PCEO Grado en Derecho / Grado en Administración y Dirección de Empresas
30605 - Mathematics II

Course 2013 - 2014

Curso: 1, Semestre: 2, Créditos: 6.0

Basic information

Teachers

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Recommendations to attend this course

The students should have a good command of all the contents of the subject Mathematics I, taught during the first semester of the first year. They must, in any case, know the meaning and implications of the differentiability of a function and be skilled in the calculus of partial derivatives. The students also have to know how to determine the sign of a quadratic form. They must also be able to present and support an argument with a logical sequence and to connect various mathematical aspects previously learnt.

Course Schedule and Deadlines

1. Presentation of the subject in the first session of the semester, in accordance with the timetable established by the Faculty.
2. Continual attendance at, and productive use of, theoretical and practical classes.
3. Attendance at practical classes P6, which may include computer practice if the global schedule allows it.
4. Midterm exams, scheduled in accordance with the academic calendar.
5. Final exam, on the day established by the Faculty.

Home

Learning outcomes that define this course

The student, in order to pass the course, will have to show her/his competence in the following skills:

1:
1. To have gained good skills in using mathematical language, both in comprehension and writing.
2. To be able to identify the fundamental elements of an optimisation problem: variables, objective function and constraints.
3. To be able to formulate static optimisation problems: unconstrained, and with equality and/or inequality constraints.
4. To know how to solve an optimisation problem by the graphical method, when that is possible.
5. To be able to evaluate whether or not a mathematical programme meets the conditions that allow it to be solved by the techniques learnt.
6. To be able to distinguish between critical points and extrema (optima).
7. To be able to discriminate between local and global optima.
8. To be able to distinguish between necessary conditions and sufficient conditions for local optimality.
9. To be able to calculate the critical points by solving the system of equations obtained by applying the first-order conditions for local optimality, both for unconstrained cases and for problems with equality constraints.
10. To know how to classify the obtained critical points by using the second-order conditions, both for unconstrained optimisation programmes and for problems with equality constraints.
11. To be able to apply the conditions which guarantee that an optimum is global.
12. To be able to interpret economically the Lagrange multipliers obtained in an optimisation problem with equality constraints.
13. To be able to evaluate whether a mathematical programme is linear. If it is, they must know how to solve it by the graphical method (when that is possible) and by the simplex algorithm.
14. When varying a parameter of a linear optimisation programme, the students must be able to analyse how the solution changes, without solving the new problem.
15. To be able to use some computer programmes to find the solution to an optimisation problem and to be able to interpret the results obtained.
16. To be able to identify a dynamic process in an economic scenario and be able to represent this process (when possible) by an ordinary differential equation.
17. To understand the concept of the solution of an ordinary differential equation and to be able to distinguish between general solution and particular solution.
18. To be able to discriminate between a first-order differential equation and a linear differential equation of order n.
19. To be able to identify whether a first-order differential equation is with separable variables, homogeneous, exact, or of linear type, and to know how to solve the equation by the appropriate method.
20. For a linear differential equation with constant coefficients, they must be able to write the complementary (homogeneous) equation and obtain its general solution.
21. To be able to find a particular solution of a linear differential equation with constant coefficients.
22. To have the know-how to calculate the general solution of a linear differential equation with constant coefficients.
23. To be able to work out the solution of a linear differential equation of order n with constant coefficients, given n initial conditions.

Introduction

Brief presentation of the course

Mathematics II is a basic-training subject with a value of 6 ECTS credits and it is taught during the second semester of the first year. It is based on and complements Mathematics I, a subject of the first semester of the first year.

The subject Mathematics II consists of two different parts: Mathematical Programming and Dynamical Analysis, which, respectively, apply to two different points of view of economic reality. After learning the first part, the students will be able to formulate and solve a wide variety of classical optimisation problems: both linear and non-linear, whether unconstrained or with equality and/or inequality constraints. In the case of optimisation programmes where both the objective function and the constraints are linear, the solving technique used is the simplex algorithm. This topic may be used to connect the traditionally-taught solving methods with the use of computer software, which simplifies the calculations and introduces students to professional practice.

The second part, Dynamical Analysis, is concerned with solving differential equations and with the analysis of the solutions. Its inclusion in the syllabus is necessary because many of the processes that Economic Analysis deals with are non-static. Some examples of these dynamic processes are: optimal economic growth, optimal management of renewable and non-renewable resources, optimal long-term investment, etc.

Competences

General aims of the course

The expected results of the course respond to the following general aims

The general objectives of the mathematical subjects in this Degree are included in the following two main goals: (1) Mathematical education, (2) Training to apply Mathematics to the challenges that the students will encounter in their careers.

The subject Mathematics II supposes a step forward in these objectives which Mathematics I also dealt with. Education in Mathematics is significant not only because of the transmission of new concepts, but also because the students gain a rigorous and accurate perspective, as well as the capacity for abstraction and the scientific method that characterise Mathematics. Regarding the second goal, this subject introduces students to modelling, using the mathematical analysis approach through two different ways: classical optimisation, and dynamical analysis.

Context/Importance of the course for the master degree

As Mathematics is a tool and a support for other subjects that are essential in the education of the students (Microeconomics, Macroeconomics, Econometrics, etc.), Mathematics II continues the line of work of Mathematics I by bringing Mathematics closer to problems in economic scenarios, which will undoubtedly facilitate a deeper comprehension of and, as a consequence, better skills in applying Mathematics.

After passing the mathematical subjects in the Degree, the students will have worked towards attaining one of the most important goals of mathematical theory: to formulate models that explain the real world. Prospective graduates will be able to use the language of science and to understand the role played by Mathematics in the development of their thinking skills, given that the students' logical reasoning, accuracy, rigour, capacity for abstraction and skills in interpreting results will be improved. This is why the subjects of Mathematics are indispensable tools which allow the designing of appropriate models that are used for researching, describing, understanding and thinking about the realities of companies

After completing the course, the student will be competent in the following skills:

1:
Problem solving

2:
Analysis and synthesis

3:

Decision-making.

4:
Applying knowledge to practice

Relevance of the skills acquired in the course

They permit the comprehension of theoretical concepts and models that are part of the contents of other related subjects studied in the Degree. Mathematics is most important in this goal because it facilitates the analysis and discussion of the models and concepts studied. In this regard, it is worth mentioning that Optimisation techniques allow the laying of the foundations of the two basic paradigms of Microeconomics, namely, the theory of consumer choice and the production theory. The concepts of convex set and concave/convex function, whose economic interpretations are, respectively, the diversity in consumption and the law of diminishing marginal returns, have important applications. Linear Programming is very useful in production planning problems and it allows the solving of some simple exercises of comparative statics. Different techniques are required for the analysis of dynamic processes in continuous time, which is essential, for example, in models of economic growth. The theory of differential equations provides the necessary tools to deal with some key concepts such as trajectory over time, evolution of the system, stability, etc.

Evaluation

Assessment tasks

The student will prove that he/she has achieved the expected learning results by means of the following assessment tasks:

1:

In the first sitting (June), the students have the possibility of passing the subject Mathematics II by continuous assessment which consists of two midterm exams.

The first midterm test is worth 5 points, and will take place between Thursday 24 April and Wednesday 30 April 2014, inclusive. The second midterm exam is worth 5 points and will take place during the last class week of the semester or on one of the dates scheduled by the Faculty.

Both midterm tests will take the form of a written exam. The list of contents to be assessed in each test will be supplied during class hours and/or on the virtual learning platforms used by the teacher. In order to pass the continuous assessment, **the students must take both exams** and must obtain a minimum mark of 25% in each test. The final mark by continuous assessment will be calculated as the sum of the marks obtained in the two midterm exams.

The students that do not take the continuous assessment, do not pass it or want to obtain a higher mark, can take the final exam on the date scheduled by the Faculty. The final mark will be whichever is the highest one obtained in the continuous assessment or in the final exam.

Any student who has not yet passed may take the second sitting in September. This will take the form of a written final exam. The test will take place within the period established by the Governing Council of the University.

The final tests will take the form of a written exam worth 10 points. These exams will assess many of the stated learning outcomes and they will correspond to the teaching given during the class period.

All the tests performed for this subject will assess the students' level of knowledge of the theoretical and practical contents that were taught during the lessons, their correct use of mathematical language and their command of the problem-solving skills taught by the teacher.

Students taking their exams at their fifth or sixth opportunity will be marked following the rules established under the Governing Council Agreement on 22 December 2010, which sets out the assessment regulations in the University of Zaragoza.

Activities and resources

Course methodology

The learning process that has been designed for this course is based on the following activities:

The objective of this subject is that the students should develop the analytical skills, rigour and intuition needed for using mathematical concepts and results and that they should be able to apply these abilities to the analysis of problems of an economic nature. Therefore, the teaching should aim to provide students with a solid mathematical knowledge and to train them in a way of reasoning that will allow them thereafter to successfully solve a wide variety of questions in an economic scenario.

Outline of the Programme

The programme offered to the students to help them achieve the learning results includes the following activities :

1:

Theoretical lessons which will be based on lectures to present the concepts and results corresponding to the contents. At the same time, some exercises will be solved with the participation of the students to help them comprehend the theoretical concepts presented. These classes are face-to-face and will be given to the full group.

Time allotted: 1.2 ECTS credits (30 hours).

2:

Practical lessons, in which the students will apply the theoretical results in order to solve, with the teacher's help, more complete exercises, and problems of an economic nature. Problem sheets will be available for the students and the teacher will announce in advance the problems that will be solved in each practical lesson so that the students can prepare them beforehand. These classes are face-to-face and will be given separately to each subgroup.

Time allotted: 1.2 ECTS credits (30 hours each subgroup).

3:

Seminars (practical classes P6), which may consist of a number of different activities designed to support the learning process, including: follow-up of some simple projects that had been assigned to small teams of students and the presentation of these projects; answering questions that students may have regarding some of the contents taught; solving problems of an economic nature by using some of the mathematical tools taught during the classes, etc. These seminars may also be devoted to the teaching of more advanced topics, intended for the students interested in learning some further mathematical tools that would allow them to deal with more general problems. In this way, the students are shown that both Mathematics and Economics are vibrant sciences with many facets to be studied.

Time allotted: 3 hours each subgroup

4:

Out of class work: 3.6 ECTS credits.

Course planning

Calendar of actual sessions and presentation of works

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1.3.3 Convex programmes.

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2.2.1 First-order conditions for local optima.

2.2.2 Second-order conditions for local optima.

2.3 Global optima. Convex programmes.

Unit 3: Programming with equality constraints

3.1 Form of the problem.

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3.2.1 First-order conditions for local optima.

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3.3 Global optima. Convex programmes and Weiertrass theorem.

3.4 Economic interpretation of Lagrange multipliers.

Unit 4: Linear programming

4.1 Form of the problem.

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4.3 The simplex algorithm.

4.4 Duality.

4.5 Post-optimal analysis.

PART II. Dynamical analysis

Unit 5: Introduction to ordinary differential equations.

5.1 Introduction to dynamical analysis.

5.2 Concepts of differential equation, solution, and types of solutions.

5.3 First-order ordinary differential equations:

5.3.1 Differential equations with separable variables.

5.3.2 Homogeneous differential equations.

5.3.3 Exact differential equations.

5.3.4 Linear first-order differential equations.

5.4 Linear differential equations of order n with constant coefficients:

5.4.1 Basic definitions and fundamental theorems.

5.4.2 General solution of the complete equation.

Bibliographic references of the recommended readings

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