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Monetary Policy in the Long-run Perspective

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"No very deep knowledge of economics is usually needed for grasping the immediate effects of a measure; but the task of economics is to foretell the remoter effects, and so to allow us to avoid such acts as attempt to remedy a present ill by sowing the seeds of a much greater ill for the future".

The Theory of Money and Credit.

Ludwig von Mises, 1934

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Introduction

This thesis consists of three interconnected essays on monetary policy which emphasize the long-term perspective. The neutrality of monetary policy in the long run has been extensively studied; however, we simultaneously consider some elements that have never been merged in the literature in order to analyze the relationship between the cycle and the trend. Our main contribution is the extension of knowledge about the role played by the long-term elements included in the widely-used monetary policy rules, generally called Taylor rules, in the determination of the long-run path. We are also interested in the performance of the financial variables and the effects of the frictions present in credit markets on the long-term dynamics.

Although the three chapters could be treated independently, as they are self-contained, this dissertation has a clearly defined storyline. We first construct the benchmark theoretical model, then we extend it in order to interpret the natural rate of interest and, to conclude, we empirically evaluate the main conclusions of the two previous chapters.

The first chapter lays the foundations of the rest of the dissertation as it develops the workhorse model and raises some issues that will be discussed thereafter. We extend the New Keynesian DSGE macroeconomic model by incorporating endogenous growth, non-zero trend inflation and frictions in the financial markets. Endogenous growth is guided by the spill-over effects generated by capital accumulation, trend inflation is set by the central bank in the monetary policy rule and financial frictions are included following an agency problem as in Gertler and Karadi (2011). In this context, we study the long-lasting effects that conventional and unconventional monetary policies generate on real and financial variables. We show that financial variables play a key role in the determination of the steady-state growth rate. Moreover, the so-called financial accelerator affects the long-term path of the real variables, given the value of the trend inflation. In the calibrated model, we find that long-run growth rate, welfare, investment and financial wealth are maximized at a trend inflation of 1.7% while the leverage, external finance premium and marginal gain of the financial intermediaries are minimized. The set of interactions present in the long-run equilibrium are characterized by a clear non-linear behavior. With respect to short-term dynamics, responses of real variables such as growth rate and investment to conventional monetary policies are influenced by the level of trend inflation. Finally, we show that, in crisis periods, unconventional monetary policies not only reduce the effects of the decline in asset valuation, but could also extend the disturbance impact to the long-run horizon.

In the second chapter, we take the theoretical model previously developed as our basis to analyze the problem posed by the unobservable character of the natural rate of interest to central banks if they mismeasure it. Our definition of the natural rate of interest indicates that financial parameters are involved in its determination, which was implicitly seen in the previous chapter. Furthermore, we raise the possibility of a mismatching of the natural rate of interest and the intercept of the Taylor rule. Our main result reveals that, if the central bank incorrectly estimates this rate, long-run growth could be affected because the trend inflation would differ from its target and, hence, long-run equilibrium would become dependent on both the natural rate of interest estimation and the long-run inflation target. Afterwards, we design an iterative mechanism to monitor the accuracy of the estimation of the natural rate of interest and the position of the inflation target relative to the optimal rate in order to improve the results of the monetary policy from the growth rate perspective. To this end, the central bank would use instruments such as the intercept of the Taylor rule and the parameter that reacts to inflation deviations from the steady-state value.

In the third chapter, we empirically test the theoretical outcomes obtained in the previous chapters for the United States over the period 1960-2013. The semi-structural econometric model based on our benchmark model allows us to analyze the main relationships established in the long term. Our results corroborate the non-linear relationship between the long-run growth rate and the trend inflation found in Chapter 1. Through the estimation of a quadratic equation, we find a hump-shaped relation between these two variables. Nevertheless, a quantile regression which differentiates among levels of trend inflation suggests that some segments of this relationship are not statistically significant. On the basis of the same approach, we also confirm the negative impact that financial frictions have on the long-run growth rate when such rigidities surpass a certain value, which occurs for extreme levels of trend inflation. For intermediate levels, such effects are not statistically significant or even positive. Moreover, our outcomes show that mismeasurement errors of the natural rate of interest affect the trend inflation deviations from its target since monetary policy is conducted through Taylor rules which react severely and preemptively against inflation deviations, thus verifying the conclusion obtained in Chapter 2.

To analyze these long-term relationships between unobservable variables, we have jointly estimated the long-run growth rate, the trend inflation and the natural rate of interest by applying the Kalman filter following Laubach and Williams (2003). All these unobservable variables are specified as time-varying and we also account for financial frictions in the estimation procedure.

Introducción

Esta tesis consiste en tres ensayos sobre política monetaria que enfatizan la perspectiva del largo plazo. La neutralidad de la política monetaria en el largo plazo ha sido ampliamente estudiada, pero este trabajo combina elementos que nunca han sido analizados conjuntamente en la literatura con el fin de profundizar en la relación entre el ciclo y la tendencia económica. La principal aportación de este trabajo es una ampliación del conomiento sobre el papel que desempeñan los elementos de largo plazo incluidos en las reglas de la política monetaria más comunes, generalmente denominadas reglas de Taylor, en la determinación del equilibrio estacionario. Además, esta tesis también se centra en los efectos que las fricciones presentes en los mercados financieros pueden originar en la dinámica de largo plazo.

Aunque los tres capítulos se pueden considerar de forma independiente, esta tesis tiene una línea argumental claramente definida. En primer lugar se construye el modelo teórico de referencia, a continuación se extiende con el fin de interpretar el tipo de interés natural y finalmente se evalúan empíricamente las principales conclusiones de los dos capítulos anteriores.

El primer capítulo amplía el modelo macroeconómico DSGE neokeynesiano mediante la incorporación de crecimiento económico endógeno, inflación de equilibrio estacionario no nula y fricciones financieras. El crecimiento endógeno está originado por los efectos externos positivos que genera el stock total de capital, la inflación de largo plazo la fija el Banco Central mediante la regla de política monetaria y las fricciones financieras se introducen a través de un problema de agencia como en Gertler y Karadi (2011). En este contexto, se estudian los efectos de largo plazo que las políticas monetarias convencionales y no convencionales ejercen sobre las variables reales y financieras. Se demuestra que las variables financieras juegan un papel clave en la determinación de la tasa de crecimiento de equilibrio estacionario y que el acelerador financiero afecta a la trayectoria de largo plazo de las variables reales, dado el valor de la inflación de equilibrio estacionario. En el modelo calibrado se concluye que, en el estado estacionario, la tasa de crecimiento, el bienestar, la inversión y la riqueza financiera se maximizan para una inflación de equilibrio del 1,7 %, mientras que el apalancamiento, la prima de financiación externa y la ganancia marginal de los intermediarios financieros alcanzan su valor mínimo. El conjunto de interacciones existentes en el equilibrio de largo plazo se caracterizan por un claro comportamiento no lineal. En la dinámica de corto plazo las respuestas de variables como la tasa de crecimiento o la inversión se ven influenciadas por el nivel de inflación de equilibrio estacionario. Por último se muestra que, en los períodos de crisis, la política monetaria no convencional reduce los efectos negativos que actúan sobre la valoración de los activos, pero también puede prolongar el impacto de las perturbaciones sobre la economía mucho más que sin la implementación de dichas políticas.

El segundo capítulo se basa en el modelo teórico desarrollado en primero con el propósito de profundizar en la problemática que genera la inobservabilidad de la tasa natural de interés cuando los bancos centrales la estiman incorrectamente. Nuestros resultados revelan que los parámetros que caracterizan la estructura del sector financiero desempeñan un papel clave en la determinación de la tasa natural de interés, lo cual se advertía implícitamente en el primer capítulo. Además, planteamos la posibilidad de que la tasa natural de interés y la constante de la regla de Taylor puedan no coincidir. De nuestros resultados se deduce que los errores en la estimación de la tasa natural de interés afectan a la tasa de crecimiento de largo plazo mediante la modificación de la inflación de equilibrio estacionario, que difiere, en tal situación, de la tasa objetivo fijada por el banco central. En este escenario, la tasa de crecimiento de largo plazo dependerá también de los objetivos de inflación y de la estimación de la tasa natural de interés. Finalmente se diseña un mecanismo dirigido a supervisar la adecuación de la estimación de la tasa natural de interés y la posición del objetivo de inflación respecto a la tasa óptima desde el punto de vista del crecimiento con el fin de mejorar los resultados de la política monetaria. El banco central tendría que usar como instrumentos la constante de la regla de Taylor y el parámetro de respuesta a las desviaciones de la inflación respecto a su valor de equilibrio estacionario.

En el tercer capítulo se verifica empíricamente la validez de los principales resultados teóricos para el caso de Estados Unidos durante el periodo 1960-2013. El modelo econométrico semiestructural basado en nuestro modelo de referencia permite examinar las relaciones entre las variables de largo plazo. Los resultados corroboran la relación no lineal existente entre la tasa de crecimiento de largo plazo y la inflación de estado estacionario establecida en el Capítulo 1. A través de una ecuación cuadrática se prueba que la relación resultante tiene forma de U invertida. No obstante, la estimación de una regresión cuantílica que diferencia entre niveles de inflación de estado estacionario sugiere que dicha relación no es significativa o difiere de los resultados anteriores para alguno de los tramos considerados. Basándonos en las mismas técnicas se confirma el impacto negativo que las fricciones financieras ejercen sobre la tasa de crecimiento de largo plazo cuando tales rigideces superan cierto valor, lo cual ocurre para niveles de inflación de estado estacionario extremos. Para niveles intermedios tales efectos son estadísticamente insignificativos e incluso positivos. Por último, nuestros resultados sugieren que los errores en la estimación de la tasa natural de interés afectan a las desviaciones de la inflación de equilibrio estacionario respecto de la tasa objetivo desde que la política monetaria se conduce a través de reglas de Taylor que reaccionan preventiva y agresivamente contra la inflación, conclusión coherente con los resultados obtenidos en el Capítulo 2.

Con el fin de analizar estas conexiones de largo plazo entre variables inobservables se estiman conjuntamente la tasa de crecimiento de largo plazo, la inflación de equilibrio estacionario y la tasa natural de interés mediante la aplicación del filtro de Kalman siguiendo el procedimiento de Laubach y Williams (2003). Se especifican todos los componentes no observables como variables endógenas y, asimismo, se incluyen las fricciones financieras en la estimación.

Chapter 1

Monetary Policy and Growth with Trend Inflation and Financial Frictions

1.1 Introduction

The recent economic crisis, triggered in mid-2007, has highlighted the severe and long-lasting consequences that financial frictions have on the economies. After the outbreak of the crisis, the power of the credit markets to provide liquidity to the economies has been weakened, which is delaying the economic recovery in most developed countries. Since the natural mechanisms of the credit markets are not operating flexibly and, as a result, countries' output is being seriously affected, central banks have adopted a two-pronged strategy. On the one hand, monetary authorities have implemented a very expansive monetary policy through near zero nominal interest rates and, on the other, they have provided direct lending to private markets in order to ensure the necessary liquidity to satisfy the real demand. However, neither the lax monetary policy nor the direct liquidity injections appear to be appeasing the short-term negative effects of the crisis. This fact leads us to think that the recovery may take some time because the effects of the financial crisis could be reaching the long-term dynamics. Hence, we need economic models of the highest accuracy that connect the short and long term in order to provide a deeper perspective. Further study of the key macroeconomic roles of the financial and monetary markets is crucial not only to explain business cycles, but also to better understand the long-run path of the economies.

So far, the literature has not considered endogenous economic growth and trend inflation together in models with sticky prices and frictions in the financial markets. The analysis of the factors that explain economic growth is a well-known field of study, and the inclusion of financial rigidities in macro models is not unusual nowadays, especially since the outbreak of the financial crisis. Nevertheless, to the best of our knowledge, the two analyses have never been merged. It is uncommon to combine economic growth with monetary issues, even less so with models with financial frictions that usually address short-term relationships. Economic growth is considered a long-term phenomenon that should not be taken into account when analyzing the short-term behavior. However, short-run dynamic macroeconomic models are built around a trend which must be consistent with economic growth. If none of the short-term elements affects the steady state, so that the latter acts only as a trend reference, this distinction would not be ignoring anything important.

By contrast, if any feature of the short-term behavior alters the steady state, then the situation is quite different. Disregarding long and short-term interactions would then cause a misunderstanding of the macroeconomic behavior, not only in the short but also in the long run. This chapter widens the macroeconomic benchmark model by considering a range of issues that have not yet been addressed simultaneously in order to derive some key implications for the monetary policy performance. We design a theoretical model which combines elements that may be critical for the interactions between the short and long term by modifying the DSGE New Keynesian framework.

To adopt the long-term perspective, we include endogenous economic growth driven by the stock of knowledge generated by capital accumulation. In addition, we consider financial frictions in the credit markets. We also allow the central bank to implement unconventional monetary policy in crisis periods by direct lending to non-financial firms. Finally, we remove the assumption of zero inflation in the steady state. With all these elements, we focus on the relationship between real and financial variables and the performance of monetary policy in the short and the long run.

Macroeconomic research has developed accurate models to evaluate monetary policy including nominal rigidities, among them, Christiano *et al.* (2005) and Smets and Wouters (2003, 2007). However, the assumption that financial markets are completely flexible has been prevalent in the academic literature. Bearing in mind the facts discussed above, it seems that the rigidities in the credit markets, the latter being strategic for the economic system, have amplified and extended the impact of the crisis. The critical importance of these markets for the performance of the monetary policy has attracted interest since Bernanke, Gertler and Gilchrist published their seminal paper in 1999 in which they introduce the so-called financial accelerator. The basic idea of this approach is the amplifying effects that credit markets have on business cycles due to the rigidities in this sector which are based on the limitation of non-financial firms in obtaining investment funds as a result of their balance sheet constraints. Other works that highlight the importance of financial rigidities are Kiyotaki and Moore (1997), Holmstrom and Tirole (1997) and Carlstrom and Fuerst (1997). Recently, Kiyotaki and Moore (2008), Christensen and Dib (2008), Brunnermeier (2009) and Christiano *et al.* (2010) have examined several topics related to this issue from different angles. An excellent survey of past and recent work is Brunnermeier *et al.* (2012). These studies conclude that the effects of monetary policy shocks are larger and more persistent when taking financial frictions into account.

Given the recent altered flow of credit and the resulting slowdown in the economic recovery, as well as the reaching of the zero lower bound of nominal interest rates, one of the various proceedings taken by central banks to solve the financial distress has been to implement unconventional monetary policy measures by issuing direct loans or by injecting immediate liquidity into financial institutions. Such policies are designed to soften the negative effects of disruptions related to the valuation of assets, but the study of their consequences is still at an early stage, as noted by Joyce *et al.* (2012). These authors also point out the lack of effectiveness and accuracy of this type of policy. A deeper study of credit policy and unconventional monetary policy from a macroeconomic point of view is, therefore, worth considering. Curdia and Woodford (2011) compute the effects of this credit policy examining several instruments, heterogeneity across agents and different monetary policy rules. In Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), the response under different intensities of credit policy is studied. However, unlike these business cycles studies, we will try to identify the consequences of this kind of unconventional policies in the short and long-term by adapting the structure of financial intermediaries proposed by Gertler and Karadi (2011). In this work, an agency problem is introduced whereby financial intermediaries are restricted in their leverage position to providing funds. What we obtain when we consider economic growth in this framework is that the effect of these measures is not restricted to the business cycles, but also affects the long-run scenario.

We introduce endogenous growth into the New Keynesian framework. There are few precedents in this field, among them, Hiroki (2009), Rannenberg (2009), Amano *et al.* (2009, 2012), Inoue and Tsuzuki (2010), Annicchiarico *et al.* (2011) and Vaona (2012). However, none of these studies consider the financial sector. Some works combining endogenous economic growth and the financial sector analyze the effects of the degree of financial development in neoclassical growth models, as in Levine (1997), but do not include nominal rigidities or monetary policy rules.

Short-term interest rate has been revealed as an effective and quick instrument of monetary policy, but also quite sensitive. As a consequence, the probability of taking a non-optimal decision is high. The choice of variables to be considered when setting the objectives and instruments of the decision-making process and their correct definition is of great importance, both in the short and in the long term. For this reason, one element that we consider essential to address the long-term analysis is the non-null inflation rate in the steady state, that is, trend inflation (hereafter TI). Mainstream monetary policy literature supposes that TI is zero, arguing analytical convenience or that it represents an optimal solution in cashless models. However, evidence shows that this is not a realistic assumption, since most central banks set positive targets on inflation rate in the medium-term, and the estimations of the long-run inflation rate are always greater than zero. Several contributions, including Ascari (2004), Hornstein and Wolman (2005), Sahuc (2006), Amano *et al.* (2007), Kiley (2007), Bakhshi *et al.* (2007), Cogley and Sbordone (2008), Ascari and Ropele (2007, 2009) and Coibion and Gorodnichenko (2011), have examined this issue by focusing on the validity of the conclusions about monetary policy in the New Keynesian models when modifying this assumption. They find that its relaxation changes the short and medium-term properties of the models. Among other outcomes, these studies conclude that the results obtained when including zero TI are not, in general, robust. In particular, the slope of the New Keynesian Phillips curve decreases with the TI and the output gap has less influence in determining the inflation rate. The only works that have combined TI and long-term growth have been Amano *et al.* (2009, 2012), who find a great loss in the steady-state output arising from the presence of TI, and Vaona (2012), who finds a non-linear relationship between inflation and economic growth in a context in which monetary policy is not established by setting the nominal interest rate but through the quantity of money. The presence of financial frictions is omitted in all these papers.

Once trend inflation, financial frictions and economic growth are simulaneously considered in a New Keynesian model, our results show that the current monetary policy generates interactions between real and financial variables that are reflected in a direct link between the long-term economic growth rate and the marginal gain from expanding assets in the financial system, given the value of the TI. As this value is given by the monetary policy rule, where the nominal short-term interest rate is decided, monetary policy disrupts the connection between economic growth and the profitability of the financial sector not only in the cycle, but also in the trend. In this context, both financial and real variables are sensitive to the level of TI in the long run and there is a set of interesting non-linear relationships between them in the steady state. In a calibrated model based on the one developed in Gertler and Karadi (2011), the main finding is that the long-run growth rate, welfare, normalized investment and financial wealth are maximized at a trend inflation of 1.7%, whilst leverage, the external finance premium and the marginal gain of financial intermediaries are minimized.

The long-run relevance of TI disappears whenever there are not price rigidities but the connection between the growth rate and the financial variables is not affected. Alternatively, this last connection disappears without financial frictions, remaining the non-linear relationship between the growth rate and TI. Consequently, the growth rate is independent of TI and financial variables without price rigidities and financial frictions.

After the steady state is studied, we analyze the response of the model to a monetary and a technology shock, showing that the level of TI affects the amplitude of the reactions for the first shock. Moreover, if we trigger a simulated crisis, the unconventional monetary policy alters the magnitude and persistence of the effect of the shock, extending its effects to the long term.

As noted before, some features of our macroeconomic model have been studied previously. However, the fundamental result we find is innovative as it combines economic growth, TI and financial frictions in a New Keynesian model. The main finding is the influence of the TI, set by the monetary policy rule, in determining the rate of economic growth through the marginal gain of the financial system. These three variables, which ultimately synthesize the long-term economic performance and are also important for the short term, are closely linked in both time perspectives.

The rest of the chapter is organized as follows. In Section 2, the theoretical model is developed, as well as the study of the main relationships in the steady state. Section 3 is devoted to calibrating the model, to analyzing the steady state numerically and to subjecting the model to monetary policy and technology shocks in different scenarios depending on the level of TI considered. In Section 4, we analyze the response of the variables to a capital quality shock under different intensities of credit policy. Finally, Section 5 summarizes the main findings. Appendix A1, A2 and A3 displays the normalized model, the steady-state equations and the log-linear model, respectively.

1.2 The model

The model proposed is a modification of the standard DSGE New Keynesian macroeconomic model which combines price rigidities, endogenous capital accumulation and spill-over effects *à la* Romer (1986) as the source of economic growth. Financial frictions have been included following Gertler and Karadi (2011), henceforth GK, and non-zero trend inflation is allowed. The model considers the presence of six types of agents in the economy: households, intermediate good firms, capital producers, retail firms, financial intermediaries and the central bank. The agents are characterized as follows:

1. Household members work, consume and save, holding their deposits in financial intermediaries.

2. Intermediate goods firms operate in perfect competition markets. These firms buy capital and rent labor force in order to produce their goods. They are financed with their own funds, but the purchase of capital is funded through bank loans.

3. Capital producers, whose behavior is characterized by an investment function that includes adjustment costs, sell their production to intermediate goods firms.

4. Retail firms acquire and differentiate intermediate goods and sell them to households, setting their prices *à la* Calvo.

5. Financial intermediaries' liabilities are the households' deposits, whilst net wealth and loans granted to intermediate goods producers are their assets. The granting of loans has an upper limit which depends on the intermediaries' leverage.

6. The central bank implements monetary policy both conventional, through the modification of the short-term nominal interest rate following a Taylor rule, and non-conventional, by direct lending to intermediate goods firms.

1.2.1 Agents

Households

Households are composed of infinite horizon individuals uniformly distributed in a continuum $[0, 1]$. Each household has a fraction σ of its members bankers and a fraction $(1 - \sigma)$ workers. Each banker manages a financial institution and transfers the profits to his household. The workers produce goods earning the competitive wage. Households consume and allocate their savings as bonds and deposits in the financial intermediaries. Their expected utility is defined as follows:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\log C_{t+i} - \chi \frac{N_{t+i}^{1+\varphi}}{1+\varphi} \right] \tag{1.1}$$

where $\beta \in (0, 1)$ is the subjetive discount factor, C_t is the consumption, N_t is the labor supply, $\chi > 0$ is the relative utility weight of labor and $\varphi > 0$ determines the intertemporal estasticity of the labor supply (inverse of Frisch elasticity).

Additionally, households must fulfill the budget constraint, which does not allow the present value of the expenditures to exceed the sum of the income and the value of the initial assets:

$$C_t + \frac{D_t}{R_t} = D_{t-1} + \Gamma_t + W_t N_t - T_t \tag{1.2}$$

where D_t are real one-period life deposits and nominally riskless discount bonds that households hold in their portfolios, R_t is the real gross interest rate, Γ_t are real firms profits and payouts from firms and financial intermediaries, W_t is the real wage and T_t are the lump sum taxes.

Moreover, we add the following restriction to avoid Ponzi schemes (Galí, 2008):

$$\lim_{T\to\infty} E_t \{D_T\} \geq 0 \tag{1.3}$$

Solving the households' utility maximization problem, we obtain the labor supply optimality condition and the Euler equation:

$$W_t = C_t \chi N_t^{\varphi} \tag{1.4}$$

$$E_t \Lambda_{t,t+1} R_t = 1 \tag{1.5}$$

with

$$\Lambda_{t,T} = \beta^{T-t} \frac{C_t}{C_T} \qquad \text{where} \qquad T = t+1 \tag{1.6}$$

Intermediate goods firms

Each intermediate goods producer is indexed by $j \in [0,1]$ and obtains the production at time t by incorporating the capital acquired at the end of period $t-1$ and by renting labor force to the households. The markets of both productive factors are competitive. The firms have a Cobb-Douglas production function, common to all of them, that generates economic growth *à la* Romer (1986):

$$Y_{jt}^i = e^{a_t} \left(e^{\xi_t} K_{jt}\right)^{\alpha} (K_t N_{jt})^{1-\alpha} \qquad \text{where} \;\; 0 < \alpha < 1 \tag{1.7}$$

Y_{jt}^i is the production obtained by firm j with a capital stock K_{jt} and labor N_{jt}.

The index $K_t = \int_0^1 K_{jt}dj$ is the stock of knowledge generated by capital accumulation, which firms take as given, and will be the source of economic growth driving the total factor productivity. ξ_t and a_t are shocks common to all firms. The first is the capital quality shock and the second the aggregate productivity shock, both following first order autoregressive processes of the type:

$$a_t = \rho_a a_{t-1} + u_t^z \tag{1.8}$$

$$\xi_t = \rho_\xi \xi_{t-1} + u_t^\xi \tag{1.9}$$

where $\rho_a, \rho_\xi \in [0, 1)$ measure the degree of persistence of the shocks and u_t^a, u_t^ξ are random errors. By aggregating the production functions of the firms, assuming that they are identical and the capital-labor ratio is common across them, we have:

$$Y_t^i = e^{a_t} \left(e^{\xi_t}\right)^\alpha K_t N_t^{1-\alpha} \tag{1.10}$$

Moreover, these firms fund capital purchases by issuing financial claims in period t (S_t), whose relative prices will be the capital price (Q_t):

$$Q_t S_t = Q_t K_{t+1} \tag{1.11}$$

The real wage (1.12) can be obtained by minimizing costs. Moreover, given that intermediate good producers do not obtain profits and return the used capital to capital producers with a relative price equal to unity, the yield of the financial claims is equivalent to the expected capital return (R_t^q):

$$W_t = P_t^i (1-\alpha) \frac{Y_t^i}{N_t} \tag{1.12}$$

$$E_t \left\{ R_{t+1}^q \right\} = \frac{\frac{P_{t+1}^i \alpha Y_{t+1}^i}{e^{\xi_{t+1}} K_{t+1}} + Q_{t+1} - \delta}{Q_t} e^{\xi_{t+1}} \tag{1.13}$$

where P_t^i is the relative price of intermediate goods and $0 < \delta < 1$ the depreciation rate.

Capital producers

The physical capital stock, whose net investment is produced with adjustment costs, is defined as follows[1]:

$$K_{t+1} = K_t + I_t^n \tag{1.14}$$

$$I_t^n = I_t - \delta K_t \tag{1.15}$$

I_t^n being the net investment and I_t the gross investment. At the beginning of each period, capital producers convert the used capital, which has been acquired from intermediate goods producers, into new capital and resell it to them, along with the newly-created capital. The refurbished capital does not entail adjustment costs, but only the net investment. If we formulate the investment decision problem, which is common to all capital

[1] We have not included the shock that affects the quality of capital in these equalities, as we maintain that it should only be included in the equations relating to the productive sector of the economy.

producers, we can obtain the capital price. This problem is the following:

$$\max E_t \sum_{T=t}^{\infty} \Lambda_{t,T} \left\{ Q_T I_T^{n,k} - \left[I_T^{n,k} + f\left(\frac{I_T^{n,k} + I^k}{I_{T-1}^{n,k} + I^k} \right) \left(I_T^{n,k} + I^k \right) \right] \right\} \tag{1.16}$$

where $I_t^{n,k} = \frac{I_t^n}{K_t}$ and $I^k = \frac{I}{K}$ is the value of the gross investment-capital ratio in the steady state. The functional form of the adjustment costs is:

$$f\left(\frac{I_T^{n,k} + I^k}{I_{T-1}^{n,k} + I^k} \right) = \frac{\varsigma}{2} \left(\frac{I_T^{n,k} + I^k}{I_{T-1}^{n,k} + I^k} - 1 \right)^2 \tag{1.17}$$

$$\text{where} \quad \varsigma > 0, \; f(1) = f'(1) = 0 \quad \text{and} \quad f''(1) > 0$$

The price of the new capital is obtained from the first order condition:

$$Q_t = 1 + f + \frac{I_t^{n,k} + I^k}{I_{t-1}^{n,k} + I^k} f' - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}^{n,k} + I^k}{I_t^{n,k} + I^k} \right)^2 f' \tag{1.18}$$

Retail firms

Each retailer differentiates a unit of intermediate good by re-packaging it. The final output Y_t is composed of a continuum of retail final goods:

$$Y_t = \left[\int_0^1 Y_{st}^{(\epsilon-1)/\epsilon} ds \right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.19}$$

where Y_{st} is the output of retailer s. If users of the final output minimize costs:

$$Y_{st} = \left(\frac{P_{st}}{P_t}\right)^{-\epsilon} Y_t \tag{1.20}$$

$$P_t = \left[\int_0^1 P_{st}^{1-\epsilon} ds\right]^{\frac{1}{1-\epsilon}} \tag{1.21}$$

where P_{st} is the price of Y_{st} and P_t is the price index of the final output.

In order to include nominal rigidities, we follow the model of Calvo (1983). Consequently, we assume that retailers will adjust their prices each period with an exogenous probability $(1-\theta)$, which is constant and common to all of them. Thus, the maximization problem of the firms, assuming zero TI, can be stated as follows:

$$\max_{P_t^*} \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} E_t \left\{ Y_{st+i} \left(\frac{P_t^*}{P_{t+i}} - P_{t+i}^i \right) \right\} \tag{1.22}$$

$$s.t. \qquad Y_{st+i} = \left(\frac{P_t^*}{P_{t+i}}\right)^{-\epsilon} Y_{t+i}$$

where P_t^* is the price set by those firms which change it at time t. Expression (1.22) can be rewritten by substituting the demand curve into the objective function in order to eliminate Y_{st+i}. This leads us to obtain the following first order condition:

$$E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} E_t \left\{ (1-\epsilon) \left(\frac{P_t^*}{P_{t+i}}\right)^{-\epsilon} \frac{Y_{t+i}}{P_{t+i}} + \epsilon P_{t+i}^i \left(\frac{P_t^*}{P_{t+i}}\right)^{-\epsilon} \frac{Y_{t+i}}{P_t^*} \right\} = 0 \tag{1.23}$$

We can solve this equation for P_t^* and, after some rearrangements, arrive at the expression of the optimal price which is set by all firms:

$$P_t^* = \mu \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(P_{t+i}\right)^{\epsilon} Y_{t+i} P_{t+i}^i}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(P_{t+i}\right)^{\epsilon-1} Y_{t+i}} \tag{1.24}$$

where $\mu = \frac{\epsilon}{\epsilon-1}$ is the markup. Additionally, the general price level follows this path:

$$P_t = \left[(1-\theta)\left(P_t^*\right)^{1-\epsilon} + \theta\left(P_{t-1}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{1.25}$$

This is the standard derivation of the optimal price and the general price level without trend inflation. If we now abandon the $\Pi = 1$ assumption, Π being the gross inflation rate in the steady state, the price equations (1.24) and (1.25) should be modified accordingly. We can now define $X_t = \frac{P_t^*}{P_t}$ and $\frac{P_t}{P_{t+i}} = \frac{1}{\prod_{k=1}^{i} \Pi_{t+k}}$ and obtain the expressions:

$$X_t = \mu \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k}\right)^{\epsilon} P_{t+i}^i Y_{t+i}}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k}\right)^{\epsilon-1} Y_{t+i}} \tag{1.26}$$

$$X_t = \left[\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right]^{\frac{1}{\epsilon-1}} \tag{1.27}$$

Financial intermediaries

The structure of our financial sector is based on the one developed by GK, who modify the original idea of Bernanke, Gertler and Gilchrist (1999). Since this section is a replication of the GK model, we will only make a brief presentation in order to state notation and describe the main relationships.

Financial intermediaries obtain funds from households and lend them to intermediate goods firms. Thus, financial intermediaries are the link between savers and investors. The balance of each intermediary f is represented as:

$$Q_t S_{ft} = F_{ft} + D_{ft} \tag{1.28}$$

F_{ft} is the net wealth held by the intermediary at the end of period t, D_{ft} are the deposits created by households which are remunerated at an interest rate R_{t+1}, S_{ft} is the number of financial claims issued by goods producers that the intermediary has in its portfolio and Q_t is the price of each claim. R^q_{t+1} is the yield obtained by the financial intermediary derived from these claims. Therefore, the evolution of the bank's wealth depends on the external finance premium $\left(R^q_{t+1} - R_{t+1}\right)$:

$$F_{ft+1} = \left(R^q_{t+1} - R_{t+1}\right) Q_t S_{ft} + R_{t+1} F_{ft} \tag{1.29}$$

Financial intermediaries provide funds if and only if they do not obtain losses with their operations, that is, if the external finance premium is equal to or greater than zero:

$$E_t \Lambda_{t,t+1+i} \left(R^q_{t+1+i} - R_{t+1+i}\right) \geq 0 \qquad i \geq 0 \tag{1.30}$$

This condition always holds with equality under the assumption of frictionless financial markets. However, if these markets are imperfect, this relationship could be positive. In this way, the bankers maximize their expected wealth:

$$V_{ft} = \max E_t \sum_{i=0}^{\infty} (1-\gamma)\, \gamma^i \Lambda_{t,t+1+i} \left[\left(R^q_{t+1+i} - R_{t+1+i}\right) Q_{t+i} S_{ft+i} + R_{t+1+i} F_{ft+i}\right] \tag{1.31}$$

where γ is the probability of survival of the bankers. Furthermore, we introduce the GK agency problem in order to limit the expansion of assets, which would occur if the inequality (1.30) is positive. The bankers have the opportunity to divert a proportion λ of the available funds towards their households at the beginning of each period. However, if this occurs, the depositors can force bankruptcy and recover the proportion $(1-\lambda)$ of available funds. Therefore, the depositors are willing to lend their funds to the bankers whenever the following equation holds:

$$V_{ft} \geq \lambda Q_t S_{ft} \tag{1.32}$$

meaning that the gain from diverting a fraction λ of assets is lower than the loss of doing it. We can write:

$$V_{ft} = \upsilon_t Q_t S_{ft} + h_t F_{ft} \tag{1.33}$$

where υ_t is the marginal gain of the banks derived from expanding their assets, $Q_t S_{ft}$, maintaining their net wealth fixed. It can be expressed as:

$$\upsilon_t = E_t \left\{ (1-\gamma) \Lambda_{t,t+1} \left(R^q_{t+1} - R_{t+1} \right) + \Lambda_{t,t+1} \gamma x_{t,t+1} \upsilon_{t+1} \right\} \tag{1.34}$$

h_t is the expected value of having an additional unit of F_{ft}, assuming that S_{ft} remains constant, with the following definition:

$$h_t = E_t \left\{ (1-\gamma) + \Lambda_{t,t+1} \gamma t_{t,t+1} h_{t+1} \right\} \tag{1.35}$$

$x_{t,t+i} = \frac{Q_{t+i}S_{ft+i}}{Q_tS_{ft}}$ is the growth rate of assets and $t_{t,t+i} = \frac{F_{ft+i}}{F_{ft}}$ is the growth rate of wealth. Constraint (1.32) can also be expressed as follows:

$$h_tF_{ft} + v_tQ_tS_{ft} \geq \lambda Q_tS_{ft} \tag{1.36}$$

When this constraint binds we have an equality. With $F_{ft} > 0$ and $v_t > 0$ the bankers obtain profits by expanding their assets and we have $v_t < \lambda$. Thus, the maximum amount of funds that the intermediaries can raise depends on their wealth, which can be stated as:

$$Q_tS_{ft} = \frac{h_t}{\lambda - v_t}F_{ft} = \phi_t^pF_{ft} \tag{1.37}$$

where ϕ_t^p can be interpreted as the private leverage ratio. This ratio increases with v_t and its limit is the point at which the gain of diverting funds is offset by its cost, since the increase of this variable in turn augments the opportunity cost of being forced into bankruptcy. We can now rewrite (1.29) as follows:

$$F_{ft+1} = \left[\left(R_{t+1}^q - R_{t+1}\right)\phi_t^p + R_{t+1}\right]F_{ft} \tag{1.38}$$

If we now redefine the variables $t_{t,t+1}$ and $x_{t,t+1}$, we obtain that:

$$t_{t,t+1} = \frac{F_{ft+1}}{F_{ft}} = \left(R_{t+1}^q - R_{t+1}\right)\phi_t^p + R_{t+1} \tag{1.39}$$

$$x_{t,t+1} = \frac{Q_{t+1}S_{ft+2}}{Q_tS_{t+1}} = \frac{\phi_{t+1}^p}{\phi_t^p}\frac{F_{ft+1}}{F_{ft}} = \frac{\phi_{t+1}^p}{\phi_t^p}t_{t,t+1} \tag{1.40}$$

Given that the banks' total demand does not depend on firm-specific factors, we can aggregate it, which leads us to the following equation:

$$Q_t S_t = \phi_t^p F_t \tag{1.41}$$

If we now distinguish between the wealth of the new (F_t^n) and the old (F_t^o) bankers, the total wealth can be stated as:

$$F_t = F_t^o + F_t^n \tag{1.42}$$

where

$$F_t^o = \gamma \left[\left(R_t^q - R_t \right) \phi_{t-1}^p + R_t \right] F_{t-1} \tag{1.43}$$

Finally, the initial funds[2] of the new bankers are defined as the ratio $\frac{\omega}{1-\gamma}$ of the old bankers' wealth, which corresponds to $(1-\gamma) Q_t S_{t-1}$. This leads us to the following expression:

$$F_t^n = \omega Q_t S_{t-1} \tag{1.44}$$

Combining (1.43) and (1.44), we obtain the evolution of the net wealth:

$$F_t = \gamma \left[\left(R_t^q - R_t \right) \phi_{t-1}^p + R_t \right] F_{t-1} + \omega Q_t S_{t-1} \tag{1.45}$$

[2] These funds are transferred by the households.

Central bank

The central bank is responsible for implementing monetary policy. It takes decisions about the short-term nominal interest rate (R_t^{st}) in each period following a Taylor rule that is specified below:

$$R_t^{st} = R\Pi \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} e^{\eta_t} \tag{1.46}$$

where R is the intercept reflecting the structural factors in the reaction function of the central bank (which can also be interpreted as the natural interest rate), Π is the steady-state gross inflation or target, Y is the steady-state level of output consistent with Π, ϕ_π, ϕ_y are, respectively, the parameters that measure the central bank's reaction to inflation and output deviations from their steady-state levels, and the monetary policy shock η_t is defined as an AR(1) process:

$$\eta_t = \rho_\eta \eta_{t-1} + u_t^\eta \tag{1.47}$$

where $\rho_\eta \in [0, 1)$ and u_t^η is the random error.

Finally, the relationship between the real and nominal interest rate is set by the Fisher equation:

$$R_t^{st} = R_t E_t \Pi_{t+1} \tag{1.48}$$

During specific periods in which the private financial system is unable to provide the necessary liquidity to non-financial firms due to balance sheets constraints, the central bank can act as a direct lender to the intermediate goods producers[3]. S_t^{cb} denotes the amount of loans issued by the central bank, assessed at the price of capital, and S_t^p the financial claims intermediated by financial intermediaries, so the total amount of loans can be stated as follows:

$$Q_t S_t = Q_t S_t^p + Q_t S_t^{cb} \tag{1.49}$$

To support these credit measures, the central bank issues riskless debt D_t^{cb}, purchased by households, and pays the market lending rate R_{t+1}. Although these operations are not restricted by intermediaries' balance sheets, they do have associated efficiency costs τ. The central bank is willing to pay a proportion ψ_t of all claims, obtaining a profit of $D_t^{cb}\left(R_{t+1}^q - R_{t+1}\right)$:

$$D_t^{cb} = Q_t S_t^{cb} = \psi_t Q_t S_t \tag{1.50}$$

Therefore, we can rewrite equation (1.49) to incorporate this lending mechanism:

$$Q_t S_t = \phi_t^p F_t + \psi_t Q_t S_t = \phi_t^T F_t \tag{1.51}$$

where ϕ_t^T is the leverage ratio of total intermediated funds:

$$\phi_t^T = \frac{1}{1-\psi_t}\phi_t^p \tag{1.52}$$

[3]Model results do no change if the central bank indirectly lends to non-financial firms via financial intermediaries, as GK point out.

The cost of intervention is funded with taxes and profits from financial intermediation. This is reflected in the following equation:

$$\tau\psi_t Q_t K_{t+1} = T_t + D^{cb}_{t-1}\left(R^q_t - R_t\right) \tag{1.53}$$

We analyze the effects of this kind of monetary policy specifically in Section 1.4.

1.2.2 Equilibrium conditions

The aggregate equilibrium of the economy is defined as follows:

$$Y_t = C_t + I_t + f\left(\frac{I^{n,k}_t + I^k}{I^{n,k}_{t-1} + I^k}\right)\left(I^n_t + I\right) + \tau\psi_t Q_t K_{t+1} \tag{1.54}$$

For the sake of simplicity, we assume that there are no other public expenditures than those derived from the efficiency costs of the unconventional monetary policy. The total output of the economy weighted by the price dispersion $\Delta_t = \int\limits_0^1 \left(\frac{P_{st}}{P_t}\right)^{-\epsilon} ds$ is equivalent to the intermediate goods firms' output.

$$Y^i_t = \Delta_t Y_t \tag{1.55}$$

Assuming that the price distribution between firms that do not change their price is the same as the full price distribution in period t, we can obtain that:

$$\Delta_{t+1} = \theta\Pi^{\epsilon}_{t+1}\Delta_t + \left(1-\theta\right)X^{-\epsilon}_{t+1} \tag{1.56}$$

Including these three equations, the model is closed.

1.2.3 Steady-state equilibrium

Since our aim is to analyze the long-term behavior of the model, it is necessary to precisely define the steady state and the key relationships that emerge in this situation. Because our model incorporates economic growth, some of the variables grow in the steady state, so the model must be normalized to show constant steady values. In order to do this, the normalization of the growing variables $(w_t, C_t, K_t, I_t, I_t^n, Y_t, Y_t^i, F_t, F_t^o, F_t^n)$ by the capital is needed. Economic growth is represented by the gross growth rate of capital $G_t = \frac{K_t}{K_{t-1}}$. We detail the normalized model in Appendix A1, where normalized variables are denoted with a superscript k. The equations of the normalized model evaluated in the steady state are reported in Appendix A2, where the variables without a time subscript are the steady-state values. The solution of the system is characterized by the pair of values of G and v that, given the TI, satisfy the following two relationships:

$$G = \frac{(1-\gamma)^2 \omega}{(1-\psi)\left[\lambda(1-\gamma) - v\right]\left[1 - \gamma \frac{G}{\beta} \frac{\lambda}{\lambda - v}\right]} \tag{1.57}$$

$$G = \frac{1}{1+\tau\psi} \left\{ \frac{\mu \Upsilon \Theta}{\alpha X \Psi} \left[\frac{1}{\Delta} - \frac{(1-\alpha) X \Psi}{\chi \mu \Upsilon} \left(\frac{\mu \Upsilon \Theta}{\alpha \Psi X} \right)^{\frac{-(1+\varphi)}{1-\alpha}} \right] + (1-\delta) \right\} \tag{1.58}$$

where $\Theta = \left[\left(1 + \frac{[\lambda(1-\gamma)-v]v}{(1-\gamma)(\lambda-v)} \right) \frac{G}{\beta} - (1-\delta) \right]$, $\Upsilon = (1 - \theta\beta\Pi^{\epsilon-1})$, $\Psi = (1 - \theta\beta\Pi^{\epsilon})$ and $X = \left[\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right]^{\frac{1}{\epsilon-1}}$.

It is clear that $\Upsilon = \Psi = X = 1$ whenever $\theta = 0$. Then, G is independent of TI without price rigidities but is affected by v. If there is not financial frictions $R^q = R$ and, hence, $v = 0$. Thus, (1.57) is not relevant and G in (1.58) is independent of the financial variables, provided $\tau = \psi = 0$, but the dependence on TI remains. Consequently, G is independent of TI and the financial variables without price rigidities and financial frictions.

Equations (1.57) and (1.58) are obtained as equations (A2.29) and (A2.32) in Appendix A2. The first contains the pair of values (G, v) compatible with equilibrium in the financial sector and the second in the goods market, given the value of TI. They determine the steady-state equilibrium in the plane $\{G, v\}$ given the value of Π targeted by the monetary policy. We should note that, unlike GK, we have endogeneized all the steady-state values including the private leverage ratio. The existence of this relationship between the economic growth rate and the marginal gain of the financial intermediaries for expanding their assets (which determines key factors such as the private leverage ratio), indicates that financial variables affect economic growth in the long term. This is a key result of our analysis. The variable v which, in turn, depends positively on the dynamics of the stock of credit accumulated and on the discounted expected external finance premium (the two components of the expected profits of the financial intermediaries) is the essential link through which the financial system affects the growth rate in the long run. Thus, the credit available in the economic system and its expected returns will determine the pace and the trend of the real activity. Moreover, the fact that the relationship between G and v in the steady state depends on TI establishes a leading role for the monetary policy through the influence of the financial activity in the growth process and its dependence on the external finance premium, not only in the performance of the financial system but also in the growth of the economy. We will study this mechanism more deeply in the calibration section.

Table 1.1: Parameter values

Parameter	Interpretation	Value
β	Discount term	0.99
α	Participation of capital	0.332
δ	Depreciation rate	0.03
χ	Relative utility weight of labor	14.1
θ	Probability of keeping prices fixed	0.779
ς	Investment adjustment costs	5.85
γ	Survival rate of the bankers	0.97
λ	Fraction of bank assets that can be diverted	0.382
ω	Wealth proportion of the new bankers	0.002
φ	Elasticity of labor supply	0.276
ϵ	Elasticity of substitution	4.167
ϕ_π	Coefficient of inflation in the Taylor rule	2.05
ϕ_y	Coefficient of output gap in the Taylor rule	0.5/4
ρ_a	Technology shock persistence	0.9
ρ_e	Monetary shock persistence	0.5
τ	Central bank efficiency costs	0.01
ψ	Steady state value of credit policy	0.001

1.3 Calibration, steady state and shocks

In this section we evaluate the model numerically in order to ascribe the values of the variables in the steady state and their responses to some selected shocks. Table 1.1 shows the values assigned to the parameters.

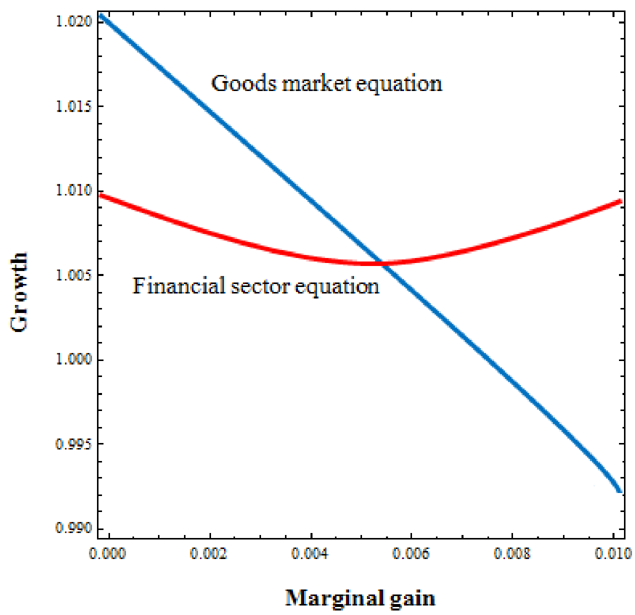
The values of the parameters β, α, δ are standard in the literature. The remaining parameter values like $\epsilon, \varphi, \theta, \varsigma, \phi_\pi, \phi_y$ and those related to financial variables such as λ, γ and ω are taken from GK. The exception is the parameter that reflects the relative utility weight of labor, calibrated to get an annual growth of 2.5% in our benchmark scenario where annual TI is also 2.5%. This calibration ensures a stable steady state and reasonable rates of economic growth, interest rate spread and private leverage at any level of trend inflation without reducing the consistency of the model.

With these values as a reference, the steady state is analyzed in a first subsection and the effects of two types of shocks are described in a second one.

1.3.1 Steady-state analysis

Given the values of the parameters, we can find the steady-state solution in the plane $\{v, G\}$. In the benchmark scenario, this point is $\{v = 0.0052, G = 1.0062\}$. Accordingly, when the incentive constraint (1.36) binds with $F^k > 0$ we have $0 < v < \lambda$. The two main equations which determine the steady state are plotted in Figure 1.1, where the red line corresponds to equation (1.57) and the blue line to equation (1.58). The crossing point determines the values of G and v that make the equilibrium in the financial sector and in the goods market simultaneously possible. We see that, while the function containing the equilibrium points in the financial sector is not monotonous, the equilibrium points in the goods market show an inverse relationship between G and v in a relevant range of values ($0 < v < \lambda$). This U-shaped relationship between G and v in the financial sector is an interesting structural characteristic of the model that provides remarkable subsequent results related to the TI. These outcomes follow from the fact that, while the U-shaped relationship does not depend on the TI, the other function does.

Figure 1.1: Steady-state equilibrium



Taking the pair of values mentioned above as our point of departure, from the equations defined in Appendix A2 we can obtain the values of the remaining variables. The private leverage ratio is 4.85, a level consistent with the results of GK, and the external finance premium is 117 basis points, a reasonable value given the TI considered.

However, these results depend on the value targeted for the TI. If we consider the scenarios within a wide range $[-5\%, 8\%]$ of the annualized rate of TI, one of the most interesting results obtained concerns the non-linear relationship between the long-run growth rate and the TI, as we show in the first plot of Figure 1.2. It can be seen how the long-term growth rate and the TI are positively related up to a maximum value located around 1.7% of the annualized TI[4], after which the growth rate declines faster. Normalized investment and normalized financial wealth also peaks at TI =1.7%, whilst the marginal gain of the financial intermediaries, the external finance premium and the private leverage reach their minimum. Regarding other variables, normalized final output, normalized consumption and real wage are highest when TI=0.5%, while the price of intermediate goods reaches its maximum at TI =1%. Although normalized variables reach their respective maximums for different levels of trend inflation, Figure 1.2 shows in the last row that, at a point in time far enough away, which would be equivalent to the steady state, the households' welfare specified in equation (1.1) is maximized when TI =1.7%. The last plot displays the deviations from the values[5] when TI=1.7% for the main variables.

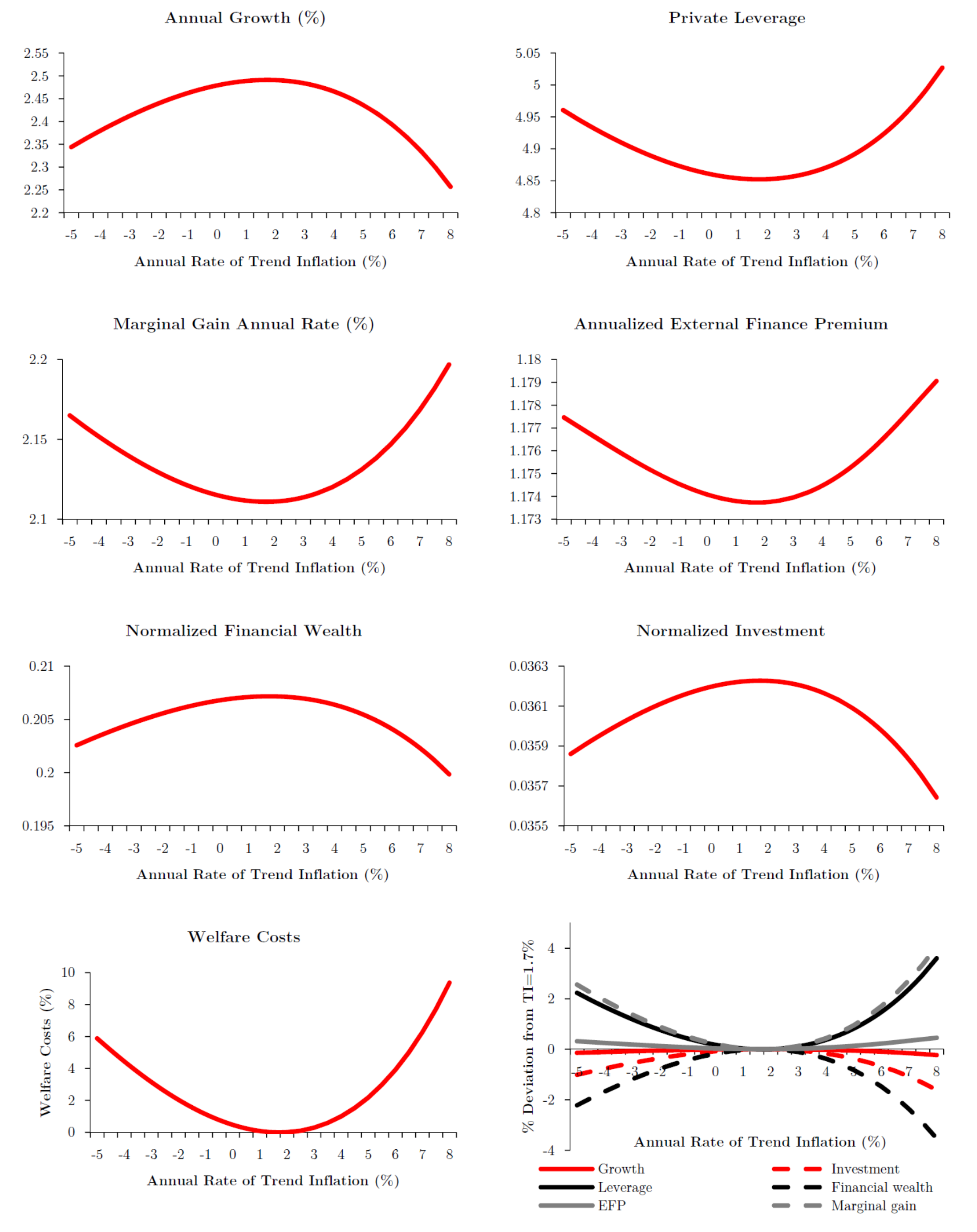
These findings suggest that there is an optimal level of TI that maximizes growth and welfare, as is also concluded in Amano *et al.* (2009). The difference in our conclusion is that the optimal value of TI does not imply deflation. Coibion, Gorodnichenko, and Wieland (2010) do not include endogenous growth in their model, but also find an optimal rate for positive but low levels of TI.

Thus, a shift in the TI alters the steady-state values of the real variables. The relationship between TI and growth suggests that the monetary policy goal of stabilizing inflation around a certain level could be conditioning economic growth and output level in the long term[6]. Therefore, given that monetary policy affects TI through the specification and targets of the monetary policy rules and the way those rules react to deviations in the inflation rate, monetary policy not only affects business cycles, but also modifies

[4] The exact value is 1.72%.
[5] Growth is plotted as percentage points differential.
[6] The relationship is due to price rigidities. All the conclusions are unchanged for reasonable values of θ.

the long-run equilibrium. This result highlights that monetary policy is not neutral in the long term, confirming conclusions recently reached by, among others, Amano *et al.* (2009) and Vaona (2012) in different and more restricted contexts as they do not consider the role played by the financial sector.

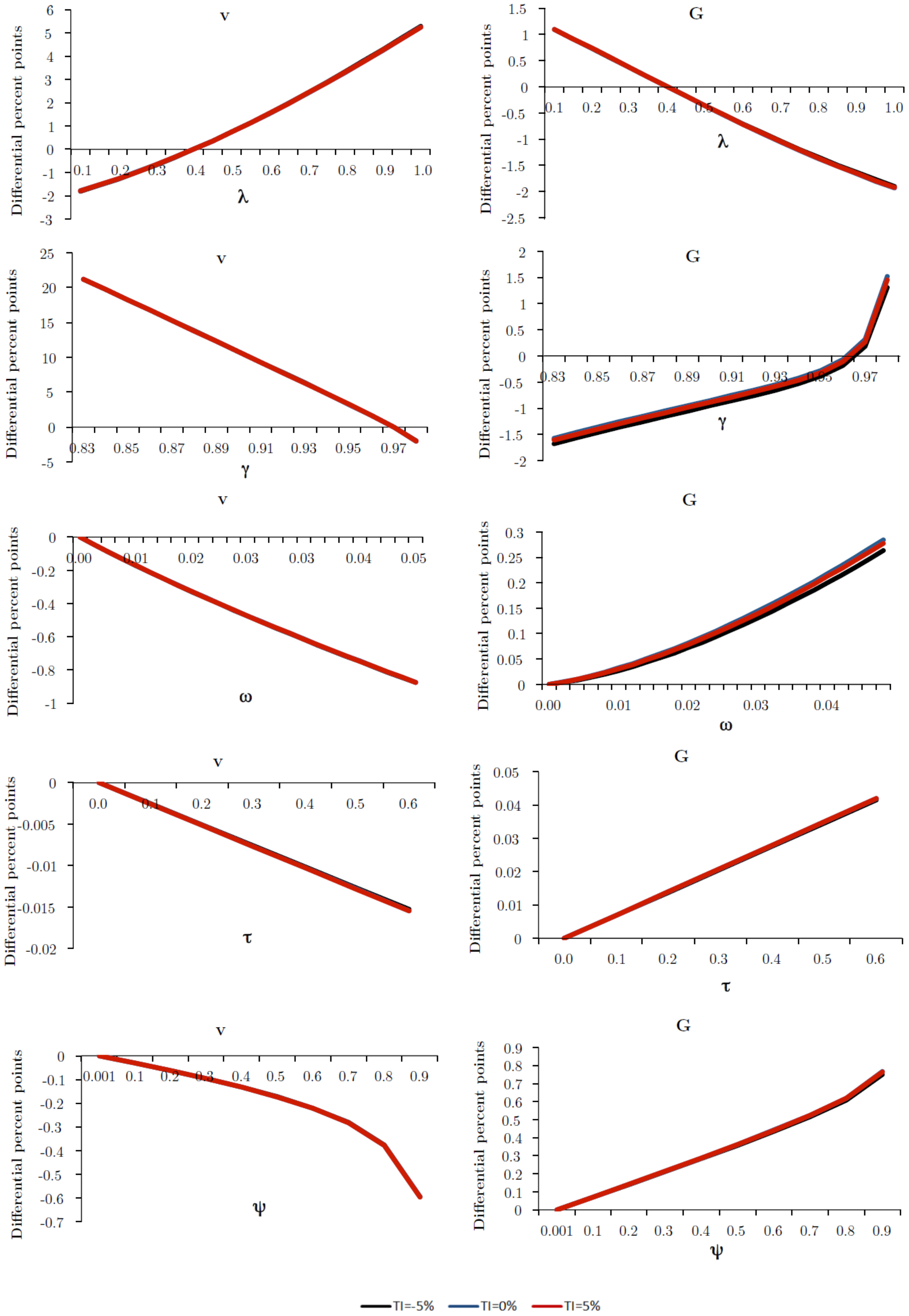
Figure 1.2: Steady-state values



To understand the mechanism described above is crucial in order to examine the behavior of the variables pertaining to the financial sector. Figure 1.2 shows the relationship between the TI and some essential financial variables. The relationship between the TI and the external finance premium is non-linear since, for low rates, it decreases but, from the level of TI that maximixes the growth rate, it increases. After that level, the shortening of the capital return is lower than the reduction of the real interest rate, so the marginal gain for the financial intermediaries to expand their assets goes up and, accordingly, the private leverage ratio increases. The increase in this ratio lowers the net wealth held by the intermediaries and, consequently, capital growth falls. The coincidence of the TI level that maximixes the growth rate with the lowest values of the marginal gain of the financial intermediaries and the external finance premium is an outstanding result in our model. In fact, the real interest rate and the capital return are also maximum with this value of TI. This is a consistent set of symmetrical results that strengthen our approach.

We now carry out a sensitivity analysis in order to check the robustness of our results as well as to determine the impact on the long-run equilibrium of changes in the parameters that reflect the structure of the financial system. Figure 1.3 displays the differential in percentage points related to the benchmark steady state of our two key variables, v and G, for a wide range of values of the financial sector parameters $\lambda, \gamma, \omega, \tau$ and ψ. First, we should note that the effects of the parameters are opposite on the marginal gain of the financial intermediaries and on the economic growth rate, whilst the level of TI barely affects the equilibrium shifts for different parameter values. Marginal gain rises whilst economic growth diminishes with the fraction of capital that bankers can divert. This means that the proportion of the potential unjustified economic losses suffered by the financial system affects positively the financial marginal gain and negatively the real

Figure 1.3: Sensitivity analysis



activity in the long run. Comparing the two extreme cases considered, the marginal gain rate increases by 8 percentage points and growth decreases by 3. As regards the survival rate of the bankers, the parameter that could represent the dynamic process of entry and exit of intermediaries in the financial system, economic growth increases with its value. For example, if $\gamma = 0.91$ instead of 0.97, which would be equivalent to an easier entry, the steady-state growth rate would decrease one percentage point. This suggests that mobility in the composition of the financial system discourages long-term growth. Furthermore, the higher the amount of funds that the new bankers begin with, the higher the long-term growth. The efficiency costs of the credit policy do not seem to significantly affect either the financial marginal gain or the economic growth. Finally, the intensity of the unconventional monetary policy that the central bank carries out in the steady state is positively related to growth. In the extreme case in which almost all funds were mediated by the central bank in the steady state, economic growth would increase by 0.8 percentage points over the benchmark case in which the central bank intermediates only 0.1% of total funds, which is a very low proportion. As expected, the marginal gain of private financial intermediaries would fall more strongly the fewer funds they intermediate.

1.3.2 Shocks

In Appendix A3 we linearize the equations of the normalized model in order to analyze the trajectories of the variables under the proposed shocks. The definition of the Phillips curve has a special importance as it includes, in our case, two such uncommon elements as TI and growth. We follow the procedure of Bakhsi *et al.* (2007), which lead us to obtain the following expression[7]:

[7] The accented lowercase letters are logarithmic deviations from steady state values.

$$\tilde{\pi}_t = \kappa_1 E_t \tilde{\pi}_{t+1} + \kappa_2 E_t \tilde{\pi}_{t+2} + \kappa_3 \tilde{p}_t^i + \kappa_4 E_t \tilde{p}_{t+1}^i + \kappa_5 E_t \tilde{g}_{t+1}^y \tag{1.59}$$

where $\tilde{g}_t^y = \tilde{y}_t^k - \tilde{y}_{t-1}^k + \tilde{g}_t$ are the output growth rate deviations and the coefficient values are:

$$\kappa_1 = \beta \left[\left(1 - \theta \Pi^{\epsilon-1}\right) \left(\epsilon \left(\Pi - 1\right) + 1\right) + \theta \Pi^{\epsilon} \left(1 + \Pi^{-1}\right) \right]$$

$$\kappa_2 = -\theta \beta^2 \Pi^{\epsilon}$$

$$\kappa_3 = \frac{\left(1 - \theta \beta \Pi^{\epsilon}\right)\left(1 - \theta \Pi^{\epsilon-1}\right)}{\theta \Pi^{\epsilon-1}}$$

$$\kappa_4 = -\beta \left(1 - \theta \Pi^{\epsilon-1}\right) \left(1 - \theta \beta \Pi^{\epsilon}\right)$$

$$\kappa_5 = \beta \left(\Pi - 1\right) \left(1 - \theta \Pi^{\epsilon-1}\right)$$

Thus, inflation depends on the inflation expectations, the marginal costs of retailers and the output gap growth rate. Note that, if $\Pi = 1$, this expression is reduced to the standard New Keynesian Phillips curve.

After linearization, to evaluate the model response we analyze two standard types of shocks: a monetary policy shock and a technology shock. For each possible scenario, Figures 1.4 and 1.5 show the deviations from each steady-state value of the main variables expressed in percentage points across 80 quarters. The darker lines represent higher TI and the lighter ones lower TI. We have reduced the considered range of TI to $[0\%, 5\%]$ in order to focus on the most probable scenarios as well as to appreciate the behavior above and below 1.7%, the value that maximizes the growth rate in the steady state.

Firstly, to assess the impact and persistence of monetary policy on the dynamics of the variables, we will submit the model to a standard monetary shock defined in equation (1.47). Regarding the size of the shock, we will force an increase of 25 basis points in the quarterly short-term nominal interest rate, which corresponds to an annual rate of 1% not explained by the variables included in the monetary policy rule.

Figure 1.4: Responses to a monetary policy shock. % Deviation from the steady state

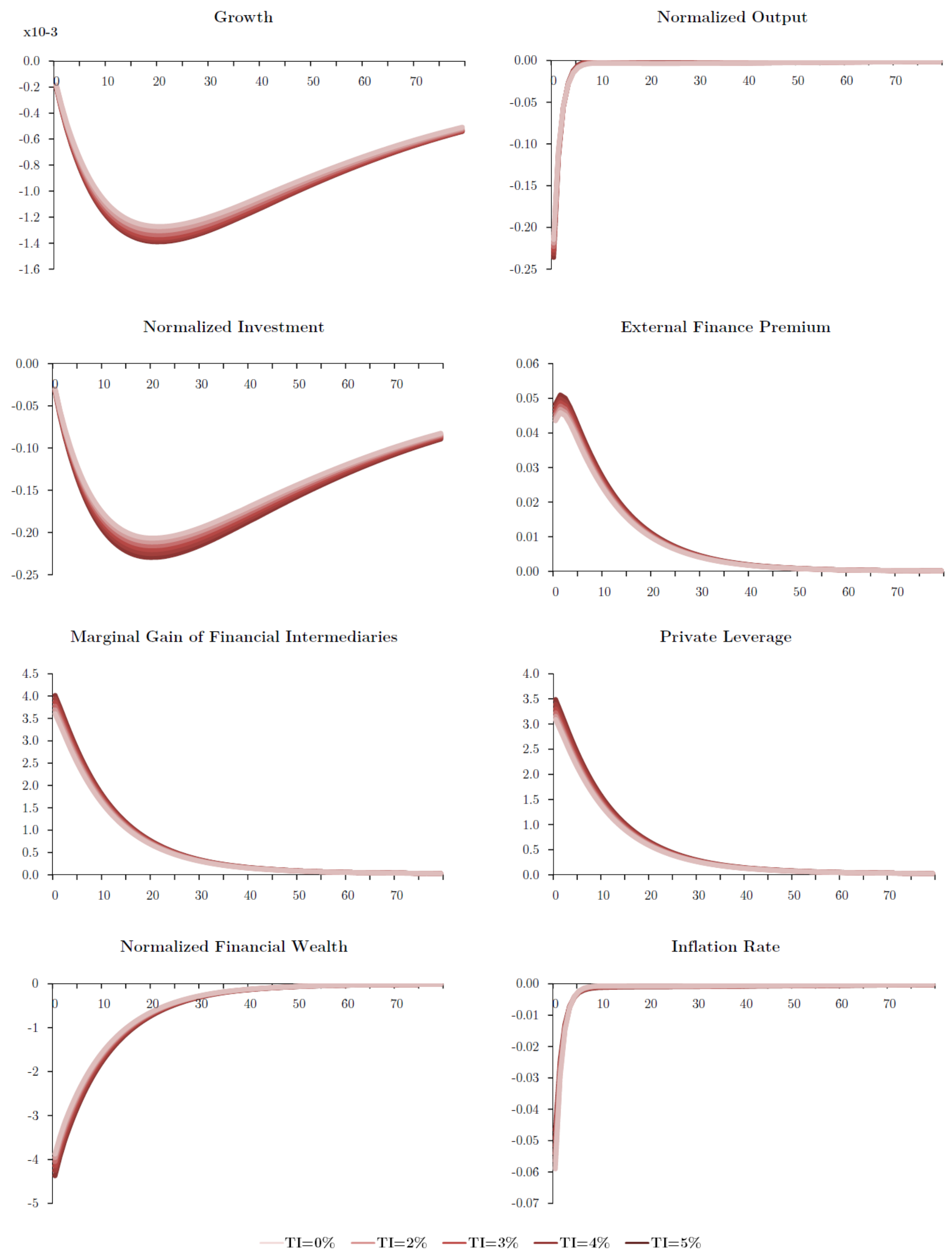
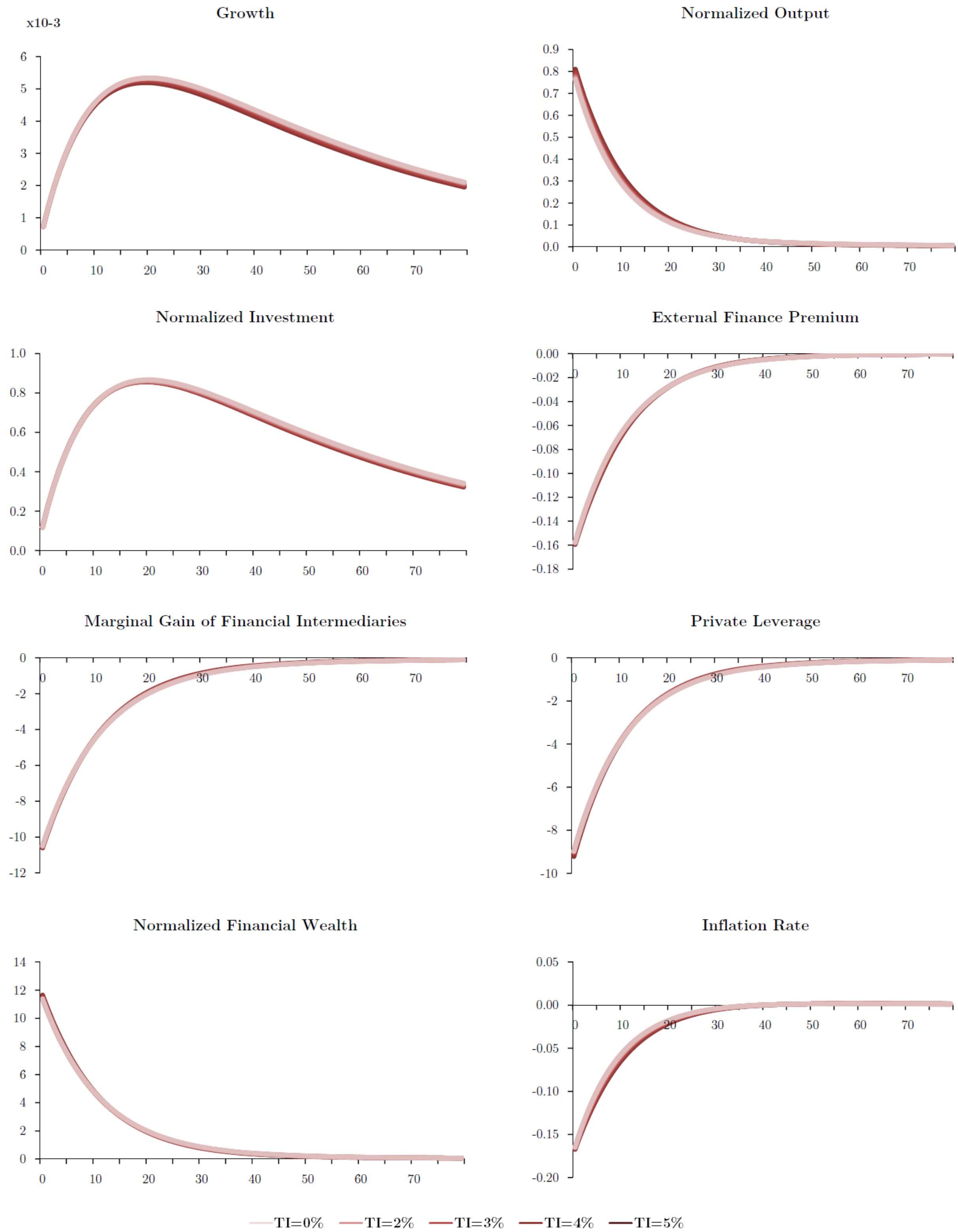


Figure 1.4 shows the responses of both financial and real variables for the different values of TI. The growth rate decreases with more intensity when the steady-state inflation rate is higher and, even though the effect is slight, the difference between extreme scenarios is wide. The effect diminishes gradually but remains for up to 20 years after the shock, which is longer than in the standard New Keynesian model due to the financial accelerator mechanism. Normalized net investment response and its relationship with TI is similar, though broader, to the growth rate response. Normalized output is less dependent on the level of TI, although the initial impact of the shock produces a response 9% higher when TI=5% than when TI=0%.

So far as the financial variables are concerned, the effects on the marginal gain for expanding financial assets, external finance premium, private leverage and the net wealth of financial intermediaries are larger when TI is higher. The sequence of the events is as follows. The rise of the nominal interest rate results in an increase of the interest rate differential. This spread pushes up the marginal gain to expand the assets for the intermediaries. Therefore, the private leverage ratio grows nearly 3% in the benchmark scenario. The result of these movements is a decrease of the intermediary normalized net wealth of 4% which finally causes a decline in the capital growth. The inflation rate falls as expected, without a clear effect of the TI.

Therefore, it has been shown that an exogenous shock in the monetary policy rule causes effects on the real and financial variables that depend on the TI level. These effects are greater for all variables the higher the TI level (except the inflation rate, which is not affected by TI in terms of deviations from the steady-state level).

Figure 1.5: Responses to a technology shock. % Deviation from the steady state



Secondly, we will study the effects of the technology shock in (1.8). The disturbance is a positive deviation of 1% in the total factor productivity included in the production function. Figure 1.5 displays the impulse response functions of the variables.

Contrary to the monetary shock experiment, the response of the variables to a technology shock hardly depends on the TI considered. At the begining, the technology shock slightly increases the growth rate without differences but the increase is greater later the lower is TI. Normalized investment increases up to 0.9% and these effects remain for 20 years after the shock. Normalized output increases around 0.8%, and more with high TI, and the response remains after 7 years. The effects on the financial variables are similar for any TI. As a result of the rise in the external finance premium, asset prices increase and the financial intermediaries balance sheets improve. These movements lead to a lowering of the marginal gain of the intermediaries and, therefore, to a deleveraging process up to 10%. The 12% increase of the net wealth derived from the previous shifts increases the growth rate and, lastly, the inflation rate drops.

So far, we have analyzed conventional shocks, concluding that the magnitude and persistence of the effects of the monetary shock have been amplified by the financial accelerator, but also depend on the TI level. In the next section, we will force a crisis in the economy in order to study the unconventional monetary policy.

1.4 Unconventional monetary policy

Hitherto we have assumed that the central bank only implements monetary policy by setting the short-term nominal interest rate, that is, through conventional monetary policy. Now let us assume that the central bank can also implement unconventional monetary policy by acting as a direct lender. Although the central bank has to bear efficiency costs in these interventions and, thus, is less efficient than private intermediaries, it does not face any restriction. The central bank has the power to act as a financial intermediary, but we assume, as GK do, that it only uses this power, to any extent, during crisis periods. The crisis is characterized by a sudden drop in the quality of capital, causing a rise in the external finance premium. These movements are consistent with what has been seen during the financial crisis. The evolution of the unconventional monetary policy responds to the following pattern:

$$\psi_t = \psi + b\left[E_t\left(R^q_{t+1} - R_{t+1}\right) - \left(R^q - R\right)\right] \tag{1.60}$$

where b is the policy parameter that represents the degree of central bank reaction to deviations in the external finance premium from its steady-state value and ψ the steady-state level of the unconventional monetary policy. We assume that $\psi = 0.001$, a positive value but close to zero. The shock, defined in (1.9), is a 5% decline in the capital quality with a persistence degree parameter of 0.66. The responses of the main variables in percentage deviations from the steady-state level along 80 quarters are shown in Figure 1.6. We distinguish four degrees of intervention represented by parameter b ($b = 0$ or no intervention, $b = 10$ or low intervention, $b = 50$ or medium intervention and $b = 100$ or high intervention).

The consequences of this disturbance for the optimal TI and without the credit policy are represented by the black line in Figure 1.6 and are characterized as follows[8]. The reduction of the effective level of capital generates a decline in the total assets value.

[8] We only present the results for the optimal level of trend inflation, TI=1.7% for the sake of clarity and because there are slight variations between scenarios for this type of shock.

In addition, the deterioration of the position of the financial intermediaries' balance sheets diminishes the demand for capital and, therefore, its price. Thus, the decline in investment and its price causes a further deterioration in the balance sheets that is amplified by the private leverage. Due to the fall in effective capital, the interest rate spread increases and, therefore, there is an increase in the marginal gain and drops in both the normalized output and the normalized net investment. Inflation increases initially but, after five quarters, it falls below the steady-state level.

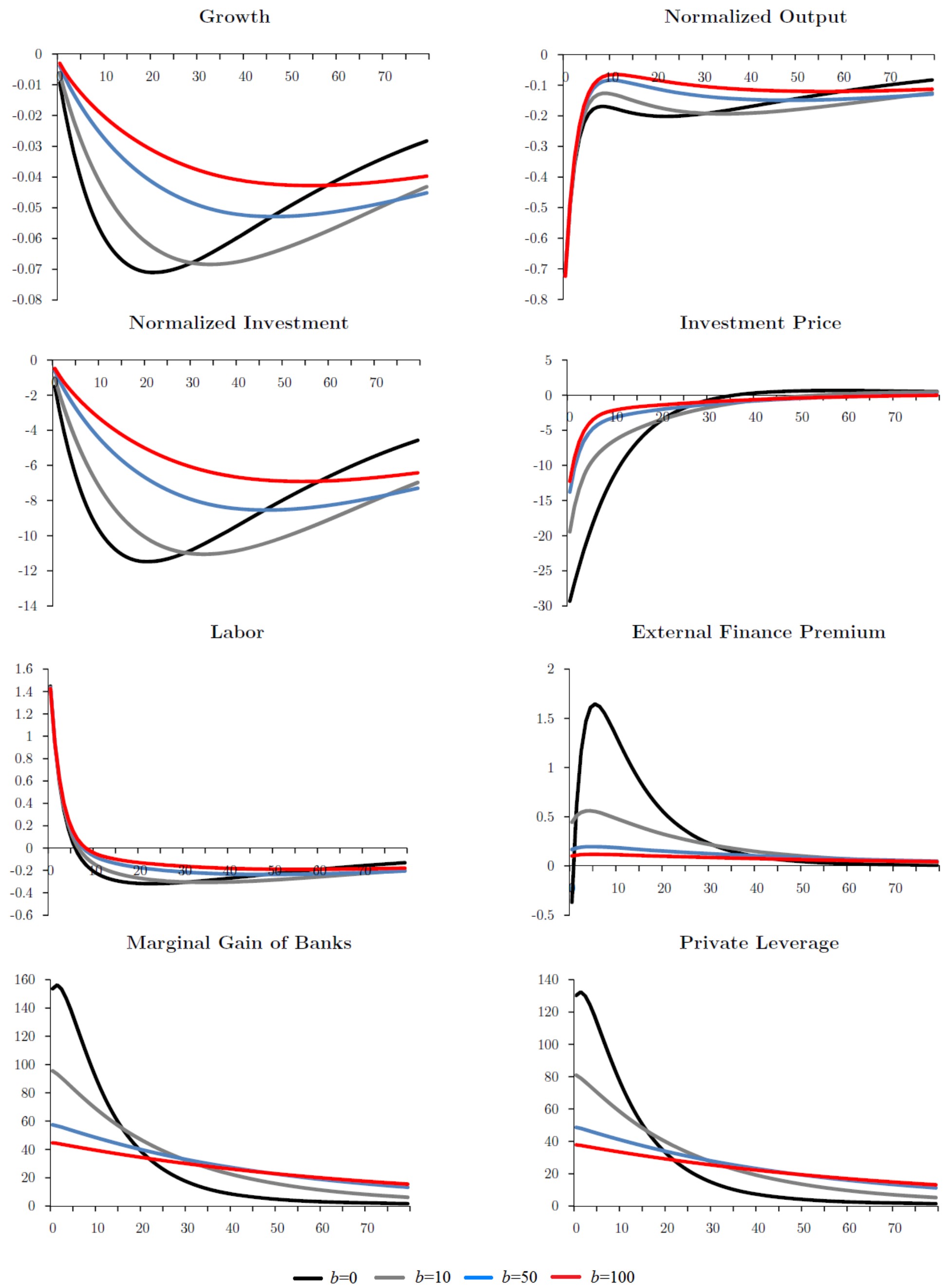
We now focus on the differences in the model response to the intensity of credit policy. As can be seen in Figure 1.6, the response of the variables to this kind of shock depends on the degree of the central bank intervention. Real variables, such as growth rate, normalized output, normalized investment and its price, go down, as expected. Inflation and labor increase in the very initial periods to drop afterwards. The responses are clearly dampened by central bank policies.

As regards financial variables, the marginal gain of the financial intermediaries depends largely on the unconventional actions of the central bank. This variable increases by 150% if the monetary authorities do not interfere, and only by 45% if they do so with a high degree of intensity. In response to the shift in the marginal gain, private leverage will also increase, in this case by more than 130% without intervention and only by 40% with a high intervention. This movement is contrary to the predictions made by GK, who describe a deleveraging process. In our model the private leverage ratio increases considerably, as seen in the U.S. data[9], where it can be observed how this ratio has increased about 25% from 2008 to 2013 despite the substantial liquidity injections made by the Federal Reserve. Finally, the normalized financial wealth of intermediaries also depends on the intensity of the policy. As in the previous section, nominal interest rate

[9] Source: SNL Financial. Core Capital as a percent of average total assets minus ineligible intangibles.

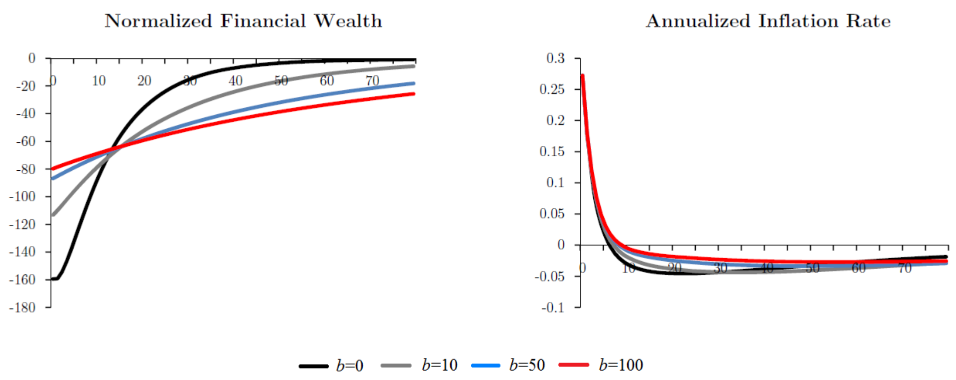
does not reach its zero lower bound.

Figure 1.6: Responses to a capital quality shock and credit policy if Π=1.7%



% Deviation from the steady state

Figure 1.6: Responses to a capital quality shock and credit policy if Π=1.7% (cont.)



% Deviation from the steady state

Another topic of interest is the persistence of the shock in terms of the intensity of the unconventional monetary policy. As can be seen in Figure 1.6, this type of policies mainly reduces the initial impact of the shock on the financial variables. However, the more intense the intervention, the greater the persistence of the effects on all variables. For example, in the case of intermediaries' wealth, if the central bank does not interfere, the effects disappear within 10 years of the shock. Conversely, if the degree of intervention is medium or high, the effects endure for more than 20 years after the shock. Hence, the general conclusion to be drawn is that the greater the intensity of the credit policy, the lower the negative initial impact of the crisis but the longer the response of the variables to a decrease in the value of capital.

Therefore, monetary authorities should adopt a long-term perspective weighting the magnitude and persistence of the effects on real and financial variables in order to choose the intensity of the unconventional monetary policy. The consideration of this perspective could increase the accuracy of the policies that central banks implement in crisis periods.

1.5 Conclusions

This chapter extends the standard New Keynesian DSGE model with endogenous economic growth, financial frictions and non-zero trend inflation in order to get a suitable framework to analyze the long-lasting effects of monetary policy on real and financial variables. In the analysis of the steady state and the dynamics of the model, we find that both the financial accelerator and the level of trend inflation affect economic growth in the short and the long term.

After calibrating the model, we have studied the long-run behavior. Our model shows the existence of a key link in the long run between the growth rate and the marginal gain for the expansion of the financial assets, given the value of the trend inflation. A non-linear relationship between the long-term growth rate and the trend inflation is one of our outstanding results where, in a model based on the one used in Gertler and Karadi (2011), growth rate, normalized investment and normalized financial wealth reach a maximum when annualized trend inflation is 1.7%, whilst the leverage, the external finance premium and the marginal gain of the financial intermediaries reach their minimum. This is a symmetric and coherent set of results that show the strength of the long-run connections between economic growth, the profitability of the financial sector and monetary policy.

The long-run relevance of the trend inflation disappears whenever there is not price rigidities, but the connection between the growth rate and the financial variables is not affected. Alternatively, this last connection disappears without financial frictions, remaining the non-linear relationship between the growth rate and the trend inflation. Consequently, the growth rate is independent of the trend inflation and the financial variables without price rigidities and financial frictions.

So, financial variables affect the steady-state equilibrium growth rate, especially the marginal gain for expanding the financial assets, given the level of trend inflation. Through the sensitivity analysis of the financial parameters, we have found that the structure of the financial sector has a well established set of implications for the long-run equilibrium, keeping the trend inflation constant. On the one hand, the rate of growth decreases with the fraction of capital the bankers can divert. On the other hand, growth rate increases with the rate of survival of the bankers, the initial capital of the new bankers and the intensity of the unconventional monetary policy. The effect on the marginal gain is the opposite.

Then, we have focused on the dynamics of the model following the emergence of different disturbances. The response of the main variables to a monetary policy shock is higher when the trend inflation is high. In the case of a technology shock, the dynamics of the real and financial variables are less dependent on trend inflation.

Finally, with the aim of assessing the effects of an unconventional monetary policy, we have simulated a crisis by forcing a decline in the quality of capital. It has been shown that real variables such as economic growth rate, output and investment are affected by the credit policy. Likewise, the response of financial variables such as the marginal gain, the leverage ratio or the financial wealth of intermediaries also depends on the intensity of the credit policy. Furthermore, when the credit policy is of higher intensity, although the negative effects of this shock are smaller at first, mainly on the financial variables, their persistence is higher, lasting at least 20 years after the disturbance. Thus, we demonstrate that more aggressive credit policies may lead to a prolongation of the effects of crises beyond the medium term.

In sum, it has been shown that, if the monetary authorities wish to encourage the long-term economic growth, they should take care of the long-run non-linear connections between economic growth, the profitability of the financial sector and the inflation target of the monetary policy.

Chapter 2

Natural Rate of Interest, Monetary Policy and Growth with Trend Inflation and Financial Frictions

2.1 Introduction

The long-run real rate of interest, also known as the natural rate of interest, has been studied by academics since Knut Wicksell introduced it in the late nineteenth century. Although there are various definitions of this concept, the most extended is the rate that ensures aggregate price stability and the reaching of the potential output in the absence of exogenous shocks or, as defined by Woodford (2003), the equilibrium real rate of return when prices are fully flexible. Nevertheless, its definition and importance have been modified over time due to the progress in the understanding of the economy and the changes in the tools used by the monetary authorities.

The way that monetary policy is currently conducted by central banks, leaving aside the monetary aggregates and using the nominal interest rate as the main instrument, recovers a key role for the natural rate of interest. The link between monetary policy and the natural rate of interest stems from the rules through which the macroeconomic mainstream, the New Keynesian models, considers that monetary policy is conducted. In these models, the nominal interest rate is set by rules in which the natural rate is the intercept and somehow characterizes the stance of the monetary policy. In this context, it is assumed that this rate is known exactly. However, the natural rate of interest is unobservable, so central banks must estimate it. During this process, monetary authorities could over or infraestimate the true value. A proof of this lack of accuracy is the gap between the observed interest rates and those that would result from the rules, as noted by Judd and Rudebusch (1998) and Orphanides (2003), who relate these gaps to the uncertainty associated with the estimation of the unobservable variables. Therefore, a deeper analysis of the fundamentals of the natural rate and the consequences of its incorrect estimation becomes convenient.

The academic literature has made progress in the understanding of the natural rate of interest in the context described above. Woodford (2001) discusses the fluctuations of this rate and its implications for the monetary policy. He argues that the natural rate of interest varies across time in response to real disturbances, but is always supposed to be accurately known. Woodford does not inquire about the possible implications of the use of a rate different to the endogenous one. By contrast, Orphanides and Williams (2002) consider that the estimated value introduced into the policy rule may differ from the endogenous rate. They show that the cost of underestimating the natural rate of interest is larger than the cost of overestimating it, considering the consequences in terms of stabilizing the economy. Meanwhile, Tristani (2009) analyzes the determinants of the natural rate of interest and also raises the possibility that the value estimated by the monetary authority may differ from the correct value. This author concludes that, if there were such a differential, the inflation target would never be reached[10]. Moreover, even if monetary authorities were able to know the exact law of motion of the natural rate of interest, delays in obtaining reliable information could lead central banks to carry out inaccurate policies in real time, as pointed out earlier in Levin, Wieland, and Williams (1999) and recently in Arestis and Sawyer (2008) and Neri and Ropele (2012). Accordingly, the unobservability of the natural rate of interest, the lags in obtaining precise information and other drawbacks have led many authors to question its usefulness for the accuracy of the monetary policy, as asserted by[11] Clark and Kozicki (2005) and Weber, Lemke and Worms (2008).

Nevertheless, and despite the difficulty of estimating the natural rate of interest, its importance is decisive for policymakers since this variable is the element integrated into the widely-used monetary rules which reflects the information about changes in the forces that guide the economy. Taylor and Williams (2010) analyze the optimal monetary policy rules and conclude that the optimal coefficients assigned to variables such as the deviations of inflation or the output gap change depending on the mismeasurements in the natural rate of interest. Cúrdia *et al.* (2011) assert that the optimal monetary policy rules have to integrate this rate and Canzoneri, Cumby and Diba (2012) show how the rules perform better if the monetary authority can track the natural rate of interest[12].

Besides, some relevant information may be overlooked in the estimation process or,

[10] Other works focus on connected issues such as the relationship between money and the natural rate of interest (Andres, Lopez-Salido and Nelson, 2009) and the impact of misunderstandings in this rate on the zero lower bound of the nominal interest rates (Williams, 2009).

[11] Alternative but related research studies the effects of mismeasurements of the output gap on the optimal monetary policy, such as Orphanides et al. (2000), Rudebusch (2001), McCallum (2001), Smets (2002) and Orphanides (2003), among others.

[12] However, this finding depends on the framework employed because other approaches as Laxton and Pesenti (2003) argue that inflation-forecast-based rules perform better in small open economies.

as Arestis and Sawyer (2008) pointed out, the theoretical assumptions imposed may not be valid. Amato (2005) indicates that some features of financial markets, such as the existence of external finance premiums or agency problems, may affect the natural rate of interest by generating a wedge between the actual long-term interest rates and the long-term natural rate. In addition, as outlined in that work, the literature has always assumed that the steady-state inflation is zero. However, many studies such as Ascari (2004) and Cogley and Sbordone (2008), among others, have proposed that trend inflation is positive, in general, so the relaxation of this assumption could shift the equilibrium and, thus, the natural rate of interest. Hence, the definition of the natural rate should be modified. We can no longer understand it as the rate that ensures price stability, but the rate which provides inflation stability around a long-term level.

The need to know the value of the natural rate of interest has led the empirical literature to estimate it and to analyze the gap between this rate and the short-term rate. These estimates go beyond simple historical averages as they are not precise, particularly in periods of a high variability of output gap and inflation. One of the first articles that address this issue is Bomfim (1997), who derives the natural rate of interest of the U.S. by assessing the IS and the potential output curves. Laubach and Williams (2003) estimate the time-varying natural rate of interest for the U.S., concluding that its underestimation worsens macro stabilization. Moreover, as also noted by Tristani (2009), they suggest that, if the natural rate included in the Taylor rule is not correctly estimated, the inflation target is never achieved. But they do not delve into the determination of the fundamentals of this rate nor go beyond the mechanism which can be applied to verify whether the monetary authority is deviated from the correct value.

The present chapter carries out an analysis of the determination of the natural rate of interest as well as of the impact of the wrong identification of this rate, emphasizing the long-term perspective. The theoretical context is the modified New Keynesian model with endogenous growth, financial frictions and trend inflation developed in Chapter 1. Firstly, we obtain the steady state of the economy and the expression for the natural rate of interest, identified as the long-run real interest rate, which depends on the long-run growth rate and on the structure of the financial system. Then, we conduct a sensitivity analysis of the natural interest rate to changes in the parameters of the financial sector, determining how are the responses to shifts in the financial structure.

Furthermore, when the central bank inaccurately estimates the natural rate of interest, its value depends on both the wrong value and the inflation target. Under this assumption, it follows that the value estimated for the natural rate of interest is non-neutral to monetary policy. The reason is found not only in the gap generated between the targeted and the actual long-run inflation when the central bank misunderstands the natural rate, as Tristani (2009) and Laubach and Williams (2003) stated, but also in its influence on the long-run growth rate.

Delving into the relationship between the long-run growth rate and the potential error in the natural rate estimate, we prove that it depends on the inflation target. As our model predicts that the long-term growth rate is maximized when the central bank approaches a certain inflation rate in the long run, this relationship will depend on the differential between this optimal inflation rate and the inflation target. We show that the natural interest rate target can serve as a policy instrument. Specifically, we demonstrate that, in order to stimulate the long-term growth rate, if the inflation target is below the optimal one, the natural interest rate target should be lower than the endogenous rate.

Finally, we develop a procedure to monitor the accuracy of the estimate of the natural rate based on our conclusions to verify whether the central bank is wrong and how to follow a path towards a more convenient or the true value.

This chapter is organized as follows. In Section 2, the theoretical model used is briefly presented. Section 3 is devoted to displaying the steady-state equations and to introducing our definition of the natural rate of interest, submitting this rate to a sensitivity analysis. Section 4 presents the impact on the economy's trend that mismeasurements of the natural rate of interest would trigger. In Section 5, we design a procedure to verify and correct the deviations from the true value of this rate. Finally, Section 6 summarizes the main findings.

2.2 Theoretical model

The theoretical setup used to derive the natural rate of interest and its characteristics is fully developed in Chapter 1 so, here, we briefly present it in order to make this chapter self-contained. It is an adaptation of the benchmark New Keynesian DGSE model which incorporates spill-over effects as the source of economic growth. Following Gertler and Karadi (2011), financial frictions have been included and, finally, trend inflation is allowed. The model considers the presence of six types of agents in the economy: households, capital producers, intermediate goods firms, financial intermediaries, retail firms and the central bank.

Households Households consume and use part of their savings to create deposits in the financial intermediaries. Each household has a fraction σ of its members who are bankers, who manage one financial institution each and transfer the profits to their households, and a fraction $(1-\sigma)$ who are workers, who produce goods and earn the competitive wages. Their preferences are given by:

$$E_t \sum_{t=0}^{\infty} \beta^i \left[\log C_{t+i} - \chi \frac{N_{t+i}^{1+\varphi}}{1+\varphi} \right] \tag{2.1}$$

where $\beta \in (0,1)$ is the discount rate, N_t the labor supply, $\chi > 0$ the relative utility weight of labor, φ the intertemporal estasticity of labor supply and C_t the consumption of final good. The households' budget constraint is:

$$C_t + \frac{D_t}{R_t} = D_{t-1} + \Gamma_t + W_t N_t - T_t \tag{2.2}$$

where D_t are one-period life deposits and public debt, R_t is the real gross interest rate, Γ_t are firms' profits, W_t is the real wage and T_t are lump sum taxes. The labor supply and the Euler equation are:

$$W_t = \chi C_t N_t^{\varphi} \tag{2.3}$$

$$E_t \Lambda_{t,t+1} R_t = 1 \tag{2.4}$$

with

$$\Lambda_{t,T} = \beta^{T-t} \frac{C_t}{C_T} \qquad \text{where} \qquad T = t+1 \tag{2.5}$$

Intermediate goods firms Intermediate goods producers obtain their output by using the capital acquired and the labor force hired from the households. These firms have a common production function that yields economic growth following Romer (1986):

$$Y_{jt}^i = e^{a_t} K_{jt}^{\alpha} \left(K_t N_{jt} \right)^{1-\alpha} \tag{2.6}$$

Y_{jt}^{i} being the production obtained by firm j with j pertaining to the interval [0,1], K_{jt} the capital stock and N_{jt} the labor used. The index $K_t = \int_0^1 K_{jt}dj$ represents the stock of knowledge generated by capital accumulation by all the intemediate producers, taken as given by firms. a_t is the AR(1) productivity shock common to all firms. Aggregating the firms' production functions and assuming that they are all identical:

$$Y_t^i = e^{a_t} K_t N_t^{1-\alpha} \tag{2.7}$$

Furthermore, goods producers fund capital purchases by issuing financial claims (S_t) at a price Q_t:

$$Q_t S_t = Q_t K_{t+1} \tag{2.8}$$

Denoting by P_t^i the price of the intermediate good and δ the depreciation rate, the wage that minimizes costs and the capital return (R_t^q) are:

$$W_t = P_t^i (1-\alpha) \frac{Y_t^i}{N_t} \tag{2.9}$$

$$E_t \left\{ R_{t+1}^q \right\} = \frac{\frac{P_{t+1}^i \alpha Y_{t+1}^i}{K_{t+1}} + Q_{t+1} - \delta}{Q_t} \tag{2.10}$$

Capital producers We define the capital accumulation, affected by adjustment costs, as follows:

$$K_{t+1} = K_t + I_t^n \tag{2.11}$$

where $I_t^n = I_t - \delta K_t$ is the net investment and I_t the gross investment. Capital producers refurbish depreciated capital adquired from intermediate goods producers and resell it, along with newly-created capital, to goods producers. The solution to the investment problem, common to all capital producers, leads to the relative price:

$$Q_t = 1 + f + \frac{I_t^{n,k} + I^k}{I_{t-1}^{n,k} + I^k} f' - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}^n + I^k}{I_t^n + I^k} \right)^2 f' \tag{2.12}$$

where $I_t^{n,k} = \frac{I_t^n}{K_t}$, $I^k = \frac{I}{K}$ is the value of gross investment relative to capital in the state steady and $\Lambda_{t,t+1}$ is the discount rate derived from the households' decision.

Retail firms These firms make use of intermediate goods, transforming them into differentiated final goods following the Dixit-Stiglitz technology. In order to include nominal rigidities, we follow the model of Calvo (1983). If we abandon the zero trend inflation assumption and define $X_t = \frac{P_t^*}{P_t}$ and $\frac{P_t}{P_{t+i}} = \frac{1}{\prod_{k=1}^{i} \Pi_{t+k}}$, P_t being the final good price index, P_t^* the optimal price in t and Π_t the gross inflation rate, the equations related to fixing prices are:

$$X_t = \mu \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k} \right)^{\epsilon} Y_{t+i} P_{t+i}^i}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k} \right)^{\epsilon - 1} Y_{t+i}} \tag{2.13}$$

$$X_t = \left[\frac{1 - \theta}{1 - \theta \Pi_t^{\epsilon - 1}} \right]^{\frac{1}{\epsilon - 1}} \tag{2.14}$$

where θ is the probability of not changing the price, $\mu = \frac{\epsilon}{\epsilon - 1}$ is the mark-up of the retailers and ϵ is the parameter of the aggregate Dixit-Stiglitz index.

Financial intermediaries The structure of our financial market follows the approach of Gertler and Karadi (2011). Financial intermediaries raise funds from households remunerated at R_{t+1} and lend them to intermediate goods firms yielding a rate R^q_{t+1}. Their objective function maximizes their expected wealth (V_{ft}):

$$V_{ft} = \max E_t \sum_{i=0}^{\infty} (1-\gamma)\gamma^i \Lambda_{t,t+1+i} \left[\left(R^q_{t+1+i} - R_{t+1+i} \right) Q_{t+i} S_{ft+i} + R_{t+1+i} F_{ft+i} \right] \quad (2.15)$$

where γ is the probability of survival of the bankers and F_{ft} is the net wealth held by intermediary f at the end of period t. Financial intermediaries provide funds if they do not obtain losses or, analogously, the discounted external finance premium is greater than zero. Furthermore, the bankers have the chance to divert a fraction λ of the disposable funds to their households. But, if this happens, the depositors could force the bankruptcy of the financial intermediary and rescue the proportion $(1-\lambda)$ of the available funds. Thus, the depositors are willing to lend their funds to the bankers whenever the following equation holds:

$$h_t F_{ft} + v_t Q_t S_{ft} \geq \lambda Q_t S_{ft} \quad (2.16)$$

where v_t is the marginal gain of bankers derived from expanding their assets $Q_t S_{ft}$ but maintaining the net wealth fixed and h_t is the expected value of having an additional unit of F_{ft}, supposing that S_{ft} remains constant:

$$v_t = E_t \left\{ (1-\gamma) \Lambda_{t,t+1} \left(R^q_{t+1} - R_{t+1} \right) + \Lambda_{t,t+1} \theta x_{t,t+1} v_{t+1} \right\} \quad (2.17)$$

$$h_t = E_t \left\{ (1-\gamma) + \Lambda_{t,t+1} \theta t_{t,t+1} h_{t+1} \right\} \tag{2.18}$$

where $x_{t,t+i} = \frac{Q_{t+i} S_{ft+i}}{Q_t S_{ft}}$ and $t_{t,t+i} = \frac{F_{ft+i}}{F_{ft}}$ are the growth rates of assets and wealth, respectively. Hence, the total funding that banks can obtain depends on their wealth:

$$Q_t S_{ft} = \frac{h_t}{\lambda - v_t} F_{ft} = \phi_t^p F_{ft} \tag{2.19}$$

with ϕ_t^p being the private leverage ratio. We define the banker's wealth as follows:

$$F_{ft+1} = \left[\left(R_{t+1}^q - R_{t+1} \right) \phi_t^p + R_{t+1} \right] F_{ft} \tag{2.20}$$

Given that the banks' total wealth is independent of firm-specific factors, we can aggregate it to obtain:

$$Q_t S_t = \phi_t^p F_t \tag{2.21}$$

Distinguishing between the wealth of the new (F_t^n) and the old (F_t^o) bankers, and given that the initial funds of the new bankers are defined as the ratio $\frac{\omega}{1-\gamma}$ of the old bankers' wealth $(1-\gamma) Q_t S_{t-1}$, the total wealth can be stated as:

$$F_t = F_t^o + F_t^n \tag{2.22}$$

where

$$F_t^o = \gamma \left[\left(R_t^q - R_t \right) \phi_{t-1}^p + R_t \right] F_{t-1} \tag{2.23}$$

$$F_t^n = \omega Q_t S_{t-1} \tag{2.24}$$

Central bank The central bank implements monetary policy by setting the short-term nominal interest rate (R_t^{st}) following a Taylor rule:

$$R_t^{st} = R_t^* \Pi_t^e \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y^*}\right)^{\phi_y} \exp\left(\eta_t\right) \tag{2.25}$$

where R_t^* is the time-varying natural rate of interest estimated by the central bank, Π_t^e is the expected inflation rate, Π^* is the steady-state gross inflation target, Y^* is the natural level of output consistent with Π^*, and ϕ_π, ϕ_y are the parameters that measure the central bank's reaction to inflation and output deviations from its target levels, respectively. The monetary policy shock η_t is an AR(1) process. Moreover, the relationship between real and nominal interest rates is defined by the Fisher equation $R_t^{st} = R_t E_t \Pi_{t+1}$.

We also contemplate the possibility that the central bank intermediates a small part of the total claims (S_t^{cb}) at price Q_t carrying efficiency costs τ. In order to simplify the model dynamics, but without ignoring the effects that this kind of intermediation could generate on the natural rate of interest, we consider that this proportion (ψ) is fixed and corresponds to the steady-state value. This lending mechanism is supported by taxes and by issuing riskless debt acquired by households which is remunerated at the rate R_t, and yields R_t^q. Hence, the structure of the financial intermediation of the economy is fully described by the equation:

$$Q_t S_t = \phi_t^p F_t + \psi Q_t S_t = \phi_t^T F_t \tag{2.26}$$

with $\phi_t^T = \frac{1}{1-\psi}\phi_t^p$ being the leverage ratio of the economy.

Equilibrium The aggregate equilibrium of the economy is given by:

$$Y_t = C_t + I_t + f\left(\frac{I_t^{n,k} + I^k}{I_{t-1}^{n,k} + I^k}\right)(I_t^n + I) + \tau\psi Q_t K_{t+1} \tag{2.27}$$

Finally, the total output of the economy is equivalent to the intermediate goods firms' output weighted by the inverse of the price dispersion of the retailers (Δ_t):

$$Y_t = Y_t^i \Delta_t^{-1} \tag{2.28}$$

$$\Delta_{t+1} = \theta \Pi_{t+1}^{\epsilon} \Delta_t + (1-\theta) X_{t+1}^{-\epsilon} \tag{2.29}$$

2.3 The natural rate of interest

In this section we introduce the equations that characterize the steady state. Then, we present the definition of the natural rate of interest that emerges from the model developed, carrying out a sensitivity analysis in order to evaluate the shifts in this rate induced by changes in the financial parameters.

2.3.1 Steady-state equations

First of all, in order to find a well-defined endogenous and stable steady state, we have to normalize the growing variables. We choose the capital stock as the normalization variable because it is the source of economic growth, whose gross growth rate is defined as $G_t = \frac{K_t}{K_{t-1}}$. All the normalized variables has the superscript k, whilst the variables evaluated in the steady state have no subscript. Below we show the main equations distinguishing among the different blocks of the model.

From households, intermediate goods producers, capital producers and retail firms, we can obtain the following expressions:

$$\chi C^k N^\varphi = (1-\alpha)\frac{X}{\mu}\frac{1-\theta\beta\Pi^\epsilon}{1-\theta\beta\Pi^{\epsilon-1}}N^{-\alpha} \tag{2.30}$$

$$\frac{1}{R} = \frac{\beta}{G} \tag{2.31}$$

$$R^q = P^i \alpha N^{1-\alpha} + 1 - \delta \tag{2.32}$$

$$P^i = \frac{X\left(1-\theta\beta\Pi^\epsilon\right)}{\mu\left(1-\theta\beta\Pi^{\epsilon-1}\right)} \tag{2.33}$$

$$X = \left[\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right]^{\frac{1}{\epsilon-1}} \tag{2.34}$$

From financial intermediaries:

$$v = (1-\gamma)\,\Lambda\,(R^q - R) + \gamma\Lambda t v \tag{2.35}$$

$$h = (1-\gamma) + \gamma\Lambda t h \tag{2.36}$$

$$\phi^p = \frac{h}{\lambda - v} \tag{2.37}$$

$$t = (R^q - R)\,\phi^p + R \tag{2.38}$$

$$G = \frac{1}{1-\psi}\phi^p F^k \tag{2.39}$$

$$F^k = \gamma\left[(R^q - R)\,\phi^p + R\right] F^k + (1-\gamma)\,\omega \tag{2.40}$$

And, from the equilibrium of the economy:

$$\frac{N^{1-\alpha}\,(1-\theta\Pi^{\epsilon})}{(1-\theta)\,X^{-\epsilon}} = C^k + G - (1-\delta) + \tau\psi G \tag{2.41}$$

In order to analyze the consequences of the incorrect estimate of the natural rate of interest by the central bank, we contemplate the possibility of $R \neq R^*$, where R is the endogenous natural rate of interest. Thus, the Taylor rule in the long run is defined as follows:

$$R^n = R^* \Pi \left(\frac{\Pi}{\Pi^*} \right)^{\phi_\pi} \tag{2.42}$$

To derive this expression, we have made use of the equivalence in the steady state $\Pi^e = \Pi$. Also, we have assumed $Y = Y^*$ but, for reasons given below, we can no longer assume that $\Pi = \Pi^*$. We should note that, if $R = R^*$, this expression is reduced to the standard relationship in the steady state between real and nominal interest rate, i.e. $R^n = R\Pi$.

Substituting the left-hand side of (2.42) by the Fisher equation:

$$R = R^* \left(\frac{\Pi}{\Pi^*} \right)^{\phi_\pi} \tag{2.43}$$

Hence, the natural rate of interest in the steady state depends on the effective inflation rate, but also on the targets (R^*, Π^*). Rearranging terms:

$$\Pi = \Pi^* \left(\frac{R}{R^*} \right)^{\frac{1}{\phi_\pi}} \tag{2.44}$$

Thus, we have endogeneized the effective inflation rate in the steady state, which is now related to the inflation target and the real interest rate deviation weighted by the parameter of reaction to deviations in the inflation rate included in the Taylor rule. This expression, in logs, coincides with that indicated by Laubach and Williams (2003):

$$\pi = \pi^* + \frac{1}{\phi_\pi} \left(r - r^* \right) \tag{2.45}$$

Therefore, the long-run endogenous inflation rate and the target rate will match if and only if the central bank correctly estimates the natural rate of interest. This finding is in accordance with the conclusion of Tristani (2009). From (2.44) and (2.31), we obtain the expression:

$$\Pi = \Pi^* \left(\frac{G}{\beta R^*} \right)^{\frac{1}{\phi_\pi}} \tag{2.46}$$

So the effective steady-state inflation rate depends on the target of both inflation and the natural rate of interest, as well as on the long-term growth.

If we draw on (2.30) and (2.41), we can solve for G:

$$G = \frac{1}{1+\tau\psi} \left[\frac{1-\theta\Pi^\epsilon}{(1-\theta) X^{-\epsilon}} N^{1-\alpha} - \frac{X(1-\alpha)\Psi}{\chi\mu\Upsilon} N^{-\varphi-\alpha} + (1-\delta) \right] \tag{2.47}$$

where $\Upsilon = (1-\theta\beta\Pi^{\epsilon-1})$ and $\Psi = (1-\theta\beta\Pi^\epsilon)$.

Furthermore, from (2.35)-(2.38) we obtain that:

$$(R^q - R) = \frac{\lambda(1-\gamma) - v}{\lambda - v} \frac{Rv}{(1-\gamma)} \tag{2.48}$$

Now, relying on (2.31), (2.37), (2.39) and (2.40):

$$G = \frac{(1-\gamma)\omega v R}{(1-\psi)(R^q - R)(\lambda - v)\left[1 - \gamma\frac{\lambda R}{\lambda - v}\right]} \tag{2.49}$$

Replacing (2.48) in (2.49):

$$G = \frac{(1-\gamma)^2 \omega}{(1-\psi)\left[\lambda(1-\gamma)-v\right]\left[1-\frac{G}{\beta}\frac{\gamma\lambda}{\lambda-v}\right]} \tag{2.50}$$

From (2.31), and (2.32):

$$(R^q - R) = \frac{\alpha X \Psi}{\mu \Upsilon} N^{1-\alpha} + (1-\delta) - R \tag{2.51}$$

Equating (2.48) and (2.51) taking into account (2.31):

$$N = \left\{ \frac{\mu\Upsilon}{\alpha X \Psi} \left[\left(1 + \frac{\left[\lambda(1-\gamma)-v\right]v}{(\lambda - v)}\right) \frac{G}{\beta} - (1-\delta) \right] \right\}^{\frac{1}{1-\alpha}} \tag{2.52}$$

And, introducing the last expression into (2.47):

$$G = \frac{1}{1+\tau\psi} \left\{ \frac{\mu\Upsilon\Theta}{\alpha X\Psi} \left[\frac{1-\theta\Pi^{\epsilon}}{(1-\theta)X^{-\epsilon}} - \frac{(1-\alpha)X\Psi}{\chi\mu\Upsilon} \left(\frac{\mu\Upsilon\Theta}{\alpha\Psi X}\right)^{\frac{-(1+\varphi)}{1-\alpha}} \right] + (1-\delta) \right\} \tag{2.53}$$

where $\Theta = \left[\left(1 + \frac{[\lambda(1-\gamma)-v]v}{(1-\gamma)(\lambda-v)}\right)\frac{G}{\beta} - (1-\delta)\right]$.

Summing up, the steady state is determined by (2.50) and (2.53), along with the expressions of the relative optimal price (2.34) and the effective inflation rate in the steady state (2.46). These equations indicate that the equilibrium is determined by the effective steady-state inflation rate, which, in turn, coincides with its target if the central bank correctly estimates of the natural rate. However, if $R \neq R^*$, the long-term inflation rate is determined not only by its target in the monetary policy rule but also by the error in the estimation of the natural rate of interest through relationship (2.44). So, if this constraint holds, the deviation in the estimation of the natural rate of interest will be non-neutral in the long run, a key outcome in our analysis. In Section 5, we delve into this question.

2.3.2 Calibration

The parameter values used in the calibration of the model are reported in Table 1.1 and correspond to the baseline model in Chapter 1. The values of most conventional parameters are standard in the literature, the policy and financial parameter values are taken from Gertler and Karadi (2011), whilst the value of χ is set so that the values of the long-run annual growth rate (2.5%), the annualized external finance premium (1.17%) and the leverage ratio (4.8) are admissible for an annual inflation rate of 2.5%. Using these values, we find the solution of equations (2.50) and (2.53) in the plane $\{v, G\}$ once the expressions of X and Π are substituted from (2.34) and (2.46).

2.3.3 Definition of the natural rate of interest

According to (2.31), the endogenous natural rate of interest depends on the long-run growth rate of the economy[13], which, in turn, depends on the financial parameters from (2.50) and on the effective steady-state inflation rate from (2.53). From these three expressions we can write:

$$R = \frac{(1-\gamma)^2 \omega}{(1-\psi)\left[\lambda(1-\gamma) - v\right]\left[\beta - \frac{G\gamma\lambda}{\lambda - v}\right]} = \tag{2.54}$$

[13] This result contradicts the finding of Weber, Lemke and Worms (2008), who argue that there is no a desirable level for the natural rate. Clark and Kozicki (2005) also empirically analyze the link between this rate and the long-run growth rate and conclude that it is weak.

$$= \frac{1}{\beta(1-\tau\psi)} \left\{ \frac{\mu\Upsilon\Theta}{\alpha X\Psi} \left[\frac{1-\theta\Pi^{\epsilon}}{(1-\theta)\,X^{-\epsilon}} - \frac{(1-\alpha)\,X\Psi}{\chi\mu\Upsilon} \left(\frac{\mu\Upsilon\Theta}{\alpha\Psi X} \right)^{\frac{-(1+\varphi)}{1-\alpha}} \right] + (1-\delta) \right\}$$

Hence, in this type of models with financial frictions, the natural rate of interest depends on the structure of the financial sector[14] as in the definition proposed by De Fiore and Tristani (2011), even assuming that the central bank correctly estimates it. Moreover, if the central bank does not match the long-term interest rate estimate with the endogenous one, this rate will also depend on the monetary policy parameters (Π^*, R^*, ϕ_π) because the efective long-run inflation is a function of these values as has been shown in (2.46). This finding leads us to maintain that, assuming $R \neq R^*$, the deviations affect the value of the effective natural rate and the steady-state inflation rate.

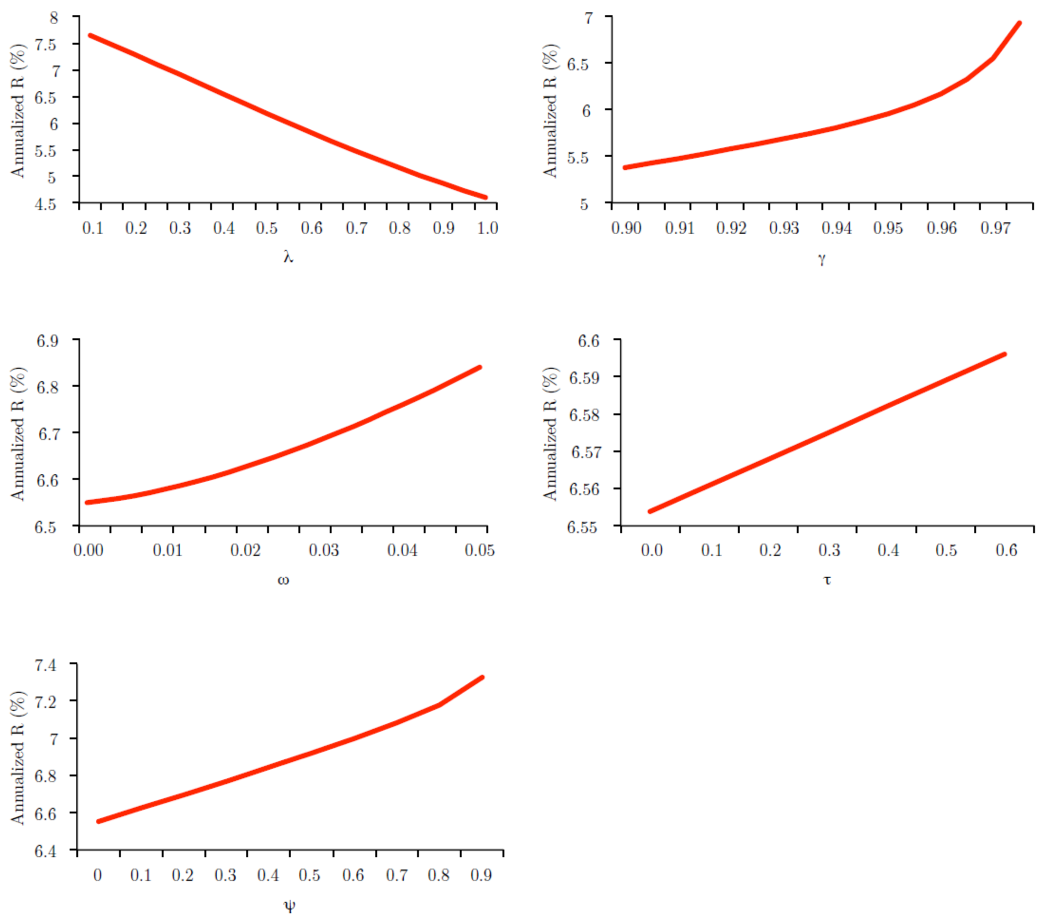
We now explore the relationships among the financial parameters and the natural rate of interest assuming $R = R^*$ since differences among alternative scenarios in terms the magnitude are mild. The natural rate of interest and the steady-state growth rate are positively related, so all parameter changes that increase the natural rate of interest also stimulate the long-term growth rate.

First, we analyze the effects of shifts in the financial parameters λ,γ, ω and, then, the effects of variations in the steady-state unconventional monetary policy parameters τ,ψ. Figure 2.1 displays the results for the annualized natural rate of interest. The first plot shows how the natural rate drops when the funds that bankers can divert increase. This means that, when the financial system profits can be diverted away and not reinvested, the trend of the deposits' yield offered by financial intermediaries is smaller. Meanwhile,

[14] In line with this result, previous works such as Borio, Disyatat and Juselius (2013) support the inclusion of financial variables in the definition of long-run references such as the potential output.

the effects of increments in the survival rate of the bankers and in the funds with which the new bankers start on the natural rate of interest are positive. The lower the rotation of the intermediaries in the financial system and the more funds the new financial intermediaries own, the higher the risk-free rate of return in the steady state. The most prominent result is perhaps the effect of a change in γ, given that it indicates that the renovation of the intermediaries is not positive for the natural interest rate and, consequently, for the long-run economic growth.

Figure 2.1: Endogenous natural rate of interest and financial parameters if $\Pi = 2.5\%$



As regards the unconventional monetary policy parameters, both efficiency costs and the proportion of the credit policy in the steady state increase the endogenous natural rate of interest. The first parameter weakly affects the natural rate of interest, while the more funds the central bank intermediates in the steady state, the greater the natural rate of interest.

2.3.4 Stabilization

As discussed above, some previous work, such as Orphanides and Williams (2002), reveals that the cost of underestimating the natural rate of interest in terms of economic stabilization is greater than the cost of overestimating it. In order to assess the stabilization costs of potential estimation errors of the natural rate of interest, we have submitted the model to the standard types of perturbations, a monetary policy shock and a technological shock. We have considered three possible scenarios corresponding to $R < R^*$, $R = R^*$ and $R > R^*$. Under the two kind of shocks, the responses of the variable do not depend on the accuracy of the estimation of the natural rate[15]. Therefore, our results do not support the finding of Orphanides and Williams (2002), because the effects of the errors are undifferentiated, at least for this sort of perturbations. Nevertheless, this does not mean that these effects are not significant on the trend, since the values around which these deviations are obtained change depending on the different assumptions about the errors. We delve into this issue in the next section.

2.4 Steady-state effects

In this section we evaluate the impact of errors in the estimation of the natural rate of interest on the trend of the economy, focusing on the effects on economic growth

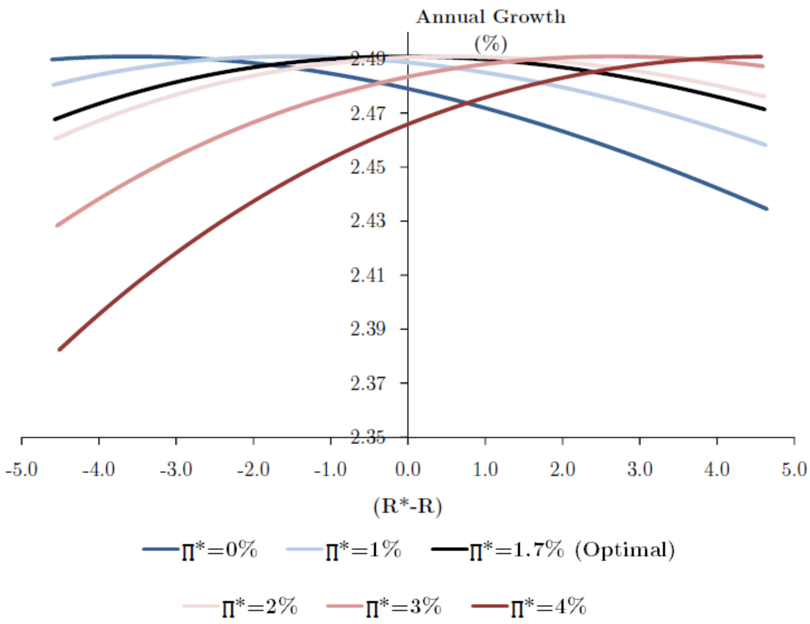
[15] We do not display the plots because the response variable is standard and indistinguishable among scenarios.

and examining their impact on the marginal gain of banks and on the gap between the effective and the targeted inflation. To obtain a complete perspective, we will asses the equilibrium value of our key variables for a wide range of the differential $(R^* - R)$ and the inflation rate target. We should note that the value of R is endogenously fixed by (2.31), so we modify the value of the interest rate differential by adjusting the interest rate targeted by the central bank.

Firstly, we analyze the effect of the differential $(R^* - R)$ on the long-run economic growth rate. As shown in Figure 2.2, where all variables are expressed in annualized rates, the behavior depends on Π^*. We can distinguish two differentiated types of effects. The first for values of Π^* below 1.7% (the rate which provides the maximum steady-state growth, denoted as Π^{op} and represented by the black line) and the second for higher rates. The cause of the existence of this specific threshold is evidenced in Chapter 1, since $\Pi^* = \Pi^{op} = 1.7\%$ is the rate at which the long-run growth rate is maximized in the absence of estimation errors, as shown in the vertical axis points (when $R^* = R$). In the particular case $\Pi^* = \Pi^{op}$, the behavior is symmetrical around the maximum for the two scenarios $R < R^*$ and $R > R^*$.

When the inflation rate target is below Π^{op}, economic growth decreases when $R > R^*$, so the overestimation of the natural interest rate (right-hand side of the graph) discourages the long-run growth rate. For example, if $\Pi^* = 0\%$ and R^* exceeds the natural rate by 200 basis points, equilibrium growth decreases more than 0.015 percentage points with respect to the scenario $R^* = R$. This long-term conclusion under the assumption $\Pi^* = 0$ is in line with the short-term finding of Laubach and Williams (2003), who defined the monetary policy as contractionary when the real rate of interest is above the natural rate.

Figure 2.2: Annual growth and interest rate differential

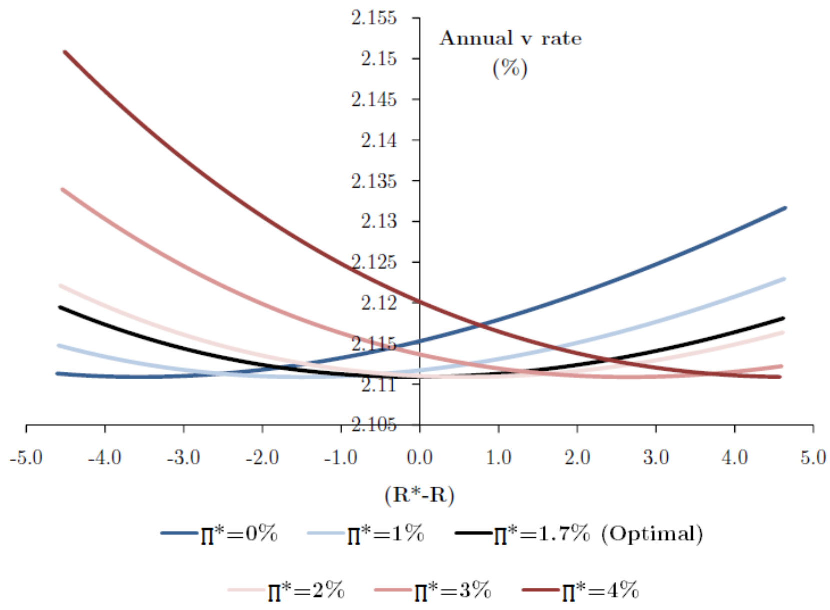


On the other hand, if $\Pi^* > \Pi^{op}$, the underestimation of the natural rate of interest has a negative effect on the long-term growth rate. If $\Pi^* = 4\%$, the growth rate decreases 0.03 percentage points when R^* is 200 basis points below R.

It is noteworthy that such differences are accentuated the farther away the central bank is from the optimal inflation.

However, we underline that, when the natural rate of interest is underestimated and $\Pi^* < \Pi^{op}$ or is overestimated and $\Pi^* > \Pi^{op}$, the conclusions are not univocal. In these scenarios the relationship between $(R^* - R)$ and G is hump-shaped. This can be explained by the fact that, for any inflation target, when $R^* \neq R$ there is an attainable maximum value for the long-run growth rate, which is almost identical to that maximum corresponding to the case of $R^* = R$ and $\Pi^* = \Pi^{op}$ but slightly lower. These maximum steady-state growth points are located on the left-hand side where $R^* < R$ for $\Pi^* < \Pi^{op}$ and on the right-hand side where $R^* > R$ for $\Pi^* > \Pi^{op}$. Moreover, the relationship between $(R^* - R)$ and G approximates the scenario $R^* = R$ the closer the inflation target is to the optimal rate.

Figure 2.3: Marginal gain rate and interest rate differential

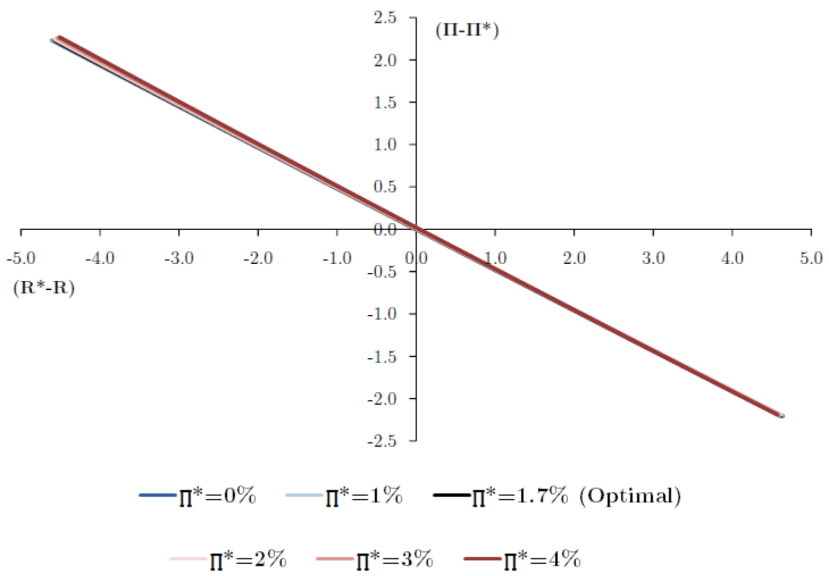


Secondly, in Figure 2.3, we explore the relationship between the marginal gain of the financial intermediaries from expanding their assets and the interest rate differential $(R^* - R)$. As in the previous exercise, the results depend on the inflation target although, in this case, the direction is the opposite due to the negative relationship[16] between G and v. When the inflation target does not overcome threshold Π^{op}, the expected earnings of the financial system, which depend on the steady-state external finance premium $(R^* - R)$, rise if $R^* > R$. Contrarily, when $\Pi^* > \Pi^{op}$, the financial gain increases if $R^* < R$ and, for the particular case of $\Pi^* = \Pi^{op}$, the behavior is symmetric around the minimum when $R^* = R$. Analogously to the case of G, for the scenarios $(\Pi^* < \Pi^{op}, R^* < R)$ and $(\Pi^* > \Pi^{op}, R^* > R)$, the relationship has a mixed behavior because it is U-shaped.

[16] This relationship is non-linear although, for reasonable levels of trend inflation, it is negative.

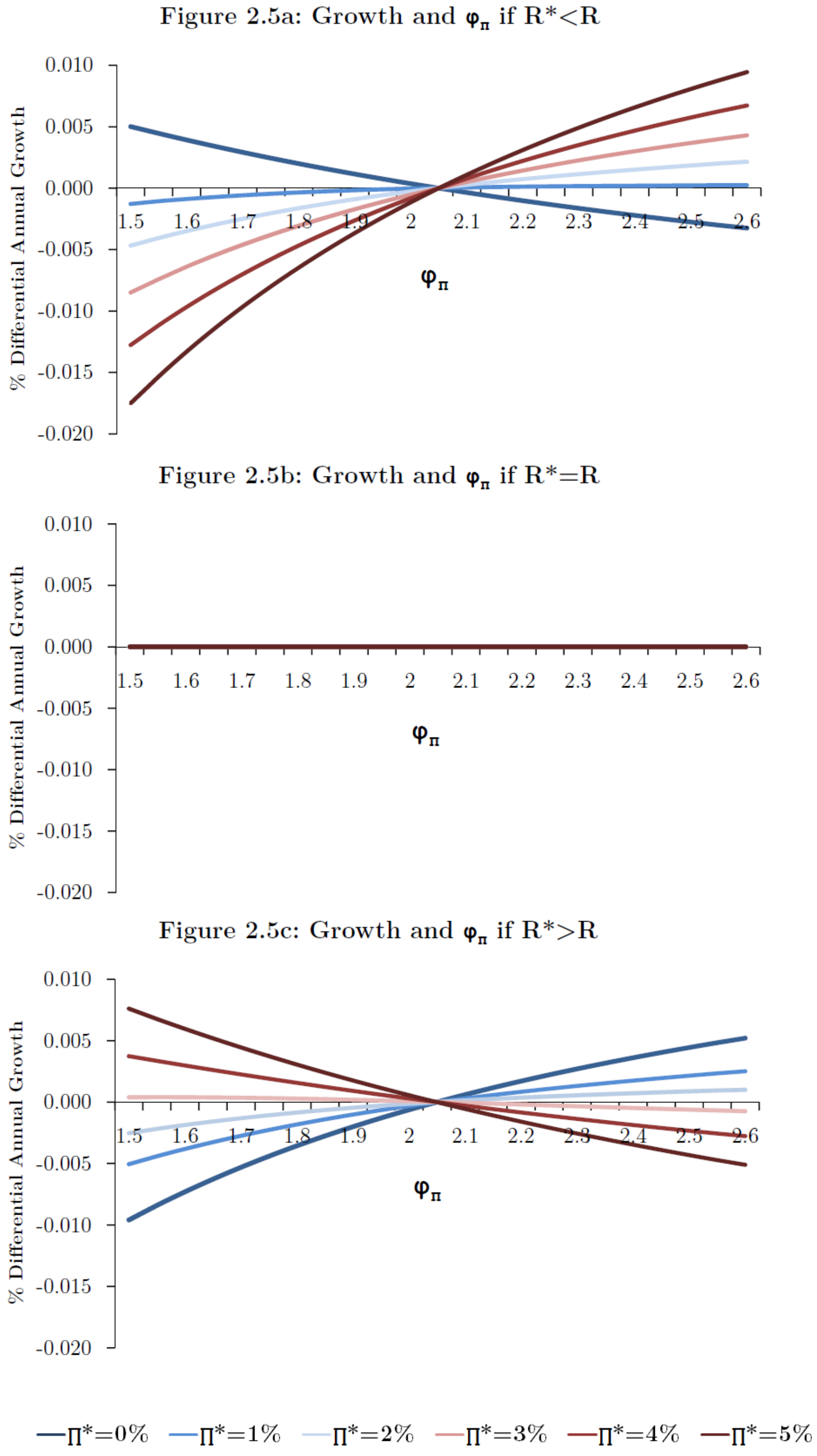
Finally, the inflation rate differential is studied in Figure 2.4. In this case, Π^* is fixed and set by the monetary authority whilst Π is endogenously determined by (2.46). When $R^* = R$, the differential is zero, when $R^* < R$, it is positive and, when $R^* > R$, it is negative. Thus, the differentials $(\Pi - \Pi^*)$ and $(R^* - R)$ have a negative linear relationship across the whole range, disclosing a very slight difference for any Π^*.

Figure 2.4: Inflation differential and interest rate differential



The interpretation of these results suggests that, when the inflation target is below the optimal rate, a lax monetary policy that reduces R^* stimulates the long-run growth rate because the effective inflation rate in the steady state approaches the optimal inflation rate up to a threshold value. By contrast, for high levels of the inflation target $\Pi^* > \Pi^{op}$, a contractionary monetary policy $(R^* > R)$ leads to an actual long-term inflation closer to the optimum, again up to a limit, so the growth rate increases. Therefore, if the central bank wants to stimulate the long-run growth, monetary policy must always reduce the long-term inflation gap $|\Pi^{op} - \Pi^*|$. This can be implemented in two different ways. The first, as we have seen, by modifying the estimate of the natural rate of interest. This policy will not be useful when $R^* < R$ and $\Pi^* < \Pi^{op}$ or when $R^* > R$ and $\Pi^* > \Pi^{op}$.

Figure 2.5: Growth and ϕ_π



But the objective of increasing the long-run growth rate can also be reached through the parameter of the Taylor rule that reacts to deviations of inflation, ϕ_π. Although the central bank interprets this parameter as a regulator of the short and medium-term nominal interest rate, if the estimated natural rate of interest does not match the endogenous one, it will also influence the long-run growth rate according to (2.46), which is a prominent result of our work. In order to analyze this relationship, we now again assume three scenarios, $R^* < R$, $R^* = R$ and $R^* > R$, setting the difference between interest rates $(R^* - R)$ at 200 basis points above and below zero and changing the value of the parameter ϕ_π to observe the effects. As shown in Figure 2.5, the effects of shifts in ϕ_π depend on both the inflation target and the differential $(R^* - R)$. If $R^* < R$, an increase in ϕ_π triggers a reduction of the long-run growth rate if and only if the inflation target is below 1%. However, if the inflation target is greater than 1%, the relationship is positive. If, instead, the central bank sets a long-run interest rate equal to the endogenous value, there are no differences in the growth rate for any variation of ϕ_π regardless of the value of Π^*. But, if $R^* > R$, the long-run growth rate decreases with ϕ_π for inflation targets higher than 3% and increases with lower ones.

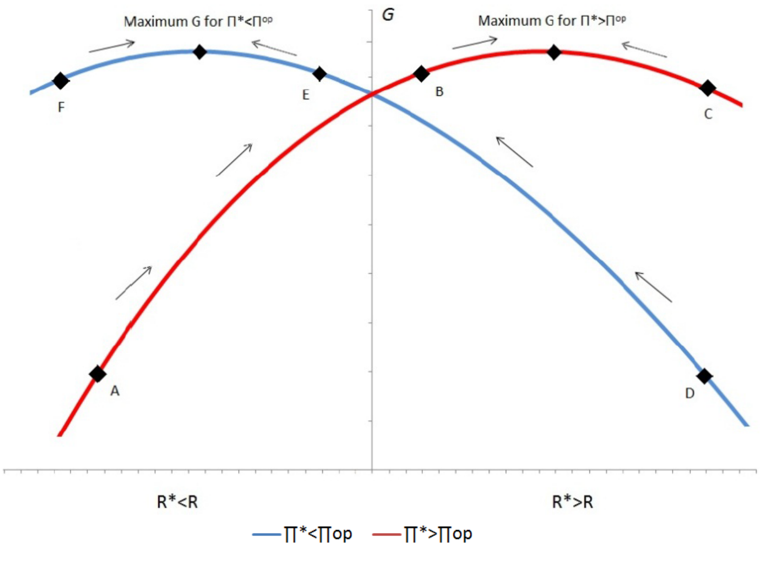
This section contains a wealthy set of conclusions about the many different situations that could be happening in the long run. The necessary information to know where an economy is situated will be crucial for reaching the objectives of the monetary policy. In the following section we use these results in order to identify where is the economy in a given point of time.

2.5 Verification

Although the exact knowledge of the natural rate of interest in real time is a challenging task, some of the conclusions drawn from our previous analysis could assist the monetary authorities in this objective. With the exercise we propose in this section, the central bank would be able to find out both its monetary policy stance regarding the inflation rate that maximizes long-run growth and the accuracy of the estimate of the natural rate of interest. In this way, monetary authorities could monitor the performance of monetary policy in relation to long-term growth.

To clarify the iterative mechanism we have designed, in Figure 2.6 we propose a stylized outline of Figure 2.2.

Figure 2.6: Verification process. The long run



Let us suppose that the central bank is located at an unknown point of Figure 2.6 in which the only known long-run variables are the targets included in the monetary policy rule (Π^*, R^*). There are two possible strategies; the first is the aim of maximizing the long-term growth rate and the second widens this goal by including the correct estimate of the natural rate of interest. Below, we describe the steps required to achieve both goals. We study the changes in the steady state that would generate the modification of some policy tools and, then, we provide some guidelines for interpreting the transition path in the short term.

Maximizing G

The success of the first strategy is very straightforward to achieve. The central bank only has to move the estimate of the natural rate of interest until the long-run growth rate stops increasing. Assuming[17] that $\Pi^* \neq \Pi^{op}$, there are six possible types of initial points (A, B, C, D, E, F), marked on Figure 2.6, and two optimal final points depending on whether $\Pi^* < \Pi^{op}$ or $\Pi^* > \Pi^{op}$. The direction of the correct shifts of R^* depends on the position of the initial point with respect to the global maximum located at a specific value of $(R^* - R)$, which differs for each of the displayed inflation targets. As the central bank does not possess this information, the first movement tests the proper adjustment of R^* and the subsequent changes iteratively modify R^* up to the maximum long-run growth rate in the direction indicated by the arrows. To illustrate this method, let us assume that the initial point is $(\Pi^* = 0\%; R^* = 4.5\%)$. The aim of the central bank is to maximize G, but it does not know either Π^{op} or R. Keeping Π^* constant, the first random move of the policymaker might be to increase R^*, but this would bring down G. Hence, the central bank has ascertained that it has to reduce R^* in order to increase G, though it still does not know if the starting point was C, D or E. This process will make G reach its maximum at the point $(\Pi^* = 0\%; R^* = 3\%)$.

This exercise would be simple if G were known. Unfortunately, monetary authorities

[17] Otherwise the procedure would be the same.

do not know the long-term growth rate exactly, but they perform estimates based on provisional information. Therefore, we now relax the assumption of the correct knowing[18] of G and we suppose that the central bank can only perceive the direction of changes in G_t, that is, the short-run growth rate. Although some authors such as Orphanides and Van Norden (2002) have criticized the confidence of real-time calculations, we consider that this assumption is not unreasonable.

Above, we have computed the steady-state values of the endogenous variables for the different levels of $(R^* - R)$, so the exercise now builds on the evaluation of the transition path between scenarios in the absence of other shocks. In order to do this, we propose a simulation in which a permanent change in the estimate of the natural rate of interest is imposed. Thus, our model loses its stochastic quality, which is explained by the anticipated nature of this kind of alteration of the model.

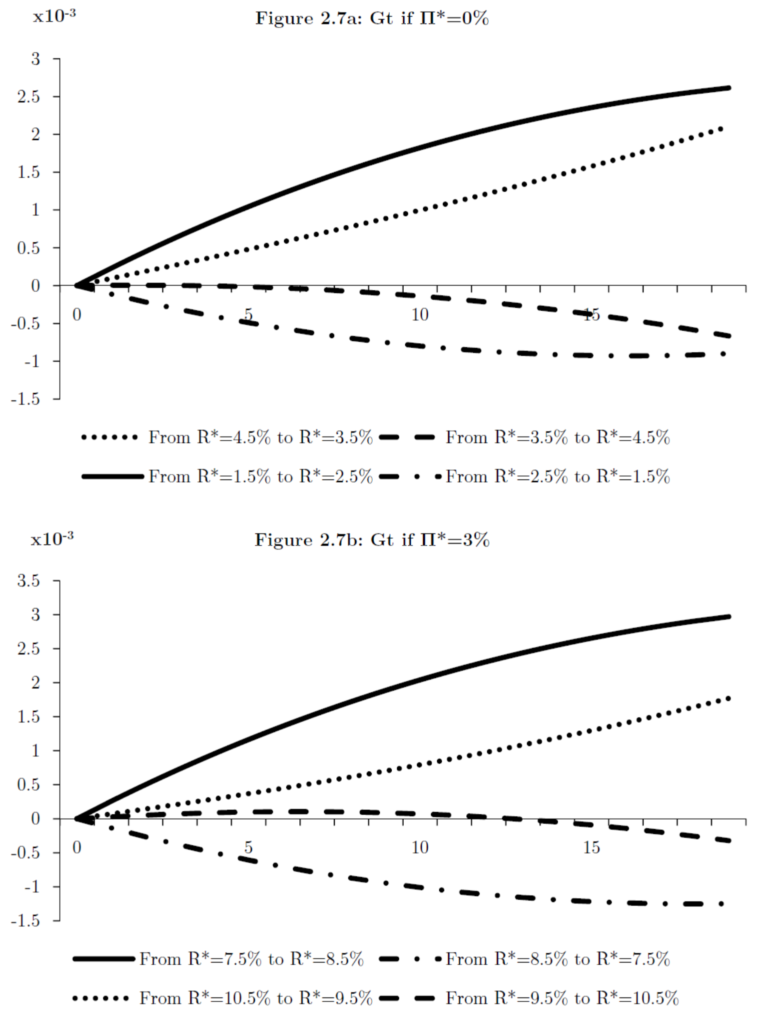
As in the previous example, we suppose that the initial target is $\Pi^* = 0\%$, the case of the blue line in Figure 2.6. Figure 2.7a displays the response of G_t across 20 quarters to a change in R^*. As expected, when the central bank reduces the intercept in the Taylor rule from 4.5% to 3.5%, the short-term growth rate increases. Conversely, if the central bank raises the natural rate estimate from 3.5% to 4.5%, the growth rate diminishes after some periods. However, this consequence is valid if and only if the estimate of the natural rate is above the rate that ensures the maximum long-run growth rate (points[19] C, D and E in Figure 2.6), which corresponds to $R^* = 3\%$ when $\Pi^* = 0\%$. If, for instance, the central bank raises the estimate from $R^* = 1.5\%$ to $R^* = 2.5\%$ (point F), G_t does

[18] Much academic literature has empirically addressed related issues with different perspectives, such as the estimation of the potential output and its trend, as in Edge *et al.* (2007). There are some earlier works, such as Kuttner (1994), that suppose this rate is constant, but later works suppose a time-varying rate as in Laubach and Williams (2003), who jointly estimate the natural rate of interest and the trend growth rate.

[19] We include point C because the central bank does not know Π^{op}.

not decrease but, as can be seen in the solid line in Figure 2.7a, increases. Analogously, a change from $R^* = 2.5\%$ to $R^* = 1.5\%$ causes a drop in G_t.

Figure 2.7: Verification process. The short run I



% Deviation from the steady state

The conclusions are the same if the target inflation rate exceeds the optimal rate. Figure 2.7b shows the transition path of the short-term growth rate between steady states if $\Pi^* = 3\%$. As in the previous simulation, G_t increases when the central bank approximates to the estimate of the natural rate that maximizes G (in this case $R^* = 9\%$) and decreases otherwise.

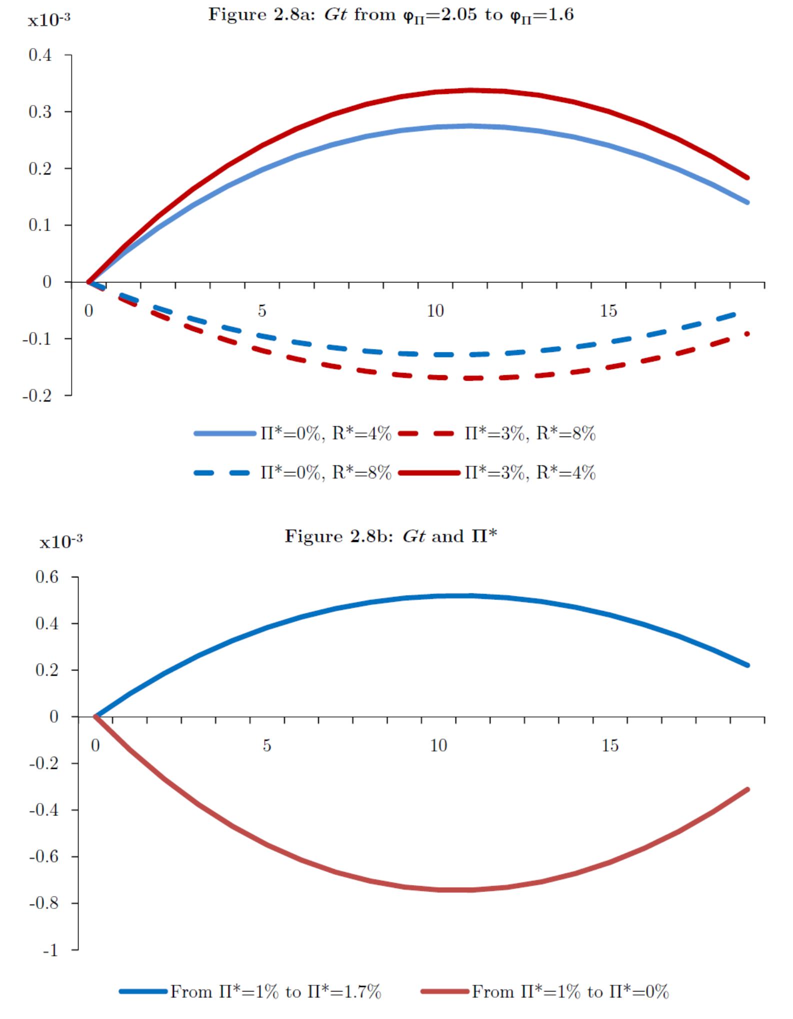
Therefore, even if both the optimal inflation rate and the natural rate of interest were unknown, central banks would have a tool that allows them to maximize the long-term growth rate through the modification of the intercept in the Taylor rule.

Maximizing G and reaching $R^* = R$

The second strategy, which includes the reaching of the maximum long-term growth and the correct estimate of R, requires a multiple-step process. Firstly, the policymaker has to move ϕ_π to assess the sign of the differential $(R^* - R)$. Figure 2.8a plots the responses of the short-term growth rate to a decrease in ϕ_π from $\phi_\pi = 2.05$ to $\phi_\pi = 1.6$ for several combinations of Π^* and R^*. Figure 2.5 showed that the shifts in the long-run growth rate from a modification in ϕ_π depend on both $(R^* - R)$ and $(\Pi^* - \Pi^{op})$. However, in the short-term horizon, G_t rises when ϕ_π decreases if and only if $R^* < R$ whatever the value of Π^* and vice versa. Given that R approaches 6.5% for both levels of Π^* considered, the state $R^* < R$ is represented in Figure 2.8a by the solid lines and $R^* > R$ by dashed lines[20]. The justification is based on that, as long as ϕ_π penalizes deviations in Π_t from Π^*, its reduction means a more expansive policy which enhances the short-term growth rate if and only if the intercept set in the Taylor rule does not constrain the monetary policy or, equivalently, if $R^* < R$. Thus, if G_t rises when ϕ_π increases, the central bank can discard, following Figure 2.6, points B, C and D. At this point, the central bank has the information derived from the first step of this mechanism about the sign of $(R^* - R)$. Once this information is known, policymakers have to move R^* until arriving to the $R^* = R$ scenario, that is, when G_t does not change when ϕ_π varies. In our example, where $\Pi^* = 0\%$, this happens when $R^* = R = 6.5\%$.

[20]For the sake of clarity, we have shown only these cases, although the behavior described holds for all other scenarios.

Figure 2.8: Verification process. The short run II



% Deviation from the steady state

If the central bank additionally wants to maximize G, shifts in Π^* will reveal the sign of $(\Pi^* - \Pi^{op})$. As shown in Figure 2.8b, and supposing that the starting inflation target is 1%, if the central bank lowers Π^* to zero and G_t decreases, this means that $\Pi^* < \Pi^{op}$, so the achieving of the double goal involves the increase of Π^* until G_t stops growing, that is, when $\Pi^* = 1.7\%$.

Hence, policymakers have all the information required to approach the desirable long-run equilibrium. The possibilities shown in this exercise can, therefore, provide the central banks with enough information to achieve their goals and to adjust the precision of its estimate of the natural rate of interest.

2.6 Conclusions

In this chapter we have analyzed, from a monetary policy point of view, the role of the natural rate of interest in a New Keynesian model with endogenous growth, financial frictions and trend inflation. First, we have introduced the DSGE model and we have delved into the steady state obtaining a definition of the natural interest rate. We have shown that this rate depends on the financial structure and have carried out a sensitivity analysis to determine the nature of this dependence. Furthermore, if the central bank does not use the correct natural rate of interest in the monetary policy rule, the long-term inflation rate does not match the target. As a result, the natural rate of interest becomes dependent on the monetary policy in two ways, since it is sensitive to its own estimate error and to the inflation target.

As regards the short-term economic stabilization, we have analyzed a monetary and a technology shock and have found that the errors in the natural rate used in the monetary policy rule do not affect for the responses of the variables and that there is no asymmetry between under- and over-estimation.

Moreover, we have determined the relationship between the potential errors in the estimation of the natural rate and the long-run growth rate. We have proved that this relationship depends on the inflation target because, if it is below (above) the optimal long-run inflation rate, long-term growth decreases when the estimation of the natural rate is above (below) the endogenous one. The opposite happens with the relationship between the marginal gain for expanding the assets of the banks and the error in the estimation. However, in the other possible scenarios, the relationship is hump-shaped, so the conclusions are not univocal. Furthermore, the parameter included in the Taylor rule that reacts to deviations of inflation in the short term will also generate shifts in the long-run growth if the natural rate estimate is not the endogenous one. Therefore, we can conclude that, according to these results, the errors in the estimation of the natural rate of interest are not neutral in the long run.

Finally, on the basis of our results, we have developed a mechanism to verify the stance of the monetary policy with respect to the optimal inflation from the growth perspective and the accuracy of the natural rate estimation, which can lead to maximizing the long-run growth. The central bank has to find out the sign of the differential between the estimated and the endogenous natural rate of interest as well as the position of the inflation target with respect to the rate that maximize the long-run growth. The monetary authority first makes use of the parameter that reacts to inflation deviations in the monetary policy rule. Then, by modifying the intercept of the Taylor rule and the inflation target, the central bank will be able to determine the position of both the inflation target with respect to the optimal rate and the estimated natural rate of interest with respect to the endogenous one.

Chapter 3

Non-linear Effects of the U.S. Monetary Policy in the Long Run

3.1 Introduction

The existence of rigidities and frictions in the markets leads to non-neutral monetary policies in the long run and non-linear effects of some key variables according to New Keynesian dynamic models with endogenous growth (Chapters 1 and 2). These two features are especially clear when monetary policy is conducted following some type of inflation targeting using the short-term interest rate as the instrument or, in other words, following a Taylor rule (Taylor, 1993). In this chapter, we search for these nonlinearities for the case of the U.S. monetary policy. In particular, we are interested in three non-linear effects of the monetary policy in the long run: the non-linear relationships between the trend inflation and the growth rate, between the trend inflation and the external finance premium and between the growth rate and the error in the estimation of the natural rate of interest. But the possibility of finding this type of evidence is hindered by the problem of the non-observable character of the long-term variables.

In fact, the relevance of the long-term variables is crucial for the performance of the monetary policy carried out by central banks. All specifications of Taylor rules include one or more long-term variables, whose unobservability is an intrinsic characteristic. The long-term variables which are usually incorporated into the monetary policy rules are the natural interest rate, the inflation target and the potential output[21]. As a result of the importance of these variables for the policy design, many contributions have been made on their estimation. Nevertheless, this task is not straightforward.

There are several approaches to estimating unobservable variables. The simplest techniques are the univariate filters such as that of Hodrick-Prescott, but these methods are only based on the statistical properties of the series and ignore the connections with other variables. Equilibrium models can also be built in order to estimate unobserved series as is done in, among others, Neiss and Nelson (2003), Smets and Wouters (2003), Giammaroli and Valla (2004) and Andrés, López-Salido and Nelson (2009), but the resulting estimates are based on subjective assumptions and are prone to be more volatile (Edge *et al.*, 2008). As an alternative to the foregoing methods, and admitting that this routine has been the object of some criticism[22], our approach is based on the Kalman filter applied to a semi-structural econometric model. This procedure has been implemented by many studies to estimate long-term values of several economic variables. However, most of the papers that follow this technique do not jointly estimate all the long-term variables involved in the monetary policy rules[23] nor emphasize the long-term perspective. By contrast, our approach simultaneously includes the natural rate of interest, the

[21] We consider these variables as long-term variables instead of medium-term references.

[22] As discussed in Weber, Lemke and Worms (2008).

[23] An exception is Bjørnland, Leitemo and Maih (2011), but they combine Bayesian and Kalman filter procedures. Benati and Vitale (2007) also obtain estimates of all the variables, but their purposes are far from ours as we focus on the long-term perspective.

long-run growth rate and the steady-state inflation in the estimation process in order to capture all the long-run interactions we are interested in.

The first long-run variable we estimate is the natural rate of interest defined as the long-run real rate of interest that ensures inflation stability and the reaching of the potential output. The estimation of this variable has attracted the interest of the literature since central banks conduct monetary policy through rules with this rate as the intercept. Moreover, the gap between the natural rate of interest and the actual real rate is very useful because it measures the monetary policy stance and has predictive power for future inflation. Many empirical studies have tried to assign a value to this rate, which initially was considered constant over time. Afterwards, in a seminal paper, Laubach and Williams (2003) drop the assumption of a fixed value[24] and estimate the time-varying natural rate of interest (TVNRI) for the U.S. by applying the Kalman filter. The papers that have followed this methodology are not few. Crespo-Cuaresma *et al.* (2004), Mésonnier and Renne (2007) and Garnier and Wilhelmsen (2009) estimate the TVNRI for the euro zone, Larsen and McKeown (2003) for the U.K., Manrique and Marques (2004) for the U.S. and Germany, Basdevant *et al.* (2004) for New Zealand, Brzoza-Brzezina (2006) for Poland and, recently, Bouis, *et al.* (2013) for Canada, the euro zone, Japan, Sweden, Switzerland, the U.K. and the U.S. Combining Bayesian methods with the Kalman filter, Edge, Kiley and Laforte (2008) and Bjørnland, Leitemo and Maih (2011) estimate the TVNRI for the U.S. [25].

We also focus on the implications generated by the potential error that central banks could commit in the estimation of the natural rate of interest. This issue has been

[24] They argue that this rate changes in response to shifts in preferences and in the trend growth rate of output. Trehan and Wu (2007) compare the implications of considering the natural rate of interest to be fixed or variable.

[25] Other methods have been applied in Christensen (2002), Vitek (2005) and Horváth (2009).

theoretically studied in Chapter 2 and in other works such as Orphanides and Van Norden (2002), Orphanides and Williams (2002) and Tristani (2009). We approximate the gap between the correct and the estimated TVNRI and compute its effects on long-term inflation dynamics.

Estimating the TVNRI requires the estimation of the long-run trend of the potential output because, in the theoretical dimension, both variables are closely related. In addition, this estimation is necessary because monetary policy rules are specified in terms of output deviations from the steady state level. Therefore, potential output and, consequently, its growth rate, is the second unobservable variable we estimate.

But the TVNRI and the potential output are not the only variables involved in the long run that are relevant for monetary policy. Trend inflation is another long-term variable that plays an important role in its design and in its outcomes[26]. Like the potential output, it serves as a reference in the deviation measure of the inflation rate as long as it coincides with the inflation target of the monetary policy rule. Moreover, as it is pointed out in Chapter 2, the potential incorrect estimation of the TVNRI generates a gap between the inflation target and its steady-state value that sets off distortions in the long-run equilibrium. Therefore, by including this variable in the estimation process, an increase in the robustness of the analysis is expected as well as an expansion of the range of conclusions. In this line, Leigh (2008) estimates the TVNRI for the U.S. through the Kalman filter, but all other unknown variables are overlooked. Moreover, a relevant topic we want to study is the relationship between the long-run growth rate and the trend inflation. Previous theoretical literature mentioned in Chapter 1, such as Amano *et al.* (2009), shows the existence of a non-linear relationship between these two long-

[26] In this regard, Ascari (2004) and Cogley and Sbordone (2008), among others, analyze the effects of non-zero trend inflation on short-term dynamics.

term variables. And, despite the fact that the Kalman filter provides a linear estimation, we use the outcomes of the model to check the kind of connection between them through a quadratic and a quantile regression.

Another issue we want to discuss is the role of financial frictions in the determination of the main long-term variables. Chapter 1 shows a connection between financial frictions, the growth rate and the trend inflation in the long run with some non-linear relationships. Öğünç and Batmaz (2011) follow the Laubach and Williams (2003) procedure and include the risk premia for Turkey. They conclude that the long-run evolution of this spread determines the natural rate of interest. This link between financial frictions and the TVNRI is also studied empirically in Archibald and Hunter (2001) for the New Zealand case.

Our database comprises time series for the U.S. during the period 1960:Q1-2013:Q2. The evolution of the estimates of the TVNRI, the long-run growth rate of the economy and the steady-state inflation rate are in line with foregoing results. Our estimates prove the negative effect that financial frictions would cause on the long-run growth rate, that potential misunderstandings of the TVNRI would deviate the trend inflation from the inflation target and that the relationship between the long-run growth rate and the trend inflation is described as a nearly hump-shaped curve.

The remainder of the chapter is organized as follows. The second section describes the methodology applied. In the third section, we present the estimation results, carry out a quadratic and a quantile analysis of the relationships among the long-run growth rate, the trend inflation and financial frictions, and study the effects of misunderstandings of the natural rate of interest. Finally, Section 4 summarizes the main conclusions. The Kalman filter procedure is detailed in Appendix B1 and the state-space form of the model is explained in Appendix B2.

3.2 Estimation methodology for the unobservable variables

To achieve the objectives stated in the previous section about the estimation of the long-run unobservable variables, we extend the Laubach and Williams (2003) semi-structural model by making some modifications. The core of the procedure, a state-space model devoted to implementing the Kalman filter, remains unchanged. However, we include a new state variable, the trend inflation. This extension complicates the model but adds robustness to the whole estimation since it jointly estimates all relevant variables in the long-term horizon. We also introduce some changes into the model specification in order to improve the consistency of the long-run implications and to check some theoretical outcomes.

This empirical model is a small-scale simplification of the New Keynesian macroeconomic model developed in Chapters 1 and 2, where the main findings we want to evaluate are the relationships among the long-run economic growth rate, the trend inflation and financial frictions, as well as other relevant conclusions like the effects of potential errors in the estimation of the natural rate of interest. The connections established among these variables show the non-linear effects of the monetary policy in the long run.

The first equation of the model corresponds to the Phillips curve and describes the evolution of the inflation rate (π_t), an observable value. We define the inflation rate as the core consumer price index, which includes all items except food and energy, and take the data from the Bureau of Labor Statistics. The quarterly series is obtained as the monthly average value and then is seasonally adjusted with the Tramo/Seats methodology. Once we have computed the quarterly inflation rate, the data is annualized. We consider the inflation rate as a function of its own lags, the output gap (z_t), the trend inflation (Π_t) and a serially uncorrelated error term ε_t^π. In this way, we ensure the consistency of the model in the long term because inflation rate would equal its steady-state value, the trend inflation. The resulting equation is the following:

$$\pi_{t+1} = \rho^\pi(L)\pi_t + \beta^z z_t + (1 - \rho^\pi(L))\,\Pi_t + \varepsilon_{t+1}^\pi \tag{3.1}$$

where $\rho^\pi(L)$ is a lag-polynomial and β^z is interpreted as the slope of the Phillips curve.

The next relationship is a state equation equivalent to the reduced form of the IS curve that explains the output gap, the percentage deviation of the real output from its potential level. This variable depends on its own lags and on the measurement error of the natural rate of interest, defined as the difference between the ex-ante real interest rate (R_t) and the natural rate of interest (R_t^n). In turn, real interest rate is gauged by subtracting the inflation expectations ($E_t\pi_{t+1}$) from the short-term nominal interest rate R_t^{st}, which is obtained from the Federal Reserve System database. Inflation expectations are computed by an 8-quarters forward-moving average and nominal interest rate is equivalent to the federal funds effective rate. Again, a serially uncorrelated error term ε_t^z is included. This relationship is also consistent with the long run because, in the absence of shocks and mismeasurement problems, the output gap would be zero in the steady state:

$$z_{t+1} = \rho^z(L)z_t + \beta^r\left(R_t - R_t^n\right) + \varepsilon_{t+1}^z \tag{3.2}$$

where $\rho^z(L)$ is a lag-polynomial. We assume, following Mésonnier and Renne (2007), a definition of the natural interest rate based on standard optimal growth models. However, the specification is slightly different and follows Bouis *et al.* (2013), where the natural rate of interest is related to the long-run growth rate (g_t) corrected by a parameter[27] ∂ and augmented by ϵ, the inverse of the intertemporal elasticity of substitution in consumption, also interpreted as the relative risk aversion. An intercept ϱ is also included, which represents the time preference of consumers:

$$R_t^* = \varrho + \epsilon\,(g_t - \partial) \tag{3.3}$$

The long-run growth rate, equivalent to the growth of the potential output y_t^*, is explained as a function of its first lag, financial frictions (f_t) and the trend inflation. Financial frictions are proxied by the spread between the average majority prime rate charged by banks on short-term loans to business and the 3-month Treasury bill rate, both series collected from the Federal Reserve System database. This external finance premium is a standard simple measure of the frictions present in the financial markets. An intercept and a serially uncorrelated error term are also included. In the steady state, the growth rate would depend on a fixed value ∂ and also on the trend inflation, as is theoretically shown in Chapter 1:

$$g_t = \partial\,(1 - \rho^g) + \rho^g g_{t-1} + \zeta f_t + \varkappa \Pi_{t-1} + \varepsilon_t^g \tag{3.4}$$

As can be seen, one main difference between our specification and those of Laubach and Williams (2003) and Mésonnier and Renne (2007) is that growth of the potential

[27] In terms of the Ramsey model, parameter ∂ could be interpreted as a measure of the effects of the average productivity and population growth rates.

output is defined as a function of state and observed variables instead of a simple AR(1) process. Moreover, our hypothesis regarding the order of integration of both R_t^n and g_t follows the approach of Mésonnier and Renne (2007) assuming highly persistent but stationary variables driven by unobservable processes which capture common low-frequency variations in R_t^n and g_t as well as idiosincratic fluctuations of g_t.

Trend inflation is determined by its first lag and the inflation expectations. As noted in equation (3.1), trend inflation would equal the inflation rate in the long run:

$$\Pi_{t+1} = \rho^{\Pi}\Pi_t + \left(1 - \rho^{\Pi}\right) E_t\pi_{t+1} + \varepsilon_{t+1}^{\Pi} \tag{3.5}$$

Finally, the last equation is the identity that defines the output gap as the difference between the output (y_t), built as the log of the real chain-weighted GDP in billions of chained 2009 dollars taken from the Bureau of Economic Analysis, and the log of its potential level:

$$z_t = y_t - y_t^* \tag{3.6}$$

Summing up, the unobservable variables that we jointly estimate are $(y_t^*, g_t, R_t^n, \Pi_t)$, whilst the observed variables are $(\pi_t, R_t^{st}, y_t, f_t)$. We should note that shocks $\left(\varepsilon_t^{\pi}, \varepsilon_t^{z}, \varepsilon_t^{g}, \varepsilon_t^{\Pi}\right)$ are independently and normally distributed and their variances are $\left(\sigma_\pi^2, \sigma_z^2, \sigma_g^2, \sigma_\Pi^2\right)$, respectively.

Having introduced the equations of the semi-structural model, we have to articulate the state-space model. Appendix A1 is devoted to presenting the state-space representation which consists of the measurement equation and the transition equation. Afterwards, we are able to implement the Kalman algorithm (Kalman, 1960). The basic intuition behind this procedure follows two steps. In the first, the system makes a prediction based on the information available at a specific point of time. In the next period, the filter corrects this prediction by uploading the new information. The maximum likelihood method is used to estimate the conditionally unbiased and efficient estimators of the state variables. In Appendix A2, we formally detail the Kalman filter mechanism[28].

3.3 Results for the U.S. economy

We now estimate the model specified in the previous section. The quarterly data set we have used refers to the United States in 1960:Q1-2013:Q2. It should be noted that the Kalman filter is very sensitive to the initial conditions. The technique we have implemented to overcome this issue consists of several steps. Firstly, we carry out a univariate estimation of each unobserved variable. To that end, we have applied the Hodrick-Prescott filter to the inflation rate, the real GDP and its growth rate in order to obtain preliminary estimates of the trend inflation, the potential output and the long-run growth rate, respectively. Secondly, we have estimated each equation including the series provided by the HP filter with the purpose of assigning the initial values to the parameters. This first estimation of the Kalman filter generates variances biased towards zero[29] and, therefore, unsatisfactory results outside the acceptable values for the unobservable variables. Thus, we have to use the common method of restricting some coefficients by calibrating the following parameters:

- Following Bouis *et al.* (2013), one of the best candidates to gauge ϱ is the average of the actual real interest rate because it measures the trend value of the natural

[28] Good references to understand this procedure are Harvey (1989) and Hamilton (1994).
[29] Because of the pile-up problem described in Stock (1994).

rate of interest approximately. This approach is also used to set the value of the intercept of the long-run growth rate equation ∂, equating it to the sample average of the real output growth rates.

- Due to the lack of consensus about the value of the parameter ϵ, we choose the value 4.167, used in our theoretical model of reference developed in Chapter 1.

- Finally, we calibrate σ^2_Π so that trend inflation accounts for 50% of the inflation rate fluctuations.

Table 3.1: Coefficient estimates

ρ_1^π	0.814	(12.72)
β^z	0.158	(2.43)
ρ_1^z	0.901	(14.06)
β^r	-0.179	(-4.33)
ρ^g	0.813	(8.20)
ζ	-0.020	(-1.69)
$\varkappa$	0.009	(1.69)
ρ^Π	0.883	(13.55)
σ^2_π	1.746	(12.84)
σ^2_z	0.3456	(8.72)
σ^2_g	0.033	(7.70)
LF	-633.2	

z-Statistic in parenthesis. LF: Likelihood function.

We now explore the results of the model by analyzing the estimated coefficients shown in Table 3.1. As can be seen, all the coefficients have the expected sign and are statistically significant. Regarding the lags included for the inflation rate in (3.1), we impose order 1 for the $\rho^{\pi}(L)$ lag-polynomial, whose coefficient is ρ_1^{π}. Otherwise, the coefficient associated with Π_t in (3.1) loses weight and, consequently, the state estimates become distorted. The significativity criterion reveals that the lag-polynomial of the output gap in (3.2) is of order 1. Both the slope of the Phillips curve β^z and the coefficient β^r, which drives the output gap in accordance with fluctuations in the difference between the actual interest rate and its natural level, are higher than those estimated by Laubach and Williams (2003) and Bouis *et al.* (2013)[30], but remain within reasonable values. Financial frictions negatively affect the long-run growth rate, which can be seen from the negative value of ζ. Trend inflation exerts the opposite effect because coefficient $\varkappa$ has a positive sign, though the size is very low. As the statistical significance of these two linear effects is at the limit of 10%, we go deeper into this issue in the last part of this section when we pose the question of the non-linear effects.

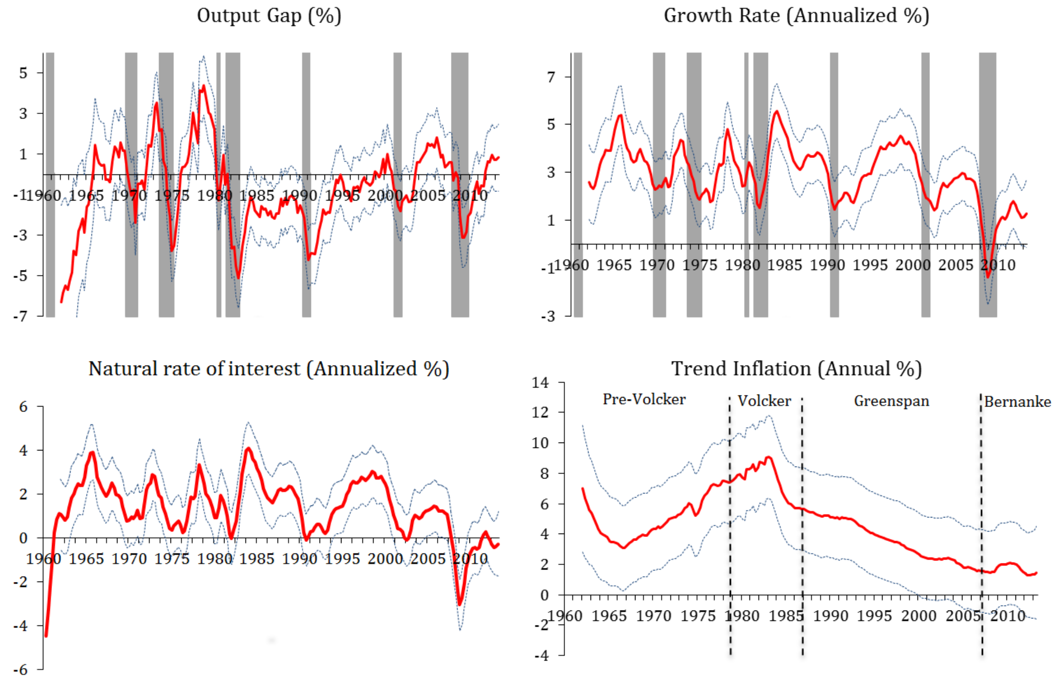
The estimation of the model with the features described above yields the evolution of the unobserved variables displayed in Figure 3.1. We should clarify that these series are two-sided estimates or smoothed estimates, that is, to compute them, the Kalman algorithm has used the information of the full sample. In addition, we have discarded the first few quarters because the estimates are outside the admissible range.

Table 3.2 displays the statistical properties of the unobserved variables. Output gap shows an expected path in the range (-6%,5%) with eight slowdowns corresponding to the official recession dates of the U.S. economy[31], which also can be appreciated in the long-run growth rate trajectory. This first inference seems to verify the accuracy of the estimates.

[30] Other values of the literature are summarized in Mésonnier and Renne (2007).

[31] The crisis periods are 1960:Q2-1961:Q1, 1969:Q4-1970:Q4, 1973:Q4-1975:Q1, 1980:Q3, 1981:Q3-1982-Q4, 1990:Q3-1991:Q1, 2001:Q1-2001:Q4 and 2007:Q4-2009:Q2.

Figure 3.1: Estimates of the unobserved variables



Grey bars refer to the official recession dates provided by the National Bureau of Economic Research. Dashed lines represents the 90% confidence interval.

The sharpest declines of the output gap are situated at the beginning of the sample and in the early 1980s recession, whilst the long-run growth rate reaches its minimum value in 2008 during the financial crisis. The trajectory of the natural rate of interest is obviously analogous to the long-run growth rate evolution because the former is defined as a linear combination of the latter. Values of the long-run growth rate and the natural rate of interest in 2008:Q4 seem to be atypical since the troughs of both series are anomalously low for long-term references. Finally, the trend inflation rises sharply from the late sixties to 1980, during the Pre-Volcker era. After reaching its peak in the middle of Volcker's presidency of the Federal Reserve System, the trend inflation has constantly decreased leading to the so-called Volcker disinflation. From the late-nineties, under the leadership of Greenspan, the trend inflation has stabilized around a value of approximately 2%, level at which it is assumed that FED locates its inflation rate target for the medium and long term.

Table 3.2: Statistical properties of the estimated series

	Mean	Standard deviation	Minimum	Maximum
Output Gap	-0.70	1.95	-6.32	4.40
			(1962:Q1)	(1978:Q4)
Growth Rate	2.91	1.21	-1.38	5.57
			(2008:Q4)	(1984:Q1)
Natural Rate of Interest	1.38	1.24	-3.06	4.07
			(2008:Q4)	(1984:Q1)
Trend Inflation	4.45	2.08	1.30	9.09
			(2012:Q2)	(1983:Q1)

3.3.1 Searching for nonlinearities

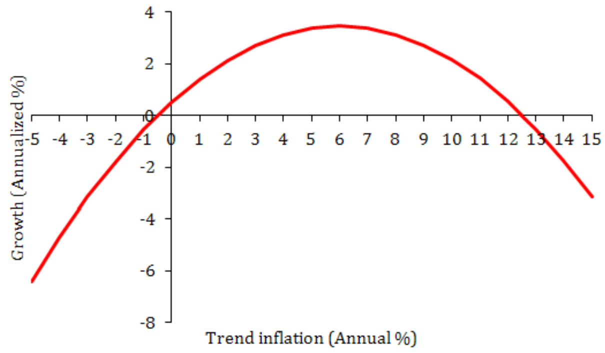
We have seen, in Table 3.1, that the estimation of the coefficient $\varkappa$ is near zero and that its corresponding p-value is slightly lower than 10%, so the linear relationship between the long-run growth rate and the trend inflation is very weak. The same remark can be made about the relationship between the external finance premium and the growth rate. These are not two counterintuitive results in the light of the findings of Chapter 1, because these outcomes may not mean the absence of a relationship between the trend inflation and the long-run growth rate and between the latter and the external finance premium, but perhaps the model specification used is veiling relevant bivariate movements. The theoretical results we have proposed to test in this chapter are the presence of nonlinearities between these two pairs of variables but, unfortunately, in the model to which the Kalman filter is applied, non-linear specifications can not be included. Although the Extended Kalman filter can integrate such specifications, its operation is very complex, so we have opted for a two-step analysis. The first, already done, is to obtain estimates of the state variables. In the second, we use these estimates of the unobservable long-run variables to test the hypothesis established in Chapter 1, a hump-shaped relationship between the long-run growth rate and the trend inflation, on the one hand, and a U-shaped relationship between the trend inflation and the external finance premium, on the other.

A simple way to look for non-linear relationships is to define a quadratic equation. By doing so, the result obtained for the regression is the folowing:

$$\hat{g}_t = \underset{(1.42)}{0.52} + \underset{(5.88)}{0.98\hat{\Pi}_t} - \underset{(-4.84)}{0.08\hat{\Pi}_t^2} + \hat{u}_t^g \tag{3.7}$$

where $\hat{\Pi}_t$ and $\hat{g}_t$ are the trend inflation and the long-run growth rate series estimated by the Kalman filter, $\hat{u}_t^g$ refers to the residuals and the t-ratios are presented in parentheses. In line with the foregoing theoretical results, the coefficient values show a hump-shaped relationship between $\hat{g}_t$ and $\hat{\Pi}_t$ plotted in Figure 3.2. This very significant non-linear relationship indicates that, for low levels of trend inflation, the long-run growth rate increases until $\hat{\Pi}_t = 6\%$ and, after this value, the growth rate decreases with the trend inflation, reaching zero when it is 12.7% and -0.5%. The annualized maximum potential growth is near 4%. But this outcome varies depending on the sample considered. If we contemplate a period of inflation stability, such as the subsample beginning in 1994 during which the Federal Reserve has reacted preemptively against deviations of inflation from its target, the level of trend inflation for which estimated growth is maximized drops markedly to $\hat{\Pi}_t = 4\%$.

Figure 3.2: Quadratic relationship between $\hat{g}_t$ and $\hat{\Pi}_t$

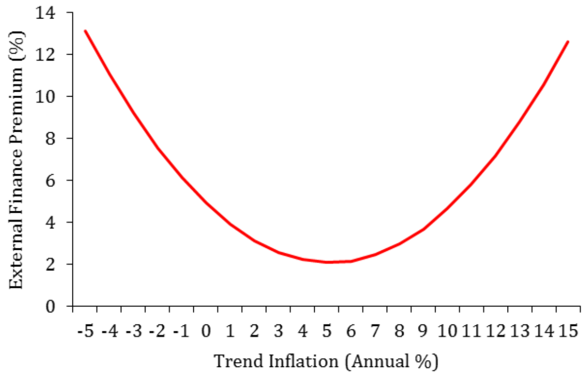


As in our previous exercise, we again perform this analysis to test for the link between financial frictions and the trend inflation in the long run. Chapter 1 conclude that this connection has a U shape. Equation (3.8) presents the estimated coefficients, where it can be seen that the U-shaped relationship found in that chapter, displayed in Figure 3.3, is corroborated. It should be noted that the level of trend inflation for which estimated growth is maximized nearly matches the minimum value of financial frictions, as is concluded in Chapter 1.

$$f_t = \underset{(16.04)}{4.90} - \underset{(-7.96)}{1.11\hat{\Pi}_t} + \underset{(7.69)}{0.11\hat{\Pi}_t^2} + \hat{u}_t^f \tag{3.8}$$

where $\hat{u}_t^f$ are the residuals.

Figure 3.3: Quadratic relationship between f_t and $\hat{\Pi}_t$



Another way of searching for the nonlinearties we are interested in is to take into account that the state estimates have been obtained using a specific linear model. Hence, maintaining this specification we analyze the relationship of both the trend inflation and financial frictions with the long-term growth rate, looking for possible nonlinearities. In doing so, we again try to verify if the theoretical results obtained in Chapter 1 are validated. We take the sample 1962:Q1-2013:Q2 in order to avoid the initial distorted observations of the state estimates. The methodology we adopt is based on a quantile regression but, in contrast to the common practice of ordering observations according to the endogenous variable, we arrange the quantiles according to an exogenous variable, the trend inflation. Thus, the specification of our equation of interest, which relates the long-term growth rate, financial frictions and the trend inflation, is the same[32] as (3.4), although levels of the explanatory variable $\hat{\Pi}_t$ are distinguished. We opt for six quantiles because this is the largest number that provides an acceptable number of observations in each quantile. Table 3.3 shows the lower and the upper limit of each quantile, the estimated coefficients, the variance and the coefficient of determination.

We expect a positive sign of $\varkappa$ in the lower quantiles and a negative one in the higher,

[32] We no longer include the trend inflation as a lag because that was a specific constraint of the Kalman filter.

what would resemble the inverted parabola obtained previously. For all the quantiles except the last one and, consequently, for most of the sample, the relationship can be described as an inverted U curve. However, when trend inflation is above 6.67% (last quantile), which occurs between 1973 and 1984, the estimated coefficient is positive. Nevertheless, this scenario could be considered as an anomalous pattern since, during those years, the two economic recoveries after strong downturns were attached to a monetary policy that did not react severely to inflation deviations.

Table 3.3: Quantile regression

	Quantiles					
	1st	2nd	3rd	4th	5th	6th
Minimum $\hat{\Pi}_t$	1.30%	2.09%	3.42%	4.35%	5.22%	6.67%
Maximum $\hat{\Pi}_t$	2.06%	3.40%	4.31%	5.21%	6.52%	9.09%
$\partial\left(1-\rho^g\right)$	0.324	-0.262	0.085	0.227	0.263	-0.454
	(1.68)	(-2.77)	(0.26)	(1.10)	(1.74)	(-2.22)
ρ^g	0.973	0.804	1.054	0.947	0.905	0.921
	(25.77)	(14.91)	(9.86)	(10.34)	(20.83)	(15.06)
ζ	-0.161	0.026	0.006	-0.024	-0.014	-0.046
	(-4.02)	(2.60)	(0.70)	(-1.11)	(-1.27)	(-3.31)
$\varkappa$	0.120	0.118	-0.035	-0.026	-0.031	0.085
	(2.23)	(2.82)	(-0.56)	(-0.52)	(-1.21)	(3.21)
σ^2	0.004	0.002	0.003	0.005	0.003	0.009
R^2	0.96	0.97	0.92	0.88	0.94	0.90

t-ratios are reported in parentheses.

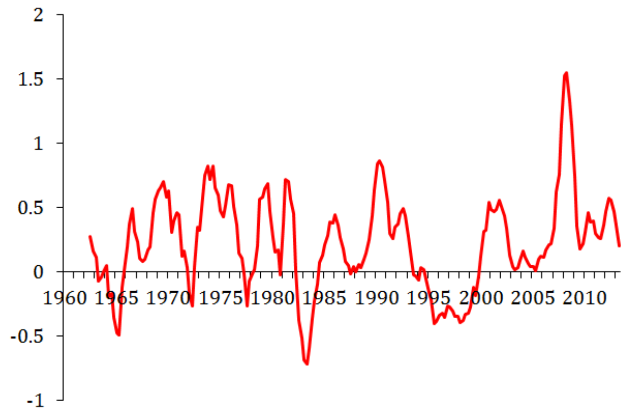
With respect to the relationship between financial frictions and the long-run growth rate, coefficient ζ is statistically significant at 5% only for the extreme lower and higher values of the trend inflation, so the influence is coherent with the U-shaped relationship between the trend inflation and the external finance premium. For medium inflation rate levels, when the external finance premium does not reach the upper levels and, therefore, credit markets operate flexibly, their fluctuations do not affect, or positively affect, the long-term growth rate. However, when the degree of financial frictions increases, long-run growth could be negatively affected by such rigidities.

3.3.2 Long-run effects of natural rate of interest mismeasurements

In order to capture other non-neutral and non-linear effects of the monetary policy in the long run, we approximate the real-time gap between the estimated and the correct value of the natural rate of interest as the difference between the one-sided ($\hat{R}_t^*$, filtered) and the two-sided ($\hat{R}_t^n$, smoothing) estimates following Mésonnier and Renne (2007). This is a proxy of the mismeasurement gap since the former estimation takes into account the information available at the time of the estimation, as central banks do, and the latter uses the full sample information of the signal variables, which approaches the true value. In Figure 3.4, the evolution of the mismeasurement gap is displayed.

This gap reaches a substantial size and, even if we do not consider the highest deviations, the gap moves around values of $(-0.5\%, 1\%)$. When the mismeasurement gap takes positive values, monetary policy tends to be more contractionary since the intercept of the rule is higher than the endogenous value. Analogously, when the gap is negative, monetary policy is more expansive. The average of the gap is near 0.25% meaning that, on average, Federal Reserve implements a restrictive monetary policy through the overestimation of the natural rate of interest.

Figure 3.4: Gap $(\hat{R}_t^* - \hat{R}_t^n)$

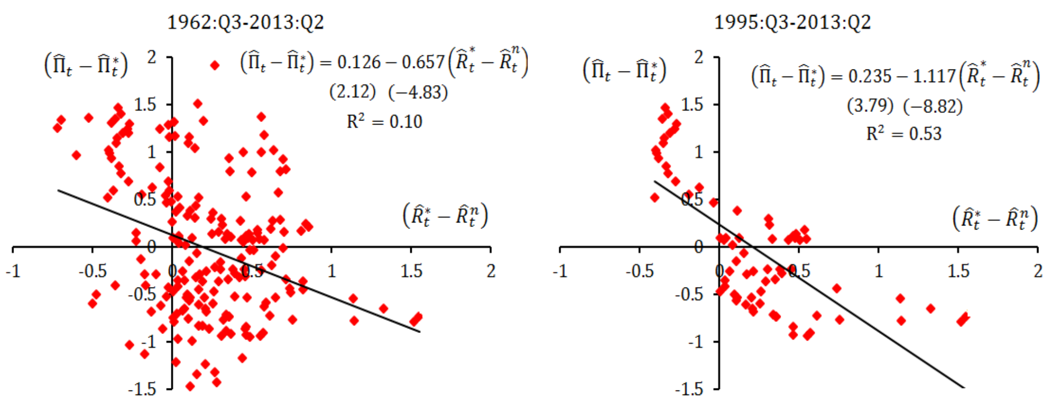


Plot of the 2-quarter central moving average of the gap between the filtered $(\hat{R}_t^*)$ and the smoothing $(\hat{R}_t^n)$ estimation of the TVNRI

We now explore the potential implications of the existence of this gap from the long-run perspective. This exercise is based on the theoretical work developed in Chapter 2, where it is concluded that central banks' misunderstandings in the estimation of the natural rate of interest affect the long-run equilibrium by deviating the steady-state inflation rate from its target. When the natural rate of interest estimated by the central bank is higher than the correct value, trend inflation is below its target and vice versa. Accordingly, the relationship between the deviation of the natural rate and the gap of the trend inflation is negative.

Firstly, in order to carry out this analysis, we have to transform the trend inflation series to be comparable with the mismeasurement gap. Hence, we have to calculate the deviations of the trend inflation from the estimated target, the latter proxied by its statistical mean. However, the evolution of the estimated trend inflation exhibits a clear non-linear pattern of behavior. To verify this point, we have estimated the mean of this variable by way of the Bai-Perron methodology, which allows for the presence of structural changes (see Bai and Perron, 1998, 2003) throughout the sample 1962:Q1-2013:Q2. The application of this methodology leads us to observe the existence of 5 differentiated periods in the evolution of the trend inflation with the following break points: 1970:Q3, 1979:Q2, 1986:Q4 and 1995:Q2. The estimated annualized values for the mean of the trend inflation in the five subperiods are 4.1%, 6.1%, 7.5%, 4.9% and 2.3%, respectively. Secondly, with these averages, we can compute the deviations of the trend inflation from the estimated target and relate them to the mismeasurement gap. In order to smooth both series, characterized by strong fluctuations, we construct the 2-quarter central moving averages for the trend inflation and the mismeasurement gap. Figure 3.5 shows the scatter plot of the trend inflation deviations $\left(\hat{\Pi}_t - \hat{\Pi}_t^*\right)$, where $\hat{\Pi}_t$ is the trend inflation estimate and $\hat{\Pi}_t^*$ is the target, and the mismeasurement gap $\left(\hat{R}_t^* - \hat{R}_t^n\right)$, where $\hat{R}_t^*$ is the estimated intercept of the Taylor rule and $\hat{R}_t^n$ is the estimated natural rate of interest:

Figure 3.5: Trend inflation deviations from its target and mismeasurement gap



Scatter plot between the 2-quarter central moving average of $(\hat{\Pi}_t - \hat{\Pi}_t^*)$ and $(\hat{R}_t^* - \hat{R}_t)$.

t-ratios in parentheses

This analysis reveals that the relationship is negative for the full sample as the theoretical analysis predicts. However, if we divide the total sample into the specified periods detailed above, the results are not homogeneous. In the final period, which is the largest one of the time intervals considered, the negative relationship is much clearer than in the whole sample. These results lead us to think that, when monetary policy was conducted through monetary aggregates and the implicit or explicit estimation of the natural rate of interest was not required, or monetary authorities do not react against inflation deviations tightly enough, the mismeasurement gap was not so relevant and was not so clearly transferred to the long-term inflation. So, we can conclude that, with policies derived from Taylor rules whose priority is the inflation stability, the natural rate of interest and its measurement become essential for the monetary policy because measurement errors could influence the long-term equilibrium. These preliminary results seem to support this intuition and, therefore, verify the negative relationship between the two gaps specially for the period in which the Federal Reserve conducts monetary policy through inflation targeting rules which react severely and preemptively against inflation deviations.

3.4 Conclusions

In this chapter, we analyze the long-run interactions of the U.S. unobservable variables included in the Taylor rules. Firstly, we look for the existence of nonlinearities between the long-run growth rate, the trend inflation and financial frictions in the long run. Through quadratic equations, we confirm the existence of a hump-shaped connection between the long-run growth rate and the trend inflation and a U-shaped relationship between the latter and financial frictions. Then, by estimating a quantile regression which distinguishes among levels of trend inflation, we corroborate that hump-shaped connection if the trend inflation does not exceeds an upper threshold. Moreover, especially for low and high levels of trend inflation, financial frictions negatively affect the long-run growth rate.

Furthermore, we approximate the gap between the real-time estimate and the correct value of the natural rate of interest and study its effects on the trend inflation deviations from the target level. We prove that, for the whole sample, there is a negative relationship between the error in the estimation of the natural rate of interest and the gap of the actual trend inflation and its target, as is concluded in the theoretical models developed in Chapters 1 and 2. This negative relationship is especially clear and significant when monetary policy reacts aggressively against inflation deviations. In turn, deviations of the trend inflation from its target could also affect financial frictions and the long-run growth rate, since have been demonstrated the interactions among these three key variables.

In order to obtain all these results, we have jointly estimated the natural rate of interest, the potential output and the trend inflation using U.S. data for the 1960:Q1-2013:Q2 period. Our procedure extends the methodology of Laubach and Williams (2003), who implement the Kalman filter to a semi-structural econometric model in order to obtain the unobservable series, by including the trend inflation as a state variable and financial frictions as an exogenous factor. The state estimates show that the long-run growth rate and the natural rate of interest have experienced an unprecedented decline during the financial crisis triggered in 2007, a pattern that is not observed for the output gap. Meanwhile, the trend inflation has stabilized since the mid-nineties around a value of 2%.

Conclusions

In the foregoing chapters, we have delved into the transmission mechanism of monetary policy in the long run. Although the idea of the neutrality of monetary policy in the long run has been widely accepted by most economic doctrines, this property is being revised recently given the current policy rules and the existence of rigidities and frictions in some markets. With the aim of studying this issue, we have abandoned some unrealistic assumptions concerning the Taylor rule that New Keynesian models usually introduce. Furthermore, there are reasons for thinking that rigidities in financial markets could affect the long-term behavior, so we have included a financial sector with frictions in our baseline model. To conclude, we have evaluated the main outcomes obtained in the theoretical developments for the United States.

Although central banks set positive inflation targets, most macroeconomic models are built around the assumption of null inflation in the steady state for reasons of analytical convenience. Therefore, in the first chapter, we have removed the assumption of zero inflation in the steady state. We have proved that the conclusions drawn in the New Keynesian models are sensitive to the elimination of this assumption. In addition, we have incorporated frictions in the financial markets through an agency problem between financial intermediaries and their depositors. As we have seen, the combination of these two elements causes several changes in the results of the model when economic growth is introduced.

Firstly, we have found nonlinearities between the long-run growth rate and the financial parameters given the value of the trend inflation, which is set by the central bank in the monetary policy rule. It should be noted that the dependence of the long-run growth rate and the trend inflation remains as long as price frictions persist, the equilibrium remaining dependent on the financial structure unless the rigidities in this market disappear. We have found a hump-shaped relationship between the long-run growth rate and the trend inflation, which implies the existence of an optimal level of trend inflation that maximizes the long-run growth rate, welfare, investment and financial wealth. The reasons behind this behavior are closely linked to the financial sector. For the optimal level of trend inflation, the external finance premium reaches its minimum value as well as the yield of financial intermediaries'claims and the marginal gain from expanding the assets. Moreover, when the financial structure changes, there is a trade-off in the long-run between the growth rate and the marginal gain of financial intermediaries.

With regard to the short-term dynamics, the triggering of a monetary shock shows that trend inflation also affects the stabilization of the economy after a transitory perturbation, since the higher the trend inflation, the greater the response of the real variables, such as the growth rate and investment, or the financial elements such as the external finance premium. Nevertheless, this behavior is not appreciated in the case of a productivity disturbance such as a technological shock.

To conclude the first chapter, we have simulated a crisis in order to evaluate the reaction of our baseline model under several intensities of credit policy. The crisis consists of a decline in capital quality, which causes an increase in the external finance premium, whilst the credit policy is implemented through direct lending to non-financial firms. The impulse-response exercise reveals that this kind of unconventional monetary policy affects both real and financial variables by mitigating the reaction in the initial periods after the crisis, but prolongs the impact on most variables to the long term. This finding, along with the relevance of the unconventional monetary policy parameters for the long-term equilibrium, confirms the caution that central banks should take when implementing such measures.

The second chapter extends our benchmark model by, again, eliminating a critical assumption. In this case, we no longer assume that the natural rate of interest, included in the Taylor rule as the intercept, is correctly estimated by central banks. In an initial step, we have obtained a definition of the natural rate of interest, which depends on the financial sector for any hypothesis regarding the accuracy of its estimation. Furthermore, we have determined that, if the central bank mismeasures the natural rate of interest, this rate becomes dependent on its own estimate and on the inflation rate target.

Although the short-term reactions to transitory perturbations do not change with errors in the estimation of the natural rate of interest, long-run references do. The relationship between the error and the long-run growth rate depends on the inflation target. If the target surpasses the optimal inflation rate, identified in Chapter 1, and the estimate of the natural rate of interest is under the endogenous value, the long-run growth rate declines when this estimate drops. However, if the inflation target is under the optimal value, the relationship is hump-shaped. The relation between the error and the marginal gain of the financial intermediaries is the opposite whilst the deviations of the long-term inflation rate from its target are always negatively related to the errors for any target. Furthermore, we have studied the interactions between the parameter included in the Taylor rule that reacts to inflation deviations from its target and the long-run growth rate in the presence of estimation errors.

We have developed a guideline for testing the accuracy of the estimation of the natural rate of interest and the position of the inflation target with respect to the optimal rate. We have taken our previous outcomes as our basis and have evaluated the transition dynamics between steady states in order to assess the short-term paths and, in this way, to determine the stance of the monetary policy. It has been shown that, by using the estimate of the natural rate of interest, the inflation target and the parameter that reacts against inflation deviations, it is possible to maximize the long-run growth rate and to correctly estimate the natural rate of interest after a sequence of changes.

In the last chapter of the dissertation, we empirically evaluate the findings obtained in the previous work for the United States for the sample 1960:Q1-2013:Q2. With the aim of searching for potential nonlinearities between the long-run growth rate and the trend inflation, we estimate a quadratic equation to describe the relationship between these two variables and find that this relation is hump-shaped, as the theoretical results predict. Nevertheless, through a quantile regression which divides the sample among the levels of trend inflation, we find that the estimates of the coeffcients, except for the highest level of trend inflation, support the hump-shaped relationship but, for some quantiles, the relation is not statistically significant.

Then, we have estimated the mismeasurement error of the natural rate of interest. We relate this gap to the deviations of the trend inflation from its target, as is outlined in Chapter 2, and find that, since monetary policy reacts severely against inflation deviations in the United States, trend inflation surpasses its target if the central bank overestimates this rate, and vice versa.

In order to perform this analysis, we have solved the problem of the unobservability of the long-run variables. With this purpose, we have jointly estimated all the long-term references included in the Taylor rule, namely, the natural rate of interest, the potential output and the trend inflation. We apply the Kalman filter to a stylized representation of our benchmark model in order to obtain the unobservable series and, thereby, are able to study the long-term connections. The estimated series of the long-run growth rate and the natural rate of interest suffer their maximum decline during the financial crisis, but the highest output gap decrease occurs at the beginning of the sample. It is interesting to note that, from the mid-nineties, the trend inflation has stabilized around 2%.

Conclusiones

A lo largo de los tres capítulos de esta tesis hemos profundizado en el mecanismo de transmisión de la política monetaria en el largo plazo. Aunque la idea de neutralidad de la política monetaria ha sido ampliamente aceptada por la mayoría de las doctrinas económicas, esta propiedad se está revisando recientemente a la luz de las reglas de política monetaria actuales y de la existencia de rigideces y de fricciones en determinados mercados. Con el propósito de profundizar en esta cuestión, se han abandonado algunos de los supuestos poco realistas que se plantean en los modelos neokeynesianos respecto a la regla de Taylor. Además, como existen argumentos que señalan a que las rigideces de los mercados financieros pueden afectar al equilibrio de largo plazo, hemos incluido en nuestro modelo de referencia un sector financiero con fricciones. Para concluir, hemos evaluado los principales resultados teóricos obtenidos para el caso de Estados Unidos.

Aunque los bancos centrales fijan un objetivo de inflación positivo, los modelos macroeconómicos se construyen habitualmente en base a la hipótesis de que la inflación es nula en el estado estacionario por razones de conveniencia analítica. En el primer capítulo hemos eliminado este supuesto y hemos probado que las conclusiones especificadas en los modelos neokeynesianos son sensibles a su supresión. Además, hemos incorporado fricciones en los mercados financieros a través de un problema de agencia entre intermediarios financieros y depositantes. Hemos comprobado que la combinación de ambos elementos origina cambios relevantes en los resultados del modelo si se introduce crecimiento económico endógeno.

En primer lugar, se detectan no linealidades entre la tasa de crecimiento de largo plazo y los parámetros financieros, dado el valor de la inflación de estado estacionario que fija el banco central en la regla de política monetaria. Además, se observa que la relación entre la tasa de crecimiento de largo plazo y la inflación de estado estacionario prevalece siempre y cuando las rigideces de los precios persistan, resultando el equilibrio dependiente de la estructura financiera salvo que las rigideces en estos mercados desaparezcan. Se demuestra que dicha relación tiene forma de U invertida. Por lo tanto, hay un nivel óptimo de inflación de estado estacionario para el cual se maximizan la tasa de crecimiento, el bienestar, la inversión y la riqueza financiera en el largo plazo. Las razones que se hallan detrás de este comportamiento se deben buscar en el sector financiero. Para dicho nivel óptimo de inflación de estado estacionario, la prima de financiación externa alcanza su valor mínimo y, por tanto, también alcanzan su suelo el rendimiento de los activos de los intermediarios financieros y la ganancia marginal derivada de expandirlos. Asimismo, cuando la estructura financiera cambia, se produce un intercambio entre la tasa de crecimiento y la ganancia marginal de los intermediarios en el largo plazo.

El el corto plazo la simulación de un shock en la regla de política monetaria muestra que la inflación de equilibrio estacionario también afecta a la estabilización de la economía tras una perturbación transitoria dado que, cuanto más alta es, mayor es la respuesta de las variables. Sin embargo, este comportamiento no se aprecia para el caso de una perturbación en las variables reales, como es el caso de un shock exógeno en la productividad.

Para concluir el primer capítulo se ha simulado una crisis con el fin de evaluar la dinámica del modelo ante distintas intensidades de política crediticia. La crisis consiste en un descenso de la calidad del capital, que provoca un aumento de la prima de financiación externa, siendo la política crediticia no convencional implementada a través de préstamos directos a las empresas productoras de bienes intermedios. El estudio de las funciones impulso-respuesta revela que este tipo de política monetaria no convencional afecta tanto a las variables reales como a las financieras mitigando su reacción en los periodos iniciales posteriores a la crisis, pero prolonga el impacto sobre la mayoría de las variables reales y financieras, cuya respuesta llega a alcanzar el largo plazo. Esta conclusión, junto con la relevancia de los parámetros de la política monetaria no convencional en el equilibrio de largo plazo, ratifica la precaución que deben tener los bancos centrales cuando implementan este tipo de medidas.

El segundo capítulo extiende el modelo excluyendo, de nuevo, un supuesto crítico. En este caso, ya no se asume que la tasa natural de interés incluida en la regla de Taylor como constante está bien estimada por parte del banco central. Se establece la definición de tasa natural de interés, que depende del sector financiero para cualquier hipótesis respecto a la precisión de la estimación. Asimismo, se determina que, si el banco central estima incorrectamente la tasa natural de interés, esta tasa queda en función de su propia estimación y del objetivo de inflación.

Respecto a la dinámica de corto plazo, los efectos de los shocks coyunturales no se ven afectados por los errores de estimación del tipo de interés natural, pero sí las variables de largo plazo. La relación entre el error de estimación y la tasa de crecimiento de largo plazo depende del objetivo de inflación. Si la tasa objetivo supera la tasa óptima, identificada en el Capítulo 1, y la estimación de la tasa natural de interés es menor que el valor correcto, la tasa de crecimiento de largo plazo desciende cuando dicha estimación se revisa a la baja. Sin embargo, si el objetivo de inflación es inferior a la tasa óptima la relación tiene forma de U invertida. Por su parte, la relación entre el error de estimación y la ganancia marginal de largo plazo de los intermediarios financieros es la contraria, mientras que las desviaciones de la tasa de inflación de equilibrio estacionario respecto a su objetivo siempre están relacionadas negativamente con los errores de estimación cualquiera que sea el objetivo. Además, se han estudiado las interacciones que se establecen entre el parámetro de respuesta a las desviaciones de la inflación incluido en la regla de Taylor y la tasa de crecimiento de largo plazo en presencia de errores de estimación.

Se ha desarrollado finalmente una guía para verificar la adecuación de la estimación de la tasa natural de interés y la posición de la inflación objetivo respecto a la tasa óptima. En base a las conclusiones previas se han evaluado las dinámicas de transición entre estados estacionarios para calcular las trayectorias de corto plazo y, de esta manera, determinar la posición de la política monetaria. A través del uso de la estimación de la tasa natural de interés, del objetivo de inflación y del parámetro de respuesta a las desviaciones de la inflación es posible maximizar la tasa de crecimiento de largo plazo e igualar la estimación de la tasa natural de interés a su valor endógeno mediante aproximaciones sucesivas.

En el último capítulo se evalúan empíricamente los resultados teóricos para el caso de Estados Unidos durante el periodo 1960:Q1-2013:Q2. Con el objetivo de localizar las posibles no linealidades existentes entre la tasa de crecimiento de largo plazo y la tasa de inflación de estado estacionario, estimamos una ecuación cuadrática para describir la conexión entre ambas variables y encontramos que dicha relación tiene forma de U invertida, tal y como los resultados teóricos establecían. No obstante, mediante una regresión cuantílica que divide la muestra entre niveles de inflación de estado estacionario, se observa que las estimaciones de los coeficientes, exceptuando el cuantil que representa los niveles de inflación de estado estacionario más elevados, apoyan dicha relación aunque, para algunos cuantiles, la significatividad estadística es muy baja.

A continuación, se ha estimado el error de estimación de la tasa natural de interés. Poniendo esta brecha en relación a las desviaciones de la inflación de estado estacionario respecto de su objetivo se encuentra que, desde que en Estados Unidos la política monetaria se lleva a cabo mediante reglas que fijan el tipo de interés nominal y que reaccionan agresivamente contra la inflación, si el banco central sobreestima la tasa natural, la inflación de equilibrio estacionario supera su objetivo y viceversa.

Para poder realizar estos análisis, se ha tenido que solventar el problema de la inobservabilidad de las variables de largo plazo incluidas en la regla de Taylor, es decir, la tasa natural de interés, la producción potencial y la tasa de inflación de equilibrio estacionario. Aplicando el filtro de Kalman a una representación estilizada de nuestro modelo teórico, se han estudiado las conexiones de largo plazo. La serie estimada de la tasa de crecimiento de largo plazo y de la tasa natural de interés sufren su máxima disminución durante la crisis financiera, pero el output gap protagoniza el mayor descenso al principio de la muestra. Es interesante señalar que, desde mediados de los noventa, la inflación de equilibrio estacionario se ha estabilizado en torno a un valor anualizado del 2 %.

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Appendix A

Appendix for the First Chapter

A.1 Normalized model

$$W_t^k = \chi C_t^k N_t^{\varphi} \tag{A1.1}$$

$$E_t \Lambda_{t,t+1} R_t = 1 \tag{A1.2}$$

$$\Lambda_{t,t+1} = \beta \frac{C_t^k}{C_{t+1}^k G_{t+1}} \tag{A1.3}$$

$$W_t^k = (1-\alpha) P_t^i \frac{Y_t^{i,k}}{N_t} \tag{A1.4}$$

$$E_t \left\{ R_{t+1}^q \right\} = \frac{P_{t+1}^i \alpha \frac{Y_{t+1}^{i,k}}{e^{\xi_{t+1}}} + Q_{t+1} - \delta}{Q_t} e^{\xi_{t+1}} \tag{A1.5}$$

$$Y_t^{i,k} = e^{a_t} \left(e^{\xi_t}\right)^{\alpha} N_t^{1-\alpha} \tag{A1.6}$$

$$I_t^{n,k} = I_t^k - \delta \tag{A1.7}$$

$$G_{t+1} = 1 + I_t^{n,k} \tag{A1.8}$$

$$Q_t = 1 + f + \frac{I_t^{n,k} + I^k}{I_{t-1}^{n,k} + I^k} f' - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}^{n,k} + I^k}{I_t^{n,k} + I^k} \right)^2 f' \tag{A1.9}$$

$$Y_t^k = C_t^k + I_t^k + f\left(\frac{I_t^{n,k} + I^k}{I_{t-1}^{n,k} + I^k} \right) \left(I_t^{n,k} + I^k \right) + \tau \psi_t Q_t G_{t+1} \tag{A1.10}$$

$$Y_t^k \Delta_t = Y_t^{i,k} \tag{A1.11}$$

$$v_t = E_t \left\{ (1-\gamma)\, \Lambda_{t,t+1} \left(R_{t+1}^q - R_{t+1} \right) + \gamma \Lambda_{t,t+1} x_{t,t+1} v_{t+1} \right\} \tag{A1.12}$$

$$h_t = E_t \left\{ (1-\gamma) + \gamma \Lambda_{t,t+1} t_{t,t+1} h_{t+1} \right\} \tag{A1.13}$$

$$\phi_t^p = \frac{h_t}{\lambda - v_t} \tag{A1.14}$$

$$t_{t,t+1} = \left(R^q_{t+1} - R_{t+1}\right)\phi^p_t + R_{t+1} \tag{A1.15}$$

$$x_{t,t+1} = \frac{\phi^p_{t+1}}{\phi^p_t} t_{t,t+1} \tag{A1.16}$$

$$Q_t G_{t+1} = \phi^T_t F^k_t \tag{A1.17}$$

$$\phi^T_t = \frac{1}{1 - \psi_t}\phi^p_t \tag{A1.18}$$

$$\psi_t = v\left[E_t\left(R^q_{t+1} - R_{t+1}\right) - \left(R^q - R\right)\right] \tag{A1.19}$$

$$F^k_t = F^{o,k}_t + F^{n,k}_t \tag{A1.20}$$

$$F^{o,k}_t = \gamma\left[\left(R^q_t - R_t\right)\phi^p_{t-1} + R_t\right] F^k_{t-1} \tag{A1.21}$$

$$F^{n,k}_t = (1 - \gamma)\,\omega Q_t e^{\xi_t} \tag{A1.22}$$

$$X_t = \mu \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k}\right)^{\epsilon} P^i_{t+i} Y^k_{t+i}}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k}\right)^{\epsilon-1} Y^k_{t+i}} \tag{A1.23}$$

$$X_t = \left[\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right]^{\frac{1}{\epsilon-1}} \tag{A1.24}$$

$$\Delta_{t+1} = \theta\Pi_{t+1}^{\epsilon}\Delta_t + (1-\theta)\,X_{t+1}^{-\epsilon} \tag{A1.25}$$

$$R_t^{st} = R\Pi\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{Y_t^k}{Y^k\xi_t}\right)^{\phi_y} e^{\eta_t} \tag{A1.26}$$

$$R_t^{st} = R_t E_t \Pi_{t+1} \tag{A1.27}$$

A.2 Steady-state equations

$$w^k = \chi C^k N^{\varphi} \tag{A2.1}$$

$$\Lambda R = 1 \tag{A2.2}$$

$$\Lambda = \frac{\beta}{G} \tag{A2.3}$$

$$w^k = (1-\alpha)\,P^i \frac{Y^{i,k}}{N} \tag{A2.4}$$

$$R^q = \frac{P^i \alpha Y^{i,k} + Q - \delta}{Q} \tag{A2.5}$$

$$Y^{i,k} = N^{1-\alpha} \tag{A2.6}$$

$$Q = 1 \tag{A2.7}$$

$$G = (1 - \delta) + I^k \tag{A2.8}$$

$$Y^k = C^k + I^k + \tau\psi QG \tag{A2.9}$$

$$Y^k \Delta = Y^{i,k} \tag{A2.10}$$

$$v = (1 - \gamma)\,\Lambda\,(R^q - R) + \gamma\Lambda x v \tag{A2.11}$$

$$h = (1 - \gamma) + \gamma\Lambda t h \tag{A2.12}$$

$$\phi^p = \frac{h}{\lambda - v} \tag{A2.13}$$

$$t = (R^q - R)\,\phi^p + R \tag{A2.14}$$

$$x = t \tag{A2.15}$$

$$QG = \phi^T F^k \tag{A2.16}$$

$$F^k = F^{o,k} + F^{n,k} \tag{A2.17}$$

$$F^{o,k} = \gamma\left[(R^q - R)\,\phi^p + R\right] F^k \tag{A2.18}$$

$$F^{n,k} = (1-\gamma)\,\omega Q \tag{A2.19}$$

$$X = P^i \mu \frac{1 - \theta\beta\Pi^{\epsilon-1}}{1 - \theta\beta\Pi^{\epsilon}} \tag{A2.20}$$

$$X = \left[\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right]^{\frac{1}{\epsilon-1}} \tag{A2.21}$$

$$\Delta = \frac{(1-\theta)\,X^{-\epsilon}}{1-\theta\Pi^{\epsilon}} \tag{A2.22}$$

$$R^{st} = R\Pi \tag{A2.23}$$

$$\phi^{T} = \frac{1}{1-\psi}\phi^{p} \tag{A2.24}$$

$$\psi = \psi \tag{A2.25}$$

These are the normalized equations evaluated in the steady state. We must now define the reduced system in order to find the endogenous equilibrium. If we equalize (A2.1) and (A2.4) and we additionally use (A2.6), (A2.8), (A2.9), (A2.20) and (A2.21), we can solve for G:

$$G = \frac{1}{1+\tau\psi}\left[\frac{N^{1-\alpha}}{\Delta} - \frac{X\left(1-\alpha\right)\Psi}{\chi\mu\Upsilon}N^{-\varphi-\alpha} + (1-\delta)\right] \tag{A2.26}$$

where $\Upsilon = \left(1-\theta\beta\Pi^{\epsilon-1}\right), \Psi = \left(1-\theta\beta\Pi^{\epsilon}\right)$. Furthermore, from (A2.11)-(A2.15) we obtain that:

$$\left(R^{q}-R\right) = \frac{\lambda\left(1-\gamma\right)-v}{\lambda-v}\frac{Gv}{\beta\left(1-\gamma\right)} \tag{A2.27}$$

Now, from (A2.2), (A2.3), (A2.7), (A2.13), and (A2.16)-(A2.19):

$$1 = \frac{\left(1-\gamma\right)\omega v}{(1-\psi)\beta\left(R^{q}-R\right)\left(\lambda-v\right)\left[1-\gamma\frac{G}{\beta}\frac{\lambda}{\lambda-v}\right]} \tag{A2.28}$$

Replacing (A2.27) in (A2.28):

$$G = \frac{(1-\gamma)^2 \omega}{(1-\psi)\left[\lambda(1-\gamma)-v\right]\left[1-\gamma\frac{G}{\beta}\frac{\lambda}{\lambda-v}\right]} \tag{A2.29}$$

From (A2.2), (A2.3), (A2.5), (A2.6), (A2.7) and (A2.20):

$$(R^q - R) = \frac{\alpha X \Psi}{\mu \Upsilon} N^{1-\alpha} + (1-\delta) - \frac{G}{\beta} \tag{A2.30}$$

Equating (A2.27) and (A2.30):

$$N = \left\{ \frac{\mu \Upsilon}{\alpha X \Psi} \left[\left(1 + \frac{\left[\lambda(1-\gamma)-v\right] v}{(\lambda - v)} \right) \frac{G}{\beta} - (1-\delta) \right] \right\}^{\frac{1}{1-\alpha}} \tag{A2.31}$$

Introducing the last expression in (A2.26):

$$G = \frac{1}{1+\tau\psi} \left\{ \frac{\mu \Upsilon \Theta}{\alpha X \Psi} \left[\frac{1}{\Delta} - \frac{(1-\alpha) X \Psi}{\chi \mu \Upsilon} \left(\frac{\mu \Upsilon \Theta}{\alpha \Psi X} \right)^{\frac{-(1+\varphi)}{1-\alpha}} \right] + (1-\delta) \right\} \tag{A2.32}$$

where $\Theta = \left[\left(1 + \frac{[\lambda(1-\gamma)-v]v}{(1-\gamma)(\lambda-v)} \right) \frac{G}{\beta} - (1-\delta) \right]$

A.3 Log-linearized model

The accented variables refer to the logarithmic deviation with respect to its steady state value.

$$\tilde{w}_t^k = \tilde{c}_t^k + \varphi \tilde{n}_t \tag{A3.1}$$

$$\tilde{\Lambda}_{t,t+1} + \tilde{r}_t = 0 \tag{A3.2}$$

$$\tilde{\Lambda}_{t,t+1} = \tilde{c}_t^k - \tilde{c}_{t+1}^k - \tilde{g}_{t+1} \tag{A3.3}$$

$$\tilde{w}_t^k = \tilde{p}_t^i + \tilde{y}_t^{i,k} - \tilde{n}_t \tag{A3.4}$$

$$R^q \tilde{r}_{t+1}^q = \alpha Y^{i,k} P^i \left(\tilde{p}_{t+1}^i + \tilde{y}_{t+1}^{i,k}\right) + \tilde{q}_{t+1} - R^q \tilde{q}_t + (1-\delta)\,\xi_{t+1} \tag{A3.5}$$

$$\tilde{y}_t^{i,k} = a_t + \alpha \xi_t + (1-\alpha)\,\tilde{n}_t \tag{A3.6}$$

$$\tilde{\imath}_t^{n,k} = \frac{I^k}{I^{n,k}} \tilde{\imath}_t^k \tag{A3.7}$$

$$\tilde{g}_{t+1} = \frac{I^{n,k} \tilde{\imath}_t^{n,k}}{G} \tag{A3.8}$$

$$\tilde{q}_t = \frac{\varsigma}{I^{n,k}} \left[\left(\tilde{\imath}_t^{n,k} - \tilde{\imath}_{t-1}^{n,k}\right) - \tilde{\Lambda}_{t,t+1}\left(\tilde{\imath}_{t+1}^{n,k} - \tilde{\imath}_t^{n,k}\right)\right] \tag{A3.9}$$

$$\tilde{y}_t^k = \frac{C^k}{Y^k} \tilde{c}_t^k + \frac{I^k}{Y^k} \tilde{\imath}_t^k \tag{A3.10}$$

$$\tilde{y}_t^{i,k} = \tilde{\Delta}_t + \tilde{y}_t^k \tag{A3.11}$$

$$\tilde{v}_t = \tilde{\Lambda}_{t,t+1} + \frac{(1-\gamma)\,\Lambda}{v}\left(R^q \tilde{r}_{t+1}^q - R\tilde{r}_{t+1}\right) + \gamma\Lambda x\left(\tilde{x}_{t,t+1} + \tilde{v}_{t+1}\right) \tag{A3.12}$$

$$\tilde{h}_t = \gamma t\Lambda\left(\tilde{\Lambda}_{t,t+1} + \tilde{t}_{t,t+1} + \tilde{h}_{t+1}\right) \tag{A3.13}$$

$$\tilde{\phi}_t^p = \tilde{h}_t + \frac{v}{\lambda - v}\tilde{v}_t \tag{A3.14}$$

$$\tilde{t}_{t,t+1}t = R^q\phi^p\tilde{r}_{t+1}^q + R\left(1-\phi^p\right)\tilde{r}_{t+1} + \phi^p\left(R^q - R\right)\tilde{\phi}_t^p \tag{A3.15}$$

$$\tilde{x}_{t,t+1} = \tilde{\phi}_{t+1}^p - \tilde{\phi}_t^p + \tilde{t}_{t,t+1} \tag{A3.16}$$

$$\tilde{q}_t + \tilde{g}_{t+1} = \tilde{\phi}_t^T + \tilde{F}_t^k \tag{A3.17}$$

$$\tilde{\phi}_t^T = \tilde{\phi}_t^p \tag{A3.18}$$

$$\tilde{\psi}_t = 0 \tag{A3.19}$$

$$\tilde{F}_t^k = \frac{F^{o,k}}{F^k}\tilde{F}_t^{o,k} + \frac{F^{n,k}}{F^k}\tilde{F}_t^{n,k} \tag{A3.20}$$

$$\tilde{F}_t^{o,k}F^{o,k} = \gamma F^k\left[\phi^p R^q \tilde{r}_t^q + R\left(1-\phi^p\right)\tilde{r}_t + \phi^p\left(R^q - R\right)\tilde{\phi}_{t-1}^p\right] + F^{o,k}\tilde{F}_{t-1}^{o,k} \tag{A3.21}$$

$$\tilde{F}_t^{n,k} = \tilde{q}_t + \xi_t \tag{A3.22}$$

$$\tilde{\pi}_t = \beta\left[\left(1-\theta\Pi^{\epsilon-1}\right)\left(\epsilon\left(\Pi-1\right)+1\right) + \theta\Pi^{\epsilon}\left(1+\Pi^{-1}\right)\right]E_t\tilde{\pi}_{t+1}$$

$$-\theta\beta^2\Pi^{\epsilon}E_t\tilde{\pi}_{t+2} + \frac{\left(1-\theta\beta\Pi^{\epsilon}\right)\left(1-\theta\Pi^{\epsilon-1}\right)}{\theta\Pi^{\epsilon-1}}\tilde{p}_t^i$$

$$-\beta\left(1-\theta\Pi^{\epsilon-1}\right)\left(1-\theta\beta\Pi^{\epsilon}\right)E_t\tilde{p}_{t+1}^i + \beta\left(\Pi-1\right)\left(1-\theta\Pi^{\epsilon-1}\right)E_t\tilde{g}_{t+1}^y \tag{A3.23}$$

$$\tilde{\Delta}_{t+1} = \theta\Pi^{\epsilon}\left(\epsilon\tilde{\pi}_{t+1} + \tilde{\Delta}_t\right) - \frac{\left(1-\theta\right)\epsilon X^{-\epsilon}}{\Delta}\tilde{X}_{t+1} \tag{A3.24}$$

$$r_t^{st} = r^n + \pi + \phi_\pi\tilde{\pi}_t + \phi_y\tilde{y}_t^k + \eta_t \tag{A3.25}$$

$$r_t^{st} = r_t + E_t\tilde{\pi}_{t+1} \tag{A3.26}$$

Appendix B

Appendix for the Third Chapter

B.1 State-space form of the model

To implement the Kalman filter procedure, equations (3.1-3.6) have to be expressed in the state-space form. This appendix describes the state-space model respecting the notation in the main text.

The measurement equation describes how the observations are derived from the internal state vectors:

$$\begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ 0 & \beta & 0 & 1-\rho_1^\pi & 0 \end{bmatrix} \begin{bmatrix} z_t \\ z_{t-1} \\ \Pi_t \\ \Pi_{t-1} \\ g_t \end{bmatrix} + \begin{bmatrix} 0 \\ \rho_1^\pi \end{bmatrix} \pi_{t-1} + \begin{bmatrix} 0 \\ \varepsilon_t^\pi \end{bmatrix} \tag{B1.1}$$

where ρ_1^π is the first element of the $\rho^\pi(L)$ lag-polynomial. The representation of the state equation indicates that the new state vector is modeled as a linear combination of the previous state and an error process:

$$\begin{bmatrix} z_t \\ z_{t-1} \\ \Pi_t \\ \Pi_{t-1} \\ g_t \end{bmatrix} = \begin{bmatrix} \rho_1^z & -\beta^r \epsilon & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho^\Pi \\ 0 & 0 & 1 \\ 0 & \rho^g & \varkappa \end{bmatrix} \begin{bmatrix} z_{t-1} \\ g_{t-1} \\ \Pi_{t-1} \end{bmatrix} + \begin{bmatrix} \beta^r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1-\rho^\Pi \\ 0 & 0 & 0 \\ 0 & \zeta & 0 \end{bmatrix} \begin{bmatrix} r_{t-1} \\ f_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \beta^r (\epsilon \partial - \varrho) \\ 0 \\ 0 \\ 0 \\ \partial (1-\rho^g) \end{bmatrix} + \begin{bmatrix} \varepsilon_t^z \\ 0 \\ \varepsilon_t^\Pi \\ 0 \\ \varepsilon_t^g \end{bmatrix} \tag{B1.2}$$

B.2 The Kalman filter

The Kalman filter is a semi-structural method to estimate unobserved variables. It is a suitable procedure because it provides a Minimum Mean Squared Error estimator if the observed variables and the noises are jointly Gaussian. To show the operation of the filter, let us define o_t as an $n \times 1$ vector, where o_t is an observable variable. This time series is a function of an $m \times 1$ vector, u_t, whose value and variance are unobservable. In order to simulate the latent variable, we have to specify a model as follows:

$$o_t = \delta_{1,t} + \delta_{2,t} u_t + \varepsilon_t^o \tag{B2.1}$$

$$u_{t+1} = \delta_{3,t} + \delta_{4,t} u_t + \varepsilon_t^u \tag{B2.2}$$

where $\delta_{i,t}$ are vectors and $\varepsilon_t^o, \varepsilon_t^u$ are vectors of Gaussian noises. The first equation (B2.1) is the measurement or observation equation whilst equation (B2.2) is the state or transition equation. Disturbance errors $\varepsilon_t^o, \varepsilon_t^u$ are serially independent, with the following variance structure:

$$\Omega_t = \text{var}\begin{bmatrix} \varepsilon_t^o \\ \varepsilon_t^u \end{bmatrix} = \begin{bmatrix} H_t & J_t \\ J_t' & B_t \end{bmatrix} \quad \text{(B2.3)}$$

where H_t is an $n \times n$ symmetric variance matrix, B_t is an $m \times m$ symmetric variance matrix, and J_t is an $n \times m$ matrix of covariances.

The smoothing procedure generates the estimates of the state variables $\hat{u}_t \equiv E_T\left(u_t\right)$ with variance $V_t^u \equiv \text{var}_T\left(u_t\right)$ and estimates of the signal variables $\hat{o}_t \equiv E\left(o_t \mid \hat{u}_t\right) = \delta_{1,t} + \delta_{2,t}\hat{u}_t$. The one-step ahead prediction error is $\breve{\varepsilon}_t^o = \varepsilon_{t|t-1}^o \equiv o_t - \breve{o}_{t|t-1}$ and the prediction error variance is $\breve{V}_t^o = V_{t|t-1}^o \equiv \text{var}\left(\varepsilon_{t|t-1}^o\right) = \delta_{2,t}P_{t|t-1}^o\delta_{2,t}' + H_t$, where $P_{t|t-1}^o$ is the mean square error of the one-step ahead mean.

The Kalman filter updates the one-step ahead estimate of the state mean and variance with new information and computes the one-step ahead estimates of the state and the associated mean square error matrix, the contemporaneous or filtered state mean and variance and the one-step ahead prediction, prediction error and prediction error variance.

Appendix C

Glossary of Variables and Parameters

Table C.1: Glossary of variables

C_t	Consumption
N_t	Labor supply
D_t	Real one-period life deposits and nominally riskless discount bonds
R_t	Real gross interest rate
Γ_t	Real firms profits and payouts
W_t	Real wage
T_t	Lump sum taxes
$\Lambda_{t,T}$	Discount rate
Y_t^i	Intermediate output
K_t	Capital stock
ξ_t	Capital quality shock
a_t	Aggregate productivity shock

Table C:1: Glossary of variables (cont.)

S_t	Financial claims
Q_t	Capital price
R_t^q	Capital return
P_t^i	Relative price of intermediate goods
I_t^n	Net investment
I_t	Gross investment
Y_t	Final output
P_t	Price index of the final output
P_t^*	Price set by those firms which change it
D	Steady-state gross inflation rate
Π_t	Real gross interest rate
X_t	Relative optimal price
F_t	Net wealth of financial intermediaries
V_t	Expected wealth of financial intermediaries
v_t	Marginal gain of the banks from expanding their assets
h_t	Expected value of having an additional unit of net wealth
$x_{t,T}$	Growth rate of assets
$t_{t,T}$	Growth rate of wealth
ϕ_t^p	Private leverage ratio
F_t^n	Financial wealth of the new bankers
F_t^o	Financial wealth of the old bankers
R_t^{st}	Short-term nominal interest rate
η_t	Monetary policy shock
S_t^{cb}	Loans issued by the central banks

Table C.1: Glossary of variables (cont)

S_t^p	Financial claims intermediated by financial intermediaries
D_t^{cb}	Riskless debt issued by the central bank
ψ	Proportion of claims intermediated by the central bank
ϕ_t^T	Leverage ratio of total intermediated funds
Δ_t	Price dispersion
G_t	Gross growth rate of capital
G_t^y	Gross growth rate of output
Π^*	Steady-state inflation rate target
R^*	Steady-state interest rate target
Π^{op}	Optimal inflation rate
z_t	Output gap
ε_t	Serially uncorrelated error term
R_t^n	Time varying natural rate of interest
f_t	Financial frictions
$\hat{u}_t^g$	Residuals quadratic function 3.7
$\hat{u}_t^f$	Residuals quadratic function 3.8

Table C.2: Glossary of parameters

σ	Proportion of households' members who are bankers
β	Discount term
χ	Relative utility weight of labor
φ	Elasticity of labor supply
α	Participation of capital
ρ_z	Technology shock persistence
ρ_ξ	Capital quality shock persistence
δ	Capital depreciation rate
ς	Investment adjustment costs
ϵ	Elasticity of substitution
θ	Probability of keeping prices fixed
γ	Survival rate of the bankers
λ	Fraction of bank assets that can be diverted
ω	Wealth proportion of the new bankers
ϕ_π	Coefficient of inflation in the Taylor rule
ϕ_y	Coefficient of output gap in the Taylor rule
ρ_η	Monetary shock persistence
τ	Central Bank efficiency costs
ψ	Steady state value of credit policy
μ	Retailers Markup
Θ	Auxiliary parameter
Υ	Auxiliary parameter
Ψ	Auxiliary parameter
$\kappa_1 - \kappa_1$	Coefficients of log-linear Phillips curve

Table C.2: Glossary of parameters (cont.)

b	Intensity of credit policy
ρ^{π}	Autoregressive parameter of inflation
β^{z}	Slope of the Phillips curve
ρ^{z}	Autoregressive parameter of output gap
β^{r}	Interest rate error coefficient
ϱ	Intercept in natural rate of interest equation
∂	Degree the productivity and population growth rates
ρ^{g}	Autoregressive parameter of output growth
ζ	Coefficient of financial frictions
$\varkappa$	Coefficient of trend inflation
ρ^{Π}	Autoregressive parameter of trend inflation
$\sigma^2_{\cdot}$	Variances

Table C.3: Glossary of superscripts and subscripts

Superscripts	t	Time
	j	Intermediate goods firms
	s	Retailers
	f	Financial intermediaries
Subscripts	k	Normalized variable
	$*$	Target

Variables without subscript t are the steady-state values.

Lower case variables are the log of the original variable.

Accented lower case variables are the log-deviations of the original variables from their steady-state levels.