identification, which is one of the most common units of analysis for number sense in the primary years (Ghazali et al. 2021).

Some authors (Dehaene 2001; Núñez 2017, Nelwan et al. 2022) argue that, alongside number sense, there is approximate number sense. The latter allows the estimation of magnitudes without the use of language or symbols, thus building the basis for the later development of the number-symbol association. In fact, according to some authors (van Hoogmoed and Kroesbergen 2018), estimation is even more important than determining the actual number of elements within a visual stimulus. Approximate number sense is usually developed through point estimation and quantity comparison tasks and can form different strategies (Dietrich et al. 2015). It is also important to note that the development of number sense is not only related to the ideas and concepts around numbers, but also on how they are arrived at and what connections can be made (Anghileri 2006).

Number patterns have been highlighted in recent research as a number sense, which is a key predictor of later arithmetic and algebraic reasoning; early sensitivity to repeating or growing sequences supports children in noticing structure and developing more efficient calculation strategies (e.g., Mulligan and Mitchelmore 2009; Mulligan et al. 2020). Incorporating this strand therefore completes the framework used in the present study and aligns it with contemporary evidence.

General Development Aspects and Content in Textbooks

The developmental needs of students also play a significant role in how mathematics content should be presented in textbooks. Jordan et al. (2008) explored how children develop number sense, a fundamental skill in early arithmetic that includes the ability to understand number relationships and manipulate them flexibly. They argue that textbooks should be structured in a developmentally appropriate way, allowing children to build on prior knowledge and gradually develop both procedural fluency and conceptual understanding. This view is supported by Mersin and Karabörk (2021), who discuss how technological tools, such as calculators and dynamic software, are distributed across grade levels to match students' developmental stages. They highlight the importance of aligning educational content with students' cognitive development to maintain engagement and promote effective learning.

Auliya and Widjajanti (2023) compared the structural organization of mathematics textbooks from Singapore and Japan, focusing on how content is sequenced to aid student learning. They found that Singaporean textbooks typically begin chapters with summaries of prerequisite knowledge, followed by new concepts, examples, and exercises, while Japanese textbooks often begin with illustrations and essential questions, followed by summaries and enrichment activities. This approach aligns with Jordan et al. (2008) argument that a well-organized curriculum that builds on prior knowledge is crucial for supporting cognitive development. After analyzing the differences in content structure, visual aids, and teaching strategies, Chegini et al. (2023) highlight the need for more integrated approaches to developing critical thinking skills.

Representation in Mathematics Textbooks

The role of representations in fostering students' mathematical understanding has been a key focus in educational research. Hiebert and Carpenter (1992) suggest that incorporating various representations in textbooks leads to more effective learning outcomes. They argue that representations should align with the learning objectives and mathematical content to help students grasp complex concepts and develop problem-solving skills. Lesh et al. (1987) further support this idea, explaining that multiple representations allow students to explore connections between different mathematical ideas, deepening their understanding of core concepts. They emphasize the importance of activities that encourage students to translate between different types of representations, such as visual and symbolic forms, to foster flexibility in thinking and application.

Especially in first-grade mathematics education, the concept of "bundling in groups of 10" is crucial (Herzog et al. 2019). At this stage, helping learners understand numbers based on units of ten forms is the foundation for comprehending number structures and serves as the basis for future skills in calculation and problem-solving. For example, when numbers are thought of in terms of bundles of ten, it becomes easier to break down and combine numbers, making concepts like addition and subtraction more intuitive.

To effectively teach the concept of base ten, the use of visual representations in textbooks is of utmost importance. Models such as blocks and beads, number lines, and diagrams help students grasp the structure of numbers visually and intuitively. Furthermore, practice exercises are essential. Beyond visual models, when students actively use the concept of base ten during calculations and problem-solving exercises, it solidifies their understanding. Practice exercises offer students opportunities to apply and reinforce their learning at their own pace. Research has shown that when the concept of base ten is presented through various methods in textbooks, students' understanding of numbers deepens (Alkhateeb 2019). Specifically, textbooks that incorporate diverse visual representations and hands-on activities tend to help students grasp number concepts more effectively. On the other hand, textbooks

that include fewer visual aids or representations may make it more difficult for students to fully comprehend these concepts. This highlights the importance of selecting effective representations in textbooks to enhance students' understanding of number structures and problem-solving abilities.

In general, a balanced inclusion of these different forms allows students to approach mathematics from multiple angles, facilitating a more comprehensive understanding. Similarly, a wide range of representations encourages critical thinking and deeper conceptual understanding (Mersin and Karabörk 2021). Conversely, reliance on symbolic representations may limit students' ability to develop critical thinking skills. Evidence from higher-level courses points in the same direction: a comparative study of fifth-grade courses conducted by Golafshani (2025) showed that Japanese textbooks offer richer images, multiple formats of representation, and more open-ended tasks than their Canadian counterparts, which encourages higher-level reasoning.

Methods

This research is conducted using a qualitative approach, with an exploratory and descriptive character based on previously established categories and variables, which have been presented and justified in the literature review section. The analysis focuses on the activities that are centered on the numerical sense—specifically, those activities within each analyzed textbook that are directly related to numbers and operations.

Sample

The object of the study in question is the comparison of two mathematics textbooks for the first year of primary school from two different countries, Japan and Spain. The choice for the comparison is not arbitrary, but the books most used at present in each of these countries are selected. In Japan, the textbook analyzed was from the Tokyo Shoseki textbook series (Fujii and Majima 2020), which has been identified as the most widely used textbook source in Japan's elementary mathematics curriculum (Watanabe 2024). In Spain, the textbook publisher that sells the most copies at primary school level is Anaya, according to data provided by the ISBN Agency (MEC 2020). This textbook is used in accordance with the new legislation (Carvajal and de la Rosa 2022).

It should be noted that the Japanese book is organized in two volumes, the first of which is shorter and focuses on the numbers from zero to ten, supported in a very clear way by manipulative and graphic materials. The Spanish book is arranged in a single volume. Both books increase in complexity as the contents and didactic units progress.

Instrument

For the analysis, we have developed an instrument (see Table 1) that unifies the criteria established by the research team and facilitates the systematization of the research results (see Table 1).

Table 1: Categories and Variables Used

<i>Categories</i>	<i>Variables</i>
Meaning	Cardinal Ordinal Measure Code or meaningless
General development aspects and content (Andrews and Sayers 2015; Carpenter et al. 1999; Sowder 1992)	Systematic counting Simple arithmetic Decomposition of numbers in a natural way Understanding of the base ten positional numbering system Estimation Writing numbers or number recognition Quantity discrimination Number patterns
Representation (Mainali 2021)	Symbolic Verbal Manipulative material Graphics

When comparing textbook problems, the three elements of “meaning,” “general development aspects and content,” and “representation” are essential for promoting a deep understanding of numbers and operations (Figure 1).

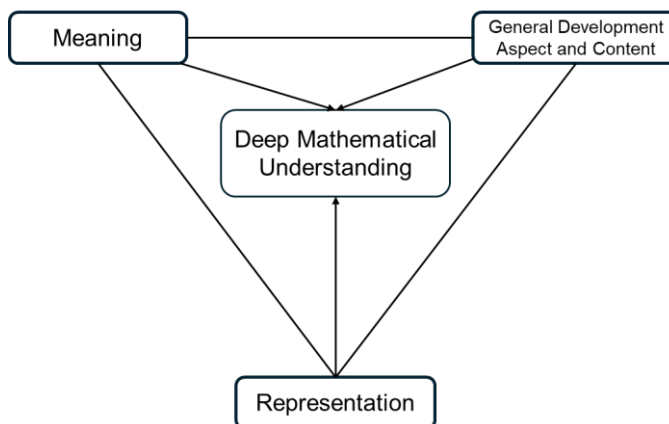


Figure 1: Comparative Elements to Deep Understanding of Numbers and Operations

First, “Meaning” is crucial for fostering the ability to understand the meaning of numbers and their connections with concrete situations. Second, “General development aspects and content” organizes the foundational and advanced elements of mathematics learning, providing a basis for problem-solving. This element aims to promote a flexible and creative approach to problem-solving by understanding the decomposition of numbers and the relationships between operations. Finally, “representation” is vital for learners to express

numbers and mathematical concepts in various ways, deepening their understanding. They argue that a balanced inclusion of these different forms allows students to approach mathematics from multiple angles, facilitating a more comprehensive understanding.

Each of the variables considered for each category is developed. For the category “Meaning,” we will consider the variables *cardinal*, in the sense of against a quantity of objects; *ordinal*, in the sense of ordinals; *measure*, when there is a relationship a magnitude; and *code* or *meaningless*, when there is no context to link the number to any of the three previous variables, for example, to present the spelling of the number or to perform certain operations.

The General development aspects and content category are an adaptation of the categories of FoNS (Andrews and Sayers 2015), along with some of the categories previously considered in the work of Carpenter et al. (1999) and Sowder (1992). These are detailed as follows:

- Systematic counting (Andrews and Sayers 2015). Counting systematically, both forwards and backwards, and from arbitrary starting points. In this variable, the considerations of Andrews and Sayers (2015) on learning the one-to-one correspondence between the name of a number and the quantity it represents have been incorporated, without attending to the variable Relating number to quantity, by taking the textbook and not the use that can be made of it.
- Simple arithmetic (Andrews and Sayers 2015). Perform simple addition and subtraction operations. Jordan et al. (2008) consider this same category, called operations with numbers.
- Decomposing numbers in a natural way (Carpenter et al. 1999). We consider a particular case of the previous category when a number is decomposed into two or more parts.
- Understanding of the positional number system in base ten (Sowder 1992). Understanding of place value. In addition, the presentation of tens and ones is also taken into account.
- Estimation (Andrews and Sayers 2015). Estimating either the size of a set or an object
- Number writing or number recognition (Andrews and Sayers 2015). Identify a particular number symbol from a collection of number symbols and name a number when shown its symbol.
- Quantity discrimination (Andrews and Sayers 2015). Compare magnitudes and use language such as “greater than” or “less than.”

Initially, the variable number patterns (Andrews and Sayers 2015) were considered but were not found in either textbook. For this reason, it will not appear in the results of this article.

Finally, the representation is taken from the organization provided by Mainali (2021), excluding the algebraic register, as it does not take place at the educational stage of the analysis carried out. These variables are symbolic, the syntax and semantics of mathematics; verbal, spoken words; manipulative material, elements such as arithmetic bars, base ten blocks, etc.,

when they are presented explicitly; graphical, including images, diagrams, coordinate planes, and other figurative representations. These representations can be identified uniquely and also in relation to each other. These categories seek to connect how the curriculum is written within textbooks and the opportunity to structure number sense (Figure 2).

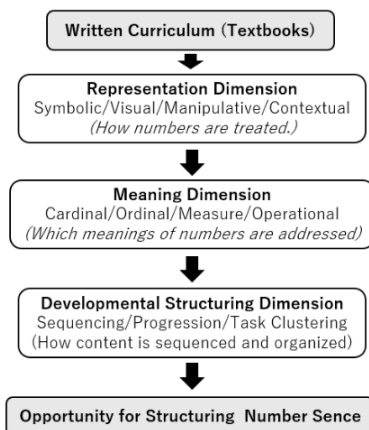


Figure 2: Diagram Showing the Logical Relationships Between the Analytical Categories and Linking the Study to the Written Curriculum

Analysis Procedure

The process followed for this analysis is presented in this section. First of all, the research team discussed the categories and did a first pilot test, based on randomly selected activities, to check their functionality and fit with the characteristics of the activities to be analyzed. Once the instrument was adjusted, especially in the category “General development aspects and content,” the analysis of both books by chapters was started. Each chapter was divided into fragments. These fragments are precisely the unit of analysis used in this study. In this work, a fragment is understood to be each of the tasks, whether examples or proposed exercises, set out in each of the chapters (see Figures 3 and 4).

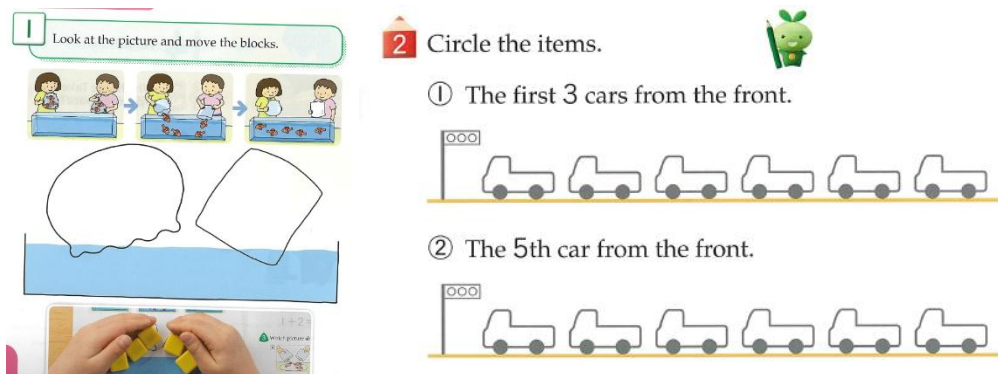


Figure 3: Two Examples of Fragments from the Analyzed Japanese Textbook
Source: Fujii and Majima 2020



Figure 4: Two Examples of Fragments from the Analyzed Spanish Textbook
 Source: Carvajal and de la Rosa 2022

The analysis is rigorous and systematic: each chapter is analyzed separately by three members of the research team, and the researcher who does not participate in the analysis reviews the contributions of the other three, pointing out the discrepancies found for later analysis by the whole research team. In this way, we ensure that each activity is analyzed by three researchers, and the criteria selected for the results of the research are unified. Once all discrepancies have been identified, there is a meeting between the four members of the research team to discuss the identification of the variables where discrepancies exist. This was repeated ten times for the analysis of the Japanese textbook, once for each of the sets of fragments into which the textbook was separated; and twelve times for the Spanish textbook. This resulted in complete agreement on the coding by the entire research team. Figure 5 and Table 2 present an example of the coding of an analyzed fragment.

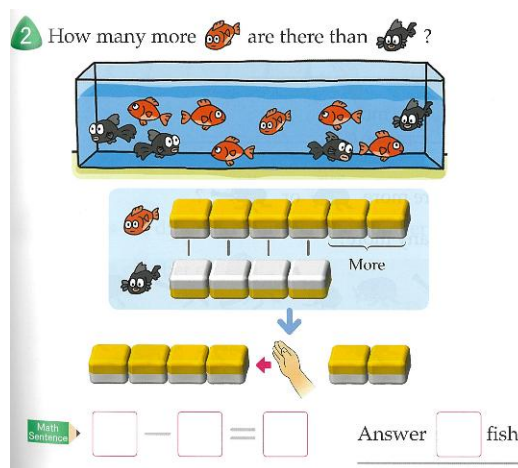


Figure 5: Example of a Fragment and Its Corresponding Encoding
 Source: Fujii and Majima 2020

Table 2: Example of Coding the Fragment Shown in Figure 5

<i>Categories</i>	<i>Variables</i>
Meaning	Cardinal
General development aspects and content (Andrews and Sayers 2015; Carpenter et al. 1999; Sowder 1992)	Counting Meaning to number and recognition of its magnitude Numbers operation Write numbers
Representation (Mainali 2021)	Symbolic Graphics Manipulative materials

This resulted in the categories of FoNS, the decomposing numbers in a natural way (Carpenter et al. 1999), and understanding of the positional number system in base ten (Sowder 1992). In addition, each fragment analyzed is accompanied by an identification sheet, which contains information specific to the book, such as school grade, didactic unit, page, and task number.

Results

The number of fragments analyzed is 527, of which 299 correspond to the Spanish book and 229 to the Japanese book. The results obtained for each of the categories: meaning, general development aspects, content, and representations are presented in the following sections.

Meaning of Number

The category of number meaning has been considered in all fragments with a single variable assigned. For this reason, the number of fragments analyzed coincides with the number of identifications of the meaning of the number. The analysis of the textbooks reveals that in the work with natural numbers in the first year, in both textbooks analyzed, cardinal meaning and no meaning predominate in a good number of fragments (see Table 3).

Table 3: Frequency of the Variables Associated with the Category “Meaning of the Number”

	<i>Cardinal</i>	<i>Ordinal</i>	<i>Measure</i>	<i>Meaningless</i>	<i>Total</i>
Spanish	146 (48.8%)	6 (2.0%)	0 (0.0%)	147 (49.2%)	299
Japanese	133 (58.1%)	9 (3.9%)	2 (0.9%)	85 (37.1%)	229

Note: Percentages are shown in parentheses.

The meaning of number as a cardinal is presented fairly uniformly throughout the course, with a slightly higher presence at the beginning of the course in both countries. While in the Japanese textbook analyzed this cardinality is considered without additional information in the hope of being able to establish comparisons between sets, in the case of the Spanish textbook analyzed, it is accompanied from the beginning by the spelling of the number (see Figure 6).



Figure 6: First Fragment Analyzed in Each of the Two Textbooks
 Source: (Left) Fujii and Majima 2020; (Right) Carvajal and de la Rosa 2022

A single meaning, or lack thereof, is identified in all fragments, with the exception of one fragment from the Japanese textbook analyzed, in which cardinal or ordinal meaning is addressed in the same task (Figure 7).

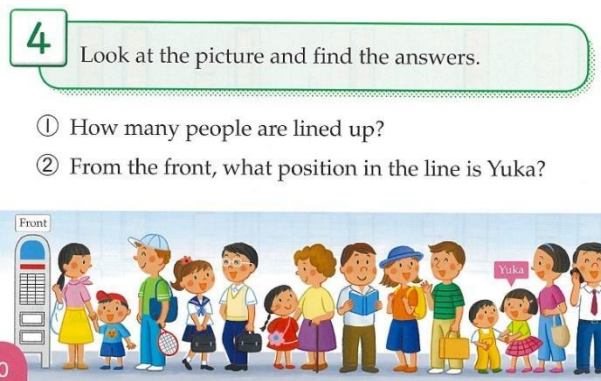


Figure 7: Ordinal and Cardinal Meaning in the Japanese Textbook
 Source: Fujii and Majima 2020

In fact, the meaning of ordinal in both books is very scarce. It appears only six times in the Spanish textbook analyzed and nine times in the Japanese one. The meaning of measure is almost non-existent. It is only found in the Japanese textbook analyzed on two occasions (Figure 8).

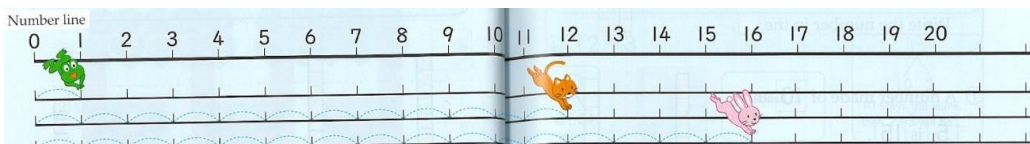


Figure 8: Meaning of Measurement in the Japanese Textbook
 Source: Fujii and Majima 2020

In addition, it is worth mentioning the high percentage of fragments in which the natural number has no meaning, as tasks are presented without any kind of context, despite the age of the target students. In the Spanish textbook analyzed, this type of fragment is presented 49% of the time, while in the Japanese textbook analyzed it corresponds to 37% of the fragments analyzed (see Figure 9).

5 Calculate the following.

① $9+5$ ② $8+4$ ③ $7+6$
 ④ $8+7$ ⑤ $7+5$ ⑥ $9+8$
 ⑦ $6+5$ ⑧ $8+8$ ⑨ $9+7$

3 ¿Qué números faltan? Escríbelos.

Figure 9: Presentation of the Meaningless Number

Source: (Top) Fujii and Majima 2020; (Bottom) Carvajal and de la Rosa 2022

General Development Aspects and Content

Another of the categories studied corresponds to the mathematical content present in the fragment. In this case, most of the fragments contain more than one mathematical content. For this reason, Table 4 presents the absolute frequency that appears in every general development aspect and content together with the percentage of the total number of fragments for each textbook, thus attempting to provide two pieces of information.

Although the fragments promoting counting appear throughout the textbook, in both countries, their frequency is higher at the beginning of the textbook. Later on, these counting activities give way to one-digit operations, followed by an understanding of the positional base ten number system and ending with two-digit operations. This means that the tasks of decomposition of a number and quantity discrimination have little presence, and those of estimation are almost non-existent.

Comparing both textbooks, it can be observed that the presence of writing numbers or number recognition is much higher in the Spanish textbook analyzed, more than 14%, while in the Japanese textbook analyzed there is a greater number of fragments which require the performance of numerical operations or the decomposition of a number, more than 7% and 8% higher respectively.

Table 4: Frequencies and Percentages of the Variables Associated with the Category “General Development Aspects and Content”

	<i>Syst. Counting</i>	<i>Simple Arithmetic</i>	<i>Decomp. of Numbers</i>	<i>Underst. of the Base Ten</i>	<i>Estim.</i>	<i>Writing Num.</i>	<i>Quantity Discrim.</i>
Spain	109 (36.5%)	137 (45.8%)	19 (6.4%)	97 (32.4%)	1 (0.3%)	224 (74.9%)	33 (11%)
Japan	83 (36.4%)	121 (53.1%)	33 (14.5%)	48 (21.1%)	0 (0%)	138 (60.5%)	23 (10.1%)
Total	192 (36.4%)	258 (49%)	52 (9.9%)	145 (27.5%)	1 (0.2%)	362 (68.7%)	56 (10.6%)

In the first part of the Spanish textbook analyzed, systematic counting tasks are closely related to writing tasks (Figure 10). Later on, these tasks start to be linked, in addition, to the performance of the first simple arithmetic (Figure 11). This is particularly noticeable in Spanish textbook analyzed.



Figure 10: Fragment of Systematic Counting and Number Writing in the Spanish Textbook Analyzed
Source: Carvajal and de la Rosa 2022

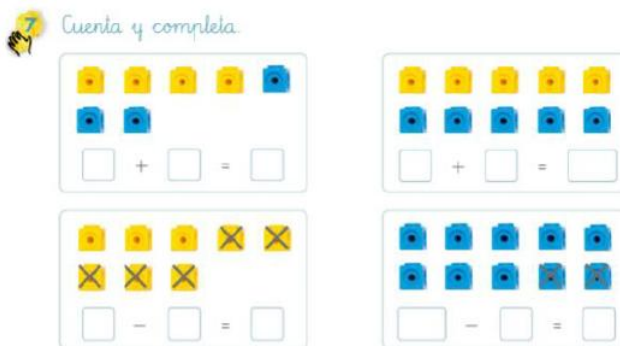


Figure 11: Fragment of Systematic Counting, Simple Arithmetic, and Number Writing in the Spanish Textbook Analyzed
Source: Carvajal and de la Rosa 2022

In the case of the Japanese textbook analyzed, simple arithmetic is not so often linked to systematic counting but is considered a pre-task to be performed. Most of these tasks are presented as the reproduction of flashcards that are currently being used by students (Figure 12).

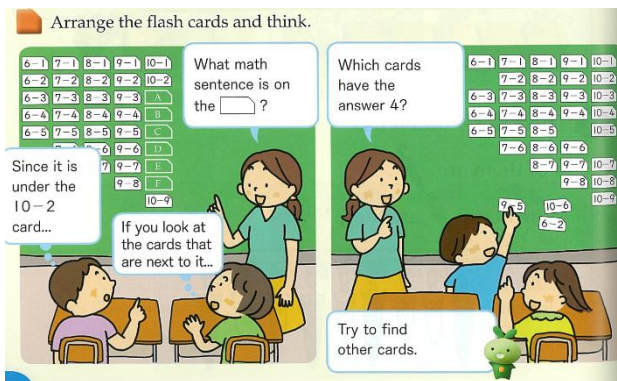
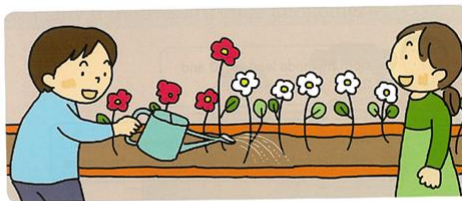


Figure 12: Fragment from Number Operations in the Japanese Textbook Analyzed
Source: Fujii and Majima 2020

In both textbooks, simple arithmetic with one-digit numbers is presented in addition and subtraction problems after sufficient familiarity with them (Figure 13).

- 11 There are 5 red flowers.
There are 8 white flowers.
How many flowers are there altogether?



2 En una tienda había 56 disfraces y llegaron 14 más. ¿Cuántos disfraces hay ahora?

Si llegaron más disfraces, ahora hay...

menos que al principio.

más que al principio.

56

→ Ahora hay disfraces.


Comprueba que ahora hay más disfraces que al principio. ✓

Figure 13: Fragments with Simple Arithmetic Presented Through an Additive Problem
Source: Top (Fujii and Majima 2020); Bottom (Carvajal and de la Rosa 2022)

While in the Spanish textbook analyzed, at the same time that the positional decimal system is presented, different operations are introduced, mainly using the addition algorithm (Figure 14), in the case of the Japanese textbook analyzed, the operations are based on the decomposition of one or both addends to bring them closer to each of the complete tens (Figure 15).

4 Suma.

No olvides sumar la decena que te llevas.



1	3
2	3
+ 3 6	
7 2	

1	7
1	1
+ 5 2	

1	6
3	0
+ 3 7	

2	4
1	1
+ 6	

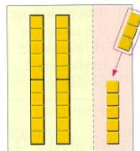
5	
1	8
+ 6 2	

2	2
2	9
+ 5 1	

Figure 14: Fragment in Which the Addition Algorithm is Used in the Spanish Textbook
 Source: Carvajal and de la Rosa 2022


4 Think about how to calculate.

① $25 + 3$




25

20 5



Misaki

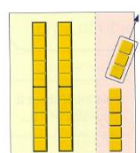
5 + 3



Kota


Math Sentence $\rightarrow 25 + 3 = \square$

② $28 - 3$




28

20 8



Riku

8 - 3



Shiho

Math Sentence $\rightarrow 28 - 3 = \square$

Figure 15: Fragment in Which an Addition Is Performed Using the Decomposition in the Japanese Textbook Analyzed
 Source: Fujii and Majima 2020

Representation Systems

As in the previous category, there are fragments with more than one representation. For this reason, Table 5 shows the absolute frequency with which each representation appears, together with the percentage of the total number of fragments for each textbook, in order to observe its representativeness in relation to the total.

The analysis of the two textbooks reveals that the work on arithmetic is mostly presented in a symbolic register. In fact, almost 90% of the fragments analyzed present a number in this register of representation. As in the previous case, different types of registers are presented simultaneously for the same fragment (see Table 5).

Table 5: Frequencies and Percentages of the Variables Associated with the Category “Representation”

	<i>Symbolic</i>	<i>Verbal</i>	<i>Manipulative Material</i>	<i>Graphics</i>
Spanish	272 (91%)	42 (14%)	2 (0.7%)	103 (34.4%)
Japanese	200 (87.7%)	38 (16.7%)	47 (20.6%)	130 (57%)
Total	472 (89.6%)	80 (15.2%)	49 (9.3%)	233 (44.2%)

In the case of the Japanese textbook analyzed, with the exception of the first fragments presenting the natural numbers, practically all of them contain a symbolic register. Sometimes alone (Figure 16) and in other cases, accompanied by other representational registers, such as the graph (Figure 17).

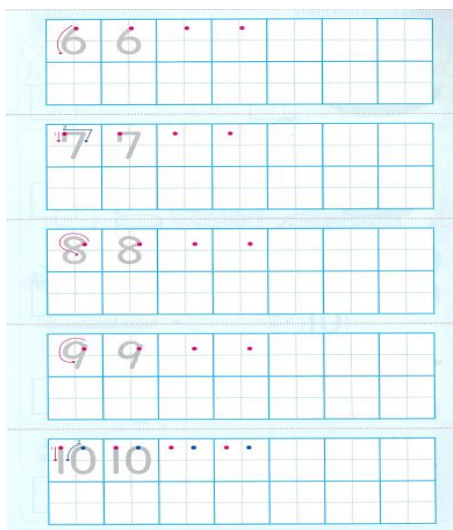


Figure 16: Fragment from the Japanese Textbook Analyzed Where Symbolic Registration Is Practiced

Source: Fujii and Majima 2020

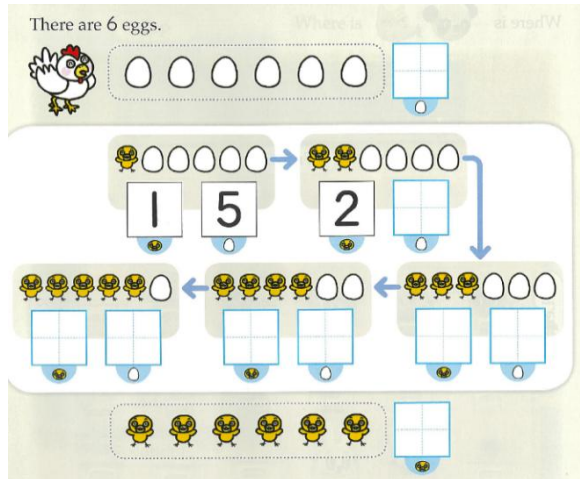


Figure 17: The Symbolic Register Is Accompanied by the Graphic Register in the Japanese Textbook
 Source: Fujii and Majima 2020

A similar situation occurs with the Spanish textbook analyzed, where more than 91% of the fragments contain a symbolic register (Figure 18). In this case, it is present from the first moment in this register, although it is true that in a third of the fragments it is accompanied by the graphic register (Figure 19).



Figure 18: Fragment from the Spanish Textbook Where Symbolic Registration Is Practiced
 Source: Carvajal and de la Rosa 2022

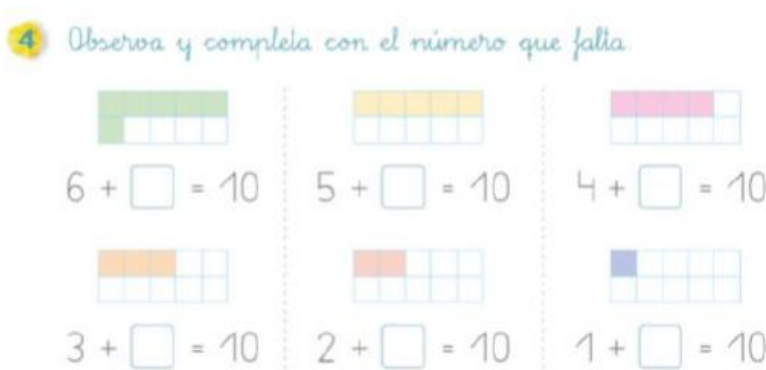


Figure 19: The Symbolic Register Is Accompanied by the Graphic Register in the Spanish Textbook
 Source: Carvajal and de la Rosa 2022

While in the Japanese textbook analyzed it is relatively common to find tasks presented in the graphic register accompanied by manipulative materials (Figure 20), in the case of the Spanish textbook analyzed the manipulative register is practically residual, only two fragments contain it (Figure 21). It is important to mention here that in Japan, students who use this textbook have the manipulative material at their disposal and use it whenever it appears in the activity. In contrast, in the Spanish textbook analyzed it is only visual in most of the exercises, and the manipulative equivalent is not available, so students who use it will not be accustomed to using them.



Figure 20: The Graphic Register Is Accompanied by the Manipulative Materials in the Japanese Textbook Analyzed
 Source: Fujii and Majima 2020

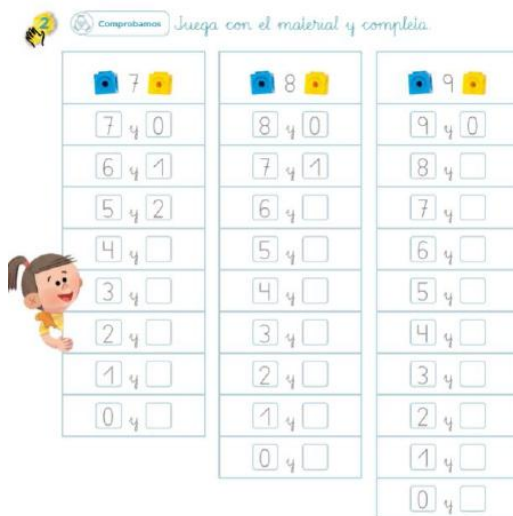


Figure 21: The Symbolic Register Is Accompanied by the Manipulative Materials in the Spanish Textbook Analyzed
 Source: Carvajal and de la Rosa 2022

In the case of the verbal register, we considered those where the words were sufficiently relevant, for example, the instruction to verbalize some aspect of the student’s resolution of the problem. We have not taken into account those statements that could be read by the teacher and where the student would receive the information orally (see Figure 22).

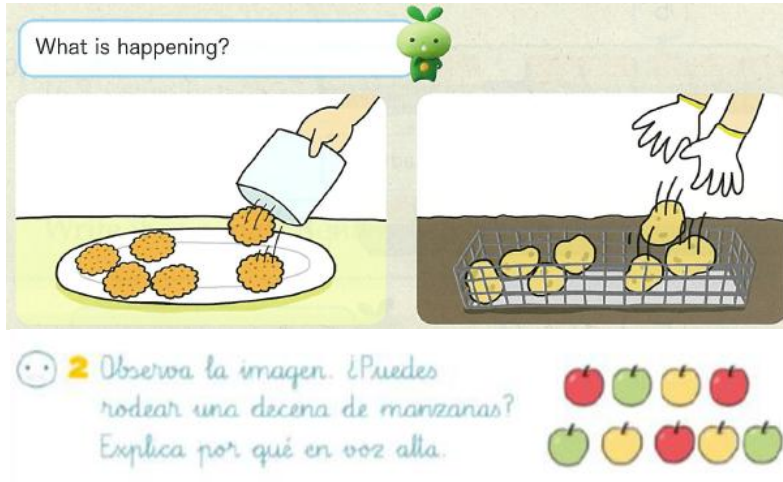


Figure 22: Fragment Where the Verbal Register Appears
 Source: Top (Fujii and Majima 2020); Bottom (Carvajal and de la Rosa 2022)

Discussion and Conclusion

A key distinction between the two textbooks is the emphasis on different mathematical processes. Spanish textbook analyzed place greater focus on number writing or number recognition (74.9%) than Japanese textbook analyzed (60.5%), indicating an emphasis on numeracy skills and numerical notation. Conversely, Japanese textbook analyzed include more exercises related to number decomposition (14.5%) and simple arithmetic (53.1%), emphasizing an understanding of number structures and relationships before procedural fluency.

The introduction of simple arithmetic also differs. The Spanish textbook analyzed often presents systematic counting tasks alongside number writing and then gradually integrates basic operations. In contrast, the Japanese textbook analyzed treats simple arithmetic as independent pre-tasks, often using structured methods such as flashcards. Moreover, while Spanish textbook analyzed introduces addition and subtraction via direct algorithms, Japanese textbook analyzed employs number decomposition techniques, breaking numbers into tens and ones to facilitate computation. In this same sense, Vicente et al. (2024) show that even word problems in recent Spanish textbooks tend to lack semantic variety and realistic contexts; most are one-step, routine arithmetic questions with few schematic aids. Their dataset, together with the results achieved from the textbook analyzed in this study, confirms that Spanish textbooks

still under-exploit contextualized tasks for meaning making. This difference reflects Japan's educational philosophy, which prioritizes conceptual understanding and problem-solving over direct procedural instruction. This focus on decomposition is strongly linked to the use of base ten, which deepens the understanding of the number (Alkhateeb 2019). Similar was the case in Cheng and Wang's (2012) study where Chinese textbooks paid more attention to the connection between number sense and operation than American textbooks. In our case, the Japanese textbook analyzed is more aligned with the Chinese textbooks from that study, while the Spanish textbook analyzed is more similar to the American ones. This difference between the two books analyzed could be one of the reasons why Japanese students perform better than Spanish students in international tests (OECD 2023).

Both textbooks predominantly use symbolic representation, demonstrating a common reliance on numerical symbols in early arithmetic instruction. However, a notable difference lies in the integration of visual and manipulative materials. Japanese textbook analyzed incorporate them significantly, while their presence in Spanish textbook analyzed is minimal. This suggests a preference for multi-representational learning approaches in Japan.

The frequent use of visual models, such as base ten blocks and diagrams, in Japanese textbook analyzed aligns with research emphasizing the role of visual aids in fostering conceptual understanding (Alkhateeb 2019; Hiebert and Carpenter 1992). The minimal presence of manipulative materials in Spanish textbook analyzed implies a stronger expectation for students to develop number sense (Figure 2) primarily through symbolic and verbal interactions rather than hands-on exploration. Furthermore, the type of representation presented could affect whether or not a particular meaning of the number appears. This difference highlights Japan's emphasis on concrete-to-abstract learning progressions, which may better support the transition from intuitive number concepts to formal arithmetic skills.

Regarding applications for mathematics education, Spanish textbook analyzed focus on presenting students how to write numbers and perform calculations, helping them develop strong numerical processing skills early on. The structured practice in writing numbers or number recognition and following set procedures makes it easier for students to gain accuracy and fluency in basic calculations, but not in their understanding. On the other hand, the Japanese textbook analyzed emphasizes understanding mathematical concepts by breaking numbers into smaller parts and introducing ideas step by step. This could have an impact on improving their problem-solving skills. These differences reflect distinct educational priorities, showing the need for a balanced approach that combines both procedural fluency and conceptual understanding.

Implementation Implications

We recommend that teachers who follow these analyzed textbooks adhere to the following country-specific guidelines. The analyzed textbook presents arithmetic symbolically in 91% of

the tasks, but combines it with concrete materials in only 0.7%. Teachers should provide a visual or manipulative model (e.g., ten frames, base ten blocks) for each new symbolic exercise even if the textbook does not provide one, emulating the richer rate of 20.6% of manipulatives found in Japan. The use of manipulatives significantly improves students' performance in mathematics, specifically in understanding numerical meaning, compared to traditional educational approaches, by making ideas tangible and visually explicit (Ahmad and Siller 2024).

Japanese textbooks, where there is less emphasis on writing tasks and algorithmic fluency (60.5% in the Japanese book analyzed compared to 74.9% in the Spanish book analyzed), could be supplemented by the teacher with short daily exercises in handwriting and vertical addition. Given that Japanese textbooks already devote 14.5% of their content to the structured decomposition of numbers, more than double that of the Spanish textbook analyzed, teachers could integrate similar "divide and make ten" activities to encourage flexible mental calculation, as this is currently lacking. Taken together, these adjustments would align procedural fluency with deep number sense and thus fulfill the document's stated goal of improving students' mathematical learning experiences.

Furthermore, the total absence of numerical patterns and the near absence of estimation in the two textbooks analyzed highlight the problem already detected by Sayers et al. (2021) in Swedish textbooks. These authors have already pointed out that the omission of estimation seems to be problematic as a determinant of subsequent arithmetic competence, especially in novel situations, where the diagnosis of children at risk of subsequent mathematical difficulties would be affected. Given these omissions in the two textbooks analyzed, teachers who use them should consider creating their own presentations outside of the textbooks used.

Given that textbooks have a major influence on educational practice in both countries: Spain (Braga Blanco and Belver Domínguez 2016) and Japan (Textbook Research Center 2024) and on student learning (López González et al. 2015), studies of this type can contribute to future lines of improvement, both in teaching practice when using these textbooks and in reflecting on possible changes to them. It is important to remember that children's activities are the most important part of the learning process (Aisyah et al. 2024); without activities, there is no learning.

Follow-Up Studies

This study is limited to a single Japanese textbook and a single Spanish first-grade textbook. Although they are the most widely used in each country, in order to draw reliable cross-national conclusions, future research should analyze a larger sample of textbooks, considering different editions. Since only student pages were examined, excluding teachers' guides, digital resources, and classroom representation, it would be useful to continue this research in this direction, to determine what learning benefits, if any, these books produce. Finally, the qualitative coding depended on researcher judgments, meaning alternative definitions of

number-sense elements might yield different interpretations. In addition, future research should focus on classroom observation when these textbooks are used and subsequent interviews with teachers.

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Informed Consent

The authors declare that informed consent was not required as there were no human participants involved.

Conflict of Interest

The authors declare that there is no conflict of interest.

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