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Beyond Special and General Relativity. Neutrinos and Cosmology.

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This PhD thesis has been written as a compendium of the articles I wrote in collaboration with my advisor, José Luis Cortes Azcoiti, with José Manuel Carmona and with Stefano Liberati. The complete references of the corresponding publications are given below.
For administrative reasons, the chapters corresponding to "Introduction" and "Summary and Conclusions" of the thesis are translated to Spanish.

- J. L. Cortes and J. Indurain, *"An extension of the cosmological standard model with a bounded Hubble expansion rate,"* Astroparticle Physics **31** (2009) 177 [arXiv:0805.3481 [astro-ph]].

- J. L. Cortes and J. Indurain, *"Linearized Treatment of Scalar perturbations in the Asymptotic Cosmological Model,"* accepted for publication in Astroparticle Physics.

- J. M. Carmona, J. L. Cortes and J. Indurain, *"Interpretation Of Neutrino Oscillations Based On New Physics In The Infrared,"* Journal of High Energy Physics **0806** (2008) 033 [arXiv:0709.2267 [hep-ph]].

- J. Indurain and S. Liberati, *"The Theory of a Quantum Noncanonical Field in Curved Spacetimes,"* Physical Review D **80** (2009) 045008 [arXiv:0905.4568 [hep-th]].

Como director de la tesis titulada "Beyond Special and General Relativity: Neutrinos and Cosmology"

AUTORIZO

a Javier Induráin a presentar dicha tesis como un compendio de publicaciones.

En Zaragoza a seis de Noviembre de 2009.

Firmado: José Luis Cortés Azcoiti

José V. García Esteve, Director del Departamento de Física Teórica de la Universidad de Zaragoza, como órgano responsable del programa de doctorado de Física,
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que, dado que el trabajo del doctorando Javier Induráin Gaspar ha dado lugar a la publicación de suficientes artículos en revistas de reconocido prestigio y que la memoria presentada reúne todos los requisitos, resulta apropiada la presentación de su tesis doctoral como compendio de publicaciones.

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A mis padres.

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Conventions and Abbreviations

Throughout this thesis and unless it is otherwise stated, we will use units in which $\hbar = c = 1$. Moreover we will use the convention in which the signature of the metric is $+ - - -$.

We will use the following abbreviations:

ACM : Asymptotic Cosmological Model

ADM : Arnowitt-Deser-Misner formalism

BAO : Baryon Acoustic Oscillations

$\beta\beta 0\nu$: Neutrinoless Double Beta Decay

CMB : Cosmic Microwave Background

CνB : Cosmic Neutrino Background

CSM : Cosmological Standard Model

DE : Dark Energy

DM : Dark matter

DSR : Doubly (or Deformed) Special Relativity

EFT : Effective Field Theory

FRW : Friedman-Robertson-Walker

GR : General Theory of Relativity

GUT : Grand Unified Theories

HDM : Hot Dark Matter, for instance massive neutrinos

HST : Hubble Space Telescope

ISW : Integrated Sachs-Wolfe effect

IR : Infrared

ΛCDM : the Concordance Model, in which the universe is dominated today by a mixture of Cold Dark Matter (CDM) and a Cosmological Constant (Λ)

Conventions and Abbreviations

LIV : Lorentz Invariance Violation

LQG : Loop Quantum Gravity

LSND : Liquid Scintillator Neutrino Detector

LSS : Large Scale Structure

MSW : Mikheev-Smirnov-Wolfenstein

NCQM : Noncommutative Quantum Mechanics

PMNS : Pontecorvo-Maki-Nakagawa-Sakata

QED : Quantum Electrodynamics

QFT : Quantum Field Theory

QNCFT : Quantum Theory of Noncanonical Fields

SM : Standard Model of particle physics

SME : Lorentz Violating Standard Model Extension

SNe : Type Ia Supernovae

SNO : Sudbury Neutrino Observatory

SR : Special Relativity

SSB : Spontaneous Symmetry Breaking

ST : String Theory (or Theories)

UV : Ultraviolet

vev : Vacuum expectation value

VSL : Varying Speed of Light

WMAP : Wilkinson Microwave Anisotropy Probe

WMAP5 : WMAP five year dataset

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1. Introduction

By the beginning of the XXI century, the General Theory of Relativity (GR) and the Standard Model of particle physics (SM) seem to offer a very accurate description of all known physical phenomena... All of them? Not really, yet neutrino oscillations and cosmology still resist a full understanding.

1.1. The Accelerating Universe

Since the discovery of GR by Albert Einstein in 1915, there has been great interest in applying its equations to determine the behavior of the universe as a whole. Einstein was the first to open the Pandora's box in 1917, presenting a model of a static universe which required the presence of a 'cosmological constant' in the field equations. Friedman found in 1922 that the equations admitted expanding universe solutions without the presence of this term. The expansion of the universe was also studied by Lemaître, Robertson and Walker, and confirmed seven years later by Hubble. Thus Einstein rejected the cosmological constant, calling it his 'greatest blunder'.

Consider the Einstein's equations of GR

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \,, \tag{1.1.1}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $g_{\mu\nu}$ is the metric of spacetime, $R = R_{\mu\nu}g^{\mu\nu}$ is the curvature scalar and $T_{\mu\nu}$ is the stress-energy tensor of the fluid filling spacetime. An isotropic and homogeneous universe is described by the Friedman-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega_3^2 \right] \,, \tag{1.1.2}$$

where $\{t, r, \theta, \phi\}$ are the comoving coordinates, $a(t)$ is the scale factor, k is the spatial curvature, and Ω_3 is the solid angle of three-dimensional space $d\Omega_3^2 = d\theta^2 + sin^2\theta d\phi^2$. The present value of the scale factor $a_0 = a(t_0)$ is usually normalized to one.

According to GR, this metric evolves following the relation (known as Friedman Equation)

$$\frac{8\pi G}{3}\rho = H^2 + \frac{k}{a^2} \,, \tag{1.1.3}$$

where ρ is the total energy density and H is the Hubble parameter, the rate of expansion of the universe ($H = \frac{da}{adt}$). Moreover the energy density of massive (matter) or massless (radiation) particles decreases with the scale factor as $\rho_m \sim a^{-3}$ or $\rho_r \sim a^{-4}$, respectively. This scenario, together with the conditions of an initially radiation-dominated universe which undergoes an adiabatic expansion, makes up the 'Cosmological Standard Model' (CSM, as named by Guth [1], by analogy with the Standard Model of particle physics). The solutions of the resulting differential equation for the scale factor are always decelerating ($\ddot{a} < 0$).

The CSM has achieved a number of successes, including the correct prediction of the relative densities of nuclei heavier than hydrogen (Nucleosynthesis) or the prediction of the Cosmic Microwave Background (CMB), the most perfect black body radiation ever measured. Since the

first observations by Hubble it has however been very difficult to directly measure the precise effect of the cosmic expansion on the motion of galaxies.

This has been connected to the difficulty of finding 'standard candles', astronomical objects with known luminosity L (energy released per unit time). Given the luminosity of an object and its spectrum, it is possible to directly measure its redshift ($z = \frac{a_0}{a} - 1$) and bolometric flux F (the energy detected per unit of area and time), where the relation between luminosity and flux is encoded in the luminosity distance d_l to the astronomical object, defined as $d_l \equiv \left(\frac{L}{4\pi F}\right)^{1/2}$. The luminosity distance vs. redshift (which is in one to one correspondence with time) relation enables us to determine the evolution of the scale factor and the Hubble parameter with time[1],

$$d_l(z) = c(1+z) \int_0^z \frac{dz'}{H(z')}, \tag{1.1.4}$$

and therefore gives us information about the relative importance of radiation, matter or spatial curvature in the evolution of the universe. As we told above, for any universe driven by a combination of those (the massless and massive particles contained in the Standard Model of particle physics (SM) or not, and the spatial curvature allowed in the FRW metric) will undergo a decelerated expansion, as gravity acts as an attractive force.

However, measurements of distant Type Ia Supernovae (SNe) (widely accepted standard candles) distance vs. redshift relation reveal that the universe is presently undergoing an accelerated expansion [5, 6], in contradiction with the decelerated expansion predicted by GR!

1.1.1. A conventional explanation: bring back the cosmological constant!

Now it is not a secret that GR admits a cosmological constant Λ, a constant term in the Einstein-Hilbert lagrangian

$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}_m\right], \tag{1.1.5}$$

where g is the determinant of the metric and $\mathcal{L}_m$ is the Lagrangian of the matter content of spacetime from which the stress-energy tensor is derived. The cosmological constant term Λ is able to produce the desired accelerated expansion, as we will explain below. It modifies the Friedman equation giving

$$\frac{8\pi G}{3}\rho = H^2 + \frac{k}{a^2} - \frac{\Lambda}{3}. \tag{1.1.6}$$

Given that the universe is expanding, at late times the effect of the energy density ρ and curvature k will be negligible and the universe will expand almost exponentially with

$$H = \sqrt{\frac{\Lambda}{3}}. \tag{1.1.7}$$

We arrive then to the Concordance Model (ΛCDM), in which the universe is homogeneous and isotropic in cosmological scales, with a spatial curvature k and a cosmological constant Λ, and is filled by a mixture of radiation (mainly photons and neutrinos), massive particles of the standard

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[1]This relation is computed in the framework of metric spacetimes with a FRW metric. For objects of sufficient size, it is possible to compare this quantity to their angular size distance $d_A \equiv l/\Delta\theta$, where l is the transverse size of the object and $\Delta\theta$ is its angular size (the angle that it subtends in the sky). In a metric spacetime, $d_l = d_A(1+z)^2$. This relation can be used to test the validity of metric theories as a good description of spacetime [2, 3]. However experimental data seem to support this relation [4]. Thus we will not leave the framework of generally covariant metric spacetimes.

model (mainly baryons, as electrons have a much smaller mass) and new massive particles which are non-baryonic and electrically neutral which we call Dark Matter (DM). If we divide (1.1.6) by H^2 and regroup terms we arrive to the cosmic balance equation,

$$\Omega_r + \Omega_b + \Omega_{DM} + \Omega_k + \Omega_\Lambda = 1 \,, \tag{1.1.8}$$

where $\Omega_i = \rho_i/\rho_c$ for $i = r, b, DM$, $\rho_c = \frac{3H^2}{8\pi G}$ is the critical density, $\Omega_k = -\frac{k}{a^2 H^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H^2}$. The Ω's represent the relative importance of each of the components of the universe at a given time, and this relative importance vary.

This simple explanation fits comfortably the data collected by the SNe searchers, from the first to the last datasets [5, 6, 7, 8, 9], which comprehends a few hundreds of SNe. It also provides a good explanation of the measured spectrum of CMB anisotropy [10, 11, 12, 13] and of the spectrum of matter anisotropy [14, 15, 16, 17] if the universe is taken to be spatially flat, $k = 0$ [18]. In particular, the joint fit of the model to the whole set of experimental data gives, in the present time:

$$H_0 = 72 \pm 3\,km/sMpc\,, \quad \Omega_r = (4.8 \pm 0.5) \times 10^{-5}\,,$$
$$\Omega_b = 0.044 \pm 0.004\,, \qquad \Omega_{DM} = 0.22 \pm 0.2\,,$$
$$-0.018 < \Omega_k < 0.008\,, \qquad \Omega_\Lambda = 0.726 \pm 0.015\,.$$

Most of the errors in the data are dominated by the error in the determination of H_0. If this is the end of the story, what is the point of the cosmic acceleration that we are not understanding?

1.1.2. The Problems of ΛCDM

The Cosmological Constant Problem Although a low value of the cosmological constant is compatible with the observations, theory predicts that it has a high value. It would not be any problem if the theory were classical, but the theory describing the source of the gravitational interaction $\langle T_{\mu\nu} \rangle$ is QFT in curved spacetime. Thus, the semiclassical Einstein equations valid for cosmology at curvatures well below the Planck scale ($E_P = 1.2209 \times 10^{19}\,GeV$ / $l_P = 1.6165 \times 10^{-35}\,m)^2$ are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R + \Lambda) = 8\pi G \langle \hat{T}_{\mu\nu} \rangle \,, \tag{1.1.9}$$

where the curvature terms are treated classically and $\langle \hat{T}_{\mu\nu} \rangle$ is the expectation value of the stress-energy tensor computed for a QFT in curved spacetime [19]. In order to renormalize the contributions of the matter Lagrangian to the total action of matter and gravitation, it is necessary to add counterterms also in the gravitational Lagrangian that cancel some of the infinities arising in the matter Lagrangian. Some of these counterterms are of the same form of the ones appearing in the classical Lagrangian (R, Λ) and some present more than two derivatives of the metric.

The term of interest is the divergent term which has the same form as the cosmological constant Λ. If we compute the value of this contribution in the frame of Effective Field Theory (EFT) [20], we assume that the SM is valid up to a high energy scale (which we will call ultraviolet (UV) scale), which we assume to be of the order of the Planck scale. Then this contribution is $\sim 10^{71} GeV^4$, that is, 120 orders of magnitude greater than the observed value, $\sim 10^{-47} GeV^4$ [21]. In order to explain the data, the quantum corrections $8\pi G\langle \hat{T}_{\mu\nu}^{vac} \rangle$ should cancel the first 120 significant figures of the classical contribution Λ, but not the next one! Such an unnatural cancelation is called the Cosmological Constant Problem.

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[2]At this scales gravity cannot be treated as a classical interaction any more, as we will explain later.

The solutions to the Cosmological Constant Problem among the quantum gravity community are not very convincing. In String Theory (ST) the natural value of the cosmological constant is negative and of order E_p^4. This has led the ST community to resort to anthropic arguments in order to save the face (see Ref. [23] for arguments and references). In Loop Quantum Gravity (LQG) it has been claimed that a positive small cosmological constant is possible in a solution called Kodama state [24]. However the dispersion relation of particles living in this spacetime have at energies well below the Planck scale a modified dispersion relation of the form

$$E^2 = \mathbf{p}^2 + m^2 + \alpha \frac{E^3}{E_p} \, , \tag{1.1.10}$$

which has been already almost ruled out by observation [25, 26, 27].

The problem is that we have quantum corrections coming from the UV behavior of a QFT to an otherwise infrared (IR) scale, i.e. of lower energy than the typical energies in particle physics, of GR. Is there any possibility that the quantum corrections coming from QFT do not stop the cosmological constant being IR? EFT relies in some basic principles, and the one responsible of the quantum corrections to the vacuum energy being of the order of the UV scale is the decoupling theorem [22]. Any mechanism able to avoid UV quantum corrections to the stress-energy tensor must violate this property. It would then be possible to have a QFT which is valid in a window of energies bounded by an IR and an UV scale, in which the main quantum corrections to the Einstein's equations come from the IR.

The Horizon Problem The present homogeneous, isotropic domain of the universe is at least as large as the present horizon scale $ct_0 \sim c/H_0$, where H_0 is the present value of the Hubble parameter. This domain has grown up with the scale factor. As inhomogeneity cannot be erased with the decelerated expansion, we can conclude that the size of the patch of the universe from which this domain originated was at $t = t_i$ at least as big as $l_i \sim ct_0 a_i/a_0$. If we compare this with the causal horizon at the same time, $l_c = ct_i$, then the ratio between the two scales is,

$$\frac{l_i}{l_c} \sim \frac{t_0}{t_i} \frac{a_i}{a_0} \, . \tag{1.1.11}$$

Thus, if we assume some random initial conditions at t_i, which we will assume to be of Planckian size because we consider that GR is valid up to that scale, and $a \sim 1/T$ under radiation domination, we get

$$\frac{l_i}{l_c} \sim \frac{10^{17} s}{10^{-43} s} \frac{2.73 K}{10^{32} K} \sim 10^{28} \, , \tag{1.1.12}$$

and therefore that patch was not causally connected, so it did not need to be homogeneous at all! This is a big problem, as the initial conditions should be very fine-tuned to reproduce our current universe. This is the Horizon Problem.

Unless there is something we are missing. Let us assume that the scale factor grows with a power of time, and therefore $\dot{a} \sim a/t$. Thus we can rewrite

$$\frac{l_i}{l_c} \sim \frac{\dot{a}_i}{\dot{a}_0} \, . \tag{1.1.13}$$

From the last expression we can read that the size of our universe was initially larger than that of a causal patch by the ratio of the corresponding expansion speeds. Only if these are of the same order can the initial conditions at t_i thermalize and give rise to an homogeneous universe. And as gravity has been decelerating this rate for a wide period of time, there must

have been a fast accelerated expansion before the decelerated one that has correctly described so much physics. This points out to new particle or gravitational physics in the UV, but below the Planck scale.

The Flatness Problem Current observations tell us that the curvature term in the Friedman equation is very close to zero. In particular, if we write the cosmic balance equation (1.1.8) and neglect Ω_Λ, the cosmological parameter $\Omega(t)$ follows

$$\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{k}{(Ha)^2} \,. \tag{1.1.14}$$

Therefore, at early times,

$$\Omega(t_i) - 1 = (\Omega_0 - 1)\frac{(Ha)_0^2}{(Ha)_i^2} = (\Omega_0 - 1)\frac{(\dot{a})_0^2}{(\dot{a})_i^2} < 10^{-56} \,. \tag{1.1.15}$$

Taking into account that this difference can be interpreted as the relative difference between kinetic and potential energy in the cosmic fluid [28], this means that the initial velocities must be fine-tuned to a degree of 10^{-28} in order to explain the current flatness of the universe! This is the flatness problem.

This fine-tuning would not be so high if the expansion rate at Planckian time would be smaller than or at least as small as the present expansion rate, which would in turn require an early accelerated expansion, as we have seen before.

1.1.3. Ingredients for an alternative to ΛCDM

The problems of ΛCDM have their origin in our attempt of extending the range of validity of our current theories of gravity and particle physics outside the region in which those theories have been experimentally tested and validated. We have no idea of what the correct theory of gravity will be at extremely high (or low) curvatures compared to the ones that have been tested; and we have arguments that point out that a quantum theory of gravity may violate some of the principles underlying GR and QFT [29].

At this point it becomes evident that one must find a way of extending either GR, QFT or both in order to explain the problems of cosmology at very early or recent times. This extensions can be performed either following a top-down or a bottom-up approach. In a top-down approach the fundamental principles of the theories are changed, a new theory is derived, and the phenomenological consequences are derived. Examples of top-down approaches to the problems of cosmology are ST [30](and its applied branch of String Cosmology) or LQG [31, 32] (from which Loop Quantum Cosmology is derived). The aforementioned approaches have departed from a solid theoretical and mathematical base but have grown up on the sidelines of the experimental results, and some of their implications for cosmology are in conflict with observational data. We will show a couple of examples below.

In contrast, a bottom-up approach is based on experimental and observational data, which are used to build up laws and models which can later on be incorporated to a well founded theory. In this PhD thesis we will try to show that it is possible to build a phenomenological extension to ΛCDM at the level of the cosmological evolution equations. This extension, which we have called the Asymptotic Cosmological Model (ACM), incorporates a modification of the laws of cosmic expansion such that an epoch of (asymptotically exponential) accelerated expansion arises both in the IR (corresponding to nowadays Hubble rate) and in the UV (corresponding to the very early universe). The model fits successfully the current observational data. The study of

the origin of the modifications is precluded, although some possibilities ar pointed out in the following.

New physics in the IR   We have mentioned that in order to solve the Cosmological Constant Problem, we need to render the UV corrections to the vacuum energy density IR. This is not possible in the frame of local EFT, due to the decoupling theorem. In fact, this is a problem not only of ΛCDM, but of GR coupled to an EFT in general. Even if there is no classical contribution to the cosmological constant/vacuum energy in the gravitational lagrangian, the renormalization of the matter lagrangian will in general induce a nonzero term. [3] The problem is that the power counting of a local EFT implies that the corrections to this term will be much greater than the observed value. Therefore a solution to the Cosmological Constant Problem will presumably require a departure from locality in QFT, which is parameterized by some new IR scale.

It has already been noticed that local EFT should fail to be a good description of nature in the presence of gravitational effects [33]. In any EFT with a UV cutoff E_{NP}, the entropy of a system is an extensive quantity and therefore grows with its volume, $S \sim L^3 E_{NP}^3$. However blackhole physics seems to reveal that the maximum entropy contained in a certain box of size L must be the Bekenstein entropy, $S < \pi E_p^2 L^2$. This means that the entropy predicted by EFT can exceed the Beckenstein bound for sufficiently large sizes. Therefore it has been argued that EFT overcounts the degrees of freedom in some way [34, 35], and that the number of degrees of freedom of a given system must saturate from an extensive behavior (proper of EFT) to a proportionality with area, when the size of the region investigated is of order $L \sim E_p^2/E_{NP}^3$. The authors of Ref. [33] propose an even more restrictive domain of validity, based on the premise that the size of the region under consideration should not exceed the Schwarzschild radius associated with their temperature $T \sim E_{NP}$ for EFT to be valid. If EFT is used to describe particles up to temperatures T, then the associated thermal energy of the system will be of order $U \sim L^3 E_{NP}^4$ and the associated entropy will be of order $S \sim L^3 E_{NP}^3$. Then the Schwarzschild radius will be $r_s \sim U/E_p^2$. If we want the system under description not to form a blackhole, then its size needs to be greater than this quantity, $L > r_s$, and therefore the momenta of the particles we are describing will present an IR cutoff of order $\lambda \sim 1/L$, where

$$\lambda \sim E_{NP}^2/E_p \, . \tag{1.1.16}$$

This would generate, aside from corrections $\propto \frac{1}{E_{NP}^n}$ to the tree level effective action, also IR corrections $\propto \lambda^2$. As a consequence, in a given experiment at energies $\sim p$ the predictions of EFT for a given quantity Q will have corrections from both irrelevant terms in the action of dimension D and gravitational physics, of the form

$$\frac{\delta Q}{Q} \sim \left[\left(\frac{p^{D-4}}{E_{NP}^{D-4}} \right) + \left(\frac{E_{NP}^4}{p^2 E_p^2} \right) \right] \, . \tag{1.1.17}$$

The maximum possible accuracy will be attained by some value E_{NP} of the maximum energy scale that can be probed. As an example given in Ref [33], if the momenta of the particles are of the order of the electroweak scale (as in the new collider experiments, like LHC), and the EFT is the standard model, new physics appears at dimension 6 operators (5 if we allow for lepton number violation), the minimum uncertainty of 10^{-13} is achieved when the maximum scale that can be probed with EFT techniques is $10^8 GeV$. The minimum scale that could be probed would then be of the order of the MeV. However if new physics appears at $10 Tev$, then

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[3]With the remarkable exception of supersymmetric theories with unbroken supersymmetry. However, the absence of exact supersymmetry in nature discards this explanation.

the SM would be valid up to that energies and down to $10^{-2}eV$, around the energy scale which is behind neutrino physics or the cosmological constant.

In the same context, if one tries to compute the quantum corrections of the matter content of the universe to the cosmological constant in the context of EFT, one should notice that for this approach to be valid at a patch of the universe of the size of the causal horizon $\sim 1/H_0$, one should introduce an IR cutoff of the order of $H_0 \sim 10^{-26}eV$ and therefore the associated UV cutoff of the EFT would be $10^{-2}eV$. As a result, quantum corrections would be of the same order of the observed value of he vacuum energy.

It has been claimed that any EFT approach to a theory including gravity will be naturally attached to a range of validity bounded by an UV and an IR scale [36]. However, if the UV scale is taken to be the Planck scale, then the IR will also equal the Planck scale and the range of validity of the EFT will be constrained to a point. It may be that the effective fundamental scale of gravity in the EFT would be decreased by the existence of large extra dimensions [37, 38, 39, 40]. The effective gravity scale has been proposed to lie in the order of magnitude of the few TeV scale, and the IR scale (corresponding to extra dimensions of the size of a millimeter) would be of the order of the scale of the cosmological constant or neutrino physics.

Anyway, if local EFT is to be modified in the IR in order to solve the Cosmological Constant problem, it is also possible that the quantum corrections induced in the gravitational Lagrangian add not only terms with a higher number of derivatives of the metric suppressed by an UV scale, as in the standard case, but also other nonlinear or even nonlocal terms suppressed by the IR scale.

In fact, there is no reason to believe that the action of the classical theory of gravity is just given by a term proportional to the curvature scalar and a constant term, as in ΛCDM. With the premises of a metric theory with general covariance and the equivalence principle, it is possible to build an infinite plethora of theories. Einstein decided to choose the simplest one for the shake of aesthetics: he required his equations to be of second order, and this is only possible with a Lagrangian of the form $\sqrt{-g}(R - 2\Lambda)$. However in modern theoretical physics we have learned that everything that is not forbidden is compulsory [41]. Additional terms in the action are not forbidden by the symmetries of the theory as long as they behave as scalars under general coordinate transformations.

Terms with a higher number of spatial derivatives can be neglected by dimensional analysis, as they must be supplied by a certain scale. If this scale is a UV scale such as the Planck scale, then the effect of these terms can be neglected at most tests, with the remarkable exception of the very early universe. Terms of these type arise naturally when one tries to renormalize a QFT in curved space [19]. In this context even if they are not originally in the classical gravitational action, terms of the following form appear in the action:

$$S_g^{UV} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[-2\Lambda + R + \frac{1}{E_{UV}^2} \left(\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \delta \Box R \right) \right],$$
$$(1.1.18)$$

and there is no reason to believe that terms with even higher derivatives of the metric appear in the gravitational action, suppressed by greater powers of the inverse of the UV scale E_{UV}.

However, if these terms are supplied by an IR scale, like the cosmological constant, their effect should be observable at cosmological distances. Aside from a cosmological constant, terms involving an infrared scale should be at least non-polynomial or nonlocal,

$$S_g = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2\Lambda + \alpha E_{IR}^2 \Box^{-1} R + \beta \frac{E_{IR}^4}{R} + ... \right], \qquad (1.1.19)$$

where the effect of these terms can naïvely be neglected in some local tests of gravity but heavily influence the cosmic evolution at the present time (provided that $\Lambda \sim E_{IR}^2$).

New physics in the UV We have also pointed out that, in order to solve the Horizon Problem and the Flatness Problem of the universe, some sort of new physics in the UV is required. As the universe expands it also cools down from an "initial" state at Planckian temperature $\sim T_p = k_B^{-1} E_p \sim 10^{32} K$, whose description requires some knowledge of the correct theory of quantum gravity. Lacking any knowledge of such a theory, the best we can do is to assume that quantum gravity provides us with some random initial conditions that can, since some initial time $t_i \gtrsim t_P$, be treated with classical gravity together with some extra corrections suppressed by an UV scale. This new physics could maybe induce an accelerated expansion just after the regime in which gravity must be described by a quantum theory.

Many models have been built in order to explain this accelerated expansion. First models resorted to the coupling of the expansion to a phase transition in the context of GUT [42, 1, 43, 44]. Later it was discovered that the details of the particle physics behind the accelerated expansion were not important [45]. The addition of terms with higher derivatives suppressed by a UV scale [46] has also been used to explain this accelerated expansion (called inflation [1]). As an additional consequence the quantum fluctuation of fields in that regime is enough to cause the fluctuations of the CMB spectrum, and therefore clusters, galaxies, stars, planets and life. As said, the details of the mechanism are unimportant, as several *a priori* very different models lead to very similar conclusions.

As an example of new physics in the UV leading to an early accelerated expansion, let us introduce the very simple model of Ref. [46]. Assume that the Einstein-Hilbert action is just a low energy approximation to a more general theory,

$$S = \int d^4x \sqrt{-g} \left[R + \Lambda - \alpha \frac{R^2}{E_p^2} + ... \right] . \tag{1.1.20}$$

It has been shown [19] that such terms appear in the renormalized action of classical GR coupled to a QFT in curved spacetime. If we consider the theory modified just by the R^2 term, it turns out to be conformally equivalent (with a conformal factor $F = 1 + 2\alpha R/E_p^2$) to a theory of GR coupled to a scalar field φ,

$$S = \int d^4x \sqrt{-g} \left[R + \Lambda + 16\pi G (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{E_p^2}{12\alpha} \varphi^2) \right] , \tag{1.1.21}$$

with α a few orders of magnitude above unity. The potential of the scalar field can be seen as an effective mass term with $m^2 = \frac{1}{6\alpha} E_p^2$ A detailed study of the dynamical system will prove that this scenario is an example of chaotic inflation [45], in which almost random initial conditions lead to an exponential expansion and to an almost flat power spectrum for the primordial fluctuations in the metric of spacetime.

Let us assume that the initial condition of the scalar field is at least locally an homogeneous condensation. The equations of motion in the homogeneous approximation (FRW metric and $\varphi \approx \varphi(t)$) can be written as:

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi \;=\; 0 \,, \tag{1.1.22}$$

$$\frac{8\pi G}{3} \left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2 \right) \;=\; H^2 \,, \tag{1.1.23}$$

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where we are neglecting the effect of the spatial curvature ($H^2 \gg k/a^2$) for the sake of simplicity. The effect of spatial curvature can be studied numerically. The system can be then reduced to a first order dynamical system for $\dot{\varphi}(\varphi)$,

$$\frac{d\dot{\varphi}}{d\varphi} = -\sqrt{-12\pi G(\dot{\varphi}^2 + m^2\varphi^2)} - m^2\frac{\varphi}{\dot{\varphi}} \tag{1.1.24}$$

It should be noticed that this dynamical system possesses an attractor to which all other solutions converge with time. The attractor behaves as $\dot{\varphi}^2 = \frac{m^2}{12\pi G}$ and $sign(\dot{\varphi}) \neq sign(\varphi)$ for $\dot{\varphi}^2 \ll m^2\varphi^2$ and then forms a double decaying orbit around the origin. If a solution meets the attractor at one of its flat branches, in this regime the field varies slowly with time and the universe expands almost exponentially, with

$$H^2 \approx \frac{8\pi G}{3}\left(\frac{1}{2}m^2\varphi^2\right) \sim const. \tag{1.1.25}$$

until $\varphi \sim \frac{1}{\sqrt{12\pi G}}$. When the value of the field reaches the Planck scale its decrease rate is high and the field rapidly decays to $\varphi = 0$, providing the exit to inflation and transferring the energy of the scalar field to the matter content of the model (reheating). As the original theory is related to the scalar-metric theory by a conformal factor which depends only on R, which is approximately constant during the inflationary regime, then in the original theory the same inflationary solution exists.

One can arbitrarily complicate this model, but the general prediction of an early accelerated expansion (in most models close to an exponential expansion) leading to a flat, thermalized universe with small perturbations obeying a scale free power spectrum is rather general. Most of the models include modifications of gravity or a set of scalar fields which mimics the previous mechanism. Noticeably, the universe is devoid of matter (or emptied by the mechanism) until the accelerated expansion stops, and then refilled by the energy released.

Agreement with observation In the past decade a vast amount of observational data has been collected, that severely constrains most of the models that have been built for cosmology, especially those derived from top-down approaches like ST or LQG. At this point, it is no longer sufficient to build models which qualitatively explain the features in the universe, but also which provide us with competitive fits of the currently gathered data and desirably with predictions to be tested by future missions.

There is a list of cosmological observations that should be accommodated in an extension of ΛCDM:

- In any sensible model of inflation quantum (or thermal) fluctuations are produced, which seed the perturbations that will later produce the large scale structure of the universe, galaxies, and so on. Most inflationary models predict that the primordial scalar fluctuations will have an almost scale invariant power spectrum $P(k)$,

$$P(k) \equiv \frac{|\Phi_k|^2 k^3}{2\pi^2} \propto k^{n_s - 1}, \tag{1.1.26}$$

  where Φ_k is the Fourier transform of the gauge-independent scalar perturbation of the FRW metric in GR without a shear, and n_s is called 'scalar spectral index' (scale invariant spectrum means $n_s \approx 1$). The Starobinsky R^2 model [46] and in general chaotic inflation [45] predict $n_s \lesssim 1$. In fact, the scalar spectral index has managed to be measured in the CMB spectrum by WMAP [13], and it turns to be $n_s = 0.963^{+0.014}_{-0.015}$.

- Models for the present accelerated expansion must be confronted with a much wealthier sample of data. Given that Type I a Supernovae (SNe) can be taken as standard candles, it has been possible to determine the luminosity distance d_L vs. redshift $z = a_0/a - 1$ relation,

$$d_l(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')} \, , \qquad (1.1.27)$$

  for a few hundred SNe [8, 9] ranging for $z < 2$. Besides supernovae, the reduced distance to the surface of last scattering R_{CMB},

$$R_{CMB} = \Omega_m^{1/2} H_0 \int_0^{1089} dz/H(z) \, , \qquad (1.1.28)$$

  can be also read from the WMAP data [47]. Both measurements involve the integral of the inverse of the Hubble parameter with respect to the redshift, but SNe observations sample the present era whereas the reduced distance R_{CMB} provides an integrated information of the evolution of the universe since its youth.

  A slightly different dependence is found in the distance parameter of Baryon Acoustic Oscillations (BAO) $A(z)$,

$$A(z) = \Omega_m^{1/2} H_0 H(z)^{-1/3} z^{-2/3} \left[\int_0^z dz'/H(z') \right]^{2/3} \, , \qquad (1.1.29)$$

  measured at redshift $z = 0.35$ by SDSS [16]. Although some other data can be confronted with cosmological models, these are the most widely used probes in cosmology.

In conclusion, GR can be extended to include new IR and UV phenomena, and it should be extended in order to satisfactorily explain both the initial conditions of the universe and the current accelerated expansion. It should also be extended in order to comprehend the new corrections arising from the renormalization of the matter lagrangian coupled to gravity, especially if this Lagrangian is also modified in the IR or the UV.

The effect of this extension in cosmological scales should be to provide us with an accelerated expansion in the very early universe and another accelerated expansion in the present age. These accelerated expansions are in most cases asymptotically exponential expansion. In this PhD thesis a cosmological model will be built, in which these to epochs of accelerated expansion will be phenomenologically incorporated. The resulting model will be used to fit the available observational data.

1.1.4. Review on alternative models to explain the accelerated expansion

This subsection is a brief review on the main alternatives proposed in order to explain the problems concerning the present accelerated expansion and the very early universe.

The main paradigm explaining the Horizon and Curvature problems in the very early universe is inflation; either occurring due to a modification of GR in the UV or the addition of new fields at high energies in extensions of QFT such as ST. The only real alternative to the inflationary paradigm is that of Varying Speed of Light (VSL) [48]. We will explain both alternatives further below.

With the increasing interest on the present accelerated expansion, a wide plethora of models has risen. Aside from modifications of gravity (such as $f(R)$-theories, higher dimensional models,

alternatives to Dark Matter explaining the smallness of the cosmological constant), these models include new forms of matter (new fields of any type, perfect fluids with exotic equations of state,...) or rejection of spatial homogeneity as a good approximation in the present universe (effective cosmological constant arising from homogenization procedure [49, 50], local void as an explanation to the apparent accelerated expansion [51, 52, 53, 54],...). Some modifications of gravity try even to unify the explanation of inflation, the present accelerated expansion and/or Dark Matter. We will briefly introduce also some models of modified gravity and of exotic sources of the gravitational field.

Scalar fields for inflation   We have already mentioned that it is possible to drive the universe into an accelerated expansion era with the help of a set of scalar fields with an appropriate potential. These scalar fields might arise as fundamental fields in a QFT beyond the standard model (like in Ref. [1]) or as new modes in a modification of general relativity (as in Ref. [46]). As noticed in Ref. [45], the particular potential behind the accelerated expansion is unimportant, provided it satisfies some very general conditions, the 'slow-roll conditions'.

Since then, there has been much effort directed to find some viable inflation mechanisms in theories of quantum gravity. In particular, the string theory community has viewed the inflationary paradigm as a playground in which their ideas can be tested. Many possible realizations of inflationary scenarios have been found.

As ST lives in 10 dimensions, it needs the compactification of 6 extra dimensions in order to describe our world. The moduli, which are the parameters describing this compactification, are stabilized dynamically in the quantum theory, producing different vacua. There are numerous such solutions, with different values of the vacuum energy. The set of these vacua is called the moduli space of supersymmetric vacua. If we expand this set to include non-supersymmetric vacua[4], we arrive to the 'landscape' [55]. It was first proposed as a way of solving the cosmological constant problem: if there are multiple vacua and the cosmological constant differs from one vacua to the other, and considering that human life is only possible if the value of this constant lies in a certain range, then we must be living in one of the 'anthropically permitted' vacua.

The landscape offers multiple options for inflationary model-building, such as hybrid inflation [56]. In this model, inflation is driven by two scalar fields, with a potential

$$V(\phi, \chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi^2\phi^2 + \frac{\lambda}{4}(\chi^2 - \frac{M^2}{4})^2\,. \tag{1.1.30}$$

The main new ingredient in the hybrid inflation as compared to standard inflation is that the corresponding quantum fluctuations are non-gaussian due to the interaction. In contrast to single field inflation, hybrid inflation naturally gives $n_s \geq 1$ [57], and thus is ruled out as the mechanism realized in nature.

Other inflationary scenarios arise in the context of Dp-branes moving through extra dimensions. In the brane-world scenario, our spacetime corresponds to one of these $D3$-branes. The SM particles live stuck to the D-brane, for they are modes of open strings whose ends lie on it. Graviton and other open string modes are in contrast not confined. In 'brane-inflation' the inflaton corresponds simply to the position of this brane [58]. A simple model is $D3$-brane/$\bar{D}3$-brane, in which brane and antibrane experiment an attractive force and finally annihilate each other, causing the end of inflation [59]. Here the inflaton φ is the relative position of brane and anti-brane and the potential $V(\varphi)$ is due to the tensions and interaction. Energy is released with the annihilation and cosmic superstrings are left as a remnant. More realistic but much more complicated is the KKLMMT scenario [60]. In general, most of these models give $n_s \geq 0.98$

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[4]We are talking about 10^{500} possible vacua!

and predict the creation of a cosmic string network which has not been observed [61], and are therefore ruled out.

However, despite the interest of string theorist, stringy scenarios are seen by phenomenologists and cosmologists as very degenerate and very little predictive. The idea of the landscape and the 'anthropic selection of vacua' is often received with skepticism and polemic.

Alternatives to inflation: VSL   For more than a decade, inflationary cosmologies have not had any real competitor. However, in Ref. [48] an alternative mechanism was first proposed under the name of Varying Speed of Light (VSL). The proposal is that the local speed of light, as measured by comoving observers, might have varied with time in the very early universe from a very large value to its current value. This implies obviously a departure from SR at very high energies, and therefore mediated by some UV scale.

Assume that in the preferred frame in which the universe is approximately homogeneous, the speed of light varies as $c(t)$. Therefore, $ds^2 = c(t)^2 dt^2 - a(t)^2 d\mathbf{x}^2$ (although $g_{00} = 1$) but the Friedman equations are still valid,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc(t)^2}{a^2}, \tag{1.1.31}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c(t)^2}\right). \tag{1.1.32}$$

If the universe is radiation dominated, $p = \rho c(t)^2/3$ and thus $a \propto t^{1/2}$ as usual. As a consequence of the violation of invariance of time translations even at the local level, the energy conservation condition is modified,

$$\dot{\rho} + 3H\left(\rho + c(t)^{-2}p\right) = \frac{3kc(t)\dot{c}(t)}{4\pi Ga^2} \tag{1.1.33}$$

The horizon problem is straightforwardly explained within this model. The ratio of the event horizon to the causal horizon at $t = t_i$ is now given by

$$\frac{l_i}{l_c} \sim \frac{c_0 t_0}{\int_0^{t_i} c(t)dt}\frac{a_i}{a_0}, \tag{1.1.34}$$

and thus can be made arbitrarily small with an appropriate choice of the function $c(t)$. A "superluminal" propagation of light at early times suffices. In the original paper [48], a phase transition between an early value of the speed of light $c_i \sim 10^{30}c_0$ and the present one c_0 at $t \lesssim t_p$ is proposed.

The flatness problem can also be understood. Taking Eqs. (1.1.31),(1.1.32),(1.1.33) into account, we get that in a universe dominated by a perfect fluid with $p = \omega\rho c(t)^2$ the cosmological parameter $\Omega(t)$ evolves as

$$\dot{\Omega} = \Omega(\Omega - 1)H(1 + 3\omega) + 2\frac{\dot{c}}{c}(\Omega - 1). \tag{1.1.35}$$

The inflationary cosmology solves the flatness problem because the inflaton field satisfies $1 + 3\omega < 0$ and thus $\Omega \approx 1$ is stable. The varying speed of light is able to solve the problem because the term proportional to $(1 + 3\omega)$ can be compensated by the term proportional to $\dot{c}$ as long as $|\frac{\dot{c}}{c}| > \Omega H$.

Some models providing concrete realizations of VSL can be found in Refs. [62, 48, 63] (see Ref. [64] for a review). A covariant theory was presented in Ref. [65], as a gauged Fock-Lorentz

symmetry theory in which c is allowed to vary as a scalar field does in the manifold, $c(x)$. Another similar covariant realization is Rainbow Geometry [66], in which the metric of the spacetime as seen by a test particle of a given energy depends on this energy. However, the interpretation of this scenario seems obscure. A more clear interpretation is found if the speed of light is wavelength dependent, $c(E)$, due to a modified dispersion relation. In the very early universe the wavelength of the photons becomes comparable to the Planck length or other UV scale and thus the global effect can be seen as VSL [67]. This approach allows a deeper description of VSL cosmology [68] and structure formation [69]. Note however that this idea is also not absent of criticism [70].

$f(R)$-**theories** There has been an interest in modifying GR since it was formulated [71], as a consequence of the success of GR in the description of the precession of the perihelion of Mercury. As a consequence of the difficulties in the renormalization of GR, extensions of the Einstein-Hilbert action with quadratic terms in the action (like in Eq. (1.1.18)) were studied; later it was proved that these corrections provided a mechanism of inflation [46], as we have explained. Nonlinear corrections in the scalar curvature to the Einstein-Hilbert action have also found motivation in ST [72, 73].

However, the motivation for extending the Einstein-Hilbert action can be much more mundane: there is no fundamental principle fixing the form of the gravitational action. Thus if we allow for new scales in a gravitational theory, new terms in the action will generally appear, which must respect general covariance. In this sense the gravitational action can be arbitrarily extended as in Eqs. (1.1.18) or (1.1.19). As a toy model one may consider actions which depend only on the curvature scalar,[5]

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{16\pi G}f(R) + L_m\Big]. \tag{1.1.36}$$

These are called $f(R)$-theories [79]. In fact, one can distinguish between three types of $f(R)$-theories depending on how the variational principle is implemented on the action. In metric $f(R)$, the only gravitational field appearing in the action is the metric $g_{\mu\nu}$ and variation with respect to the metric provides the equations of motion. In Palatini $f(R)$ the scalar curvature is the result of the contraction of the inverse of the metric of the Ricci tensor, which in turns depends only on the connnection (treated as an independent field). The matter Lagrangian is considered to depend only on the metric, but not on the conection. In metric-affine $f(R)$, not only the Ricci tensor is made of the connection, but also the matter Lagrangian contains this field. Both of the three formalisms are equivalent if $f(R) = R + \Lambda$, but differ if $f''(R) \neq 0$. In this thesis mainly metric $f(R)$-theories will be treated.

In a metric $f(R)$ theory, the equations of motion are,

$$f'(R)R_\nu^\mu - \frac{1}{2}f(R)\delta_\nu^\mu + (\delta_\nu^\mu \Box - \nabla^\mu \nabla_\nu)f'(R) = 8\pi G T_\nu^\mu, \tag{1.1.37}$$

which, particularized to a FRW metric, become

$$8\pi G\rho = -3(\dot{H} + H^2)f'(R_{(0)}) - \frac{1}{2}f(R_{(0)}) - 3Hf''(R_{(0)})\dot{R}_{(0)}, \tag{1.1.38}$$

$$-8\pi G p = -(\dot{H} + 3H^2)f'(R_{(0)}) - \frac{1}{2}f(R_{(0)}) + f^{(3)}(R_{(0)})\dot{R}_{(0)}^2 + f''(R_{(0)})\ddot{R}_{(0)}, \tag{1.1.39}$$

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[5]More general models have also been considered. For instance, models which depend on the squared curvature scalars [74, 75] or which are nonlocal [76, 77, 78].

where

$$R_{(0)}(t) = -6(\dot{H} + 2H^2) \tag{1.1.40}$$

is the curvature scalar of the Friedman-Robertson-Walker (FRW) metric. For a $f(R)$-type modification in the IR it is then possible to find mechanisms for the present acceleration of the universe [80]; in the same way in a $f(R)$-type modification in the UV it is possible to find mechanisms for inflation [46]. It has also been tried to unify inflation and the present cosmic acceleration in the context of these theories [81, 82].

Metric $f(R)$ theories are formally equivalent to scalar-tensor theories (as we explained in the context of the Starobinski model), with a potential for the scalar field

$$V(\varphi) = \frac{1}{16\pi G} \frac{Rf'(R) - f(R)}{f'(R)^2}, \tag{1.1.41}$$

where $R(\varphi)$ can be obtained from

$$\varphi \equiv \sqrt{\frac{3}{16\pi G}} log f'(R). \tag{1.1.42}$$

The scalar presents an effective mass

$$m^2 = \frac{1}{2}\frac{d^2V}{d\varphi^2} = \frac{1}{6}\left(\frac{Rf'(R) - 4f(R)}{f'(R)^2} + \frac{1}{f''(R)}\right). \tag{1.1.43}$$

Thus, $f(R)$ can be seen as theories of a propagating massless spin-2 field (the graviton) together with a (generally massive) scalar field.[6] This scalar field may act as an inflaton in the very early universe but becomes a problem when building an $f(R)$ theory consistent with solar system tests of gravity.

In order to present a viable alternative to ΛCDM, a modified gravity theory must pass a number of tests. In particular, an $f(R)$-theory must:

- possess the correct cosmological dynamics. Many models present non-smooth transitions, for instance between the radiation dominated and matter dominated era, which were naively unexpected (for instance, see Ref. [83]). It has been also pointed out that some $f(R)$ theories may possess different future time singularities [84], classified in Ref. [85]. Future time singularities can be cured by adding R^2 terms to the action, as shown in [84]. Both of these problems may also be surmounted by starting from a given evolution $a(t)$ and integrating the ordinary differential equation that relates it to the family of $f(R)$ which present that solution [86].

- not suffer from instabilities. The simple model in Ref. [80] was shortly found to have an instability [87]. However this instability was later cured by the addition of new terms to the action relevant at high curvatures [81], much like future time singularities. Metric $f(R)$-theories suffer from this instability if $f''(R) < 0$ [88]. If this is the case the energy of the corresponding scalar field may become negative. This can be understood in terms of the effective gravitational coupling appearing in metric $f(R)$ theories $G_{eff} = G/f'(R)$. If $f''(R) < 0$ then $dG_{eff}/dR > 0$. Large curvature causes gravity to become stronger, so curvature further increases, causing a positive feedback mechanism. Also an unprotected curvature singularity arises in the nonlinear regime of IR modified gravity, which makes it difficult to describe the strong gravity regime of relativistic stars [89]. In fact this

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[6]In more general modifications, more effective propagating degrees of freedom may arise.

singularities that appear in the spherically symmetric case are connected to the future singularities of the FRW spacetime mentioned above [90], and are as well cured with the addition of new terms in the action which are relevant at high curvature [84].

- be free from ghosts. Although $f(R)$ is ghost-free, more general modifications of gravity may contain ghost fields. Extensions containing also the Gauss-Bonnet scalar $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ present no ghosts [91].

- have the correct Newtonian and post Newtonian limit and be compatible with the data of CMB and LSS. These conditions have in common the fact that they depend on the behavior of inhomogeneous perturbations of a homogeneous background metric. In the conformally equivalent scalar-tensor theory, the problem is that the propagating scalar degree of freedom may spoil the behavior of this perturbations; it gives rise to a modification of the Newton's law and modifies cosmological perturbation dynamics [92]. In most models, the mass of the scalar is naturally light ($m < 10^{-3} \, eV$) and thus the effect on Solar System tests rules them out as viable gravity theories. However it must be taken into account that the effective mass of the scalar depends on the local curvature, and therefore differs in cosmological, Solar System and on Earth tests. This is known as the 'chameleon effect' [93, 94]. Thus it is possible to have scalars modifying gravity at sub-millimeter scales in the Solar System (and therefore remaining undetected) while offering the long-ranged modified cosmological dynamics. Examples can be found in Refs. [95, 96, 97, 98]. However it is not clear wether the physics of the conformally equivalent scalar-tensor theory always provides a good picture of the physics in the $f(R)$ theory [99]. The behavior of scalar perturbations may change due to the redefiniton of fields done after the conformal transformation. The behavior of cosmological perturbations in metric $f(R)$-theories is still work in progress: it has been defined in the covariant approach [100] and in the Newtonian gauge [101]. It is then clear that the evolution of the energy density involves a fourth order differential equation. A similar situation holds for Palatini $f(R)$-theories.

- have well-posed Cauchy problem. It has been shown that metric $f(R)$-theories present a well-posed Cauchy problem for several sources (perfect fluids, scalar fields) while Palatini $f(R)$-theories do not [102].

Remarkably, these problems do not only arise in modified gravity theories, but also have their equivalent in other approaches to the problem of acceleration like quintessence models or dark fluids. This can be trivially seen as one can write the effect of a mofification of gravity as a scalar-tensor theory (as shown above) or as a dark fluid model. In the same way as in singular $f(R)$ theories, the problem of singular quintessence or dark fluid models can be cured with the addition of terms with higher powers of curvature to the action, such as the ones that appear in the renormalization of field theories in curved space [103].

To sum up, $f(R)$-theories are a simple toy model which shows the difficulties of departing from GR in a consistent way and in agreement with the experimental and observational data. This is possible under certain conditions that impose a great fine-tuning of the function $f(R)$. The origin of the constraints is the presence of new effective propagating degrees of freedom, i.e. the presence of differential equations of order greater than two.

This is a general problem of theories beyond GR. We will try to surmount this problem (at least in an effective way) in the formulation of the Asymptotic Cosmological Model (ACM) in one of the works presented in this PhD thesis, and in the treatment of cosmological perturbations in the ACM.

Higher Dimensional Models   Inspired by ST, it has been proposed that our universe might be a four-dimensional hypersurface (a 3-brane) embedded in a higher dimensional bulk spacetime. The SM fields are considered to live in the brane while the gravitational field (or fields) are considered to live in the whole bulk spacetime. The question is how to recover the standard Newton's law instead of its higher dimensional version in the brane. This can be achieved through compactification of the extra dimension [37, 38, 40], whose size can be much larger than in the case of Kaluza-Klein models; thus this approach is called 'Large Extra Dimensions'. This can also be achieved by embedding the brane with positive tension into an Anti-deSitter bulk [104, 105]; this approach is called 'Warped Extra Dimensions'. In both cases, the extra dimension is finite and has a size of the order of the millimeter.

Dvali *et al.* proposed another solution: it suffices to add to the action of the brane an Einstein-Hilbert term computed with the intrinsic curvature of the brane. This term is necessary for the renormalizability of the quantum field theory in the brane interacting with the bulk gravity. Thus the standard Newton's law is recovered even for infinite size extra dimensions [106]. The case of a 3-brane embedded in a 4+1 Minkowski spacetime is of special interest. We will explain this model in the lines below.

Consider the action

$$S_{(5)} = \frac{1}{2\kappa^2} \int d^5 X \sqrt{-\bar{g}} \bar{R} + \int d^4 x \sqrt{-g} (\lambda_b + l_m) + \frac{1}{2\mu^2} \int d^4 x \sqrt{-g} R \,, \qquad (1.1.44)$$

where the first term corresponds to the Einstein-Hilbert action in 4+1 dimensions, with a five-dimensional bulk metric $\bar{g}_{AB}$ and curvature scalar $\bar{R}$; the third term corresponds to the Einstein-Hilbert action of the induced metric g_{ab} on the brane, with its corresponding intrinsic curvature scalar R; the second term corresponds to the matter action, which might include a brane tension (an effective cosmological constant) λ_b. The following quantities can be defined,

$$\kappa^2 = 8\pi G_{(5)} = M_{(5)}^{-3} \qquad (1.1.45)$$

$$\mu^2 = 8\pi G_{(4)} = M_{(4)}^{-2} \,. \qquad (1.1.46)$$

In Ref. [106], it was shown that the Newtonian potential is recovered in the brane for distances smaller than $r_0 = M_{(4)}^2 / 2M_{(5)}^3$, with an effective Newton's constant $G_N = \frac{4}{3} G_{(4)}$.

The cosmology of the model was derived by Deffayet in Ref. [107]. Consider the brane-isotropic metric

$$ds^2 = n^2(\tau, y) d\tau^2 - a^2(\tau, y) \gamma_{ij} dx^i dx^j - b^2(\tau, y) dy^2 \,, \qquad (1.1.47)$$

where $\{\tau, \mathbf{x}\}$ are coordinates on the brane ($y = 0$) and γ_{ij} is the maximally symmetric 3-metric (with spatial curvature k). The stress-energy tensor of the bulk is reduced to a possible bulk cosmological constant ρ_B, while the stress-energy tensor of the brane can be expressed as

$$T_B^A = \frac{\delta(y)}{b} diag(\rho_b, -p_b, -p_b, -p_b, 0) \,. \qquad (1.1.48)$$

Assuming that the metric $\bar{g}_{AB}$ is symmetric under the change $y \rightarrow -y$, the corresponding Friedman equation reads

$$\pm \sqrt{H^2 - \frac{\kappa^2}{6}\rho_B - \frac{C}{a_0^4} + \frac{k}{a_0^2}} = r_0 \left(H^2 + \frac{k}{a_0^2}\right) - \frac{\kappa^2}{6}\rho_b \,, \qquad (1.1.49)$$

where C is a constant of integration, the subscript 0 means evaluation at $y = 0$, and $H = \frac{\dot{a}_0}{n_0 a_0}$, with dot denoting derivation with respect to the time variable τ. The restriction of the fields to the brane also leads to the continuity equation for the brane, $\dot{\rho}_b + 3(\rho_b + p_b)\frac{\dot{a}_0}{a_0} = 0$.

If we choose the $+$ sign and neglect ρ_B, C and k for simplicity, we arrive to the modified Friedman equation

$$\frac{8\pi G_{(4)}}{3}\rho_b = H^2 - H/r_0 \,, \tag{1.1.50}$$

which produces a late accelerated expansion tending asymptotically to an exponential expansion with $H = 1/r_0$, as in the case of ΛCDM. However, this modified Friedman equation is distinguishable from the one appearing in ΛCDM,

$$\frac{8\pi G_N}{3}\rho = H^2 - \Lambda/3 \,. \tag{1.1.51}$$

Current constraints on the expansion history from supernova luminosity distances, the CMB, and the Hubble constant exclude the simplest flat DGP model at about 3σ. Even including spatial curvature, best-fit open DGP model is a marginally poorer fit to the data than flat ΛCDM. Moreover, its substantially different expansion history raises serious challenges for the model from structure formation [108].

Moreover, the DGP model is also plagued with anomalies [109, 110]. First, it was discovered that ghosts are present in perturbation theory [111]. Second, if general bulk metrics are allowed, then the energy spectrum of the solutions is unbounded from below (negative mass blackholes are allowed) and therefore the theory is unstable [110]. Furthermore there are solutions (positive mass black holes) in which 'pressure singularities' are present: pressure diverges for finite energy density. Third, the DGP vacuum is unstable as it may tunnel to alternative vacua [110].

Thus, we will not consider DGP as a viable alternative to explain the current accelerated expansion. However, it is interesting how a simple modified Friedman equation such as (1.1.51) is able to produce an accelerated expansion in the absence of strange fluids.

Quintessence　Aside from modifications of gravity such as the ones described above, it is also possible to explain the present acceleration of the universe as a consequence of the existence of a new form of matter, different form massless or massive particles, which presents at least at later cosmic times an effective negative pressure.

There are several ways of implementing this idea, but one of the most popular ones is to introduce phenomenologically a self-interacting scalar field. Extensions of GR including scalar fields have a long history [112]. This mechanism has also been used in QFT in order to explain some of the unnaturally small parameters of the SM [113, 114]. In analogy, it is possible to build dynamical scalar field actions which mimic the effect of a cosmological constant, and which could render its value small [115, 116]. However, this would imply a new force mediated by the scalar field, despite new interactions mediated by light scalar fields are highly constrained [117]. Thus there must be a fine-tuning in the action at least in the effective mass of the scalar or its coupling to observed matter.

Typically the action of one of such 'quintessence' theories has the form

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) + \mathcal{L}_m[g^{\mu\nu}, \varphi]\right] \,, \tag{1.1.52}$$

where φ is the quintessence scalar field[7], V is the effective self-interaction potential, which may be induced in a more fundamental theory, and $\mathcal{L}_m$ is the matter Lagrangian, which may include couplings between ordinary matter and the quintessence.

In the homogeneous approximation, in which the metric is FRW and the quintessence and matter fields are decoupled and depend only on the time variable, the equations of motion become

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) \quad = 0 \qquad (1.1.53)$$

$$\frac{8\pi G_N}{3}(\rho + \frac{\dot{\varphi}^2}{2} + V(\varphi)) \quad = H^2 \qquad (1.1.54)$$

$$-3(1+\omega)H\rho \quad = \dot{\rho}. \qquad (1.1.55)$$

Note that if the quintessence currently varies sufficiently slow with time ($\dot{\varphi}^2 \ll V(\varphi)$), then the quintessence is approximately frozen at its present value φ_0 and acts as an effective cosmological constant $\Lambda = 8\pi G V(\varphi_0)$; for a sufficiently slow variation, the negative effective pressure $p_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi)$ required by the accelerated expansion is guarantied [116, 118, 119].

Some specific quintessence models deserve more attention. One of this classes is 'tracker models' [120, 121], which try to explain the coincidence problem[8]. The idea underlying tracker models is the following: the energy density of quintessence component somehow tracks below the background density for most of the history of the universe, falling from a Planckian value in the very early universe with the expansion history. Only recently it grows and becomes the dominant energy density, driving the universe to its current accelerated expansion. The tracker field φ rolls down the potential $V(\varphi)$ following an attractor-like solution. Examples of tracker potentials are exponentials $V(\varphi) = M^4(e^{M_p/\varphi} - 1)$ and inverse power laws $V(\varphi) = M^{4+n}\varphi^{-n}$.

For instance for inverse power law potentials $V(\varphi) = M^{4+n}\varphi^{-n}$ it is possible to arrive to a quintessence-dominated period when $\varphi \sim M_p$. The value of the scale M must be fit by observations (typically $M \gtrsim 1\,GeV$). For a given background component with equation of state $\omega_B \equiv \rho_B/p_B$, the subdominant quintessence then adapts itself and adopts an equation of state

$$\omega_\varphi \approx \frac{n\omega_B/2 - 1}{n/2 + 1}. \qquad (1.1.56)$$

Thus for high n if quintessence is subdominant it mimics the equation of state of radiation and its energy density decreases at the same rate. When matter takes over radiation, the pressure of quintessence becomes negative and later on begins to dominate. However for these potentials the condition $\dot{\varphi}^2 \ll V(\varphi)$ never holds and thus they are not able to reproduce equation of state of a cosmological constant. Thus the model is disfavored by observations. The coincidence problem is also just moved from the choice of the value of Λ in ΛCDM to the choice of the scale M.

Other interesting type of quintessence models are k-essence models, in which the kinetic term of the scalar field is non-standard [122]. Such kinetic terms may arise in ST [123]. Typically the action of a k-essence model can be written as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G}R + \frac{1}{\varphi^2}f(X) + \mathcal{L}_m[g^{\mu\nu}, \varphi] \right], \qquad (1.1.57)$$

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[7]This scalar field is given a variety of names, depending on its role in the particular model in which it is introduced. It may be the dilaton (the Goldstone mode associated to the spontaneous breaking of dilatation symmetry), the cosmon (a particular case of the dilaton), the inflaton (if we consider a model of inflation),...
[8]Why is the value of the vacuum energy density of the same order that the matter energy density today?

where $f(X)$ is a certain function of $X \equiv g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$. The energy density and pressure of the k-essence read

$$\rho_k = (2Xf'(X) - f(X))/\varphi^2\,, \tag{1.1.58}$$

$$p_k = f(X)/\varphi^2\,. \tag{1.1.59}$$

As shown in Ref. [122], k-essence models are able to reproduce the behavior of tracker models in the early universe, while showing a behavior more similar to a cosmological constant in the present universe. However, these models are quite *ad hoc* and do not fit the data as well as an effective cosmological constant does. Moreover they do not really solve the coincidence problem, as the transition from a matter dominated to a k-essence dominated universe occurs just as in ΛCDM.

Similar tracker behaviors arise much more naturally in vector-tensor models such as the one proposed in Ref. [124]. However, such a model presents an undesirable tension when compared to SNe, CMB and BAO observations [125].

Dark Energy When phenomenologically describing the exotic component of the universe either with a self-interacting scalar field, a k-essence or other types of fields, we are making an *a priori* assumption (the field content of our model) that is going to condition our following analysis. For this reason, it may be better from a phenomenological point of view to consider just the essential features of a new matter component, independently of a field theory representation. The essential ingredient of the matter content that must be characterized in GR is just the stress-energy tensor.

In the homogeneous approximation, the only two nonzero elements of the stress-energy tensor are the energy density ρ_X and the pressure p_X. Thus, the effective description of a new fluid component amounts to the relation between this quantities, the equation of state $p_X = p_X(\rho_X)$. The most simple parametrization of this relation is a linear equation of state $p = \omega\rho$ [126]; [9] for instance radiation has $\omega = 1/3$, massive matter has $\omega = 0$ (strictly speaking $\rho \gg p$) and a vacuum energy has $\omega = -1$. Naturally, this parametrization is also the most used in data analyses, such as [11, 7, 12, 13]. In contrast, an example of a nonlinear equation of state could be a gas of strings, the Chaplygin gas $p_X = -K/\rho_X$ [127, 128], which is able to mimic the dark matter at high densities but leads to an accelerated expansion at low densities.

The relation between the pressure and the energy density does not contain all the information about the fluid if one considers the evolution of inhomogeneous perturbations [129]. Then, it is necessary to consider the scalar part of the non-diagonal terms of the stress-energy tensor, i.e.: the momentum density $(\rho_X + p_X)\boldsymbol{u}_X = (\rho_X + p_X)\boldsymbol{\nabla}\theta_X$ and anisotropic stress $\Pi_{Xij} = \partial_i\partial_j\Pi_X$. A complete description of the fluid should involve an expression relating the pressure and anisotropic stress as a function (or a functional) of the energy density and the momentum density of the fluid at each point.

This description is of course too general to be predictive, and one usually restricts oneself to the use of perfect fluids ($\Pi = 0$) with a certain $p(\rho)$ equation of state. However, the effect of some specific anisotropic stress has been investigated in the literature (see for instance Refs. [129, 130, 131]).

In light of the vast amount of concrete models, a phenomenological description of the equation of state has been proposed as a way of parameterizing the present acceleration of the universe, such as

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[9]Despite it is a flagrant abuse of language, ω is often referred to as the equation of state.

$$p_X = \left[\omega_0 + \omega_a(1 - a) + \mathcal{O}\left((1 - a)^2\right)\right] \rho_X\,, \tag{1.1.60}$$

where the normalization $a_0 = 1$ is taken. Although we agree with the bottom-up spirit of such parameterizations, this choice is very anthropocentric and sheds little light on the physics of the dark component or gravitational physics behind the expansion. Is it possible to make a better choice?

1.1.5. The Asymptotic Cosmological Model

The first thing one should note when thinking about an extension of the cosmological standard model is that it is difficult (impossible?) to distinguish by means of gravitational physics between modified gravitational physics and new forms of matter. This is due to the Einstein equation. Any modification of the Einstein tensor can be moved to the rhs of the equation and become a modification to the stress-energy tensor and viceversa. This term will have a null covariant divergence as a consequence of being derived from a scalar action, either from the gravitational or the matter part of the Lagrangian. Thus, we will consider the departure to be encoded in the gravitational part of the Lagrangian without loss of (phenomenological) generality.

Secondly, in order to build a concrete model we have to make a number of assumptions based on phenomenological grounds. We have mentioned that it is experimentally well established that gravity should be a metric theory [2, 3, 4] and thus we will assume our model to be a metric theory. We will also consider that the cosmological principle is a good approximation at sufficiently large scales, so that the FRW metric will be a good approximation to the metric of spacetime. The FRW metric depends just on the spatial curvature of spacetime, k. In ΛCDM, k is severely constrained by the observations to be zero, so in our model we will assume that $k = 0$.

In GR with no spatial curvature, the evolution of the universe in the homogeneous approximation is obtained by an algebraic relation between the Hubble rate and the energy density. Furthermore if we assume that $\dot{H} < 0$, then there is a one-to-one correspondence between t and H, and thus we can always find an algebraic relation between the energy density (of a fluid composed of SM particles and DM) and the Hubble parameter. Therefore we are going to define our model through the relation between H and ρ.

Third, we will consider that the standard relation between H and ρ, the first Friedman equation (1.1.6), will be valid just in a window limited by the presence of an UV and IR scale. From the point of view of the phenomenologist, every theory must be valid in the regime in which it has been tested, but might fail in the regime in which it has not. GR has been well tested in a range from the millimeter scale to the galactic scale and QFT has been tested from the eV to the TeV scale, and thus they might fail beyond this scales. In our model, the effect of the new UV and IR scales will be responsible of the accelerated expansion in the very early universe and the present epoch, respectively.

We will take as a basic principle that the Hubble rate is constrained to a bounded interval between an upper scale H_+ and a lower scale H_-. Therefore in the infinite past and infinite future the universe will asymptotically tend to the deSitter spacetime. We have called this model the Asymptotic Cosmological Model.

1.2. Neutrino Oscillations

The development of QFT has been the most active sector of theoretical physics in the past century. Quantum Electrodynamics (QED) has provided the most precise predictions for exper-

iments which have later confirmed those predictions figure by figure. The Standard Model of particle physics (SM), built up through a continuous interaction with experiment, has managed to provide a framework in which to understand the electromagnetic, weak and strong interactions, and to propose a mechanism to explain the apparition of mass (yet to be confirmed).

The model consists on a $SU(3) \times SU(2) \times U(1)$ gauge theory, in which the $SU(2) \times U(1)$ symmetry is "broken" through a Higgs mechanism by the vacuum expectation value (vev) of a scalar field. The spinors of the theory are divided in three families (or flavors) and can be either quarks ($SU(3)$-interacting) or leptons ($SU(3)$-noninteracting), and their masses are attained by their Yukawa interactions with the scalar field.

Let us denote the fields in the standard model by

$$\begin{aligned}
\Psi_{L\alpha} &= \begin{pmatrix} \nu_\alpha \\ l_\alpha \end{pmatrix}_L, & L_\alpha &= (l_\alpha)_R, \\
Q_{L\alpha} &= \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L, & U_\alpha &= (u_\alpha)_R, & D_\alpha &= (d_\alpha)_R, \\
\Phi &= \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, & \tilde{\Phi} &= \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}, \\
G_\mu, & & W_\mu, & & B_\mu,
\end{aligned} \tag{1.2.1}$$

where in the first line we have denoted the leptons, in the second line the quarks, in the third line the Higgs boson and its conjugate, and in the last line the gauge fields carrying the $SU(3)$, $SU(2)$ and $U(1)$ interactions respectively, whose respective field strengths will be denoted $G_{\mu\nu}$, $W_{\mu\nu}$ and $B_{\mu\nu}$. The index $\alpha = 1, 2, 3$ labels the flavor, the coupling of the gauge fields are respectively g_3, g and g'. The covariant derivatives will be denoted as ∇_μ, and $A\overleftrightarrow{\nabla}_\mu B \equiv A\nabla_\mu B - (\nabla_\mu A)B$. The Yukawa couplings will be denoted Y_L, Y_U, Y_D. Then, the Lagrangian of the SM is just the sum of the following terms:

$$\mathcal{L}_{lepton} = \frac{i}{2}\bar{\Psi}_{L\alpha}\gamma^\mu\overleftrightarrow{\nabla}_\mu\Psi_{L\alpha} + \frac{i}{2}\bar{L}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu L_\alpha, \tag{1.2.2}$$

$$\mathcal{L}_{quark} = \frac{i}{2}\bar{Q}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu Q_\alpha + \frac{i}{2}\bar{U}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu U_\alpha + \frac{i}{2}\bar{D}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu D_\alpha, \tag{1.2.3}$$

$$\mathcal{L}_{Yukawa} = -[(Y_L)_{\alpha\beta}\bar{\Psi}_{L\alpha}\Phi L_\beta + (Y_U)_{\alpha\beta}\bar{Q}_\alpha\tilde{\Phi}U_\beta + (Y_D)_{\alpha\beta}\bar{Q}_\alpha\Phi D_\beta] + h.c., \tag{1.2.4}$$

$$\mathcal{L}_{Higgs} = (\nabla_\mu\Phi)^\dagger\nabla^\mu\Phi + \mu^2\Phi^\dagger\Phi - \frac{\lambda}{3!}(\Phi^\dagger\Phi)^2, \tag{1.2.5}$$

$$\mathcal{L}_{gauge} = -\frac{1}{2}Tr(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}Tr(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \tag{1.2.6}$$

In the SM, the scalar field acquires a vev through the Higgs mechanism, and thus the Yukawa terms turn into mass terms for the quarks u, d and the electrically charged leptons l, but not the neutrinos ν. Therefore, the neutrinos are massless electrically neutral particles which only interact weakly (exchanging $W^\pm$ or Z bosons, which are combinations of the W and B bosons). In a given experiment involving a charged lepton l_α, the presence of a neutrino ν_α as a final state can be detected as a violation of energy-momentum conservation.

These particles are generally detected in deep underground laboratories, in which the presence of more intensely interacting particles has been damped by the Earth crust[10]. Their detection

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[10]This is due to the fact that at energies below the electroweak scale neutrinos interact very weakly and otherwise the signal to noise ratio is small. However it is possible to detect neutrinos in accelerator experiments in which the source is very luminous, of high energy and controllable. However, detection of accelerator neutrinos at installations far away the source also need to be underground in order to improve the signal to noise ratio

usually involves the exchange of a $W^{\pm}$ boson and the detection of the charged lepton l_{α} associated to the incoming neutrino ν_{α}.

Therefore, if a ν_{α} is produced somewhere in a process involving l_{α}, the massless and weakly interacting neutrino can propagate huge distances until being detected in another process involving the same l_{α}. This was the idea underlying the Homestake experiment headed by Raymond Davis Jr. and John N. Bahcall [132]. The experiment designed by Davis managed to measure the flux of ν_e coming from the sun as a result of the nuclear reactions happening at its core. The aim of the experiment was to confirm the theoretical predictions, derived by Bahcall, of the Standard Solar Model. However, the experiment measured around one third of the expected neutrinos! This was named the 'solar neutrino problem'. At that time it was impossible to decide whether the problem was in neutrino physics or in the current understanding of the sun.

1.2.1. A conventional explanation: neutrinos do have mass!

It was first proposed by Pontecorvo [133] that neutrinos may change their nature (flavor) along their propagation through long distances in the same sense the neutral kaons do. He named this process 'neutrino oscillations'. This explanation fitted the results of the Homestake experiment well but has profound implications [134].

This phenomenon is shocking because in the context of the SM it cannot take place. When a neutrino is produced in a certain interaction within the SM of particle physics, it is produced by definition in a state which is a flavor eigenstate of definite momentum (ν_e if the interaction involves an electron, ν_{μ} if the interaction involves a muon, or ν_{τ} if the interaction involves a τ). In the SM, the energy of these three states E_{α} is degenerate and depends only on their momentum, $E_{\alpha} = p$. Thus if a neutrino is produced in some flavor eigenstate, this is also a propagation eigenstate and it will be detected in the same flavor eigenstate, i.e.: its interaction with matter will produce the same charged lepton that was involved in its creation.

For neutrino oscillations to occur, two circumstances must concur. First, it is necessary that there exist different eigenstates of the effective Hamiltonian of the neutrinos with different energies E_i for the same linear momentum $\mathbf{p}$. Let us call the neutrino propagation eigenstates $\{\nu_1, \nu_2, \nu_3\}$ with energies $\{E_1(p), E_2(p), E_3(p)\}$. In a Lorentz invariant theory the only possibility of the neutrinos to have different energy eigenstates is that they have different masses m_i: $E_i = \sqrt{\mathbf{p}^2 + m_i^2}$. Therefore it is often stated that neutrino oscillations imply that the neutrinos are massive.

Second, it is necessary that these eigenstates do not coincide with the flavor eigenstates. The reason is that the eigenstates of the effective Hamiltonian (states of definite energy for a definite momentum) coincide with the eigenstates of the propagation, but those must not coincide with the eigenstates of definite flavor for a mixing to happen. This is quite natural in the context of QFT, and we already have an example in the flavor physics of quarks, parameterized by the Cabibbo-Kobayashi-Maskawa matrix connecting mass eigenstates of negatively charged quarks with their "flavor" eigenstates, defined as the eigenstates resulting of their charged current weak interaction with a positively charged quark. In analogy, a unitary matrix (called the Pontecorvo-Maki-Nakagawa-Sakata or PMNS matrix [135]) connects the mass eigenstates of neutrinos ν_i with the flavor eigenstates ν_{α},

$$\nu_{\alpha} = U_{\alpha i} \nu_i \,, \tag{1.2.7}$$

which are defined as the states resulting of the charged current weak interaction of a charged lepton l_{α}. This state is a linear combination of states with definite and different energies, which propagate in a definite way.

However, if there exists different states with different energies, it is no longer true that the neutrino is produced in a flavor eigenstate, due to energy-momentum conservation (unless the laws of energy-momentum conservation are only an approximation for sufficiently high energy-momentum transfers). If we reject this possibility, what happens is that in the production, a ν_1, ν_2 or ν_3 can appear with different probabilities of sum one, which depends of the nature of the charged lepton involved in the production. Similarly, any of this neutrinos will produce, in the interaction with a detector, one of any of the three charged leptons, with different probabilities depending on the nature of the neutrino which has been detected.

It is assumed that this mass is an infrared (IR) scale, i.e.: it is much lower than the energy of the neutrino, and therefore the difference of energy between two mass eigenstates will be much lower than the total energy of any of these states, and the oscillations are assumed to take a significant place in neutrinos propagating through very long distances. Also, the effect of the mass in the total energy of the neutrino is often neglected, and it is commonly stated that neutrinos are produced and detected in states of definite flavor.

When we define a particle in a theory with a symmetry group, it is characterized by the eigenvalue of the invariant (under the symmetry group) operators that can be defined combining elements of the symmetry group. In the Lorentz invariant case, these operators are the mass and the spin of a particle. The particles also have definite charges under the gauge group. In the case of neutrinos, these states are not the flavor eigenstates, as they do not have a definite mass, but the propagation eigenstates $\{\nu_1, \nu_2, \nu_3\}$. Therefore, although flavor eigenstates can formally be defined in the context of a theory exhibiting neutrino oscillations it is an abuse of notation to call them particles.

Therefore it has been necessary to extend the SM of particle physics to include at least mass effects. The Standard Model of Neutrino oscillations assumes that there exist a new field, the sterile neutrino ν_i^R, which is a singlet of the entire gauge group and which is necessary to include perturbatively renormalizable mass terms for the neutrinos in the Lagrangian (see Ref. [136] for a review). The new terms that are added to the SM Lagrangian are:

$$-\mathcal{L}_{\nu_R} = \bar{\nu}_i^R \gamma^\mu \partial_\mu \nu_i^R + Y_{i\alpha}^D \tilde{\Phi}^\dagger \bar{\nu}_i^R \gamma^\mu \partial_\mu \Psi_\alpha^L + m_{ij}^M \bar{\nu}_i^R \nu_j^{R,c} + h.c., \qquad (1.2.8)$$

where ν^R is the sterile neutrino singlet, with flavor index i running from 1 to n; Y^D is an adimensional matrix of coefficients in flavor space, which will give rise to Dirac mass terms after the Higgs acquires its expectation value; and m^M is a Majorana mass matrix allowed by all the symmetries (the only of such terms in the whole lagrangian).

This terms lead to the so-called 'see-saw mechanism'. Let us assume that sterile neutrinos are a signal of new physics at some UV scale E_{NP} a few orders of magnitude below the Planck scale. Then, the m^M matrix has components naturally of the same order of this scale, while the Dirac mass terms associated to neutrinos will be naturally of the order of the vev of the scalar field. Then, the diagonalization of the resulting total mass matrix lead to $3+n$ eigenstates with different masses. n of them are mostly sterile, have a mass comparable to the UV scale, and are very weakly coupled to the charged leptons. The other three are mostly lefthanded, have a mass of order v^2/E_{NP}. [11] All of these particles are Majorana fermions, i.e.: they are their own antiparticles, in contrast with Dirac fermions like the charged leptons or quarks.

In this case the SM with Majorana neutrinos is a good effective field theory (EFT) [20] at energies below the almost sterile neutrino mass. In this case operators of dimension greater than 4 cannot be omitted, but they are corrected with an appropriate negative power of the UV scale.

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[11]This idea fits well in the frame of GUT theories with symmetry group $SU(5)$, in which the GUT scale is $M_{GUT} \sim 10^{15}\, GeV$; however, the absence of proton decay [137] within the predicted window has ruled out the simplest of these theories.

Then the first corrections to the SM should come from terms of dimension 5. If we compute all the 5-order terms compatible with the Poincaré and gauge symmetry, the only new term which is added is

$$- \mathcal{L}_{NP} = \frac{Z_{ij}}{E_{NP}} \bar{\Psi}_i^L \tilde{\Phi} \tilde{\Phi}^T \Psi_j^{L,c} + h.c.. \tag{1.2.9}$$

In fact, this result tells us that if we are expecting any new physics at a scale E_{NP} we should expect that this new physics (not necessarily sterile neutrinos) provides the neutrinos with an effective mass term.

The proposal that massive neutrinos should suffice to explain the solar neutrino problem in terms of neutrino oscillations in the propagation of neutrinos from the Sun to the earth needed only to be contrasted with more experimental data. The so-called solar neutrino problem was further studied with similar detectors as Kamiokande, SAGE, GALLEX, Superkamiokande and the Sudbury Neutrino Observatory (SNO). SNO was the first experiment that proved that neutrino oscillations were behind the solar neutrino problem [138], although neutrino oscillations were first detected by Superkamiokande [139]. In the latter case, the oscillation was produced in the neutrinos produced by muon decay in the atmosphere (atmospheric neutrinos). Neutrino oscillations produced in the propagation of neutrinos generated in artificial nuclear reactors [140] and in nuclear decays in the Earth's crust and mantle (geo-neutrinos) [141] have been also measured by KamLAND.

The aforementioned experiments have produced a vast amount of data about neutrino oscillations. Solar neutrino oscillation are affected also by the so-called MSW effect (neutrino oscillations due to its interaction with matter), so they do not provide a clean picture of vacuum neutrino oscillations. Atmospheric neutrino experiments have observed ν_μ disappearance with $\Delta m^2 \sim 3 \times 10^{-3} eV^2$ [139], while reactor antineutrino experiments have measured $\bar{\nu}_e$ disappearance at $\Delta m^2 \sim 8 \times 10^{-5} eV^2$ [140, 141]. The minimum joint description of solar, atmospheric and reactor neutrinos requires Dirac neutrinos with different masses

$$m_1^2 - m_2^2 = 7.9 \pm 1.1 \times 10^{-5} eV^2 \tag{1.2.10}$$
$$|m_1^2 - m_3^2| = 2.6 \pm 0.6 \times 10^{-3} eV^2 \tag{1.2.11}$$

and the four independent degrees of freedom in the PMNS matrix[12], which have been constrained [136]. In fact, the phenomenology involving massive Dirac neutrinos depends only in the squared absolute value of the elements of this matrix, which we will call populations and denote by

$$r_{i\alpha} = |U_{\alpha i}|^2 = \begin{pmatrix} 0.69 \pm 0.03 & 0.31 \pm 0.03 & < 0.01 \\ 0.16 \pm 0.07 & 0.36 \pm 0.11 & 0.46 \pm 0.11 \\ 0.14 \pm 0.08 & 0.30 \pm 0.10 & 0.52 \pm 0.12 \end{pmatrix}. \tag{1.2.12}$$

The population $r_{i\alpha}$ represent the portion of a state ν_α in a state ν_i. Although there exist nine populations in a three-mixing problem, the sum-one condition makes only four of them linearly independent. In the Standard Model of Neutrino Oscillations these parameters are constants, but in a more general oscillation problem they could be functions of the momentum **p**.

It seems that massive neutrinos fit the current experimental data well. What is then the problem of this explanation?

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[12]This is true for Dirac neutrinos. In the case of Majorana neutrinos there are six independent parameters in the PMNS matrix.

1.2.2. Doubts arising with massive neutrinos

The LSND anomaly and other suggestive experimental results However, not all the ongoing experiments have their results well described by this simple picture. The Liquid Scintillator Neutrino Detector (LSND) reported evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ at $\Delta m^2 \sim 1 eV^2$ [142, 143]. This results have later been confirmed by the same group [144, 145], which has spent a huge effort in finding sources of experimental error. The final sample tests both $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillations in a window of $20\,MeV < p < 200\,MeV$.

In order to confirm or deny the results of the LSND collaboration, the KARMEN and Mini-BOone collaborations have been running similar experiments. The MiniBOone collaboration has reported no evidence of $\nu_\mu \to \nu_e$ oscillations for neutrino beams with $p > 475\,MeV$ [146, 147] but has found a tiny excess of ν_e in beams with $200\,MeV > p > 475\,MeV$ [147] (although this excess is incompatible with the oscillation mechanism proposed by LSND). The same group has also performed a $\bar{\nu}_\mu \to \bar{\nu}_e$ search at $200\,Mev < p < 3000\,MeV$, but no significant signal was found [148]. These results have been qualified as 'inconclusive' [148].

Furthermore, if we consider that the neutrino mass arises as a consequence of EFT with new physics in the UV (maybe close to $10^{15}\,GeV$), then the corresponding dimension 5 operator giving mass to the neutrinos is a Majorana mass term which violates lepton number by two units. This should lead to new currently unobserved phenomena related to lepton-nonconservation such as neutrinoless double-beta decay ($0\nu\beta\beta$). These experiments are also interesting because, if $0\nu\beta\beta$ is observed, it will provide a measurement of the absolute value of the neutrino mass scale (in contrast to the relative values provided by neutrino oscillations). The search for these phenomena has been performed in the last years and has become quite controversial.

A subgroup of the Heidelberg-Moscow collaboration led by H. V. Klapdor-Kleingrothaus published a series of controversial papers claiming to have observed neutrinoless double-beta decay in ^{76}Ge with 6σ CL [149, 150, 151]. This "evidence" has been hotly argued [152], and is not accepted by most of the scientific community. Their claim, in which the expectation value of the mass matrix in the ν_e state is found to be $m_\beta \sim 0.2 - 0.5\,eV$ has yet to be confirmed by other experiments. Even if the 'Klapdor's evidence' turns out to be wrong, $0\nu\beta\beta$ experiments would give us a hint on the absolute value of neutrino masses in the (near?) future if neutrinos are Majorana particles: it would be just a matter of being sensitive to lower masses.

However if these experiments persist without any positive signal of $0\nu\beta\beta$, then a solution to the absence of lepton number violating phenomena is to impose lepton number conservation by hand on the model (lepton number conservation is just an accidental symmetry of the standard model, but it stops to be a symmetry if we allow for dimension 5 operators or other neutrino mass terms), so that $m^M = 0$. If $n = 3$, the sterile neutrinos can be seen as the right handed components of a Dirac spinor (therefore sterile Dirac neutrinos are called 'righthanded neutrinos'). Then, processes violating lepton number would be forbidden. However, the reason of the smallness of the neutrino mass would be precluded, worsening the hierarchy problem of the SM.

Apart from $\beta\beta0\nu$, beta decay experiments in general are also able to determine the absolute scale of neutrino masses. However, the expected scale of neutrino masses are well below the sensitivities of most experiments. Recently, a handful of experiments such as Troitsk Neutrino Mass Experiment [153] and the Mainz Neutrino Mass Experiment [154] have tried to measure the endpoint region of the tritium beta decay with enormous precision. These experiments rely on the low energy exchange in the beta decay of light nuclei ($18.6\,keV$ in the case of tritium). Both experiments have set $m_\beta < 2.3\,eV$, but, surprisingly, the midpoint of the confidence regions lie in the negative mass squared sector. In particular, the Troitsk Experiment gives $m_\beta = -2.3 \pm 2.5(stat.) \pm 2.0(syst.)\,eV$, while the Mainz experiment gives $m_\beta = -0.6 \pm 2.2(stat.) \pm 2.1(syst.)\,eV$. In order to work out a more precise result, the Karl-

sruhe Tritium Neutrino Experiment (KATRIN) [155], which expects to achieve a sensitivity of $m_\beta > 0.2\,eV$, has already begun to gather data.

Are massive neutrinos a too mild assumption?   There have been attempts of conciling the data of LSND and MiniBOone with other neutrino experiments simply by adding more sterile neutrinos to the model. At this point one might wonder, if this is how one should proceed. Taking into account that sterile neutrinos are almost undetectable particles, for which we have little evidence, is it not just a new problem of adding "epicycles" to the SM? Or is it just that we are driving the SM beyond its range of validity and that we should look for radically new particle physics?

In the previous section about the Cosmological Constant Problem we have argued that QFT should be modified in the IR in order to be consistent with results such as the Bekenstein entropy or the smallness of the cosmological constant. In agreement with this point of view, there is a striking coincidence that should not be overlooked. If we consider the cosmological constant as the vacuum energy of some field theory, the value of this vacuum energy is roughly $\rho_v \sim (10^{-2.5}\,eV)^4$. In comparison, the scale of neutrino oscillations (in this case, the scale driving solar and reactor neutrino oscillations in the SM with massive neutrinos) is of the order of $\Delta m_\odot^2 \sim (10^{-2.5}\,eV)^2$.

We wonder if these numbers are unrelated or if they are derived from the same extension of local relativistic QFT. It might be that the same IR physics which causes the quantum corrections to the vacuum energy to be of the order of an IR scale $\lambda \sim 10^{-2.5}eV$ also causes a modification in the neutrino propagator of the same order. If this is true, we have argued that the corresponding departure from QFT should be nonlocal, and therefore it might even be that the description of neutrino oscillations requires some kind of non-locality.

1.2.3. Ingredients for an alternative to mass-driven oscillations

Alternative mechanisms to mass-driven neutrino oscillations must share a basic characteristic: they must include some new ingredient, which must have different eigenvalues for different neutrino species, and the corresponding eigenstates must not coincide with the flavor eigenstates, i.e. the coupling of these neutrino species to the charged leptons via W exchange must be non-diagonal.

We will assume that the mechanism responsible of the phenomena we have listed above is some oscillation mechanism during the propagation of neutrinos. Other possible mechanisms would be nonstandard couplings to either the gravitational potential (violation of the equivalence principle), a torsion field or even matter.

Lorentz Invariance Violation   Let us think about how one could go beyond the mechanism of mass terms driving the oscillation mechanism. We have stated that an oscillation mechanism can appear only if different neutrino families have different energies $E_i(p)$ for the same momentum p. Lorentz invariance restricts these different energies to be a consequence of different masses. Therefore if we want to go beyond the mechanism of mass-driven oscillations, this has to be done at the cost of abandoning Lorentz invariance. Given that the CPT theorem assures that CPT is a symmetry of any local field theory with Lorentz invariance, then CPT could also be violated. Although problematic from the point of view of self-consistency, CPT violation could be very handy in order to explain the discrepancies between LSND and MiniBOone.

It has been argued that Lorentz Invariance Violation (LIV) is an expectable phenomenon in a theory of quantum gravity. It arises naturally in LQG at scales at which the discreteness of spacetime cannot be neglected; Lorentz Invariance is restored as an approximated symmetry at

low energies. In this case the departure from Lorentz symmetry is characterized by the Planck scale. The dispersion relation of particles reflects the scale of the spacetime fabric, and takes the form [156]

$$E^2 = \mathbf{p}^2 + m^2 + \eta \mathbf{p}^2 \left(\frac{E}{E_p} \right)^n , \qquad (1.2.13)$$

where m is the mass of the tested particle, η is a phenomenological parameter, presumably of order 1, and $n > 0$ is assumed to be an integer. This is the kind of models that are being contrasted with experimental tests of Lorentz invariance. They are explicitly rotation invariant, as it is boost invariance what is usually assumed to be broken by quantum gravity. The reason is the presence of a new very high energy scale (typically the Planck scale E_p, although this might vary in particular models) which is either an invariant of the theory or a quantity in whose value every observer should agree.

Violations of Lorentz Invariance may also arise in ST as a result of Spontaneous Symmetry Breaking (SSB). In order for Lorentz SSB to occur, it would be required that some Lorentz tensor would acquire an effective negative mass term. In relativistic perturbatively renormalizable quantum field theories, it is impossible to arrive to an effective negative mass term for any vector field.

However it has been proven that this is not the case in string theory [157]. For instance, in the open bosonic string, it is possible that a vector field B_μ acquires a nonzero vacuum expectation value due to an effective potential V in the Lagrangian $\mathcal{L}_B$ of the type

$$\mathcal{L}_B = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - V(B_\mu B^\nu \pm b^2) - B_\mu J^\mu , \qquad (1.2.14)$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the field strength, $V(x) = \frac{1}{2}\lambda x^2$ is the potential and J^μ is the matter current. Depending on the sign appearing in the argument of the potential, the vacuum expectation value of the Bumblebee will be timelike or spacelike (the case of a lightlike expectation value is ill-defined). It is usual to take the vacuum expectation value of the vector field, b^μ, as independent of spacetime coordinates so that the broken symmetries are global Lorentz symmetries, although solitonic solutions are also plausible.

In the same sense in which Lorentz symmetry can be spontaneously broken in string theories, it is also possible to find models in which CPT symmetry is also spontaneously broken [158, 159]. Taking to account this spontaneous symmetry breaking, it is reasonable to consider that the low energy limit of such a quantum theory of gravity admits a description as an ordinary QFT including departures from Lorentz invariance and can be studied with the techniques of EFT.

In the Standard Model Extension (SME) [160, 161], the starting point is the minimal SM of particle physics and its associated scale, the electroweak scale m_W. New Lorentz- and possibly CPT-violating terms[13] arising from a spontaneous breaking of Lorentz invariance in a more fundamental theory are assumed to be suppressed at least by the ratio $\frac{m_W}{E_p} \approx 10^{-17}$. The new terms also respect the gauge invariance of the SM. A minimal standard model extension can be defined to include just power-counting renormalizable terms (as in Ref. [161]), while terms of dimensions greater than four can be included using the standard EFT technics [163].

Both in the case of an explicit or a spontaneous Lorentz Symmetry Breaking, the symmetry group is reduced and thus a preferred frame arises. These frame is often identified with the frame in which the CMB is almost isotropic [164]. However our laboratory frame does not coincide with

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[13] The CPT theorem ensures that a local Lorentz invariant theory preserves CPT symmetry [162]. Thus, a local theory violating CPT must violate Lorentz invariance. The converse is not true; Lorentz violating but CPT conserving theories exist.

the CMB rest frame; an obvious consequence is the need of removing the dipole in the observed CMB spectrum, as it only expresses the boost that relates both frames [10]. This boost between the preferred frame (in which the laws of physics are assumed to be isotropic) and the laboratory frame introduces a new directional effect, and therefore an anisotropy in the laws of physics in the laboratory frame. Thus one needs to consider the anisotropic couplings of the SME in order to build a realistic model.

Fortunately, the rotational and translational motions of the Earth are essentially non relativistic in the approximately inertial Sun-centered frame relevant for neutrino experiments, so in practice any direction-dependent effects are suppressed to parts in a thousand or more [165]. Anisotropic effects would also be excluded if instead of a Lorentz violation a new symmetry principle (DSR) would replace Lorentz invariance [166, 167, 168, 169].

The possibility of neutrino oscillations driven or corrected by LIV has been considered in several works, which we will review below.

Agreement with observational and experimental data   As it should always be in any field of science, it will be our priority to find alternatives to mass driven oscillations which are fully consistent with the experimental data. Many models of LIV driving neutrino oscillations have been proposed, although few of them have managed to explain the energy behavior that has been experimentally observed.

In order to fit the existing experimental data (aside from the controversial results of LSND and $0\nu\beta\beta$), it is sufficient for these models to meet one property: that the new effect be a correction proportional to the square of an infrared scale $\lambda \sim 1 - 10^{-3} eV$ at energies $E \gg \lambda$.[14] This is due to the fact that all the neutrino experiments that have been carried out involve high energy neutrinos (compared to the scale λ).

It has not been easy for researchers of LIV in neutrinos to find this energy dependence in their models. In one of the works presented in this PhD thesis, we will show that it is possible to find such a behavior if we invoke nonlocal physics in the IR, as we suggested as a solution to the Cosmological Constant Problem.

Also cosmology will impose important bounds on any alternative to neutrino masses, as well as it imposes a competitive bound on the sum of the neutrino masses in the case of massive neutrinos [171, 172].

In order to have some information about the low energy behavior of neutrinos, it would be necessary to get information directly or indirectly about neutrinos of energies close to this scales, such as tritium decay close to the endpoint of the spectrum or detection of the Cosmic Neutrino Background (CνB), which has not yet been observed. These two experiments are completely unfeasible in the near future. However the effect of the CνB in the cosmic matter power spectrum of large scale structure might be observable.

In conclusion, although neutrino masses are a simple extension of the SM perfectly able to describe the current experimental results on neutrino oscillations (aside of the poorly understood LSND and MiniBOone), it is not the only possibility. An alternative mechanism to neutrino masses requires LIV, but it must also be consistent with the experimental and observational data. A sufficient condition is that the deformation of the dispersion relation of neutrinos involves the square of some IR energy scale.

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[14]Surprisingly, the tandem model presented in Ref. [170] manages to fit the experimental data without resorting to this mechanism.

1.2.4. Review on alternatives to neutrino masses

In this subsection I want to review some of the attempts to offer an alternative description of neutrino oscillations with massless neutrinos. As mentioned this requires to go beyond Special Relativity (SR). Neutrino oscillations have been suggested to be a good probe of physics beyond SR in Ref. [164], maybe arising in the low energy limit of a theory of quantum gravity. A folkloric example is a quantum black hole swallowing a neutrino of a given flavor and evaporating into a neutrino of a different one [173].

In particular, it has been stated that the addition of Lorentz violating but CPT conserving terms to the SM Lagrangian would induce differences in the maximum attainable velocities of the neutrinos which may depend on their nature and helicity. It was also first stated that massless neutrinos could oscillate in vacua in the presence of LIV. More complicated models have been built in order to fit the vast amount of experimental data that has been collected in the past decade.

Velocity-driven oscillations Coleman and Glashow introduced a simple model of oscillation between electron and muon neutrino [164],

$$\nu_\mu = \cos\theta\nu_1 + \sin\theta\nu_2 \,, \tag{1.2.15}$$

$$\nu_e = \cos\theta\nu_2 - \sin\theta\nu_1 \,, \tag{1.2.16}$$

where ν_1 and ν_2 are propagation eigenstates with different velocities and dispersion relations

$$E_1 = p(1 + v_1) \,, \ E_2 = p(1 + v_2) \,, \tag{1.2.17}$$

and θ is a phase characterizing the mixing. The differences between the speed of the neutrino and the speed of light v_i, are assumed to be $|v_i| \ll 1$. The populations of the flavor eigenstates in the propagation eigenstates are

$$r_{1\mu} = r_{2e} = \cos^2\theta \,, \ r_{1e} = r_{2\mu} = \sin^2\theta \,, \tag{1.2.18}$$

and the probability of an electron neutrino of traversing a distance R without changing its flavor is

$$P_{e \to e}(R) = r_{1e}\sin^2\left((v_1 - v_2)pR/2\right) \,, \tag{1.2.19}$$

while the same probability, if the oscillation mechanism is driven by mass differences, is

$$P_{e \to e}(R) = r_{1e}^m \sin^2\left((m_1^2 - m_2^2)R/4p\right) \,. \tag{1.2.20}$$

It is also possible that both masses and difference between velocities are behind the neutrino oscillations. In this case, the diagonalization problem would be more complicated and momentum dependent. The effective Hamiltonian would be of the type

$$\mathcal{H}_{\alpha\beta} = p\left(\delta_{\alpha\beta} + V_{\alpha\beta}\right) + \frac{1}{2p}M_{\alpha\beta}^2 \tag{1.2.21}$$

in the base of flavor eigenstates. The diagonalization of the effective Hamiltonian should be performed for any given momentum p of the neutrino, resulting in a mixing of the type

$$P_{e \to e}(R) = r_{1e}(p)\sin^2\left((E_1(p) - E_2(p))R/2\right) \,. \tag{1.2.22}$$

The limits of low and high momentum of this expression should coincide with the expressions for the mass driven (1.2.20) and velocity driven oscillations (1.2.19), respectively.

The clear distinction between the behavior with momentum p between (1.2.19) and (1.2.20) has enabled the experimentalists to clearly favor mass driven oscillations over velocity driven oscillations [136], at least if the velocity of the neutrinos is independent of their frequency. In general, a deviation from the energy behavior of (1.2.20) would be a smoking-gun signal of LIV effects [174].

This has therefore led most of the studies of Lorentz violation in neutrino physics to consider Lorentz violation as a subleading mechanism that can be tested within the framework of mass driven oscillations. In some of these studies [175, 174, 176, 177, 178] the LIV has been studied at the level of a phenomenological deformation of the dispersion relation of the type of (1.2.13).

The Standard Model Extension and Neutrino Oscillations   As we mentioned before, the Minimal SME includes all the Lorentz non-invariant terms with or without CPT violation which are perturbatively renormalizable and respect the gauge symmetries of the SM. The new terms can be divided into CPT preserving (CPT-even) or CPT violating (CPT-odd). The new terms are supposed to be suppressed with respect to the Lorentz conserving terms of the same kind by powers of the electroweak scale over the Planck scale. In this sense, some of the dimensionful terms can be interpreted as new IR scales.

As an example, we will give the explicit form of the terms in the fermion sector. In the lepton sector, the new terms are:

$$\mathcal{L}_{lepton}^{CPT-even} = \frac{i}{2}(c_L)_{\mu\nu\alpha\beta}\bar{\Psi}_{L\alpha}\gamma^\nu\overleftrightarrow{\nabla}_\mu\Psi_{L\beta} + \frac{i}{2}(c_R)_{\mu\nu\alpha\beta}\bar{L}_\alpha\gamma^\nu\overleftrightarrow{\nabla}_\mu L_\beta \,, \qquad (1.2.23)$$

$$\mathcal{L}_{lepton}^{CPT-odd} = -(a_L)_{\mu\alpha\beta}\bar{\Psi}_{L\alpha}\gamma^\mu\Psi_{L\beta} - (a_R)_{\mu\alpha\beta}\bar{L}_\alpha\gamma^\mu L_\beta \,. \qquad (1.2.24)$$

All the coefficients c and a are understood to be hermitian matrices in flavor space as well as Lorentz tensors. These new terms are of dimension 4 and 3, respectively. Therefore the coefficients c_L and c_R are dimensionless and may have symmetric and antisymmetric parts but can be assumed to be traceless. The coefficients a_L and a_R have dimensions of energy. If we consider that these vectors are timelike, in the preferred frame of reference their only nonzero component will be $(a_L)_0$ and $(a_R)_0$, and will play the role of an IR scale.

In the quark sector the new terms are analogous to those appearing in the lepton sector, with coefficient matrices c_Q, c_U, c_D, ... Other Lorentz violating terms appear in the Yukawa, Higgs and gauge sectors of the model.

The Minimal SME has been constrained by a huge amount of experiments, among which we could stress high precision tests of QED, neutrino and neutral meson oscillations (for a living review on tests of Lorentz Invariance see Ref. [26], for constraints on the SME parameters see Ref. [179]). Despite the long list of experiments, a vast sector of the Minimal SME remains yet unexplored. Meson oscillation experiments have set the CPT-violating coefficients of the quark sector $a_\mu < 10^{-20}GeV$. The most stringent bounds come from precision tests in QED. Bounds on a spacelike vector vev b_μ are of order $10^{-25}GeV$ to $10^{-31}GeV$ depending of the experiment and the kind of particle.

The lepton sector has also been constrained with the data provided by neutrino oscillations. The possibility of LIV-induced neutrino oscillations arises in the model without sterile neutrinos, and the SME has also been extended to include the righthanded Dirac neutrinos by Kostelleckỳ and Mewes [180]. The complete SME with righthanded neutrinos has many parameters and thus several simpler models have been built within this framework in order to gain predictivity. These

models include neutrinos with [180, 181, 170, 178] or without [180, 182, 183, 181] mass terms, in which the Lorentz-violating coefficients of the SME are treated as subleading terms or as driving the oscillation phenomenon, respectively.

Aside from the aforementioned anomalous energy dependence of the oscillation phenomena, more new effects are expected if the SME with righthanded neutrinos is taken into account [180]. Anisotropic terms induce periodic variations in the oscillations: those induced by the Earth's rotation and sidereal translation. These effects can be averaged away by integrating the data over time, but anisotropic effects would not be erased, as asymmetries between the east-west and north-south directions or between the ecliptic plane and the ecliptic north-south would appear. Other possible effects arising from the SME would be discrete symmetry violations, like effects violating lepton number (as neutrino-antineutrino oscillations), CPT (deviations from the relationship $P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$ or both. CPT violation is extremely interesting as it could provide an explanation to the discrepancy between the LSND [145] and MiniBoONE [147] experiments.

The most general modified Dirac equation of the neutrino fields within the SME with righthanded neutrinos would be [180]

$$\left(i\Gamma^\nu_{\bar{\alpha}\bar{\beta}} \partial_\nu - M_{\bar{\alpha}\bar{\beta}} \right) \nu_{\bar{\beta}} = 0 \,, \tag{1.2.25}$$

where $\bar{\alpha}$ and $\bar{\beta}$ run for the three possible neutrino flavors and their charge conjugates and

$$\Gamma^\nu_{\bar{\alpha}\bar{\beta}} \quad := \quad \gamma^\nu \delta_{\bar{\alpha}\bar{\beta}} + c^{\mu\nu}_{\bar{\alpha}\bar{\beta}} \gamma_\mu + d^{\mu\nu}_{\bar{\alpha}\bar{\beta}} \gamma_5 \gamma_\mu + e^\nu_{\bar{\alpha}\bar{\beta}} + if^\nu_{\bar{\alpha}\bar{\beta}} \gamma_5 + \frac{1}{2} g^{\lambda\mu\nu}_{\bar{\alpha}\bar{\beta}} \sigma_{\lambda\mu} \,, \tag{1.2.26}$$

$$M_{\bar{\alpha}\bar{\beta}} \quad := \quad m_{\bar{\alpha}\bar{\beta}} + im_{5\bar{\alpha}\bar{\beta}} \gamma_5 + a^\mu_{\bar{\alpha}\bar{\beta}} \gamma_\mu + b^\mu_{\bar{\alpha}\bar{\beta}} \gamma_5 \gamma_\mu + \frac{1}{2} H^{\mu\nu}_{\bar{\alpha}\bar{\beta}} \sigma_{\mu\nu} \,. \tag{1.2.27}$$

The coefficients m and m_5 are a linear combination of the righthanded and lefthanded neutrino mass matrices m_R and m_L, and thus Lorentz invariant. The coefficients c, d, H are CPT conserving but Lorentz violating, while a, b, e, f, g are CPT violating. Hermiticity implies that all the coefficients are Hermitian in flavor space.

The effective Hamiltonian for the neutrinos in the Minimal SME reads

$$(\mathcal{H}_{eff})_{\bar{\alpha}\bar{\beta}} = \mathcal{H}_0 - \frac{1}{2} \left\{ \gamma^0 \delta\Gamma^0, \mathcal{H}_0 \right\} - \gamma^0 (i\delta\Gamma^j \partial_j - \delta M) \,, \tag{1.2.28}$$

where $\mathcal{H}_0 = -\gamma^0(i\gamma^j \partial_j - M_0)$ is the general Lorentz conserving Hamiltonian, M_0 is the Lorentz conserving part of M and $\delta\Gamma$, δM are the Lorentz violating parts of Γ, M. The general treatment is possible but rather horrible, due to the vast amount of parameters and the fact that the coefficient matrices do not necessarily commute.

Therefore the model has to be simplified in order to gain some predictive power. It is standard to get rid off righthanded neutrinos and present a model for oscillations among active (lefthanded) neutrinos and their antiparticles. Further simplification can be achieved if oscillations between neutrinos and antineutrinos $\nu \leftrightarrow \bar{\nu}$ are highly suppressed. Then, the effective Hamiltonian for the lefthanded ultrarelativistic neutrinos reads

$$(h_{eff})_{\alpha\beta} = p\delta_{\alpha\beta} + \frac{1}{2p} \left(m^2_L \right)_{\alpha\beta} + \frac{1}{p} \left[(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu \right]_{\alpha\beta} \tag{1.2.29}$$

and its complex conjugate for antineutrinos, with the opposite sign for the CPT-odd coefficient a_L. In the previous formula m_L is the combination of m and m_5 relevant for lefthanded neutrinos (see [180] for the details), $c_L = c + d$ and $a_L = a + b$. Even simpler models can be built in order to explain the oscillations data.

1. Introduction

'Fried-chicken' models [180] are those in which spatial isotropy is imposed,

$$\left(h_{eff}^{FC}\right)_{\alpha\beta} = p\delta_{\alpha\beta} + \left[\frac{1}{2p}\left(m_L^2\right) + (a_L)^0 - \frac{4}{3}\left(c_L\right)^{00} p\right]_{\alpha\beta} \tag{1.2.30}$$

so that the study of modified dispersion relations is not complicated by direction dependent effects. Most models of Lorentz-violating neutrino oscillations fall in this class. However, isotropy in the laboratory frame is difficult to motivate in a Lorentz violating theory.

Anisotropic models, in contrast, need a well defined spacetime reference frame, which is usually taken to be the Sun-centered celestial equatorial frame of coordinates $\{X, Y, Z, T\}$, in order to report the values of its coefficients. An interesting model of this class is the so called 'Bicycle Model' presented in Ref. [182], in which most of the oscillations data can be explained only with two free parameters and no mass term.

It must be remembered that the a_L coefficient has dimensions of mass and represents some infrared scale, presumably of order $\frac{m_W^2}{E_p}$. If the dimensionless coefficient c_L is of order $\frac{m_W}{E_p}$, then for momenta well below the electroweak scale (and this happens in every single neutrino oscillations experiment), the dominant source of the oscillation in a massless model ($m_L = 0$) is the a_L term. The argument of the oscillation's sine function will be $\sim \Delta a_L L$ instead of the standard $\Delta m^2 L/2E$. This behavior is disfavored by the experimental data.

However, this naïve energy behavior was surmounted by the Bicycle Model. In this model, the coefficients $c_{Lee}^{TT} = \frac{3}{2}\check{c}$ and $a_{Le\mu}^Z = a_{Le\tau}^Z = \frac{\check{a}}{\sqrt{2}}$ are the only nonzero coefficients. The resulting neutrino oscillation probabilities are

$$
\begin{aligned}
P_{\nu_e \to \nu_e} &= 1 - 4\sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right), & (1.2.31)\\
P_{\nu_e \to \nu_\mu} &= P_{\nu_e \to \nu_\tau} = 2\sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right), & (1.2.32)\\
P_{\nu_\mu \to \nu_\mu} &= P_{\nu_\tau \to \nu_\tau} = 1 - \sin^2\theta\sin^2\left(\Delta_{21}L/2\right) & \\
&\quad - \sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right) - \cos^2\theta\sin^2\left(\Delta_{32}L/2\right), & (1.2.33)\\
P_{\nu_\mu \to \nu_\tau} &= \sin^2\theta\sin^2\left(\Delta_{21}L/2\right) & \\
&\quad - \sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right) + \cos^2\theta\sin^2\left(\Delta_{32}L/2\right), & (1.2.34)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_{21} &= \sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2} + \check{c}p, & (1.2.35)\\
\Delta_{31} &= \sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2}, & (1.2.36)\\
\Delta_{32} &= \sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2} - \check{c}p, & (1.2.37)\\
sin^2\theta &= \frac{1}{2}\left[1 - \check{c}p/\sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2}\right], & (1.2.38)
\end{aligned}
$$

and Θ is the angle between the celestial north pole and the direction of propagation. The same probabilities are valid for antineutrinos (which are obtained by changing $\check{a} \to -\check{a}$).

In this model, electron neutrino propagating at low energies compared with the scale $\check{a}/\check{c}$ oscillate into muon and tau neutrinos. However, at greater energies electron neutrinos do not oscillate, but there is a maximal mixing between muon and tau neutrinos, which oscillate with a wavelength $\sim \frac{\check{c}E}{\check{a}^2\cos^2\Theta}$. Thus, high energy oscillations mimic mass driven oscillations with a direction-dependent effective mass squared difference $\Delta m_\Theta^2 = \check{a}^2\cos^2\theta/\check{c}$. The model failed to explain the LSND anomaly, but did quite well in fitting the then-current neutrino data with

$\check{c} = 10^{-19}$ and $\check{a} = 10^{-20} GeV$. More recent tests using crossed data have ruled out the bicycle model and some simple extensions [184].

The big difference between the predictions of this model and the standard mass driven oscillations mechanism is the directional dependence which induce small semiannual variations in the neutrino flux that could be observed and 'compass' asymmetries. Directional variations have been tightly constrained by MINOS [185]. Nevertheless, the model can be modified to have no directional dependence by changing a_L^Z to a_L^T with a small impact on the fits [182].

An extension of the Bicycle Model, the 'Tandem Model' [170] was proposed to fit also the anomalous data of LSND. The model includes similar parameters to the ones in the Bicycle Model and a single mass term $m_{\tau\tau}$. The effective Hamiltonian can be written in the flavor basis as

$$h_{TM}^\nu = \begin{pmatrix} (1 + \check{c})p & \check{a} & \check{a} \\ \check{a} & p & \check{a} \\ \check{a} & \check{a} & p + \check{m}^2/2p \end{pmatrix} , \qquad (1.2.39)$$

where $\check{c} = (c_L)_{ee}^{TT}$ and $\check{a} = (a_L)_{e\mu}^T = (a_L)_{e\tau}^T = (a_L)_{\mu\tau}^T$. The corresponding effective Hamiltonian for antineutrinos is obtained by changing $\check{a} \to -\check{a}$.

The Tandem Model provides a good description of existing neutrino-oscillations data with

$$\begin{aligned} \frac{1}{2}\check{m}^2 &= 5.2 \times 10^{-3} eV^2 , \\ \check{a} &= -2.4 \times 10^{-19} GeV , \\ \check{c} &= 3.4 \times 10^{-17} . \end{aligned} \qquad (1.2.40)$$

Solar neutrino oscillations can be fit without the need of any MSW effect [186, 187]. Atmospheric neutrino oscillations can be fit as in the Bicycle Model, as it reduces to it at high energies. Long-baseline experiments like KamLAND [188] lie in the low energy regime in which the $\check{c}$ term can be neglected. In this regime, the CPT violation is manifest as a difference in the ν_e and $\bar{\nu}_e$ survival probabilities which can be tested. Short-baseline experiments like LSND [145] and MiniBOone [147] can be fit due to this CPT violation. The effect lies on the edge of experimental sensitivity due to the short paths that are being tested. In fact, the tandem model predicted a low energy excess in the MiniBOone experiment prior to its discovery, although the observed excess is qualitatively greater. This discovery remarks the interest of the search for hybrid mass and LIV-driven oscillation mechanisms. Nevertheless, the observations of LSND and MiniBOone must be confirmed by other ongoing or planned short-baseline experiments.

Finally, the case of mass-driven neutrino oscillation modified by subleading LIV-effects has also been studied in the framework of the SME. The constraints of the parameters of the model have been gathered in Ref. [178].

Neutrino Oscillations arising from a noncanonical quantization procedure? Motivated by the apparition of noncommutative spacetime [189] in ST [190, 191, 192], it has been proposed that the most general extension of Noncommutative Quantum Mechanics (NCQM) [193, 194, 195] to field theory is to make the fields, which are the degrees of freedom in a field theory, noncommutative. The field noncommutativity is introduced in a scalar quantum field theory at the level of the commutation relations that define the way from the classical theory to the QFT.

In this scalar quantum 'noncommutative' or noncanonical field theory (QNCFT) [196, 197], the (equal-time) commutation relations read

$$
\begin{aligned}
\left[\Phi(\mathbf{x}), \Phi^\dagger(\mathbf{x}')\right] &= \theta\delta^{(3)}(\mathbf{x}-\mathbf{x}'), \\
\left[\Phi(\mathbf{x}), \Pi^\dagger(\mathbf{x}')\right] &= i\delta^{(3)}(\mathbf{x}-\mathbf{x}'), \\
\left[\Pi(\mathbf{x}), \Pi^\dagger(\mathbf{x}')\right] &= B\delta^{(3)}(\mathbf{x}-\mathbf{x}'),
\end{aligned}
\tag{1.2.41}
$$

while the free Hamiltonian remains unmodified.[15] The resulting free theory can be fully solved and proves it to be a theory of particles a and antiparticles b with Lorentz and CPT violating dispersion relations

$$
\frac{E_a(\mathbf{p})}{\omega_{\boldsymbol{p}}} = \sqrt{1+\left(\frac{B/\omega_{\boldsymbol{p}}-\theta\omega_{\boldsymbol{p}}}{2}\right)^2} + \frac{B/\omega_{\boldsymbol{p}}+\theta\omega_{\boldsymbol{p}}}{2}, \tag{1.2.42}
$$

$$
\frac{E_b(\mathbf{p})}{\omega_{\boldsymbol{p}}} = \sqrt{1+\left(\frac{B/\omega_{\boldsymbol{p}}-\theta\omega_{\boldsymbol{p}}}{2}\right)^2} - \frac{B/\omega_{\boldsymbol{p}}+\theta\omega_{\boldsymbol{p}}}{2}, \tag{1.2.43}
$$

where $\omega_{\boldsymbol{p}} = \sqrt{\mathbf{p}^2+m^2}$. Note that the parameters introduced in the commutation relations act as an UV and an IR scale. Thus, relativistic QFT is recovered in the limit in which the energies of the particles are between but far from these values.

QNCFT was extended in Ref. [199] to spinor fields. In this case, Hamiltonian of the spinor fields is the standard massless one, but the anticommutation relations are changed. In their model a massless spinor field with just two flavors is considered. The theory is quantized by imposing the (equal-time) anticommutation relations

$$
\left\{\Psi_\alpha^i(\mathbf{x}), \Psi_\beta^{j\dagger}(\mathbf{x}')\right\} = \mathcal{A}_{\alpha\beta}\delta^{ij}\delta^{(3)}(\mathbf{x}-\mathbf{x}'), \tag{1.2.44}
$$

where i,j are the spinor indices and $\mathcal{A}$ is an hermitian matrix. The new parameters in $\mathcal{A}$ are in this case adimensional, and therefore the theory amounts to be a field theory description of the model introduced in Ref. [164] of velocity driven oscillations or of the SME with just a c_L^{00} nonzero coefficient, and therefore is ruled out by experiment.

The authors of [199] try to surmount this problem by making the eigenvalues of $\mathcal{A}$ momentum-dependent, but this has not much sense as the anticommutation is just a function of the spacetime coordinates. We will try to solve this problem in one of the papers presented in this thesis.

1.2.5. Neutrino Physics and new physics in the IR

In this thesis we want to present an alternative to neutrino oscillation driven by neutrino masses which does not modify the field content of the SM, respects the gauge invariance principle and does not imply lepton number violating phenomenon. We also want this alternative to be consistent with all the present experimental and observational data and physically distinguishable from the standard mechanism with future data.

We will seek a realistic model in the framework of QNCFT, for it is a simple extension of QFT. We will try to find a modification of the dispersion relation of neutrinos which involves the square of an IR scale at momenta well above this scale.

The solution appears to invoke a nonlocal extension of the anticommutation relations (1.2.44), similar to the one in Ref. [198] and to the nonlocal extension of QFT in the IR that was suggested in the previous section. We will refer to this modified neutrino physics as noncanonical neutrinos.

---

[15]This commutation relations have also been extended in Ref. [198], in which the Dirac deltas have been turned into Gaussian distributions. Note that this change implies that the theory becomes nonlocal.

1.2.6. Probing new neutrino physics in cosmology

The aforementioned alternative mechanism for neutrino oscillations will be difficult to be distinguished from the mass mechanism by means of particle physics experiments. The reason is that particle physics experiments always probe neutrinos of energies well above the IR scale, and in this regime the behavior of both mechanism is the almost the same.

In order to distinguish between massive neutrinos and noncanonical neutrinos, it will be necessary to probe neutrinos of momenta comparable to or be below the IR scale, and such neutrinos can be found in the CνB. However in order to consistently explore the cosmological effect of such a Lorentz violating, nonlocal neutrinos, it will be necessary to extend the concept of QFT in curved spacetimes [19] to more general field theories.

Doing this consistently is a difficult task and is beyond the scope of this PhD thesis. However, as an first step, we will explore how the scalar QNCFT presented in Refs. [196, 197] may couple to gravity. The symmetry properties of this theory have been studied in Ref. [200]. A more detailed introduction to QNCFT and its symmetry properties have been included in this work. The requirement of keeping this symmetry properties in the curved spacetime case will be essential for determining the precise form of the coupling.

1.3. Objectives

We have tried to argue that problems in cosmology might be solved by an extension of QFT in the IR and the UV, and that the effect of gravity might be behind a modification of QFT responsible of ill-understood phenomena such as neutrino oscillations. Although others have considered top-down approaches to these problems, we think that tracking the experimental data from a bottom-up perspective is the key of scientific progress. Thus we will try to explore departures from GR and QFT in the UV and the IR that might be behind the present problems in cosmology and the recently explored physics of neutrinos.

In the first of the papers presented in this thesis, we will build a cosmological model based on a phenomenological departure from the cosmological standard model both in the UV and the IR, the Asymptotic Cosmological Model. The model will be defined in the homogeneous approximation. The kinematics of the universe will be determined. The horizon problem will be automatically solved. Through a simulation using Monte Carlo Markov Chains, the model will be contrasted with the available observational data which do not depend on the behavior of inhomogeneous perturbations. Also particular realizations of the model in the homogeneous approximation will be considered among the most popular theories and models in cosmology.

We will also try to depart from the homogeneous approximation in the Asymptotic Cosmological Model. In the second paper presented in this thesis, the linearized treatment of scalar perturbations in the model will be derived with just one assumption based on phenomenological consistence. The resulting model will turn out to be distinguishable from the most popular theories and models in cosmology. The effect of the departure from the standard cosmological model in the CMB spectrum will be computed using a modification of the CAMB code [201].

An alternative to neutrino masses as a mechanism for neutrino oscillations which is compatible with all the experimental data within the framework of QNCFT will be presented in the third of the papers presented in this thesis. The extension will not add new fields to the SM and respect all its gauge and discrete symmetries, but will violate Lorentz invariance and locality. This violation will be mediated by a new IR scale controlling the modification of the dispersion relation and the presence of new physics in the IR.

In order to build a theory of quantum fields in curved spacetime for particles with modified

dispersion relations, it will also be important to distinguish if Lorentz symmetry is violated or deformed, as we will see in the fourth of the works presented in this PhD thesis. This distinction could be crucial in order to derive the quantum corrections to the cosmological constant, for instance. In the fouth of the papers presented in this compendium we will explain how the symmetry properties determine the formulation of an extension of QFT to curved spacetime. For the sake of simplicity we will choose the case of a QNCFT of a scalar field in the $B = 0$ limit.

A comment on the implications of this mechanism of neutrino oscillations in cosmology will be included in Appendix A. Here a naïve calculation of the effect of a modified dispersion relation of neutrinos in the expansion of the universe is presented.

2. An extension of the cosmological standard model
with a bounded Hubble expansion rate

The possibility of having an extension of the cosmological standard model with a Hubble expansion rate H constrained to a finite interval is considered. Two periods of accelerated expansion arise naturally when the Hubble expansion rate approaches to the two limiting values. The new description of the history of the universe is confronted with cosmological data and with several theoretical ideas going beyond the standard cosmological model.

2.1. Introduction

According to General Relativity, if the universe is filled with the particles of the Standard Model of particle physics, gravity should lead to a deceleration of the expansion of the universe. However, in 1998 two independent evidences of present accelerated expansion were presented [5, 6] and later confirmed by different observations [10, 11, 14]. On the other hand, measurements of large scale structure [15] and CMB anisotropy [10] also indicate that the universe evolved through a period of early accelerated expansion (inflation).

There is no compelling explanation for any of these cosmic accelerations, but many intriguing ideas are being explored. In the case of inflation, the origin of the accelerated expansion can be either a modification of gravity at small scales [46] or a coupling of the expansion of the universe to the progress of phase transitions [202, 42, 1, 43, 44]. In the case of the present accelerated expansion these ideas can be classified into three main groups: new exotic sources of the gravitational field with large negative pressure [126, 120, 122] (Dark Energy), modifications of gravity at large scales [203, 204] and rejection of the spatial homogeneity as a good approximation in the description of the present universe [52, 49, 50].

Different models (none of them compelling) of the source responsible for each of the two periods of accelerated expansion have been considered. Einstein equations admit a cosmological constant Λ, which can be realized as the stress-energy tensor of empty space. This Λ, together with Cold Dark Matter, Standard Model particles and General Relativity, form the current cosmological model (ΛCDM). However, quantum field theory predicts a value of Λ which is 120 orders of magnitude higher than observed. Supersymmetry can lower this value 60 orders of magnitude, which is still ridiculous [21]. In order to solve this paradox, dynamical Dark Energy models have been proposed.

This has also lead to explore the possibility that cosmic acceleration arises from new gravitational physics. Also here several alternative modifications of the Einstein-Hilbert action at large and small curvatures [205, 80, 82, 96, 206, 81, 98], or even higher dimensional models [107, 106], producing an accelerated expansion have been identified. All these analysis include an *ad hoc* restriction to actions involving simple functions of the scalar curvature and/or the Gauss-Bonnet tensor. This discussion is sufficient to establish the point that cosmic acceleration can be made compatible with a standard source for the gravitational field, but it is convenient to consider a

more general framework in order to make a systematic analysis of the cosmological effects of a modification of general relativity.

In this paper we parametrize the evolution of the universe (considered isotropic and homogeneous) with the Hubble parameter H. One finds that it is possible to restrict the domain of H to a bounded interval. This restriction naturally produces an accelerated expansion when the Hubble expansion rate approaches any of the two edges of the interval. Therefore, we find a new way to incorporate two periods of cosmic acceleration produced by a modification of general relativity. But the dependence on the two limiting values of H can be chosen independently. One could even consider a Hubble expansion rate constrained to a semi-infinite interval with a unique period of accelerated expansion. In this sense the aim to have a unified explanation of both periods of accelerated expansion is only partially achieved. This simple phenomenological approach to the problem of accelerated expansion in cosmology proves to be equivalent at the homogeneous level to other descriptions based on modifications of the Einstein-Hilbert action or the introduction of exotic components in the matter Lagrangian.

In the next section we will review how a general modification of the gravitational action leads to a generalized first Friedman equation at the homogeneous level. In the third section we will present a specific model (ACM) based on the simplest way to implement a bounded interval of H. In the fourth section we will contrast the predictions of ACM for the present acceleration with astrophysical observations. In the fifth section we will show that it is always possible to find modified gravitational actions which lead to a given generalized first Friedman equation, and present some simple examples. In the sixth section we will show that it is also always possible to find Dark Energy models which are equivalent to a given generalized first Friedman equation, and present some simple examples. The last section is devoted to summary and conclusions.

2.2. Action of the cosmological standard model extension

The spatial homogeneity and isotropy allow to reduce the gravitational system to a mechanical system with two variables $a(t)$, $N(t)$ which parametrize the Robertson-Walker geometry

$$ds^2 = N^2(t)dt^2 - a^2(t)\Delta_{ij}dx^i dx^j \tag{2.2.1}$$

$$\Delta_{ij} = \delta_{ij} + \frac{kx_i x_j}{1 - k\mathbf{x}^2} . \tag{2.2.2}$$

Invariance under parameterizations of the time variable imply that the invariant time differential $N(t)dt$ must be used. Also a rescaling of the spatial variables $x^i \rightarrow \lambda x^i$ together with $a(t) \rightarrow \lambda^{-1}a(t)$ and $k \rightarrow \lambda^{-2}k$ is a symmetry that must be kept in the Lagrangian. The action of the reduced homogeneous gravitational system can be then written as

$$I_g = \int dt N L \left(\frac{k}{a^2}, H, \frac{1}{N}\frac{dH}{dt}, \frac{1}{N}\frac{d}{dt}\left(\frac{1}{N}\frac{dH}{dt}\right), ... \right) , \tag{2.2.3}$$

with $H = \frac{1}{aN}\frac{da}{dt}$.

If we keep the standard definition of the gravitational coupling and the density (ρ) and pressure (p) of a cosmological homogeneous and isotropic fluid as a source of the gravitational field, we have the equations of the reduced system

$$\left(\frac{8\pi G_N}{3} \right) \rho = -\frac{1}{a^3}\frac{\delta I_g}{\delta N(t)} \tag{2.2.4}$$

$$- (8\pi G_N)\, p \;=\; -\frac{1}{N a^2}\frac{\delta I_g}{\delta a(t)}\,. \tag{2.2.5}$$

It is possible to choose a new time coordinate t' such that

$$\frac{dt'}{dt} \;=\; N(t)\,. \tag{2.2.6}$$

This is equivalent to set $N = 1$ in the action (2.2.3) of the gravitational system and the evolution equations of the cosmological model reduce to a set of equations for the scale factor $a(t)$. If we introduce the notation

$$H^{(i)} \;=\; \left(\frac{d}{dt}\right)^i H \tag{2.2.7}$$

$$(\delta_i L)^{(j)} \;=\; \left(\frac{d}{dt}\right)^j \left[a^3 \partial_i L\right]\,, \tag{2.2.8}$$

with $\partial_i L$ denoting the partial derivatives of the Lagrangian as a function of the variables $(k/a^2, H, H^{(1)}, H^{(2)}, ...)$ we have

$$\left(\frac{8\pi G_N}{3}\right)\rho \;=\; -L + \sum_{i=2}^{\infty}\sum_{j=0}^{i-2}\frac{(-1)^{i-j}}{a^3}H^{(j)}\,(\delta_i L)^{(i-j-2)} \tag{2.2.9}$$

$$- (8\pi G_N)\, p \;=\; -3L + \frac{2k}{a^2}\partial_1 L + \sum_{i=2}^{\infty}\frac{(-1)^i}{a^3}(\delta_i L)^{(i-1)}\,. \tag{2.2.10}$$

In the homogeneous and isotropic approximation, the vanishing of the covariant divergence of the energy-momentum tensor leads to the continuity equation

$$\frac{d}{dt}\left(\rho a^3\right) \;=\; -p\frac{d}{dt}a^3\,. \tag{2.2.11}$$

In the radiation dominated era one has

$$\rho \;=\; 3p \;=\; \frac{\sigma}{a^4}\,, \tag{2.2.12}$$

where σ is a constant parameterizing the general solution of the continuity equation. In a period dominated by matter one has a pressureless fluid and then

$$p = 0\,, \qquad\qquad \rho \;=\; \frac{\eta}{a^3}\,, \tag{2.2.13}$$

with constant η. When these expressions for the energy density and pressure are plugged in (2.2.9-2.2.10), one ends up with two compatible differential equations for the scale factor $a(t)$ which describe the evolution of the universe. From now on we will use the more common notation $\frac{dH}{dt} \equiv \dot{H}$,...

2.3. The Asymptotic Cosmological Model

Let us assume that the Lagrangian L of the gravitational system is such that the evolution equations (2.2.9-2.2.10) admit a solution such that $a(t) > 0$ (absence of singularities), $\dot{a} > 0$ (perpetual expansion) and $\dot{H} < 0$. In that case, one has a different value of the scale factor a

and the Hubble rate H at each time and then one has a one to one correspondence between the scale factor and the Hubble rate. Since the continuity equation (together with the equation of state) gives a relation between the scale factor and the density, one can describe a solution of the evolution equations of the generalized cosmological model through a relation between the energy density and the Hubble rate, i.e. through a generalized first Friedman equation

$$\left(\frac{8\pi G_N}{3}\right)\rho \,=\, g(H)\,,\tag{2.3.1}$$

with g a smooth function which parametrizes the different algebraic relations corresponding to different solutions of different cosmological models. Each choice for the function $g(H)$ defines a phenomenological description of a cosmological model. Then one can take it as a starting point trying to translate any observation into a partial information on the function $g(H)$ which parametrizes the cosmological model.

Eq. (2.3.1) is all one needs in order to reconstruct the evolution of the universe at the homogeneous level. The generalized second Friedman equation is obtained by using the continuity equation (2.2.11) and the expression for ρ as a function of H. One has

$$-\,(8\pi G_N)\,p \,=\, 3g(H) + \frac{g^{'}(H)}{H}\dot{H}\,.\tag{2.3.2}$$

In the matter dominated era one has

$$\frac{\dot{H}}{H^2} \,=\, -3\frac{g(H)}{Hg'(H)}\tag{2.3.3}$$

and then the assumed properties of the solution for the evolution equations require the consistency conditions

$$g(H) > 0\,,\qquad g^{'}(H) > 0\,.\tag{2.3.4}$$

In the period dominated by radiation one has

$$\frac{\dot{H}}{H^2} \,=\, -4\frac{g(H)}{Hg'(H)}\tag{2.3.5}$$

instead of (2.3.3) and the same consistency conditions (2.3.4) for the function $g(H)$ which defines the generalized first Friedman equation.

We introduce now a phenomenological cosmological model defined by the condition that the Hubble rate has an upper bound H_+ and a lower bound H_-. This can be implemented through a function $g(H)$ going to infinity when H approaches H_+ and going to zero when H approaches H_-. We will also assume that there is an interval of H in which the behavior of the energy density with the Hubble parameter is, to a good approximation, scale-free i.e. $g(H) \propto H^2$. The source of the gravitational field will be a homogeneous and isotropic fluid composed of relativistic and non-relativistic particles. The Cosmological Standard Model without curvature is recovered in the limit $H_-/H \to 0$ and $H_+/H \to \infty$ which is a good approximation for the period of decelerated expansion.

Notice that this interpretation is independent of the underlying theory of gravitation. The Hubble parameter can be used to parametrize the history of universe as long as $\dot{H} \neq 0\,\forall\,t$. The total density can be thus expressed as a function of H. If H is bounded, then $\rho(H)$ will have a pole at $H = H_+$ and a zero at $H = H_-$. Far from these scales, the behavior of $\rho(H)$ can be assumed to be approximately scale-free.

Under these conditions, we can parametrize the dependence of the cosmological model on the lower bound H_- by

$$g(H) = H^2 h_- \left(\frac{H_-^2}{H^2} \right) \tag{2.3.6}$$

and similarly for the dependence on the upper bound H_+

$$g(H) = \frac{H^2}{h_+ \left(\frac{H^2}{H_+^2} \right)} , \tag{2.3.7}$$

where the two functions $h_\pm$ satisfy the conditions

$$\lim_{x \to 0} h_\pm(x) = \beta^{\pm 1} , \qquad \lim_{x \to 1} h_\pm(x) = 0 . \tag{2.3.8}$$

β is a constant allowed in principle by dimensional arguments. If $\beta \neq 1$ then it can be moved to the lhs of the Friedman equation, turning $G_N \to \beta G_N$, and can be interpreted as the ratio between an effective cosmological value of the gravitational coupling and the value measured with local tests. But $\beta \neq 1$ would be in conflict with Nucleosynthesis, through the relic abundances of ^4He and other heavy elements (for 3 neutrino species) [28], so we set $\beta = 1$. Therefore

$$\lim_{x \to 0} h_\pm(x) = 1 , \qquad \lim_{x \to 1} h_\pm(x) = 0 . \tag{2.3.9}$$

The consistency conditions (2.3.4) result in

$$h_\pm(x) > 0 , \qquad h_\pm(x) > x h_\pm^{'}(x) \tag{2.3.10}$$

for the two functions $h_\pm$ defined in the interval $0 < x < 1$. Thus, we can divide the cosmic evolution history into three periods. In the earliest, relativistic particles dominate the energy density of the universe and the generalized first Friedman equation shows a dependence on the upper bound H_+. There is also a transition period in which the effect of the bounds can be neglected and the rhs of the first Friedman equation is scale-free; this period includes the transition from a radiation dominated universe to a matter dominated universe. In the third present period, non-relativistic particles dominate the energy density of the universe but the dependence on the lower bound H_- must be accounted for in the generalized first Friedman equation.

From the definition of the Hubble parameter, one has

$$\frac{\dot{H}}{H^2} = -1 + \frac{a \ddot{a}}{\dot{a}^2} . \tag{2.3.11}$$

Then, in order to see if there is an accelerated or decelerated expansion, one has to determine whether $\dot{H}/H^2$ is greater or smaller than -1.

In the period dominated by radiation one has

$$\frac{\dot{H}}{H^2} = -2 \frac{1}{1 - \frac{x h_+^{'}(x)}{h_+(x)}} , \tag{2.3.12}$$

with $x = H^2/H_+^2$, where we have used (2.3.7) assuming that only the dependence on the upper bound (H_+) of the Hubble parameter is relevant. A very simple choice for this dependence is given by

$$g(H) = \frac{H^2}{\left(1 - \frac{H^2}{H_+^2} \right)^{\alpha_+}} , \tag{2.3.13}$$

with α_+ a (positive) exponent which parametrizes the departure from the standard cosmological model when the Hubble rate approaches its upper bound. With this simple choice one has a transition from an accelerated expansion for $H^2 > H_+^2/(1 + \alpha_+)$ into a decelerated expansion when $H^2 < H_+^2/(1 + \alpha_+)$, which includes the domain of validity of the standard cosmological model ($H \ll H_+$).

In the period dominated by matter (which corresponds to lower values of the Hubble rate) we assume that only the dependence on the lower bound (H_-) of the Hubble rate is relevant. Then one has

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} \frac{1}{1 - \frac{xh'_-(x)}{h_-(x)}} \, , \tag{2.3.14}$$

with $x = H_-^2/H^2$. We can also consider a dependence on H_- parametrized simply by an exponent α_-

$$g(H) = H^2 \left(1 - \frac{H_-^2}{H^2}\right)^{\alpha_-} \, . \tag{2.3.15}$$

With this choice one has a transition from a decelerated expansion for $H^2 > H_-^2(1 + \alpha_-/2)$ leaving the domain of validity of the standard cosmological model and entering into an accelerated expansion when the Hubble rate approaches its lower bound for $H^2 < H_-^2(1 + \alpha_-/2)$.

The possibility of describing ρ as a function of H is independent of the existence of spatial curvature k. However, in the kinematics of observables in the expanding universe we do need to specify the value of k. In the rest of the paper we will assume that the universe is flat ($k = 0$), although the same analysis could be done for arbitrary k.

The properties of the expansion obtained in this simple example (a period of decelerated expansion separating two periods of accelerated expansion) are general to the class of phenomenological models with a generalized first Friedman equation (2.3.1) with $g(H)$ satisfying the consistency conditions (2.3.4) and a Hubble rate constrained to a finite interval. The specific part of the example defined by (2.3.13,2.3.15) is the simple dependence on the Hubble rate bounds and the values of the Hubble rate at the transitions between the three periods of expansion. From now on we will name this description the Asymptotic Cosmological Model (ACM). With respect to the late accelerated expansion, ACM can be seen as a generalization of ΛCDM, which can be recovered by setting $\alpha_- = 1$. It also includes an early period of exponential expansion which can be seen as a phenomenological description of the evolution of the universe at inflation in the homogeneous approximation.

2.3.1. Horizon problem

One can see that the horizon of a radiation-dominated universe can be made arbitrarily large as a consequence of an upper bound on the Hubble parameter and in this way one can understand the observed isotropy of the cosmic microwave background at large angular scales.

Let us consider the effect of the modification of the cosmological model on the calculation of the distance $d_h(t_f, t_i)$ of a source of a light signal emitted at time t_i and observed at time t_f

$$d_h(t_f, t_i) = c \, a(t_f) \int_{t_i}^{t_f} \frac{dt}{a(t)} \, . \tag{2.3.16}$$

We have

$$\frac{dt}{a} = \frac{da}{a^2 H} = \left(\frac{8\pi G_N \sigma}{3}\right)^{-1/4} \frac{g'(H)dH}{4Hg(H)^{3/4}} \, , \tag{2.3.17}$$

where in the first step we have used the definition of the Hubble expansion rate H and in the second step we have used the relation between the scale factor a and H as given by (2.2.12-2.3.1). We are considering both times t_i and t_f in the radiation dominated period.

The distance d_h is then given by

$$d_h(H_f, H_i) = \frac{1}{4g(H_f)^{1/4}} \int_{H_f}^{H_i} \frac{dH \, g'(H)}{H g(H)^{3/4}} \, . \tag{2.3.18}$$

If H_i is very close to H_+ (i.e. if we choose the time t_i when the light signal is emitted well inside the period of accelerated expansion) then the integral is dominated by the region around H_i which is very close to H_+. Then one can approximate in the integrand

$$g(H) \approx \frac{H_+^2}{\left(1 - \frac{H^2}{H_+^2}\right)^{\alpha_+}} \, . \tag{2.3.19}$$

On the other hand if the observation is made at a time t_f within the domain of validity of the cosmological standard model ($H_-^2 \ll H_f^2 \ll H_+^2$) then the factor $g(H_f)^{1/2}$ in front of the integral can be approximated by H_f and then one has

$$d_h(H_f, H_i) \approx \frac{\alpha_+}{4\sqrt{H_f H_i}} B_{\frac{H_i^2}{H_+^2}} \left(1/2, -\alpha/4\right) , \tag{2.3.20}$$

where $B_z(m, n)$ is the incomplete Beta function, and it can be made arbitrarily large by choosing H_i sufficiently close to H_+. In this way we see that a cosmological model with a finite interval of variation for H solves the horizon problem.

2.4. Constraints of ACM by Observations

In this section we will carry out a more technical analysis about how the astrophysical observations constrain the parameter space of ACM in the matter dominance period. This analysis is based on the use of (assumed) standard candles, basically Type Ia Supernovae [8] and CMB [12]. These observations constrain the parameter space to confidence regions in which the combination $\Omega_m \equiv (1 - H_-^2/H_0^2)^{\alpha_-}$ is constrained to be around one quarter. The consideration of both Type Ia SNe and CMB together favor $\alpha_- > 1.5$. The results of this analysis can be seen in figures (FIG. 1-3). A reader not interested in technical details might well skip this section.

We center our discussion of the experimental tests of the Asymptotic Cosmological Model in the late accelerated expansion produced when H approaches its lower bound H_-. The vast amount of supernovae data collected by [8] and [7], the data from the SDSS Baryon Acoustic Oscillation [16], the mismatch between total energy density and total matter energy density seen at CMB anisotropies [12] and the measurements of present local mass density by 2dF and SDSS [15, 17] compared with the measurements of H_0 from the HST Cepheids [207] show that the universe undergoes a surprising accelerated expansion at the present time.

We will firstly confront the model with the Supernovae Ia data from Riess *et al.* and SNLS collaboration. The usefulness of the Supernovae data as a test of Dark Energy models relies on the assumption that Type Ia SNe behave as standard candles, i.e., they have a well defined environment-independent luminosity $\mathcal{L}$ and spectrum. Therefore, we can use measured bolometric flux $\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}$ and frequency to determine luminosity distance d_L and redshift z. The luminosity distance is given now by

ACM with dataset gold
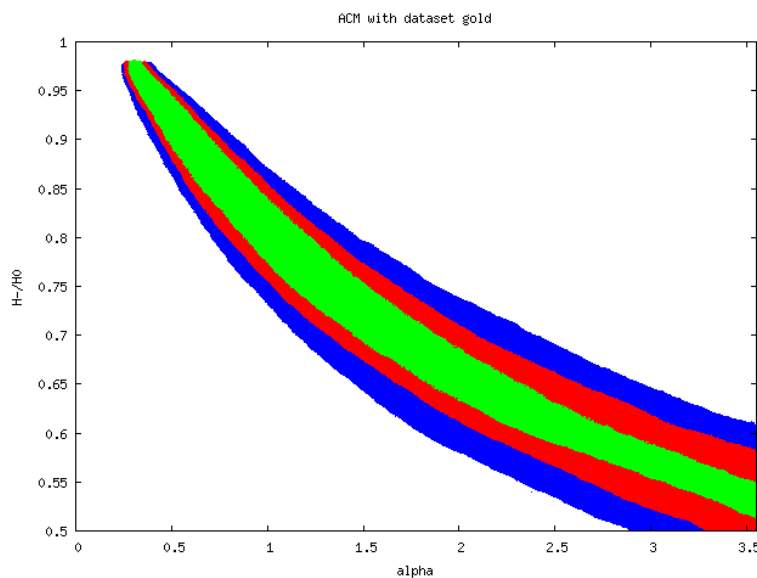
$$d_L(z) = c(1+z) \int_0^z \frac{dz'}{H(z')} = \frac{c(1+z)}{3} \int_{H_0}^{H(z)} \frac{g'(H)dH}{Hg^{1/3}(H_0)g^{2/3}(H)} \,. \qquad (2.4.1)$$

The computed value must be compared with the one obtained experimentally from the measured extinction-corrected distance moduli ($\mu_0 = 5log_{10}(\frac{d_L}{1Mpc}) + 25$) for each SN. The SNe data have been compiled in references [8], and we have limited the lowest redshift at $cz < 7000km/s$ in order to avoid a possible "Hubble Bubble" [208, 209]. Therefore our sample consists on 182 SNe. We will determine the likelihood of the parameters from a χ^2 statistic,

$$\chi^2(H_0, H_-, \alpha_-) = \sum_i \frac{(\mu_{p,i}(z_i; H_0, H_-, \alpha_-) - \mu_{0,i})^2}{\sigma_{\mu_{0,i}}^2 + \sigma_v^2} \,, \qquad (2.4.2)$$

where σ_v is the dispersion in supernova redshift due to peculiar velocities (we adopt $\| \mathbf{v_p} \| = 400km/s$ in units of distance moduli), $\sigma_{\mu_{0,i}}$ is the uncertainty in the individual measured distance moduli $\mu_{0,i}$, and $\mu_{p,i}$ is the value of μ_0 at z_i computed with a certain value of the set of parameters H_0, H_-, α_-. This χ^2 has been marginalized over the nuisance parameter H_0 using the adaptive method in reference [210]. The resulting likelihood distribution function $e^{-\chi^2/2}$ has been explored using Monte Carlo Markov Chains. We get a best fit of ACM at $\alpha_- = 0.36$ and $\frac{H_-}{H_0} = 0.95$, for which $\chi^2 = 157.7$. In contrast, fixing $\alpha_- = 1$, we get $\Omega_\Lambda \equiv \frac{H_-^2}{H_0^2} = 0.66$ and $\chi^2 = 159.1$ for the best fit ΛCDM. The confidence regions are shown in Fig. 1 (top).

We can add new constraints for the model coming from measurements of the present local matter energy density from the combination of 2dF and SDSS with HST Cepheids, rendering $\Omega_m = 0.28 \pm 0.03$; and the distance to the last scattering surface from WMAP, which leads to $r_{CMB} \equiv \sqrt{\Omega_m} \int_0^{1089} \frac{H_0 dz'}{H(z')} = 1.70 \pm 0.03$ [8]. Including these priors we get a best fit of ACM at $\alpha_- = 3.54$ and $\frac{H_-}{H_0} = 0.56$ (our simulation explored the region with $\alpha_- < 3.6$), for which $\chi^2 = 164.1$. In contrast, we get $\Omega_\Lambda = 0.73$ and $\chi^2 = 169.5$ for the best fit ΛCDM, which is outside the 2σ confidence region shown in Fig. 1 (bottom). The fits of the best fit ΛCDM and ACM taking into account the priors to the SNe data are compared in Fig. 2. The information which can be extracted from the data is limited. This can be seen in Fig. 3, in

Figure 2.1.: Confidence regions in parameter space of the Asymptotic Cosmological Model
(ACM) with (bottom) and without (top) priors from CMB and estimations of present
matter energy density at 1σ (green), 2σ (red) and 3σ (blue). The ΛCDM is inside
the 1σ region if we consider no priors, but is moved outside the 2σ region when
we confront the SNe data with CMB and estimations from present matter energy
density. In this case a value $\alpha_- > 1.5$ is favored.

which it is explicit that the data constrain mainly the value of the present matter energy density,
$\Omega_m \equiv \frac{\rho_m}{\rho_C} = (1 - \frac{H_-^2}{H_0^2})^{\alpha_-}$.

2.5. Generalized First Friedman Equation and $f(R)$ gravity

The extension of the cosmological model considered here could be compared with recent works
on a modification of gravity at large or very short distances [205, 80, 82, 96, 206, 81, 98]. It has
been shown that, by considering a correction to the Einstein Hilbert action including positive
and negative powers of the scalar curvature, it is possible to reproduce an accelerated expansion
at large and small values of the curvature in the cosmological model. Some difficulties to make
these modifications of the gravitational action compatible with the solar system tests of general
relativity have lead to consider a more general gravitational action, including the possible scalars
that one can construct with the Riemann curvature tensor [74], although the Gauss-Bonnet
scalar is the only combination which is free from ghosts and other pathologies. In fact, there is
no clear reason to restrict the extension of general relativity in this way. Once one goes beyond
the derivative expansion, one should consider scalars that can be constructed with more than
two derivatives of the metric and then one does not have a good justification to restrict in this
way the modification of the gravitational theory. It does not seem difficult to find an appropriate
function of the scalar curvature or the Gauss-Bonnet scalar, which leads to a cosmology with a
bounded Hubble expansion rate.

One may ask if a set of metric f(R) theories which include ACM as an homogeneous and
isotropic solution exists. The answer is that a bi-parametric family of f(R) actions which lead to
an ACM solution exists. In the following section we will derive them and we will discuss some
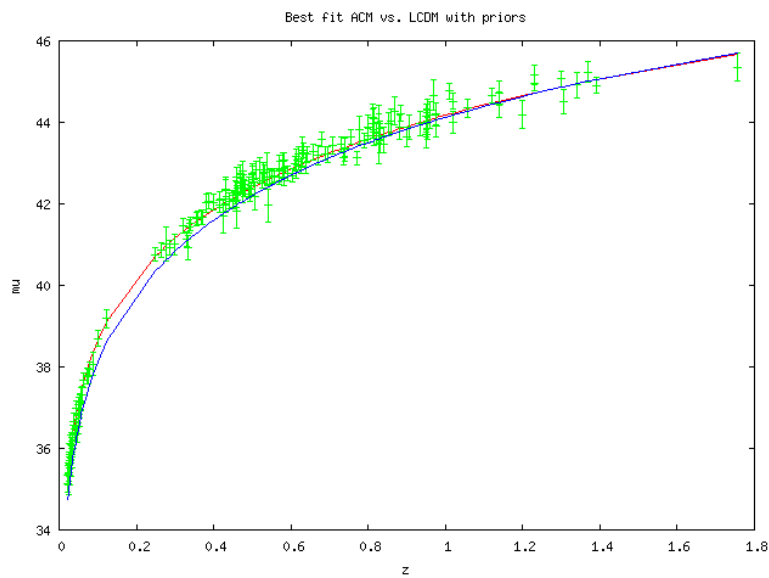
Figure 2.2.: Distance Moduli vs. Redshift comparison between the best fits of ACM ($H_0 = 66 \, Km/sMPc$, red) and ΛCDM ($H_0 = 65 \, Km/sMPc$, blue) to the SNe data with priors. ACM fits clearly better the medium redshift SNe.
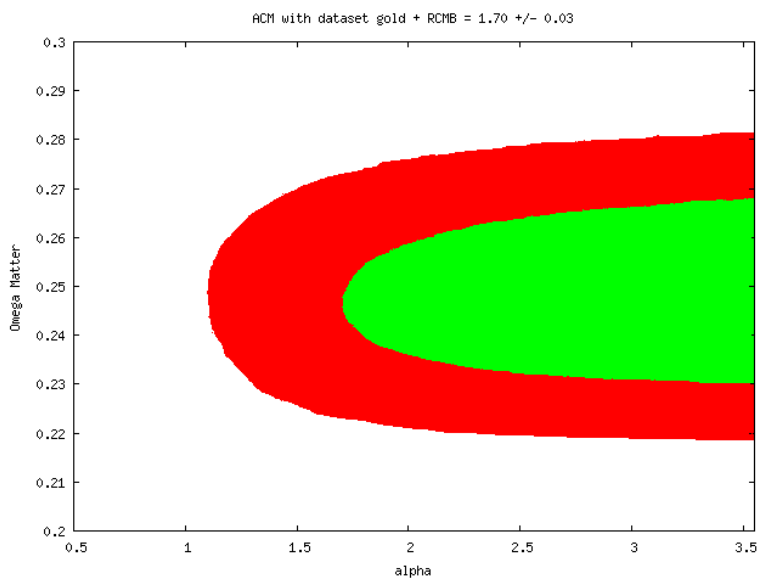
Figure 2.3.: Ω_m vs. α confidence regions with the CMB prior only. The confidence regions show $\Omega_m = 0.25 \pm 0.03$ and $\alpha > 1.15$ at 2σ level.

examples. The derivation follows the same steps of modified f(R)-gravity reconstruction from any FRW cosmology [211].

We start from the action

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}[f(R) + 16\pi G L_m], \tag{2.5.1}$$

which leads to the "generalized first Friedman equation"

$$-18(H\ddot{H} + 4H^2\dot{H})f''(R) - 3(\dot{H} + H^2)f'(R) - \frac{1}{2}f(R) = (8\pi G_N)\rho, \tag{2.5.2}$$

where

$$R = -6(\dot{H} + 2H^2). \tag{2.5.3}$$

If the energy density of the universe is mainly due to matter (as in the late accelerated expansion), we can use (2.3.3) and (2.3.1) to express ρ, R, $\dot{H}$ and $\ddot{H}$ as functions of H. Thus we get

$$[H^3gg'^2 - 3H^2g^2g' + 3H^3g^2g'']f''(R) + \frac{[3Hgg'^2 - H^2g'^3]}{18}f'(R) - \frac{g'^3}{108}f(R) = \frac{gg'^3}{18}, \tag{2.5.4}$$

where

$$R(H) = 18Hg(H)/g'(H) - 12H^2. \tag{2.5.5}$$

$H(R)$ can be obtained from (2.5.5) and set into (2.5.4); then we get an inhomogeneous second order linear differential equation with non-constant coefficients. Therefore, there will always be a bi-parametric family of $f(R)$ actions which present ACM as their homogeneous and isotropic solution. The difference between these actions will appear in the behavior of perturbations, which is not fixed by (2.3.1). Some of these actions are particularly easy to solve. If $g(H) = H^2$ then $R = -3H^2$ and the differential equation becomes

$$6R^2f''(R) - Rf'(R) - f(R) + 2R = 0. \tag{2.5.6}$$

Its general solution is

$$f(R) = R + c_1 R^{\frac{1}{12}(7-\sqrt{73})} + c_2 R^{\frac{1}{12}(7+\sqrt{73})}, \tag{2.5.7}$$

which will give (2.3.1) as First Friedman equation as long as the radiation energy density can be neglected. In general the differential equation (2.5.4) will not be solvable analytically, and only approximate solutions can be found as power series around a certain singular point R_0 (a value of R such that $H(R)$ cancels out the coefficient of $f''(R)$ in (2.5.4)). These solutions will be of the form $f(R) = f_p(R) + c_1 f_1(R) + c_2 f_2(R)$ with

$$\frac{f_i(R)}{R_0} = \sum_{m=0}^{\infty} a_m^{(i)}\left(\frac{R - R_0}{R_0}\right)^{s_i+m}, \tag{2.5.8}$$

where $i = p, 1, 2$, $a_0^{(1)} = a_0^{(2)} = 1$ and the series will converge inside a certain radius of convergence. An interesting choice of R_0 is $R_0 = -12H_-^2 \equiv R_-$, which is the value of R at H_-. One can also find the approximate solution of the differential equation for $\mid R \mid \gg \mid R_- \mid$, which will be of the form

$$\frac{f_i(R)}{R} = \sum_{m=0}^{\infty} a_m^{(i)}\left(\frac{R_-}{R}\right)^{s_i+m}. \tag{2.5.9}$$

One can in principle assume that the action (2.5.9) could be considered as valid also in the region in which radiation begins to dominate, but this action reproduces (2.3.1) only if matter

dominates. However, some of the new terms appearing in (2.5.9) will be negligible against R when radiation begins to dominate. The others can be canceled out by setting to zero the appropriate integration constant and therefore the Cosmological Standard Model will be recovered as a good approximation when radiation begins to dominate.

$f(R)$-theories are not the only modified gravity theories studied in the literature; $f(G)$-theories [212] are also a popular field of research. In these theories, the Einstein-Hilbert action is supplemented by a function of the Gauss-Bonnet scalar $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ (not to be confused with the gravitational coupling G_N).The same approach can be used to answer what $f(G)$ actions are able to reproduce ACM homogeneous evolution, once more in analogy with the modified f(G)-gravity reconstruction of a FRW cosmology [213]. The form of the action will be

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}[R + f(G) + 16\pi G_N L_m]\,, \qquad (2.5.10)$$

from which we derive the modified Friedman equation

$$(8\pi G_N)\,\rho = 3H^2 - \frac{1}{2}Gf'(G) + \frac{1}{2}f(G) + 12f''(G)\dot{G}H^3\,. \qquad (2.5.11)$$

Following the same procedure as in the case of $f(R)$-theories we arrive to the differential equation

$$3g(H) = 3H^2 - \frac{1}{2}Gf'(G) + \frac{1}{2}f(G) + \frac{864H^6g}{g'^3}[9gg' - Hg'^2 - 3Hgg'']f''(G)\,, \qquad (2.5.12)$$

where

$$G = 24H^2(H^2 - 3Hg/g')\,. \qquad (2.5.13)$$

For a given $g(H)$, one can use (2.5.13) to get $H(G)$, then set it into (2.5.12) and solve the second order linear differential equation. As in the previous case, there will be a bi-parametric family of solutions for $f(G)$ which will have (2.3.1) as their homogeneous isotropic solution as long as matter dominates. The parameters will need to be fixed in order to make the contribution of undesired terms in the Friedman equation to be negligible when radiation dominates. Again $g(H) = H^2$ is an example which can be solved analytically. The differential equation turns to be

$$12G^2 f''(G) - Gf'(G) + f(G) = 0\,, \qquad (2.5.14)$$

which has the solution

$$f(G) = c_1 G + c_2 G^{1/12}\,. \qquad (2.5.15)$$

In most cases this procedure will not admit an analytical solution and an approximate solution will need to be found numerically.

A similar discussion can be made to explain the early accelerated expansion (inflation) as a result of an $f(R)$ or $f(G)$ action, taking into account that in this case the dominant contribution to the energy density is radiation instead of matter. The analogue to (2.5.9) will be now a solution of the form

$$\frac{f_i(R)}{R} = \sum_{n=0}^{\infty} a_n^{(i)} \left(\frac{R}{R_+}\right)^{r_i+n}\,, \qquad (2.5.16)$$

for $|R| \ll |R_+|$ with $R_+ = -12H_+^2$. Terms in the action which dominate over R when R becomes small enough should be eliminated in order to recover the Standard Cosmological model before matter starts to dominate.

Both accelerated expansions can be described together in the homogeneous limit by an $f(R)$ action with terms $(\frac{R_-}{R})^{s_i+m}$ with $s_i > 0$ coming from (2.5.9) and terms $(\frac{R}{R_+})^{r_i+n}$ with $r_i > 0$ coming from (2.5.16). The action will contain a term, the Einstein-Hilbert action R, which will be dominant for $H_- \ll H \ll H_+$ including the period when matter and radiation have comparable energy densities.

2.6. Alternative Descriptions of the ACM

In general, by virtue of the gravitational field equations, it is always possible to convert a modification in the gravitational term of the action to a modification in the matter content of the universe. In particular, at the homogeneous level, it is possible to convert a generalized first Friedman equation of the type (2.3.1) to an equation in which, apart from the usual matter term, there is a dark energy component with an unusual equation of state $p = p(\rho)$ and in which General Relativity is not modified.

The trivial procedure is the following. Assume that the source of the gravitational field in the modified gravity theory behaves as $p_d = \omega \rho_d$ (matter or radiation). This is the case when the modification of gravity is relevant. We can then define a dark energy or effective gravitational energy density as

$$\rho_g = \frac{3}{8\pi G_N} \left(H^2 - g(H) \right) . \tag{2.6.1}$$

The continuity equation (2.2.11) allows to define a pressure for this dark energy component as $p_g = -\rho_g - \dot{\rho}_g/3H$. The equation of state of the dominant content of the universe leads to

$$\dot{H} = -3(1+\omega)\frac{H g(H)}{g'(H)} , \tag{2.6.2}$$

which can be used to express $\dot{\rho}_g$ as a function of H, and one gets the final expression for p_g,

$$p_g = \frac{-3}{8\pi G_N} \left(H^2 - 2(1+\omega)H\frac{g(H)}{g'(H)} + \omega g(H) \right) . \tag{2.6.3}$$

Then, for any given $g(H)$ we can use (2.6.1) to get $H = H(\rho_g)$ and substitute in (2.6.3) to get an expression of $p_g(\rho_g; H_\pm, \omega)$ which can be interpreted as the equation of state of a dark energy component. In the simple case of a matter dominated universe with ACM, $\alpha_- = 1$ gives obviously a dark energy component verifying $p_g = -\rho_g$. Another simple example is $\alpha_- = 2$, for which

$$\left(\frac{8\pi G_N}{3H_-^2} \right) p_g = \frac{2}{\frac{8\pi G_N}{3H_-^2} \rho_g - 3} . \tag{2.6.4}$$

The inverse procedure is also straightforward. Suppose we have a universe filled with a standard component $p_d = \omega \rho_d$ and a dark energy fluid $p_g = p_g(\rho_g)$. Using the continuity equation of both fluids one can express their energy densities as a function of the scale factor a and then use this relation to express ρ_g as a function of ρ_d. Then the Friedman equation reads

$$H^2 = \frac{8\pi G_N}{3}(\rho_d + \rho_g(\rho_d)), \tag{2.6.5}$$

which, solving for ρ_d, is trivially equivalent to (2.3.1). This argument could be applied to reformulate any cosmological model based on a modification of the equation of state of the dark energy component [214] as a generalized first Friedman equation (2.3.1).

Until now we have considered descriptions in which there is an exotic constituent of the universe besides a standard component (pressureless matter or radiation). In these cases, pressureless matter includes both baryons and Dark Matter. However, there are also descriptions in which Dark Matter is unified with Dark Energy in a single constituent of the universe. One of these examples is the Chaplygin gas $p_g = -1/\rho_g$ [127, 128]. The use of the continuity equation leads to

$$\rho_g = \sqrt{A + Ba^{-6}}, \tag{2.6.6}$$

where A and B are integration constants. The previous method can be used to find the generalized first Friedman equation for the baryon density in this model,

$$\frac{8\pi G_N}{3}\rho_b = \frac{H^2}{k}\left(\sqrt{1 + k(1 - \frac{H_-^4}{H^4})} - 1\right),\qquad(2.6.7)$$

where $H_- = \sqrt{\frac{8\pi G}{3}}A^{1/4}$, $k = B\rho_{b0}^{-2} - 1$, and ρ_{b0} is the present value of ρ_b. In the $H \gg H_-$ limit this model can be interpreted as a universe filled with baryons and dark matter or as a universe filled with baryons and with a higher effective value of G_N. Equation (2.6.7) does not fulfill (2.3.9) because it describes the behavior of just baryon density. In the $H \gtrsim H_-$ limit, the model can be interpreted as a universe filled with baryons and a cosmological constant or as an ACM model with $\alpha_- = 1$ and filled only with baryons.

A similar example in the early period of accelerated expansion would be a universe filled with a fluid which behaves as a fluid of ultra-relativistic particles if the energy density is low enough but whose density has an upper bound

$$\rho_X = \frac{\sigma}{a^4 + C}.\qquad(2.6.8)$$

This dependence for the energy density in the scale factor follows from the equation of state

$$p_X = \frac{1}{3}\rho_X - \frac{4C}{3\sigma}\rho_X^2.\qquad(2.6.9)$$

This model turns out to be equivalent to a universe filled with radiation following eq. (2.3.13) with $\alpha_+ = 1$ and $H_+^2 = \left(\frac{8\pi G_N}{3}\right)\frac{\sigma}{C}$.

Another equivalent description would be to consider that the universe is also filled with some self interacting scalar field φ which accounts for the discrepancy between the standard energy-momentum and GR Einstein tensors. Given an arbitrary modified Friedman equation (2.3.1) a potential $V(\varphi)$ can be found such that the cosmologies described by both models are the same. The procedure is similar to the one used to reconstruct a potential from a given cosmology [215]. From the point of view of the scalar field, the cosmology is defined by a set of three coupled differential equations

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) \quad = 0\qquad(2.6.10)$$

$$\frac{8\pi G_N}{3}(\rho_d + \frac{\dot{\varphi}^2}{2} + V(\varphi)) \quad = H^2\qquad(2.6.11)$$

$$-3(1 + \omega)H\rho_d \quad = \dot{\rho}_d.\qquad(2.6.12)$$

The solution of these equations for a certain $V(\varphi)$ will give $H(t)$ and $\rho_d(t)$, and therefore $\rho_d(H)$ which is $g(H)$ up to a factor $\frac{8\pi G_N}{3}$. In this way one finds the generalized first Friedman equation associated with the introduction of a self interacting scalar field. Alternatively, given a function $g(H)$ in a generalized first Friedman equation (2.3.1) for the density ρ_d, one can find a scalar field theory leading to the same cosmology in the homogeneous limit. By considering the time derivative of (2.6.11) and using (2.6.10) and (2.6.12) one gets

$$\dot{\varphi}^2 = -(1 + \omega)\rho_d - \frac{\dot{H}}{4\pi G_N},\qquad(2.6.13)$$

where we can use (2.6.2) and (2.3.1) to get $\dot{\varphi}^2$ as a function of H. Setting this on (2.6.11) we get $V(\varphi)$ as a function of H. On the other hand, $\varphi'(H) = \dot{\varphi}/\dot{H}$, so

$$\frac{8\pi G_N}{3}(\varphi'(H))^2 = \frac{(2H - g'(H))g'(H)}{9(1 + \omega)H^2 g(H)}\qquad(2.6.14)$$

and

$$\frac{8\pi G_N}{3} V(\varphi(H)) = H^2 - g(H) + \frac{(1+\omega)(2H - g'(H))g(H)}{2g'(H)} \,. \qquad (2.6.15)$$

If $g(H)$ is such that the rhs of (2.6.14) is positive definite, it can be solved and the solution $\varphi(H)$ inverted and substituted into (2.6.15) in order to get $V(\varphi)$. This is the case of a function of the type (2.3.15) with $\alpha_- > 1$. For the particular case of an ACM expansion with $\alpha_- = 1$, the solution is a flat potential $V = V_0$ and a constant value of $\varphi = \varphi_0$.

If $g(H)$ is such that the rhs of (2.6.14) is negative definite, as it happens in (2.3.15) with $\alpha_- < 1$ or in (2.3.13), the problem can be solved by changing the sign of the kinetic term in (2.6.11). The result is a phantom quintessence in the case of (2.3.15) with $\alpha_- < 1$. The case (2.3.13) is more complicated because the energy density of the associated inflaton turns out to be negative. Moreover, it is of the same order as the energy density of ultra-relativistic particles during the whole period of accelerated expansion. Therefore, it seems that this model is inequivalent to other inflation scenarios previously studied.

In summary, there are many equivalent ways to describe the discrepancy between observed matter content of the universe and Einstein's General Relativity. At the homogeneous level, it is trivial to find relations among them. The possibility to establish the equivalence of different descriptions is not a peculiarity of the description of this discrepancy in terms of a generalized first Friedman equation (2.3.1). The same relations can be found among generalized equations of state, scalar-tensor theories and f(R) modified gravity [216].

2.7. Summary and discussion

It may be interesting to go beyond ΛCDM in the description of the history of the universe in order to identify the origin of the two periods of accelerated expansion. We have proposed to use the expression of the energy density as a function of the Hubble parameter as the best candidate to describe the history of the universe. In this context the late time period of accelerated expansion and the early time inflation period can be easily parametrized.

We have considered a simple modification of the cosmological equations characterized by the appearance of an upper and a lower bound on the Hubble expansion rate. A better fit of the experimental data can be obtained with this extended cosmological model as compared with the ΛCDM fit. Once more precise data are available, it will be possible to identify the behavior of the energy density as a function of the Hubble parameter and then look for a theoretical derivation of such behavior.

We plan to continue with a systematic analysis of different alternatives incorporating the main features of the example considered in this work. Although the details of the departures from the standard cosmology can change, we expect a general pattern of the effects induced by the presence of the two bounds on H. We also plan to go further, considering the evolution of inhomogeneities looking for new consequences of the bounds on H.

The discussion presented in this work, which is based on a new description of the periods of accelerated expansion of the universe, can open a new way to explore either modifications of the theory of gravity or new components in the universe homogeneous fluid. Lacking theoretical criteria to select among the possible ways to go beyond ΛCDM, we think that the phenomenological approach proposed in this work is justified.

2. An extension of the cosmological standard model with a bounded Hubble expansion rate

3. Linearized Treatment of Scalar perturbations in the Asymptotic Cosmological Model

In this paper the implications of a recently proposed phenomenological model of cosmology, the Asymptotic Cosmological Model (ACM), on the behavior of scalar perturbations are studied. Firstly we discuss new fits of the ACM at the homogeneous level, including fits to the Type Ia Supernovae UNION dataset, first CMB peak of WMAP5 and BAOs. The linearized equations of scalar perturbations in the FRW metric are derived. A simple model is used to compute the CMB temperature perturbation spectrum. The results are compared with the treatment of perturbations in other approaches to the problem of the accelerated expansion of the universe.

3.1. Introduction

According to General Relativity (GR), if the universe is filled with the particles of the Standard Model of particle physics, gravity should lead to a decelerated expansion of the universe. However, in 1998 two independent evidences of present accelerated expansion were presented [5, 6] and later confirmed by different observations [10, 11, 14].

There is no compelling explanation for this cosmic acceleration, but many intriguing ideas are being explored. These ideas can be classified into three main groups: new exotic sources of the gravitational field with large negative pressure [126, 120, 122] (Dark Energy), modifications of gravity at large scales [203, 204] and rejection of the spatial homogeneity as a good approximation in the description of the present universe [52, 49, 50].

Different models (none of them compelling) for the source responsible of acceleration have been considered. Einstein equations admit a cosmological constant Λ, which can be realized as the stress-energy tensor of empty space. This Λ together with Cold Dark Matter, Standard Model particles and General Relativity form the current cosmological model, ΛCDM. However, quantum field theory predicts a value of Λ which is 120 orders of magnitude higher than observed. Supersymmetry can lower this value 60 orders of magnitude, which is still ridiculous [21]. In order to solve this paradox, dynamical Dark Energy models have been proposed.

This has also lead to explore the possibility that cosmic acceleration arises from new gravitational physics. Again here several alternatives for a modification of the Einstein-Hilbert action at large and small curvatures [79], or even higher dimensional models [107, 106], producing an accelerated expansion have been identified. All these analyses include an *ad hoc* restriction to actions involving simple functions of the scalar curvature and or the Gauss-Bonnet tensor. This discussion is sufficient to establish the point that cosmic acceleration can be made compatible with a standard source for the gravitational field but it is convenient to consider a more general framework in order to make a systematic analysis of the cosmological effects of a modification of general relativity.

The Asymptotic Cosmological Model (ACM) was presented [217] as a strictly phenomenological generalization of the Standard Cosmological Model including a past and a future epoch of

accelerated expansion. It follows from the assumptions that GR is not a fundamental theory, but only a good approximation when the Hubble rate H is between but far away from two fundamental scales H_- and H_+, which act as bounds on H. A general covariant metric theory of gravity without spatial curvature is assumed. The model is well defined in the homogeneous approximation and includes ΛCDM as a particular case.

In next section we review the ACM and we provide new fits to the Type Ia Supernovae UNION dataset, first acoustic peak of CMB of WMAP5 and BAOs. In the third section we derive the linearized equations of the scalar perturbations of the metric, following from general covariance and a single new assumption on the perturbations. In the fourth section we will consider how to solve the system of equations for adiabatic perturbations in a given fluid. In the fifth section we derive the CMB spectrum in the ACM. In the sixth section we will compare the treatment of the scalar perturbations in the ACM with other models which are equivalent in the homogeneous approximation. The last section is devoted to the summary and conclusions.

3.2. The Asymptotic Cosmological Model: Homogeneous Background

The Asymptotic Cosmological Model (ACM) was introduced in Ref. [217]. In this model the universe is filled with photons and neutrinos (massless particles), baryons (electrically charged massive particles) and Dark Matter (electrically neutral massive particles), but General Relativity (GR) is only a good approximation to the gravitational interaction in a certain range of the Hubble rate H, between but far from its two bounds, H_- and H_+. General Covariance and the absence of spatial curvature are assumed.

The gravitational part of the action might include derivatives of the metric of arbitrarily high order, and therefore arbitrarily high derivatives of the scale factor should appear in the Friedman Equations. We should start by considering a generalized first Friedman equation

$$8\pi G \rho(t) = 3f(H(t), \dot{H}(t), ..., H^{(n)}(t), ...) , \qquad (3.2.1)$$

and the corresponding equation for the pressure will be derived using the continuity equation

$$\dot{\rho} = -3H(\rho + p) . \qquad (3.2.2)$$

However, as the resulting differential equations should be solved and only one of its solutions deserves interest (the one describing the evolution of the universe), we can use the one to one correspondence between time t and Hubble parameter H (assuming $\dot{H} < 0$) to write the modified First Friedman Equation as a bijective map linking the total energy density ρ with H in our universe

$$8\pi G \rho = 3g(H) . \qquad (3.2.3)$$

The use of the continuity equation (3.2.2) enables us to write the modified Friedman Equation for the pressure evaluated at the solution corresponding to the cosmic evolution

$$-8\pi G p = 3g(H) + g'(H)\dot{H}/H . \qquad (3.2.4)$$

The evolution of the universe will be determined by the concrete form of the function $g(H)$, which we assume to be smooth. However, the most significant features of the evolution at a given period can be described by some simple approximation to $g(H)$ and the matter content. Those are a pole at $H = H_+$ of order α_+ and a zero of order α_- at $H = H_-$. The energy density of the universe has a contribution from both massless particles (radiation) and massive ones (matter).

For the sake of simplicity, the history of the universe can be then divided into three periods. In the first period, $H \gg H_-$ so the effect of the lower bound can be neglected and the universe is radiation dominated. The universe undergoes (and exits from) an accelerated expansion which we call inflation. The simplest parametrization is

$$g(H) \approx H^2 \left(1 - \frac{H^2}{H_+^2}\right)^{-\alpha_+} . \tag{3.2.5}$$

In the second period we cannot neglect neither the effect of radiation nor the effect of nonrelativistic matter, but $H_+ \gg H \gg H_-$. Then GR offers a good description of the gravitational interaction, $g(H) \approx H^2$ in this region, and the universe performs a decelerated expansion.

In the third period, $H_+ \gg H$ so the effect of the upper bound can be neglected, and the universe is matter dominated. This period corresponds to the present time in which the universe also undergoes an accelerated expansion. The simplest choice is

$$g(H) \approx H^2 \left(1 - \frac{H_-^2}{H^2}\right)^{\alpha_-} . \tag{3.2.6}$$

We will assume that the transitions between these three periods are smooth and that the details about these transitions are unimportant.

This model preserves the successes of the Cosmological Standard Model, while giving a description of the early accelerated expansion (inflation) and of the present one, including ΛCDM as a particular case ($\alpha_- = 1$).

Without any knowledge of the evolution of perturbations in this model, the background evolution at late times can be used to fit the Type Ia Supernovae UNION dataset [9], the first acoustic peak in the Cosmic Microwave (CMB) Background [13], and the Baryon Acoustic Oscillations (BAOs) [16], via the parameters H_0 (the present Hubble parameter), H_- and α_-.

We use Monte Carlo Markov Chains to explore the likelihood of the fit of the supernovae UNION dataset [9] to the ACM. The dataset provides the luminosity distance

$$d_l(z) = c(1+z) \int_0^z dz'/H(z') \tag{3.2.7}$$

and redshift of 307 supernovae. A χ^2 analysis have been performed, where χ^2 has been marginalized over the nuisance parameter H_0 using the method described in [210]. The resulting parameter space is spanned by the values of α_- and H_-/H_0 (FIG. 1).

The constraints from the CMB data follow from the reduced distance to the surface of last scattering at $z = 1089$. The reduced distance R is often written as

$$R = \Omega_m^{1/2} H_0 \int_0^{1089} dz/H(z) . \tag{3.2.8}$$

The WMAP-5 year CMB data alone yield $R_0 = 1.715 \pm 0.021$ for a fit assuming a constant equation of state ω for the dark energy [47]. We will take this value as a first approximation to the fit assuming ACM. We can define the corresponding χ^2 as $\chi^2 = [(R - R_0)/\sigma_{R_0}]^2$, and find the confidence regions of the joint constraints (FIG. 2).

BAO measurements from the SDSS data provide a constraint on the distance parameter $A(z)$ at redshift $z = 0.35$,

$$A(z) = \Omega_m^{1/2} H_0 H(z)^{-1/3} z^{-2/3} \left[\int_0^z dz'/H(z')\right]^{2/3} . \tag{3.2.9}$$

Figure 3.1.: Confidence regions in parameter space of the Asymptotic Cosmological Model (ACM) from the fit of the supernovae UNION dataset without priors at 1σ, 2σ and 3σ. The ΛCDM is inside the 1σ region. The best fit to ACM lies in $\alpha_- = 0.35$, $H_-/H_0 = 0.97$ ($\chi^2 = 310.5$) in contrast to the best fit to ΛCDM, which lies in $\alpha_- \equiv 1$, $H_-/H_0 = 0.84$ ($\chi^2 = 311.9$).

Figure 3.2.: Confidence regions in parameter space of the Asymptotic Cosmological Model (ACM) from the fit of the supernovae UNION dataset (red), of the distance to the surface of last scattering from WMAP-5 (blue) and from the joint fit (magenta) at 1σ, 2σ and 3σ. Significantly, the ΛCDM is still inside the 1σ region, unlike in our previous study. This is due to the change in the Supernovae dataset. The best fit to ACM lies in $\alpha_- = 1.50$, $H_-/H_0 = 0.77$ ($\chi^2 = 312.5$) in contrast to the best fit to ΛCDM, which lies in $\alpha_- \equiv 1$, $H_-/H_0 = 0.86$ ($\chi^2 = 313.2$).
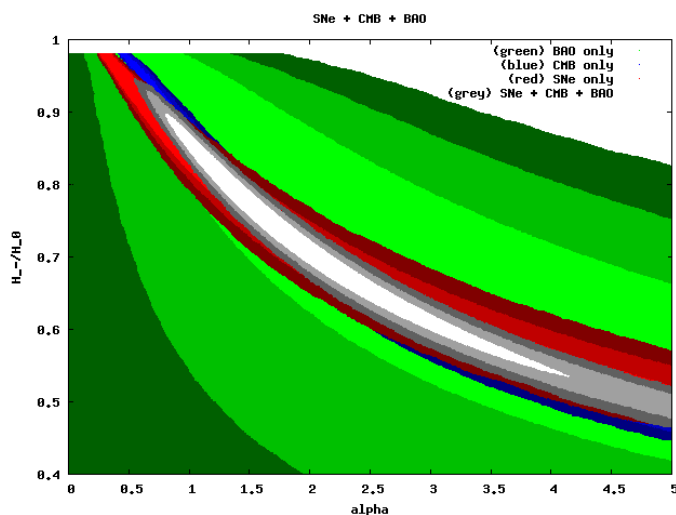
Figure 3.3.: Confidence regions in parameter space of the Asymptotic Cosmological Model (ACM) from the fit of the supernovae UNION dataset (red), of the distance to the surface of last scattering from WMAP-5 (blue), of the Baryon Acoustic Oscillations peak (green) and from the joint fit (grey) at 1σ, 2σ and 3σ. Measurements of the BAOs peak do not add significant information with their present precision. The best fit to ACM lies in $\alpha_- = 1.50$, $H_-/H_0 = 0.77$ ($\chi^2 = 312.7$) in contrast to the best fit to ΛCDM, which lies in $\alpha_- \equiv 1$, $H_-/H_0 = 0.86$ ($\chi^2 = 313.7$).

Ref. [16] gives $A_0 = 0.469 \pm 0.17$. We can define the corresponding χ^2 as $\chi^2 = [(A(z = 0.35) - A_0)/\sigma_{A_0}]^2$. The confidence regions resulting from adding this constraint are shown in FIG. 3.

In our previous work the use of the supernovae Gold dataset [8] and of the WMAP-3 data [12] led us to the conclusion that the ΛCDM was at 3σ level in the parameter space of the ACM. The position of the confidence regions seems to depend very tightly on the dataset that is being used. However, the value of the combination of parameters

$$\Omega_m \equiv \left(1 - \frac{H_-^2}{H_0^2}\right)^{\alpha_-} \tag{3.2.10}$$

does not depend much neither on the value of α_- or the dataset used (FIG. 4).

Moreover, we can conclude from the figures that BAO's do not provide much information in order to constrain the confidence regions of the ACM, unlike in other models such as ΛCDM with nonzero spatial curvature.

3.3. Scalar Perturbations

The lack of an action defining the ACM is a serious obstacle in the derivation of the equations governing the behavior of the perturbations. Given a background behavior described by the ACM, what can be said about the evolution of perturbations on top of this background? We will find that general covariance together with an additional assumption fixes completely the set of equations for the scalar perturbations in the linearized approximation and in the region close to H_-.
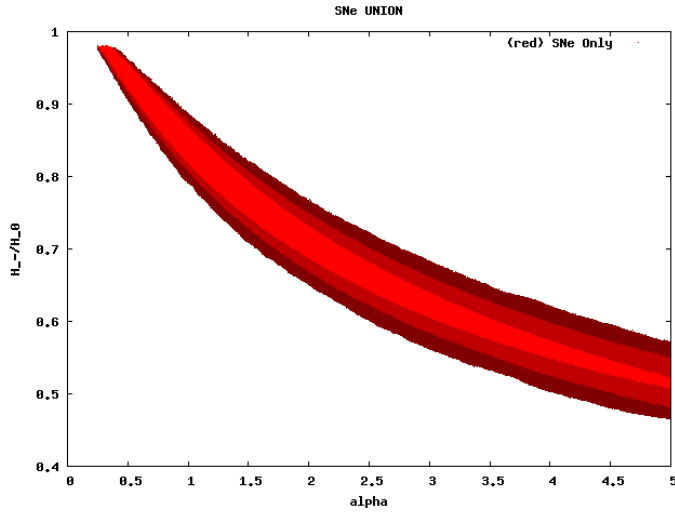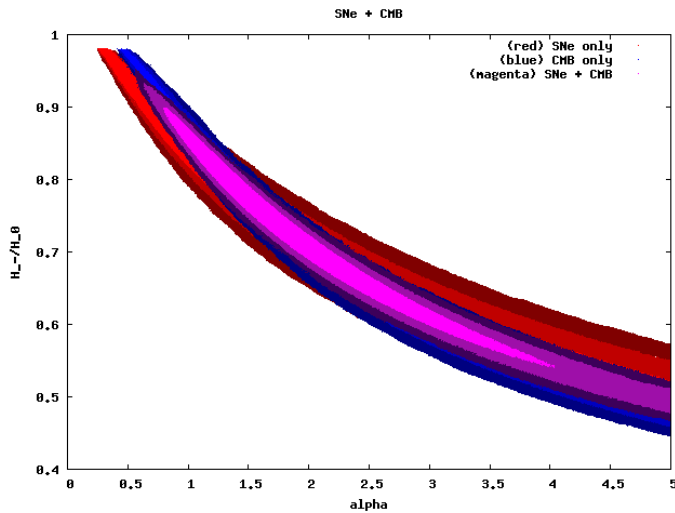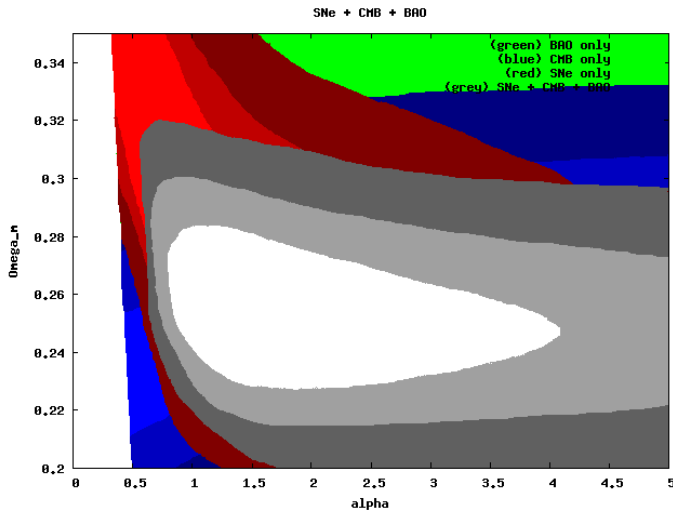
Figure 3.4.: Confidence regions in parameter space of the Asymptotic Cosmological Model (ACM) from the fit of the supernovae UNION dataset (red), of the distance to the surface of last scattering from WMAP-5 (blue), of the Baryon Acoustic Oscillations peak (green) and from the joint fit (grey) at 1σ, 2σ and 3σ. The plot shows α_- in the horizontal axis and Ω_m in the vertical axis. Noticeably, $\Omega_m = 0.26 \pm 0.04$ almost independently of the value of α_-.

The key point will be the following. Knowing the exact Friedman equations in the homogeneous approximation gives us a clue on the form of the equations for the scalar perturbations. In particular, we can formally describe perturbations over the FRW metric which do not depend on the spatial coordinates, $\{\phi(\mathbf{x}, t) = \phi(t), \psi(\mathbf{x}, t) = \psi(t)\}$. Thus the perturbed metric becomes the FRW metric written in a new coordinate frame. With a change of coordinates one can derive, starting from the Friedman equations, the terms containing only time derivatives of the scalar perturbations. Next an assumption on the validity of the GR description of scalar perturbations when $H_- \ll H \ll H_+$, together with the relations, valid for any general covariant theory, between terms with time derivatives and those involving spatial derivatives, allow to derive the evolution of scalar perturbations.

After this short sketch, we will perform the derivation of the equations for the scalar perturbations in detail. We can write the metric of spacetime with scalar perturbations in the Newtonian gauge,

$$ds^2 = (1 + 2\phi(\mathbf{x}, t))dt^2 - a^2(t)(1 - 2\psi(\mathbf{x}, t))d\mathbf{x}^2 \,, \tag{3.3.1}$$

and the stress-energy tensor of the source fields will be

$$
\begin{aligned}
T^0_0 &= \rho_{(0)}(t) + \delta\rho(\mathbf{x}, t) \,, \\
T^0_i &= (\rho_{(0)}(t) + p_{(0)}(t))\partial_i\theta(\mathbf{x}, t) \,, \\
T^i_j &= -(p_{(0)}(t) + \delta p(\mathbf{x}, t))\delta^i_j + \partial^i\partial_j\Pi(\mathbf{x}, t) \,,
\end{aligned}
\tag{3.3.2}
$$

where θ is the velocity potential of the fluid, Π is the anisotropic stress tensor (shear) potential, $\delta\rho$ and δp are small perturbations on top of the background homogeneous density $\rho_{(0)}$ and pressure $p_{(0)}$, respectively, and $\phi \sim \psi \ll 1$. We have used the Newtonian gauge because the gauge invariant scalar perturbations of the metric (Φ and Ψ) and gauge invariant perturbations in the stress energy tensor coincide with the perturbations explicitly written in this gauge.

The general form of the equations for the perturbations in a metric theory with arbitrarily high derivatives is

$$8\pi G \delta\rho = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\frac{a_{nm}}{H^{2n+m-2}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \phi + \frac{b_{nm}}{H^{2n+m-2}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \psi\right], \qquad (3.3.3)$$

$$8\pi G (\rho_{(0)} + p_{(0)})\partial_i\theta = \partial_i \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\frac{c_{nm}}{H^{2n+m-1}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \phi + \frac{d_{nm}}{H^{2n+m-1}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \psi\right], \qquad (3.3.4)$$

$$8\pi G (-\delta p\, \delta_j^i + \partial_i\partial_j\Pi) = \delta_j^i \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\frac{e_{nm}}{H^{2n+m-2}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \phi + \frac{f_{nm}}{H^{2n+m-2}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \psi\right]$$
$$+ \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{1}{a^2}(\delta_j^i\Delta - \partial_i\partial_j)\left[\frac{g_{nm}}{H^{2n+m}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \phi + \frac{h_{nm}}{H^{2n+m}}\left(\frac{\Delta}{a^2}\right)^n \partial_t^m \psi\right] \qquad (3.3.5)$$

The coefficients $a_{nm}, ..., h_{nm}$ are adimensional functions of the Hubble parameter an its time derivatives, and can be turned into functions of just the Hubble parameter using the bijection explained in the beginning of the previous section. We have mentioned we are going to be able to determine exactly the terms with only time derivatives, that is, the precise form of the coefficients a_{0m}, b_{0m}, e_{0m} and f_{0m}. This shows that there is much freedom of choosing a covariant linearized theory of cosmological perturbations, even for a given solution of the homogeneous equations.

However, under a single assumption, it is possible to greatly reduce this freedom. The assumption is that the standard linearized equations of the perturbations of General Relativity are effectively recovered in the limit $H_+ \gg H \gg H_-$ for all the Fourier modes with $H_+ \gg k > H_-$ (subhorizon modes in the present time).

Let us work out the implications of this assumption. In the equation for the perturbation of the energy density in General relativity,

$$8\pi G \delta\rho = -6H^2\phi - 6H\dot{\psi} + \frac{2}{a^2}\Delta\psi, \qquad (3.3.6)$$

there is only a term proportional to $\Delta\psi$. However, for subhorizon modes $(k \gg H)$ the dominant terms are those with the highest number of spatial derivatives. If we demand the terms with spatial derivatives of order greater than 2 not to spoil the behavior of these modes in the period in which GR is a good approximation, they must be negligible at least for the observable modes.

For instance, they should be negligible for the modes responsible of the acoustic peaks of the CMB spectrum, and therefore for the modes which have entered the horizon after recombination (which have even lower k). It is possible that these terms are suppressed by inverse powers of the UV scale H_+, becoming irrelevant for current tests of gravity, although they may be relevant for the physics of quantum fluctuations in the very early universe.

Therefore, at times when $H \ll H_+$, the equations can be approximated by

$$8\pi G\delta\rho = \sum_{m=0}^{\infty}\left[\frac{a_{0m}}{H^{m-2}}\partial_t^m\phi + \frac{b_{0m}}{H^{m-2}}\partial_t^m\psi + \frac{a_{1m}}{H^m}\frac{\Delta}{a^2}\partial_t^m\phi + \frac{b_{1m}}{H^m}\frac{\Delta}{a^2}\partial_t^m\psi\right] \quad (3.3.7)$$

$$8\pi G(\rho_{(0)}+p_{(0)})\partial_i\theta = \partial_i\sum_{m=0}^{\infty}\left[\frac{c_{0m}}{H^{m-1}}\partial_t^m\phi + \frac{d_{0m}}{H^{m-1}}\partial_t^m\psi\right] , \quad (3.3.8)$$

$$8\pi G(-\delta p\,\delta_j^i + \partial_i\partial_j\Pi) = \delta_j^i\sum_{m=0}^{\infty}\left[\frac{e_{0m}}{H^{m-2}}\partial_t^m\phi + \frac{f_{0m}}{H^{m-2}}\partial_t^m\psi\right]$$

$$+ \sum_{m=0}^{\infty}\frac{1}{a^2}(\delta_j^i\Delta - \partial_i\partial_j)\left[\frac{g_{0m}}{H^m}\partial_t^m\phi + \frac{h_{0m}}{H^m}\partial_t^m\psi\right] , \quad (3.3.9)$$

where general covariance has been used to exclude the terms with coefficients e_{1m}, f_{1m} which are not compatible with (3.3.7), (3.3.8).

We must also take into account that the modes which are responsible of the acoustic peaks of the CMB undergo a phase in which they oscillate as sound waves, i.e.: $\partial_t\phi \sim k\phi$. In order to explain the acoustic peaks of the spectrum of the CMB it is required that the acoustic oscillations of the modes of the gravitational potentials, $\phi_{\mathbf{k}}$ and $\psi_{\mathbf{k}}$, which lead to the acoustic peaks of the CMB spectrum, have a frequency $\sim k$. When radiation dominates and we consider modes well inside the horizon, two of the solutions of the system of differential equations of arbitrary order have this property [1].

However, in order for nondecaying superhorizon modes to evolve into oscillating subhorizon modes with frequency $\sim k$, a fine-tuning of coefficients is required unless the equations are of order two in time derivatives. Then the acoustic peaks in the CMB spectrum are reproduced when terms with more than two derivatives in the equations for the perturbations can be neglected. Then

$$8\pi G\delta\rho = \sum_{m=0}^{2}\left[a_{0m}H^{2-m}\partial_t^m\phi + b_{0m}H^{2-m}\partial_t^m\psi\right] + a_{10}\frac{\Delta}{a^2}\phi + b_{10}\frac{\Delta}{a^2}\psi \quad (3.3.10)$$

$$8\pi G(\rho_{(0)}+p_{(0)})\partial_i\theta = \partial_i\sum_{m=0}^{1}\left[c_{0m}H^{1-m}\partial_t^m\phi + d_{0m}H^{1-m}\partial_t^m\psi\right] , \quad (3.3.11)$$

$$8\pi G(-\delta p\,\delta_j^i + \partial_i\partial_j\Pi) = \delta_j^i\sum_{m=0}^{2}\left[e_{0m}H^{2-m}\partial_t^m\phi + f_{0m}H^{2-m}\partial_t^m\psi\right]$$

$$+ \frac{1}{a^2}(\delta_j^i\Delta - \partial_i\partial_j)\left[g_{00}\phi + h_{00}\psi\right] , \quad (3.3.12)$$

and the arbitrariness in the evolution equations for the scalar perturbations has been reduced to twenty undetermined dimensionless coefficients at this level.

The equations (3.3.10),(3.3.11),(3.3.12) are in principle valid for any perturbation mode $\{\phi_{\mathbf{k}},\psi_{\mathbf{k}}\}$ as long as $k \ll H_+$. In particular, it must be valid for the mode $\mathbf{k} = \mathbf{0}$, which corresponds formally to a perturbation with no spatial dependence. In practice, we will be able to neglect the spatial dependence of perturbation whose spatial dependence is sufficiently smooth, i.e.: its

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[1] As we will see below, if the term with the highest time derivative is of order D, $a_{1,D-2} = -e_{0D}$ as a consequence of general covariance. Well inside the horizon these are the dominant terms appearing in the wave equation resulting when calculating the adiabatic perturbations in the radiation domination epoch.

wavenumber k is sufficiently low. In GR it suffices for a mode to be superhorizon $k \ll H$ in order to neglect its spatial dependence in a first approximation.

If we consider the FRW metric perturbed by one of these modes we have,

$$ds^2 = (1 + 2\phi(t))dt^2 - a^2(t)(1 - 2\psi(t))d\mathbf{x}^2 \,. \tag{3.3.13}$$

By means of the invariance under time reparameterizations we can introduce a new time variable $dt' = (1 + \phi(t))dt$, a new scale factor $a'(t') = a(t)(1 - \psi(t))$ and a new energy density $\rho'(t') = \rho_0(t) + \delta\rho(t)$ leading us back to the ACM in a homogeneous background. The Hubble rate $H = \frac{\dot{a}}{a}$ and its time derivatives $H^{(n)}$ will change as

$$H^{(n)'} = \frac{d^n}{dt'^n}\left[\frac{1}{a'(t')}\frac{da'}{dt'}\right] = H^{(n)} + \delta H^{(n)},$$
$$\delta H^{(n)} = -\psi^{(n+1)} - \sum_{m=0}^{n} \begin{pmatrix} n+1 \\ m+1 \end{pmatrix} H^{(n-m)}\phi^{(m)}. \tag{3.3.14}$$

Noticeably, this procedure must be applied to the generalized Friedman equation before one of its homogeneous solutions is used to write the time variable t as a function of H. Therefore we should start by considering a generalized first Friedman equation in primed coordinates

$$8\pi G\rho'(t') = 3f(H'(t'), \dot{H}'(t'), ..., H^{(n)'}(t'), ...) \,, \tag{3.3.15}$$

and the corresponding equation for the pressure will be derived using the continuity equation. The linearized equations for the sufficiently smooth scalar perturbations in the Newtonian gauge (3.3.13) can then be derived with the help of (3.3.14). The result is

$$8\pi G\delta\rho(t) = \sum_{n=0}^{\infty} g_{|n}(H(t))\delta H^{(n)}(t), \tag{3.3.16}$$

$$-8\pi G\delta p = \frac{1}{H}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} g_{|nm}(H)H^{n+1}\delta H^{(m)}$$
$$+ \sum_{n=0}^{\infty} g_{|n}(H)\left(\frac{\delta H^{(n+1)}}{H} - \frac{H^{(n+1)}}{H^2}\delta H\right)$$
$$+ 3\sum_{n=0}^{\infty} g_{|n}(H)\delta H^{(n)}, \tag{3.3.17}$$

where

$$g_{|n}(H) = \left[\frac{\partial f}{\partial H^{(n)}}\right]_{H^{(n)}=H^{(n)}(t(H))}, \tag{3.3.18}$$

$$g_{|nm}(H) = \left[\frac{\partial^2 f}{\partial H^{(n)}\partial H^{(m)}}\right]_{H^{(n)}=H^{(n)}(t(H))}, \tag{3.3.19}$$

and the time dependence of H is derived from the homogeneous evolution (3.2.3). This procedure fixes exactly the coefficients a_{0m}, b_{0m}, e_{0m} and f_{0m} in (3.3.7),(3.3.8),(3.3.9), i.e. all the terms with no spatial derivatives in the equations for the scalar perturbations, as functions of $f(H, \dot{H}, ...)$ and its partial derivatives.

3. Linearized Treatment of Scalar perturbations in the Asymptotic Cosmological Model

We can now count the number of functional degrees of freedom in the superhorizon modes of the linearized theory. Let us assume that there is a maximum number D of time derivatives in the equations for the perturbations [2]. The number of derivatives, with the use of (3.3.14),(3.3.16),(3.3.17), fixes f to be a function of at most $H^{(D-2)}$. When particularized to a solution of the homogeneous Friedman equations, f and its first and second partial derivatives become functions of the Hubble rate H: $g(H)$, $g_{|n}(H)$ and $g_{|nm}(H)$ respectively. That makes $1 + (D-1) + D(D-1)/2 = D(D+1)/2$ functional degrees of freedom. The first derivative of $g(H)$ can be written in terms of the $g_{|n}(H)$, and the first derivative of the later can be also written in terms of the $g_{|nm}(H)$. That makes $1 + (D-1)$ conditions, so the result depends on $D(D-1)/2$ independent functions of the Hubble parameter H in the terms without any spatial derivatives.

Notice however that if we restrict the number of derivatives appearing in the equations for the perturbations to two as in Eqs. (3.3.10),(3.3.11),(3.3.12), then Eqs. (3.3.14),(3.3.16),(3.3.17) tell us that we must consider just functions $f(H, \dot{H}, \ddot{H}, ...) = f(H) = g(H)$, at least as an approximation at times $H \ll H_+$ and modes $k \ll H_+$. Therefore the number of functional degrees of freedom in the terms with no spatial derivatives is just one: the homogeneous evolution $f(H) = g(H)$.

Our assumption might be relaxed and we could impose that general relativity should be valid for all the modes $H_- < k < k_{obs}$ that have been observed in the spectrum of CMB and matter perturbations. This could lead to new terms in the equations for the perturbations (suppressed not necessarily by the UV scale H_+) which have been negligible for the observed modes but that could lead to ultraviolet deviations from the spectrum derived in the general relativistic cosmology which have not yet been observed. However, terms with more than two spatial derivatives will be very tightly constrained by solar system experiments.

In this article we will restrict ourselves to the system of second order differential equations (3.3.10), (3.3.11), (3.3.12). Let us now derive the linearized equations for the rest of the modes under this assumption. In the $(t', \mathbf{x})$ coordinate system, the metric is Friedman Robertson Walker, and we know that for this metric, we can use equation (3.2.3). Thus,

$$8\pi G \rho' = 3g(H'), \tag{3.3.20}$$

with

$$H' = \frac{1}{a'}\frac{da'}{dt'} = H(1 - \phi) - \dot{\psi} \tag{3.3.21}$$

at linear order in perturbations. Therefore, we can deduce from (3.2.3) that in the $(t, \mathbf{x})$ coordinate system

---

[2]We are forgetting here the restriction on the number of derivatives required in order to reproduce the acoustic peaks of CMB.

$$8\pi G\delta\rho = -3g'(H)(H\phi) + \dot\psi) + a_{10}\frac{\Delta}{a^2}\phi + b_{10}\frac{\Delta}{a^2}\psi\,, \qquad (3.3.22)$$

$$8\pi G(\rho_{(0)} + p_{(0)})\partial_i\theta = \partial_i\sum_{m=0}^{1}\left[\frac{c_{0m}}{H^{m-1}}\partial_t^m\phi + \frac{d_{0m}}{H^{m-1}}\partial_t^m\psi\right]\,, \qquad (3.3.23)$$

$$8\pi G(-\delta p\,\delta_j^i + \partial_i\partial_j\Pi) = \delta_j^i\left[g'(H)\left(3H\phi + 3\dot\psi - \frac{\dot H}{H^2}\dot\psi + \frac{\dot H}{H}\phi + \dot\phi + \frac{1}{H}\ddot\psi\right)\right.$$
$$\left. + \frac{\dot H}{H}g''(H)(H\phi + \dot\psi)\right]$$
$$+ \frac{1}{a^2}(\delta_j^i\Delta - \partial_i\partial_j)[g_{00}\phi + h_{00}\psi]\,, \qquad (3.3.24)$$

and the twelve coefficients of terms with no spatial derivatives are fixed by the function $g(H)$ which defines the homogeneous cosmological model.

The last requirement comes again from general covariance. If the tensor $T_{\mu\nu}$ comes from the variation of a certain matter action S_m with respect to the inverse of the metric $g^{\mu\nu}$, and S_m is a scalar under general coordinate transformations, then T_ν^μ must be a 1-covariant, 1-contravariant divergenceless tensor, i.e. $\nabla_\mu T_\nu^\mu = 0$. As the stress-energy tensor is proportional to G_ν^μ, the later must also be divergenceless. In the linearized approximation $G_\nu^\mu = G_{(0)\nu}^\mu + \delta G_\nu^\mu$ and the Christoffel symbols $\Gamma_{\mu\nu}^\lambda = \Gamma_{(0)\mu\nu}^\lambda + \delta\Gamma_{\mu\nu}^\lambda$, and we can write the linearized version of this requirement as

$$\delta G_{\mu|0}^0 + \delta G_{\mu|i}^i + \delta G_\mu^\lambda\Gamma_{(0)\lambda\nu}^\nu - \delta G_\nu^\lambda\Gamma_{(0)\lambda\mu}^\nu$$
$$+ G_{(0)\mu}^\lambda\delta\Gamma_{\lambda\nu}^\nu - G_{(0)\nu}^\lambda\delta\Gamma_{\lambda\mu}^\nu = 0\,. \qquad (3.3.25)$$

Let us study this condition in order to see if we can further limit the number of independent coefficients of the equations for scalar perturbations. For $\mu = i$ the term $\delta G_{i|0}^0$ will give at most a term proportional to $d_{01}\ddot\psi_{|i})$, which will not be present in other terms except $\delta G_{i|j}^j$. This term will give at most a term $g'\ddot\psi_{|i}/H$ which fixes the exact value of d_{01} for all $H \ll H_+$. For the same reason, the term $c_{01}\ddot\phi_{|i}$ in $\delta G_{i|0}^0$ can not be canceled and the term $c_{00}H\dot\phi_{|i}$ can only be canceled by the term $g'\dot\phi_{|i}$ in $\delta G_{i|j}^j$, which means that $c_{01} = 0$ and fixes the exact value of c_{00} for all $H \ll H_+$. The value of d_{00} can be fixed in terms of d_{01} and the terms proportional to $\dot\psi$ in (3.3.24). This fixes (3.3.23) completely in terms of the function describing the homogeneous evolution, $g(H)$:

$$8\pi G(\rho_0 + p_0)\partial_i\bar\theta = \frac{g'(H)}{H}\partial_i\left(H\Phi + \dot\Psi\right)\,. \qquad (3.3.26)$$

In the previous equation and from now on we will refer directly to the gauge invariant counterparts of the variables in the Newtonian gauge, which we will denote by $\Phi, \Psi, \bar\theta, \bar\delta\rho$ and $\bar\delta p$ (Π is already gauge invariant). Now we can use the divergenceless condition for $\mu = 0$. The term $\delta G_{0|0}^0$ will give at most a term proportional to $a_{10}\Delta\dot\phi$ and a term proportional to $b_{10}\Delta\dot\psi$, which will not be present in other terms except $\delta G_{0|j}^j$. This term will give at most a term proportional to $\Delta\phi$ and a term proportional to $\Delta\dot\psi$. Therefore $a_{10} = 0$ and the value of b_{10} is set completely in terms of $g(H)$:

$$8\pi G\bar\delta\rho = -3g'(H)(H\Phi + \dot\Psi) + \frac{g'(H)}{a^2 H}\Delta\Psi\,. \qquad (3.3.27)$$

The complete knowledge of (3.3.27) and (3.3.26) when $H \ll H_+$ gives a complete knowledge of (3.3.24) in terms of $g(H)$ when $H \ll H_+$:

$$8\pi G(\bar{\delta p}\,\delta_{ij} + \partial_i\partial_j\bar{\Pi}) = \delta_{ij}\left\{g'(H)\left[3H\Phi + 3\dot{\Psi} - \frac{\dot{H}}{H^2}\dot{\Psi} + \frac{\dot{H}}{H}\Phi + \dot{\Phi} + \frac{1}{H}\ddot{\Psi}\right] + \frac{\dot{H}}{H}g''(H)\left[H\Phi + \dot{\Psi}\right]\right\}$$
$$+ \frac{1}{a^2}\left(\delta_{ij}\Delta - \partial_i\partial_j\right)\left(\frac{g'(H)}{2H}(\Phi - \Psi) + \frac{\dot{H}}{2H^3}(g'(H) - Hg''(H))\Psi\right) \quad ,$$

$$(3.3.28)$$

and we have finally a set of equations for the scalar perturbations (3.3.27), (3.3.26), (3.3.28) which are determined by the function $g(H)$ which defined the ACM in the homogeneous approximation.

Had we relaxed our assumption, we could work with the system of equations (3.3.3),(3.3.4),(3.3.5) restricted to a maximum number of derivatives D as an approximation. Together with the condition $G^\mu_{\nu;\mu} = 0$, the system of equations for the perturbations define a set of coupled differential equations for the coefficients of the terms with at least one spatial derivative. The set of equations coming from $G^\mu_{i;\mu} = 0$ define the terms proportional to e_{mn} and f_{mn} in (3.3.5) as a function of the terms in (3.3.4). The set of equations coming from $G^\mu_{0;\mu}$ define the rest of the terms in (3.3.5) as a function of the terms in (3.3.4) and (3.3.3). Therefore, the only freedom, for what concerns linearized perturbations, is that of choosing the set of functions $\{a_{nm}, b_{nm}, c_{nm}, d_{nm}\}$, with some of them fixed by the homogeneous dynamics (3.2.1). An interesting case is found if $D = 4$ is imposed. This includes a description of $f(R)$-theories [79], bigravity theories [218], and other of the most studied modified gravity theories. We will study further this case in the subsection devoted to the comparison of the model with $f(R)$ theories.

The rhs of the equations derived are the equivalent to the components of the linearized Einstein tensor derived in a general covariant theory whose Friedman equation is exactly (3.2.3). Aside from General Relativity with or without a cosmological constant, i.e. for $\alpha_- \neq 1, 0$, it will be necessary to build an action depending on arbitrarily high derivatives of the metric in order to derive a theory such that both the equations in the homogeneous approximation and the linearized equations for the scalar perturbations are of finite order.

Noticeably, the nonlinear equations for the perturbations will include derivatives of arbitrary order of the metric perturbations, with the exceptional case of General Relativity with a cosmological constant. This makes problematic the consistency of the ACM beyond the linear approximation. This issue will be further studied in a future work.

To summarize, what we have shown in this section is that the behavior of scalar perturbations in a general covariant theory is intimately connected to the background evolution. Except in the very early universe, the linearized equations for the scalar perturbations are determined by the equations in the homogeneous approximation if one assumes that there are no terms with more than two spatial derivatives. This assumption could be relaxed in order to include more general theories.

There will be a subset of general covariant theories with a background evolution given by (3.2.3) that verify these equations for the perturbations. We may be able to distinguish among these at the linearized level by means of the vector and tensor perturbations.

3.4. Hydrodynamical Perturbations

Vector perturbations represent rotational flows which decay very quickly in the General Relativistic theory. As we expect a small modification of the behavior of perturbations just in the vicinity of the lower bound H_-, we will assume that the vector perturbations have decayed to negligible values when the scale H_- begins to play a role and therefore they will be ignored.

Tensor perturbations correspond to gravitational waves, which at the present have been not observed, their effect being far beyond the resolution of current observations.

Present measurements restrict their attention to perturbations in the photon sector (CMB) and the matter sector (matter power spectrum, both dark and baryonic). These observations can be computed taking into account only the effect of scalar perturbations. Therefore, we can use the result of the previous section to study the deviations from ΛCDM in the spectrum of CMB. A comprehensive study of the matter power spectrum predicted by the ACM would require a detailed knowledge of the nonlinear regime, which we lack at present.

Let us start by deriving the behavior of scalar perturbations in a universe whose gravitation is described by the ACM and filled with a perfect fluid ($\Pi = 0$) in (3.3.2). Therefore, (3.3.28) with $i \neq j$ fixes Φ as a function of Ψ,

$$\Phi = \left\{ 1 + \frac{\dot{H}}{2H^2} \left(1 - \frac{H g''(H)}{g'(H)} \right) \right\} \Psi \,. \tag{3.4.1}$$

Then, this expression can be used to substitute Φ in the remaining three differential equations for $\bar{\delta\rho}$, $\bar{\delta p}$ and $\bar{\theta}$. Given the equation of state of the fluid $p = p(\rho, S)$, with S the entropy density, the perturbation of the pressure can be written as

$$\bar{\delta p} = c_s^2 \bar{\delta\rho} + \tau \delta S \,, \tag{3.4.2}$$

where $c_s^2 \equiv (\partial p/\partial\rho)_S$ is the square of the speed of sound in the fluid and $\tau \equiv (\partial p/\partial S)_\rho$. Substituting the pressure and the energy density perturbations for the corresponding functions of Ψ and its derivatives into (3.4.2), we arrive at the equation for the entropy density perturbation δS. The perturbations we are interested in are adiabatic, i.e.: $\delta S = 0$ and therefore, the equation for the entropy perturbations turns into an equation of motion of the adiabatic perturbations of the metric. If we want to consider the effect of the lower bound of the Hubble rate, H_-, on the evolution of perturbations, we must consider a matter dominated universe ($c_s^2 = 0$),

$$g'(H) \left[3H\Phi + 3\dot{\Psi} - \frac{\dot{H}}{H^2}\dot{\Psi} + \frac{\dot{H}}{H}\Phi + \dot{\Phi} + \frac{1}{H}\ddot{\Psi} \right]$$
$$+ \frac{\dot{H}}{H} g''(H) \left[H\Phi + \dot{\Psi} \right] = 0 \,. \tag{3.4.3}$$

This equation is a second order linear differential equation with non-constant coefficients. Thus it is useful to work with the Fourier components of the metric perturbation Ψ_k. The equation of motion for the Fourier components is just a second order linear ODE with non-constant coefficients, which will be analytically solvable just for some simple choices of $g(H)$. However, in general, it will be mandatory to perform a numerical analysis.

3.5. The CMB Spectrum in ACM versus ΛCDM

The main observation that can be confronted with the predictions of the theory of cosmological perturbations at the linearized level is the spectrum of temperature fluctuations of the Cosmic Microwave Background (CMB), from which WMAP has recorded accurate measurements for five years [47].

The temperature fluctuations $\frac{\delta T}{T}$ are connected to the metric perturbations via the Sachs-Wolfe effect [219], which states that, along the geodesic of a light ray $\frac{dx^i}{dt} = l^i(1 + \Phi + \Psi)$, characterized by the unit three-vector l^i, the temperature fluctuations evolve according to
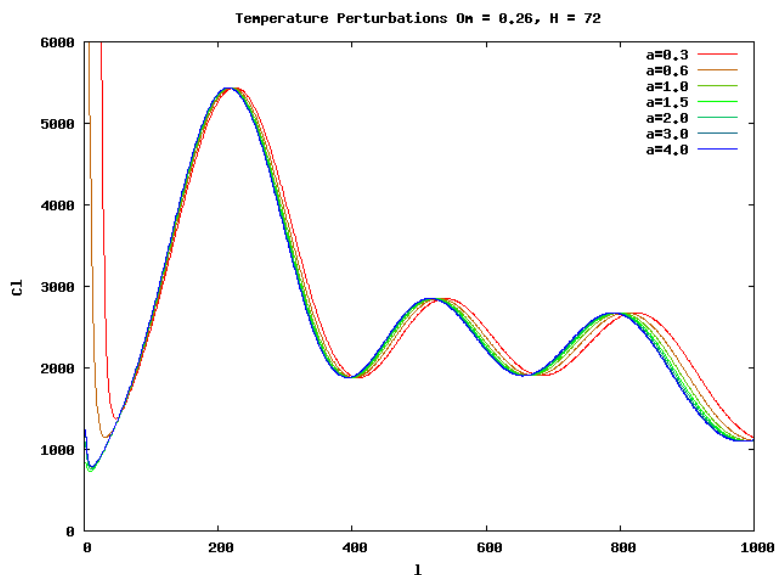
Figure 3.5.: Spectrum of temperature fluctuations of the Cosmic Microwave Background for $\Omega_m = 0.26$, $H_0 = 72 km/sMpc$ and varying $\alpha_- = 0.3$ (red), 0.6, 1.0, 1.5, 2.0, 3.0, 4.0 (blue). The lowest values of α_- show extreme ISW effects and therefore will be ruled out also by the CMB spectrum. Unfortunately, the cosmic variance masks the effect of $1.0 < \alpha_- < 4.0$ at large angular scales and therefore we can obtain little information about the ACM from the late ISW effect.

$$\left(\frac{\partial}{\partial t} + \frac{l^i}{a}\frac{\partial}{\partial x^i}\right)\left(\frac{\delta T}{T} + \Phi\right) = \frac{\partial}{\partial t}(\Psi + \Phi). \tag{3.5.1}$$

Neglecting a local monopole and dipole contribution, taking recombination to be instantaneous at a certain time and assuming that the reionization optical depth is negligible, the present temperature fluctuation of a distribution of photons coming from a given direction of the sky l^i can be related to the temperature perturbation and metric perturbation Ψ in the last scattering surface plus a line integral along the geodesic of the photons of the derivatives of Φ and Ψ (Integrated Sachs Wolfe effect, ISW).

These relations are purely kinematical and remain unchanged in the ACM. In order to take into account all the effects involved in the calculation of the CMB spectrum the code of CAMB [201] has been adapted to the ACM. (see the appendix for details).

We have compared the late time evolution of the perturbations described by the ACM (3.2.6) for several values of α_- (taking into account that $\alpha_- = 1$ corresponds to ΛCDM) for constant H_0 and Ω_m defined in Eq. (3.2.10). We have assumed a Harrison-Zel'dovich scale invariant spectrum of scalar perturbations as initial condition ($n_s = 0$) in the region in which the universe is radiation dominated but $H \ll H_+$. Thus the study of the effect of the scale H_+ is postponed. We have taken $H_0 = 72 \pm 8 \, km/sMpc$ from the results of the Hubble Key Project [207][3] and $\Omega_m = 0.26 \pm 4$ from our previous analysis of the evolution at the homogeneous level. The effect of ACM as compared to ΛCDM is twofold (FIG. 5).

There is a shift in the peak positions due to the difference in the distance to the last scattering surface for different values of α_-. This shift increases with the peak number. In particular, more precise measurements of the position of the third peak could be used to estimate the value of α_-.

There is also an increase of the lower multipoles due to the late Integrated Sachs-Wolfe effect, which is particularly extreme for $\alpha_- \lesssim 1.0$ (which corresponds to $H_- \simeq H_0$ for constant Ω_m). This large deviation is clearly not present in the experimental data. The much smaller deviation for $\alpha_- \gtrsim 1.0$ is within the error bands due to the cosmic variance, and therefore the value of the lower multipoles cannot be used to exclude values of α_- greater than but of the order of one.

3.6. Comparison with the treatment of perturbations in other models

3.6.1. Perturbations in f(R) theories

In our previous work [217] we found that given an expansion history parameterized by a modified Friedman equation in a universe filled by a given component, it was always possible to find a biparametric family of $f(R)$ theories which had the same homogeneous evolution as a solution of their equations of motion. We wonder now if these theories have also the same linearized equations for the scalar perturbations of the metric.

An $f(R)$ theory is defined by its action and therefore the equations for the perturbations are uniquely determined. The action is given by

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}[f(R) + 16\pi G L_m] \,, \tag{3.6.1}$$

where R is the curvature scalar and $f(R)$ is an arbitrary function of this scalar. The equations governing the evolution of the perturbations will be derived from the Einstein equations of the $f(R)$ theory,

$$f'(R)R^\mu_\nu - \frac{1}{2}f(R)\delta^\mu_\nu + (\delta^\mu_\nu \Box - \nabla^\mu \nabla_\nu)f'(R) = 8\pi G T^\mu_\nu \,. \tag{3.6.2}$$

At the homogeneous level, it is possible to write the Friedman equations of the $f(R)$ theory,

$$8\pi G \rho_{(0)} = -3(\dot{H} + H^2)f'(R_{(0)}) - \frac{1}{2}f(R_{(0)}) - 3Hf''(R_{(0)})\dot{R}_{(0)} \,, \tag{3.6.3}$$

$$-8\pi G p_{(0)} = -(\dot{H} + 3H^2)f'(R_{(0)}) - \frac{1}{2}f(R_{(0)}) + f^{(3)}(R_{(0)})\dot{R}^2_{(0)} + f''(R_{(0)})\ddot{R}_{(0)} \,, \tag{3.6.4}$$

where

$$R_{(0)}(t) = -6(\dot{H} + 2H^2) \tag{3.6.5}$$

is the curvature scalar of the Friedman-Robertson-Walker (FRW) metric (zeroth order in perturbations). The solution of the system of equations (3.6.3),(3.6.4) together with the equation of state of the dominant component of the stress-energy tensor, defines $R_{(0)}$ and its derivatives as functions of the Hubble parameter H [217].

If we expand (3.6.2) in powers of the scalar perturbations of the FRW metric (3.3.1), the first order term gives the linearized equations for the perturbations. This will be a system of differential equations of fourth order, but it will be possible to turn it into a system of differential equations of second order, as we will see below.

If the stress-energy tensor has no shear, the Einstein equation for $\mu = i \neq \nu = j$ reduces to

$$f''(R_{(0)})\delta R + f'(R_0)(\Phi - \Psi) = 0 \,, \tag{3.6.6}$$

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[3]Recently the SHOES team has provided a more precise measurement, $H_0 = 74.2 \pm 3.6 \, km/sMpc$,[220]

where δR is the perturbation of the curvature scalar

$$\delta R = 12(\dot{H} + 2H^2)\Phi + 6H\dot{\Phi} + 24H\dot{\Psi}$$
$$+6\ddot{\Psi} + \frac{2}{a^2}\Delta(\Phi - 2\Psi)).$$

(3.6.7)

Eq. (3.6.6) is a second order differential equation for the scalar perturbations except in the case of General Relativity ($f'' = 0$) where it becomes an algebraic equation ($\Phi = \Psi$). Eq. (3.6.6) can be used to turn the remaining components of the equations of $f(R)$ theories (3.6.2) into second order equations.

In the case of adiabatic perturbations in a matter dominated epoch one has

$$-8\pi G\bar{\delta p} = f'(R_{(0)})(a^{-2}\Delta(\Phi - 2\Psi) + \ddot{\Psi} + 6H\dot{\Psi} + H\dot{\Phi} + 2(\dot{H} + 3H^2)\Phi - \frac{1}{2}\delta R)$$
$$+f''(R_{(0)})(-(\dot{H} + 3H^2)\delta R - 2\dot{R}_{(0)}\dot{\Psi} - 8H\dot{R}_{(0)}\Psi - 4H\dot{R}_{(0)}\Phi - 2\ddot{R}_{(0)}\Phi) - 2f^{(3)}(R_{(0)})\dot{R}_{(0)}^2\Phi$$
$$-\partial_t^2\left[f'(R_{(0)})(\Phi - \Psi)\right] - 2H\partial_t\left[f'(R_{(0)})(\Phi - \Psi)\right] = 0.$$

(3.6.8)

If we compare the equations governing the behavior of adiabatic perturbations in a $f(R)$ theory (3.6.6), (3.6.8) with those coming from the ACM, (3.4.1),(3.4.3), we find that they describe very different behaviors. In one case we have a system of two coupled second order differential equations for Φ and Ψ. In the ACM case we got a single second order differential equation and an algebraic equation between the two gravitational potentials. For a given $f(R)$ theory with an associated $g(H)$ homogeneous behavior, we find that the behavior of linearized perturbations differs from the one defined by the linearized perturbations in the ACM. This means that $f(R)$ theories break in general our assumption in the third section (there are terms with more than two derivatives in the equations for the perturbations, and therefore the behavior of perturbations in General Relativity is not recovered when $H_+ \gg H \gg H_-$ for all modes).

Let us see this in a simple example. We will choose the easiest biparametric family of $f(R)$ theories: the one that, under matter domination, gives the same background evolution as General Relativity without a cosmological constant [217],

$$f(R) = R + c_1|R|^{\frac{1}{12}(7-\sqrt{73})} + c_2|R|^{\frac{1}{12}(7+\sqrt{73})}.$$

(3.6.9)

These kind of models were introduced in Ref. [81].

Let us first check if the solutions of the equations for the perturbations in General Relativity in the presence of pressureless matter,

$$\Psi = \Phi = const,$$
$$\Psi = \Phi \propto t^{-5/3},$$

(3.6.10)

are solutions of the equations of motion in the case of the biparametric family of $f(R)$ theories (3.6.9). The easiest is to verify if Eq. (3.6.2) with $\mu = i \neq \nu = j$,

$$2f''(R_{(0)})(3H^2\Phi + 3H\dot{\Phi} + 12H\dot{\Psi} + 3\ddot{\Psi} + \frac{1}{a^2}\Delta(\Phi - 2\Psi))$$
$$+f'(R_{(0)})(\Phi - \Psi) = 0,$$

(3.6.11)

where $R_{(0)}(H) = -3H^2$ is the value of the homogeneous curvature scalar as a function of H in this family of theories, is also fulfilled by the solutions (3.6.10). The result is obviously not.

The second order differential equations for the perturbations in the $f(R)$ theories (3.6.9) in the presence of matter can be found by substituting (3.6.11) and its derivatives into the other equations of the system (3.6.2). The equation for the pressure perturbations then gives

$$-f'(R_{(0)})[\ddot{\Psi} + \ddot{\Phi} + 4H\dot{\Psi} + 4H\dot{\Phi} + 3H^2\Phi]$$
$$-27H^3f''(R_{(0)})\dot{\Phi} + \frac{f(R_{(0)})}{2}(\Psi - 3\Phi) = 8\pi G\bar{\delta p} = 0.$$

(3.6.12)

It is obvious that the $i \neq j$ equation of the cosmologic perturbations in General Relativity,

$$\Phi - \Psi = 0 \tag{3.6.13}$$

is not recovered from (3.6.6) in the $H \gg H_-$ limit for all $k \ll H_+$. In fact the deviation would be significant for modes with $k \gtrsim Ha \left(\frac{H}{H_-} \right)^{\frac{1}{12}(7-\sqrt{73})} \sim Ha$. Therefore, the assumption we have made in order to derive the equations for the perturbations in the ACM is broken.

It is a known problem that $f(R)$ theories are unable to pass cosmological and astrophysical tests involving perturbations [92], unless the function involved is properly fine-tuned. In particular, if the conformal equivalence between $f(R)$ theories and scalar-tensor theories is used, it is necessary that the effective mass acquired by the new scalar degree of freedom is unnaturally large [95]. Some $f(R)$ theories that pass cosmological and solar system tests have been proposed [96, 97, 98]. However, it is also subject of debate if the conformal equivalence can be used in order to extract physical predictions from this models, especially predictions which involve perturbations [99]. A mathematically rigorous treatment of perturbations in $f(R)$-gravity can be found in Ref. [100].

3.6.2. Quintessence Models

Another class of models used to describe the acceleration of the universe are those in which a so called quintessence field, typically of scalar type φ, is added as a component of the universe [116]. The only functional degree of freedom in most models is just the scalar potential $V(\varphi)$, which can be tuned to fit the homogeneous expansion of the universe. In our previous work [217] we found the correspondence between a given homogeneous evolution parameterized by $g(H)$ and the potential for the quintessence which drives under General Relativity this evolution. We now wonder if the behavior of perturbations in quintessence models also resembles the behavior in ACM.

The field action is given by

$$S_\varphi = \int d^4 x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right] . \tag{3.6.14}$$

Let us assume that the field can be split in two components: one which is only time dependent and which drives the homogeneous evolution of the universe, $\varphi_{(0)}(t)$, and a small perturbation which is inhomogeneous, $\delta\varphi(\mathbf{x}, t)$. We will consider linearized perturbations of the metric, the field, and the other components of the universe. The resulting linearized stress-energy tensor of the field is

$$\delta T^\varphi_{00} = \dot{\varphi}_{(0)} \dot{\delta\varphi} - \Phi V(\varphi_{(0)}) + \frac{1}{2} V'(\varphi_{(0)}) \delta\varphi , \tag{3.6.15}$$

$$\delta T^\varphi_{0i} = \dot{\varphi}_{(0)} \partial_i \delta\varphi , \tag{3.6.16}$$

$$\delta T^\varphi_{ij} = a^{-2} \delta_{ij} \left[\dot{\varphi}^2_{(0)} (\Phi - \Psi) + \dot{\varphi}_{(0)} \dot{\delta\varphi} \right.$$
$$\left. + \Psi V(\varphi_{(0)}) - \frac{1}{2} V'(\varphi_{(0)}) \delta\varphi \right] . \tag{3.6.17}$$

By virtue of the Einstein's equations, it is always possible to turn a modification in the Einstein tensor into a new component of the stress-energy momentum tensor of the sources of the gravitational field. We wonder if it is possible to account for the modification of the effective Einstein tensor described in the third section with a new component described by this quintessence field.

However, it is not hard to see that the effective Einstein tensor that we are proposing has a modified $i \neq j$ component, while the $i \neq j$ component of the stress-energy tensor of the quintessence field φ is zero. Therefore, it is not possible to describe the evolution of perturbations in the ACM as driven by an effective scalar field component (3.6.14).

3.6.3. Dark Fluid Models

A deformation of the gravitational physics can be also made equivalent to the addition of a non-standard fluid component to the cosmic pie at the homogeneous level. The fluid component used to explain the present accelerated expansion of the universe is typically taken to be a perfect fluid with large negative pressure.

One of the most popular parameterizations of this fluid is the so called equation of state $\omega = p/\rho$ [126]. The simplest models are $\omega = -1$, which is equivalent to a cosmological constant, or a constant ω, but recently a possible time dependence of ω has been considered [221]. Needless to say, these *ad hoc* parameterizations are subsets of the more general equation of state of a perfect fluid, $p = p(\rho, s)$, where s is the entropy density. It is also questionable why must we restrict ourselves to perfect fluids and not include a possible anisotropic stress tensor, possibly depending on the energy and entropy densities [130].

As in the previous subsection, it is always possible to use the Einstein's equations to turn a modification in the gravitational physics into a new fluid component of the universe. In the case of the equations derived in the third section, the equivalent Dark Energy Fluid would have the following properties:

$$8\pi G \bar{\delta\rho}_X = (2H - g') \left[\frac{1}{a^2 H} \Delta\Psi - 3(H\Phi + \dot{\Psi}) \right], \tag{3.6.18}$$

$$8\pi G \bar{\theta}_X = \frac{(2H - g')}{H} (H\Phi + \dot{\Psi}), \tag{3.6.19}$$

$$8\pi G \Pi_X = \frac{1}{a^2} \left\{ \frac{2H - g'}{2H} (\Phi - \Psi) - \frac{\dot{H}}{2H^3} (g' - Hg'')\Psi \right\}, \tag{3.6.20}$$

$$8\pi G \bar{\delta p}_X = (2H - g')(3H\Phi + 3\dot{\Psi} - \frac{\dot{H}}{H^2}\dot{\Psi} + \frac{\dot{H}}{H}\Phi + \dot{\Phi} + \frac{1}{H}\ddot{\Psi})$$

$$+ \frac{\dot{H}}{H} (2 - g'')(H\Phi + \dot{\Psi}) - 8\pi G \Delta\Pi_X. \tag{3.6.21}$$

The resulting fluid is not an ideal fluid ($\Pi_X = 0$) or even a Newtonian fluid ($\Pi_X \propto \bar{\theta}_X$). With the use of (3.2.3), (3.4.1) and (3.4.3), it will be possible to write $\bar{\delta\rho}_X$ and $\bar{\delta p}_X$ as a combination of $\bar{\theta}_X$, Π_X and their derivatives,

$$\bar{\delta\rho}_X = f_1(\rho_{(0)X})\Delta\Pi_X + f_2(\rho_{(0)X})\bar{\theta}_X, \tag{3.6.22}$$

$$\bar{\delta p}_X = f_3(\rho_{(0)X})\Pi_X - \Delta\Pi_X + f_4(\rho_{(0)X})\bar{\theta}_X. \tag{3.6.23}$$

These very non-standard properties show that, although it is formally possible to find a fluid whose consequences mimic the ones of such a modification of the gravitational physics, this fluid is very exotic.

3.7. Summary and Conclusions

In view of recent data, an updated comparison of cosmological observations with a phenomenological model proposed in a recent work has been presented.

An extension of this phenomenological model (ACM) beyond the homogeneous approximation has been introduced allowing us to describe the evolution of scalar perturbations at the linear level.

A comparison with the spectrum of thermal fluctuations in CMB has been used to explore the possibility to determine the parameters of the ACM through its role in the evolution of scalar perturbations. The results of this comparison does not further restrict the parameters of the model, due to the masking of the associated late ISW effect by the cosmic variance. However, better measurements of the position of the third acoustic peak should improve the constraints significantly.

It has been shown that the equivalence of different formulations of the accelerated expansion of the universe in the homogeneous approximation is lost when one considers inhomogeneities. In particular we have shown that the general structure of the evolution equations for scalar perturbations in the ACM differs from the structure of the equations corresponding to modified $f(R)$ theories of gravity, to quintessence models or to a dark fluid with standard properties.

The possibility of going beyond the linearized approximation for the scalar perturbations and to consider vector and tensor perturbations will be the subject of a future work.

Appendix: Covariant Perturbation Equations for the ACM

The code of CAMB [201] makes use of the equations for the perturbations of the metric in the covariant approach. The quantities can be computed in a given "frame", labeled by a 4-velocity u^μ. In particular, CAMB uses the dark matter frame, in which the velocity of the dark matter component is zero (the dark matter frame). Furthermore, it parameterizes the time evolution with the conformal time $a(\tau)d\tau = dt$.

In order to apply CAMB to the ACM model it is necessary to identify frame invariant quantities, and then relate them to gauge invariant quantities [222]. The following comoving frame quantities are used in the CAMB code: η (the curvature perturbation), σ (the shear scalar), z (the expansion rate perturbation), A (the acceleration, $A = 0$ in the dark matter frame), ϕ (the Weyl tensor perturbation), $\chi^{(i)}$ (the energy density perturbation of the species i), $q^{(i)}$ (the heat flux of the species i), and $\Pi^{(i)}$ (the anisotropic stress of the species i). All of these quantities are defined in [223]. These variables are related to the gauge invariant variables via the following dictionary,

$$
\begin{aligned}
-\frac{\eta}{2} - \frac{\mathcal{H}\sigma}{k} &\equiv \Psi\,, \\
-A + \frac{\sigma' + \mathcal{H}\sigma}{k} &\equiv \Phi\,, \\
\chi + \frac{\rho'\sigma}{k} &\equiv \bar{\delta\rho}\,, \\
\rho q + (\rho + p)\sigma &\equiv \frac{k}{a}(\rho + p)\bar{\theta}\,,
\end{aligned}
\tag{3.7.1}
$$

where prime denotes derivatives with respect to conformal time, except when acting on $g(H)$ where it denotes a derivative with respect to the Hubble rate H, and $\mathcal{H} = a'/a = aH$. On the other hand $z = \sigma + \frac{3}{2k}(\eta' + 2\mathcal{H}A)$ and $\phi = (\Phi + \Psi)/2$. The anisotropic stress Π is already frame invariant.

Written in the dark matter frame, the equations for the scalar perturbations read

$$
\begin{aligned}
g'\left(\tfrac{k^2\eta}{2\mathcal{H}} + kz\right) &= 8\pi Ga\Sigma_i\chi^{(i)}\,, \\
\tfrac{g'k^2}{3\mathcal{H}}(\sigma - z) = -\tfrac{g'k}{2\mathcal{H}}\eta' &= 8\pi Ga\Sigma_i\rho_i q^{(i)}\,, \\
-\mathcal{H}kg'(z' + \mathcal{H}z) &= 8\pi Ga\Sigma_i\left[\mathcal{H}^2(1 + 3c_s^{(i)2}) - (\mathcal{H}' - \mathcal{H})^2(1 - \tfrac{\mathcal{H}g''}{ag'})\right]\chi^{(i)}\,, \\
\tfrac{g'}{\mathcal{H}}\left(\tfrac{\sigma' + \mathcal{H}\sigma}{k} - \phi\right) - \tfrac{\mathcal{H}' - \mathcal{H}^2}{2\mathcal{H}^3}(g' - \mathcal{H}g''/a)\left(\tfrac{\eta}{2} + \tfrac{\mathcal{H}\sigma}{k}\right) &= -8\pi Ga\Sigma_i\Pi^{(i)}/k^2\,.
\end{aligned}
$$

$$(3.7.2)$$

The following combination of the constraint equations is also useful:

$$
k^2\phi = -\frac{8\pi Ga\mathcal{H}}{g'}\Sigma_i\left[\Pi^{(i)} + (1 - \frac{\mathcal{H}' - \mathcal{H}^2}{2\mathcal{H}^2}(1 - \frac{\mathcal{H}g''}{ag'}))(\chi^{(i)} + 3\mathcal{H}\rho_i q^{(i)}/k)\right]\,. \qquad (3.7.3)
$$

These equations are plugged into the Maple files provided with the CAMB code and run to get the ISW effect [201].

4. Interpretation of neutrino oscillations based on new physics in the infrared

An interpretation of neutrino oscillations based on a modification of relativistic quantum field theory at low energies, without the need to introduce a neutrino mass, is seen to be compatible with all observations.

4.1. Introduction

The now well-established observation of deficit of solar neutrinos, atmospheric neutrinos, and neutrinos from reactors and accelerators finds a coherent interpretation in terms of neutrino oscillations between three neutrino flavors of different masses [136]. In the minimal standard model (SM), and in contrast with the rest of the matter particles, the neutrino is assumed to be a zero mass, left-handed fermion. Therefore neutrino oscillations is our first glimpse of physics beyond the SM.

Massive neutrinos are introduced in extensions of the SM which normally invoke new physics at high energies. In particular, one can consider a Majorana mass term for the neutrino, generated by a five-dimensional operator in the SM Lagrangian which would be suppressed by the inverse of a certain high-energy scale. Another possibility is to enlarge the field content of the SM with a right-handed neutrino, which allows mass to be generated by the usual Higgs mechanism. One has to account however for the smallness of the neutrino mass, which is achieved by the see-saw mechanism [224, 225, 226, 227, 228], again invoking a grand-unification scale.

The presence of new physics at high energies has been explored in several attempts to find alternatives to the standard neutrino oscillation mechanism. This new physics might include Lorentz and/or CPT violations. These two low-energy symmetries are being questioned at very high energies in the framework of quantum gravity and string theory developments [229, 230, 231, 232, 233, 234, 235], and in fact simple models with Lorentz and/or CPT violations are able to generate neutrino oscillations, even for massless neutrinos [164, 180, 182]. Some of them are considered in the context of the Standard Model Extension (SME) [160, 161], which is the most general framework for studying Lorentz and CPT violations in effective field theories.

However, all these alternative mechanisms involve new energy dependencies of the oscillations which are in general disfavored over the standard oscillation mechanism by experimental data, which also put strong bounds on the contribution of new physics to this phenomenon [236, 176]. This seems to indicate that our understanding of neutrino oscillations as driven by mass differences between neutrino flavors is indeed correct.

In this letter we want to argue that this might not be the case. We will present an example of new physics to the SM able to generate neutrino oscillations and which is essentially different from previously considered models in one or several of the following aspects: it does not necessarily add new fields to those present in the SM, it may be completely indistinguishable from the standard oscillation mechanism in the energy ranges where the phenomenon has been studied (for neutrinos of medium and high energies), so that automatically satisfies all constraints which are already

fulfilled by the standard mechanism, and finally, it predicts new physics in the infrared, so that future neutrino low energy experiments could distinguish this mechanism from the standard one.

Our example will be based on the so-called theory of noncommutative quantum fields, which has recently been proposed as an specific scheme going beyond quantum field theory [196, 197, 237, 198]. The consequences for neutrino oscillations of a simple model with modified anticommutators for the neutrino fields, which can be identified as an example of a SME in the neutrino sector, has very recently been explored in Ref. [199]. We will see however that it is possible to introduce a generalization of the anticommutation relations of fields in a more general way than that studied in Ref. [199], going beyond the effective field theory framework of the SME, which is the key to reproduce the oscillation results without the need to introduce a neutrino mass, and with new consequences at infrared energies.

4.2. Noncanonical fields and neutrino oscillations

The theory of noncommutative fields was first considered in Refs. [196, 197]. It is an extension of the usual canonical quantum field theory in which the procedure of quantification of a classical field theory is changed in the following way: the quantum Hamiltonian remains the same as the classical Hamiltonian, but the canonical commutation relations between fields are modified. In the case of the scalar complex field this modification leads to the introduction of two new energy scales (one infrared or low-energy scale, and another one ultraviolet or high-energy scale), together with new observable effects resulting from the modification of the dispersion relation of the elementary excitations of the fields [197]. If one is far away from any of these two scales the theory approaches the canonical relativistic quantum field theory with corrections involving Lorentz invariance violations which can be expanded in powers of the ratios of the infrared scale over the energy and the energy over the ultraviolet scale. By an appropriate choice of the two new energy scales one can make the departures from the relativistic theory arbitrarily small in a certain energy domain.

We will now explore the relevance of the extension of relativistic quantum field theory based on noncanonical fields in neutrino oscillations. In particular, we will show that it is possible to obtain oscillations with the observed experimental properties just by considering a modification of the anticommutators of the fields appearing in the SM, without the need to introduce a right-handed neutrino or a mass for this particle.

With the left-handed lepton fields of the SM

$$\Psi_{L\alpha} = \begin{pmatrix} \nu_\alpha \\ l_\alpha \end{pmatrix}_L , \tag{4.2.1}$$

where α runs the flavor indices, $(\alpha = e, \mu, \tau)$, the simplest way to consider an analog of the extension of the canonical quantum field theory for a complex scalar field proposed in Refs. [196, 197] is to introduce the modified anticommutation relations

$$\{\nu_{L\alpha}(\mathbf{x}), \nu_{L\beta}^\dagger(\mathbf{y})\} = \{l_{L\alpha}(\mathbf{x}), l_{L\beta}^\dagger(\mathbf{y})\} = [\delta_{\alpha\beta} + A_{\alpha\beta}] \, \delta^3(\mathbf{x} - \mathbf{y}). \tag{4.2.2}$$

A particular choice for the matrix $A_{\alpha\beta}$ in flavor space which parametrizes the departure from the canonical anticommutators corresponds to the new mechanism for neutrino oscillations proposed in Ref. [199] which, however, is not compatible with the energy dependence of the experimental data.

In order to reproduce the observed properties of neutrino oscillations [136] one has to go beyond this extension and consider an anticommutator between fields at different points. This can be

made compatible with rotational and translational invariance and with $SU(2) \times U(1)_Y$ gauge symmetry by making use of the Higgs field

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \tilde{\Phi} = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}. \tag{4.2.3}$$

The modified anticommutators of the left-handed lepton fields that we consider in this work are

$$\{\Psi_{L\alpha}(\mathbf{x}), (\Psi_{L\beta})^\dagger(\mathbf{y})\} = \delta_{\alpha\beta}\,\delta^3(\mathbf{x} - \mathbf{y}) + \tilde{\Phi}(\mathbf{x})\tilde{\Phi}^\dagger(\mathbf{y})B_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|), \tag{4.2.4}$$

where $B_{\alpha\beta}$ are now functions of $|\mathbf{x} - \mathbf{y}|$ instead of constants. Note that Eq. (4.2.4) is compatible with gauge invariance since $\tilde{\Phi}(\mathbf{x})$ has the same $SU(2) \times U(1)_Y$ quantum numbers as $\Psi_{L\alpha}(\mathbf{x})$.

After introduction of spontaneous symmetry breaking ($\langle\varphi^0\rangle = v/\sqrt{2}$), and neglecting effects coming from the fluctuation of the scalar field which surely is a good approximation for neutrino oscillations, the only anticommutators that are changed are those of the neutrino fields

$$\{\nu_{L\alpha}(\mathbf{x}), \nu_{L\beta}^\dagger(\mathbf{y})\} = \delta^3(\mathbf{x} - \mathbf{y})\,\delta_{\alpha\beta} + C_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|), \tag{4.2.5}$$

where

$$C_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|) = \frac{v^2}{2}B_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|). \tag{4.2.6}$$

One can suspect that the new anticommutation relations Eq. (4.2.5) introduce a source of mixing between flavors that will affect neutrino oscillations. We will see in the next Section that this is indeed the case.

4.3. Solution of the free theory

In order to study the neutrino oscillations induced by the modified anticommutators of fields in the neutrino sector, one needs to solve the free theory given by the Hamiltonian

$$H = \sum_\alpha \int d^3\mathbf{x} \left[i\,\nu_{L\alpha}^\dagger\,(\boldsymbol{\sigma} \cdot \boldsymbol{\nabla})\,\nu_{L\alpha} \right] \tag{4.3.1}$$

(where $\boldsymbol{\sigma}$ are the 2×2 Pauli matrices), and the anticommutation relations showed in Eq. (4.2.5).

Let us introduce a plane wave expansion for the neutrino field

$$\nu_{L\alpha}(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\frac{1}{\sqrt{2p}}\,\sum_i \left[b_i(\mathbf{p})\,u_{L\alpha}^i(\mathbf{p})\,e^{i\mathbf{p}\cdot\mathbf{x}} + d_i^\dagger(\mathbf{p})\,v_{L\alpha}^i(\mathbf{p})\,e^{-i\mathbf{p}\cdot\mathbf{x}} \right], \tag{4.3.2}$$

where $p = |\mathbf{p}|$, and $(b_i(\mathbf{p}), d_i^\dagger(\mathbf{p}))$ are the annihilation and creation operators of three types of particles and antiparticles (expressed by subindex i) with momentum $\mathbf{p}$. We now use the following *ansatz* for the expression of the Hamiltonian (4.3.1) as a function of the creation-annihilation operators:

$$H = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\sum_i \left[E_i(p)\,b_i^\dagger(\mathbf{p})\,b_i(\mathbf{p}) + \bar{E}_i(p)\,d_i^\dagger(\mathbf{p})\,d_i(\mathbf{p}) \right]. \tag{4.3.3}$$

This corresponds to the assumption that the free theory describes a system of three types of free particles and antiparticles for each value of the momentum, with energies $E_i(p), \bar{E}_i(p)$, respectively.

Now, computing $[H, \nu_{L\alpha}]$ by two procedures: firstly by using Eq. (4.3.1) for the Hamiltonian and the anticommutators (4.2.5), and secondly, by using the expressions (4.3.2) and (4.3.3), and equalling both results, one obtains the following simple result for the energies and the coefficients in the plane wave expansion of the field:

$$
\begin{aligned}
E_i(p) &= \bar{E}_i(p) = p\left[1 + \tilde{c}_i(p)\right], & (4.3.4)\\
u^i_{L\alpha}(\mathbf{p}) &= v^i_{L\alpha}(\mathbf{p}) = e^i_\alpha(p)\,\chi^i(\mathbf{p}), & (4.3.5)
\end{aligned}
$$

where $\chi^i(\mathbf{p})$ is the two component spinor solution of the equation

$$
(\boldsymbol{\sigma}\cdot\mathbf{p})\,\chi^i(\mathbf{p}) = -p\,\chi^i(\mathbf{p}) \qquad (4.3.6)
$$

with the normalization condition

$$
\chi^{i\dagger}(\mathbf{p})\,\chi^i(\mathbf{p}) = 2E_i(p), \qquad (4.3.7)
$$

$\tilde{c}_i(p)$ are the three eigenvalues of $\tilde{C}_{\alpha\beta}(p)$, the Fourier transform of $C_{\alpha\beta}(|\mathbf{x}-\mathbf{y}|)$ in Eq. (4.2.5), and $e^i_\alpha(p)$ are the components of the normalized eigenvectors of $\tilde{C}_{\alpha\beta}(p)$.

From Eq. (4.3.4), we see that the model presented here contains violation of Lorentz invariance, but preserves CPT symmetry.

4.4. New IR physics and neutrino oscillations

Since in the free theory solution there are three types of particles and antiparticles with different energies, and a mixing of creation and annihilation operators of different kinds of particle-antiparticle in the expression of each field, it is clear that the nonvanishing anticommutators of different fields will produce neutrino oscillations, even for massless neutrinos. This observation was already present in Ref. [199]. The probability of conversion of a neutrino of flavor α produced at $t=0$ to a neutrino of flavor β, detected at time t, can be directly read from the propagator of the neutrino field (4.3.2). This probability can be written as

$$
\mathcal{P}\left(\nu_\alpha(0) \to \nu_\beta(t)\right) = \left|\sum_i e^i_\alpha(p)^* \, e^i_\beta(p)\, e^{-i\,E_i(p)t}\right|^2. \qquad (4.4.1)
$$

This is the standard result for the oscillation between three states with the unitary mixing matrix elements U^i_α replaced by the coefficients $e^i_\alpha(p)$ of the plane wave expansion of the non-canonical neutrino fields and the energy of a relativistic particle $\sqrt{p^2 + m_i^2}$ replaced by the energy $E_i(p)$ of the particle created by the noncanonical neutrino fields.

Let us now make the assumption that the modification of the anticommutators is a footprint of new physics at low energies, parametrized by an infrared scale λ. Although the introduction of corrections to a quantum field theory parametrized by a low-energy scale has not been so well explored in the literature as the corrections produced by ultraviolet cutoffs, there are several phenomenological and theoretical reasons that have recently lead to think on the necessity to incorporate a new IR scale to our theories [37, 38, 39, 40, 238, 33, 36, 239].

If the modifications of the anticommutation relations are parametrized by an infrared scale λ then it is reasonable to assume an expansion in powers of λ^2/p^2 so that

$$
\tilde{C}_{\alpha\beta}(p) \approx \tilde{C}^{(1)}_{\alpha\beta}\,\frac{\lambda^2}{p^2} \quad \text{for } p^2 \gg \lambda^2, \qquad (4.4.2)
$$

and then

$$\tilde{c}_i(p) \approx \tilde{c}_i^{(1)} \frac{\lambda^2}{p^2}, \quad e_\alpha^i(p) \approx e_\alpha^{i(1)}, \tag{4.4.3}$$

where $e_\alpha^{i(1)}$ $(\tilde{c}_i^{(1)})$ are eigenvectors (eigenvalues) of $\tilde{C}_{\alpha\beta}^{(1)}$, independent of p.

But in this approximation, the description of neutrino oscillations produced by the new physics is completely undistinguishable from the conventional description based on mass differences (Δm_{ij}^2) with a mixing matrix (U_α^i) between flavor and mass eigenstates, just by making the correspondence

$$\Delta m_{ij}^2 = 2\,(\tilde{c}_i^{(1)} - \tilde{c}_j^{(1)})\,\lambda^2, \quad U_\alpha^i = e_\alpha^{i(1)}. \tag{4.4.4}$$

One should note that when $\lambda \neq 0$ the different energies for different states select a basis in the Fock space and the mixing of Fock space operators in the fermionic fields is unavoidable. It is only when one considers the energy splitting of the different particles that one has a physical consequence of the mixing of different creation-annihilation Fock space operators in each fermionic field. On the other hand, in the case $\lambda = 0$ (corresponding to unmodified anticommutation relations) there is an arbitrariness in the construction of the Fock space. One could make use of this arbitrariness to choose a basis such that each fermionic field contains only one annihilation and one creation operator (which is equivalent to saying that the $e_\alpha^i(p)$ are indeed the $\delta_{\alpha i}$) and therefore no oscillation phenomenon is produced.

4.5. Conclusions

We have seen in the previous Section that the observations of neutrino oscillations are compatible with their interpretation as a footprint of new physics in the infrared. As far as we are aware, this is the first time that an interpretation of neutrino oscillations coming from new physics, without the need to introduce neutrino masses, and compatible with all experimental results, is presented.

This is achieved because of the indistinguishability of the new mechanism from the conventional one in the range of momenta $p^2 \gg \lambda^2$. In order to reveal the origin of the oscillations it is necessary to go beyond the approximation Eq. (4.4.2), which requires the exploration of the region of small momenta $(p^2 \approx \lambda^2)$. To get this result it has been crucial to introduce a new infrared scale through a nonlocal modification of the anticommutation relations of the neutrino field. Gauge invariance forbids a similar nonlocal modification for the remaining fields due to the choice of quantum numbers for the fermion fields in the standard model. In fact the possibility to have the modified anticommutators (4.2.5) for the neutrino fields is related to the absence of the right-handed neutrino field.

The model of noncanonical fields presented in this work has to be considered only as an example of the general idea that new infrared physics may be present in, or be (partially) responsible of, neutrino oscillations, and that the conventional interpretation may be incomplete. In fact an extension of relativistic quantum field theory based on the modification of canonical anticommutation relations of fields might not be consistent. We have not examined the associated problems of unitarity or causality beyond the free theory. But it seems plausible that the consequences that we have obtained in the neutrino sector will be valid beyond this specific framework.

In conclusion, in this work we have shown that the experimentally observed properties of neutrino oscillations do not necessarily imply the existence of neutrino masses. In fact, future experiments attempting to determine the neutrino mass, such as KATRIN [155], may offer a window to the identification of new physics beyond relativistic quantum field theory in the IR. At this level it is difficult to predict specific observational effects due to the lack of criteria to select

a choice for $\tilde{C}_{\alpha\beta}(p)$ in this specific model. A simple example, however, would be the presence of negative eigenvalues of this matrix, which could be reflected in an apparent negative mass squared for the neutrino (see Eq. (4.4.3)) in the fits from the tail of the tritium spectrum. Effects on cosmology could also be possible, again depending on the exact modification of the neutrino dispersion relation in the infrared. All we can say is that if the origin of neutrino oscillations is due to new physics in the infrared then experiments trying to determine the absolute values of neutrino masses and/or cosmological observations might have a reflection of the generalized energy-momentum relation Eq. (4.3.4) for neutrinos.

5. The Theory of a Quantum Noncanonical Field in Curved Spacetimes

Much attention has been recently devoted to the possibility that quantum gravity effects could lead to departures from Special Relativity in the form of a deformed Poincarè algebra. These proposals go generically under the name of Doubly or Deformed Special Relativity (DSR). In this article we further explore a recently proposed class of quantum field theories, involving noncanonically commuting complex scalar fields, which have been shown to entail a DSR-like symmetry. An open issue for such theories is whether the DSR-like symmetry has to be taken as a physically relevant symmetry, or if in fact the "true" symmetries of the theory are just rotations and translations while boost invariance has to be considered broken. We analyze here this issue by extending the known results to curved spacetime under both of the previous assumptions. We show that if the symmetry of the free theory is taken to be a DSR-like realization of the Poincaré symmetry, then it is not possible to render such a symmetry a gauge symmetry of the curved physical spacetime. However, it is possible to introduce an auxiliary spacetime which allows to describe the theory as a standard quantum field theory in curved spacetime. Alternatively, taking the point of view that the noncanonical commutation of the fields actually implies a breakdown of boost invariance, the physical spacetime manifold has to be foliated in surfaces of simultaneity and the field theory can be coupled to gravity by making use of the Arnowitt-Deser-Misner prescription.

5.1. Introduction

Quantum gravity has historically suffered from lack of observational support and problems with conceptual issues common to all of the most studied proposals [240, 241, 31, 32, 242, 243] (e.g. the "problem of time" and the "background-independence problem" [244]). However, over the past few years there has been a growing interest in possible low energy, observable, effects of quantum gravity scenarios, ranging from TeV-scale quantum gravity to high energy departures from Local Lorentz invariance of spacetime (see e.g. the discussion in Ref. [26, 156]). In particular, it was recently suggested that the Planck length L_p ($L_p \sim 10^{-33}$cm) should be taken into account in describing the rotation/boost transformations between inertial observers.

In alternative to the standard approach of considering Planck suppressed Lorentz (and possibly CPT) invariance violations in effective field theory (EFT) (see e.g. Ref. [26, 245]), it was conjectured that the relativity principle could be preserved via a deformation of special relativity, in the form of the so-called "Doubly or Deformed special relativity" (DSR) [166, 246, 247]. While this proposal is formulated in momentum space, a coordinate space definitive formulation is still lacking (and several open issues are still unresolved, see e.g. Ref. [248, 249]).

Although some interpretation of DSR in commutative spacetime have been proposed [249, 250, 251], there has been a growing activity [252] toward connecting it with noncommutative spacetime [253, 254, 255], which also arise in 3d-Quantum Gravity [256] and String Theories [189]. However, the connection between quantum gravity and noncommutative spacetime is in principle problematic: in fact the latter seem to entail a "background-independence problem"

given that the commutation relations among spacetime coordinates promote them to the role of operators, despite they should be treated as mere labels in a background independent theory (see e.g. Ref. [257]).

An alternative to modify the commutation relations between coordinates, while preserving background independence, is to modify the commutation relations of field operators on a spacetime manifold. If the manifold is flat spacetime, the Quantum Theory of Noncanonical Fields (QNCFT) has been studied [196, 197][1] and it can be seen as a natural extension to field theory of Non Commutative Quantum Mechanics [193, 194, 195]. Noticeably, it has been shown that there is a link between the one particle sector of QNCFT in flat spacetime and DSR [200] .

Here we are going to use the results of Ref. [200] to study the case of the QNCFT in Curved Spacetimes. In the Section II we will review the main results of Ref. [197]. In the Section III we will discuss the connection between QNCFT and DSR as derived in Ref. [200]. In the Section IV we will derive the Lagrangian, the equation of motion, and the internal product of solutions of the QNCFT in Curved Spacetimes in different cases depending on which are the symmetries of the free theory in flat spacetime that we want to promote to gauge symmetries of the curved spacetime. The Section V is devoted to some closing remarks.

5.2. QNCFT in Flat Spacetime

Let us consider the theory of a complex scalar field, given by the Hamiltonian density:

$$\mathcal{H} = \Pi^\dagger(\mathbf{x})\Pi(\mathbf{x}) + \nabla\Phi^\dagger(\mathbf{x}) \cdot \nabla\Phi(\mathbf{x}) + m^2\Phi^\dagger(\mathbf{x})\Phi(\mathbf{x}), \tag{5.2.1}$$

the Hamiltonian being the integral to spacelike coordinates of $\mathcal{H}$. The field is quantized by imposing the noncanonical commutation relations:

$$\begin{aligned}
\left[\Phi(\mathbf{x}), \Phi^\dagger(\mathbf{x}')\right] &= \theta\delta^{(3)}(\mathbf{x} - \mathbf{x}'), \\
\left[\Phi(\mathbf{x}), \Pi^\dagger(\mathbf{x}')\right] &= i\delta^{(3)}(\mathbf{x} - \mathbf{x}'), \\
\left[\Pi(\mathbf{x}), \Pi^\dagger(\mathbf{x}')\right] &= B\delta^{(3)}(\mathbf{x} - \mathbf{x}'),
\end{aligned} \tag{5.2.2}$$

where θ and B are considered to be "sufficiently small", so that the canonical quantum field theory should be seen as a good approximation to this theory for a certain range of energies or momenta. It should be noticed that $1/\theta$ plays the role of an UV scale and B plays the role of an IR scale [197]. We will forget for the moment about the IR scale and set $B = 0$ in our calculations (the effect of the scale B turns out to be, when $\theta = 0$, a redefinition of the mass m and a constant energy gap between n-particle and (n+1)-particle states) [197]. We shall see now that the just given set of definitions completely specifies the theory.

Let us start by noticing that the momentum operator

$$\mathbf{P} = \int d^3\mathbf{x} \left[\Pi^\dagger(\mathbf{x})\nabla\left(\Phi(\mathbf{x}) - \frac{i\theta}{2}\Pi(\mathbf{x})\right) + h.c.\right], \tag{5.2.3}$$

commutes with the Hamiltonian, and hence the field can be expanded in terms of creation-

---

[1]These references named the theory Quantum Theory of Noncommutative Fields, but Quantum Theory of Noncanonical Fields provides indeed a more precise description.

annihilation operators of particles and antiparticles with a given momentum.

$$\Phi(x) \quad = \quad \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(U_\mathbf{p}(\mathbf{x},t)\, a_\mathbf{p} + V_\mathbf{p}^*(\mathbf{x},t)\, b_\mathbf{p}^\dagger \right) , \tag{5.2.4}$$

$$U_\mathbf{p}(\mathbf{x},t) \quad = \quad \sqrt{\frac{E_a}{E_b\,(E_a + E_b)}}\, e^{-i(E_a t - \mathbf{p}\cdot\mathbf{x})} , \tag{5.2.5}$$

$$V_\mathbf{p}(\mathbf{x},t) \quad = \quad \sqrt{\frac{E_b}{E_a\,(E_a + E_b)}}\, e^{-i(E_b t - \mathbf{p}\cdot\mathbf{x})} . \tag{5.2.6}$$

Equations (5.2.1) and (5.2.2) describe a theory of particles with energy-momentum relation

$$\frac{E_a(\mathbf{p})}{\omega_\mathbf{p}} = \sqrt{1 + \left(\frac{\theta\omega_\mathbf{p}}{2}\right)^2} + \frac{\theta\omega_\mathbf{p}}{2} , \tag{5.2.7}$$

and their antiparticles, with energy-momentum relation

$$\frac{E_b(\mathbf{p})}{\omega_\mathbf{p}} = \sqrt{1 + \left(\frac{\theta\omega_\mathbf{p}}{2}\right)^2} - \frac{\theta\omega_\mathbf{p}}{2} , \tag{5.2.8}$$

where $\omega_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$. The difference of energies between particles and antiparticles with the same momentum is a consequence of the CPT violation of the theory. Notice also that $E_a(-\theta) = E_b(\theta)$, as it should be, due to the symmetry $a \leftrightarrow b$, $\Phi \leftrightarrow \Phi^\dagger$, $\theta \to -\theta$. Therefore $V_\mathbf{p}(\mathbf{x},t;\theta) = U_\mathbf{p}(\mathbf{x},t;-\theta)$.

The equation of motion of the field can be obtained by using the Heisenberg equation for the evolution of operators in the Heisenberg picture. The result is

$$L(x,\partial)\Phi(x) = \left[\partial_0^2 + (m^2 - \Delta)(1 + i\theta\partial_0)\right]\Phi(x) = 0 . \tag{5.2.9}$$

If we define the following internal product in the space of solutions of (5.2.9),

$$(\varphi_1(\theta),\varphi_2(\theta)) \quad = \quad -i \int d^3\mathbf{x}\left(\varphi_1(\theta)\frac{1}{1 - i\theta\partial_0}\partial_0\varphi_2^*(\theta)\right.$$
$$\left. -\varphi_2^*(-\theta)\frac{1}{1 - i\theta\partial_0}\partial_0\varphi_1(-\theta)\right) , \tag{5.2.10}$$

then $U_\mathbf{p}(\mathbf{x},t)$ and $V_\mathbf{p}^*(\mathbf{x},t)$ form an orthonormal (in the sense of Ref. [19]) basis of solutions of the equation of motion.

As a side remark let us notice that equations (5.2.7), (5.2.8), (5.2.9), show for $m = 0$ a striking similarity with the dispersion relation

$$\omega = \pm\sqrt{c^2 k^2 - \left(\frac{2\nu k^2}{3}\right)^2} - i\frac{2\nu k^2}{3} , \tag{5.2.11}$$

and equation of motion

$$\partial_t^2\psi_1 = c^2\nabla^2\psi_1 + \frac{4}{3}\nu\partial_t\nabla^2\psi_1 \tag{5.2.12}$$

of sound waves propagating through a viscous fluid of viscosity ν at rest ($\vec{v}_0 = 0$) [258]. They turn out to be the same, taking into account that the frequency associated to a b-mode of energy E_b is $-E_b$, and redefining $i\theta = \frac{4}{3}\nu$. The meaning of a purely imaginary viscosity is far from clear, although the main consequence is obviously that the dissipative effects associated to viscosity are turned into dispersive ones, the dispersion relation becoming real.

5.3. Symmetries of QNCFT in Flat Spacetime

A symmetry of a theory is defined as a transformation which leaves the action of the theory invariant. A symmetry also has the property of transforming solutions of the equation of motion into solutions of the equations of motion. Thus, if we want to search for the symmetries of the theory, a good starting point would be to find the operators $\mathcal{O}(x, \partial, ...)$ such that if $\Phi(x)$ is a solution of (5.2.9), then also $\mathcal{O}(x, \partial, ...)\Phi(x)$ is. This implies that the action of the commutator of L and $\mathcal{O}$ on solutions of the field equations has to vanish

$$[L(x, \partial, ...), \mathcal{O}(x, \partial, ...)] \, \Phi(x) = 0 \,. \tag{5.3.1}$$

The above program is carried out more easily if one Fourier transforms the field $\Phi(x) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \Phi(p)$. Then, the equation of motion turns out to be [259],

$$L\Phi(p) = \left\{ -p_0^2 + (m^2 + \mathbf{p}^2)(1 + \theta p_0) \right\} \Phi(p) = 0 \,, \tag{5.3.2}$$

and the operators verifying the property (5.3.1) are only

$$\begin{align} \mathcal{P}_i &= p_i \,, \tag{5.3.3} \\ \mathcal{P}_0 &= p_0 \,, \tag{5.3.4} \\ \mathcal{M}_{ij} &= p_i \frac{\partial}{\partial p^j} - p_j \frac{\partial}{\partial p^i} \,, \tag{5.3.5} \\ \mathcal{M}_{0i} &= \frac{p_0}{\sqrt{1 + \theta p_0}} \frac{\partial}{\partial p^i} - p_i \frac{(1 + \theta p_0)^{3/2}}{1 + \theta p_0/2} \frac{\partial}{\partial p^0} \,, \tag{5.3.6} \end{align}$$

or arbitrary functions of them. Therefore the previous operators can be considered the generators of the symmetry group.

It is now evident that while the generators of translations $\mathcal{P}_i, \mathcal{P}_0$ and rotations $\mathcal{M}_{ij}$ are left unchanged, a new set of generators for the boosts $\mathcal{M}_{0i}$ is induced by the noncanonical commutation of the field. Notice however, that if one restricts p_0 so that $p_0 > -1/\theta$, then under the change of variables

$$\tilde{p}_0 = \frac{p_0}{\sqrt{1 + \theta p_0}} \,, \qquad \tilde{p}_i = p_i \,, \tag{5.3.7}$$

the generators of the symmetry group can be rewritten as:

$$\begin{align} \mathcal{P}_i &= \tilde{p}_i \,, \tag{5.3.8} \\ \mathcal{P}_0 &= \tilde{p}_0 \left(\sqrt{1 + \theta^2 \tilde{p}_0^2/4} + \theta \tilde{p}_0/2 \right) \,, \tag{5.3.9} \\ \mathcal{M}_{ij} &= \tilde{p}_i \frac{\partial}{\partial \tilde{p}^j} - \tilde{p}_j \frac{\partial}{\partial \tilde{p}^i} \,, \tag{5.3.10} \\ \mathcal{M}_{0i} &= \tilde{p}_0 \frac{\partial}{\partial \tilde{p}^i} - \tilde{p}_i \frac{\partial}{\partial \tilde{p}^0} \,. \tag{5.3.11} \end{align}$$

This shows that the generators of rotations and deformed boosts are isomorphic to the generators of the Lorentz group and it is only the action of the boosts on the four-momentum that it is deformed due to the presence of θ (reducing to its standard form in the limit of $\theta \to 0$). This is in striking similarity with what is normally conjectured in DSR scenarios [166, 246, 247] [2] and hence it is no surprise that also in this case the whole group of symmetries can be made

---

[2] Note that in the DSR literature as well as in Ref. [200] Greek letters (π_0, π) are used to refer to the auxiliary variables $(\tilde{p}_0, \tilde{p}_i)$. We chose here a different notation in order to render the expressions in the rest of the paper more understandable.

isomorphic to the Poincaré group when $\mathcal{P}_0$ is substituted by $\tilde{\mathcal{P}}_0 = \tilde{p}_0$, which is also the generator of a symmetry. This clearly hints at the possibility to recast the theory in an auxiliary spacetime associated with momenta $(\tilde{p}_i, \tilde{p}_0)$ in which the action of the Poincaré group will be standard. We shall explore this possibility in what follows but before a few remarks about the nature of the just found symmetry are in order.

Symmetries in field theories can be of two types. Internal symmetries are transformations in the field space which do not mix fields at different spacetime points. These transformations will have the form $\Phi(x) \to \Phi'(x)$ with $\Phi'(x) = \mathcal{O}(x)\Phi(x)$. We will call instead external symmetries those symmetries which do mix fields at different spacetime points. For example this can happen if the transformation affects also spacetime itself. Indeed, spacetime symmetries are transformations in the spacetime background which affect the field. These transformations will have the form $x \to x'(x)$ and $\Phi(x) \to \Phi'(x')$ where, in the case of the scalar field, $\Phi'(x') = \Phi(x)$. Therefore spacetime symmetries are external symmetries. However, the opposite is not always true. In particular the symmetry associated to (5.3.3), (5.3.4), (5.3.5), (5.3.6), is not a proper spacetime symmetry because of its non-local nature.

Coming back to our study of the symmetries of the theory in flat spacetime we now want to explore the possibility to recast the whole field theory in a auxiliary spacetime in which the Poincaré group acts in the usual way. We shall do so by looking for transformations which leave the action invariant. While a rigorous treatment can be found in Ref. [200] we shall provide here a more concise derivation for the sake of simplicity.

The first step would be to find the action S, which will be the spacetime integral of some Lagrangian density $\mathcal{L}$. In order to achieve this, we will write (5.2.1) in terms of field variables whose commutation relations are canonical,

$$\Phi_c = \Phi - \frac{i\theta}{2}\Pi \,. \tag{5.3.12}$$

Then, writing the Hamiltonian in these variables and performing the canonical transformation

$$\begin{aligned}
\bar{\Phi} &= \left(1 - \frac{i\theta}{2}\sqrt{m^2 - \Delta}\right)^{-1}\Phi_c \,, \\
\bar{\Pi} &= \left(1 - \frac{i\theta}{2}\sqrt{m^2 - \Delta}\right)\Pi \,,
\end{aligned}$$

the action can be written, up to boundary terms, as

$$\begin{aligned}
S &= \int d^4x \mathcal{L}(x) \\
&= \int d^4x \left(\Pi^\dagger \dot{\Phi}_c + \dot{\Phi}^\dagger{}_c\Pi - \mathcal{H}(\Phi(\Phi_c), \Pi)\right) \\
&= \int d^4x \left(\bar{\Pi}^\dagger \dot{\bar{\Phi}} + \dot{\bar{\Phi}}^\dagger\bar{\Pi} - \mathcal{H}(\Phi(\bar{\Phi}), \Pi(\bar{\Pi}))\right) \\
&= \int d^4x \, \bar{\Phi}^\dagger \left(-\partial_0^2 - (1 + i\theta\partial_0)(m^2 - \Delta)\right)\bar{\Phi} \,, \tag{5.3.13}
\end{aligned}$$

where in the last line we have substituted the momentum as a function of the field and its derivatives via the Hamilton-Jacobi equations.

We want now to show that the action (5.3.13) can be expressed as a standard scalar field action in flat spacetime. We can start by introducing the field redefinition

$$\phi(x) = \sqrt{1 + i\theta\partial_0}\,\bar{\Phi}(x) \,, \tag{5.3.14}$$

so that the action takes the simpler form,

$$S = \int d^4x \, \phi^\dagger(x) \left(\frac{-\partial_0^2}{1 + i\theta\partial_0} - (m^2 - \Delta) \right) \phi(x) \,. \tag{5.3.15}$$

Let us look at this action in momentum space by using the Fourier transform of $\phi(x)$, $\phi(p)$

$$S = \int \frac{d^4p}{(2\pi)^4} \phi^\dagger(p) \left(\frac{p_0^2}{1 + \theta p_0} - (m^2 + \mathbf{p}^2) \right) \phi(p) \,, \tag{5.3.16}$$

where the integral over p_0 goes from $-\infty$ to $+\infty$. Now we can split the action into two terms S^θ and $\bar{S}^\theta$, with the integral over p_0 ranging respectively from $-1/\theta$ to $+\infty$ and from $-\infty$ to $-1/\theta$. All the solutions of the classical equations of motion lay in S^θ, but the $\bar{S}^\theta$ term can affect the quantum behavior of the theory [200]. Changing in S^θ the integration variable p_0 for $\tilde{p}_0$, defined in (5.3.7), this can be rewritten as

$$S^\theta = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{d\tilde{p}_0}{2\pi} \frac{\partial p_0}{\partial \tilde{p}_0} \phi^\dagger(p(\tilde{p})) \left(\tilde{p}_0^2 - (m^2 + \mathbf{p}^2) \right) \phi(p(\tilde{p})) \,. \tag{5.3.17}$$

Finally we can define the auxiliary field

$$\tilde{\phi}(\tilde{p}) = \sqrt{\frac{\partial p_0}{\partial \tilde{p}_0}} \, \phi(p(\tilde{p})) \,, \tag{5.3.18}$$

and inverse Fourier transform, so to arrive to the standard Klein-Gordon-action in some "auxiliary spacetime".

$$S^\theta = \int d^3\mathbf{x} \, d\tilde{x}_0 \, \tilde{\phi}^\dagger(\tilde{x}) \left(-\eta^{\mu\nu} \partial_\mu \partial_\nu - m^2 \right) \tilde{\phi}(\tilde{x}) \,. \tag{5.3.19}$$

Now the properties of the symmetry group of the theory can be understood. When written in the auxiliary variables $(\tilde{x}_0, \mathbf{x})$ (auxiliary frame), the symmetry group is the Poincaré one associated to the flat auxiliary spacetime symmetries of S^θ. Reversely, if we assume that the Poincaré group acts trivially on $\bar{S}^\theta$, its action on the real spacetime and the original field variables (original frame) can be derived by pulling back the operations that we have applied above. In this case the action of the group is found to be realized in some non-local way. Indeed, this setting is similar to the one proposed in the DSR framework [166].[3]

However, it has to be noticed that the necessary splitting of the action into two terms implies that our initial theory is not exactly equivalent to a relativistic field theory in flat space, at the quantum level [200]. As a matter of fact, one might argue that the symmetry we have found is a fake one, because it invokes non-locality (see e.g. the discussion about symmetries of Maxwell equations in [260]).

Indeed spacetime symmetries are local because the transformed field in a spacetime point is a function of the value of the field in just one spacetime point. In the case we are considering, the $\mathcal{M}_{0i}$ are generating a family of external transformations which cannot be interpreted as spacetime symmetries, at least in the usual way, due to their intrinsic non-locality. Indeed such a symmetry transforms a whole field configuration, which is a solution of the equation of motion, into a different field configuration which is also a solution.

Finally, one must also take into account that the Hamiltonian of the theory, the space integral of Eq. (5.2.1), does not belong to the set of generators of this Poincaré-like symmetry group,

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[3]Note that the above treatment does not depend on the detailed form of the field dispersion relation nor on its origin. In principle, the discussion may be applied to more general dispersion relations.

although time translations are a symmetry of the free theory. The problem is that the commutator of the Hamiltonian with the generator of boosts would end up defining a new generator and hence the algebra would not be not closed.

An infinite set of generators of the symmetry group of the free theory in flat spacetime may be obtained by iteration of this procedure. However, the whole symmetry group forbids the addition of new terms in the action or the appearance of a spacetime dependence of the coefficients of the existing terms. Therefore the symmetry group must be reduced (broken into subgroups) in order to extend the theory.

These considerations force us to take a decision. We either try to generalize the theory to curved spacetime by promoting the non-local implementation of the Poincaré group to a gauge symmetry, and therefore time translations are taken as an accidental symmetry of the theory in flat spacetime (or at low energies where θ corrections are supposed to be negligible), or we can take an alternative point of view and consider as the physical symmetries only those realized in a local way while boost invariance is broken.

The first approach has been shown to suffice in the construction of a renormalizable interacting theory in flat spacetime [200], but, as previously discussed, some of the intuitive notions of what a symmetry is can be spoiled by the non-locality of the implementation. While more intuitive, the second approach will imply a preferred time direction and preferred slices of simultaneity in spacetime.

5.4. QNCFT in Curved Spacetimes

The aforementioned alternative points of view regarding the actual symmetries imply that two different approaches in constructing a QNCFT in Curved spacetimes can be taken depending on which is the symmetry of the free field theory in flat spacetime we would like to promote to a gauge symmetry of the spacetime. Hence, we will perform them separately in the following subsections.

5.4.1. A spacetime with a gauge DSR-like Invariance?

One of the ways of formulating General Relativity (GR) consists in turning the global Poincaré symmetry of Minkowski spacetime into a gauge symmetry of any curved geometry.[4] This procedure provides also a link between Quantum Field Theory (QFT) in flat spacetimes and QFT in curved backgrounds. We will try to follow the same steps by gauging the symmetry we have found for the noncanonical field.

A global symmetry in the noncanonical field has been found, whose generators are:

$$
\begin{aligned}
\mathcal{P}_i &= -i\partial_i\,, \\
\tilde{\mathcal{P}}_0 &= i\frac{\partial_0}{\sqrt{1+i\theta\partial_0}}\,, \\
\mathcal{M}_{ij} &= x_i\partial_j - x_j\partial_i\,, \\
\mathcal{M}_{0i} &= x_i\frac{\partial_0}{\sqrt{1+i\theta\partial_0}} - x_0\partial_i\frac{(1+i\theta\partial_0)^{3/2}}{1+i\theta\partial_0/2}\,.
\end{aligned}
\tag{5.4.1}
$$

These operators satisfy the Poincaré Algebra. We want to generalize our considerations about the symmetry to curved spacetime without loosing this connection. We can do this by introducing

---

[4] Indeed the Strong Equivalence Principle (which selects GR among relativistic theories of gravitation with spin-2 gravitons) entails Local Lorentz invariance as a fundamental postulate (together with universality of free fall and local position invariance of experiments, including gravitational ones)[261].

the tetrad field formalism i.e. by constructing a set of normal coordinates y_X^a at each point X of the spacetime manifold (for example for time-like observers one can pick up as a preferred axis the one associated to the tangent vector to the observer worldline and the other three axis as an orthogonal basis in the spacelike hypersurface orthogonal to such vector).

The relationship between these coordinates and the general coordinates in the manifold defines the tetrad field $e_\mu^a(x) = \frac{\partial y_X^a}{\partial x^\mu}|_{x=X}$. In terms of the y_X^a the metric at X is simply η_{ab} (which is used to lower latin indices), and can be transformed to the metric expressed in terms of the general coordinates $g_{\mu\nu}(x)$ (which is used to lower greek indices) with the use of the tetrad.

In the standard case, we must introduce the covariant derivative, an operator which turns global Poincaré and Lorentz invariant tensors into local Poincaré and general coordinate transformations invariant tensor.

$$D_\mu = \partial_\mu - e_\mu^a(x)\mathcal{P}_a - \frac{1}{2}\omega_\mu^{ab}(x)\mathcal{M}_{ab}\,, \qquad (5.4.2)$$

where $e_\mu^a(x)$ and $\omega_\mu^{ab}(x)$ (the spin connection) play the role of gauge fields. This leads from QFT in Flat Spacetime to QFT in Curved Spacetimes when partial derivatives ∂_a are replaced by covariant derivatives $D_a = e_a^\mu D_\mu$ and Lorentz tensors are replaced by generalized tensors built up with the tetrad.

However, if this procedure is applied to the case of the noncanonical field, changing the partial derivatives by covariant derivatives in the action (5.3.13) and inserting the tetrad field does not make the gauge transformation (5.4.1) a gauge symmetry. The connection between the QNCFT and the relativistic QFT (5.3.19) is broken.

In order to see this, let us try to follow the same procedure as in the standard case. The action will have the form

$$
\begin{aligned}
S \;=\; & \int d^4x \sqrt{-g(x)} \Big[\Pi^\dagger e_0^\mu(x)(D_\mu\Phi) + e_0^\mu(x)(D_\mu\Phi^\dagger)\Pi \\
& -\Pi^\dagger\Pi - \delta^{ij} e_i^\mu(x)(D_\mu\Phi^\dagger)e_j^\nu(x)(D_\nu\Phi) - m^2\Phi^\dagger\Phi \\
& -\frac{i\theta}{2}\left(\Pi^\dagger e_0^\mu(x)(D_\mu\Pi) - e_0^\mu(x)(D_\mu\Pi^\dagger)\Pi \right) \Big].
\end{aligned}
\qquad (5.4.3)
$$

This action is explicitly general covariant and in the flat spacetime limit does reproduce the second line of Eq. (5.3.13). However, it is easy to see that there is no transformation enabling one to write (5.4.3) as the curved spacetime generalization of the last line of Eq. (5.3.13) due to the explicit dependence of the tetrad field on the spacetime coordinates. Indeed, the derivation of Eq. (5.3.13), required infinite integrations by parts. If these were done in the curved spacetime case, an infinite series of new terms would appear in the action, involving covariant derivatives of the tetrad fields. Furthermore, it is also possible to check that the infinitesimal gauge transformations generated either by $\tilde{\mathcal{P}}_0$ or by $\mathcal{M}_{0i}$ do not leave the Lagrangian associated with (5.4.3) invariant even at linear order in θ.

These evident problems lead us to the conclusion that there seems to be a fundamental obstruction in gauging external symmetries generated by non-local operators and in particular in making the symmetry generated by (5.4.1) a gauge symmetry of the field action.

However, in the previous section we have shown that the non-locally implemented symmetry can be seen as a proper (i.e. local) spacetime symmetry of a new auxiliary $\tilde{x}$-spacetime. Though non-locality prevents the gauging of symmetries in x-spacetime, the procedure of gauging spacetime symmetries and its results are well known. Therefore we can argue that the proper way to proceed is to gauge global DSR-like symmetries in $\tilde{x}$-spacetime. Of course, the procedure of

$\tilde{x}$-gauging the global Poincaré symmetry leads to the standard QFT in curved spacetime, but now the curved spacetime is the auxiliary one.

In those regions in which the auxiliary spacetime is asymptotically flat, the mapping between auxiliary and noncanonical fields (5.3.18) can be used and the propagator in the noncanonical field, in physical spacetime, can be found. However, a connection between the auxiliary field and the noncanonical field in a point of auxiliary spacetime in which the metric is not asymptotically flat seems precluded, at least in the present treatment.

Let us then reformulate all the theory in the auxiliary frame, in which everything is standard. The part of the action which contains the solutions of the equations of motion in this DSR-like approach would be

$$S^\theta = \int d^4\tilde{x} \sqrt{\tilde{g}(\tilde{x})} \tilde{\phi}(\tilde{x}) \left(-\tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu - m^2 + \xi \tilde{R}(\tilde{x}) \right) \tilde{\phi}(\tilde{x}) , \qquad (5.4.4)$$

where $\tilde{\nabla}$ means covariant derivative, $\tilde{R}(\tilde{x})$ is the Ricci scalar, and the line element in the $\tilde{x}$-spacetime is given by

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} d\tilde{x}_\mu d\tilde{x}_\nu .$$

The theory in the auxiliary variables is ordinary QFT in Curved Spacetime [19]. The equation of motion of the field in auxiliary spacetime is,

$$\left(\tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu + m^2 + \xi \tilde{R}(\tilde{x}) \right) \tilde{\phi}(\tilde{x}) = 0 . \qquad (5.4.5)$$

Its solutions are spanned by a basis $\{ \tilde{u}_{\mathbf{p}}(\tilde{x}), \tilde{u}_{\mathbf{p}}^*(\tilde{x}) \}$ which is orthonormal under the internal product

$$(\varphi_1, \varphi_2) = -i \int_{\tilde{\Sigma}} d\tilde{\Sigma} \sqrt{-\tilde{g}_{\tilde{\Sigma}}(\tilde{x})} \tilde{n}_\mu \left(\varphi_1 \tilde{\partial}_\mu \varphi_2^* - \varphi_2^* \tilde{\partial}_\mu \varphi_1 \right) , \qquad (5.4.6)$$

where $\tilde{\Sigma}$ is some spacelike hypersurface, $\tilde{n}$ the future-oriented timelike vector orthonormal to it. The value of the internal product is independent of the choice of $\tilde{\Sigma}$.

From the above discussion the standard approach to QFT in curved spacetime straightforwardly follows. In particular standard problems like particle creation from the vacuum can be approached with the usual Bogoliubov techniques. Of course, one may wonder how far one could trust such calculations given that the correspondence between the original, physical, spacetime and the auxiliary one (in which one ends up working) is only via the S^θ part of the real action. In this sense it is important to notice that the virtual modes belonging to the $\bar{S}^\theta$ term of the action do not affect the Bogoliuvov transformation between modes in the S^θ term, as these virtual modes "do not see" the auxiliary spacetime.

The conclusion is that working in this framework the resulting theory will give then the same results as the standard QFT in curved spacetime, at least insofar one is working in the auxiliary spacetime. It still remains open the issue of mapping back the results to the original spacetime which, as we said, seems doable only in asymptotically flat regions and hence not generically.

5.4.2. Foliating spacetime with noncanonical commutation relations

Let us adopt the alternative point of view that meaningful symmetries are local implementations of a certain symmetry group, in this case rotations in space and translations in space and time, and that the DSR-like Poincaré group is just an accidental symmetry of the free action in flat spacetime. Therefore, we would like to curve physical spacetime in a way such that just spatial rotations and spacetime translations are preserved. It should be clear that in this case, a

preferred arrow of time emerges in which the fields appearing in the commutation relations are simultaneous. Each set of simultaneous events defines a spacelike hypersurface in the manifold of spacetime, foliating it, while the orthogonal, timelike direction defines the evolution in time. In those hypersurfaces no spacelike coordinates are preferred, and thus the commutation relations of the field in flat spacetime (5.2.2) have to be suitably generalized in order to make them covariant under general coordinate transformations on the spacelike hypersurfaces,

$$
\begin{aligned}
\left[\Phi(\mathbf{x},t),\Phi^\dagger(\mathbf{x}',t)\right] &= \frac{\theta}{\sqrt{h(t)}}\delta^{(3)}(\mathbf{x}-\mathbf{x}') \\
\left[\Phi(\mathbf{x},t),\Pi^\dagger(\mathbf{x}',t)\right] &= \frac{i\hbar}{\sqrt{h(t)}}\delta^{(3)}(\mathbf{x}-\mathbf{x}') \\
\left[\Pi(\mathbf{x},t),\Pi^\dagger(\mathbf{x}',t)\right] &= 0
\end{aligned}
\tag{5.4.7}
$$

where $h(t)$ is the determinant of the three-metric $h_{ij}(t)$ associated to each spacelike hypersurface of simultaneity. This foliation of spacetime into slices of simultaneity naturally leads to the Arnowitt Deser Misner description of GR (ADM) [262, 263]. We briefly review it here for completeness and for fixing the notation.

In the ADM formalism one takes a foliation of spacetime in spacelike hypersurfaces, which implies a split of the whole spacetime metric, the four-metric, into its spacelike-spacelike components $g_{ij}(\mathbf{x},t)$ — which coincide in this frame of reference with the ones of the induced metric on the hypersurfaces (the three-metric h_{ij}) — a three-vector shift function $N^i(\mathbf{x},t)$ and a three-scalar lapse function $N(\mathbf{x},t)$. The components of the four-metric in this frame are then

$$
g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N^2 - N^i N^k h_{ik} & -h_{kj}N^k \\ -h_{ik}N^k & -h_{ij} \end{pmatrix}.
\tag{5.4.8}
$$

It has to be noticed that the formalism is still covariant under general spacelike coordinate transformations. Spacelike indices $i,j,k,\dots$ are lowered with the three-metric h_{ij} and raised with the inverse of the three-metric, h^{ij}. The inverse of the four-metric turns out to be

$$
g^{\mu\nu} = \begin{pmatrix} 1/N^2 & -N^j/N^2 \\ -N^i/N^2 & -h^{ij} + N^i N^j/N^2 \end{pmatrix},
\tag{5.4.9}
$$

and the determinant of the four-metric is $g = -N^2 h$. The timelike vector which is orthonormal to the spacelike hypersurfaces is

$$
n^\mu = (1/N, -N^i/N).
\tag{5.4.10}
$$

The covariant four derivative is constructed in the usual way with the affine connections which are furthermore assumed to be the Christoffel symbols $\Gamma^{(4)\lambda}_{\mu\nu}$ associated with the four-metric $g_{\mu\nu}$. The extrinsic curvature K_{ij} is defined with the covariant four-derivative of the normal to the hypersurfaces ($K_{ij} = N\Gamma^{(4)0}_{ij}$). The covariant three-derivative is defined as the projection of the covariant four-derivative on the spacelike hypersurface, $\nabla^{(3)}_i V^j \equiv V^j_{|i} = \partial_i V^j + \Gamma^{(3)j}_{ik}V^k$, where $\Gamma^{(3)j}_{ik} = \Gamma^{(4)j}_{ik} + K_{ik}N^j/N$ coincides with the Christoffel symbols built up with the three-metric. (From now on the superscript (3) can be dropped in order to simplify the notation.) The intrinsic curvature R^i_{jkl} is built in the usual way with the three-metric h_{ij} and its derivatives, and can be related to the four-curvature and the extrinsic curvature.

Finally, this notation has also a link to the tetrad notation introduced in the previous subsection. The tetrad e^μ_a associated with the metric (5.4.8) is

$$
\begin{aligned}
e^0_0 &= 1/N & e^i_0 &= N^i/N \\
e^0_i &= 0 & e^i_j &= e^{(3)j}_i
\end{aligned},
\tag{5.4.11}
$$

where $e_a^{(3)i}$ is the dreibein associated to the h_{ij} three-metric.

We have now to consider how can we couple the noncanonical field to the physical spacetime metric (5.4.8). The result was already given in the previous subsection in Eq. (5.4.3). Written in the notation of the ADM prescription with the use of (5.4.11), the expression of the action, Lagrangian density and Hamiltonian density of the field are

$$S_\Phi = \int d^4x \sqrt{h} \mathcal{L}_\Phi \, , \tag{5.4.12}$$

$$\mathcal{L}_\Phi = \Pi^\dagger \dot{\Phi}_c + \dot{\Phi}_c^\dagger \Pi - N \mathcal{H}_\Phi \, , \tag{5.4.13}$$

$$\mathcal{H}_\Phi = \Pi^\dagger \Pi + h^{ij} \partial_i \Phi^\dagger \partial_j \Phi + m^2 \Phi^\dagger \Phi$$
$$+ \left(\frac{N^i}{N} \Pi^\dagger \partial_i \Phi_c + h.c. \right) \, . \tag{5.4.14}$$

where Φ_c is given by (5.3.12). The stress energy tensor is defined as

$$T_{\mu\nu} = \frac{2}{N\sqrt{h}} \frac{\delta S_\Phi}{\delta g^{\mu\nu}} \, , \tag{5.4.15}$$

and turns out to be

$$
\begin{aligned}
T_{00} &= N\Pi^\dagger \dot{\Phi}_c + NN^i \Pi^\dagger \partial_i \Phi_c + N^i N^j \partial_i \Phi^\dagger \partial_j \Phi + h.c. \\
&\quad -(N^2 - N_i N^i) \mathcal{L}_\Phi \\
T_{0i} &= N^j \partial_i \Phi^\dagger \partial_j \Phi + N\Pi^\dagger \partial_i \Phi_c + h.c. + N_i \mathcal{L}_\Phi \\
T_{ij} &= \partial_i \Phi^\dagger \partial_j \Phi + h.c. + h_{ij} \mathcal{L}_\phi \, .
\end{aligned} \tag{5.4.16}
$$

Now we have all the information required about the coupling of the noncanonical field to the gravitational potential in curved physical spacetime. In particular, we can derive from the action (5.4.12) the equation of motion of the field. These equations can be rewritten in a more compact way if we write them in terms of the Lie derivative of the fields along the timelike direction (5.4.10),

$$\pounds_n \Phi = \frac{1}{N} \dot{\Phi} - \frac{N^i}{N} \partial_i \Phi \, . \tag{5.4.17}$$

Varying the action with respect to $\Pi^\dagger$ we get

$$N \pounds_n \Phi - i\theta N \pounds_n \Pi - N\Pi - \frac{i\theta}{4} \frac{\dot{h}}{h} \Pi + \frac{i\theta}{2} N_{|i}^i \Pi = 0 \tag{5.4.18}$$

and varying the action with respect to $\Phi^\dagger$,

$$- N \pounds_n \Pi - \frac{\dot{h}}{2h} \Pi - Nm^2 \Phi + (N\partial_i \Phi)^{|i} + N_{|i}^i \Pi = 0 \, . \tag{5.4.19}$$

These equations define a system of coupled differential equations which should be solved provided the metric of spacetime is known and we neglect the back-reaction of the field on the metric. We should remark that the general covariance of the whole spacetime is broken by the preferred choice of a time variable, but the general covariance on the spacelike hypersurfaces is guaranteed by construction. This setting reminds the context of analogue gravity models [264].

The formulation of the quantum theory of a field in curved spacetime requires the definition of a scalar product in the space of solutions of the field equations. We are now able to define an inner product in the space of solutions. The inner product essentially carries the information of

the commutation relations. Given two solutions, $\Phi = \varphi_{1,\theta}$ and $\Phi = \varphi_{2,\theta}$, of the equations of the field (5.4.18), (5.4.19) we can define the internal product $(,)$ as

$$(\varphi_1, \varphi_2) = -i \int d^3\mathbf{x}\sqrt{h} \left[\varphi_{1,\theta}\varphi_{2,\theta}^{\Pi\,*} - \varphi_{2,-\theta}^*\varphi_{1,-\theta}^{\Pi} \right] , \tag{5.4.20}$$

where the integral is evaluated at any of the spacelike hypersurfaces in which spacetime becomes foliated and

$$\varphi_{r,\theta}^{\Pi} = \left[1 + \frac{i\theta}{N} \left(N\mathcal{L}_n + \frac{\dot{h}}{4h} + \frac{N_{|i}^i}{2} \right) \right]^{-1} \mathcal{L}_n \varphi_r(\theta) , \tag{5.4.21}$$

is the conjugate momentum solution $\Pi = \varphi_{r,\theta}^{\Pi}$ associated to the field solution $\Phi = \varphi_{r,\theta}$ with $r = 1, 2$. This internal product preserves the symplectic structure of the theory and is independent of the choice of the hypersurface by construction [265].

Let $\{\Phi = U_\mathbf{p}(\mathbf{x}, t; \theta)\}$ be an orthonormal basis of solutions of the system of differential equations (5.4.18), (5.4.19) with positive frequency with respect to the preferred time direction (5.4.10), labeled by the index $\mathbf{p}$. Let $\{\Phi = V_\mathbf{p}^*\}$ be an orthonormal basis of solutions of negative frequency of the same system of differential equations. Then the field can be expanded in terms of annihilation operators of the $U_\mathbf{p}(\mathbf{x}, t; \theta)$ and creation operators of the $V_\mathbf{p}^*(\mathbf{x}, t; \theta)$

$$\Phi(\mathbf{x}, t) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[U_\mathbf{p}(\mathbf{x}, t; \theta)\, a_\mathbf{p} + V_\mathbf{p}(\mathbf{x}, t; \theta)^*\, b_\mathbf{p}^\dagger \right] . \tag{5.4.22}$$

If $V_\mathbf{p}^*$ is a solution of negative frequency of the system of field equations (5.4.18), (5.4.19), then $V_\mathbf{p}$ is a solution of positive frequency of the complex conjugate of the system, which coincides with the result of changing $\theta \to -\theta$ in the system. Therefore $V_\mathbf{p}(\mathbf{x}, t; \theta) \propto U_\mathbf{p}(\mathbf{x}, t; -\theta)$. A more careful analysis of the commutation relations shows that $V_\mathbf{p}(\mathbf{x}, t; \theta) = U_\mathbf{p}(\mathbf{x}, t; -\theta)$. This is due to the symmetry $\theta \to -\theta$, $a \leftrightarrow b$, $\Phi \leftrightarrow \Phi^\dagger$, $\Pi \leftrightarrow \Pi^\dagger$.

As usual, there is non-uniqueness in the choice of an orthonormal basis $\{U_\mathbf{p}, V_\mathbf{p}^*\}$ of solutions of the system of field equations if the metric induces a loss of the time translation symmetry. We must then resort to Bogoliubov techniques in order to relate the associated non equivalent vacua. Let us expand the field in terms of two different basis of solutions of the equation of motion

$$\begin{aligned} \Phi(x) &= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[U_\mathbf{p}(x)\, a_\mathbf{p} + V_\mathbf{p}^*(x)\, b_\mathbf{p}^\dagger \right] \\ &= \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \left[\bar{U}_{\mathbf{p}'}(x)\bar{a}_{\mathbf{p}'} + \bar{V}_{\mathbf{p}'}^*(x)\bar{b}_{\mathbf{p}'}^\dagger \right] . \end{aligned} \tag{5.4.23}$$

As both sets of solutions of the equation of motion are basis, we can define the Bogoliubov transformation as a change of basis in the space of solutions of the equation of motion,

$$U_\mathbf{p} = \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \left[\bar{U}_{\mathbf{p}'}(x)\alpha_{\mathbf{p}'\mathbf{p}} + \bar{V}_{\mathbf{p}'}^*(x)\beta_{\mathbf{p}'\mathbf{p}} \right] , \tag{5.4.24}$$

where

$$\alpha_{\mathbf{p}'\mathbf{p}} = \left(U_\mathbf{p}, \bar{U}_{\mathbf{p}'} \right) , \qquad \beta_{\mathbf{p}'\mathbf{p}} = -\left(U_\mathbf{p}, \bar{V}_{\mathbf{p}'}^* \right) . \tag{5.4.25}$$

We define a state $|0\rangle$ as the vacuum in the $\{U_\mathbf{p}, V_\mathbf{p}^*\}$ basis. We want to compute the expectation value in this state of the number operators $\bar{N}_{a,\mathbf{p}'}$, $\bar{N}_{b,\mathbf{p}'}$ defined in the $\{\bar{U}_{\mathbf{p}'}, \bar{V}_{\mathbf{p}'}^*\}$ basis. Inserting

(5.4.24) in (5.4.23) in order to get the Bogoliubov transform of the creation-annihilation operators and plugging them in the expression of the number operators, the result will be

$$\langle 0|\bar{N}_{a,\mathbf{p}'}|0\rangle \;\; = \;\; \int d^3\mathbf{p}\,|\beta_{\mathbf{p}'\mathbf{p}}(-\theta)|^2\,, \tag{5.4.26}$$

$$\langle 0|\bar{N}_{b,\mathbf{p}'}|0\rangle \;\; = \;\; \int d^3\mathbf{p}\,|\beta_{\mathbf{p}'\mathbf{p}}(\theta)|^2\,. \tag{5.4.27}$$

Noticeably, the internal product (5.4.20) has two terms which differ not only in sign but also in magnitude, unlike in the standard QFT in curved spacetime or the QNCFT in Curved auxiliary Spacetime. This two terms will in general be complex and oscillating with different phases. Hence, when computing the number of particles and antiparticles observed by a detector in the vacuum of some other detector (like for example in the case of Rindler observers in flat spacetime or static observers at infinity in a black hole spacetime), one then finds that the corresponding spectrum follows the pattern of interference of the terms in the Bogoliubov coefficient β. This result is essential for example in the studying black hole evaporation by Hawking radiation, something which we plan to address elsewhere.

5.5. conclusions

Recent works [200] have offered a mechanism to identify new symmetries in the theory of QNCFT [197]. This mechanism goes beyond this particular example and can be applied in principle to more general theories with modified dispersion relations. The new symmetry transformations will be non-local, but the action can be rewritten in terms of an auxiliary spacetime in which the symmetry transformations are local and the action is Poincaré invariant. There is a link between the conjugate four-momenta in physical and auxiliary spacetimes, which reminds the DSR paradigm [247]. As a result a non-local implementation of the Poincaré group is found to be a symmetry group of the theory in flat spacetime. However, the generators of the symmetries of this group do not form a closed algebra with the generator of the symmetry of translations in time. When the theory is tried to be extended, it is unclear which of the symmetries are to be kept and which will be treated as accidental. Some works [260] do not give much credit to non-locally implemented symmetries. On the other hand, in the absence of the Poincaré symmetry, the possibility of constructing a perturbatively renormalizable interacting theory of the noncanonical field is also unclear [200].

Two different paths have been followed in order to derive a QNCFT in Curved spacetimes depending on the symmetries that are required to be gauged: either the DSR-like symmetries of the free theory, though some of its elements are implemented non-locally, or the group of symmetries of the free theory that are locally implemented, i.e. translations and rotations.

We find that curving the physical spacetime breaks the DSR-like symmetry. This maybe due to an obstruction in gauging symmetries generated by non-local operators. Consequently, if one wants to preserve the DSR-like implementation of the Poincaré group, one must take the auxiliary spacetime as the one to be curved. This leads to a relativistic QFT in Curved Auxiliary Spacetime, which is very similar to the standard one. The correspondence between the fields in the auxiliary frame and fields in the physical spacetime can be trivially done in regions in which the auxiliary spacetime is asymptotically flat. Strictly speaking, the two theories are not really equivalent due to the presence of $\bar{S}^\theta$ part of the physical action. The role of the $\bar{S}^\theta$ modes, which cannot be mapped to auxiliary spacetime, is to fix time ordering in the original frame but they do not seem to have any observable influence on the auxiliary frame, for example they do not affect the Bogoliubov transformations.

On the other hand, if only the symmetries that are implemented locally are required to characterize spacetime at short scales, the physical spacetime of the QNCFT can be made curve using the ADM prescription. The commutation relations must be adapted to fit in a general covariant frame. The action of the field coupled to the three-metric, lapse and shift functions can be written unambiguously. General coordinate invariance is broken by the commutation relations into invariance under general coordinate transformations on the slices of simultaneity, and reparameterizations of time. The resulting QNCFT in Curved Physical Spacetime seems to have the same structure as the ordinary one, but new phenomena related to the UV scale may appear. The main difference between this theory and the standard QFT in Curved spacetimes is that, due of the different energies of particles and antiparticles, the two terms appearing in the internal product (5.4.20) become different not only in sign, but also in magnitude.

We conclude that the theories introduced offer clearly distinguishable outcomes for related phenomenology. This will be further studied in future work.

6. Summary and Conclusions

The General Theory of Relativity (GR) and the Standard Model of particle physics (SM) have offered a very accurate description of all known phenomena with two remarkable exceptions, the neutrino oscillations and the accelerated expansion of the universe in the early universe and the present age. The neutrino oscillations and the present acceleration of the universe accept a quite conventional explanation in terms of neutrino masses and a cosmological constant/vacuum energy, respectively. However, these explanations are not by themselves very satisfactory because:

- they introduce an enormous naturalness problem,

- they introduce a new ingredient in the theory that is hard or impossible to measure aside from in the phenomena that caused their proposal,

- they are unable to explain the striking coincidence in scale between the effective energy density attached to the cosmological constant ($\rho_{vac} \sim (10^{-3}\,eV)^4$) and the lowest neutrino mass difference ($\Delta m^2 \sim (10^{-3}\,ev)^2$),

- the present experimental and observational data are still unable to rule out some of the alternative proposals to these explanations.

Despite these questions that have been raised, the neutrino masses and the cosmological constant have been taken as a confirmed fact by many because they are the simplest explanation in agreement with the experimental and observational data, and because they do not compromise any of the basic principles underlying GR or the SM.

However nothing is sacred in science, aside from the proof of experiment. Simplicity is also important, but aesthetics cannot be used as a substitute of experimental verification of a prediction. The physical principles that are taken as basic must be rejected in case they conflict a single experiment or observation, and thus must always be subject of scrutiny.

Therefore we would like to build models that are alternatives to neutrino masses and the cosmological constant, which are consistent with all the known phenomena, and that could be contrasted with the more conventional explanations in the light of future experiments and surveys. We will not fear to violate the basic principles underlying GR and the SM, as long as the departures from them is small enough to be consistent with the available data.

For instance, neutrino masses are the only explanation to neutrino oscillations in vacuum within Special Relativity (SR), and thus we have considered a small Lorentz Invariance Violation (LIV) in neutrinos. On the other side, the too high quantum corrections to a vacuum energy are computed in the framework of Effective Field Theory (EFT), which is based in the Decoupling Theorem, which in turn is based in locality, so we have considered a small departure from locality.

Not always it is required to violate the underlying principles in a theory in order to go beyond it, as is the case of GR. We have considered more general actions than the Einstein-Hilbert action in the search for new physics that can explain the accelerated expansion.

In both cases, the departure from the standard framework has been characterized by energy scales which determine the onset of new physics. These scales can be either ultraviolet (UV) or infrared (IR), depending on the requirements of the phenomena we want to explore. Neutrino

oscillations as well as the present accelerated expansion of the universe require the introduction of a new IR scale, whereas the explanation of the horizon and flatness problems requires the introduction of a UV scale. It is possible that the two IR scales are the same, as it is suggested by the coincidence mentioned above, but we have not been able to explain this coincidence within a certain consistent model.

Instead of resorting to a top-down approach like String Theories (ST), Loop Quantum Gravity (LQG) or Grand Unified Theories (GUT), which have grown far from the contact with experiment, we have prefer to adopt a bottom-up approach in which we have started from simple models that offer a description of the data and may grow in the future if they are not ruled out by future data first.

In order to explain the two epochs of accelerated expansion of the universe, we have introduced a phenomenological extension of the Cosmological Standard Model (CSM) based on GR. The model is similar to other phenomenological descriptions that have been proposed, which relate the expansion of the universe with more "anthropocentric" parameters such as the redshift z. The Asymptotic Cosmological Model (ACM) is based (as GR) in a metric theory of spacetime in the homogeneous approximation, and it is defined through the relation between the Hubble rate H and the energy density of the matter content of the universe defined by the SM particles and the Dark Matter. The effect of new physics is encoded in the dependence of these relation on an UV and an IR scales which act as bounds of the open interval to which the Hubble rate is constrained. For dimensional arguments, in the limit in which the Hubble rate is far from these bounds, the standard behavior predicted by GR is recovered, whereas the universe tends asymptotically to a deSitter universe in the infinite past and the infinite future when the bounds are approached. The CSM supplied with a cosmological constant (ΛCDM) is included as a particular case.

The ACM is built with four free parameters, two of them characterizing the early accelerated expansion and two of them characterizing the present accelerated expansion. We have constrained the two of them relevant for the present accelerated expansion, contrasting the results derived from simulations using Monte Carlo-Markov Chains with the observational data from Type Ia Supernovae (SNe), the distance to the recombination surface deduced from the Cosmic Microwave Background (CMB), direct measurements of the energy density and the distance parameter of Baryon Acoustic Oscillations (BAO). We have found that there are better fits to the expansion of the universe than ΛCDM, but that the quality of the fit depends too crucially on the dataset used. Thus an improvement on the datasets is important in order to favor ΛCDM or one of its competitors.

The homogeneous behavior of the ACM can be realized in the framework of modified gravity (for instance $f(R)$-theories), scalar-tensor theories or phenomenological Dark Energy models indistinguishably. Differences between these descriptions may be found in the behavior of perturbations.

The lack of an action for the ACM is a serious obstacle in building a theory of perturbations in the ACM. However, under an additional assumption based on phenomenological consistency (the differential equations being of order two), the homogeneous behavior of the ACM determines univocally the linearized behavior of scalar perturbations.

With the assumption of a Harrison-Zel'dovich scale invariant spectrum from the very early universe, this linear treatment suffices to derive the CMB spectrum deduced from the ACM. The comparison between this prediction and the observed spectrum imposes similar bounds on parameter space than the homogeneous analysis. This is due to the fact that the Integrated Sachs-Wolfe effect (ISW) of the ACM for the parameters lying within the confidence regions of our homogeneous analysis is masked by the cosmic variance, i.e.: the intrinsic statistical error associated to the lowest multipoles (cosmic censorship).

The treatment of linear perturbations is able to distinguish the ACM from the most naïve Dark Energy forms, such as scalar-tensor theories or f(R)-theories. These examples usually present fine-tuning problems such as the requirement of a chameleon mechanism for a fifth long-range interaction to disappear.

In the future we plan to explore further the consequences of this model, such as the nonlinear behavior of perturbations, the behavior of vector and tensor modes, or the weak gravity limit.

It is often stated that the neutrino oscillation in vacuum experiments prove that neutrinos are massive particles. We have shown that this statement is false. Neutrino oscillations can be also explained in terms of oscillations between particles with different dispersion relations, wether this difference relies on different masses or not.

Neutrinos with different constant velocities or dispersion relations modified by a UV scale are certainly ruled out, with the exception of a single very fine-tuned model. However, neutrino oscillation experiments only probe the high energy (few MeV to few GeV) regime of neutrinos, and therefore *any* modification of the dispersion relation in the IR, such that the high energy effect of the modification amounts to a term proportional to the square of the IR scale, can naturally explain neutrino oscillations in vacuum.

It is difficult to find such a modification of the dispersion relation different from a mass term in QFT. We have found a model in which this is possible also respecting the gauge symmetry and lepton number conservation of the standard model. However, we have had to sacrifice not only Lorentz invariance but also locality. We plan to study a modification of this model allowing for lepton number violation in the future.

In order to distinguish between massive neutrinos or other IR modification of the standard model, it will be necessary to probe the low energy regime (few meV to few eV) of the neutrinos. The Cosmic Neutrino Background (CνB) predicted by cosmology offers the possibility of measuring such neutrinos in the future.

However, in order to study the effect of a Lorentz non-invariant theory in cosmology it is important to build a quantum field theory in curved spacetime which preserves the symmetry properties of such a theory. Although this is a very complicated task, outside the scope of this thesis, as a first step we have explored the possibility of building the scalar Quantum Theory of Noncanonical Fields (QNCFT) in a curved spacetime.

Respecting either the non-locally implemented Poincaré group of symmetries or the locally implemented Euclidean group of symmetries of the theory in flat spacetime leads to two different ways of defining the theory in curved spacetime.

If one chooses to gauge the non-locally implemented Poincaré group of symmetries by the standard procedure, an obstruction is found. The solution is to introduce curvature in the auxiliary spacetime in which the Poincaré symmetry can be interpreted as a spacetime symmetry, instead of curving the physical spacetime. The resulting theory is nonlocal in time. In the limit in which the parameter θ tends to zero both spacetimes coincide and one recovers the standard QFT in curved spacetime. The consequences of this theory will be similar to the ones of standard QFT in curved spacetime but the particles will have a modified dispersion relation.

Alternatively, one may curve the physical spacetime, but then the non-locally implemented symmetry of the theory in flat spacetime is broken. In the resulting theory a preferred reference frame arises, and spacetime is foliated in spacelike simultaneity hypersurfaces. New phenomena related to the UV scale θ may arise, which were not present in the standard QFT in curved spacetime.

We plan to study how the Hawking radiation is affected by the noncanonical commutation relation in both theories. We would also like to go further in determining what are the symmetries of in the model of noncanonical neutrinos and how they can be coupled to gravity.

With a naïve assumption of about how the modified dispersion relation of the noncanonical neutrinos can be introduced in cosmology, we have computed the effect of a IR modified dispersion which mimics a cosmological constant in cosmology. Using the bounds on 'squared neutrino mass differences' and on the 'spatial curvature', the resulting "effective cosmological constant" turns out to be three orders of magnitude above the observed value.

If there exist a connection between the IR scale pointed out by neutrino oscillations and by the present acceleration of the universe, much work is still to be done to uncover this relation. We hope that this PhD thesis will serve as inspiration for future research in this direction.

We will now proceed to enunciate the results of this thesis

- We have built a cosmological model (ACM) defined by the relation between the energy density and the Hubble parameter in the homogeneous approximation. The model contains the ΛCDM as a particular case. This model suffices to explain the horizon problem and the present accelerated expansion. Through a simulation, the late time behavior of the model has been constrained. Although a cosmological constant is permitted at 1σ, the observations favor other regions of the parameter space. However, the parameter determination depends strongly on the dataset which is being used.

- Assuming order two differential equations for the perturbations, we have found that it is possible to find a unique set of equations for the scalar perturbations over a FRW universe whose homogeneous behavior departs from GR. In contrast to common belief, this is doable without resorting to higher order equations or new degrees of freedom, at least at the linearized level. The model is distinguishable from other proposals such as $f(R)$-gravity, scalar tensor theories or the most naïve dark fluid models.

- We have used the equations for the scalar perturbations associated to the ACM to derive the spectrum of the CMB in a universe following an expansion governed by the ACM. The fit to observations does not add new information with respect to the homogeneous fits, due to a masking of the late ISW effect by the cosmic variance at low multipoles.

- It is often stated that neutrino oscillation experiments imply that neutrinos have mass. We have found a counter example to this statement. We have built a model in which the effect of new physics governed by an IR scale is able to explain all the neutrino oscillation experiments. Thus neutrino oscillations imply that neutrinos are sensitive to new physics in the IR. The most promising observation that would be able to distinguish between massive and noncanonical neutrinos is the cosmological effect of neutrinos, and in particular the CνB.

- As a first step towards determining the cosmological effect of noncanonical neutrinos, we have built two possible quantum theories of scalar noncanonical fields in curved spacetimes, depending on which symmetry of the theory in flat spacetime is broken when curving spacetime. In one of them there is no preferred reference frame and particles with modified dispersion relations are allowed, but it has an unclear interpretation. The other involves a preferred time direction and new effects may arise, but the renormalizability of the QFT is not guaranteed.

- Our work has tried to explore two phenomena which seem to point to new physics in the IR: neutrino oscillations and the present cosmic acceleration. If these phenomena are related, there is still much work to do to uncover this relation.

A. Implications of noncanonical neutrinos on cosmology

Violating Lorentz invariance in QFT through a deformation of the anticommutation rules of quantum fields yields the possibility of the neutrinos having a IR modified dispersion relation such that mimics the role of the cosmological constant in the present cosmic acceleration. This possibility is further studied.

A.1. Motivation

Our last work on LIV in neutrino physics caused by a deformation of the anticommutation rules of the fermion fields opens the possibility of the neutrinos having a dispersion relation very different from the relativistic one. Experimental data from high energy physics have shown that the neutrino dispersion relation differs from that of a relativistic massless particles at low energies. The scale of energy of that deviation is $\lambda < 1eV$ and the differences between the dispersion relation of the different neutrino species can be parameterized up to first order corrections as

$$E_i(p) = p + \frac{m_i^2}{2p} + \mathcal{O}(\frac{\lambda^3}{p^2}), \tag{A.1.1}$$

where we suppose that $m_i \sim \lambda$, with $|\Delta m_{12}^2| = 7.9^{+0.27}_{-0.28} \times 10^{-5}(eV)^2$ and $|\Delta m_{13}^2| = 2.6 \pm 0.2 \times 10^{-3}(eV)^2$ as shown by neutrino oscillation experiments.

However, a modification in the dispersion relation of the neutrinos should have a visible impact on cosmology, due to the effective temperature of the cosmic neutrino background being at least one or two orders of magnitude below the scale λ ($T_\nu = 1.95\,K = 1.68 \times 10^{-4}\,eV$). Moreover cosmological observations seem to imply the existence of a cosmological constant with energy density

$$\rho_\Lambda = \rho_c \Omega_\Lambda = \frac{3H_0 \Omega_\Lambda}{8\pi G} = \frac{3\Lambda}{8\pi G} = 3.0 \times 10^{-11}\,(eV)^4. \tag{A.1.2}$$

The similarity between the energy scale of the cosmological constant and the energy scale of neutrino masses seem to suggest that both phenomena are connected. Might neutrinos be the Dark Energy?

Let us check the order of magnitude of a cosmological constant-like contribution to the energy density of the universe. Neutrinos would mimic a cosmological constant if their dispersion relation satisfies $E(p_d \frac{a_d}{a})n_\nu = const$, where n_ν is the number density of neutrinos ($n_\nu \propto a^{-3}$, $n_{\nu,now} \sim n_{\gamma,now} \sim 10^{-12}\,(eV)^3$), p_d is the momentum of a neutrino at neutrino decoupling ($T_d \sim 1.5\,MeV$) and a_d is the scale factor at neutrino decoupling. Therefore, the contribution of neutrinos to the energy density will be constant if they have a dispersion relation $E(p) \sim \frac{\lambda^4}{p^3}$ for $p \ll \lambda$. If we assume that the average momentum of the neutrinos at neutrino decoupling is similar to the decoupling temperature and that the number of photons and neutrinos is more or less conserved during the expansion of the universe after they decouple, we get, for $\lambda^2 \sim \Delta m_{12}^2$:

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$$\Omega_{\nu,now} \approx 0.68\Omega_\gamma \frac{E_\nu(T_\nu)}{T_\nu} \sim 0.68\Omega_\gamma \frac{\lambda^4}{T_\nu^4} \sim 0.68 \times 10^{-5} \frac{(10^{-5}\,(eV)^2)^2}{(10^{-4}\,eV)^4} \sim 1\,. \tag{A.1.3}$$

It is known that $\Omega_\Lambda = 0.73$ in ΛCDM and the contribution of this neutrinos seems to be of the same order, so maybe we can explain what is the Dark Energy by considering some specific neutrino dispersion relation.

We will now see how a polynomial dispersion relation in the ratio λ/p is able to reproduce the effect of a cosmological constant. However, a detailed computation will show that the resulting effective cosmological constant is too high.

A.2. A polynomial dispersion relation

We will consider a universe composed by the following:

- **Photons**: Their contribution to the total energy density today is $\Omega_\gamma = 4.92 \times 10^{-5}$. They form a Plankian spectrum with temperature $T_\gamma = 2.73\,K = 2.35 \times 10^{-4}\,eV$, which implies an energy denisity of $\rho_\gamma = 4.64 \times 10^{-34}\,g/cm^3 = 2.01 \times 10^{-15}\,(eV)^4$.

- **Baryons**: By baryons it is meant ordinary mater: nuclei and electrons (the contribution of electrons to the energy density is negligible compared to that of nulei). $\Omega_b = 0.0415 \pm 0.0016$.

- **Dark Matter**: It doesn't interact with photons, it is neither baryonic. $\Omega_{DM} = 0.236^{+0.016}_{-0.024}$.

- **Primordial Neutrinos**: Consider 3 families of neutrinos whose abundance would be $\rho_{\nu_i} \sim \frac{0.68}{3}\rho_\gamma$ if they were massless relativistic particles. They form a Plankian distribution with temperature $T_\nu = 1.95\,K = 1.68 \times 10^{-4}\,eV$. In this section we will consider that their energy dispersion relation is

$$E_{\nu_i}(p) = p\left(1 + \frac{c_i}{p^2}\right)^2\,. \tag{A.2.1}$$

We can obtain more information about the coeficient c_i from neutrino oscillation experiments: $|c_2 - c_1| \equiv \frac{\Delta m_{12}^2}{4} = (1.98 \pm 0.07) \times 10^{-5}(eV)^2$, $|c_3 - c_1| \equiv \frac{\Delta m_{13}^2}{4} = (6.5 \pm 0.5) \times 10^{-4}(eV)^2$.

A.3. Equal momenta approximation

As a first approximation we will consider that today the momentum of every neutrino is equal to the temperature T_ν. Taking into account the dispersion relation (A.2.1), the contribution from each of the neutrino species to the total energy density will be:

$$\Omega_{\nu_i} = \frac{\rho_{\nu_i}}{\rho_c} \sim \frac{0.68}{3}\Omega_\gamma\left(1 + \frac{c_i}{T_\nu^2}\right)^2 \sim 1.12 \times 10^{-5} + 793\frac{c_i}{(1eV)^2} + 1.40 \times 10^{10}\frac{c_i^2}{(1eV)^4}\,. \tag{A.3.1}$$

The first term equals the contribution of massless relativistic neutrinos to the energy density today. The second term plays the role of an effective spatial curvature contribution to the energy density $\equiv -\frac{k}{(aH_0)^2}$. The coefficients c_i must be chosen to make this contribution compatible with the upper bounds on a spatial curvature term from WMAP. The last term mimics the contribution of a cosmological constant to the total energy density.

It is easy to see that the condition for $|c_2 - c_1|$ is easy to satisfy consistently with all the experimental data. However, the condition for $|c_3 - c_1|$ forces at least one of the c_i to be of order $10^{-4}(eV)^2$, and this causes the last term in (A.3.1) to be too large compared to the experimental data for Ω_Λ.

A particular example is

$$c_1 = -2.23 \times 10^{-4}(eV)^2\,,$$
$$c_2 = -2.03 \times 10^{-4}(eV)^2\,,$$
$$c_3 = +4.27 \times 10^{-4}(eV)^2\,,$$
$$\Rightarrow \Omega_\Lambda \sim 3800 \gg 0.73\,.$$

A.4. Taking into account the Planckian spectrum

It might be possible that this disease is cured by taking into account that the momenta of the neutrinos are distributed forming a Planckian spectrum. We must have into account that there are two degrees of freedom (ν and $\bar{\nu}$) per neutrino species. The distribution in thermal equilibrium at temperature T is then

$$\frac{dn_i(p)}{dp} = \frac{1}{\pi^2} \frac{p^2}{e^{(E_i(p) - \mu_i)/T} + 1}\,. \tag{A.4.1}$$

The neutrinos were in thermal equilibrium before they decoupled. Then they keep their distribution with $p = p_d \frac{a_d}{a}$. The chemical potential can be neglected since

$$\frac{\mu_i}{T} = \frac{n_{\nu_i} - n_{\bar{\nu}_i}}{n_\gamma} \equiv L_i < 10^{-9}\,. \tag{A.4.2}$$

At energies close to decoupling ($T_d \sim 1.5\,MeV$) we can use the approximation $E_d \approx p_d$. Therefore, at neutrino decoupling we have:

$$n_i \approx \frac{1}{\pi^2} \int dp\, p^2 (e^{p/T} + 1)^{-1} = \frac{3T^3 \zeta(3)}{2\pi^2} \approx 0.1827 \times T^3\,, \tag{A.4.3}$$

$$\langle p \rangle_i \approx \frac{1}{n_i} \frac{1}{\pi^2} \int dp\, p^3 (e^{p/T} + 1)^{-1} = \frac{7\pi^2 T^4}{120 n_i} \approx 3.1514 \times T\,, \tag{A.4.4}$$

$$\left\langle \frac{1}{p} \right\rangle_i \approx \frac{1}{n_i} \frac{1}{\pi^2} \int dp\, p (e^{p/T} + 1)^{-1} = \frac{T^2}{12 n_i} \approx \frac{0.4561}{T}\,. \tag{A.4.5}$$

$\langle p^{-3} \rangle_i$ diverges due to the contribution of small momenta to the integral, unless we take into account that $E(p) = p \left(1 + \frac{c_i}{p^2}\right)^2$. This integral cannot be solved analytically, so we will solve it numerically using the values for the c_i in the previous example:

$$\begin{aligned}
\langle p^{-3} \rangle_i &= \frac{1}{n_i} \frac{1}{\pi^2} \int \frac{dp}{p} (e^{p\left(1 + \frac{c_i}{p^2}\right)^2/T_d} + 1)^{-1} \\
&\approx \begin{cases}
\frac{1.23609}{n_i} \approx 6.76 T_d^{-3} & \text{for } c_i = -2.23 \times 10^{-4}\,(eV)^2\,, \\
\frac{1.23926}{n_i} \approx 6.78 T_d^{-3} & \text{for } c_i = -2.03 \times 10^{-4}\,(eV)^2\,, \\
\frac{1.21415}{n_i} \approx 6.64 T_d^{-3} & \text{for } c_i = 4.27 \times 10^{-4}\,(eV)^2\,.
\end{cases}
\end{aligned} \tag{A.4.6}$$

The coefficients coming out of this strict derivation are of order one, which means the dispersion relation (A.2.1) gives a too large constant contribution to the total energy density. In fact,

$$\Omega_\nu \equiv \sum_i \Omega_{\nu_i}^{(\gamma)} + \Omega_{\nu_i}^{(k)} + \Omega_{\nu_i}^{(\Lambda)} \,, \tag{A.4.7}$$

where $\sum_i \Omega_{\nu_i}^{(\gamma)} = 3.37 \times 10^{-5}$, $\sum_i \Omega_{\nu_i}^{(k)} < 0.02$, $\sum_i \Omega_{\nu_i}^{(\Lambda)} \sim 8100$. This means that the simplest energy-momentum dispersion relation which we have thought is ruled out by the cosmological data.

A.5. Discussion

Although it is possible to find *ad hoc* dispersion relations for the neutrinos that mimic the effect of the Dark Energy and are consistent with the cosmological data, the simplest examples are ruled out. This indicates that maybe our assumption that neutrinos were driving the present accelerated expansion was too ambitious.

Thus cosmology imposes great constraints in the low energy behavior of the dispersion relation of neutrinos. Any nontrivial behavior should be non-polynomial in λ/p in order not to be ruled out by cosmological observations. Otherwise it would enter in conflict with the cosmological bounds on Hot Dark Matter (HDM)[1], spatial curvature, or a cosmological constant, and we have proven that this bounds are tight.

The effect of neutrinos with a modified dispersion relation in the IR will be further studied in future work. In particular, the effect of a modified dispersion relation in the distance to neutrino decoupling surface or the matter power spectrum will be addressed elsewhere.

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[1] If the neutrino dispersion relation mimics that of a massive particle in the IR, then one should obviously choose the simplest explanation and work with massive neutrinos.

B. Introducción

A comienzos del siglo XXI, la Teoría General de la Relatividad (GR) y el Modelo Estándar de física de partículas (SM) parecen ofrecer una descripción muy precisa de todos los fenómenos físicos conocidos... ¿De todos ellos? En realidad no, puesto que las oscilaciones de neutrinos y la cosmología todavía se resisten a un entendimiento completo.

B.1. El Universo se acelera

Desde el descubrimiento de GR por Albert Einstein en 1915, ha habido un gran interés en aplicar sus ecuaciones para determinar el comportamiento del universo como un todo. Einstein fue el primero en abrir al caja de Pandora en 1917, presentando un modelo de un universo estático que requería la presencia de una 'constante cosmológica' en las ecuaciones del campo gravitatorio. Friedman encontró en 1922 que las ecuaciones admitían soluciones en forma de universo en expansión sin la presencia de este término. La expansión del universo fue también estudiada por Lemaître, Robertson y Walker, y confirmada siete años después por Hubble. Por tanto Einstein rechazó la constante cosmológica, refiriéndose a ella como su 'peor error'.

Considérense las ecuaciones de Einstein de GR

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}\,, \tag{B.1.1}$$

donde $R_{\mu\nu}$ es el tensor de curvatura de Ricci, $g_{\mu\nu}$ es la métrica del espacio-tiempo, $R = R_{\mu\nu}g^{\mu\nu}$ es la curvatura escalar y $T_{\mu\nu}$ es el tensor de energía-impulso correspondiente al fluido que llena el espaciotiempo. Un universo homogéneo e isótropo viene descrito por la métrica de Friedman-Robertson-Walker (FRW)

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega_3^2 \right]\,, \tag{B.1.2}$$

donde $\{t, r, \theta, \phi\}$ son las coordenadas comóviles, $a(t)$ es el factor de escala, k es la curvatura espacial, y Ω_3 es el ángulo sólido del espacio tridimensional $d\Omega_3^2 = d\theta^2 + sin^2\theta d\phi^2$. El valor actual del factor de escala $a_0 = a(t_0)$ se normaliza habitualmente a uno.

Según GR, esta métrica evoluciona de acuerdo con la relación (conocida como ecuación de Friedman)

$$\frac{8\pi G}{3}\rho = H^2 + \frac{k}{a^2}\,, \tag{B.1.3}$$

donde ρ es la densidad de energía total y H es el parámetro de Hubble, la velocidad de expansion del universo ($H = \frac{da}{adt}$). Más aún, la densidad de partículas masivas (materia) o sin masa (radiación) decrece con el factor de escala como $\rho_m \sim a^{-3}$ ó $\rho_r \sim a^{-4}$, respectivamente. Estos ingredientes, junto con las condiciones de un universo inicialmente dominado por radiación que se expande adiabáticamente, constituyen el 'Modelo Cosmológico Estándar' (CSM, llamado así por Guth [1], por analogía con el SM). Las soluciones de la ecuación diferencial resultante para el factor de escala son siempre deceleradas ($\ddot{a} < 0$).

El CSM ha logrado un cierto número de éxitos, incluyendo la predicción correcta de la cantidad relativa de nucleos más pesados que el protio (Nucleosíntesis) o la predicción del Fondo Cósmico de Microondas (CMB), la radiación de cuerpo negro más perfecta medida hasta la fecha. Desde las primeras observaciones por Hubble, ha sido sin embargo muy difícil medir directamente el efecto preciso de la expansión del universo en el movimiento de las galaxias.

Esta dificultad está conectada con la dificultad de encontrar 'candelas estándar', objetos astronómicos con luminosidad conocida L (energía radiada por unidad de tiempo). Dada la luminosidad de un objeto y su escpectro, es posible medir directamente su desplazamiento al rojo ($z = \frac{a_0}{a} - 1$) y su flujo bolométrico F (la energía detectada por unidad de área y tiempo), donde la relación entre luminosidad y flujo bolométrico está reflejada en la distancia luminosa d_l al objeto astronómico, definida como $d_l \equiv \left(\frac{L}{4\pi F}\right)^{1/2}$. La relación entre la distancia luminosa y el desplazamiento al rojo (que está en correspondencia uno a uno con el tiempo) permite determinar la evolución del factor de escala y el parámetro de Hubble con el tiempo,[1]

$$d_l(z) = c(1+z) \int_0^z \frac{dz'}{H(z')}\,, \tag{B.1.4}$$

y por lo tanto da información sobre la importancia relativa de la radiación, la materia, o la curvatura espacial en la evolución del universo. Como se ha dicho, para cualquier universo sujeto a una combinación de los anteriores (pertículas masivas o sin masa, contenidas en el SM o no, más la curvatura espacial permitida en la métrica FRW) experimentará una expansión decelerada, ya que la gravedad actúa como una fuerza atractiva.

Sin embargo, medidas de la relación entre la distancia luminosa y el desplazamiento al rojo de Supernovas de tipo Ia (SNe) lejanas (consideradas ampliamente como candelas estándar) revelan que el universo está actualmente experimentando una expansión acelerada [5, 6], ¡en contradicción con la expansión decelerada predicha por GR!

B.1.1. Una explicación convencional: ¡qué vuelva la constante cosmológica!

Hoy en día no es un secreto que GR admite una constante cosmológica Λ, un término constante en el Lagrangiano de Einstein-Hilbert

$$S = \int d^4x \sqrt{-g}[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}_m]\,, \tag{B.1.5}$$

donde g es el determinante de la métrica y $\mathcal{L}_m$ es el Lagrangiano del contenido de materia del espacio-tiempo, del que se deriva el tensor de energía-impulso. La constante cosmológica Λ es capaz de producir la deseada expansión acelerada, como se explica bajo estas líneas. La ecuación de Friedman se modifica de la siguiente manera,

$$\frac{8\pi G}{3}\rho = H^2 + \frac{k}{a^2} - \frac{\Lambda}{3}\,. \tag{B.1.6}$$

Dado que el universo se expande, a tiempos tardíos el efecto de la densidad de energía ρ y la curvatura k se diluye y el universo se expandirá casi exponencialmente con

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[1]Esta relación se calcula en el marco de espacio-tiempos métricos con una métrica FRW. Para objetos de suficiente tamaño, es posible comparar esta cantidad con su distancia de tamaño angular $d_A \equiv l/\Delta\theta$, donde l es la dimensión transversal del objeto y $\Delta\theta$ es el su dimensión angular (el ángulo que subtiende en el cielo). En un espacio-tiempo métrico, $d_l = d_A(1+z)^2$. Esta relación puede usarse para examinar la validez de las teorías métricas como una buena descripción del espacio-tiempo [2, 3]. Los datos experimentales parecen corroborar esta relación [4]. Por lo tanto no abandonaremos el marco de teorías covariantes generales en espaciotiempos métricos.

$$H = \sqrt{\frac{\Lambda}{3}}\,. \tag{B.1.7}$$

Se llega así al Modelo de Consenso [2] (ΛCDM), en que el universo es homogéneo e isótropo a escalas cosmológicas, con una curvatura espacial k y una constante cosmológica Λ, y está lleno de una mezcla de radiación (sobre todo fotones y neutrinos), partículas masivas del SM (sobre todo bariones, ya que los electrones tienen una masa mucho menor) y un nuevo tipo de partículas masivas no bariónicas y sin carga eléctrica que llamaremos Materia Oscura (DM). Si se divide (B.1.6) por H^2 y se reagrupan los términos se llega a la ecuación de equilibrio cósmico,

$$\Omega_r + \Omega_b + \Omega_{DM} + \Omega_k + \Omega_\Lambda = 1\,, \tag{B.1.8}$$

donde $\Omega_i = \rho_i/\rho_c$ para $i = r, b, DM$, $\rho_c = \frac{3H^2}{8\pi G}$ es la densidad crítica, $\Omega_k = -\frac{k}{a^2 H^2}$ y $\Omega_\Lambda = \frac{\Lambda}{3H^2}$. Las Ω representan la importancia relativa de cada componente en el universo a un tiempo dado, y esta importancia relativa puede variar con el tiempo.

Esta simple explicación se ajusta cómodamente a los datos reunidos por los investigadores de SNe, desde los primeros hasta los últimos conjuntos de datos [5, 6, 7, 8, 9], que comprenden unos pocos cientos de SNe. También proporciona una buena explicación del espectro de las anisotropías del CMB medido por varios grupos [10, 11, 12, 13] y del espectro de anisotropía de la materia [14, 15, 16, 17] si el universo se considera espacialmente plano, $k = 0$ [18]. En particular, el ajuste del modelo a los datos conjuntos de todas estas observaciones da, actualmente,

$$\begin{aligned} H_0 &= 72 \pm 3\,km/sMpc\,, & \Omega_r &= (4{,}8 \pm 0{,}5) \times 10^{-5}\,, \\ \Omega_b &= 0{,}044 \pm 0{,}004\,, & \Omega_{DM} &= 0{,}22 \pm 0{,}2\,, \\ -0{,}018 &< \Omega_k < 0{,}008\,, & \Omega_\Lambda &= 0{,}726 \pm 0{,}015\,. \end{aligned}$$

Muchos de los errores en estas cantidades están dominados por el error en la determinación de H_0. Si este es el final de la historia, ¿qué es exactamente lo que no se entiende de la aceleración cósmica?

B.1.2. Los problemas de ΛCDM

El Problema de la Constante Cosmológica   Aunque un valor pequeño de la constante cosmológica es compatible con las observaciones, la teoría predice que tiene un valor grande. No existiría ningún problema de tratarse de una teoría clásica, pero la teoría que describe la fuente de la interacción gravitatoria $\langle T_{\mu\nu} \rangle$ es una Teoría Cuántica de Campos (QFT) en espacio-tiempo curvo. Por tanto, las ecuaciones de Einstein semiclásicas válidas para la cosmología a curvaturas por debajo de la escala de Planck ($E_P = 1{,}2209 \times 10^{19}\,GeV \ / \ l_P = 1{,}6165 \times 10^{-35}\,m$)[3] son

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R + \Lambda) = 8\pi G \langle \hat{T}_{\mu\nu} \rangle\,, \tag{B.1.9}$$

donde los términos de curvatura son tratados clásicamente y $\langle \hat{T}_{\mu\nu} \rangle$ es el valor esperado del tensor de energía-impulso calculado en una QFT en espacio-tiempo curvo [19]. Para renormalizar las contribuciones del Lagrangiano del sistema material al total de la acción, es necesario añadir contratérminos también en la parte gravitatoria de la acción que cancelen algunos de los infinitos que surgen en el Lagrangiano cuántico del sistema material. Algunos de estos contratérminos son

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[2] *Concordance Model* en el original. (N. del T.)
[3] A estas escalas la gravedad no puede tratarse comi una teoría clásica, tal y como se explicará más adelante.

de la misma forma que los que aparecen en el Lagrangiano clásico (R, Λ) y otros presentan más de dos derivadas de la métrica.

El término de interés es el término divergente que tiene la misma forma que la constante cosmológica Λ. Si se calcula el valor de esta contribución en el marco de Teorías de Campos Efectivas (EFT) [20], estamos suponiendo que el SM es válido hasta una escala de altas energías (a la que llamaremos escala ultravioleta (UV)), pongamos orden de la escala de Planck. Entonces esta contribución es $\sim 10^{71} GeV^4$, es decir, 120 órdenes de magnitud mayor del valor observado, $\sim 10^{-47} GeV^4$ [21]. ¡Para explicar los datos, las correcciones cuánticas $8\pi G \langle \hat{T}^{vac}_{\mu\nu} \rangle$ deberían cancelar las primeras 120 cifras significativas de la contribución clásica a Λ, pero no las siguientes! Esta cancelación tan poco natural se conoce como el problema de la constante cosmológica.

Las soluciones al problema de la constante cosmológica propuestas por la comunidad de gravedad cuántica no son muy convincentes. En Teorías de Cuerdas (ST) el valor natural de la constante cosmológica es negativo y del orden de la escala de Planck E_p^4. Esto ha llevado a la comunidad de ST a recurrir a argumentos antrópicos para poder salvar los muebles (ver Ref. [23] para argumentos y referencias). En Teoría Cuántica de Bucles se ha encontrado que una constante cosmológica pequeña es posible en una solución llamada estado de Kodama [24]. Sin embargo, la relación de dispersión de partículas en dicho estado sufre una modificación a bajas energías de la forma

$$E^2 = \mathbf{p}^2 + m^2 + \alpha \frac{E^3}{E_p} \,, \tag{B.1.10}$$

que ya ha sido casi refutada por las observaciones [25, 26, 27].

El problema es que las correcciones cuánticas que tienen su origen en el comportamiento UV de la QFT contribuyen a una cantidad de GR que de otro modo sería infraroja (IR), es decir, de energía menor que las energías típicas de física de partículas. ¿Existe la posibilidad de que las correcciones cuánticas procedentes de QFT no estropeen el carácter IR de la constante cosmológica? EFT se basa en unos principios básicos, y el responsable de que las correcciones cuánticas a la energía del vacío sean UV es el teorema del desacoplo [22]. Cualquier mecanismo capaz de impedir que las correcciones cuánticas sean UV debe violar esta propiedad. Entonces sería posible tener una QFT válida en una ventana de energías acotada por una escala IR y otra escala UV, y en la que las correcciones principales a las ecuaciones de Einstein provengan del comportamiento de la teoría cerca de la escal IR.

El Problema de Horizonte La porción homogénea e isótropa de universo que actualmente observamos es al menos tan grande como la escala del horizonte $ct_0 \sim c/H_0$, donde H_0 es el valor actual del parámetro de Hubble. Esta porción ha crecido con el factor de escala. Como la inhomogeneidad no puede borrarse en una expansión decelerada, podemos concluir que el tamaño de la porción de universo de la que se originó todo aquello que vemos era en $t = t_i$ al menos tan grande como $l_i \sim ct_0 a_i / a_0$. Si se compara esta cantidad con el horizonte causal al mismo tiempo, $l_c = ct_i$, entonces la razón de las dos escalas es,

$$\frac{l_i}{l_c} \sim \frac{t_0}{t_i} \frac{a_i}{a_0} \,. \tag{B.1.11}$$

Por tanto, si suponemos condiciones iniciales aleatorias en t_i, que suponemos de orden Planckiano porque consideramos que GR es válida hasta esa escala, y como $a \sim 1/T$ cuando domima radiación, obtenemos

$$\frac{l_i}{l_c} \sim \frac{10^{17}s}{10^{-43}s} \frac{2{,}73K}{10^{32}K} \sim 10^{28} \,. \tag{B.1.12}$$

Por tanto la porción no estaba causalmente conectada, ¡así que no tenía por qué haber sido homogénea! Este es un gran problema, ya que las condiciones iniciales deberían de ajustarse muy finamente para poder reproducir el universo que observamos. Éste es el problema del horizonte.

A no ser que haya algo que se nos escapa. Supongamos que el factor de escala crece como una potencia del tiempo, y por tanto $\dot{a} \sim a/t$. Se puede reescribir

$$\frac{l_i}{l_c} \sim \frac{\dot{a}_i}{\dot{a}_0} \, . \tag{B.1.13}$$

De la última expresion se infiere que el tamaño del universo era inicialmente más grande que un dominio causal por una razón correspondiente al cociente de las distintas velocidades de expansión. Sólo si éstas son del mimso orden han podido las condiciones iniciales aleatorias en t_i termalizar y dar lugar a un universo homogéneo. Y como la gravedad ha estado decelerando la expansión por un largo periodo de tiempo, debe de haber habido un periodo de expansión acelerada previo a la expansión decelerada subsiguiente que ha podido explicar tanta física. Esto apunta a la aparición de nueva física de partículas o gravitatoria en el UV, pero por debajo de la escala de Planck.

El Problema de Planitud   Las observaciones recientes nos dicen que el término de curvatura en la ecuación de Friedman es muy próximo a cero. En particular, si se escribe la ecuación de equilibrio cósmico (B.1.8) y se desprecia Ω_Λ, el parámetro cosmológico $\Omega(t)$ cumple

$$\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{k}{(Ha)^2} \, . \tag{B.1.14}$$

Por tanto, a tiempos primitivos

$$\Omega(t_i) - 1 = (\Omega_0 - 1)\frac{(Ha)_0^2}{(Ha)_i^2} = (\Omega_0 - 1)\frac{(\dot{a})_0^2}{(\dot{a})_i^2} < 10^{-56} \, . \tag{B.1.15}$$

Teniendo en cuenta que esta diferencia puede interpretarse como la diferencia relativa entre la energía potencial y cinética en el fluido cósmico [28], ¡esto indica que las velocidades iniciales deben ajustarse muy finamente con una precisión de 10^{-28} para poder explicar la actual planitud del universo! Éste es el problema de planitud.

Este ajuste fino no sería tan extremo si la velocidad de expansión a un tiempo Planckiano fuese menor que o al menos tan pequeño como la velocidad actual de expansión, lo que a cambio requeriría un periodo de expansión acelerada en el universo primitivo, tal y como hemos sugerido en el apartado anterior.

B.1.3. Ingredientes para una alternativa a ΛCDM

Los problemas de ΛCDM tienen su origen en nuestro intento de extender el rango de validez de nuestras actuales teorías de la gravedad y la física de partículas más allá de la región en que dichas teorías han sido comprobadas experimentalmente. No tenemos ni idea cuál será la teoría de gravedad a curvaturas extremadamente grandes (o pequeñas) comparadas con aquellas que hemos medido; incluso tenemos indicios de que una teoría de gravedad cuántica debería violar algunos de los principios en los que nuestras actuales teorías se sustentan [29].

Llegados a este punto se hace evidente que debe encontrarse una generalización de GR, QFT o ambas para poder explicar los problemas de cosmología a tiempos muy primitivos o recientes. Estas generalizaciones se pueden hallar bien mediante un método deductivo o inductivo. Siguiendo un método deductivo, los principios fundamentales de una teoría se modifican, se formula una

nueva teoría y se derivan sus consecuencias fenomenológicas. Ejemplos de métodos deductivos para resolver los problemas de cosmología son ST [30](y su rama aplicada de Cosmología de Cuerdas) o LQG [31, 32] (de la que se deduce la Cosmología Cuántica de Bucles). Las teorías mencionadas han partido de una sólida base teórica y matemática pero han crecido al margen de los resultados experimentales, y algunas de sus implicaciones en cosmología están en contra con los datos observacionales. Mostraremos algunos ejemplos más adelante.

Por contra, el método inductivo se basa en los datos experimentales y observacionales, que se usan para construir leyes y modelos que pueden más tarde incorporarse a una teoría bien fundamentada. En esta tesis doctoral trataremos de mostrar cómo es posible construir una ampliación fenomenológica de ΛCDM al nivel de las ecuaciones de evolución cosmológicas. Esta ampliación, a la que hemos llamado Modelo Cosmológico Asintótico (ACM), incorpora una modificación de las leyes de la expansión cósmica tal que una época de expansión acelerada (asintóticamente exponencial) surge tanto en el IR (correspondiendo con la expansión acelerada actual) como en el UV (correspondiendo con la expansión acelerada primitiva). El modelo ajusta con éxito los datos observacionales disponibles. El estudio del origen de esta modificación se reserva para trabajos posteriores, aunque algunas posibilidades se señalarán más adelante.

Nueva física en el IR   Hemos mencionado que para poder resolver el problema de la constante cosmológica, se necesita convertir las correcciones UV a la energía del vacío en IR. Esto no es posible en el marco de una EFT local, debido al teorema del desacoplo. De hecho, éste es un problema no sólo de ΛCDM, sino de GR acoplada a EFT en general. Incluso si no existen contribuciones clásicas a la constante cosmológica/energía de vacío en el Lagrangiano gravitatorio, la renormalización del Lagrangiano del sistema material inducirá en general un término distinto de cero. [4] El problema está en que el contaje de potencias en una EFT local implca que las correcciones cuánticas a este término van a ser mucho más grandes que el valor observado. Por tanto una solución al problema de la constante cosmológica requiere un abandono de la localidad en QFT, que está parametrizado por una nueva escala IR.

Se ha reconocido que una EFT local no debería ofrecer una buena descripción de la naturaleza en presencia de fenómenos gravitatorios [33]. En cualquier EFT con una cota superior UV E_{NP}, la entropía del sistema es una cantidad extensiva y por tanto crece con el volumen, $S \sim L^3 E_{NP}^3$. Por contra la física de agujeros negros parece indicar que la máxima entropía contenida en un cierto volumen de tamaño L debe ser propocional a su área, $S < \pi E_p^2 L^2$. Esto quiere decir que la entropía predicha por EFT puede exceder el límite de Bekenstein para regiones suficientemente grandes. Por tanto se ha argumentado que EFT de alguna manera tiene en cuenta demasiados grados de libertad [34, 35], y que el número de grados de libertad en un sistema dado debe saturar desde un comportamiento extensivo (propio de EFT) a un valor proporcional al área, cuando el tamaño de la región sea del orden de $L \sim E_p^2 / E_{NP}^3$. Los autores de Ref. [33] proponen un rango de validez aún más estricto, basándose en la premisa de que el tamaño de la región considerada no debería ser inferior al radio de Schwarschild asociado a su temperatura $T \sim E_{NP}$ para que EFT sea válida. Si EFT se usa para describir partículas hasta una temperatura T, entonces la energía térmica asociada al sistema será del orden de $\sim L^3 E_{NP}^4$ y la entropía asociada será del orden de $S \sim L^3 E_{NP}^3$. Entonces el radio de Schwarzschild será $r_s \sim U/E_p^2$. Si queremos que el sistema que describimos no forme un agujero negro, entonces necesitamos que el tamaño del sistema sea mayor que esta cantidad, $L > r_s$, y por tanto el momento de las partículas que estamos describiendo presenta una cota inferior IR del orden de $\lambda \sim 1/L$, donde

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[4]Con la notable excepción de las teorías supersimétricas en las que la supersimetría no está espontáneamente rota. Sin embargo, la ausencia de supersimetría exacta en la naturaleza descarta esta explicación.

$$\lambda \sim E_{NP}^2/E_p \,. \tag{B.1.16}$$

Esto generaría, además de correcciones $\propto \frac{1}{E_{NP}^n}$ a la acción efectiva a nivel arból, también correcciones $\propto \lambda^2$. Como consecuencia, a energías $\sim p$ las predicciones de EFT para una magnitud dada Q tendrán correcciones tanto de términos irrelevantes en la acción de dimensión D como de física gravitacional, de la forma

$$\frac{\delta Q}{Q} \sim \left[\left(\frac{p^{D-4}}{E_{NP}^{D-4}} \right) + \left(\frac{E_{NP}^4}{p^2 E_p^2} \right) \right] \,. \tag{B.1.17}$$

La máxima precisión que se alcanzará para algún valor E_{NP} de la máxima energía que puede experimentarse con dicha EFT. Como ejemplo dado en Ref [33], si los momentos de las partículas son del orden de la escala electrodébil (como en los experimentos de aceleradores, tipo LHC), y la EFT es el SM, nueva física aparece en operadores de dimensión 6 (5 si se permite violación del número leptónico), la mínima incertidumbre de 10^{-13} se alcanza cuando la escala máxima a la que se puede usar la EFT es $10^8\,GeV$. Entonces la cota mínima que aparece es del orden del MeV. Sin embargo, si la nueva física aparece a $10\,Tev$, entonces el SM sería válido desde energías del orden de $10^{-2}\,eV$, en torno a la escala de energía detrás de la física de neutrinos y la constante cosmológica.

En el mismo contexto, si se trata de calcular las correcciones cuánticas del contenido material del universo a la constante cosmológica en el marco de EFT, se debe notar que para que esta estrategia sea válida en una porción de universo del orden del horizonte causal $\sim 1/H_0$, debería introducirse una cota IR del orden de $H_0 \sim 10^{-26}\,eV$ y por tanto la cota UV asociada a la EFT sería $10^{-2}\,eV$. Como resultado, las correcciones cuánticas deberían ser del mismo orden que el valor observado de la energía del vacío.

Se dice que cualquier acercamiento desde EFT a una teoría que incluya gravedad estará ligado a un rango de validez acotado por una escala IR y una escala UV [36]. Sin embargo, si la escala UV se toma del orden de la escala de Planck, entonces la escala IR también coincidirá con la escala de Planck y dicho dominio de validez se convertirá en un punto. Puede ocurrir que la escala fundamental de gravedad en una EFT disminuya debido a la presencia de nuevas dimensiones extensas [37, 38, 39, 40]. Se ha propuesto que la escala efectiva de gravedad sea del orden de magnitud de unos pocos TeV, y que la escala IR (correspondiente a las nuevas dimensiones de un tamaño del orden del milímetro) sea del orden de magnitud de la energía del vacío o las masas de neutrinos.

En cualquier caso, si la EFT local debe modificarse en el IR para poder resolver el problema de la constante cosmológica, también es posible que las correcciones cuánticas inducidas en el Lagrangiano gravitatorio añadan no sólo términos con un número mayor de derivadas de la métrica suprimidas por una escala UV, como en el caso estándar, sino también nuevos términos no polinómicos o no locales suprimidos por una escala IR.

De hecho, no hay motivo para creer que la acción de la teoría clásica de la gravedad viene dada sólo por un término proporcional al escalar de curvatura y un término constante, como en ΛCDM. Con las premisas de una teoría métrica con covariancia general y el principio de equivalencia, es posible construir una plétora de teorías. Einstein decidió escoger la más simple de ellas por motivos estéticos: requirió que sus ecuaciones fuesen de segundo orden, y esto sólo es posible con un Lagrangiano de la forma $\sqrt{-g}(R - 2\Lambda)$. Sin embargo en física teórica moderna hemos aprendido que todo lo que no está prohibido es obligatorio [41]. No está prohibido por ninguna simetría añadir términos adicionales en la acción siempre y cuando se comporten como escalares bajo transformaciones generales de coordenadas.

Términos con un número mayor de derivadas pueden despreciarse por análisis dimensional, ya que deben suplirse con una cierta escala. Si esta escala es una escala UV como la escala de Planck, entonces el efecto de estos términos puede despreciarse en muchos tests, con la notable excepción del universo primitivo. Términos de este tipo surgen de manera natural cuando se trata de renormalizar una teoría cuántica en espaciotiempo curvo [19]. En este contexto, incluso si originalmente no están en la acción gravitatoria clásica, términos con la forma siguiente aparecen en la acción:

$$S_g^{UV} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[-2\Lambda + R + \frac{1}{E_{UV}^2} \left(\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \delta \Box R \right) \right] ,$$
$$(\text{B.1.18})$$

y no hay razón para creer que términos con derivadas aún mayores no aparecerán en la acción gravitatoria, suprimidos por potencias aún mayores del inverso de la escala UV, E_{UV}.

Sin embargo, si estos términos están suprimidos por una escala IR, como la constante cosmológica, el efecto debería ser observable a distancias cosmológicas. Además de una constante cosmológica, términos que incluyan una escala IR deberían ser al menos no polinómicos o no locales,

$$S_g = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2\Lambda + \alpha E_{IR}^2 \Box^{-1} R + \beta \frac{E_{IR}^4}{R} + ... \right] , \qquad (\text{B.1.19})$$

donde el efecto de estos términos puede despreciarse ingenuamente en algunos tests de gravedad pero influyen fuertemente en la evolución cósmica en la actualidad (dado que $\Lambda \sim E_{IR}^2$).

Nueva física en el UV   Hemos señalado también que, para poder resolver los problemas del horizonte y planitud, algún tipo de nueva física en el UV es necesario. Al expandirse, el universo también se enfría de un estado "inicial" a temperatura Planckiana $\sim T_p = k_B^{-1} E_p \sim 10^{32} K$, cuya descripción requiere de algún conocimiento acerca de la teoría correcta de gravedad cuántica. Al carecer de cualquier conocimiento sobre esta teoría, lo mejor que se puede hacer es suponer que la gravedad cuántica proporciona unas condiciones iniciales que podemos considerar aleatorias y que, desde un tiempo inicial $t_i \gtrsim t_P$, pueden tratarse con gravedad clásica corregida por una serie de términos suprimidos por una escala UV. Esta nueva física podría quizá inducir una expansión acelerada justo después del régimen en el que la gravedad debe describirse mediante una teoría cuántica.

Muchos modelos se han construido para poder explicar esta fase de expansión acelerada. Los primeros modelos intentaban asociar la expansión a un acoplo entre la expansión y una transición de fase en el contexto de Teorías de Gran Unificación (GUT) [42, 1, 43, 44]. Después se descubrió que los detalles de la física de partículas que están detrás de la expansión acelerada no eran importantes [45]. La inclusión de términos con derivadas adicionales suprimidos por una escala UV [46] también se ha usado para explicar esta expansión acelerada (llamada 'inflación' [1]). Como una consecuencia adicional, las fluctuaciones cuánticas de los campos en este régimen son suficientes para provocar las fluctuaciones el especrtro de CMB, y por lo tanto los cúmulos, galaxias, estrellas, planetas y la vida. Como se a dicho, los detalles del mecanismo no son importantes, ya que modelos *a priori* muy diferentes llevan a conclusiones muy similares.

Como ejemplo de nueva física en el UV que provoca una expansión acelerada primitiva, introduzcamos el modelo simple de la Ref. [46]. Supongamos que la acción de Einstein-Hilbert es simplemente una aproximación a una teoría más general,

$$S = \int d^4x \sqrt{-g} \left[R + \Lambda - \alpha \frac{R^2}{E_p^2} + ... \right] . \tag{B.1.20}$$

Se ha demostrado [19] que dichos términos aparecen en la acción renormalizada de GR clásica acoplada a una QFT en espacio-tiempo curvo. Si se considera una teoría modificada tan sólo por el término R^2, resulta ser conformemente equivalente (con un factor conforme $F = 1 + 2\alpha R/E_p^2$) a una teoría de GR acoplada a un campo escalar φ,

$$S = \int d^4x \sqrt{-g} \left[R + \Lambda + 16\pi G(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{E_p^2}{12\alpha}\varphi^2) \right] , \tag{B.1.21}$$

con α unos órdenes de magnitud por encima de la unidad. El potencial del campo escalar puede verse como un término de masa efectiva con $m^2 = \frac{1}{6\alpha}E_p^2$. Un estudio detallado del sistema dinámico demostrará que este modelo es un ejemplo de inflación caótica [45], en el que unas condiciones iniciales casi aleatorias llevan a una expansión exponencial y a un espectro en potencias casi plano para las fluctuaciones primordiales en la métrica cel espacio-tiempo.

Supóngase que la condición inicial para el campo escalar es al menos localmente un condensado homogéneo. Las ecuaciones del movimiento en la aproximación homogénea (métrica FRW y $\varphi \approx \varphi(t)$) pueden escribirse como:

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0 , \tag{B.1.22}$$

$$\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2 \right) = H^2 , \tag{B.1.23}$$

donde se desprecia el efecto de la curvatura espacial ($H^2 \gg k/a^2$) por simplicidad. El efecto de la curvatura espacial puede estudiarse numéricamente. El sistema puede reducirse a un sistema dinámico de primer orden para $\dot{\varphi}(\varphi)$,

$$\frac{d\dot{\varphi}}{d\varphi} = -\sqrt{-12\pi G(\dot{\varphi}^2 + m^2\varphi^2)} - m^2\frac{\varphi}{\dot{\varphi}} \tag{B.1.24}$$

Debería notarse que este sistema dinámico posee un atractor al cual convergen todas las soluciones con el tiempo. El atractor se comporta como $\dot{\varphi}^2 = \frac{m^2}{12\pi G}$ y $sign(\dot{\varphi}) \neq sign(\varphi)$ para $\dot{\varphi}^2 \ll m^2\varphi^2$ y después forma una doble órbita que decae en torno al origen. Si una solución encuentra al atractor en una de sus ramas, en este régimen el campo varía lentamente con el tiempo y el universo se expande casi de manera exponencial, con

$$H^2 \approx \frac{8\pi G}{3}\left(\frac{1}{2}m^2\varphi^2 \right) \sim const . \tag{B.1.25}$$

hasta que $\varphi \sim \frac{1}{\sqrt{12\pi G}}$. Cuando el valor del campo alcanza la escala de Planck, su velocidad es alta y el campo decae rápidamente a $\varphi = 0$, produciendo una salida del periodo inflacionario y transfiriendo la energía del campo escalar al sistema material del modelo (recalentamiento). Como la teoría original está relacionada con la teoría escalar tensor por un factor conforme que depende sólo de R, que es aproximadamente constante durante el periodo inflacionariom, en la teoría original también existe el mismo periodo inflacionario.

Se puede complicar arbitrariamente este modelo, pero la predicción general de una expansión acelerada inicial (en muchos modelos cercana a una expansión exponencial) que lleva a un universo plano, termalizado y con pequeñas fluctuaciones que obedecen un espectro de potencias

invariante de escala, es bastante general. Muchos de estos modelos incluyen modificaciones de gravedad o un conjunto de campos escalares que imitan el mecanismo anterior. Notablemente, el universo está vacío de materia (o se vacía durante el periodo inflacionario) hasta que la expansión acelerada para, y la energía liberada genera de nuevo el contenido material.

Acuerdo con la Observación   En la década pasada se ha reunido una gran cantidad de datos observacionales que restringe severamente muchos de los modelos que han sido construidos para la cosmología (especialmente aquellos procedentes de aproximaciones deductivas al problema, como ST ó LQG). Llegados a este punto no basta ya con encontrar modelos que expliquen cualitativamente los rasgos del universo, sino también que proporcionen ajustes competitivos a los datos que se han reunido, y de manera deseable también predicciones para ser comprobadas o refutadas en futuras misiones.

He aquí una lista de observaciones cosmológicas que deberían venir descritas por una generalización de ΛCDM:

- En cualquier modelo sensato de infación se producen fluctuaciones cuánticas (o térmicas), que sirven de simiente para la estructura a gran escala del universo, las galaxias, etc. La mayoría de los modelos inflacionarios predicen fluctuaciones primordiales con un espectro $P(k)$ casi invariante de escala,

$$P(k) \equiv \frac{|\Phi_k|^2 k^3}{2\pi^2} \propto k^{n_s-1} \,, \tag{B.1.26}$$

  donde Φ_k es el transformado de Fourier de la perturbación invariante gauge de la métrica FRW en GR sin estrés anisótropo, y n_s se llama 'índice espectral escalar' (espectro invariante de escala quiere decir $n_s \approx 1$). El modelo R^2 de Starobinsky [46] y en general inflación caótica [45] predice $n_s \lesssim 1$. De hecho, el índice escalar espectral se ha conseguido medir en el espectro de CMB con WMAP [13], y resulta ser $n_s = 0{,}963^{+0{,}014}_{-0{,}015}$.

- Los modelos para la expansión acelerada presente deben confrontarse con una variedad de datos mucho más rica. Dado que las SNe pueden tomarse como candelas estándar, ha sido posible determinar la relación entre la distancia luminosa d_L y su desplazamiento al rojo $z = a_0/a - 1$,

$$d_l(z) = c(1+z) \int_0^z \frac{dz'}{H(z')} \,, \tag{B.1.27}$$

  para unos pocos cientos de SNe [8, 9] hasta $z < 2$. Además de las supernovas, la distancia reducida a la superficie de recombinación R_{CMB},

$$R_{CMB} = \Omega_m^{1/2} H_0 \int_0^{1089} dz/H(z) \,, \tag{B.1.28}$$

  puede leerse de los datos de WMAP [47]. Ambas medidas involucran una integral del inverso del parámetro de Hubble conrespecto al desplazamiento al rojo, pero las SNe determinan esta integral para valores pequeños de z mientras que R_{CMB} permite obtener una información integrada de la evolución del universo desde su juventud.

  Una dependencia ligeramente distinta se encuentra en el parámetro de distancia para las Oscilaciones Acústicas de Bariones (BAO) $A(z)$,

$$A(z) = \Omega_m^{1/2} H_0 H(z)^{-1/3} z^{-2/3} \left[\int_0^z dz'/H(z') \right]^{2/3} \,, \tag{B.1.29}$$

medido a $z = 0{,}35$ por SDSS [16]. Aunque algunos otros datos pueden confrontarse con modelos cosmológicos, éstos son los tests más ampliamente utilizados en cosmología.

En conclusión, GR puede generalizarse para incluir nuevos fenómenos en el IR y el UV, y debería generalizarse para poder explicar satisfactoriamente tanto las condiciones iniciales del universo como la expansión acelerada actual. Debería también generalizarse para poder incluir las nuevas correcciones que surjan en al renormalizar una QFT acoplada a gravedad en el Lagrangiano del sistema material, especialmente si este Lagrangiano se modifica también en el UV ó el IR.

El efecto de esta generalización en escalas cosmológicas debería proporcionar una expansión acelerada en el universo primitivo y otra en el universo presente. Estas expansiones aceleradas son en muchos casos asintóticamente exponenciales. En esta tesis doctoral se construye un modelo cosmológico en el cuál estas dos épocas se incorporan fenomenológicamente. El modelo resultante se contrastará con los datos observacionales disponibles.

B.1.4. Revisión de modelos alternativos para explicar la expansion acelerada

Esta subsección es una breve reseña de las principales alternativas propuestas para explicar los problemas concernientes a las expansiones aceleradas primitiva y presente.

El principal paradigma que explica los problemas de horizonte y planitud en el universo primitivo es inflación; tanto si ocurre como resultado de una modificación de GR en el UV o mediante la incorporación de nuevos campos a altas energías en ampliaciones de QFT como ST. La única verdadera alternativa al paradigma inflacionario es la de Velocidad de la Luz Variable (VSL) [48]. Explicaremos ambas alternativas más adelante.

Con el interés creciente en la expansión acelerada actual, ha surgido una amplia plétora de modelos. Además de modificaciones de gravedad (como teorías $f(R)$, modelos con dimensiones extra), estos modelos incluyen nuevas formas de materia (nuevos campos de todo tipo, fluidos perfectos con ecuaciones de estado exóticas,...) o el rechazo de la homogeneidad espacial como una buena aproximación en el tiempo presente (una constante cosmológica efectiva que surge del procedimiento de tomar valores promedio [49, 50], un vacío local como una explicación de la aparente expansión acelerada [51, 52, 53, 54],...). Algunas de estas modificaciones incluso tratan de unificar la explicación de inflación, la expansión acelerada actual, y/o la Materia Oscura. Procederemos a presentar brevemente algunos modelos de gravedad modificada y de fuentes exoticas del campo gravitatorio.

Campos Escalares para Inflación   Hemos mencionado ya que es posible que el universo entre en una época de expansión acelerada con ayuda de un conjunto de campos escalares con un potencial apropiado. Estos campos escalares podrían surgir como campos fundamentales en una QFT más allá del modelo estándar (como en la Ref. [1]) o como nuevos modos en una modificación de la GR (como en Ref. [46]). Como se apunta en Ref. [45], no es importante cuál sea el potencial concreto que esté detrás de la expansión acelerada, siempre y cuando éste satisfaga unas condiciones muy generales, las condiciones de 'caída lenta'.[5]

Desde entonces, ha habido muchos esfuerzos dirigidos a encontrar mecanismos viables de inflación en teorías de gravedad cuántica. En particular, la comunidad de ST ha visto el paradigma inflacionario como un campo en el que sus ideas pueden ser comprobadas. Se han encontrado muchas realizaciones posibles del paradigma inflacionario.

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[5] *Slow roll* en el original. Las condiciones suponen que la contribución de las derivadas temporales del campo a la densidad de energía sea mucho menor que la contribución del potencial en un periodo de tiempo suficientemente largo. (N. del T.)

Como ST predice que el universo tiene 10 dimensiones, necesita que las 6 dimensiones adicionales estén compactificadas para poder describir nuestro mundo. Los módulos, que son los parámetros que caracterizan esta compactificación, se estabilizan dinámicamente en una teoría cuántica, produciendo distintos vacíos. Hay muchas de estas soluciones con distintos valores de la energía de vacío. El conjunto de estos vacíos se denomina espacio de módulos de vacíos supersimétricos. Para incluir vacíos no supersimétricos[6] es necesario ampliar el espacio de módulos para llegar al denominado 'paisaje' [55]. Fue propuesto primero como una manera de resolver el problema de la constante cosmológica: si hay múltiples vacíos y la constante cosmológica varía de uno a otro, y considerando que la vida humana sólo es posible si el valor de esta constante yace en un cierto intervalo, entonces debemos estar viviendo en uno de los que están 'antrópicamente permitidos'.

El paisaje ofrece múltiples opciones para la construcción de modelos de inflación, como la inflación híbrida [56]. En este modelo, la inflación es llevada a cabo por dos campos escalares, con un potencial

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi^2\phi^2 + \frac{\lambda}{4}(\chi^2 - \frac{M^2}{4})^2 \,. \tag{B.1.30}$$

El principal nuevo ingrediente de la inflación híbrida, comparada con la inflación estándar, es que las fluctuaciones cuánticas son no Gaussianas debido a la interacción. En contraste con la inflación con un campo escalar, la inflaciónn híbrida da de manera natural $n_s \geq 1$ [57], y por lo tanto está descartada como el mecanismo que se realiza en la naturaleza.

Otro tipo de modelos inflacionarios surgen en el contexto de Dp-branas moviéndose a través de dimensiones adicionales. En el formalismo de 'mundo-branas', nuestro espacio-tiempo se corresponde a una de estas $D3$-branas. Las partículas del SM se corresponden a campos ligados a la D-brana, ya que son modos de la cuerda abierta cuyos extremos yacen en la D-brana. Por contra, el gravitón y otros modos de cuerda cerrada no están confinados. En 'inflación de branas', el inflatón se corresponde simplemente con la posición de esta brana [58]. Un modelo simple es inflación $D3$-brana/$\bar{D}3$-brana, en el que brana y antibrana experimentan una fuerza atractiva y finalmente se aniquilan una a otra, causando el final de inflación [59]. Aquí el inflatón φ es la distancia de la brana a la antibrana y el potencial $V(\varphi)$ se debe a las tensiones y la interacción. La energía se libera cuando las branas se aniquilan y quedan cuerdas cósmicas como vestigio. Un modelo más realista pero mucho más complicado es el modelo KKLMMT [60]. En general, muchos de estos modelos dan $n_s \geq 0,98$ y predicen la creación de una red de cuerdas cósmicas que no ha sido observada [61], y por lo tanto quedan descartados.

No obstante, a pesar del interés de los teóricos de cuerdas, los modelos de cuerdas son vistos por los fenomenólogos y cosmólogos como muy degenerados y muy poco predictivos. La idea del paisaje y la 'selección antrópica de vacíos' se recibe habitualmente con escepticismo y polémica.

Alternativas a inflación: VSL   Durante más de una década, las cosmologías inflacionarias no han tenido ningún competidor real. Sin embargo, en la Ref. [48] se propuso por primera vez un mecanismo alternativo bajo el nombre de Velocidad de la Luz Variable (VSL). La propuesta es que la velocidad de la luz local, medida por los observadores comóviles, podría haber variado con el tiempo en el universo muy primitivo de un valor muy grande a su valor actual. Esto implica obviamente una desviación de SR a muy altas energías, y por lo tanto mediada por cierta escala UV.

Supóngase que en el sistema de referencia privilegiado en el que el universo es aproximadamente homogéneo, la velocidad de la luz varía como $c(t)$. Por tanto, $ds^2 = c(t)^2 dt^2 - a(t)^2 d\mathbf{x}^2$ (aunque

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[6]¡Y hablamos de 10^{500} posibles vacíos!

$g_{00} = 1$) pero las ecuaciones de Friedman aún son válidas,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc(t)^2}{a^2}, \tag{B.1.31}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c(t)^2}\right). \tag{B.1.32}$$

Si el universo está dominado por radiación, $p = \rho c(t)^2/3$ y por tanto $a \propto t^{1/2}$ como es habitual. Como consecuencia de la violación de la invariancia bajo translaciones temporales incluso a nivel local, la condición de conservación de la energía se modifica,

$$\dot{\rho} + 3H\left(\rho + c(t)^{-2}p\right) = \frac{3kc(t)\dot{c}(t)}{4\pi G a^2} \tag{B.1.33}$$

El problema del horizonte se explica de manera directa dentro de este modelo. La razón del horizonte de sucesos sobre el horizonte causal en $t = t_i$ viene dada por

$$\frac{l_i}{l_c} \sim \frac{c_0 t_0}{\int_0^{t_i} c(t)dt}\frac{a_i}{a_0}, \tag{B.1.34}$$

y por tanto puede hacerse arbitrariamente pequeña mediante una elección apropiada de la función $c(t)$. Una propagación "superlumínica" de la luz a tiempos primitivos es suficiente. En el artículo original [48], se propone una transición de fase entre un valor primitivo de la velocidad de la luz $c_i \sim 10^{30}c_0$ y el valor presente c_0 a $t \lesssim t_p$.

El problema de planitud también puede entenderse en este contexto. Tomando en cuenta las Eqs. (B.1.31),(B.1.32),(B.1.33), se obtiene que en un universo dominado por un fluido perfecto con $p = \omega\rho c(t)^2$, el parámetro cosmológico $\Omega(t)$ evoluciona como

$$\dot{\Omega} = \Omega(\Omega - 1)H(1 + 3\omega) + 2\frac{\dot{c}}{c}(\Omega - 1). \tag{B.1.35}$$

La cosmología inflacionaria resuelve el problema porque el campo del inflatón satisface $1+3\omega < 0$ y entonces $\Omega \approx 1$ es una solución estable. VSL es capaz de resolver el problema porque el término proporcional a $(1+3\omega)$ puede compensarse por el término propocional a $\dot{c}$ siempre que $|\frac{\dot{c}}{c}| > \Omega H$.

Algunos modelos que proporcionan realizaciones concretas de VSL pueden encontrarse en las Refs. [62, 48, 63] (ver Ref. [64] para una revisión). Una teoría covariante general fue presentada en la Ref. [65], como una teoría con una simetría Fock-Lorentz symmetry gauge en la que se permite que c varíe como un campo escalar en al variedad, $c(x)$. Otra realización covariante similar es 'Geometría de Arcoiris' [66], en la que la métrica del espacio-tiempo vista por una partícula de prueba con una cierta energía depende de dicha energía. Sin embargo la interpretación de este formalismo está bastante poco clara. Una interpretación más sencilla es que la velocidad de la luz sea dependiente de la frecuencia, $c(E)$, debido a una relación de dispersión modificada. En el universo primitivo, la longitud de onde de los fotones se hace comparable a la longitud de Planck u otra escala UV y por tanto el efecto global puede verse como VSL [67]. Este camino permite una descripción más profunda de la cosmología con [68] y la formación de estructuras [69]. Nótese sin embargo que esta idea tampoco está carente de críticas [70].

Teorías $f(R)$ Ha habido mucho interés en modificar GR desde que fue formulada [71], como consecuencia del éxito de GR en la descripción de la precesión del perihelio de Mercurio. Como consecuencia de las dificultades en la renormalización de GR, se han estudiado modificaciones de la acción de Hilbert-Einstein con términos cuadráticos en los tensores de curvatura (como los

de la Eq. (B.1.18)); más tarde se demostró que estas correcciones proporcionan un mecanismo inflacionario [46], como hemos explicado. También se han encontrado correcciones no lineales en el escalar de curvatura a la acción de Einstein-Hilbert en ST [72, 73].

Sin embargo, los motivos para modificar la acción de Einstein-Hilbert pueden ser mucho más mundanos: no existe ningún principio fundamental que fije la forma de la acción de gravedad. Luego si se permite la introducción de nuevas escalas en la teoría gravitatoria, aparecerán genéricamente nuevos términos en la acción, que deben respetar la covariancia general. En este sentido la acción de gravedad puede ampliarse arbitrariamente como en las Eqs. (B.1.18) or (B.1.19). Como modelo de juguete, pueden considerarse las acciones que dependen sólo del escalar de curvatura,[7]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} f(R) + L_m \right]. \tag{B.1.36}$$

Éstas son las llamadas teorías $f(R)$ [79]. De hecho, puede distinguirse entre tres tipos de teorías $f(R)$ dependiendo de cómo se implemente el principio variacional en la acción. En las teorías métricas $f(R)$, el único campo gravitacional que aparece en la acción es la métrica $g_{\mu\nu}$ y la variación con respecto a la métrica proporciona las ecuaciones del movimiento. En las teorías $f(R)$ de Palatini el escalar de curvatura es el resultado de la contracción con el inverso de la métrica del tensor de Ricci, que a su vez se construye únicamente con la conexión (tratada como un campo independiente). Se considera que el Lagrangiano de materia depende únicamente de la métrica, no de la conexión. En las teorías $f(R)$ métricas afines, no sólo el tensor de Ricci está construido únicamente con la conexión, sino que además ésta también entra en el Lagrangiano de materia a través de las derivadas covariantes. Todos estos formalismos son equivalentes si $f(R) = R + \Lambda$, pero difieren si $f''(R) \neq 0$. En esta tesis trataremos sobre todo las teorías métricas $f(R)$.

En una teoría métrica $f(R)$, las ecuaciones del movimiento son,

$$f'(R)R_\nu^\mu - \frac{1}{2} f(R)\delta_\nu^\mu + (\delta_\nu^\mu \Box - \nabla^\mu \nabla_\nu) f'(R) = 8\pi G T_\nu^\mu, \tag{B.1.37}$$

las cuales, al particularizarse a una métrica FRW, se convierten en

$$8\pi G\rho = -3(\dot{H} + H^2)f'(R_{(0)}) - \frac{1}{2}f(R_{(0)}) - 3Hf''(R_{(0)})\dot{R}_{(0)}, \tag{B.1.38}$$

$$-8\pi Gp = -(\dot{H} + 3H^2)f'(R_{(0)}) - \frac{1}{2}f(R_{(0)}) + f^{(3)}(R_{(0)})\dot{R}_{(0)}^2 + f''(R_{(0)})\ddot{R}_{(0)}, \tag{B.1.39}$$

donde

$$R_{(0)}(t) = -6(\dot{H} + 2H^2) \tag{B.1.40}$$

es el escalar de curvatura en resultante de una métrica FRW. Para una modificación del tipo $f(R)$ en el IR, es posible encontrar mecanismos para la aceleración presente del universo [80]; en el mismo sentido que en algunas modificaciones de tipo $f(R)$ en el UV es posible encontrar mecanismos de inflación [46]. También se ha tratado de unificar la inflación y la expansión acelerada actual en el contexto de estas teorías [81, 82].

Las teorías métricas $f(R)$ son formalmente equivalentes a las teorías escalar-tensor (tal y como explicamos en el contexto del modelo de Starobinski de inflación), con un potencial para el campo escalar

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[7]También se han considerado modelos más generales. Por ejemplo, modelos que dependen de los escalares cuadráticos en el tensor de curvatura [74, 75] o con términos no locales [76, 77, 78].

$$V(\varphi) = \frac{1}{16\pi G} \frac{Rf'(R) - f(R)}{f'(R)^2}, \tag{B.1.41}$$

donde se puede obtener $R(\varphi)$ de la relación

$$\varphi \equiv \sqrt{\frac{3}{16\pi G}} \log f'(R). \tag{B.1.42}$$

El escalar presenta una masa efectiva

$$m^2 = \frac{1}{2} \frac{d^2 V}{d\varphi^2} = \frac{1}{6} \left(\frac{Rf'(R) - 4f(R)}{f'(R)^2} + \frac{1}{f''(R)} \right). \tag{B.1.43}$$

Por tanto, las teorías métricas $f(R)$ pueden verse como teorías de un campo sin masa que se propaga (el gravitón) junto con un campo escalar (generalmente masivo).[8] Este campo escalar puede verse como un inflatón en el universo muy primitivo pero se convierte en un problema cuando se trata de encontrar una teoría $f(R)$ consistente con los requisitos impuestos por las comprobaciones de GR en el sistema solar.

Para poder presentar una alternativa viable a ΛCDM, una teoría de gravedad modificada debe verificar un número de requisitos. En particular, una teoría $f(R)$ debe:

- poseer una dinámica cosmológica correcta. Muchos modelos presentan transiciones que no son suaves, por ejemplo entre el periodo dominado por radiación y el periodo dominado por materia, que ingenuamente no se esperaban (por ejemplo, ver Ref. [83]). También se ha señalado que algunas teorías $f(R)$ pueden poseer diversas singularidades en el futuro [84], clasificadas en la Ref. [85]. Las singularidades del futuro pueden eliminarse añadiendo un término R^2 a la acción, como se muetra en [84]. Estos problemas pueden superarse partiendo de una evolución cosmológica concreta $a(t)$ e integrando la ecuación diferencial ordinaria que la relaciona con la familia de teorías $f(R)$ que presentan esa solución [86].

- no padecer inestabilidades. Poco después de proponerse, se encontró que el modelo simple de la Ref. [80] poseía una inestabilidad [87]. No obstante se mostró que esta inestabilidad puede eliminarse mediante la adición de términos en la acción relevantes a altas curvaturas [81], de la misma manera que las singularidades del futuro. Las teorías métricas $f(R)$ padecen esta inestabilidad cuando $f''(R) < 0$ [88]. Si este es el caso la energía correspondiente al campo escalar puede volverse negativa. Esto puede entenderse en términos del acoplo gravitatorio efectivo que aparece en las teorías métricas de $f(R)$, $G_{eff} = G/f'(R)$. Si $f''(R) < 0$, entonces $dG_{eff}/dR > 0$. Una gran curvatura hace que la gravedad sea más fuerte, lo que hace que la curvatura a su vez aumente, provocando un mecanismo de retroalimentación. También aparece una singularidad desnuda en el régimen no lineal a altas curvaturas de una teoría de gravedad modificada en el IR, que hace difícil describir el régimen de gravedad fuerte en estrellas relativistas [89]. De hecho estas singularidades del caso esféricamente simétrico están conectadas con las singularidades del futuro de los espaciotiempos FRW mencionadas en el punto anterior [90], y también pueden eliminarse mediante la adición de nuevos términos relevantes a altas curvaturas [84].

- estar libre de fantasmas. Aunque las teorías métricas $f(R)$ no poseen fantasmas, modificaciones más generales de gravedad podrían contener campos fantasma. Se sabe que las modificaciones que también contengan sólo el escalar de Gauss-Bonnet $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ no poseen fantasmas [91].

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[8]En modificaciones más generales, pueden aparecer más grados de libertad que se propaguen.

- poseer los límites Newtoniano y post-Newtoniano correctos y ser compatible con los datos de CMB y LSS. Estas condiciones tienen en común el hecho de que dependen del comportamiento de perturbaciones inhomogéneas sobre una métrica homogénea. En la teoría escalar-tensor conformemente equivalente, el problema es que el grado de libertad escalar puede estropear el comportamiento de estas perturbaciones; da lugar a una modificación de la ley de Newton y modifica la dinámica cosmológica de perturbaciones [92]. En muchos modelos, la masa del escalar es de manera natural ligera $(m < 10^{-3}\,eV)$ y por lo tanto el efecto en experimentos gravitatorios en el sistema solar los descarta como teorías de gravedad viables. Sin embargo, debe tenerse en cuenta que la masa efectiva de los escalares depende de la curvatura local, y por lo tanto difiere en los tests cosmológicos, del sistema solar y terrestres. Esto se conoce como el 'efecto camaleón' [93, 94]. Por tanto es posible tener escalares que modifiquen la ley de gravedad a escalas menores que el milímetro en el sistema solar (y consecuentemente permanezcan indetectados) y al mismo tiempo ofrezcan una dinámica cosmológica modificada a largas distancias. Pueden encontrarse ejemplos en las Refs. [95, 96, 97, 98]. Sin embargo no está claro si la física de la teoría escalar-tensor conformemente equivalente ilustra siempre bien física de la teoría $f(R)$ [99]. El comportamiento de las perturbaciones escalares puede cambiar debido a la redefinición de campos que se hace después de la transformación conforme. El comportamiento de las perturbaciones cosmológicas en teorías métricas $f(R)$ es todavía sujeto de trabajo: se ha definido en el formalismo covariante [100] y en el convenio Newtoniano [101]. Una situación similar ocurre en el caso de las teorías $f(R)$ de Palatini.

- presentar un problema de Cauchy bien definido. Ha sido demostrado que las teorías métricas $f(R)$ presentan un problema de Cauchy bien definido para distintas fuentes en el sistema material (fluidos perfectos, campos escalares), mientras que las teorías $f(R)$ de Palatini no [102].

Resumiendo, las teorías $f(R)$ son un modelo de juguete simple que muestra la dificultad de desviarse de GR de manera consistente y de acuerdo con los datos experimentales y observacionales. Esto es posible bajo ciertas condiciones que imponen un gran ajuste fino en la función $f(R)$. El origen de los requisitos es la presencia de nuevos grados de libertad efectivos que se propagan, o lo que es lo mismo, la presencia de ecuaciones diferenciales de orden mayor que dos.

Notablemente, estos problemas no surgen sólo en teorías de gravedad modificada, sino que también tienen su equivalente en otras alternativas para el problema de la aceleración como los modelos de quintaesencia y los fluidos oscuros. Esto puede verse trivialmente ya que puede trasladarse el efecto de una modificación de gravedad a una teoría escalar tensor (como se ha mostrado) o a un fluido oscuro. Al igual que en las teorías $f(R)$ singulares, el problema de las singularidades en modelos de quintaesencia o fluidos oscuros puede eliminarse añadiendo términos con potencias más altas del escalar de curvatura a la acción gravitatoria, como los que aparecen en la renormalización de QFT en espaciotiempo curvo [103].

Este es un problema general de las modificaciones de GR. Trataremos de superar este problema (al menos de manera efectiva) en la formulación del Modelo Cosmológico Asintótico (ACM) en uno de los trabajos presentados en esta tesis doctoral, y en el tratamiento de las perturbaciones cosmológicas en el ACM.

Modelos con dimensiones adicionales   Inspirados por ST, se ha propuesto que nuestro universo podría ser una hipersuperficie tetradimensional (una 3-brana) inmersa en un espacio-tiempo macizo[9] con dimensiones adicionales. Se considera que los campos del SM son campos sobre la

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[9]*Bulk spacetime* en el original. (N. del T.)

brana mientras que el campo (o campos) gravitacionales están definidos en todo el volumen del espacio-tiempo macizo. La cuestión es cómo recuperar la ley de Newton estándar en vez de su versión para dimensión más alta en la brana. Esto puede conseguirse mediante la compactificación de las dimensiones extra [37, 38, 40], cuyo tamaño puede ser mucho mayor que en el caso de los modelos de Kaluza-Klein; por tanto este método se denomina 'dimensiones adicionales grandes'. También puede conseguirse haciendo que la brana esté inmersa en un espacio-tiempo voluminoso de tipo Anti-deSitter [104, 105]; este otro método se denomina 'dimensiones adicionales curvadas'. [10] En ambos casos, las dimensiones adicionales son finitas y tienen un tamaño del orden del milímetro.

Dvali *et al.* propusieron otra solución: basta añadir a la acción de la brana un término de Einstein-Hilbert calculado a partir de su curvatura intrínseca. Este término es necesario para la renormalizabilidad de la QFT definida para los campos sobre la brana, y que interactúan con la gravedad del espacio-tiempo macizo. Por tanto la ley de Newton se recupera incluso para dimensiones adicionales de tamaño infinito [106]. El caso de una 3-brana dentro de un espacio-tiempo de Minkowski de dimensión 4+1 es de espacial interés. Explicaremos este modelo en las siguientes líneas.

Considérese la acción

$$S_{(5)} = \frac{1}{2\kappa^2} \int d^5X \sqrt{-\bar{g}}\bar{R} + \int d^4x \sqrt{-g}(\lambda_b + l_m) + \frac{1}{2\mu^2} \int d^4x \sqrt{-g}R \,, \qquad (B.1.44)$$

donde el primer término se corresponde con la acción de Einstein-Hilbert en 4+1 dimensiones, con la métrica del volumen pentadimensional $\bar{g}_{AB}$ y la curvatura escalar $\bar{R}$; el tercer término se corresponde con la acción de Einstein-Hilbert de la métrica inducida en la brana, g_{ab}, con su correspondiente escalar de curvatura intrínseca R; el segundo término se corresponde a la acción de materia, que podría incluir una tensión de la brana (una constante cosmológica efectiva) λ_b. Pueden definirse las siguientes cantidades,

$$\kappa^2 = 8\pi G_{(5)} = M_{(5)}^{-3} \qquad (B.1.45)$$

$$\mu^2 = 8\pi G_{(4)} = M_{(4)}^{-2} \,. \qquad (B.1.46)$$

En la Ref. [106], se demostró que el potencial Newtoniano se recupera en la brana para distancias menores $r_0 = M_{(4)}^2/2M_{(5)}^3$, con una constante de Newton efectiva $G_N = \frac{4}{3}G_{(4)}$.

La cosmología del modelo fue derivada por Deffayet en la Ref. [107]. Considérese la métrica isótropa en la brana

$$ds^2 = n^2(\tau,y)d\tau^2 - a^2(\tau,y)\gamma_{ij}dx^i dx^j - b^2(\tau,y)dy^2 \,, \qquad (B.1.47)$$

donde $\{\tau, \mathbf{x}\}$ son coordenadas en la brana ($y = 0$) y γ_{ij} es la métrica tridimensional máximamente simétrica (con curavtura espacial k). El tensor de energía-impulso del espacio-tiempo macizo se reduce a una posible constante cosmológica ρ_B, mientras que el tensor de energía-impulso de la brana puede expresarse como

$$T_B^A = \frac{\delta(y)}{b}diag(\rho_b, -p_b, -p_b, -p_b, 0) \,. \qquad (B.1.48)$$

Suponiendo que la métrica $\bar{g}_{AB}$ sea simétrica bajo el cambio $y \rightarrow -y$, la correspondiente ecuación de Friedman es

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[10] *Warped extra dimensions* en el original. (N. del T.)

$$\pm\sqrt{H^2 - \frac{\kappa^2}{6}\rho_B - \frac{C}{a_0^4} + \frac{k}{a_0^2}} = r_0\left(H^2 + \frac{k}{a_0^2}\right) - \frac{\kappa^2}{6}\rho_b\,, \qquad (B.1.49)$$

donde C es una constante de integración, el subíndice 0 significa 'evaluado en $y = 0$', y $H = \frac{\dot{a}_0}{n_0 a_0}$, donde un punto denota derivación con respecto a la variable temporal τ. La restricción de los campos a la brana también conlleva una ecuación de continuidad para la brana, $\dot{\rho}_b + 3(\rho_b + p_b)\frac{\dot{a}_0}{a_0} = 0$.

Si se elige el signo $+$ y se desprecia ρ_B, C y k por simplicidad, se llega a la ecuación de Friedman modificada

$$\frac{8\pi G_{(4)}}{3}\rho_b = H^2 - H/r_0\,, \qquad (B.1.50)$$

que produce una expansión acelerada tardía que tiende asintóticamente a una expansión exponencial con $H = 1/r_0$, como en el caso de ΛCDM. Sin embargo, esta ecuación de Friedman modificada es distinguible de la que aparece ΛCDM,

$$\frac{8\pi G_N}{3}\rho = H^2 - \Lambda/3\,. \qquad (B.1.51)$$

Los límites actuales impuestos sobre la historia de la expansión del universo por las distancias luminosas a las supernovas, el CMB y el parámetro de Hubble excluyen el modelo más simple de DGP sin curvatura a cerca de 3σ. Incluso si se incluye curvatura espacial, el universo abierto que mejor ajusta los datos ofrece un ajuste un poco peor que el ajuste de ΛCDM. Más aún, las diferencias sustanciales en su historia de la expansión suponen un desafío a la hora de explicar cuantitativamente la formación de estructuras [108].

Además, el modelo DGP está plagado de patologías [109, 110]. En primer lugar, se ha descubierto que posee fantasmas en la teoría de perturbaciones [111]. Segundo, si se permiten métricas generales en el espacio-tiempo macizo, entonces el espectro de energías de las perturbaciones no está acotado por debajo (aparecen agujeros negros de masa negativa) y por lo tanto la teoría es inestable [110]. Más aún, existen soluciones (agujeros negros de masa positiva) que poseen 'singularidades de presión': la presión diverge para una densidad de energía finita. En tercer lugar, el vacío de DGP es inestable, ya que puede decaer por efecto túnel a otros vacíos [110].

Por tanto, no consideraremos DGP como una alternativa viable para explicar la expansión acelerada actual. Sin embargo, es interesante cómo una simple ecuación de Friedman modificada como (B.1.51) es capaz de reproducir una expansión acelerada en ausencia de fluidos exóticos.

Quintaesencia Además de modificaciones de gravedad como las descritas anteriormente, es posible explicar la aceleración actual del universo como consecuencia de la existencia de un nuevo tipo de materia, diferente de las partículas masivas o sin masa, que presenta al menos a tiempo cósmico tardío una presión efectiva negativa.

Existen varias maneras de implementar esta idea, pero una de las más populares consiste en introducir fenomenológicamente un campo escalar autointeractuante. Las modificaciones de GR con campos escalares tienen una larga historia [112]. Este mecanismo también ha sido utilizado en QFT para explicar algunos de los parámetros antinaturalmente pequeños del SM [113, 114]. En analogía, se pueden construir acciones con campos escalares dinámicos que imiten el efecto de una constante cosmológica, y que hagan que su valor sea pequeño [115, 116]. No obstante, esto implicaría la existencia de una nueva fuerza mediada por el campo escalar, aunque la presencia de nuevas fuerzas mediadas por campos escalares ligeros está fuertemente restringida [117]. Por

tanto debe ajustarse finamente la acción, al menos el término de masa efectivo del escalar o su acoplo al resto de los campos.

Típicamente la acción de una de estas teorías de 'quintaesencia' tiene la forma

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) + \mathcal{L}_m[g^{\mu\nu}, \varphi] \right] , \quad (B.1.52)$$

donde φ es el campo escalar de quintaesencia[11], V es el potencial efectivo de autointeracción, que puede introducirse en una teoría más fundamental, y $\mathcal{L}_m$ es el Lagrangiano del sistema material, que puede incluir acoplos entre la materia ordinaria y la quintaesencia.

En la aproximación homogénea, en la que la métrica es FRW y los campos de quintaesencia y materia están desacoplados y dependen sólo de la variable temporal, las ecuaciones del movimiento se tornan

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) \quad = 0 \quad\quad\quad\quad (B.1.53)$$

$$\frac{8\pi G_N}{3}(\rho + \frac{\dot{\varphi}^2}{2} + V(\varphi)) \quad = H^2 \quad\quad\quad\quad (B.1.54)$$

$$-3(1+\omega)H\rho \quad = \dot{\rho} . \quad\quad\quad\quad (B.1.55)$$

Nótese que si la quintaesencia varía suficientemente despacio con el tiempo ($\dot{\varphi}^2 \ll V(\varphi)$), entonces la quintaesencia está aproximadamente congelada en su valor actual φ_0 y actua como una constante cosmológica efectiva $\Lambda = 8\pi G V(\varphi_0)$; para una variación suficientemente lenta, la presión efectiva negativa $p_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi)$ requerida por la expansión acelerada está garantizada [116, 118, 119].

Algunos modelos específicos de quintaesencia merecen más atención. Una de estas clases son los 'modelos rastreadores' [12][120, 121], que intentan explicar el problema de coincidencia [13]. La idea que subyace a los modelos rastreadores es la siguiente: la densidad de energía de la componente de quintaesencia de alguna manera sigue la pista de la densidad de energía de la componente dominante desde un valor más bajo, cayendo desde un valor Planckiano en el universo primitivo a lo largo de gran parte de la historia del universo. Sólo recientemente crece y se convierte en la componente dominante de la densidad de energía, llevando al universo a la expansión acelerada actual. El campo rastreador φ cae en el potencial $V(\varphi)$ siguiendo una solución similar a un atractor. Algunos ejemplos de potenciales rastreadores son las exponenciales $V(\varphi) = M^4(e^{M_p/\varphi} - 1)$ y las leyes de inversos de potencias $V(\varphi) = M^{4+n}\varphi^{-n}$.

Por ejemplo, para los potenciales de tipo inverso de potencia $V(\varphi) = M^{4+n}\varphi^{-n}$ es posible llegar a un periodo dominado por quintaesencia cuando $\varphi \sim M_p$. El valor de la escala M debe ajustarse por medio de las observaciones (típicamente $M \gtrsim 1\,GeV$). Para una componente dominante dada con ecuación de estado $\omega_B \equiv \rho_B/p_B$, la quintaesencia subdominante se adapta y adopta la ecuación de estado

$$\omega_\varphi \approx \frac{n\omega_B/2 - 1}{n/2 + 1} . \quad\quad\quad\quad (B.1.56)$$

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[11]A este campo escalar se le otorga una variedad de nombres, dependiendo de su papel en el modelo particular en el que se introduce. Puede ser el dilatón (el bosón de Goldstone asociado a la ruptura espontánea de la simetría bajo dilataciones), el cosmón (un caso particular del dilatón), el inflatón (si se considera un modelo de inflación),...

[12]*Tracker models* en el original (N. del T.)

[13]Por qué el valor de la densidad energía de vacío es comparable al valor de la densidad de energía de la materia hoy.

Por tanto para gran n si la quintaesencia es subdominante entonces imita la ecuación de estado de radiación en el universo primitivo y su densidad de energía decrece al mismo ritmo. Cuando el universo pasa a estar dominado por materia, la presión de la quintaesencia se vuelve negativa y más tarde la quintaesencia pasa a ser la componente dominante. No obstante, para estos potenciales la condición $\dot{\varphi}^2 \ll V(\varphi)$ nunca se satisface y por tanto no son capaces de reproducir la ecuación de estado de una constante cosmológica. Por tanto el modelo ofrece un peor ajuste a los datos observacionales. El problema de coincidencia simplemente se ha desplazado de la elección del valor de Λ en ΛCDM a la elección del valor de M.

Otro tipo interesante de modelos de quintaesencia son los modelos de k-esencia, en los que el término cinético del campo escalar no es el habitual [122]. Estos términos cinéticos pueden surgir en ST ST [123]. Típicamente, la acción de un modelo de k-esencia puede escribirse como

$$ S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R + \frac{1}{\varphi^2} f(X) + \mathcal{L}_m[g^{\mu\nu}, \varphi] \right] , \qquad (B.1.57) $$

donde $f(X)$ es una cierta función de $X \equiv g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$. La densidad de energía y presión de la k-esencia son

$$ \rho_k = (2Xf'(X) - f(X))/\varphi^2 , \qquad (B.1.58) $$
$$ p_k = f(X)/\varphi^2 . \qquad (B.1.59) $$

Como se demuestra en la Ref. [122], los modelos de k-esencia son capaces de reproducir el comportamiento de los modelos rastreadores en el universo primitivo, mientras que muestran un comportamiento más similar al de una constante cosmológica en el universo actual. Sin embargo, estos modelos son bastante *ad hoc* y no se ajustan a los datos observacionales tan bien como lo hace una constante cosmológica efectiva. Además no resuelven realmente el problema de coincidencia, ya que la transición entre el periodo dominado por materia y el periodo dominado por k-esencia ocurre exactamente como en ΛCDM.

Un comportamiento rastreador similar ocurre de manera mucho más natural en modelos vector-tensor como el propuesto en la Ref. [124]. No obstante, este tipo de modelos presentan una tensión poco deseable cuando se ajusta sus parámetros a los datos de SNe, CMB y BAO [125].

Energía Oscura Cuando se trata de describir fenomenológicamente la componente exótica del universo ya sea con un campo escalar autointeractuante, una k-esencia u otro tipo de campos, se hace una suposición *a priori* (el contenido de campos de nuestro modelo) que va a condicionar todo el análisis posterior. Por este motivo, puede ser mejor desde el punto de vista fenomenológico el considerar solamente los rasgos esenciales de la nueva componente de materia, independientemente de su representación como una teoría de campos. El ingrediente esencial del nuevo componente que debe caracterizarse en GR es sólo el tensor de energía-impulso.

En la aproximación homogénea, los únicos elementos no nulos del tensor de energía-impulso son la densidad de energía ρ_X y la presión p_X. Por lo tanto, la descripción efectiva del nuevo fluido se reduce a la relación entre estas dos cantidades, la ecuación de estado $p_X = p_X(\rho_X)$. La parametrización más simple de esta relación es una ecuación de estado lineal $p = \omega\rho$ [126]; [14] por ejemplo la radiación posee $\omega = 1/3$, la materia masiva tiene $\omega = 0$ (estrictamente hablando, $\rho \gg p$) y una energía de vacío verifica $\omega = -1$. Naturalmente, esta parametrización es la que más se usa en análisis de datos, como en las Refs. [11, 7, 12, 13]. Por contra, un ejemplo de una ecuación de estado no lineal podría ser un gas de cuerdas, el gas de Chaplygin $p_X = -K/\rho_X$

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[14]A pesar de constituir un flagrante abuso del lenguaje, habitualmente se designa a ω como la ecuación de estado.

[127, 128], que es capaz de imitar a la materia oscura a densidad de energía alta pero lleva a una expansión acelerada a densidades bajas.

La relación entre la presión y la densidad de energía no contiene toda la información sobre el fluido en lo que se refiere a la evolución de las perturbaciones inhomogéneas [129]. En este caso es necesario considerar además la parte escalar de los términos no diagonales del tensor de energía-impulso, a saber, la densidad de momento $(\rho_X + p_X)\boldsymbol{u}_X = (\rho_X + p_X)\boldsymbol{\nabla}\theta_X$ y la tensión anisótropa $\Pi_{Xij} = \partial_i\partial_j\Pi_X$. Una descripción completa del fluido debería estar constituida por una expresión que relacione la presión y la tensión anisótropa como función (o funcional) de la densidad de energía y la densidad de momento del fluido en cada punto.

Esta descripción es desde luego demasiado general para ser predictiva, y habitualmente uno se limita al uso de fluídos perfectos ($\Pi = 0$) con una cierta ecuación de estado $p(\rho)$. Sin embargo, el efecto de algunas formas específicas de tensión anisótropa ha sido investigado en la literatura (ver por ejemplo las Refs. [129, 130, 131]).

A la luz de la gran cantidad de modelos concretos, se ha propuesto como una manera de parametrizar la aceleración del universo una descripción fenomenológica de la ecuación de estado del tipo

$$p_X = \left[\omega_0 + \omega_a(1-a) + \mathcal{O}\left((1-a)^2\right)\right]\rho_X\,, \tag{B.1.60}$$

donde se ha tomado la normalización $a_0 = 1$. Aunque estamos de acuerdo con el espíritu inductivo de estas parametrizaciones, la elección es un tanto antropocéntrica y arroja poca luz sobre la física de esta componente oscura o física gravitacional que está detrás de la expansión. ¿Es posible encontrar una elección mejor?

B.1.5. El Modelo Cosmológico Asintótico

Lo primero que uno debería notar cuando se piensa en una ampliación del CSM es que es difícil (¿imposible?) distinguir por medio de física gravitacional entre una modificación de la teoría de gravedad y nuevas formas de materia. Esto se debe a las ecuaciones de Einstein. Cualquier modificación del tensor de Einstein puede moverse a la parte derecha de la ecuación y convertirse en una modificación del tensor de energía-impulso y viceversa. Este término tendrá una cuadridivergencia covariante nula como consecuencia de ser derivado de una acción escalar, tanto si proviene de la parte material como si procede de la parte la gravitatoria de la acción. Por tanto, se considerará que la modificación estará contenida en la parte gravitacional de la acción sin pérdida de generalidad (fenomenológica).

En segundo lugar, para construir un modelo concreto se necesita hacer una serie de suposiciones basadas en motivos fenomenológicos. Hemos mencionado que está experimentalmente bien establecido que la gravedad debería describirse mediante una teoría métrica covariante general [2, 3, 4], y por tanto se supondrá que nuestro modelo está contenido en una teoría métrica. Se considerará también que el principio cosmológico es una buena aproximación a escalas suficientemente grandes, de manera que la métrica FRW será una buena aproximación a la métrica del espaciotiempo. La métrica FRW depende sólo de la curvatura espacial, k. En ΛCDM, el valor de k está severamente restringido por las observaciones, así que se asumirá que $k = 0$.

En GR sin curvatura espacial, la evolución del universo en la aproximación homogénea se obtiene de una relación algebráica entre el parámetro de Hubble y la densidad de energía. Más aún, si se supone que $\dot{H} < 0$, entonces hay una correspondencia inyectiva entre t y H, y por tanto siempre se puede encontrar una relación algebráica entre la densidad de energía (de un fluido compuesto por las partículas del SM y DM) y el parámetro de Hubble. Entonces se definirá el modelo a través de la relacion entre H y ρ.

En tercer lugar, se considerará que la relación estándar entre H y ρ, la primera ecuación de Friedman (B.1.6), será válida en una ventana limitada por la presencia de una escala UV y otra escala IR. Bajo el punto de vista de un fenomenólogo, toda teoría es válida sólo en el régimen en el que ha sido comprobada, pero puede fallar en cualquier régimen en el que aún no lo haya sido. GR ha sido bien comprobada en un rango que va desde la escala del milímetro hasta la escala galáctica, y QFT se ha comprobado desde el eV hasta el TeV, y por tanto podría fallar más allá de estas escalas. En nuestro modelo, el efecto the las nuevas escalas UV e IR será responsable de la expansión acelerada en el universo primitivo y actual, respectivamente.

Se tomará como principio básico que el parámetro de Hubble está restringido a un intervalo abierto acotado por una cota superior H_+ e inferior H_-. Por tanto en el pasado y futuro infinitos el universo tiende asintóticamente a un universo de deSitter. Hemos llamado a este modelo el Modelo Cosmológico Asintótico.

B.2. Oscilaciones de Neutrinos

El desarrollo de QFT ha sido uno de los sectores más activos de la física teórica a lo largo del siglo pasado. La electrodinámica cuántica (QED) ha proporcionado las predicciones más precisas de la historia de la física, que luego han sido confirmadas experimentalmente cifra a cifra. El SM, construido en constante interacción con el mundo experimental, ha conseguido proporcionar un marco en el cual entender las interacciones electromagnética, débil y fuerte, y proponer un mecanismo para explicar la aparición de masa (que aún espera confirmación).

El modelo consiste en una teoría gauge $SU(3) \times SU(2) \times U(1)$, en la que la simetría $SU(2) \times U(1)$ está "rota" por el mecanismo de Higgs por el valor esperado en el vacío (vev) de un campo escalar. Los espinores de la teoría se dividen en tres familias (o sabores) y pueden ser tanto quarks (con interacción $SU(3)$) o leptones (sin interacción $SU(3)$), y las masas se consiguen a través de las interacciones de Yukawa con el campo escalar.

Denótense los campos del SM mediante:

$$
\begin{aligned}
\Psi_{L\alpha} &= \begin{pmatrix} \nu_\alpha \\ l_\alpha \end{pmatrix}_L, & L_\alpha &= (l_\alpha)_R, \\
Q_{L\alpha} &= \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L, & U_\alpha &= (u_\alpha)_R, & D_\alpha &= (d_\alpha)_R, \\
\Phi &= \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, & \tilde{\Phi} &= \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}, \\
G_\mu, & & W_\mu, & & B_\mu,
\end{aligned}
\tag{B.2.1}
$$

donde en la primera línea hemos denotado los leptones, en la segunda línea los quarks, en la tercera línea el bosón de Higgs y su conjugado, y en la última línea los campos que portan las interacciones gauge $SU(3)$, $SU(2)$ y $U(1)$, respectivamente, cuyas respectivas intensidades de campo se denotan $G_{\mu\nu}$, $W_{\mu\nu}$ y $B_{\mu\nu}$. El índice $\alpha = 1, 2, 3$ etiqueta el sabor, el acoplo de los campos de gauge es respectivamente g_3, g y g'. Las derivadas covariantes se denotan por ∇_μ, y $A \overleftrightarrow{\nabla}_\mu B \equiv A \nabla_\mu B - (\nabla_\mu A) B$. Los acoplos de Yukawa se denotan por Y_L, Y_U, Y_D. Entonces, el Lagrangiano del SM es la suma de los siguientes términos:

$$\mathcal{L}_{lepton} = \frac{i}{2}\bar{\Psi}_{L\alpha}\gamma^\mu\overleftrightarrow{\nabla}_\mu\Psi_{L\alpha} + \frac{i}{2}\bar{L}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu L_\alpha\,, \tag{B.2.2}$$

$$\mathcal{L}_{quark} = \frac{i}{2}\bar{Q}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu Q_\alpha + \frac{i}{2}\bar{U}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu U_\alpha + \frac{i}{2}\bar{D}_\alpha\gamma^\mu\overleftrightarrow{\nabla}_\mu D_\alpha\,, \tag{B.2.3}$$

$$\mathcal{L}_{Yukawa} = -[(Y_L)_{\alpha\beta}\bar{\Psi}_{L\alpha}\Phi L_\beta + (Y_U)_{\alpha\beta}\bar{Q}_\alpha\tilde{\Phi}U_\beta + (Y_D)_{\alpha\beta}\bar{Q}_\alpha\Phi D_\beta] + h.c.\,, \tag{B.2.4}$$

$$\mathcal{L}_{Higgs} = (\nabla_\mu\Phi)^\dagger\nabla^\mu\Phi + \mu^2\Phi^\dagger\Phi - \frac{\lambda}{3!}(\Phi^\dagger\Phi)^2\,, \tag{B.2.5}$$

$$\mathcal{L}_{gauge} = -\frac{1}{2}Tr(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}Tr(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\,. \tag{B.2.6}$$

En el SM, el campo escalar adquiere un vev a través del mecanidmo de Higgs, y por lo tanto los términos de Yukawa se convierten en términos de masa para los quarks u, d y los leptones eléctricamente cargados l, pero no para los neutrinos ν. Por tanto, los neutrinos son partículas sin masa ni carga eléctrica que sólo interaccionan débilmente (intercambiando bosones $W^\pm$ ó Z, que son combinaciones de los bosones W y B). En un experimento que involucre a un leptón cargado l_α, la presencia de un neutrino ν_α como un estado final se percibirá como una violación de la conservación de energía y momento.

Estas partículas generalmenete se detectan en laboratorios subterráneos, en los que la presencia de partículas que interactúan más intensamente se amortigua en la corteza terrestre.[15] Su detección suele involucrar el intercambio de un bosón $W^\pm$ y la detección del leptón cargado l_α asociado al neutrino entrante ν_α.

Por tanto, si un ν_α se produce en algún proceso que involucre l_α, el neutrino sin masa y débilmente interactuante puede propagarse enormes distancias hasta ser detectado por otro proceso que involucre el mismo l_α. Ésta era la idea en que se basaba el experimento Homestake dirigido por Raymond Davis Jr. y John N. Bahcall [132]. El experimento diseñado por Davis consiguió medir el flujo de ν_e procedentes del sol como resultado de las reacciones nucleares que ocurren en su núcleo. El objetivo del experimento era confirmar las predicciones teóricas, derivadas por Bahcall, del modelo estándar del sol. Sin embargo, ¡el experimento midió un tercio de la cantidad esperada de neutrinos! A esto se le llamo el 'problema de los neutrinos solares'. En aquel tiempo era imposible decidir si el problema estaba en la física de neutrinos o en el entendimiento de los mecanismos solares de la época.

B.2.1. Una explicación convencional: ¡los neutrinos sí tienen masa!

Pontecorvo propuso por primera vez [133] que los neutrinos podían cambiar su naturaleza (sabor) a lo largo de su propagación a través de largas distancias en el mismo sentido en que lo hace el sistema de kaones neutros. Llamó a este proceso 'oscilaciones de neutrinos'. Esta explicación ajustaba bien los resultados de Homestake, pero tenía profundas implicaciones [134].

Este fenómeno es impactante porque no puede tener lugar en el contexto del SM. Cuando un neutrino es producido en una cierta interacción contenida en el SM, se produce por definición en un estado que es un autoestado de sabor con momento definido: (ν_e si la interaccion involucra un electrón, ν_μ si la interacción involucra un muón, o ν_τ si la interacción involucra un τ). En el SM, la energía de estos estados E_α es degenerada y depende sólo del momento, $E_\alpha = p$. Luego si un

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[15]Esto se debe a que a energías por debajo de la escala electrodébil los neutrinos interactúan muy débilmente y de otra manera la razón señal-ruido sería muy baja. Sin embargo, es posible detectar neutrinos en aceleradores, en los que la fuente es muy luminosa, muy energética y controlable. De nuevo la detección de neutrinos producidos en aceleradores en instalaciones lejos del acelerador vuelve a requerir que la instalación sea subterránea para que la razón señal-ruido mejore.

neutrino se produce en un autoestado de sabor, éste es también un autoestado de propagación y será detectado en el mismo autoestado de sabor, es decir, su interacción con la materia produce el mismo leptón cargado que estaba involucrado en su creación.

Para que ocurran oscilaciones de neutrinos, deben concurrir dos circunstancias. Primero, es necesario que existan diferentes autoestados del Hamiltoniano efectivo de los neutrinos con diferentes energías E_i para el mismo momento $\mathbf{p}$. Llámese a los autoestados de propagación de los neutrinos $\{\nu_1, \nu_2, \nu_3\}$ con energías $\{E_1(p), E_2(p), E_3(p)\}$. En una teoría invariante Lorentz la única posibilidad de que los neutrinos tengan diferentes energías para el mismo momento es que tengan diferentes masas m_i: $E_i = \sqrt{\mathbf{p}^2 + m_i^2}$. Por tanto habitualmente se dice que las oscilaciones de neutrinos implican que los neutrinos tienen masa.

Segundo, es necesario que estos autoestados no coincidan con los autoestados de sabor. La razón es que los autoestados del Hamiltoniano efectivo (estados de energía definida para un momento dado) coinciden con los autoestados de propagación , que no deben coinidir con los autoestados de sabor para que se produzca una mezcla. Esto es bastante natural en QFT; de hecho existe un ejemple en la física del sabor de los quaks, parametrizado por la matriz de Cabibbo-Kobayashi-Maskawa, que conecta los autoestados de masa de los quarks con carga negativa con sus autoestados de "sabor", definidos como los estados resultantes de la interacción por corriente cargada débil con un quark positivamente cargado. En analogía, una matriz unitaria (llamada matriz de Pontecorvo-Maki-Nakagawa-Sakata o matriz PMNS [135]) conecta los autoestados de masa de los neutrinos ν_i con sus autoestados de sabor ν_α,

$$\nu_\alpha = U_{\alpha i} \nu_i \,, \tag{B.2.7}$$

que se definen como los estados resultantes de la interacción por corriente cargada débil de un leptón cargado l_α. Este estado es una combinación lineal de estados con diferentes energías, que se propagan de manera diferente.

No obstante, si existen diferentes estados con diferentes energías, estrictamente no es cierto que el neutrino se produzca en un autoestado de sabor, debido a la conservación de energía y momento (a no ser que las leyes de conservación de energía y momento sean sólo una aproximación para tranferencias de energía-momento suficientemente grandes). Si rechazamos esta posibilidad lo que ocurre es que en la producción, un ν_1, ν_2 ó ν_3 puede aparecer con diferentes probabilidades que suman uno, y que dependen de la naturaleza del leptón cargado involucrado en la producción. Similarmente, cualquiera de estos neutrinos producirá, en su interacción con un detector, uno cualquiera de los tres leptones cargados, con diferentes probabilidades dependientes de la naturaleza del neutrino que se ha detectado.

Se supone que esta masa es una escala IR, es decir, que es mucho menor que la energía del neutrino, y por lo tanto la diferencia de energía entre dos autoestados de masa será mucho menor que la energía total de cualquiera de los estados, y que las oscilaciones tienen importancia sólo en la propagación de los neutrinos a través de muy largas distancias. También, el efecto de la masa en la energía total del neutrino normalmente se desprecia, y por tanto se dice habitualmente que los neutrinos se producen y detectan en autoestados de sabor definido.

Cuando se define una partícula en una teoría con un grupo de simetría, viene caracterizada por los autovalores de los operadores invariantes (bajo el grupo de simetría) del grupo que pueden definirse combinando elementos de ese grupo. En el caso de una teoría invariante Lorentz, estos operadores son la masa y el espín de una partícula. Las partículas también tienen cargas definidas bajo el grupo de gauge. En el caso de los neutrinos, estos estados no son los autoestados de sabor, ya que no tienen masa definida, sino los autoestados de propagación $\{\nu_1, \nu_2, \nu_3\}$. Por tanto, aunque los autoestados de sabor pueden definirse formalmente en el contexto de una teoría que exhibe oscilaciones de neutrinos, es un abuso de notación llamarlos partículas.

Por consiguiente, es necesario ampliar el SM para incluir al menos una masa para los neutrinos. El Modelo Estándar de Oscilaciones de Neutrinos supone que existe un nuevo campo, el neutrino estéril ν_i^R, que transforma bajo la representación trivial del grupo de gauge completo y que es necesario para incluir términos de masa perturbativamente renormalizable en el Lagrangiano (ver Ref. [136] para una revisión). Los nuevos términos que han sido añadidos al Lagrangiano del SM son:

$$-\mathcal{L}_{\nu_R} = \bar{\nu}_i^R \gamma^\mu \partial_\mu \nu_i^R + Y_{i\alpha}^D \tilde{\Phi}^\dagger \bar{\nu}_i^R \gamma^\mu \partial_\mu \Psi_\alpha^L + m_{ij}^M \bar{\nu}_i^R \nu_j^{R,c} + h.c., \tag{B.2.8}$$

donde ν^R es el singlete del neutrino estéril, con índice de sabor i que toma valores de 1 a n; Y^D es una matriz adimensional de coeficientes en el espacio de sabores, que dará lugar a un término de masa de Dirac después de que el Higgs adquiera su vev; y m^M es una matriz de masa de Majorana permitida por todas las simetrías (el único término de su clase en todo el Lagrangiano).

Este término lleva al llamado 'mecanismo del balancín'[16]. Supóngase que los neutrinos estériles son una señal de nueva física a alguna escala UV E_{NP} unos pocos órdenes de magnitud por debajo de la escala de Planck. Entonces, la matriz m^M tendrá componentes naturalmente de ese orden de magnitud, mientras que el término de masa de Dirac de los neutrinos será de manera natural del orden del vev del campo escalar. Entonces, la diagonalización de la matriz total resultante llevará a $3+n$ autoestados con diferentes masas. n de ellos serán prácticamente estériles, tendrán una masa comparable a la escala UV, y estarán muy débilmente acoplados a los leptones cargados. Los otros tres serán prácticamente zurdos y tendrán una masa del orden de v^2/E_{NP}. [17] Todas estas partículas son fermiones de Majorana, es decir, son sus propias antipartículas, al contrario que los fermiones de Dirac, como los leptones cargados o los quarks.

En este caso el SM con neutrinos de Majorana es una buena EFT [20] a energías por debajo de la masa del neutrino estéril. En este caso, operadores de dimensión mayor que 4 no pueden ser omitidos, pero vienen corregidos con la potencia negativa apropiada de la escala UV. Entonces las primeras correcciones al SM deberían venir de términos con dimensión 5. Si calculamos todos los términos de dimensión 5 compatibles con las simetrías gauge y Poincaré, el único término que debemos añadir al Lagrangiano es

$$-\mathcal{L}_{NP} = \frac{Z_{ij}}{E_{NP}} \bar{\Psi}_i^L \tilde{\Phi} \tilde{\Phi}^T \Psi_j^{L,c} + h.c.. \tag{B.2.9}$$

De hecho, el resultado final nos dice que si se espera nueva física a la escala E_{NP}, se debería esperar que esta nueva física proporcionase un término de masa efectivo para los neutrinos.

La propuesta de que los neutrinos masivos deberían bastar para explicar el problema de los neutrinos solares gracias al mecanismo de oscilaciones de neutrinos en su propagación desde el Sol hasta la Tierra sólo necesitaba ser contrastada con datos más recientes. El llamado problema de los neutrinos solares fue estudiado con detectores similares como Kamiokande, SAGE, GALLEX, Superkamiokande y el Sudbury Neutrino Observatory (SNO). SNO fue el primer experimento que demostró que las oscilaciones de neutrinos estaban detrás del problema de los neutrinos solares [138], aunque las oscilaciones de neutrinos fueron detectadas primero por Superkamiokande [139]. En el último caso, la oscilación se producía en los neutrinos procedentes de la desintegración de muones en la atmósfera (neutrinos atmosféricos). Las oscilaciones de neutrinos producidas en la

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[16]*Seesaw mechanism* en el original (N. del T.)

[17]La idea encaja bien en el marco de GUT con grupo de simetría $SU(5)$, en el que la escala de gran unificación es $M_{GUT} \sim 10^{15}\, GeV$; sin embargo, la ausencia de desintegración de protones [137] en la ventana predicha ha refutado las GUT más simples.

propagación de neutrinos generados en reactores nucleares artificiales [140] y en desintegraciones nucleares en la corteza terrestre (geo-neutrinos) [141] han sido medidos también por KamLAND.

Los mencionados experimentos han producido una gran cantidad de datos sobre las oscilaciones de los neutrinos. Las oscilaciones de neutrinos solares se ven afectadas por le llamado efecto MSW (oscilaciones de neutrinos debidas a su interacción con la materia), así que no proporcionan una descripción limpia de las oscilaciones de neutrinos en el vacío. Los experimentos de neutrinos atmosféricos han observado desaparición de ν_μ con $\Delta m^2 \sim 3 \times 10^{-3} eV^2$ [139], mientras que los experimentos de antineutrinos de reactor han medido desaparición de $\bar{\nu}_e$ a $\Delta m^2 \sim 8 \times 10^{-5} eV^2$ [140, 141]. La descripción conjunta mínima de neutrinos atmosféricos, solares y de reactor necesita neutrinos de Dirac con diferentes masas

$$m_1^2 - m_2^2 = 7{,}9 \pm 1{,}1 \times 10^{-5} eV^2 \qquad (B.2.10)$$

$$|m_1^2 - m_3^2| = 2{,}6 \pm 0{,}6 \times 10^{-3} eV^2 \qquad (B.2.11)$$

y los cuatro grados de libertad independientes de la matriz PMNS[18], que han sido determinados [136]. De hecho, la fenomenología de neutrinos masivos de Dirac depende únicamente de los cuadrados de los valores absolutos de los elementos de la matriz, a los que llamaremos 'poblaciones' y que denotaremos por

$$r_{i\alpha} = |U_{\alpha i}|^2 = \begin{pmatrix} 0{,}69 \pm 0{,}03 & 0{,}31 \pm 0{,}03 & < 0{,}01 \\ 0{,}16 \pm 0{,}07 & 0{,}36 \pm 0{,}11 & 0{,}46 \pm 0{,}11 \\ 0{,}14 \pm 0{,}08 & 0{,}30 \pm 0{,}10 & 0{,}52 \pm 0{,}12 \end{pmatrix} . \qquad (B.2.12)$$

La población $r_{i\alpha}$ representa la proporción del estado ν_α en un estado ν_i. Aunque existen nueve poblaciones en un problema de mezcla de tres estados, la condición de suma-uno hace que sólo cuatro de ellas sean linealmente independientes. En el Modelo Estándar de oscilaciones de neutrinos, estos parámetros son constantes, pero en un problema de oscilaciones más general podrían ser funciones del momento **p**.

Parece que los neutrinos masivos se ajustan a los datos experimentales correctamente. ¿Cuál es entonces el problema con esta explicación?

B.2.2. Las dudas que surgen con los neutrinos masivos

La anomalía LSND y otros resultados experimentales sugerentes Sin embargo, no todos los experimentos que se están llevando a cabo tienen resultados bien descritos por este modelo simple. El Liquid Scintillator Neutrino Detector (LSND) publicó una evidencia de $\bar{\nu}_\mu \to \bar{\nu}_e$ a $\Delta m^2 \sim 1 eV^2$ [142, 143]. Este resultado ha sido confirmado más tarde por el mismo grupo [144, 145], que ha invertido enormes esfuerzos en encontrar fuentes de error experimental. Su última toma de datos explora oscilaciones tanto de $\bar{\nu}_\mu \to \bar{\nu}_e$ como de $\nu_\mu \to \nu_e$ en una ventana de energía de $20\,MeV < p < 200\,MeV$.

Para poder confirmar o refutar los resultados de la colaboración LSND, las colaboraciones KARMEN y MiniBOone han construido experimentos similares. La colaboración MiniBOone ha publicado la no evidencia de oscilaciones $\nu_\mu \to \nu_e$ para haces de neutrinos con $p > 475\,MeV$ [146, 147] pero ha encontrado un diminuto exceso de ν_e en haces con $200\,MeV > p > 475\,MeV$ [147] (aunque este exceso es incompatible con el mecanismo de oscilaciones propuesto por LSND). El mismo grupo a realizado la búsqueda de $\bar{\nu}_\mu \to \bar{\nu}_e$ a $200\,Mev < p < 3000\,MeV$, pero no se ha

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[18]Esto es cierto para neutrinos de Dirac. En el caso de neutrinos de Majorana, hay seis parámetros independientes en la matriz PMNS.

encontrado una señal significativa [148]. Estos resultados han sido calificados de 'no concluyentes' [148].

Más aún, si se considera que las masas de neutrinos surgen en el marco de una EFT con nueva física en el UV (quizá cerca de $10^{15}\,GeV$), entonces el correspondiente operador de dimensión 5 que da masa a los neutrinos es un término de masa de Majorana que viola la conservación del número leptónico en dos unidades. Este resultado implica nuevos fenómenos aún inobservados relacionados con la no conservación del número leptónico, como la desintegración doble beta sin neutrinos ($0\nu\beta\beta$). Estos experimentos también son interesantes porque, si se observa $0\nu\beta\beta$, esto proporcionará una manera de determinar directamente el valor absoluto de la masa del neutrino (en contraste con los valores relativos propocionados por las oscilaciones de neutrinos). La búsqueda de este fenómeno se ha venido realizando en los últimos años rodeada de controversia.

Un subgrupo de la colaboración Heidelberg-Moscow liderado por H. V. Klapdor-Kleingrothaus publicó una serie de artículos controvertidos proclamando haber observado una desintegración doble beta sin neutrinos en ^{76}Ge con nivel de confianza de 6σ [149, 150, 151]. Esta "evidencia" ha sido discutida calurosamente [152], y no es aceptada por la amplia mayoría de la comunidad científica. Su supuesta evidencia, en la que el valor esperado de la matriz de masa de neutrinos en el estado ν_e es $m_\beta \sim 0{,}2 - 0{,}5\,eV$ todavía debe ser confirmada por otros experimentos. Incluso si la 'evidencia de Klapdor' resulta ser falsa, los experimentos de $0\nu\beta\beta$ nos darían una estimación del valor absoluto de las masas de neutrinos en el futuro (¿próximo?) si los neutrinos son partículas de Majorana: sería simplemente una cuestión de ser sensible a masas menores.

Sin embargo si estos experimentos persisten sin una señal positiva de $0\nu\beta\beta$, entonces la solución a la ausencia de fenómenos que violan el número leptónico es imponer la conservación del número leptónico a mano en el modelo (la conservación del número leptónico es sólo una simetría accidental del modelo estándar, pero deja de ser una simetría si se permiten términos de dimension mayor que 5 en el Lagrangiano), de manera que $m^M = 0$. Si $n = 3$, los neutrinos estériles pueden verse como las componentes diestras de un espinor de Dirac (y por tanto los neutrinos estériles de Dirac se llaman 'neutrinos diestros'). Entonces, los procesos que violan el número leptónico estarían prohibidos. No obstante, el motivo de la pequeñez de la masa del neutrino quedaría oculto, empeorando el problema dejerarquías presente en el SM.

Además de la $\beta\beta0\nu$, los experimentos de desintegración beta en general también son capaces de determinar la escala absoluta de las masas de los neutrinos. Sin embargo, la escala esperada de las masas de los neutrinos está muy por debajo de la sensibilidad de muchos de los experimentos. Recientemente, un puñado de experimentos como el Troitsk Neutrino Mass Experiment [153] y el Mainz Neutrino Mass Experiment [154] han tratado de medir la cola del espectro de la desintegración beta del tritio con una precisión enorme. Estos experimentos se basan en el pequeño intercambio de energía en una desintegración beta de nucleos ligeros ($18{,}6\,keV$ en el caso del tritio). Algunos experimentos han determinado que $m_\beta < 2{,}3\,eV$, pero, sorprendentemente, el mejor ajuste de las a los datos se produce en el sector de una cuadrado de la masa negativo. En particular, el Troitsk Experiment da como resultado $m_\beta = -2{,}3 \pm 2{,}5(stat.) \pm 2{,}0(syst.)\,eV$, mientras que el experimento de Mainz da $m_\beta = -0{,}6 \pm 2{,}2(stat.) \pm 2{,}1(syst.)\,eV$. Para poder trabajar con reaultados más precisos, el Karlsruhe Tritium Neutrino Experiment (KATRIN) [155], que espera alcanzar una sensibilidad de $m_\beta > 0{,}2\,eV$, ha empezado a recoger datos.

¿Son los neutrinos masivos una explicación demasiado conservadora? Ha habido varios intentos de conciliar los datos de LSND y MiniBOone con los otros experimentos de neutrinos simplemente a base da añadir más neutrinos estériles al modelo. En este punto uno debería preguntarse, si es así como uno debería proceder. Teniendo en cuenta que los neutrinos estérles son prácticamente indetectables, y que tenemos muy poca evidencia de su existencia, ¿no estamos

simplemente añadiendo "epiciclos" al SM? ¿O es que estamos llevando al SM más allá de su rango de validez y deberíamos buscar física de partículas radicalmente nueva?

En la sección anterior sobre el problema de la constante cosmológica hemos mencionado que QFT debería modificarse en el IR para poder ser consistente con los resultados de la entropía de Bekenstein o con el valor observado de la constante cosmológica. De acuerdo con este punto de vista, hay una extraña coincidencia que no debería pasarse por alto. Si consideramos que la constante cosmológica es la energía de vacío de una QFT, el valor de esta energía de vacío es aproximadamente $\rho_v \sim (10^{-2,5}\,eV)^4$. En comparación, la escala de las oscilaciones de neutrinos (en este caso, la escala detrás de las oscilaciones de neutrinos solares y de reactor en el SM con neutrinos masivos) es del orden de $\Delta m_\odot^2 \sim (10^{-2,5}\,eV)^2$.

Nos preguntamos si estos números no estarán relacionados o si se derivan de la misma ampliación de teoría de campos relativista local. Podría ser que la mísma física IR provocase que las correcciones cuánticas a la energía del vacío fuesen del orden de una escala IR $\lambda \sim 10^{-2,5}eV$ y también una modificación en el propagador de los neutrinos del mismo orden. Si esto fuese cierto, hemos argumentado que la correspondiente ampliación de QFT debería ser no local, y por lo tanto podría ocurrir incluso que la descripción de las oscilaciones de neutrinos requiriese algún tipo de no localidad.

B.2.3. Ingredientes para una alternativa a los neutrinos masivos

Los mecanismos altenativos a las oscilaciones de neutrinos debidas a masas deben compartir una característica común: deben incluir algún nuevo ingrediente, que debe tener distintos autovalores para diferentes tipos de neutrino, y sus correspondientes autoestados no deben coincidir con los autoestados de sabor, es decir, el acoplo de dichos tipos de neutrino a los leptones cargados mediante intercambio de W debe ser no diagonal.

Supóngase que el mecanismo responsable de los fenómenos que hemos detallado anteriormente es un mecanismo de oscilación durante la propagación del neutrino. Otros posibles mecanismos podrían ser acoplos no estándar al potencial gravitatorio (violación del principio de equivalencia), a una tosión, o incluso a la materia.

Violación de Invariancia Lorentz Pensemos en si puede ir más allá del mecanismo de oscilaciones debidas a masas. Hemos dicho que el mecanismo de oscilaciones puede aparecer sólo si diferentes tipos de neutrinos tienen diferentes energías $E_i(p)$ para el mismo momento p. La invariancia Lorentz restringe estas energías distintas a ser consecuencia de tener diferentes masas. Por tanto, si queremos ir más allá del mecanismo de oscilaciones debidas a masas, el precio a pagar es abandonar la invariancia Lorentz. Dado que el teorema CPT asegura que CPT es una simetría de cualquier teoría local invariante Lorentz, entonces también es posible que CPT se viole. Aunque la violación de CPT es problemática desde el punto de vista de la autoconsistencia, puede venir muy bien para explicar las discrepancias entre LSND y MiniBOone.

Se ha argumentado que la Violación de Invariancia Lorentz (LIV) es un fenómeno esperable en una teoría de gravedad cuántica. Surge de manera nautral en LQG a escalas en las que la discretización del espacio-tiempo no puede despreciarse; la invariancia Lorentz se recupera como una simetría aproximada a bajas energías. En este caso la desviación de la invariancia Lorentz viene parametrizada por la longitud de Planck. La relación de dispersión de las partículas refleja la escala del tejido espacio-temporal, y toma la forma [156]

$$E^2 = \mathbf{p}^2 + m^2 + \eta \mathbf{p}^2 \left(\frac{E}{E_p}\right)^n,$$
(B.2.13)

donde m es la masa de la partícula prueba, η es un parámetro fenomenológico, presumiblemente de orden 1, y se supone que $n > 0$ es entero. Este es el tipo de modelos que son contrastados con las observaciones en las comprobaciones de invariancia Lorentz. Son explícitamente isótropos, ya que se considera es la simetría bajo boost la que está rota en gravedad cuántica. La razón es la presencia de una escala de muy alta energía (típicamente la escala de Planck E_p, aunque esto puede variar según el modelo) que es o bien un invariante de la teoría o bien un valor en el que todos los observadores deberían estar de acuerdo.

LIV puede surgir también en ST como resultado de una ruptura espontánea de la simetría (SSB). Para que ocurra una SSB Lorentz, es necesario que un tensor Lorentz adquiera un término de masa negativo. En una teoría relativista perturbativamente renormalizable, es imposible llegar a tal término de masa efectiva negativa para ningún campo vectorial.

Sin embargo, éste no es el caso en ST [157]. Por ejemplo, en la cuerda bosónica abierta, es posible que un campo vectorial B_μ adquiera un vev debido a un potencial efectivo V en el Lagrangiano $\mathcal{L}_B$ del tipo

$$\mathcal{L}_B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - V(B_\mu B^\nu \pm b^2) - B_\mu J^\mu \,, \tag{B.2.14}$$

donde $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ es la intensidad del campo, $V(x) = \frac{1}{2}\lambda x^2$ es el potencial y J^μ es la corriente. Dependiendo del signo que aparece en el argumento del potencial, el vev del campo 'abejorro'[19] B_μ será tipo espacio o tipo tiempo (el caso de un valor esperado en el vacío de tipo luz está mal definido). Es habitual tomar el vev de este campo, b^μ, como independiente de las coordenadas espaciales para que las simetrías rotas sean simetrías Lorentz globales, aunque también son posibles soluciones solitónicas.

En el mismo sentido en el que la simetría Lorentz puede estar espontáneamente rota en ST, también es posible encontrar modelos en los que CPT esté espontáneamente rota [158, 159]. Teniendo en cuenta esta ruptura espontánea, es razonable considerar que el límite de baja energía de dicha teoría de gravedad admite una descripción como QFT ordinaria incluyendo desviaciones de la invariancia Lorentz y puede estudiarse en el marco de EFT.

En la Ampliación del Modelo Estándar (SME) [160, 161], el punto de partida es el SM mínimo y su escala asociada, la escala electrodébil m_W. Nuevos términos con LIV y posiblemente violación de CPT[20] aparecen procedentes de una SSB Lorentz en una teoría más fundamental, y se suponen suprimidos al menos por la razón $\frac{m_W}{E_p} \approx 10^{-17}$. Los nuevos términos también respetan la invariancia gauge del SM. Una ampliación mínima puede definirse incluyendo sólo términos perturbativamente renormalizables (como en la Ref. [161]), mientras que términos con dimensión mayor que cuatro pueden incluirse utilizando las técnicas de EFT [163].

Tanto en el caso de una ruptura espontánea o explícita de de la invariancia Lorentz, se reduce el grupo de simetría y por tanto aparece un sistema de referencia privilegiado. Este sistema de referencia se identifica habitualmente con el sistema de referencia en el que el CMB es casi isótropo [164]. Sin embargo el sistema de referencia de laboratorio no coincide con el sitema de referencia del CMB; una consecuencia obvia es la necesidad de deshacerse del dipolo que aparece al observar el expectro de CMB, ya que sólo aporta información sobre el movimiento relativo de ambos sistemas de referencia [10]. Este impulso entre el sistema de referencia privilegiado (en el que las leyes de la física se suponen isótropas) y el sistema de referencia del laboratorio inducen nuevos efectos direccionales, y por tanto una anisotropía de las leyes de la física en el sistema

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[19]*Bumblebee field* en el original. (N. del T.)
[20]El teorema CPT asegura que una teoría local invariante Lorentz preserva CPT [162]. Luego una teoría local que viola CPT viola automáticamente Lorentz. El inverso no es cierto; existen términos que violan Lorentz pero no CPT.

de referencia del laboratorio. Por consiguiente es necesario considerar acoplos anisótropos en la SME para poder construir modelos realistas.

Afortunadamente, los movimientos rotacional y traslacional de la Tierra son esencialmente no relativistas en el sistema de referencia aproximadamente inercial centrado en el sol, relevante para los experimentos de neutrino. Así que en la prácitca cualquier efecto dependiente de la dirección estará suprimido en una parte por millón o más [165]. Los efectos anisótropos podrían excluirse también si en vez de una LIV hubiese un nuevo principio de simetría, Relatividad Doblemente Especial (DSR), reemplazase a la invariancia Lorentz [166, 167, 168, 169].

La posibilidad de oscilaciones de neutrinos producidas o corregidas por LIV ha sido considerada en varios trabajos que se detallan a continuación.

Acuerdo con los datos experimentales Como debería ser siempre en cualquier campo de la ciencia, será nuestra prioridad encontrar alternativas a oscilaciones debidas a masas que sean completamente consistentes con los resultados experimentales. Se han propuesto muchos modelos de LIV causantes de oscilaciones de neutrinos, pero pocos de ellos han sido capaces de explicar el comportamiento con la energía que se ha observado experimentalmente.

Para poder ajustar los datos experimentales existentes (fuera de los controvertidos resultados de LSND y $0\nu\beta\beta$), es suficiente que estos modelos cumplan una propiedad: que el nuevo efecto sea una corrección proporcional al cuadrado de una escala IR $\lambda \sim 1 - 10^{-3} eV$ a energías $E \gg \lambda$.[21] Esto se debe al hecho de que todos los experimentos de neutrinos que se han llevado a cabo han utilizado neutrinos de energías enormes (comparadas con la escala λ).

Encontrar esta dependencia en la energía en sus modelos no ha sido fácil para los investigadores de LIV en neutrinos. En uno de los trabajos presentados en esta tesis doctoral se mostrará que es posible encontrar este comportamiento si se recurre a física no local en el IR, como se sugirió en el apartado de soluciones al problema de la constante cosmológica.

También la cosmología impone importantes cotas a una alternativa a neutrinos con masa, como también impone una cota competitiva a la suma de las masas de los neutrinos en el caso de neutrinos masivos [171, 172].

Para poder tener más información sobre el comportamiento a baja energía de los neutrinos, será necesario obtener información directa o indirectamente sobre neutrinos a energías cercanas a esta escala, como experimentos de desintegración del tritio cerca de la cola del espectro o la detección del Fondo Cósmico de Neutrinos (CνB), que no ha sido aún observado. Estos dos experimentos son completamente irrealizables en el futuro cercano. No obstante, quizá el efecto del CνB en el espectro de potencias de la estructura a gran escala (LSS) pueda ser observable.

Como conclusión, aunque las masas de neutrinos sean una modificación simple del SM perfectamente capaz de describir los datos de oscilaciones de neutrinos actuales (sin contar los pobremente entendidos LSND y MiniBOone), no es la única posibilidad. Una alternativa al mecanismo de oscilaciones debidas a masas requiere LIV, pero también debe ser consistente con los datos experimentales y observacionales. Una condición suficiente es que la deformación de la relación de dispersión de los neutrinos involucre el cuadrado de una escala de energía IR.

B.2.4. Revisión de alternativas a neutrinos masivos

En esta subsección se repasarán algunos de los intentos de ofrecer una descripción alternativa a las oscilaciones de neutrinos sin recurrir a términos de masa. Como se ha mencionado eso require ir más allá de la Relatividad Especial (SR). Se ha sugerido que las oscilaciones de neutrinos son

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[21]Sorprendentemente, el modelo 'tándem' presentado en Ref. [170] consigue ajustar los datos sin recurrir a este mecanismo.

un buen terreno donde explorar nueva física más allá de SR en la Ref. [164], quizá surgiendo en el límite de baja energía de una teoría de gravedad cuántica. Un ejemplo popular es el de un agujero negro cuántico que absorbe un neutrino de un sabor y emite un neutrino de un sabor diferente [173].

En particular, se ha dicho que la adición de términos que violan Lorentz pero respetan CPT al modelo estándar induciría diferencias en las máximas velocidades de propagación alcanzables por los neutrinos, que podrían depender de su naturaleza y helicidad. También se dijo inicialmente que los neutrinos sin masa podrían oscilar en el vacío en presencia de LIV. Se han construido modelos más complicados para poder ajustar la gran cantidad de datos que ha sido recogida en la última década.

Oscilaciones debidas a velocidades Coleman y Glashow presentaron un modelo simple de oscilación entre el neutrino electrónico y el neutrino muónico [164],

$$\nu_\mu = \cos\theta\nu_1 + \sin\theta\nu_2 \,, \tag{B.2.15}$$

$$\nu_e = \cos\theta\nu_2 - \sin\theta\nu_1 \,, \tag{B.2.16}$$

donde ν_1 y ν_2 son los autoestados de propagación con diferentes velocidades y relaciones de dispersión

$$E_1 = p(1 + v_1) \,, \ E_2 = p(1 + v_2) \,, \tag{B.2.17}$$

y θ es una fase que caracteriza la mezcla. Se supone que las diferencias entre la velocidad del neutrino y la velocidad de la luz, v_i, son $|v_i| \ll 1$. Las poblaciones de los autoestados de sabor en los autoestados de propagación son

$$r_{1\mu} = r_{2e} = \cos^2\theta \,, \ r_{1e} = r_{2\mu} = \sin^2\theta \,, \tag{B.2.18}$$

y la probabilidad de que un neutrino electrónico atraviese una distancia R sin cambiar de sabor es

$$P_{e\to e}(R) = r_{1e}\sin^2\left((v_1 - v_2)pR/2\right) \,, \tag{B.2.19}$$

mientras que la misma probabilidad, si el mecanismo de oscilaciones está producido por diferencias de masa, es

$$P_{e\to e}(R) = r_{1e}^m\sin^2\left((m_1^2 - m_2^2)R/4p\right) \,. \tag{B.2.20}$$

También es posible que tanto una diferencia de masas como de máxima velocidad de propagación alcanzable estén detras de las oscilaciones. En este caso, el problema de diagonalización se hace mucho más complicado y dependiente del momento. El Hamiltoniano efectivo tendría la forma

$$\mathcal{H}_{\alpha\beta} = p\left(\delta_{\alpha\beta} + V_{\alpha\beta}\right) + \frac{1}{2p}M_{\alpha\beta}^2 \tag{B.2.21}$$

en la base de autoestados de sabor. La diagonalización del Hamiltoniano efectivo debería realizarse para cada momento p del neutrino, resultando una mezcla del tipo

$$P_{e\to e}(R) = r_{1e}(p)\sin^2\left((E_1(p) - E_2(p))R/2\right) \,. \tag{B.2.22}$$

Los límites de momento bajo y alto de esta expresión deberían coincidir con las expresiones para las oscilaciones debidas a masas (B.2.20) y para las oscilaciones debidas a velocidades (B.2.19), respectivamente.

La distinción clara entre el comportamiento con el momento p de (B.2.19) y (B.2.20) ha permitido a los físicos experimentales comprobar que el mecanismo de oscilaciones debidas a masas está favorecido con respecto a las oscilaciones debidas a velocidades [136], al menos en el modelo en que la velocidad de los neutrinos es independiente de su frecuencia. En general, una desviación del comportamiento con la energía de (B.2.20) sería una señal inequívoca de LIV [174].

Es por esto que en la mayor parte de los estudios de LIV en física de neutrinos se considera la LIV como un mecanismo subdominante que puede investigarse dentro del marco de oscilaciones debidas a masas. En algunos de estos estudios [175, 174, 176, 177, 178] se ha estudiado la LIV a nivel de una deformación fenomenológica de la relación de dispersión del tipo (B.2.13).

La Ampliación del Modelo Estándar y las oscilaciones de neutrinos   Como se ha mencionado anteriormente, el modelo SME mínimo incluye todos los términos no invariantes Lorentz con o sin violación de CPT que son perturbativamente renormalizables y respetan las simetrías gauge del SM. Los nuevos términos se pueden dividir entre términos que conservan CPT (CPT-pares) y que violan CPT (CPT-impares). Se supone que los nuevos términos están suprimidos con respecto a los términos invariantes Lorentz del mismo tipo por potencias de la escala electrodébil sobre la masa de Planck. En este sentido, algunos de los nuevos términos con dimensiones pueden interpretarse como nuevas escalas IR.

Como un ejemplo, daremos la forma explícita de los términos del sector fermiónico. En el sector leptónico, los nuevos términos son:

$$\mathcal{L}_{lepton}^{CPT-even} = \frac{i}{2}(c_L)_{\mu\nu\alpha\beta}\bar{\Psi}_{L\alpha}\gamma^\nu\overleftrightarrow{\nabla}_\mu^\nu\Psi_{L\beta} + \frac{i}{2}(c_R)_{\mu\nu\alpha\beta}\bar{L}_\alpha\gamma^\nu\overleftrightarrow{\nabla}_\mu^\nu L_\beta\,, \qquad (B.2.23)$$

$$\mathcal{L}_{lepton}^{CPT-odd} = -(a_L)_{\mu\alpha\beta}\bar{\Psi}_{L\alpha}\gamma^\mu\Psi_{L\beta} - (a_R)_{\mu\alpha\beta}\bar{L}_\alpha\gamma^\mu L_\beta\,. \qquad (B.2.24)$$

Se entiende que todos los coeficientes c y a son matrices Hermíticas en el espacio de sabor además de tensores Lorentz. Estos nuevos términos son de dimensión 4 y 3, respectivamente. Por tanto los coeficientes c_L y c_R son adimensionales y pueden tener su parte simétrica y antisimétrica, aunque se puede eliminar su traza sin pérdida de generalidad. Los coeficientes a_L y a_R tienen dimensiones de energía. Si consideramos que estos vectores son tipo tiempo, en el sistema de referencia privilegiado las únicas componentes no nulas serán $(a_L)_0$ y $(a_R)_0$, y jugarán el papel de una escala IR.

En el sector quark los nuevos términos son análogos a los del sector leptónico, con matrices de coeficientes c_Q, c_U, c_D, ... Otros términos con LIV aparecen en los sectores de Yukawa, del Higgs y de los bosones de gauge del modelo.

El SME mínimo ha sido contrastado con una gran cantidad de experimentos, entre los cuales pueden destacarse los tests de alta precisión de QED, y las oscilaciones de neutrinos y de mesones neutros (ver Ref. [26] para un *Living Review* sobre cotas experimentales a LIV, ver Ref. [179] para las cotas experimentales a los parámetros del SME). A pesar de la larga lista de experimentos, un vasto sector del SME mínimo permanece aún inexplorado. Los experimentos de oscilaciones de mesones han acotado los coeficientes CPT-impares del sector quark, $a_\mu < 10^{-20}GeV$. Las cotas más restrictivas vienen de los tests de precisión de QED. Las cotas sobre un vev vectorial de tipo espacio son del orden de $10^{-25}GeV$ a $10^{-31}GeV$ dependiendo del experimento y del tipo de partícula.

El sector leptónico también ha sido contrastado con los datos procedentes de las oscilaciones de neutrinos. La posibilidad de que las oscilaciones estén producidas por una LIV surge en el modelo sin neutrinos estériles, aunque el modelo también ha sido ampliado para incluir neutrinos de Dirac con parte diestra por Kostelleckỳ y Mewes [180]. El SME completo con neutrinos diestros tiene muchos parámetros y por tanto se han construido varios modelos simples dentro de este marco para ganar predictividad. Estos modelos incluyen neutrinos masivos [180, 181, 170, 178] o sin masa [180, 182, 183, 181], en los que los coeficientes con LIV del SME son tratados como términos subdominantes o como causantes del fenómeno de oscilaciones, respectivamente.

Aparte de la mencionada dependencia anómala con la energía del fenómeno de oscilaciones, se espera que ocurran otros nuevos efectos si se tiene en cuenta el SME con neutrinos diestros [180]. Los términos anisótropos inducen variaciones periódicas en las oscilaciones, debido a los movimientos periódicos de rotación y translación de la Tierra. Estos efectos pueden eliminarse en los experimentos tomando promedios anuales, pero siempre quedaría como vestigio de la anisotropía una asimetría entre las direcciones este-oeste y norte-sur o entre el plano de la eclíptica y el eje norte-sur eclíptico. Otros posibles efectos que surgen en el SME son las violaciones de simetrías discretas, como nuevos efectos que violan la conservación del número leptónico (como oscilaciones neutrino-antineutrino), CPT (desviaciones de la relación $P_{\nu_\alpha \to \nu_\beta} = P_{\bar\nu_\beta \to \bar\nu_\alpha}$ o ambas. La violación de CPT es extremadamente interesante ya que puede proporcionar una explicación a la discrepancia entre los experimentos LSND [145] y MiniBoONE [147].

La ecuación de Dirac modificada más general para los campos de neutrinos en el SME con neutrinos diestros sería [180]

$$\left(i\Gamma^\nu_{\bar\alpha\bar\beta}\partial_\nu - M_{\bar\alpha\bar\beta}\right)\nu_{\bar\beta} = 0\,, \tag{B.2.25}$$

donde $\bar\alpha$ y $\bar\beta$ etiquetan los tres posibles sabores de neutrinos y sus conjugados de carga y

$$\Gamma^\nu_{\bar\alpha\bar\beta} \quad := \quad \gamma^\nu\delta_{\bar\alpha\bar\beta} + c^{\mu\nu}_{\bar\alpha\bar\beta}\gamma_\mu + d^{\mu\nu}_{\bar\alpha\bar\beta}\gamma_5\gamma_\mu + e^\nu_{\bar\alpha\bar\beta} + if^\nu_{\bar\alpha\bar\beta}\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}_{\bar\alpha\bar\beta}\sigma_{\lambda\mu}\,, \tag{B.2.26}$$

$$M_{\bar\alpha\bar\beta} \quad := \quad m_{\bar\alpha\bar\beta} + im_{5\bar\alpha\bar\beta}\gamma_5 + a^\mu_{\bar\alpha\bar\beta}\gamma_\mu + b^\mu_{\bar\alpha\bar\beta}\gamma_5\gamma_\mu + \frac{1}{2}H^{\mu\nu}_{\bar\alpha\bar\beta}\sigma_{\mu\nu}\,. \tag{B.2.27}$$

Los coeficientes m y m_5 son una combinación lineal de las matrices de masas de neutrinos diestros y zurdos m_R y m_L, y por tanto son invariantes Lorentz. Los coeficientes c, d, H son CPT-pares pero violan Lorentz, mientras que a, b, e, f, g son CPT-impares. La hermiticidad implica que todos los coeficientes son matrices hermíticas en el espacio de sabor.

El Hamiltoniano efectivo para los neutrinos del SME mínimo se lee

$$(\mathcal{H}_{eff})_{\bar\alpha\bar\beta} = \mathcal{H}_0 - \frac{1}{2}\left\{\gamma^0\delta\Gamma^0, \mathcal{H}_0\right\} - \gamma^0(i\delta\Gamma^j\partial_j - \delta M)\,, \tag{B.2.28}$$

donde $\mathcal{H}_0 = -\gamma^0(i\gamma^j\partial_j - M_0)$ es el Hamiltoniano general que conserva invariancia Lorentz, M_0 es la parte de M que conserva inviariancia Lorentz, y $\delta\Gamma$, δM son las partes que violan Lorentz de Γ, M. Un tratamiento general es posible pero bastante horrible, debido a la gran cantidad de parámetros y el hecho de que las matrices de coeficientes no necesariamente conmutan.

Por tanto el modelo debe simplificarse para poder ganar algo de poder predictivo. Es habitual desprenderse de los neutrinos diestros y tratar de construir un modelo para oscilaciones entre neutrinos activos (zurdos) y sus antipartículas. Se puede simplificar aún más si las oscilaciones entre neutrinos y antineutrinos $\nu \leftrightarrow \bar\nu$ están muy suprimidas. Entonces, el Hamiltoniano efectivo para los neutrinos zurdos ultrarrelativistas se reduce a

$$(h_{eff})_{\alpha\beta} = p\delta_{\alpha\beta} + \frac{1}{2p}\left(m_L^2\right)_{\alpha\beta} + \frac{1}{p}\left[(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu\right]_{\alpha\beta} \tag{B.2.29}$$

y su complejo conjugado para los antineutrinos, con el signo opuesto del coeficiente CPT-impar a_L. En la fórmula anterior m_L es la combinación de m y m_5 relevante para neutrinos zurdos (ver [180] para los detalles), $c_L = c + d$ y $a_L = a + b$. Se pueden construir modelos aún más simples para explicar los datos de oscilaciones.

Los modelos de 'Pollo Frito' (*sic*) [180] son aquellos en los que se impone isotropía espacial,

$$\left(h_{eff}^{FC}\right)_{\alpha\beta} = p\delta_{\alpha\beta} + \left[\frac{1}{2p}\left(m_L^2\right) + (a_L)^0 - \frac{4}{3}(c_L)^{00}p\right]_{\alpha\beta} \tag{B.2.30}$$

de manera que el estudio de las relaciones de dispersión modificadas no está complicado por la presencia de efectos direccionales. Muchos modelos de oscilaciones de neutrinos con LIV se incluyen en esta clase. Sin embargo, la isotropía del sistema de referencia del laboratorio es difícil de motivar en una teoría con LIV.

Por contra, los modelos anisótropos necesitan un sistema de referencia espacio-temporal bien definido, que a menudo se toma como el sistema de referencia sideral ecuatorial centrado en el Sol, con coordenadas $\{X, Y, Z, T\}$, para poder consensuar los valores de los coeficientes. Un modelo interesante de este tipo es el llamado 'Modelo Bicicleta' presentado en la Ref. [182], en el que la mayor parte de los datos de oscilaciones pueden explicarse sólo con dos parámetros libres y ningún término de masa.

Debe recordarse que el coeficiente a_L tiene dimensiones de masa y representa una escala IR, presumiblemente del orden de $\frac{m_W^2}{E_p}$. Si el coeficiente adimensional c_L es del orden de $\frac{m_W}{E_p}$, entonces para momentos muy por debajo de la escala electrodébil (y esto es válido en todos y cada uno de los experimentos de oscilaciones de neutrinos), la fuente dominante de oscilaciones en el modelo sin masas ($m_L = 0$) es el término con a_L. El argumento de la función seno que aparece en las oscilaciones será $\sim \Delta a_L L$ en vez del habitual $\Delta m^2 L/2E$. Este comportamiento está descartado por los datos experimentales.

No obstante, el comportamiento que uno ingenuamente espraría no es el que aparece en el Modelo Bicicleta. En este modelo, los coeficientes $c_{Lee}^{TT} = \frac{3}{2}\check{c}$ y $a_{Le\mu}^Z = a_{Le\tau}^Z = \frac{\check{a}}{\sqrt{2}}$ son los únicos que son distintos de cero. Las probabilidades de oscilación resultantes son

$$\begin{aligned}
P_{\nu_e \to \nu_e} &= 1 - 4\sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right), \tag{B.2.31}\\
P_{\nu_e \to \nu_\mu} &= P_{\nu_e \to \nu_\tau} = 2\sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right), \tag{B.2.32}\\
P_{\nu_\mu \to \nu_\mu} &= P_{\nu_\tau \to \nu_\tau} = 1 - \sin^2\theta\sin^2\left(\Delta_{21}L/2\right) \\
&\quad - \sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right) - \cos^2\theta\sin^2\left(\Delta_{32}L/2\right), \tag{B.2.33}\\
P_{\nu_\mu \to \nu_\tau} &= \sin^2\theta\sin^2\left(\Delta_{21}L/2\right) \\
&\quad - \sin^2\theta\cos^2\theta\sin^2\left(\Delta_{31}L/2\right) + \cos^2\theta\sin^2\left(\Delta_{32}L/2\right), \tag{B.2.34}
\end{aligned}$$

donde

$$\begin{aligned}
\Delta_{21} &= \sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2} + \check{c}p, \tag{B.2.35}\\
\Delta_{31} &= \sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2}, \tag{B.2.36}\\
\Delta_{32} &= \sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2} - \check{c}p, \tag{B.2.37}\\
\sin^2\theta &= \frac{1}{2}\left[1 - \check{c}p/\sqrt{(\check{c}p)^2 + (\check{a}\cos\Theta)^2}\right], \tag{B.2.38}
\end{aligned}$$

y Θ es el ángulo entre el polo norte sideral y la dirección de propagación. Las mismas probabilidades son válidas para los antineutrinos (se obtienen cambiando $\check{a} \to -\check{a}$).

En este modelo, los neutrinos electrónicos que se propagan a bajas energías en comparación con la escala $\check{a}/\check{c}$ oscilan a neutrinos muónicos y tau. Sin embargo, a energías mayores, los neutrinos electrónicos no oscilan, pero hay una mezcla máxima entre neutrinos muónicos y tau, que oscilan con longitud de onda $\sim \frac{\check{c}E}{\check{a}^2 \cos^2 \Theta}$. Por lo tanto, las oscilaciones de alta energía imitan el comportamiento con la energía de las oscilaciones debidas a masas con una diferencia de cuadrados de masas efectiva que depende de la dirección $\Delta m_\Theta^2 = \check{a}^2 \cos^2 \theta / \check{c}$. Este modelo no ha conseguido explicar la anomalía de LSND, pero lo hizo bastante bien en ajustar los datos experimentales vigentes cuando se propuso, con $\check{c} = 10^{-19}$ y $\check{a} = 10^{-20} GeV$. Análisis más recientes usando datos cruzados han descartado el modelo bicicleta y algunas modificaciones [184].

La gran diferencia entre las predicciones de este modelo y las del SM de oscilaciones de neutrinos debidas a masas es la dependencia direccional, que induce pequeñas variaciones semianuales en el flujo de neutrinos que pueden observarse como asímetrías entre direcciones cardinales. La presencia de variaciones direccionales ha quedado muy restringida por MINOS [185]. Sin embargo, el modelo puede modificarse para no incluir dependencias en la dirección cambiando a_L^Z por a_L^T con poco impacto en los ajustes [182].

Una modificación del Modelo Bicicleta, el 'Modelo Tándem' [170] se propuso para ajustar también los datos anómalos de LSND. El modelo incluye parámetros similares a los del Modelo Bicicleta y una única componente no nula del término de masa $m_{\tau\tau}$. El Hamiltoniano efectivo puede escribirse en la base de sabor como

$$h_{TM}^\nu = \begin{pmatrix} (1+\check{c})p & \check{a} & \check{a} \\ \check{a} & p & \check{a} \\ \check{a} & \check{a} & p + \check{m}^2/2p \end{pmatrix}, \tag{B.2.39}$$

donde $\check{c} = (c_L)_{ee}^{TT}$ y $\check{a} = (a_L)_{e\mu}^T = (a_L)_{e\tau}^T = (a_L)_{\mu\tau}^T$. El Hamiltoniano efectivo correspondiente para los antineutrinos se obtiene cambiando $\check{a} \to -\check{a}$.

El Modelo Tándem proporciona una buena descripción de los datos existentes de oscilaciones de neutrinos con

$$\begin{aligned} \frac{1}{2}\check{m}^2 &= 5{,}2 \times 10^{-3} eV^2 \,, \\ \check{a} &= -2{,}4 \times 10^{-19} GeV \,, \\ \check{c} &= 3{,}4 \times 10^{-17} \,. \end{aligned} \tag{B.2.40}$$

Las oscilaciones de neutrinos solares pueden ajustarse sin necesidad de recurrir al efecto MSW [186, 187]. Las oscilaciones de neutrinos atmosféricos pueden ajustarse como en el Modelo Bicicleta, ya que el Modelo Tándem se reduce a éste a altas energías. Los experimentos de detección de neutrinos procedentes de reactores lejanos como KamLAND [188] caen en el régimen de baja energía en el que el término con $\check{c}$ puede despresciarse. En este régimen, la violación de CPT se manifiesta como una diferencia entre las probabilidades de pervivencia de ν_e y $\bar{\nu}_e$ que puede medirse. Los experimentos de neutrinos procedentes de aceleradores cercanos como LSND [145] y MiniBooNE [147] pueden ajustarse debido a esta violación de CPT. El efecto está en los límites de la sensibilidad de los experimentos debido a lo corto de las trayectorias que han sido comprobadas. De hecho, el Modelo Tándem predijo un exceso de eventos a baja energía en el experimento de MiniBooNE antes de su descubrimiento, aunque el exceso observado es cualitativamente mayor. Este descubrimiento aumenta el interés de los modelos híbridos en los que las

oscilaciones son debidas tanto a términos de masa como a LIV. No obstante, las observaciones de LSND y MiniBooNE deben ser confirmadas por otros experimentos en planificación o en marcha antes de tomárselos en serio.

Por último, el caso de oscilaciones debidas a masas modificadas por efectos subdominantes de LIV también ha sido estudiado en el marco del SME. Las cotas a los parámetros del modelo han sido reunidas en la Ref. [178].

¿Oscilaciones de neutrinos debidas a un procedimiento de cuantización deformado? Motivado por la aparición del especio-tiempo no conmutativo [189] en ST [190, 191, 192], se ha propuesto que la generalización más natural de la Mecánica Cuántica No Conmutativa (NCQM) [193, 194, 195] a una teoría de campos es hacer que los campos, que son los grado de libertad de la teoría, sean no conmutativos. La no conmutatividad de campos se incluye en una la QFT de un campo escalar a nivel de las relaciones de conmutación que definen el paso de la teoría clásica a la cuántica.

En esta teoría cuántica del campo no conmutativo o no canónico (QNCFT) [196, 197] escalar, las reglas de conmutación (a tiempos iguales) son

$$
\begin{aligned}
\left[\Phi(\mathbf{x}), \Phi^\dagger(\mathbf{x}')\right] &= \theta \delta^{(3)}(\mathbf{x} - \mathbf{x}')\,, \\
\left[\Phi(\mathbf{x}), \Pi^\dagger(\mathbf{x}')\right] &= i \delta^{(3)}(\mathbf{x} - \mathbf{x}')\,, \\
\left[\Pi(\mathbf{x}), \Pi^\dagger(\mathbf{x}')\right] &= B \delta^{(3)}(\mathbf{x} - \mathbf{x}')\,,
\end{aligned}
\tag{B.2.41}
$$

mientras que el Hamiltoniano libre permanece inalterado.[22] La teoría libre resultante puede resolverse completamente y resulta ser una teoría de partículas a y sus antipartículas b con relaciones de dispersión que violan Lorentz y CPT,

$$
\frac{E_a(\mathbf{p})}{\omega_{\boldsymbol{p}}} = \sqrt{1 + \left(\frac{B/\omega_{\boldsymbol{p}} - \theta\omega_{\boldsymbol{p}}}{2}\right)^2} + \frac{B/\omega_{\boldsymbol{p}} + \theta\omega_{\boldsymbol{p}}}{2}\,,
\tag{B.2.42}
$$

$$
\frac{E_b(\mathbf{p})}{\omega_{\boldsymbol{p}}} = \sqrt{1 + \left(\frac{B/\omega_{\boldsymbol{p}} - \theta\omega_{\boldsymbol{p}}}{2}\right)^2} - \frac{B/\omega_{\boldsymbol{p}} + \theta\omega_{\boldsymbol{p}}}{2}\,,
\tag{B.2.43}
$$

donde $\omega_{\boldsymbol{p}} = \sqrt{\mathbf{p}^2 + m^2}$. Nótese que lo parámetros introducidos en las reglas de conmutación actúan como escalas UV e IR. Por tanto, la QFT relativista usual se recupera en el límite en el que las energías de las partículas están más allá de estos valores.

QNCFT ha sido ampliada en la Ref. [199] para incluir campos espinoriales. En este caso, el Hamiltoniano de los campos espinoriales es el estándar sin masa, pero las reglas de anticonmutación cambian. En el modelo se considera un campo espinorial sin masa con sólo dos sabores. La teoría se cuantiza imponiendo las reglas de anticonmutación (a tiempos iguales)

$$
\left\{\Psi_\alpha^i(\mathbf{x}), \Psi_\beta^{j\dagger}(\mathbf{x}')\right\} = \mathcal{A}_{\alpha\beta} \delta^{ij} \delta^{(3)}(\mathbf{x} - \mathbf{x}')\,,
\tag{B.2.44}
$$

donde i, j son los índices espinoriales y $\mathcal{A}$ es una matriz hermítica. Los nuevos parámetros en $\mathcal{A}$ son en este caso adimensionales, y por tanto la teoría acaba siendo una descripción en QFT del modelo introducido en Ref. [164] de oscilaciones debidas a velocidades o el SME con sólo coeficientes c_L^{00} distintos de cero, y por lo tanto está refutado por los datos experimentales.

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[22]Estas reglas de conmutación también han sido ampliadas en Ref. [198], en donde las deltas de Dirac han sido sustituidas por distribuciones Gaussianas. Nótese que este cambio implica que la teoría se vuelve no local.

Los autores de [199] tratan de subsanar el problema haciendo que los autovalores de la matriz $\mathcal{A}$ sean dependientes del momento, pero esto no tiene mucho sentido ya que las reglas de anticonmutación son sólo función de las coordenadas espaciales. Trataremos de resolver este problema en uno de los artículos presentados en esta tesis.

B.2.5. Física de Neutrinos y Nueva Física en el IR

En esta tesis queremos presentar una alternativa a las oscilaciones de neutrinos debidas a masas que no modifique el contenido de campos del SM, respete la invariancia gauge y no implique violación del número leptónico. También queremos que esta alternativa sea consistente con los datos experimentales y observacionales y físicamente distinguible del mecanismo estándar con datos futuros.

Se buscará un modelo realista en el marco de QNCFT, ya que es una ampliación simple de QFT. Se tratará de encontrar modificaciones de la relación de dispersión de los neutrinos que involucren el cuadrado de una escala infraroja a momentos muy por encima de dicha escala.

La solución parece invocar una generalización no local de las reglas de anticonmutación (B.2.44), similar a la que aparece en la Ref. [198] y a la modificación no local de QFT en el IR que se ha sugerido en la sección anterior. Nos referiremos a est modificación de la física de neutrinos como neutrinos no canónicos.

B.2.6. Comprobando la hipótesis los neutrinos no canónicos en cosmología

El mencionado mecanismo alternativo para explicar las oscilaciones de neutrinos será difícil de distinguir del mecanismo basado en masas por medio de experimentos de física de partículas. La razón es que los experimentos de física de partículas siempre involucran neutrinos de energías mucho más altas que la escala IR, y en ese régimen el comportamiento de ambos mecanismos es casi idéntico.

Para poder distinguir entre neutrinos masivos y neutrinos no canónicos, será necesario recurrir a neutrinos de momentos comparables a o por debajo de la escala IR, y tales neutrinos pueden encontrarse en el CνB. Sin embargo para poder explorar consistentemente el efecto cosmológico de un modelo no local y que viola invariancia Lorentz, será necesario generalizar el concepto de QFT en espacio-tiempo curvo [19] a teorías de campos más generales.

Hacer esto de manera consistente es una tarea complicada y está más allá de los objetivos de esta tesis doctoral. No obstante, como primer paso, exploraremos como la QNCFT escalar presentada en Refs. [196, 197] puede acoplarse a gravedad. Las propiedades de simetría de esta teoría han sido estudiadas en Ref. [200]. Una introducción más detallada a QNCFT y sus propiedades de simetría se ha incluído en este trabajo. El requisito de mantener estas propiedades de simetría en el caso de espacio-tiempo curvo serán esenciales para determinar cuál es la forma precisa del acoplo.

B.3. Objetivos

Hemos tratado de argumentar que los problemas de la cosmología pueden resolverse mediante una extensión de QFT en el IR y el UV, y que el efecto de la gravedad puede estar detrás de una modificación de QFT resoponsable de fenómenos poco entendidos como las oscilaciones de neutrinos. Aunque se ha atacado estos problemas utilizando métodos deductivos, pensamos que seguir el rastro a los resultados experimentales siguiendo un método inductivo es clave en el

progreso científico. Por lo tanto trataremos de exploar desviaciones de GR y QFT en el UV y el IR que puedan estar detrás de los problemas presentes en cosmología y física de neutrinos.

En el primero de los artíuclos, se construirá un modelo cosmológico basado en una modificación fenomenológica del CSM tanto en el UV como en el IR, el Modelo Cosmológico Asintótico. El modelo estará definido en la aproximación homogénea. Se determinará la cinemática del universo. El problema del horizonte quedará automáticamente resuelto. A través de una simulación mediante cadenas de Monte Carlo Markov, se contrastará el modelo con los datos observacionales que no dependen del comportamiento de las inhomogeneiodades. También se discutirán algunas realizaciones particulares del modelo.

Se intentará abandonar la aproximación homogénea en el Modelo Cosmológico Asintótico. En el segundo artículo presentado en esta tesis, el tratamiento linealizado de las perturbaciones escalares se derivará gracias a una sola suposición adicional basada en la consistencia fenomenológica. El modelo resultante resultará ser distinguible de las teorías y modelos más populares en cosmología. Se calculará el efecto de la generalización del CSM en el espectro de CMB usando una modificación del código CAMB [201].

Se presentará en el tercer artículo una alternativa al mecanismo de masas para oscilaciones de neutrinos que es compatible con todos los datos experimentales dentro del marco de QNCFT. La modificación no incluirá nuevos campos fuera del SM y respetará la simetría gauge y las simetrías discretas, pero violará la invariancia Lorentz y la localidad. Esta violación estará mediada por una nueva escala de bajas energías que controla la modificación de la relación de dispersión y la nueva física en el IR.

Para poder construir una QFT en espacio-tiempo curvo para partículas con relaciones de dispersión modificadas, será también importante distinguir si la simetría Lorentz está violada o deformada, como se verá en el cuarto artículo presentado en esta tesis. Esta distinción podría ser crucial para poder derivar las correcciones cuánticas a la constante cosmológica, por ejemplo. En el cuarto artículo presentado en este compendio, se explicará cómo las propiedades de simetría determinan la formulación de modificación de QFT en espacio-tiempo curvo. Por simplicidad escogeremos el caso de una QNCFT de un campo escalar en el límite $B = 0$.

En el apédice A se incluirá un comentario sobre las implicaciones de este mecanismo de oscilaciones de neutrinos en cosmología. Se presentará un cálculo ingenuo del efecto de una relación de dispersión modificada para los neutrinos en la expansisón del universo.

C. Resumen y Conclusiones

La Teoría General de la Relatividad (GR) y el Modelo Estándar de física de partículas (SM) han ofrecido una descripción muy precisa de todos los fenómenos conocidos con dos notables excepciones, las oscilaciones de neutrinos y la expansión acelerada del universo primitivo y presente. Las oscilaciones de neutrinos y la aceleración del universo presente admiten explicaciones bastante convencionales en términos de masas de neutrinos y una constante cosmológica/energía de vacío, respectivamente. Sin embargo, estas explicaciones no son muy satisfactorias, ya que:

- introducen enormes problemas de naturalidad,

- introducen un nuevo ingrediente en la teoría que es difícil o imposible de medir, salvo en los fenómenos que han provocado su propuesta,

- no son capaces de explicar la extraña coincidencia de escalas entre la densidad de energía asociada a la constante cosmológica ($\rho_{vac} \sim (10^{-3}\,eV)^4$) y la menor de las diferencias de masas de neutrinos ($\Delta m^2 \sim (10^{-3}\,ev)^2$),

- los datos experimentales y observacionales actuales son aún incapaces de refutar algunas de las propuestas alternativas a estas explicaciones.

A pesar de las cuestiones que se han suscitado, las masas de neutrinos y la constante cosmológica han sido consideradas un hecho confirmado por muchos porque son la explicación más simple de entre las acordes con los datos experimentales y observacionales, y porque no entran en conflicto con ninguno de los principios básicos en los que se basan GR o el SM.

No obstante nada es sagrado en ciencia, salvo la comprobación experimental. La simplicidad es importante, pero la estética no puede usarse como sustituto de la verificación experimental de una predicción. Los principios físicos que son considerados básicos deben rechazarse en caso de que entren en conflicto con un sólo experimento u observación, y deben permanecer siempre bajo constante escrutinio.

Por tanto nos gustaría construir modelos que sean alternativa a las masas de neutrinos y la constante cosmológica, que sean consistentes con todos los fenómenos conocidos, y que puedan ser contrastados con las explicaciones más convencionales a la luz de experimentos y misiones futuras. No temeremos violar los principios básicos de GR o el SM, siempre y cuando la desviación de éstos sea lo suficientemente pequeña para ser consistente con los datos disponibles.

Por ejemplo, las masas de neutrinos son la única explicación de las oscilaciones de neutrinos en el vacío dentro del marco de Relatividad Especial (SR), y por tanto debe considerarse una pequeña Violación de Invariancia Lorentz (LIV) en neutrinos. Por otro lado, unas correcciones cuánticas demasiado grandes a la constante cosmológica calculadas en el marco de Teorías de Campos Efectivas (EFT) están basadas en el Teorema del Desacoplo, que a su vez se basa en la localidad; así que hemos considerado una pequeña desviación de la localidad.

No siempre se requiere violar los principios básicos de una teoría para poder ir más allá de ella, como es el caso de GR. Hemos considerado acciones más generales que la de Einstein-Hilbert en busca de nueva física que pueda explicar la expansión acelerada.

En ambos casos, la desviación del marco estándar viene caracterizada por escalas de energía que determinan la aparición de nueva física. Estas escalas pueden ser ultravioleta (UV) o infraroja

(IR), dependiendo de los requisitos de los fenómenos que queremos estudiar. Las oscilaciones de neutrinos así como la expansión acelerada actual requieren la introducción de una escala IR, mientras que la explicación de los problemas de horizonte y planitud requiere la introducción de una escala UV. Es posible que las dos escalas IR sean la misma, como sugiere la extraña coincidencia que se ha señalado, pero no hemos sido capaces de explicar esta coincidencia dentro de un modelo consistente.

En vez de recurrir a una aproximación deductiva como Teoría de Cuerdas (ST), Teoría Cuántica de Bucles (LQG) o Teorías de Gran Unificación (GUT), que han crecido lejos del contacto con el experimento, hemos preferido adoptar una aproximación inductiva en la que hemos empezado por construir modelos simples que ofrecen una descripción de los datos y quizá puedan crecer en el futuro si no son refutados por nuevos datos experimentales.

Para poder explicar las dos épocas de expansión acelerada del universo, hemos introducido una generalización fenomenológica del Modelo Estándar Cosmológico (CSM) basado en GR. El modelo es similar a otras descripciones fenomenológicas propuestas, que relacionan la expansión del universo con parámetros más "antropocéntricos" como el desplazamiento al rojo z. El Modelo Cosmológico Asintótico (ACM) se basa (como GR) en una teoría métrica del espacio-tiempo en la aproximación homogénea, y se define mediante la relación entre el parámetro de Hubble H y la densidad de energía del contenido material del universo definido por las partículas del SM y la Materia Oscura (DM). El efecto de la nueva física viene dado por la dependencia de esta relación de una escala UV y otra escla IR que actúan como cotas del intervalo abierto al que el parámetro de Hubble está restringido. Por argumentos dimensionales, en el límite en el que el parámetro de Hubble está lejos de estas cotas, el comportamiento estándar dado predicho por GR se recupera, mientras que el universo tiende asintóticamente a un universo de deSitter en el pasado y futuro infintos, cuando H se aproxima a una de sus cotas. El CSM con constante cosmológica (ΛCDM) está incluido como caso particular.

El ACM está construido con cuatro parámetros libres, dos de ellos caracterizando la expansión acelerada primitiva y otros dos caracterizando la expansión acelerada actual. Hemos restringido los dos relevantes para la expansión acelerada actual, contrastando los resultados derivados de las simulaciones mediante cadenas de Monte Carlo Markov con los datos observacionales de Supernovas de Tipo Ia (SNe), la distancia a la superficie de recombinación deducida del Fondo Cósmico de Microondas (CMB), medidas directas de la densidad de energía y el parámetro de distancia de las Oscilaciones Acústicas de Bariones (BAO). Hemos encontrado que hay mejores ajustes a la expansisón del universo que ΛCDM, pero que la calidad de los ajustes depende demasiado fuertemente en la muestra de datos que se está utilizando. Por tanto una mejora en las muestras de datos es importante a la hora de favorecer o refutar el ΛCDM frente a sus competidores.

El comportamiento homogéneo de ACM puede encontrar una realización concreta en el marco de gravedad modificada (por ejemplo en teorías $f(R)$), teorías escalar-tensor o modelos de Energía Oscura (DE) fenomenológicos de manera indistinguible. Se pueden encontrar diferencias entre estas descripciones en el comportamiento de las perturbaciones.

La ausencia de una acción para el ACM es un obstáculo serio para poder hacer una teoría de perturbaciones en el ACM. No obstante, bajo una suposición adicional badada en la consistencia fenomenológica (que las ecuaciones diferenciales sean de orden dos), el comportamiento homogéneo del ACM determina unívocamente el comportamiento linealizado de las perturbaciones escalares.

Con la suposición de un epectro invariante de escala de Harrison-Zel'dovich para el universo primitivo, el tratamiento linealizado basta para determinar el espectro de CMB deducido del ACM. La comparación entre esta predicción y el espectro observado impone cotas similares en el espacio de parámetros que el análisis homogéneo. Esto se debe al hecho de que el efecto Sachs-

Wolfe integrado (ISW) del ACM para parámetros que yacen dentro de las regiones de confianza determinadas por nuestro análisis homogéneo está completamente enmascarado por la varianza cósmica, es decir, el error estadísitico intrínseco asociado a los multipolos más bajos (censura cósmica).

El tratamiento de perturbaciones linealizado es capaz de distinguir el ACM de las formas de DE más ingénuas, como teorías escalar-tensor o teorías $f(R)$. Estos ejemplos habitualmente presentan problemas de ajuste fino, como necesitar un 'mecanismo camaleón' que haga desaparecer una nueva interacción de largo alcance.

En el futuro planeamos explorar aún más las consecuencias de este modelo, como el comportamiento no lineal de las perturbaciones, el comportamiento de los modos vectoriales y tensoriales, y el límite de gravedad débil.

Habitualmente se dice que las oscilaciones de neutrinos en vacío demuestran que los neutrinos tienen masa. Hemos demostrado que esta afirmación es falsa. Las oscilaciones de neutrinos pueden explicarse en términos de oscilaciones entre partículas con diferentes relaciones de dispersión, tanto si esta diferencia se debe a masas como si no.

Neutrinos con diferentes velocidades o relaciones de dispersión modificadas en el UV están descartadas, con la excepción de un único modelo con un gran ajuste fino. Sin embargo, los experimentos de oscilaciones de neutrinos sólo exploran el régimen de alta energía (pocos MeV a pocos GeV) de los neutrinos, y por tanto *cualquier* modificación de la relación de dispersión en el IR, tal que el efecto a alta energía se pueda describir mediante un término proporcional al cuadrado de una escala IR, puede describir las oscilaciones de neutrinos en el vacío.

Es difícil encontrar una mpodificación así que difiera de un término de masa en QFT. Hemos encontrado un modelo en el que esto es posible también respetando la invariancia gauge y la conservación del número leptónico del SM. No obstante, hemos tenido que sacrificar no sólo la invariancia Lotentz sino también la localidad. Planeamos estudiar una modificación de este modelo que incluya violación de número leptónico en el futuro.

Para poder distinguir los neutrinos masivos u otra modificación en el IR del SM, será necesario comprobar el régimen de baja energía (de pocos meV a pocos eV) de los neutrinos. El fondo cósmico de neutrinos (CνB) predicho por la cosmología ofrece la posibilidad de medir estos neutrinos en el futuro.

Sin embargo, para poder estudiar el efecto de una teoría no invariante Lorentz en cosmología es importante construir primero una Teoría Cuántica de Campos (QFT) en espacio-tiempo curvo que preserve las propiedades de simetría de dicha teoría en espacio-tiempo plano. Aunque esto es una tarea muy compleja, fuera de los objetivos de esta tesis, como primer paso hemos explorado la posibilidad de construir la Teoría Cuántica de los Campos No Canónicos (QNCFT) escalares en un espacio-tiempo curvo.

Respetar bien el grupo de Poincaré implementado no localmente, bien el grupo Euclídeo implementado localmente de simetrías de la teoría en espacio-tiempo plano, lleva a dos maneras de definir la teoría en espacio-tiempo curvo.

Si se elige hacer local la simetría implementada no localmente por el procedimiento estandar, se encuentra una obstrucción. La solución es introducir curvatura en el espacio-tiempo auxiliar en el que la simetría Poincaré puede interpretarse como una simetría espacio-temporal. La teoría resultante es no local en el tiempo. En el límite en el que el parámetro θ tiende a cero ambos espaciotiempos coinciden y se recupera la QFT estándar en espacio-tiempo curvo. Las consecuencias de esta teoría son similares a las usuales, pero las partículas presentan una relación de dispersión modificada.

Alternativamente, uno puede curvar el espacio-tiempo físico, pero entonces la simetría implementada no localmente se rompe. En la teoría resultante aparece un sistema de referencia privilegiado, el espacio-tiempo se folia en hipersuperficies de simultaneidad de tipo espacio. Pueden

aparecer nuevos fenómenos asociados a la escala UV θ, que no estaban presentes en la QFT estándar en espacio-tiempo curvo.

Planeamos estudiar cómo se ve afectada la radiación Hawking por las relaciones de conmutación no canónicas en ambas teorías. También nos gustaría ir más lejos y determinar las simetrías del modelo de neutrinos no canónicos y ver cómo pueden acoplarse a gravedad.

Con una suposición ingenua sobre cómo puede incorporarse la relación de dispersión modificada de los neutrinos no canónicos a la cosmología, hemos calculado el efecto una relación de dispersión modificada en el IR que imita el efecto de una constante cosmológica. Utilizando las cotas de 'diferencias de cuadrados de masas de neutrinos' y de la 'curvatura espacial', la "constante cosmológica efectiva" obtenida resulta estar tres órdenes de magnitud pr encima del valor observado.

Si existe una conexión entre la escala IR indicada por las oscilaciones y por la expansión acelerada presente, ha de hacerse aún mucho trabajo para desentrañar esta relación. Esperamos que esta tesis doctoral sirva de inspiración a futuras investigaciones en esta dirección.

Ahora se procederá a enunciar los resultados obtenidos en esta tesis doctoral:

- Se ha construido un modelo cosmológico (ACM) definido mediante la relación entre la densidad de energía y el parámetro de Hubble en la aproximación homogénea. El modelo contiene a ΛCDM como caso particular. Este modelo basta para explicar el problema de horizonte y la expansión acelerada actual. A través de una simulación, el comportamiento del modelo a tiempos tardíos ha sido restringido. Aunque una constante cosmológica está permitida a 1σ, las observaciones favorecen otras regiones del espacio de parámetros. No obstante la determinación de los parámetros depende fuertemente de la muestra de datos que se utilice.

- Suponiendo ecuaciones diferenciales de orden dos para las perturbaciones, se ha encontrado que es posible encontrar un único sistema de ecuaciones para las perturbaciones escalares sobre un universo FRW cuyo comportamiento homogéneo se aleje de GR. En contra de lo que se cree, esto es factible sin recurrir a ecuaciones de orden superior ni a nuevos grados de libertad, al menos a nivel lineal. El modelo es distinguible de otras propuestas como teorías $f(R)$, teorías escalar tensor, o los modelos más ingenuos de DE.

- Hemos usado las ecuaciones de las perturbaciones escalares asociadas al ACM para derivar el espectro de CMB en un universo en el que la expansión homogénea viene determinada por el ACM. El ajuste a las observaciones no añade nueva información con respecto a los ajustes homogéneos, debido la censura cósmica del efecto ISW por parte de la varianza cósmica.

- Se dice habitualmente que las oscilaciones de neutrinos implican que los neutrinos tienen masa. Hemos encontrado un contraejemplo de esta afirmación. Se ha construido un modelo en el que el efecto de nueva física gobernada por una escala IR puede explicar todos los experimentos de oscilaciones de neutrinos. Por tanto las oscilaciones de neutrinos son sensibles a nueva física en el IR. La observación más prometedora que podría distinguir entre neutrinos masivos y no canónicos es el efecto de los neutrinos en cosmología, y en particular el CνB.

- Como un primer paso hacia la determinación del efecto cosmológico de los neutrinos no canónicos, se han construido dos posibles teorías cuánticas de un campo escalar no canónico en espacio-tiempo curvo, dependiendo de que simetrías de la teoría en espacio-tiempoo plano se rompen al curvarlo. En una de ellas no existe un sistema de referencia privilegiado

y se permiten partículas con relación de dispersión modificadas, pero su interpretación no está clara. La otra involucra una dirección temporal privilegiada y nuevos efectos pueden aparecer, pero la renormalizabilidad de la QFT no está garantizada.

- Nuestro trabajo ha tratado de explorar fenómenos que parecen apuntar a la presencia de nueva física en el IR: las oscilaciones de neutrinos y la aceleración actual del universo. Si estos fenómenos resultan estar relacionados, hay aún mucho trabajo por hacer para desentrañar esta relación.

C. Resumen y Conclusiones

D. Cartas de Aceptación de los trabajos presentados

Como corresponde en un compendio de publicaciones, declaro que el doctorando ha sido responsable de la mayor parte de las aportaciones a los trabajos presentados, dirigidos por el director Prof. José Luis Cortés, con la colaboración, en el caso del trabajo de neutrinos, del Prof. José Manuel Carmona y en el caso del trabajo realizado en la estancia breve en Italia, bajo la dirección del Prof. Stefano Liberati.

Los trabajos presentados han sido publicados en las siguientes revistas,

- Astroparticle Physics, Publisher: ELSEVIER SCIENCE BV Factor de Impacto (2008): 3.388

- Journal of High Energy Physics, Publisher: INT SCHOOL ADVANCED STUDIES Factor de Impacto (2008): 5.375

- Physical Review D, Publisher: AMER PHYSICAL SOC Factor de Impacto (2008): 5.050

Todas ellas pertenecientes a la cateogoría "Physics, Particles and Fields" del JCR, con un factor de impacto promedio de 2.015 en 2008.

En éste apéndice adjunto una copia de la carta de aceptación del artículo "Linearized treatment of scalar perturbations in the Asymptotic Cosmological Model":

"Ms. Ref. No.: ASTROPARTPHYS-D-09-00097R1 Title: Linearized Treatment of Scalar perturbations in the Asymptotic Cosmological Model Astroparticle Physics

Dear jose,

I am pleased to confirm that your paper "Linearized Treatment of Scalar perturbations in the Asymptotic Cosmological Model(ASTROPARTPHYS-D-09-00097R1)" has been accepted for publication in Astroparticle Physics. Thank you for submitting your work to this journal.

With kind regards,

S. Dodelson Receiving Editor Astroparticle Physics "

D. Cartas de Aceptación de los trabajos presentados

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