Bachelor Thesis

Identification of parameters for optimization of crash sensitive structures

Technischen Universität Berlin
Institut für Konstruktion, Mikro- und Medizintechnik
Fachgebiet Methoden der Produktentwicklung und Mechatronik

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LIST OF ABBREVIATIONS

[1]EA: Evolutionary Algorithm
[2] FEM: Finite Element Method
[3] CoC: Coefficient of correlation
[4] CoD: Coefficient of determination
[5]CoI: Coefficient of importance
[6] MOP: Metamodel of Optimal Prognosis
[7] LHS: Latin Hypercube Sampling
[8] MCS: Monte Carlo Simulation
[9] DoE: Design of Experiment
[10] ALHP: Advanced Latin Hypercube Sampling
[11] Elite_SA: Elite start design using Self adaptive mutation type
[12] Mix_SA: Mix start design using Self Adaptive mutation type
[13] Mix_ND: Mix start design using Normal distribution mutation type

1. ABSTRACT

This Thesis was developed throughout "Verkehrs und Maschinensystem" department belonging to TU-Berlin with the expectation of making an accurate inquiry in regarding to optimization methods of models based on a large quantity of parameters.

It is specially oriented on Multidisciplinary optimization methods, concretely on multi-objective optimization. The inquiry is focused on Evolutionary Algorithms (EA[1]), and how the objectives functions are maximized or minimized according to the operators applied. Identification of the effect of the chosen optimizer on the optimized design is desired, in the limits of suitable numbers of parameters and determinate the settings for the most promising solver for crash optimization

The core of this work is founded on a reduced parametrically crash box. The response of this model has been previously studied through an optimization loop updated by a parametric CATIA Crash box-Model. This structure is in charge of the joining between the bumper and the chassis. Its duty consists of cushioning the impacts and making them less significant for the craft.

The software utilized to carry the optimization process out is OptiSlang. This software is a product developed by Dynardo ANSYS GmlH, which allows whereby an intelligibly interface and graphics to read the output information and collect all the useful correlations between parameters, inputs and outputs in an intuitive way.

2. SCENARIO, STRUCTURE AND AIM

Every engineering area is being calling somehow for continuous improvements on the field they are working on. Concretely on this thesis the automotive field is aboard, in order to minimize crash consequences against automobile frontal impacts. Optimization methods on this area are being currently studied and constantly improved to decrease the damages suffered when crashes take place. However, not only the automotive field is concerned about this topic but multi-objective optimization is also related to robotics, materials improvement, aerospace engineering… Practically every field which the amount of variables and their responses are too big to be analyzed, and its improvement becomes a tedious matter, multi-objective optimization is need to carry out the tasks other engineering methodologies cannot face.

Before the model introduction, a scheme of the investigation working procedure is described below, in order to ease the optimization reasoning. Figure 2.1

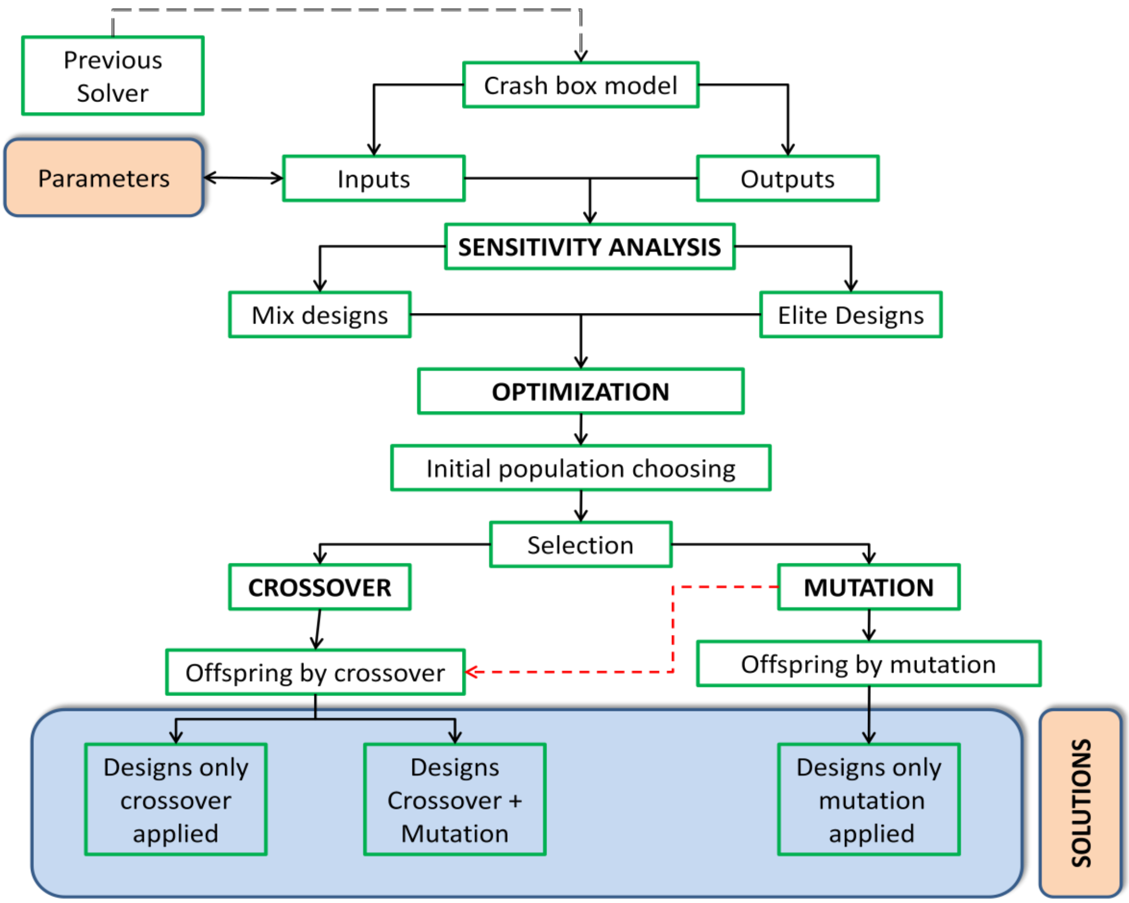


Figure 2.1: Investigation scheme

The working process starts with the sensitivity analysis, which lets to determinate the most relevant parameters according to the objectives established, as in will provide the designs that will be used as reference to initial population of the optimization process. Once the system is already pre-optimized by the sensitivity analysis, optimization can be run and finally the most suitable designs will be extracted from each method, in order to compare them and demarcate that one which optimizes the objectives the best. As can be observed in the previous scheme, crossover and mutation operator are the pillar on which the process is based. Goals of this thesis are summarize along the following points.

- Regarding to the sensitivity analysis: Knowing the most important parameters, reduction of parameters ranges if it is possible and extraction of reference designs.

- Regarding to the optimization processes: Knowing the importance of initial population, understanding of how mutation and crossover operator works. Identify the consequences of applying mutation on crossover offspring. Demarcating which the most suitable optimization solver is. Finding the best designs according to the objectives. Improving such designs as much as possible.

The investigation is divided in two main groups: Batch process and OptiSLang. Figure 2.2 shows a schematic of how the loop is carried out. Batch process gets inputs from OptiSLang and then it returns outputs to OptiSLang.

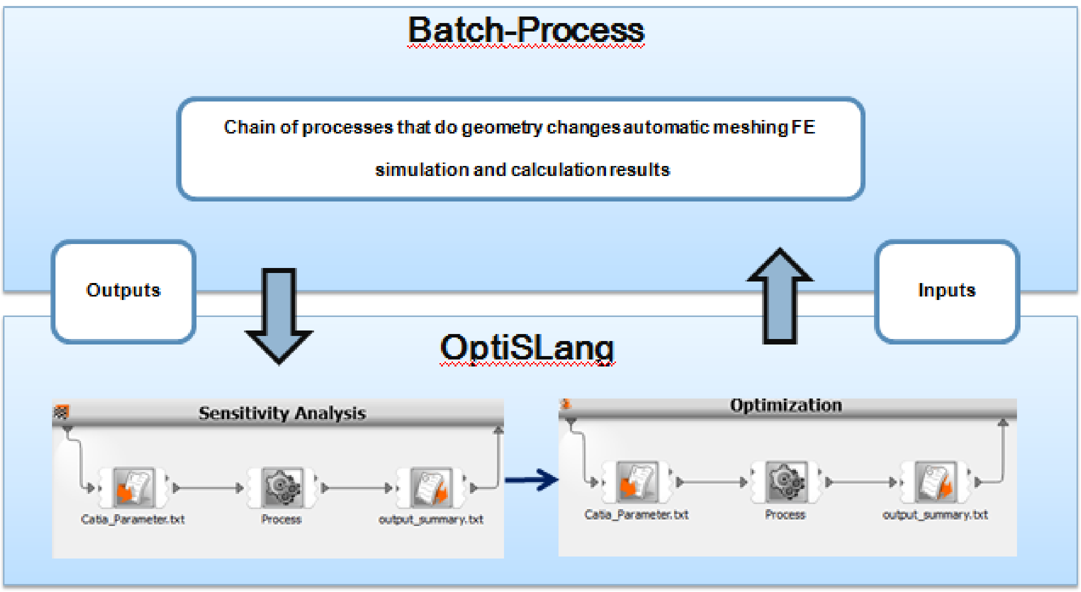
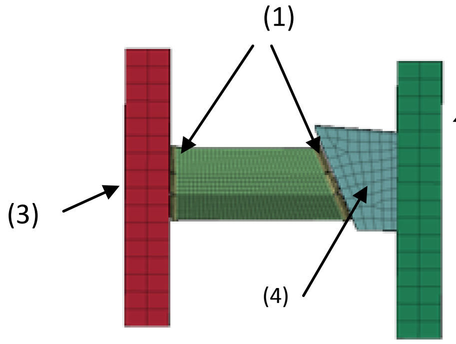
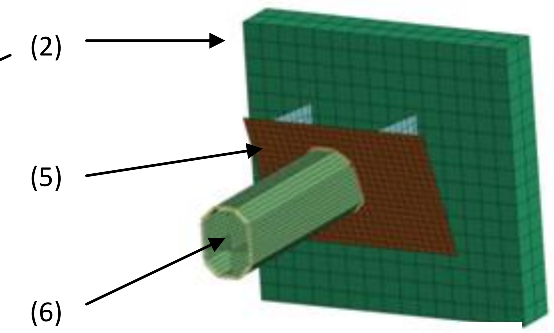


Figure 2.2:Batch-Process and OptiSLang Loop

3. INTRODUCTION TO THE MODEL

The thesis is based on crash box model. The response of this structure against impacts produced by low velocities is crucial to guarantee the security of the chassis making less significant its reparation costs. In Figure 3.1 frontal (a) and lateral (b) views of one random model design are shown. The structure is already designed and automatically FEM[2] meshed,



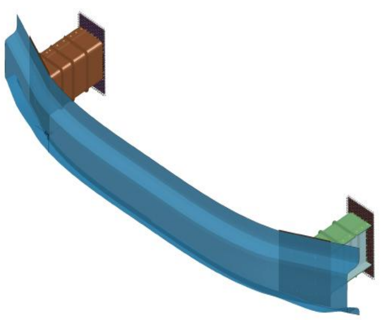


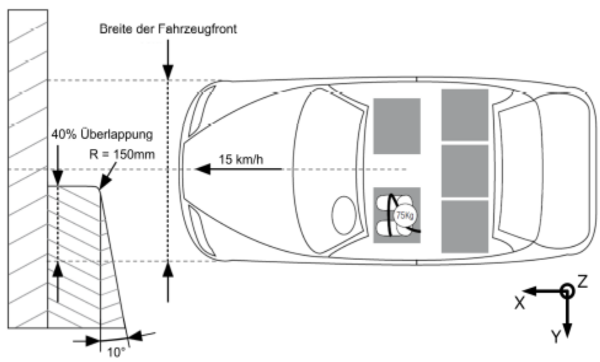
a) Frontal view

b) Lateral view

Figure 3.1: Crash box views

The bumper is hitched by two crash boxes to the chassis, through the *"Contact Shell (7000)l"*, this element is represented in Figure above by number 4. Its shape is kept. Section 3.1 delves into the different crash box parts, making a specific description and numbering of each one. In Figure 3.2 a) such coupling is illustrated. Bumper's design is not part of this thesis, although logically its shape is essential to minimize the impact on the chassis. In Figure 3.2 b) is represented the sort of collision that will be analyzed. Car drives at 15 Km/h and frontally crashes to a rigid surface at an angle of ten degrees as is shown in the image . The surface tip is round with 150mm radius





a) Bumper. Source: . (Werner, 2014)

b) Crash box nodes (RCAR, 7/2011)

Figure 3.2: Bumper and impact overview

3.1 Structure of the model

The model was designed with CATIA V5, automatically solved with LS-Dyna and the outputs are transferred back to the optimizer. It is formed by elements “Beam” and “Shell”. In order to provide a better idea of how the model is built, an itemization of it into several fractions is made. The goal of the thesis is not to clarify how the model was designed, but find the most suitable solve to optimize the crash box. Thus neither specifications of its shape nor how it was assembly, mesh or boundaries used will be detailed.

Table 3.1: Model elements

Beam elements:	Shell elements:
100001 BeamSpotWelds	3000 Contact_shells 7000 Contact_shells 10000 Contact_shells 2000120 FramebackcapL 2000121 RailowerouterL

BeamSpotWelds (100001): Welding that joints the “Railowerouter” to the front (10000) and back (7000) contact shells Figure 3.1 (1).

Contact_shells (3000): It brings the energy that will collide to the crash box Figure 3.1 (2)

Contact_shells (7000): Its function is to connect the crash box to the chassis through the “Frameback” (2000120). It is a compact assemblage that is not easy to deformed Figure 3.1 (3).

Contact_shells (10000): It is in charge of modifying the impact angle Figure 3.1 (4).

FramebackcapL (2000120): It is a thin square sheet that connects by welding the “Railowerouter” (2000121) to the “Contact_shell” (7000) Figure 3.1 (5).

The central part of the structure “RailowerouterL (2000121)” Figure 3.1 (6). is in charge of cushioning the impact. Its response against the crash is where the inquiry is focused on. Absorbing as much energy as it was possible. Following sequence of images illustrated in Figure 3.3, let make an idea of how the crash box shape can be varied.

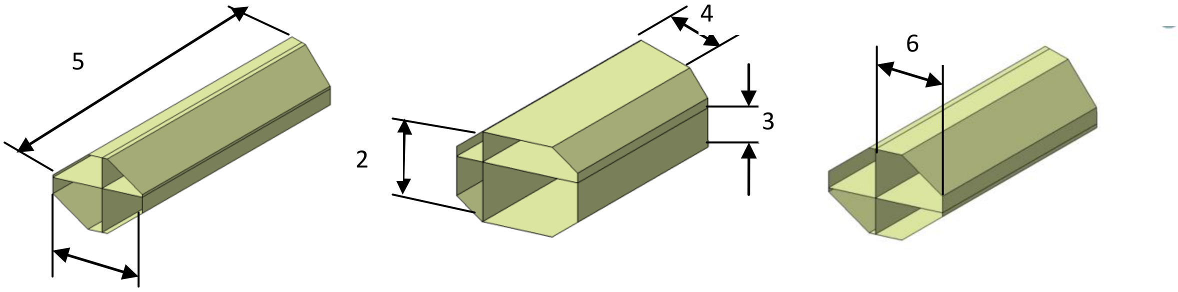


Figure 3.3: Railowerouter [2000121] designs

3.2 Parameters

Once the model has been presented and its shape is already known, it is possible to focus the investigation on the parameters which set it up. Every parameter regardless of its importance will be listed below in Table 3.2. Besides each parameter has a corresponding number assigned which demarcate its meaning in Figure 3.3.

Table 3.2: List of parameters that compounds the Crash box.

Parameter Name	Reference Value	Value Type	min	max
P_B (1)	32	REAL	20	40
P_H (2)	30	REAL	28	30
Zwischenblench vertical (3)	0,9	REAL	0,2	0,99
Blechdicke	3	REAL	2	4
Zwischenblech (4)	0,9	REAL	0,1	0,99
Verkürzung Box (5)	0	REAL	0	1
Fase (6)	0,5	REAL	0,1	0,8

Table 3.3: Parameters description

Parameter	Description
P_B	Crash box width.
P_H	Crash box height.
Zwischenblech vertical	Width of the vertical sheet metal along the crash box.
Blechdicke	Sheet metal thickness.
Zwischenblech	Width of the horizontal sheet metal along the crash box.
Verkürzung Box:	Crash box length
Fase	Cross position.

It is important to point out that range values of the parameters might be modified as the Design of the Experiment requires it. Notice that one of the goals of the sensitivity analysis is to delimitate those ranges according to the results achieved.

3.3 Inputs and Outputs

Inputs are the necessary information about the model are already known and are used to run the analysis. During this Thesis, the inputs that will be used are the characteristic parameters of the model which were described within the previous part.

By contrast, the outputs are the responses that have been obtained by a previous analysis of the model. In this case, the model was solved by LS-Dyna and results that were captured are listed in Table 3.4.

Table 3.4: List of output values

OUTPUT	VALUE	UNITS
crash_Box_efficiency_per	0,57724	-
crash_Box_efficiency_per_deformed	0,82793	-
Maximalkraft	69779,1	KN
added_mass_perc_max	0,26671	-
hourglass_max	10727,4	J

Crash Box efficiency: Ratio of absorbed energy by the crash box to the maximal energy supplied to it considering the total crash box length. (Werner, 2014)

$$\mu(\eta) = \frac{\int_{l_{CB}} F_K dl}{F_{K.max} * l_{CB}} \tag{3.1}$$

Crash Box efficiency deformed: Ratio of absorbed energy by the crash box to the maximal energy supplied to it considering the crashed length. (Werner, 2014)

$$\mu(\eta_{\mathrm{def}}) = \frac{\mu(\int_{l_{def}} F_K dl)}{\mu(F_{K.max} * l_{def.\mathrm{max}})} \tag{3.2}$$

Maximalkraft: Maximal reaction force at the end of the crash box.

Added mass max: Maximum artificial mass added to the element in comparison to the real model

Hourglass max: False deformation mode of a FEM, resulting from the excitation of zero energy degrees of freedom.

4. SENSITIVITY ANALYSIS

By definition, *Sensitivity Analysis is the study of how the uncertainty in the output of a model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of a model* (A. Saltelli, 2008).

It is necessary to delve into a more detailed explanation to make sure the procedure understanding.

4.1 Definition and aim

As a real model is tested, drawbacks come into sight and are materialized in different forms. The most frequent problem is the “curse of dimensionality”. The accuracy of a model decreases by the rise of variables number, what means that the quality of the results will be affected by the size of the model.

The model is composed by seven parameters, what make it no simple to analyze. That is the reason for a sensitivity analysis is needed to do. As the sensitivity analysis is carried out, it is possible to assess which parameters, inputs or outputs are more relevant upon the model evaluation. That enables to focus the investigation on those aspects where the optimal crash box is more likely to find.

4.2 Mathematical algorithms

OptiSlang is the software used to optimize the model objectives. The mathematical methodology on which is based is constituted for some statistical terms. *http://www.dynardo.de/en/software/optislang.html*.

4.2.1 Coefficient of Correlation (CoC^3):

It is a value that indicates the measure of relationship degree between two random variables X and Y. It can be calculated whereby the expression below.

$$\rho(x,y) = \frac{cov(xy)}{\sigma_x \sigma_y} \tag{4.1}$$

Where $cov(xy)$ means covariance of variables X and Y and σ (must be finite and nonzero) is the deviation values of each variable (Dynardo, 2014)

If the relationship between X a Y is lineal and N number of samples is supposed, the correlation can be illustrated by an easy plot. Figure 4.1.

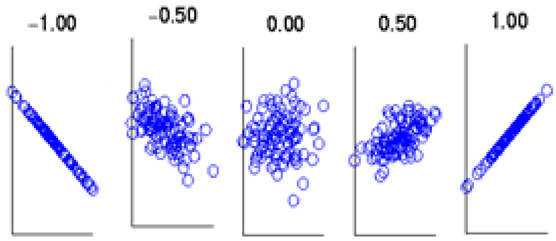


Figure 4.1: Coefficient of correlation Scatterplot. (Stattrek, 29.07.2015)

As can be seen in Figure 4.1 $-1 \leq \rho(x, y) \leq 1$, when the value tends to 1, the variables are close. By contrast, if it tends to -1, means that the variables are inverse and the increase of one of them implies a decrease of another. If the value is close to 0, indicates that a weak linear relationship exists and consequently samples are randomly plotted.

4.2.2 Coefficient of Determination (CoD[4]):

This value indicate how well detailed the quality of a polynomial regression model is. It is an important value upon knowing the accuracy of the regression and enables the designer to determinate when information ought to be neglected or saved. CoD is defined as follows:

$$R^2 = \frac{SS_{Reg}}{SS_{tot}} = 1 - \frac{SS_{res}}{SS_{tot}} \tag{4.2}$$

Where SS_{reg} indicates the variation due to the regression, SS_{tot} is the equivalent to the total variation of the output Y and SS_{res} quantifies the unexplained variation.

The interpretation of CoD is simple, when R^2 is closed to 1 the polynomial regression model has been represented with small error. By contrast if it tends to 0 the model is little accurate.

4.2.3 Coefficient of Importance (CoI[5]):

The Coefficient of Importance (CoI) was developed by Dynardo to quantify the input variable importance by using the CoD measure. Based on a polynomial model, including all investigated variables, the CoI of a single variable Xi with respect to the response Y is defined as follows:

$$CoI(x_{i,}y) = CoI(y, x_i) = R^2_{y,x} - R^2_{y,x\sim i} \tag{4.3}$$

Where $R^2_{y,x}$ indicates the CoD of the full model, including all terms of the variables in X and $R^2_{y,x\sim i}$ is the CoD of the reduced model, where all linear, quadratic and interactions terms belonging to Xi are removed from the polynomial basis (Dynardo, 2014).

It must be stand out that CoI value of a variable is close to zero, means the importance of such variable is low.

4.3 Coefficient of Prognosis and Metamodel of Optimal Prognosis

In order to solve the "curse of dimensionaly", the concept of Metamodel of Optimal Prognosis (MOP[6]) is introduced. *In this approach the optimal input variable subspace together with optimal met-model are determined with help of an objective and model independent quality, the coefficient of prognosis.* (Dynardo, 2014)

The coefficient of prognosis (CoP) was proposed by (J. Will, 2009) and is defined as it is shown below.

$$CoP = 1 - \frac{SS_E^{Prediction}}{SS_T} \tag{4.4}$$

Where $SS^{Prediction}{}_E$ is the sum of squared prediction errors and SS_T is equivalent to the total variation.

OptiSlang uses Latin Hypercube Sampling (LHS[7]) approach to detect the dependencies and correlations among the inputs. This method of sampling overcomes the problems that can be found by other variation of sampling, such as a Monte Carlo Simulation (MCS[8]), which is restricted for a small number of variables, due to its accuracy decreases radically.

Using LHS the input distributions and the specified input correlations are represented very accurately even for a small number of samples. For the minimization of the undesired correlation the method according to Iman and Conover (1982) is used (Dynardo, 2014).

4.4 Sensitivity analysis

4.4.1 Design of Experiment (DoE[9])

The sensitivity analysis is part of an iterative process that finishes when the convergence of the values is reached. The cycle follows a simple flowchart which is described below in Figure 4.2.

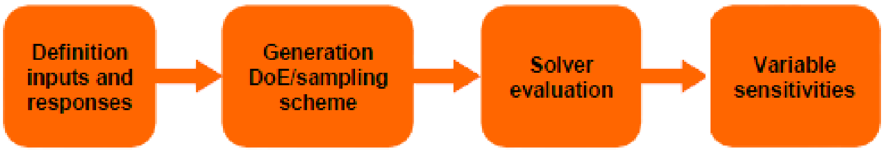


Figure 4.2: Flowchart of sensitivity analysis (Dynardo, 2014)

First of all, the inputs of the system are defined, as in which responses will be taken into consideration establishing which variables are executing the main tasks during the experiment.

The second step settles how the DoE will be created. It is needed to provide the criteria that will be analyzed. There are two essential features that must be defined in order to carry the sensitivity analysis out, objectives and constraints.

Objectives are the functions which are desired to maximize or minimize. Both criteria can be established according to the function. Objectives selected were: "Crash box efficiency" and "Crash box efficiency deformed", (equations 3.1 and 3.2 respectively), which were defined as objectives to maximize, "Difference" was the objective to minimize. It represents the difference between both efficiencies. Ideal case would be that such difference was zero, what means that all the energy would have been absorbed. Unfortunately in practice that is not feasible. On the other hand, constraints demarcate specific limits that must not be exceeded, "added mass max", "Maximalkraft and "Hourglass max" were the constraints defined.

To conclude the DoE, it is necessary to determine as which sampling type will be carried out as how many samples are analyzed.

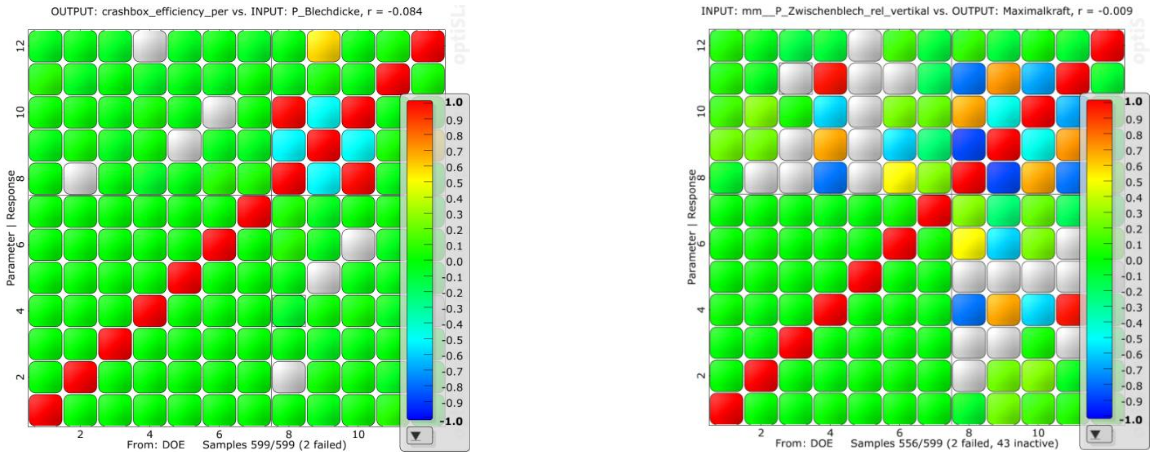
The number of samples chosen for the sensitivity analysis was 600 because it allows making an accurate pre-optimization process what is beneficial for the initial population of the optimization. Advanced Latin Hypercube Sampling (ALHP[10]) was the approach selected as sampling type, due to its advantages when working with a large number of samples in regarding to MCS, as it was mentioned within section 3.3 of this thesis.

However, at the end of this thesis the influence of the number of samples chosen for the sensitivity analysis will be analyzed. Throughout a comparison between the results obtained by the optimization calculations according to the initial population selected.

4.4.2 Running the sensitivity analysis

The first point to point out of the study is only 2 of the 600 samples were failed. This low number is regarded to good accuracy due to the parameters range. On the other hand 136 of them were concluded as “No feasible”, because of the constraints violations, forces or FE’ limits. Although this number corresponds to more than 20% of the samples, it was considered not enough to modify the range limits, due to a large number of succeeded sample could be analyzed.

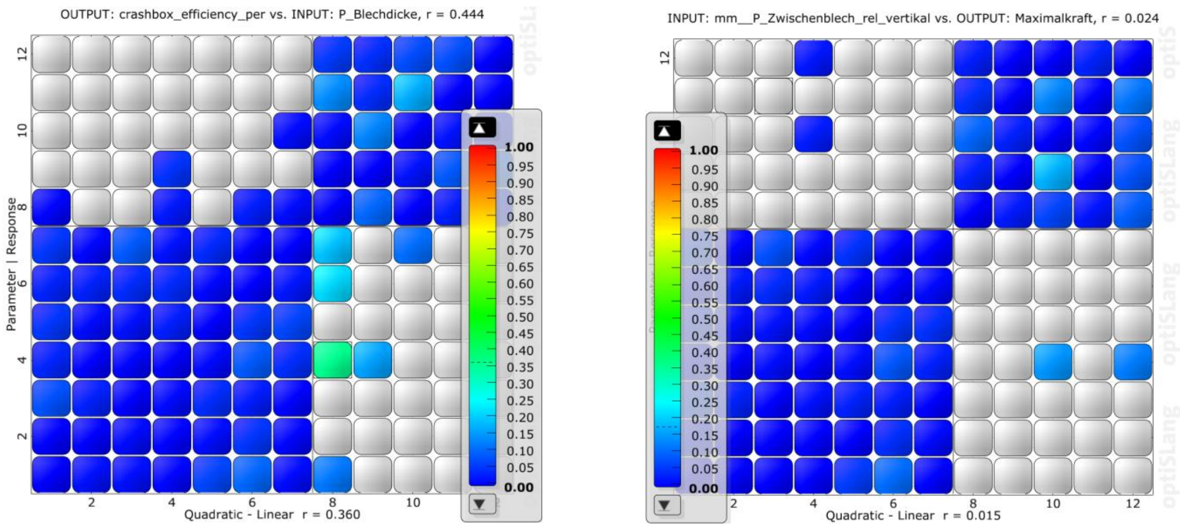
As can be observed into the graphics below (Figure 4.3 and 4.4), OptiSlang creates an osl3.bin file that shows several intuitive multicolor plots, where all the information related to correlations between inputs and outputs can be analyzed. After making a results assessment, is simple to guess that there are some points that are located distant of the theoretical curve. This can be modified by deactivating outliers which are really deflected, due to a bad program calculation. Nevertheless, such a samples dismissal cannot be made lightly. Only those that were assured that do not affect the investigation were deactivated. Changes in correlations are visible in Figure 4.3 and Figure 4.4.



a) Before nodes deactivation

b) After nodes deactivation

Figure 4.3 Linear correlation Matrix



a) Before nodes deactivation

b) After nodes deactivation

Figure 4.4 Quadratic correlation Matrix

It is possible now to notice that the sensitivity analysis has increased its accuracy after the nodes deactivation, according to some visible factors such as below is described.

-There is dependency between inputs neither on the quadratic correlation matrix nor on the linear.

-As it was expected, there are as significant nonlinear correlations as linear between inputs and outputs..

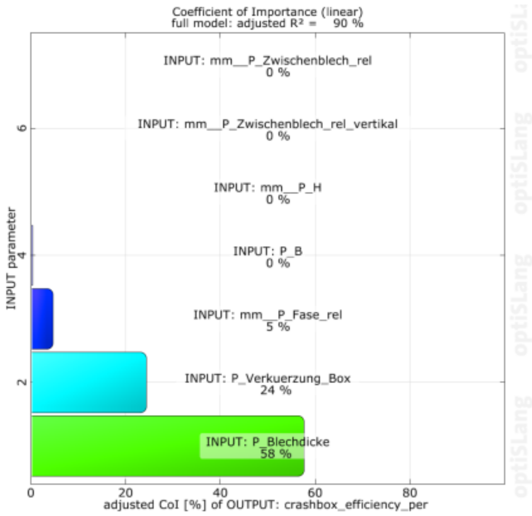
-However, the quadratic relationship numbers were decreased, because of the deflected points, which caused an imprecise plotting and diverted the curve tendency.

- It is not difficult to assume that results of the analysis are right when modifying the parameter values on the parallel coordinates plot and looking the outputs variation over. There are no contradictions while varying the values. That can be checked out in [Appendix A.1.]. Four pictures are shown, Global indicates all results trajectories. If “Blechdicke” is reduced between 2.3 and 2 values, as it is shown on the second picture, it can be observed how the trajectory lines narrow when come by the objectives. The same effect happens when “Verkürzung” and “Fase” values are comprised between 60 and 90, and 0.39 and 0.5 respectively. Parallel coordinate plots verify the strong correlations that exist between these parameters and both efficiencies.

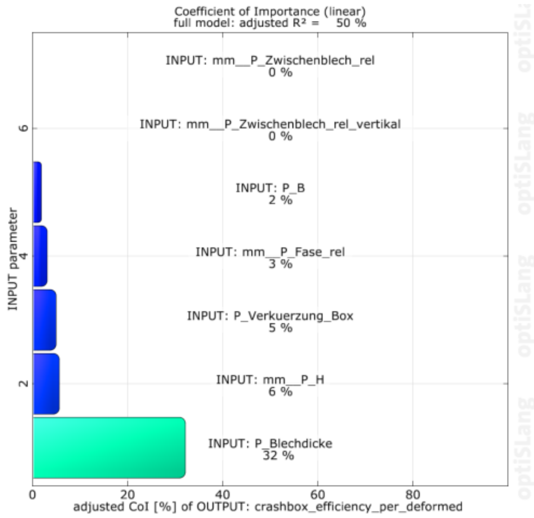
The other parameters relevance in regarding to the selected objectives is much lower as “Blechdicke” and “Verkuerzung” Hence they are concluded as the most influence parameters, and it has been done an approximate exploration of ranges that optimize the objective values established.

4.4.3 Correlations and dependencies between variables.

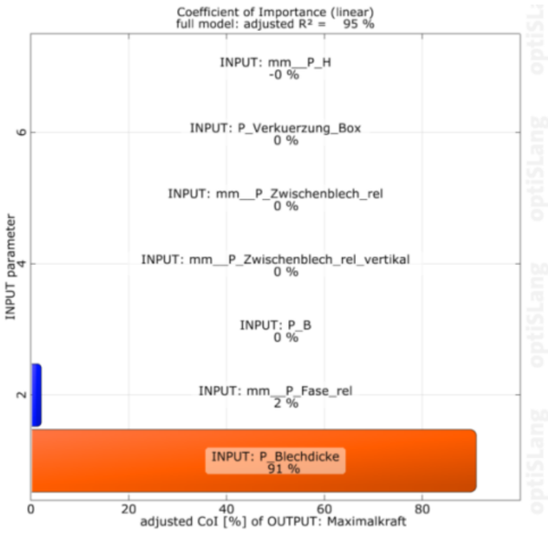
Taking as basis the results collected during the sensitivity analysis, the correlations linear and quadratic based on the CoI were analyzed. On the X axis is represented every analyzed output of the model, on the Y axis can be distinguished the different values in percentage of CoI of each input.



a) Efficiency



b) Efficiency Deformed

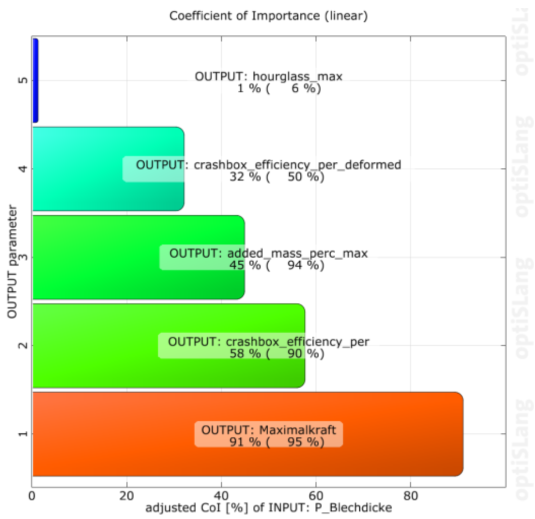


d) Maximalkraft

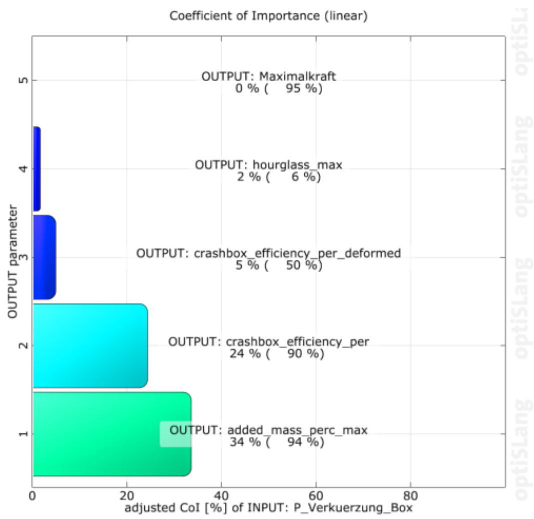
Figure 4.5 Outputs dependences according to COI

Previous plots let identify easily which parameters are more significant on objectives and force applied, and having a look over them, “Blechdicke” and “Verkuerzung” parameters are recognized as the most important ones again. Horizontal bars indicate the COI of every parameter according to the output. In Figure 4.6 a)“Blechdicke” dependences with the outputs is shown. As it can be observed, the thickness affects to every output, except for “Hourglass”, what it is not relevant due to its constraint condition. “Maximalkraft”, is the output affected the most by “Blechdicke” reaching 92% of COI follow by “Efficiency” (58%), “Added_Mass” (45%) and “Efficiency_Deformed” (32%). In Figure 4.6 b) the same information for “Verkürzung” is gathered. Crash box length affects to “Added_mass” (34%), follow by “Efficiency” (24%) and “Efficiency_Deformed” (5%).

It is obvious that “Blechdicke” COI values are higher, hence it is highly probably that the crash box response depends on the quality and properties of the material it is manufactured. It is not concern of this thesis to find out which one is the most suitable according to the impact sort. However it is suggested to analyze the crash box response for different materials, applying the conclusions obtained by this thesis.



a) Blechdicke



b) Verkürzung

Figure 4.6 Relevant Inputs dependences according to COI

4.4.4 Results

Once the influence of each parameter is already identified, the second goal of the sensitivity analysis might be aboard, collection of reference initial population for optimization. The considered criteria was the crash box efficiency, those designs that provided a higher efficiency were selected. In the table 4.1, all the data related to the designs is gathered.

Table 4.1: Elite Start designs set.

Design	P_B	Dicke	Verkürzung	Fase	P_H	Zwischen Blech	Zwischen Blech vertikal	Efficiency	Efficiency deformed
338	26	2,1	78	0.49	33	0.34	0.97	0.48965	0.85050
166	23	2	51	0.44	23	0.14	0.22	0.48274	0.81561
197	24	2,1	71	0.49	32	0.56	0.32	0.47956	0.86156
327	25	2,2	81	0.47	24	0.53	0.78	0.47269	0.77335

397	21	2,1	63	0.45	27	0.23	0.41	0.47246	0.84165
144	21	2,2	88	0.5	27	0.71	0.34	0.47165	0.74340
245	21	2,1	85	0.39	34	0.64	0.57	0.46746	0.80672
243	27	2,1	58	0.46	27	0.72	0.82	0.4654	0.83048
87	29	2	72	0.44	32	0.14	0.24	0.46282	0.76674
541	24	2	38	0.44	24	0.4	0.93	0.45310	0.84840
325	20	2,2	62	0.46	29	0.17	0.52	0.44561	0.85589
407	26	2,3	75	0.49	29	0.21	0.21	0.44198	0.83982
378	28	2,1	76	0.25	24	0.15	0.95	0.43470	0.84290
580	25	2,2	87	0.48	26	0.75	0.69	0.43462	0.68018
538	22	2,1	66	0.35	25	0.64	0.88	0.43369	0.84599
82	29	2,2	84	0.47	36	0.36	0.2	0.43295	0.82201
198	26	2,1	80	0.46	25	0.42	0.68	0.43138	0.67158

Also a mix of best, bad and an average designs were extracted from the sensitivity results table, look up Table 4.2 Therefore the optimization development can be compared with two different initial population and identify not only how important the quality of initial parents provided to the optimizer is, but also how it affects to the optimization progress velocity.

Table 4.2: Mix start designs set.

Design	P_B	dicke	Verkuerzung	Fase	P_H	Zwischen blech	Zwischen Blech Vertikal	Efficiency	Efficiency Deformed
338	26	2,2	78	0.49	33	0.34	0.97	0.48965	0.85050
166	23	2,1	51	0.44	23	0.14	0.22	0.48274	0.81561
197	24	2,1	71	0.49	32	0.56	0.32	0.47956	0.86156
327	25	2,2	81	0.47	24	0.53	0.78	0.47269	0.77335
397	21	2,1	63	0.45	27	0.23	0.41	0.47246	0.84165
144	21	2	88	0.5	27	0.71	0.34	0.47165	0.74340
41	29	2	24	0.23	23	0.76	0.85	0.20234	0.78872
535	31	3,4	11	0.19	24	0.6	0.58	0.20206	0.79170
453	29	3	9	0.32	36	0.32	0.59	0.20205	0.80278
409	27	2,9	35	0.4	33	0.68	0.41	0.20184	0.80908
11	30	3,3	46	0.29	25	0.68	0.65	0.20148	0.51975
261	28	3,3	70	0.42	37	0.79	0.47	0.20143	0.82285
531	30	3,2	10	0.36	33	0.35	0.68	0.13092	0.75683
15	31	2,4	7	0.3	22	0.35	0.69	0.12893	0.71054
450	23	3,1	2	0.39	28	0.64	0.92	0.12873	0.71904
342	30	2,1	4	0.23	33	0.79	0.33	0.12411	0.76416
97	26	2,6	4	0.38	38	0.12	0.84	0.12255	0.74721
323	21	2,1	80	0.23	36	0.48	0.27	0.10963	0.46292
342	30	3,1	4	0.23	33	0.79	0.33	0.12411	0.76416
535	31	2,3	11	0.19	24	0.6	0.58	0.20206	0.79170

4.4.5 Conclusions of Sensitivity Analysis

Keeping in mind the two important parameters (“Blechdicke” and “Verkuerzung Box”) and observing the previous tables, it is singular that the values of “Blechdicke” which provide a higher efficiency are between 2 and 2,2, however the “Verkuerzung Box” range is much wider. This fact makes assumable that the smaller values of blechdicke are, the higher the efficiency of the crash box is.

The rest of parameters seem to not having an especial influence on the objectives established, but for “Fase” whose optimized values fluctuates between 0.39 and 0.5, as it was mentioned in sensitivity analysis.

“Efficiency deformed” values must be also pointed out. It might have made sense that the higher “Efficiency”, the higher values of “Efficiency deformed”. However, only one design of those which have a high efficiency deformed value is listed in the highest efficiency designs and underline in yellow in Table 4.3.

Table 4.3: Best „Efficiency deformed“ designs

Design	Efficiecny	Efficiency deformed
403	0.4218212	0.8713147
168	0.2824578	0.8663886
203	0.3371895	0.8663656
481	0.2981972	0.8636731
255	0.402611	0.863366
74	0.3999468	0.8625083
197	0.4795601	0.8615645
418	0.371755	0.8607197
174	0.4024325	0.8591523

Finally, in Figure 4.7 “Blechdicke” and “Verkürzung“ boxplots are illustrated. Through this plots is possible to determinate in which design space area the parameters are located. Rectangle above indicates those values between the median and the third quartile, whereas the rectangle beneath represents those between the median and the second quartile. As it is observed, for Elite start designs, parameter range is really shortened, especially for “Blechdicke”. By contrast, Mix start design values fluctuates between a much wider space. This fact lets deduced that the optimization process that will be carried out by Elite is more restricted than Mix. But the exploitation of such space region will be deeper. So the space region that according to the Sensitivity Analysis is the most likely to find the most suitable solutions is smaller and parameter values combination might be more precise. That is why the election of the initial population is one of the most important criteria to develop the optimization process.

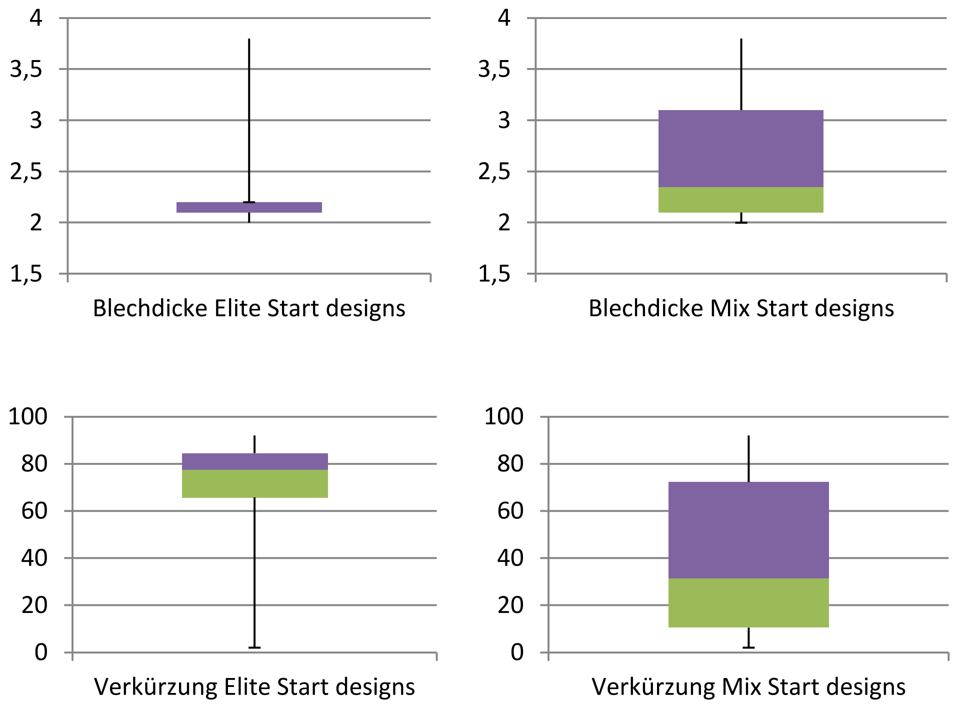


Figure 4.7: Initial population parameters Boxplots

5. OPTIMIZATION

5.1 Definition and aim

A system's optimization is basically based on the search of maximums and minimum points of a real function, starting with a random selection of values that could be one possible solution of the system. However the difficulty of this topic goes further than that simply concept. As a real structure is optimized a lot of factors play an important role along the process, such as quantity of information about the structure is already known, grade of exploration is desired to obtain, what methodology will be follow… so that the problem cannot be reduced to the searching of maximum and minimum points. Some complex procedures based on mathematical approximation and correlations are made. In order to truncate the investigation range, an inquiry of the influential parameters was done, that is the sensitivity analysis.

Throughout this thesis optimization concept is broached as a mathematical process, which looks for the most suitable solution of a problem among a large amount of possible alternatives. The model is run by a non-linear behavior. That kind of modeling involves a high level of complexity, due to the diverse subspaces generated from one variable and all the possibly solutions it entails.

The investigation is based on evolutionary algorithms, which is deeper explained in section 5.2.3. However a brief description of different optimization alternatives is following presented.

5.2 Mathematical algorithms– Optimization method

5.2.1 Gradient based

These methods search for the points where the first derivate is equal to zero and then if the second derivate is known Newton methods are used in order to overcome the problem of nonlinearity (Kelley, 1999).

Gradient based methods distinguish each other mainly in the way how the second order derivates are estimated and how additional constraint equations are considered (Dynardo, 2014)

OptiSlang uses NLPQL approach (Nonlinear Programming by quadratic Lagrangian), which is recommendable when working with low dimensions, as It is the case broached on this thesis. A good overview over the approach is given in (Schittkowski, 1986). Nevertheless, the aim was to identify how multi-objective optimization is working. That is why another optimization methodology was chosen.

5.2.2 Gradient free

- Adaptive Response Surface Method (ARSM)

The ARSM first employs an experimental strategy to generate design points in the design space, then applies either the first-order model or the second-order model to approximate the unknown system (G. Gary Wang, 2000)

In the next iteration step a new DoE scheme is built around this optimal design. Depending on the distance between the optimal designs of the current and previous iteration steps, the DoE scheme is moved, shrunken or expanded. Further details can be found in (Etman, 1996).

- Downhill simplex Method

This method is commonly used in nonlinear regression programs and is due to (Mead, 1965). It requires only function evaluations, not derivatives and is convenient to use when the computational burden is small.

The method uses the concept of a simplex, a polytope N + 1 points (or vertices) in N dimensions: a line segment in a line, a triangle in a plane, a tetrahedron in three dimensional spaces and so on.

5.2.3 Natural inspired

- Evolutionary Algorithms

Evolutionary algorithms (EA) are stochastic searching methods that mimic processes of natural biological evolution. Every population contains x individuals, which represent a group of possible solutions. So that their combination through stochastic operators such as mutation and crossover can generate a new population of individuals called offspring. The most suitable solutions according to the objectives established are used to restart the loop and create the next generation.

5.3 Optimization process

Optimization was set up on the solver after having finished the Sensitivity Analysis and interacts constantly with software installed on the Batch process, which is in charge of making the calculations and creating the geometry. Inputs and output considered into the optimization remained as was previously within section 2.3 explained. OptiSlang suggested using Evolutionary Algorithms-local for the optimization, due to the optimization characteristics. It might be useful to remind that it is being approached a multi-objective optimization without previous knowledge about the optimal solver, except for the information obtained through the Sensitivity Analysis, what can be considered as pre-optimized solution.

Above of all, the two main reasons that determined the chosen algorithm were the model type of parameters and the frequency of constraints violations during the sensitivity analyses.

The scheme of the EA will be followed is represented in the Figure 5.1. Two operators will be applied on the start designs previously obtained by the sensitivity analysis, mutation and crossover.

Mutation will be characterized by the use of two different mutation types, “Self Adaptive” and “Normal Distribution”. Finding which type is most suitable for the crash box optimization will be one of the aims at this section. In order to assess the optimization procedure, rates from 10% to 90% will be considered. In a similar way, crossover rates will be analyzed, this time from 5% to 95%. First it will be only studied one crossover type called “Simulated Binary”, results obtained by this type will be compared with those reached by “Hybrid” methodology. Definitions of every mutation and crossover type will be detailed in pages 24 and 25.

Finally, it is desired to comprehend how mutations and crossover operator interact to each other. So that, mutation operator will be introduced on crossover offspring to verify if it exists an improvement of the objective values.

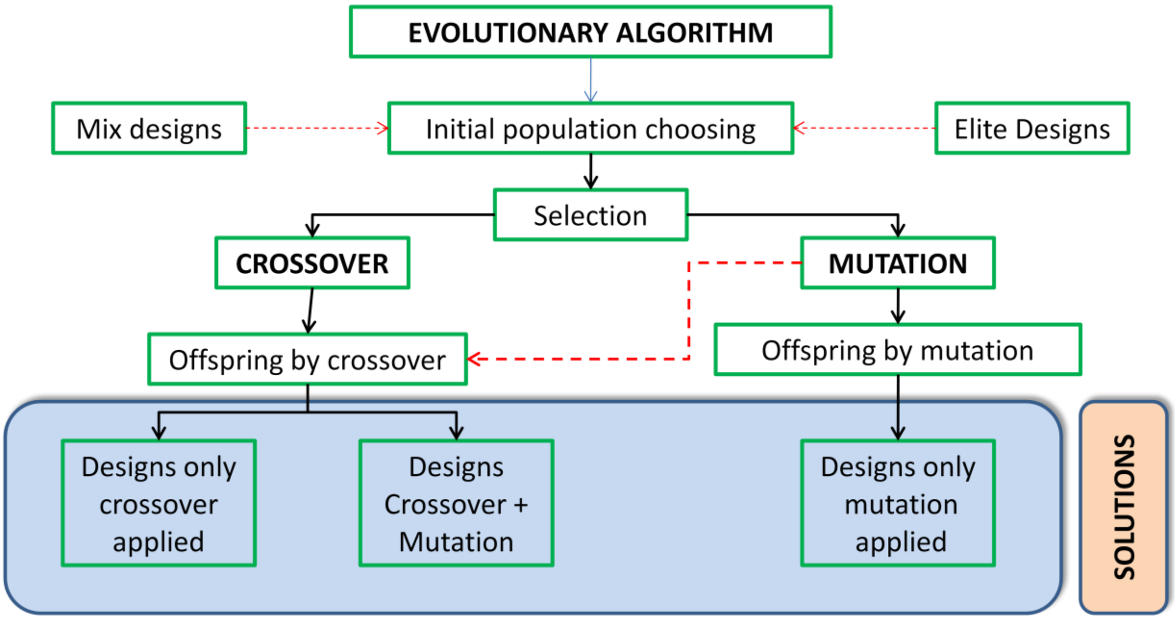


Figure 5.1 Evolutionary Algorithm investigation schematic

- **Initial population**: Two categories were considered, a selection of the most suitable designs obtained by the Sensitivity Analysis which were shown in pages 17 and 18 of the thesis and a mixture of the best, average and bad designs obtained by the Sensitivity Analysis. So that, it can be studied which are the relationships between the results produced for two different groups of start designs and verify if the elitism of the initial population is relevant to determinate the result values.
- **Selection**: This setting is used to ascertain how the new population of the next generation is created from the optimal solutions of the last loop, according to the ranking selected.

Along the inquiry the selection operators were not modified, in order to get a more specific perspective of the optimization process when varying the crossover and mutation operators. Selection features are listed below:

Ranking: Pareto, the election of this method is justified by the number of objectives to optimize. Three objectives are simple to compare by the Pareto´s 3-D frontier and the feasibility of solutions are rapidly and easily compared.

Number of parents: 18

Selection: Tournament, which is the mostly used type, due to its wide range and random election of each individual. Individuals compete between them and the algorithm chooses that one which provides the best solutions.

Tournament size: 4.This number represents the quantity of competitors that take part into the tournament selection at the same time.

- **Only mutation operators applied**: Mutation is a genetic operator which is used to vary the genetic code of a population according to a random variable that modifies arbitrarily one or more bit of the sequence chain, changing definitely the chromosome of an individual. That is applied to increase the region of possible optimal solutions.

Due to the lack of awareness of the model's response, a large range of mutation rate was analyzed. In order to figure in which region of the space the optimal could be found. Rate values 10%, 30%, 50%, 79% and 90% were simulated. As it was mentioned before, as the most suitable as a mixture of the design obtained were set up as start designs. Every percentage rate value was also analyzed with a variation of the mutation type. Self adaptive and normal distribution were the mutation types selected according to the studied area's characteristics.

Table 5.1 shows a summary of the investigation field under the criteria above described.

Table 5.1: Mutation strategy

Mutation rate	10%	30%	50%	70%	90%
ELITE	self adaptive	self adaptive	self adaptive	self adaptive	self adaptive
MIXTURE	self adaptive	self adaptive	self adaptive	self adaptive	self adaptive
MIXTURE	Normal distribution	Normal distribution	Normal distribution	Normal distribution	Normal distribution

Self adaptive mutation *"Technically, this so-called self-adaption principle combines the representation of a solution and its associated strategy parameters within each individual, and the strategy parameters are subject to mutation and recombination just as the object variables"* (Bäck, 1996)

Normal distribution: Variables are distributed forming a bell curve symmetrically in concordance to a specific statistical parameter. Further information can be found on (Waeber, 2008).

- **Only crossover applied**: Crossover is the other important genetic operator considered for the optimization. On the contrary to mutation operator, chromosomes are not randomly modified, but an offspring is created by more than one parent through information exchange, miming “sexual reproduction”.

Crossover range from 5% to 95% will be applied, so a vast extension of the space can be explored. The same shall apply to crossover operators, simulating every percentage as for mix as elite start designs.

The method selected was Simulated Binary, because it allows vary widely the crossover rate, in Table 5.2 are shown those rates which were tested by only “Simulated Binary” and those that also “Hybrid” crossover type was applied. Hybrid allows to combining two crossover operators, as it will be analyzed in section 5.5.3. The effectiveness of that tool on the crash box will be also tested.

Table 5.2: Crossover Strategy

Crossover rate	5%	25%	50%	75%	95%
ELITE	Simulated binary/Hybrid	Simulated binary	Simulated binary/Hybrid	Simulated binary	Simulated binary/Hybrid
MIXTURE	Simulated binary/Hybrid	Simulated binary	Simulated binary/Hybrid	Simulated binary	Simulated binary/Hybrid

<u>Simulated Binary</u>: The genes of the new individuals are determined from intervals defined in neighborhoods of the parent genes throughout probability distributions (K. Deb, 1995).

- **Crossover and mutation:** The last point of the researching will be to analyze the influence when the optimizer applies mutation on crossover offspring, so an exploitation of the explored space could be carried out.

This section will let to know how the optimization interacts when both criteria (mutation and crossover) are applied and its significance to optimize the model.

Mutation rates will vary from 1% to 5%. The selection of these values is due to higher values might have brought a too wide exploitation, what entails a non accurate searching and makes it a randomly variation of the parameters.

The chosen offspring are those produced by applying 5%, 50% and 95% crossover rates

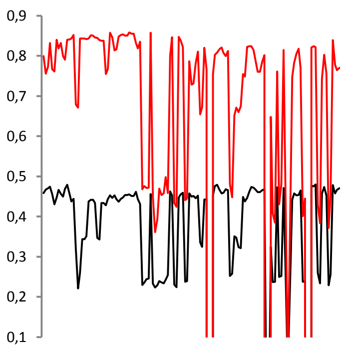
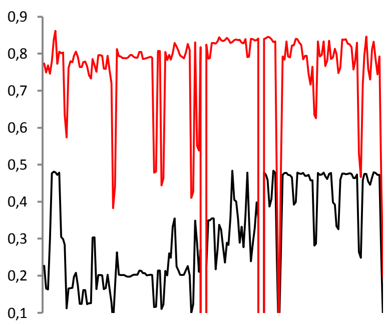
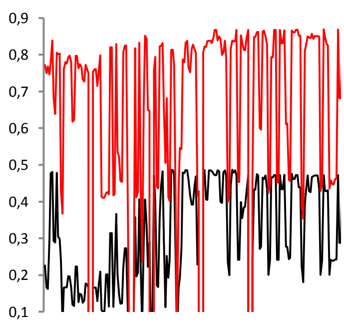
5.4 Results optimization process after applying only mutation operator

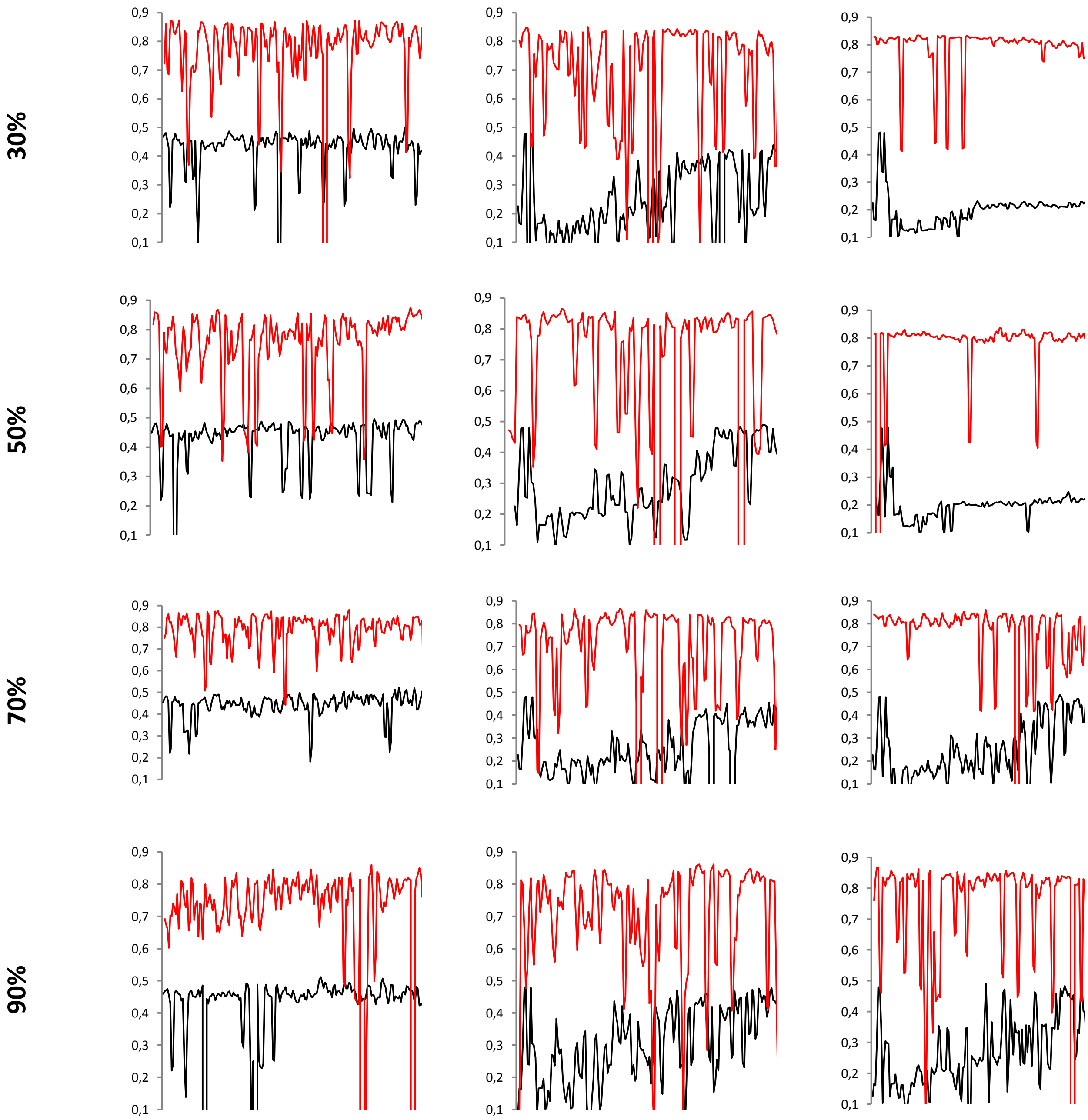
At this point the development of the different optimization methods according to their mutation rates and start designs selected will be analyzed. The goal of this approaching is to understand the exploration methods used by OptiSLang and how the variation of mutation operator affects to the designs obtained.

5.4.1 Optimization procedure Analysis

Assessing the optimizer trends is an essential factor to comprehend how the optimization process is working. Table 5.3 illustrates the tendency lines based on “moving average”, according to mutation type and rate. So that the settings applied on each one could be understood. The X axis, which is not drawn, indicates the design number from 1 to 208. As the number of selected parents was 18, designs from 1 to 18 correspond to results obtained by the Sensitivity Analysis and considered as start designs. The Y-axis indicates the “Efficiency” and “Efficiency deformed” values, in every plot the tendency line that is above belongs to “Efficiency deformed”, by contrast the tendency line beneath represents “Efficiency” objective. Such plots allow to identify the yield of the optimizer, as much irregular the trend is, the more number of false designs have been calculated. If an objective value goes under 0,1 means that this design has been not succeed.

Table 5.3: Mutation objectives distribution according to rate and mutation type applied.

	Elite_SA[11]	Mix_SA[12]	Mix_ND[13]
10%			



In Table 5.4 balances of how many succeed (green number) and false (red number) designs of every mutation methodology have been calculated, are shown below.

Table 5.4: Summary of feasible designs according to rate and mutation type applied.

Mutation rate	Elite	Mix_SA	Mix_ND
10%	8/200	21/197	20/181
30%	6/202	29/179	13/195
50%	10/198	14/194	8/200

70%	4/204	20/188	18/190
90%	9/199	19/189	14/194

There is not any relationship between the amount of false designs calculated and mutation rate. However, it is remarkable that for each rate elite got fewer false designs and Mix_SA the more.

Information of the two main objectives collected from the previous table is fully gathered on the table below.

Table 5.5: Objectives comparison according to start designs and mutation type.

	ELITE	MIX_SA	MIX_ND
Efficiency	Increases softly progressively, as the optimization process is run, due to high values of the best sensitivity designs. For middle and low mutation rates, the optimal search is irregular. By contrast, as the mutation rate grows, process get more constant	In every mutation rate is visible the effect of the "random" start designs at the beginning of the optimization process. However, the bad designs are clearly improved by the mutation operator and the efficiency results are getting bigger, as the optimization is run. The process ends with similar values as it was set, and the mutation rate does not affect the tendency.	When the mutation rates were between 10% and 50%, it is evident that the optimal searching is regular without big deviations of the objective maximum peak. Applying higher mutation rates, such regularity turns to a non uniform increase of the objective, although more optimal efficiency values are achieved.
Efficiency deformed	The optimizer did not find a clear region of the design-space where to look for the optimal. That is the reason why the trend line does not make a regular progression.	The objective is not highly improved and remains mainly constant along the optimization process, although some designs obtain values that exceed those previously reached by the sensitivity analysis. However sometimes big fluctuation appears due to those non feasible designs calculates that provide values under 1%	A similar response occurs with efficiency deformed values. As the mutation rate increases, the optimization process becomes more irregular, without a significant improvement of the objective.

5.4.2 Comparison Elite Vs. Mix start designs

One of the goals of this thesis was to know if the start designs set up on the optimizer solver (EA) was actually relevant along the process observing how the objectives were modified according to this criteria.

In order to determinate such an influence, the results of every mutation rate from 10% to 90% are compared below after applying a good result filter. Both were set using Self-adaptive mutation type. Only those designs that exceed 0.48 of efficiency and 0.85 of efficiency deformed were displayed. Every plot in Figure 5.3 represents on the X axis the design number, so from 1 to 18 are the start designs and from 190 to 200 the last generation. On the Y axis efficiencies values are represented, dispersion points above indicate “Efficiency deformed” values and points beneath “Efficiency”.

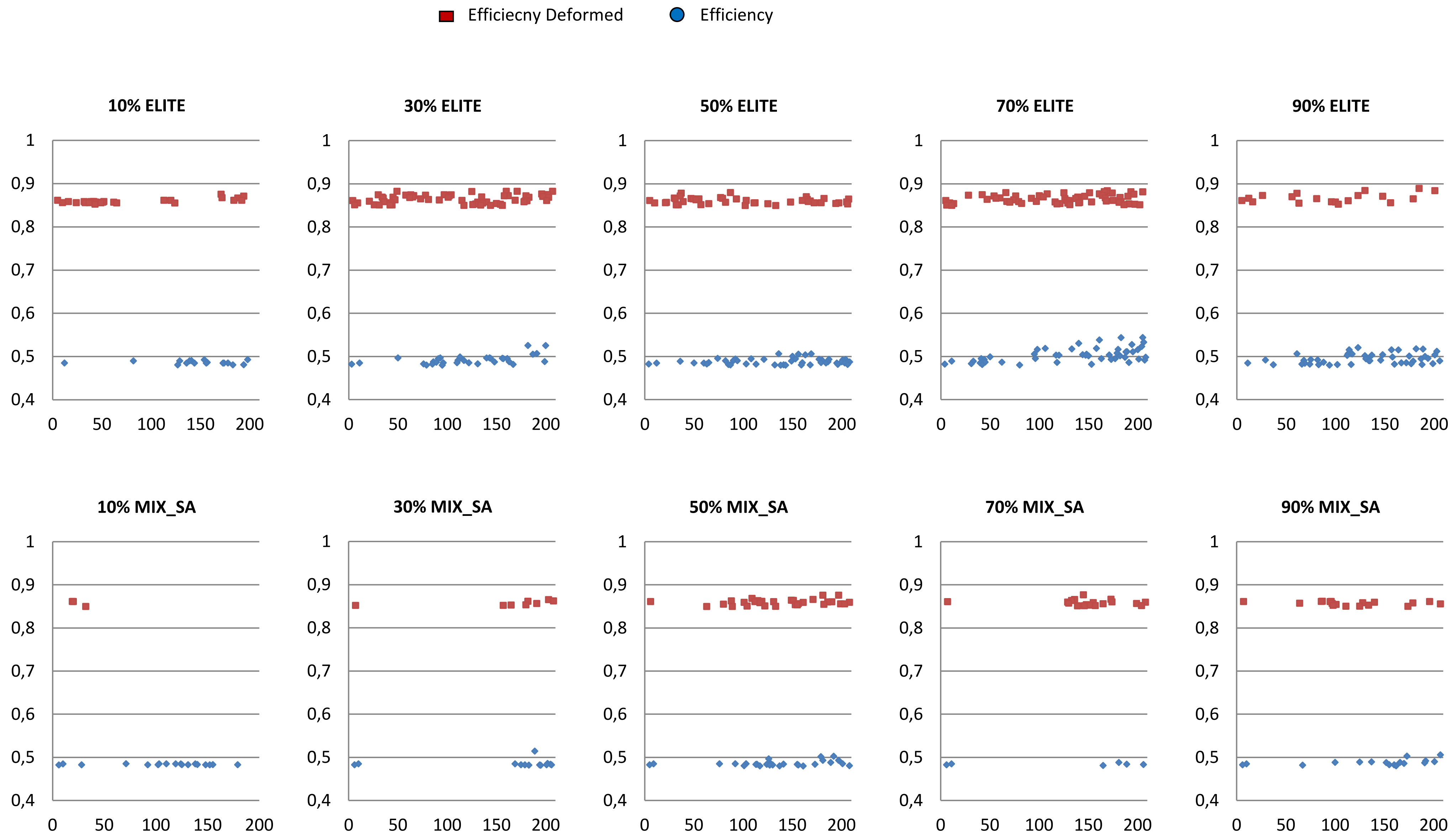


Figure 5.3: Good mutation results compared by initial population and rate

Elite start designs always obtains a bigger quantity of good results, as it can be verified observing the number of dispersion points. Moreover for every mutation rate but especially for high ones, the number of good results is not the only remarkable point, but also the values themselves. As “Efficiency” as “Efficiency deformed” objective reached higher values when Elite Start designs were used. Exceeding normally 0.50 “Efficiency” value and 0.87 “Efficiency deformed”.

5.4.3 Comparison Self-Adaptive Vs. Normal distribution

In the same way it was done to compare Elite with Mix start design, Figure 5.4 compares the number of good results, (same filter applied, page 29), according to the objectives values reached.

According to the results, using Mix, independently whether the mutation type Self Adaptive or Normal distribution is, the optimization process is slower, and good results are approximately reached from design 100 for Self-adaptive and even later for Normal Distribution. In fact, for 30% and 50% mutations rates when Normal distribution was applied, none good result could exceed the filter limits.

Despite having softly improved the objectives established can be observed how start design obtained by the Sensitivity Analysis are still displayed on the plots.

Normal distribution mutation type provides a more secure line of analysis due to its regularity, although the optimization process is slower, because of its parameter variation is established beforehand, instead of varies while the process is being developed, as self adaptive type does. However, self adaptive achieves normally higher objective values.

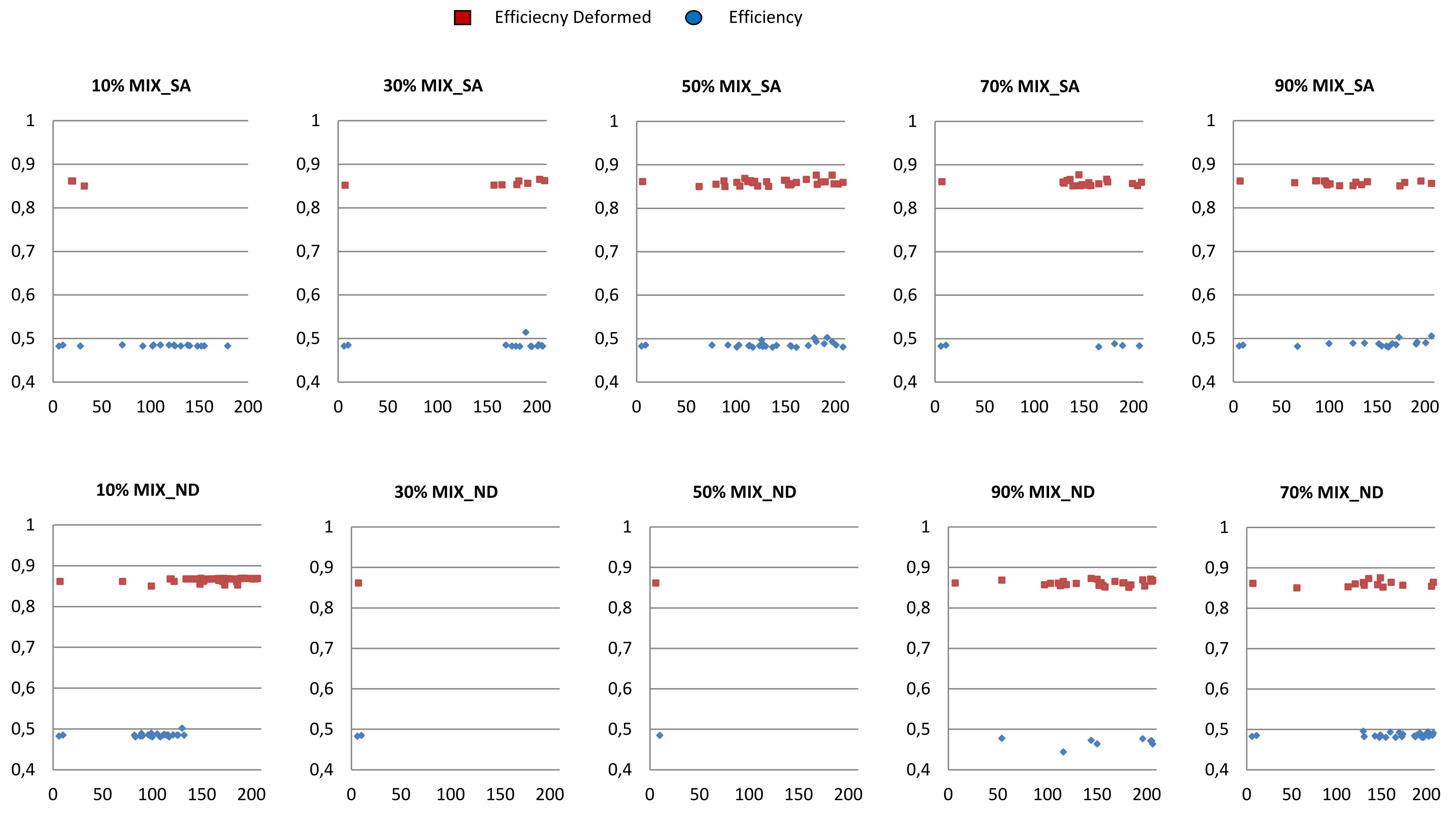


Figure 5.4: Good mutation results compared by mutation type and rate

5.4.4 Searching the maximal Optimum

In order to locate those designs which maximize the objectives that have been established, three designs from every mutation rate and type were extracted. The criteria chosen for the design election were maximal efficiency (objective 1), maximal efficiency deformed (objective 2) and the best design according to Pareto ranking. Information is detailed on [Appendix A.2.]

In most of the cases coincide that the best Pareto design is that one which "Efficiency" is the highest, curiously when normal distribution mutation type is applied for rates 30% and 70% does not occur. In order to provide a better understanding of how Pareto's front is working and to find the optimal design, plots are illustrated in Appendix A.3. It should be pointed out that "difference" as third objective is considered. Despite having not given so much relevance to this third objective before, it will acquire importance to decide which designs are most suitable. If it is desired to delve into how the different settings distribute the designs in the space, Pareto 3-D plots are included [Appendix A.4 and A.5].

Table 5.6 gives a review of those designs that maximize the objectives the best. Three objectives for every design are evaluated and only those which satisfy both criteria established by the filters were extracted for a final assessment. "Difference" is also indicated to make the designs assessment.

Efficiency	Efficiency deformed
>0.48	>0.86

Table 5.6: Most suitable designs obtained by mutation

	Design	Crashbox Efficiency	Crashbox Efficiency Deformed	Difference
Elite 30	140	0,49682	0,85824	0,36142
Elite 50	82	0,49033	0,85753	0,36719
Elite 70	205	0,5441	0,88173	0,33759
Elite 70	183	0,5439	0,8590	0,31513
Mix ND 70	130	0,49527	0,86381	0,36853

Elite 90	61	0,50654	0,87824	0,3717
Elite 90	123	0,52079	0,87316	0,3523
Elite 90	130	0,50206	0,88477	0,38271
Elite 90	148	0,5044	0,87146	0,36705
Elite 90	201	0,50392	0,88448	0,38056
Mix SA 90	207	0,50587	0,85604	0,35016

The most suitable designs were compared and it was concluded that design 205 of elite_70 provides the best objective values. Thus, there is no doubt that is the most suitable design to improve the crash box response after using mutation settings. Nevertheless if "Difference" objective is considered as the most relevant criteria, the election changes in benefit of Elite 70 – 183.

On the other hand it is interesting to notice how most suitable designs shape are. In most cases are comparable. Figure 5.5 shows how the preferable shape is a hexagonal prism with the cross on the left side of the crash box.

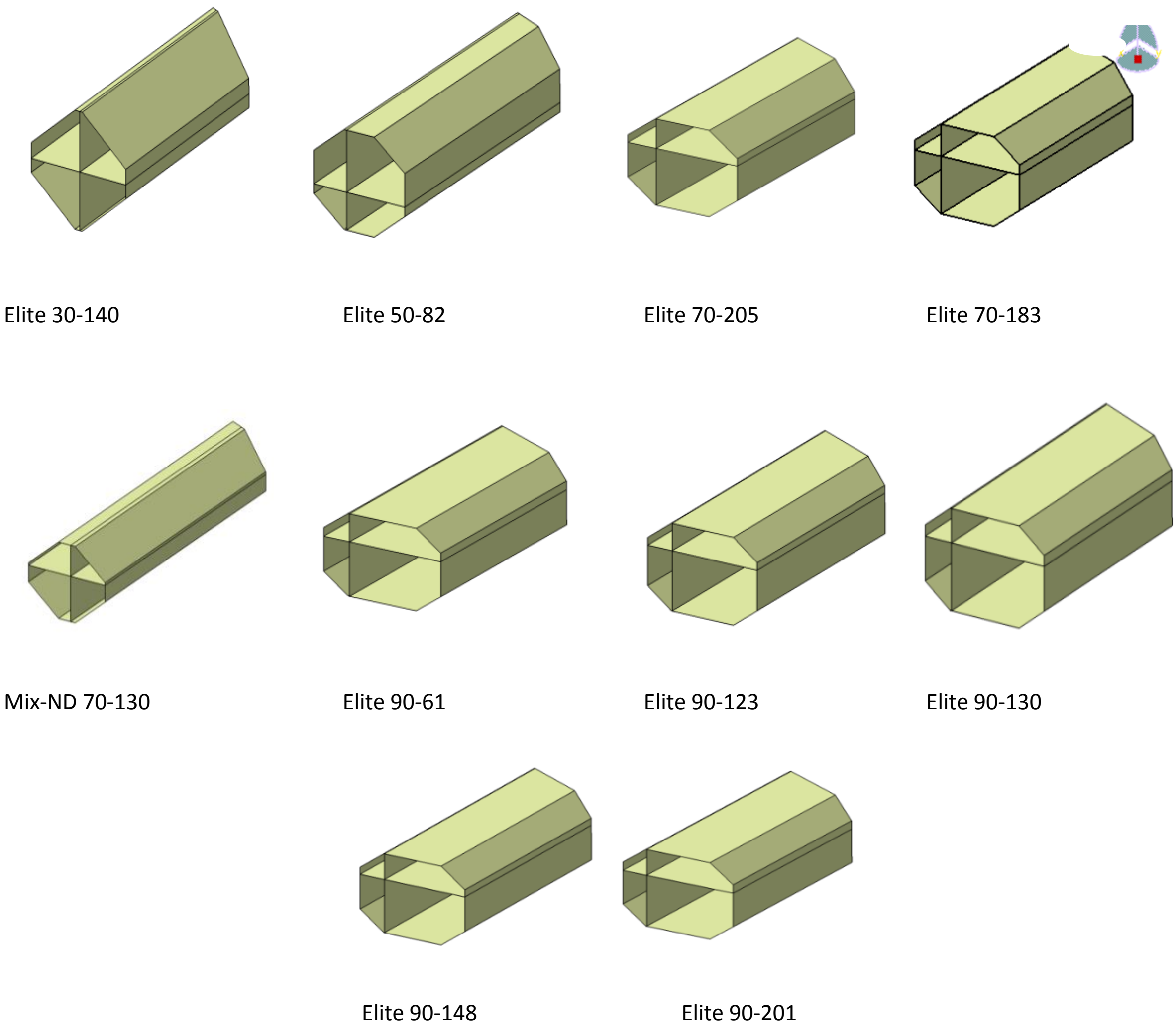


Figure 5.5: Most suitable designs compared by crash box shape

5.4.5 Mutation methodology and influence parameters

As the sensitivity analysis had predicted, “Blechdicke” and “Verkürzung” are the most important parameters for the objectives. It is simple to identify how the parameters space distribution is. In Figure 5.6 “Blechdicke” distribution is illustrated, on the other hand, Figure 5.7 shows “Verkürzung” distribution.

Black bar in boxplots indicates the entire range of values of each parameter. “Blechdicke” range fluctuates from 2 to 3.8 and “Verkürzung” from 0 to 90. Underneath box gathers every data located between the median and the second quartile and above box does it for those data between the median and the third quartile. Once the most suitable designs have been identified and is already known which range of values are the best, this sort of graphic lets deduce intuitively how every mutation methodology works on the optimal searching, It can be observed how the range is symmetrically shortened for Elite Start designs, what makes sense due to the biggest amounts of good results were found using Elite. Moreover high mutation rates provide values closer to the minimum (Blechdicke) or maximum (Verkürzung) indicating where the best parameter values are.

If Self-adaptive and Normal distribution mutation types are compared, it is visible that their distribution differences are not significant. Ranges are much wider than Elite and in some cases the symmetry is broken. More data is located on the second or third quartile and therefore distancing the searching of the goal.

Remembering boxplots illustrated in Figure 4.7 of section.4.4.5 where initial population distributions were shown. It is indispensable to point out resemblances between the beginning and the final distributions. Elite keeps a high grade of similarity, thus, it can be claimed that the sensitivity analysis 600 samples is a precise tool to approach faultless a first exploration of the global optimum.

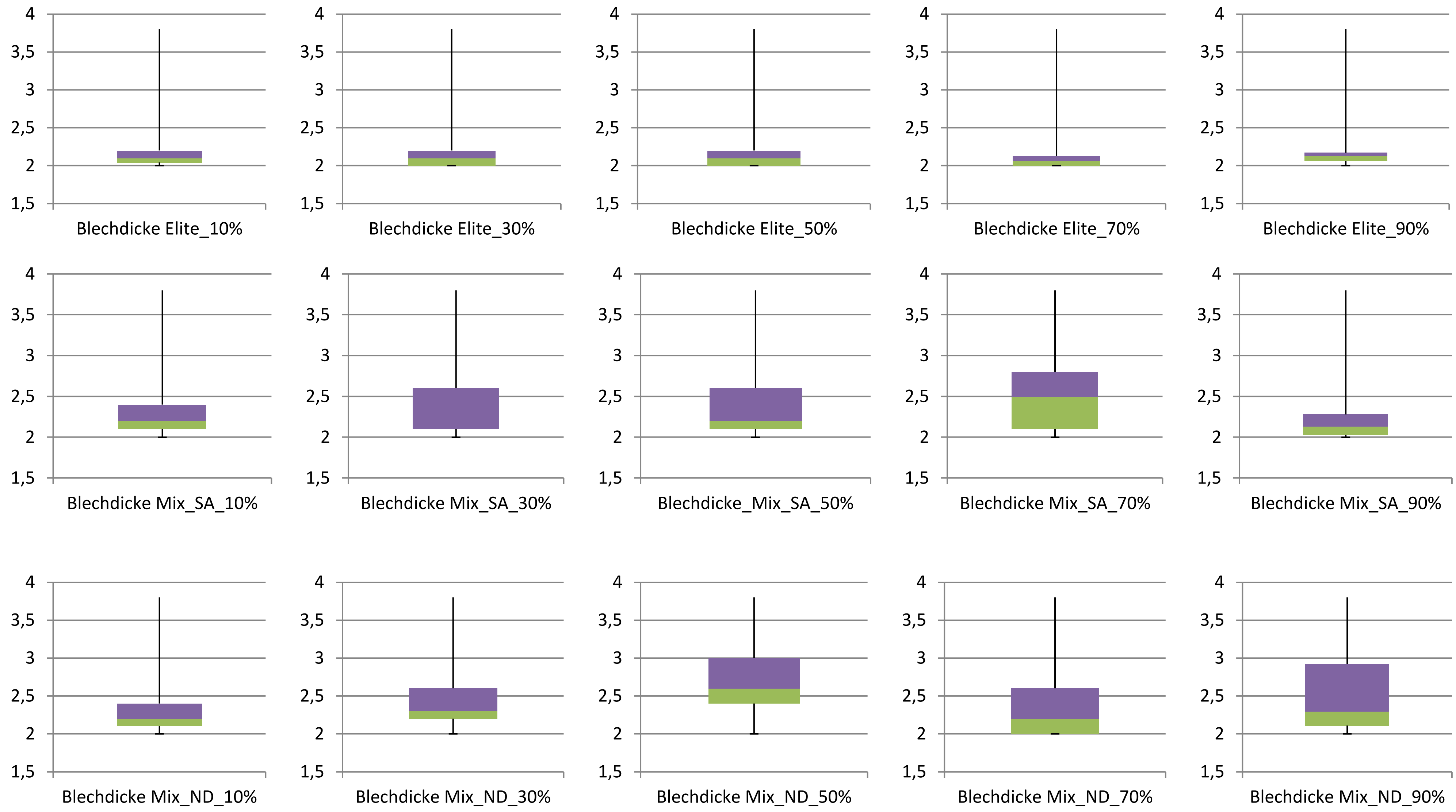


Figure 5.6: Blechdicke Boxplots for every mutation rate, type and initial population

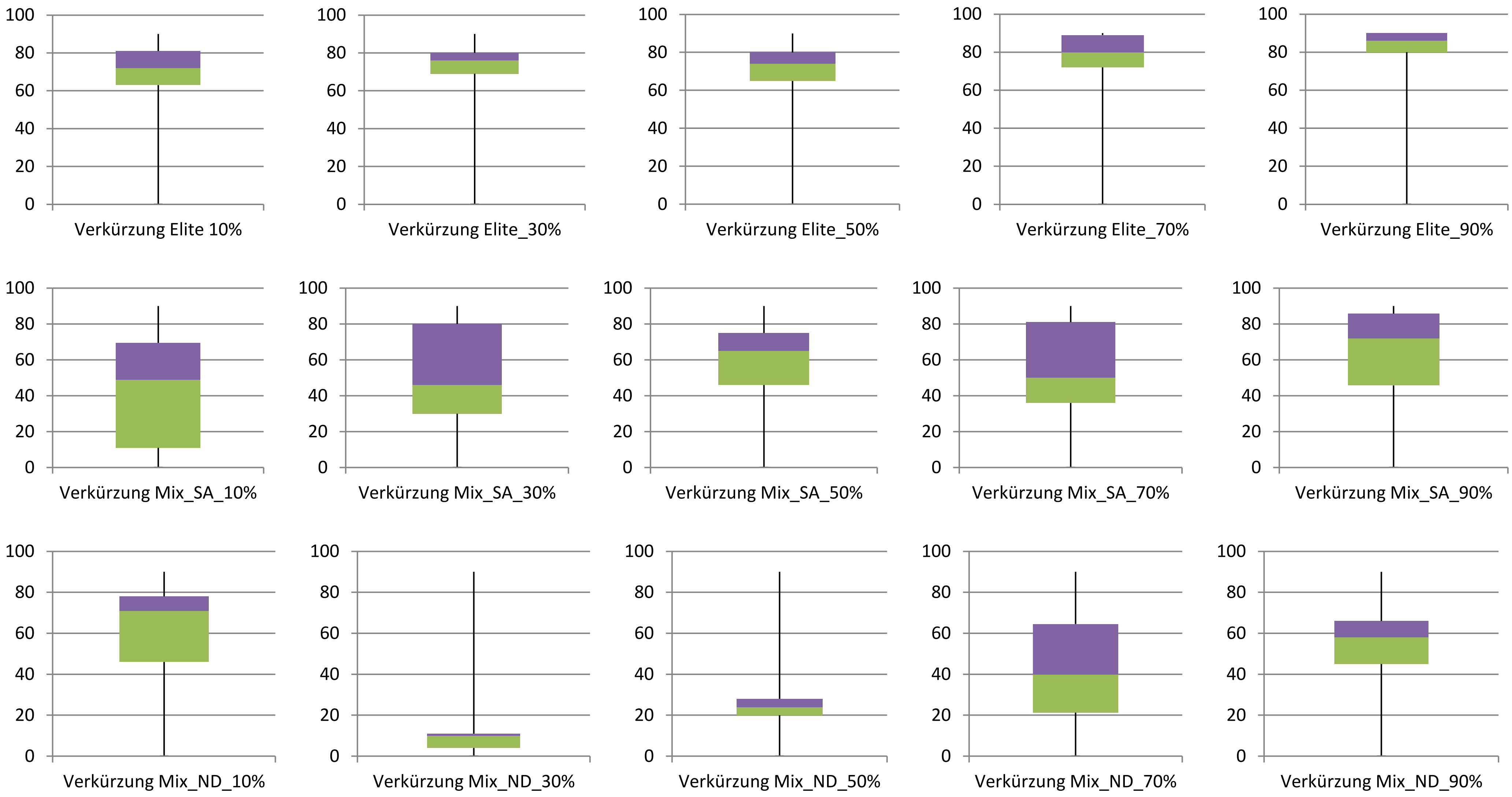


Figure 5.7: Verkürzung Boxplots for every mutation rate, type and initial population

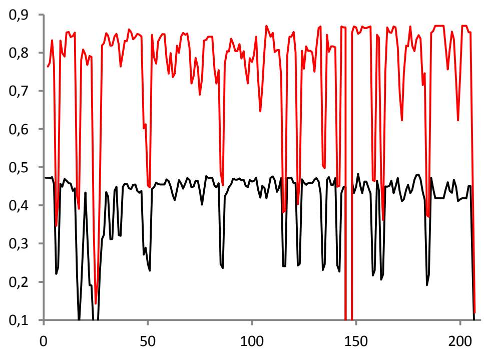
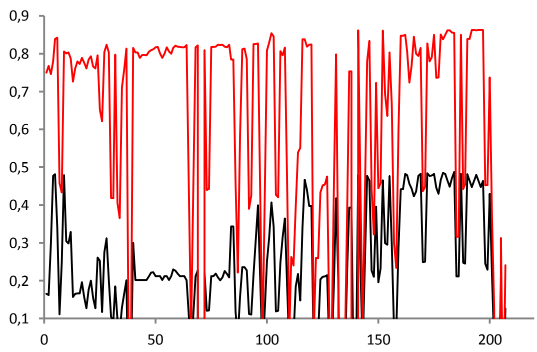
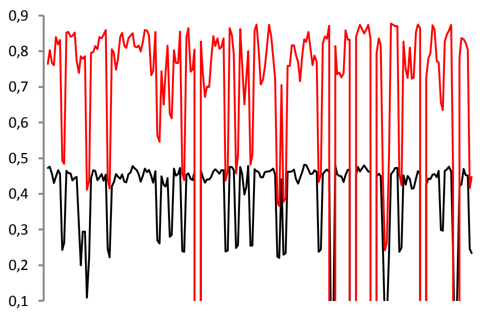
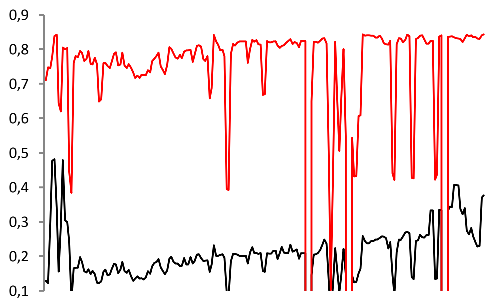
5.5 Results only crossover

In the same way as it was done during the application of mutation operator. The effect of start designs when the crossover rate is varied is following analyzed and the consequences it produces along the optimization process development. The rates that were applied fluctuate between 5% and 95% and Simulated Binary was the crossover method used.

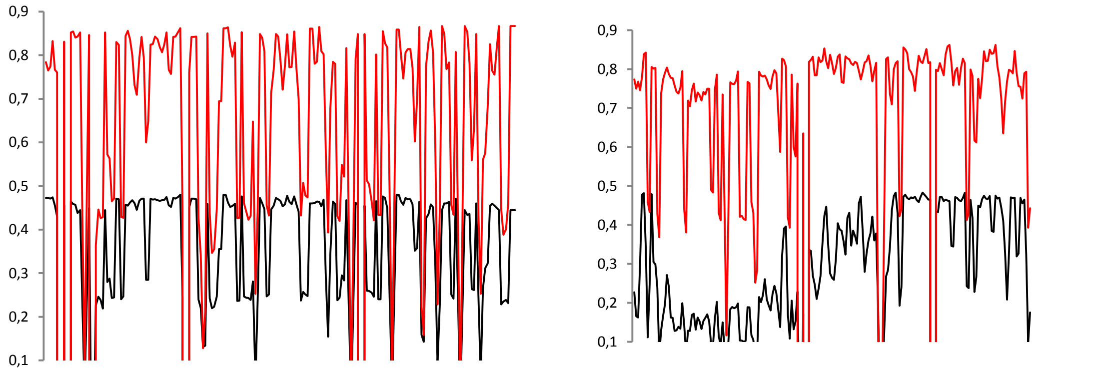
5.5.1 Optimization procedure analysis

Table 5.7 illustrates the tendency lines based on “media movil”, according to crossover rate. The X axis, which is only represented for 5%, indicates the design number from 1 to 208, the number of selected parents was 18, designs from 1 to 18 correspond to results obtained by the sensitivity analysis and considered as start designs. The Y-axis indicates the “Efficiency” and “Efficiency deformed” lines beneath and above respectively. Optimizer yield is represented as it was explained within section 3.5.1 of this thesis.

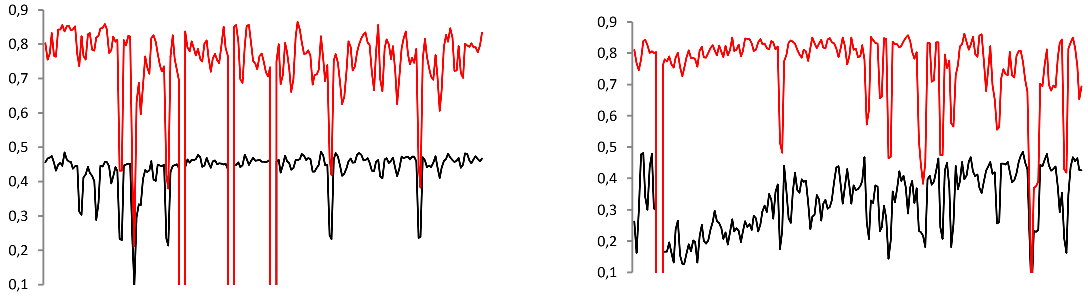
Table 5.7: Objectives development according to crossover rate and initial population.

	ELITE	MIX
5%		
25%		

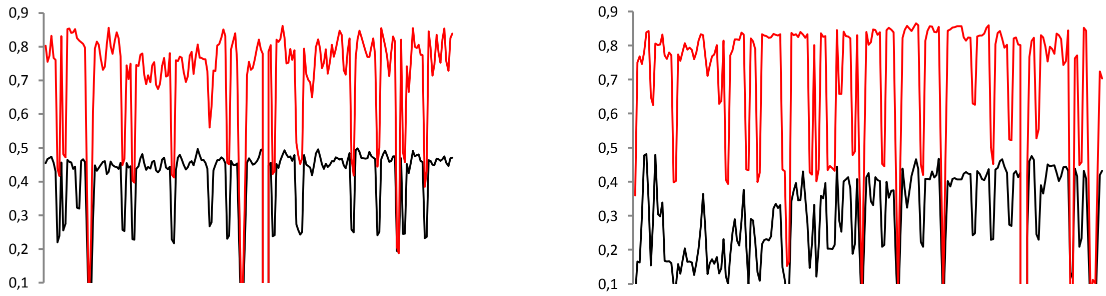
50%



75%



95%



In Table 5.8 verification of every crossover yield is shown. False design (left), succeed designs (right). As it was mentioned in section 5.4.1, a false design is considered when one of the objectives goes under 0.1.

Table 5.8: Summary of feasible designs according to rate and crossover type.

CROSSOVER RATE	ELITE	MIX_SA
5%	14/194	35/173
25%	15/193	16/192
50%	30/178	22/186
75%	6/202	12/196
95%	17/191	25/183

The lesser false designs, the more regular the graphics above are represented. Generally, elite obtains fewer false designs, but for 50% rate. It is no possible to extract a clear correlation between crossover rates, start designs election and their influence on the quantity of false designs calculated.

Information of the two main objectives collected from the previous table is fully gathered on the table below.

Table 5.9: Objectives comparison according to start designs

	ELITE	MIX_SA
Efficiency	Due to the high efficiency values at the beginning of the optimization process, the results are practically constant. Results do not fluctuate along a wide range of values, so the crossover rate is not a significant operator.	In the same way it happened with mutation operator, it is evident the use of mix start designs. Tendency of efficiency values is increased, as the process is developed. Bad offspring are rapidly eliminated however as can be observed in previous plots, the efficiency improvement is not really substantial.
Efficiency deformed	Some progresses on the results are found, but it can be appreciated that the trend is completely irregular. The optimizer did not focused on one space region, instead was searching randomly the parameters which made higher the objective, finding some good results.	

5.5.2 Comparison Elite Vs. Mix start designs

The goal of this point is to determine if there is any difference when start design are different on crossover settings application, as it was when applying mutation. In Figure 5.8 dispersion plot for both initial populations are illustrated. As can be observed “Efficiency” objective is better optimized for Elite start designs. The number of good results obtained by Mix is extremely low and additionally in most cases are those used as initial population. This fact involves that crossover operator does not work properly for Mix start designs or because of the large DoE[3] carried out by 600 designs. If “Efficiency deformed” objective is compared, the difference is not so significant, especially for high crossover rates. However, Elite gets the results faster, gradually along the optimization process, contrary to Mix, whose good results appear around design 100.

Remembering the comparison of the same criteria in mutation results, it was obvious that elite provides better objective results. Under crossover control, this fact is also clear. It is certain that elite achieves normally higher objectives values than mix start design, especially for low crossover rates, but it cannot be omitted those singular spots achieved through the Mix application. At the end one of the main goals is to demarcate the most suitable design, so the omission of these possibilities would be mistake.

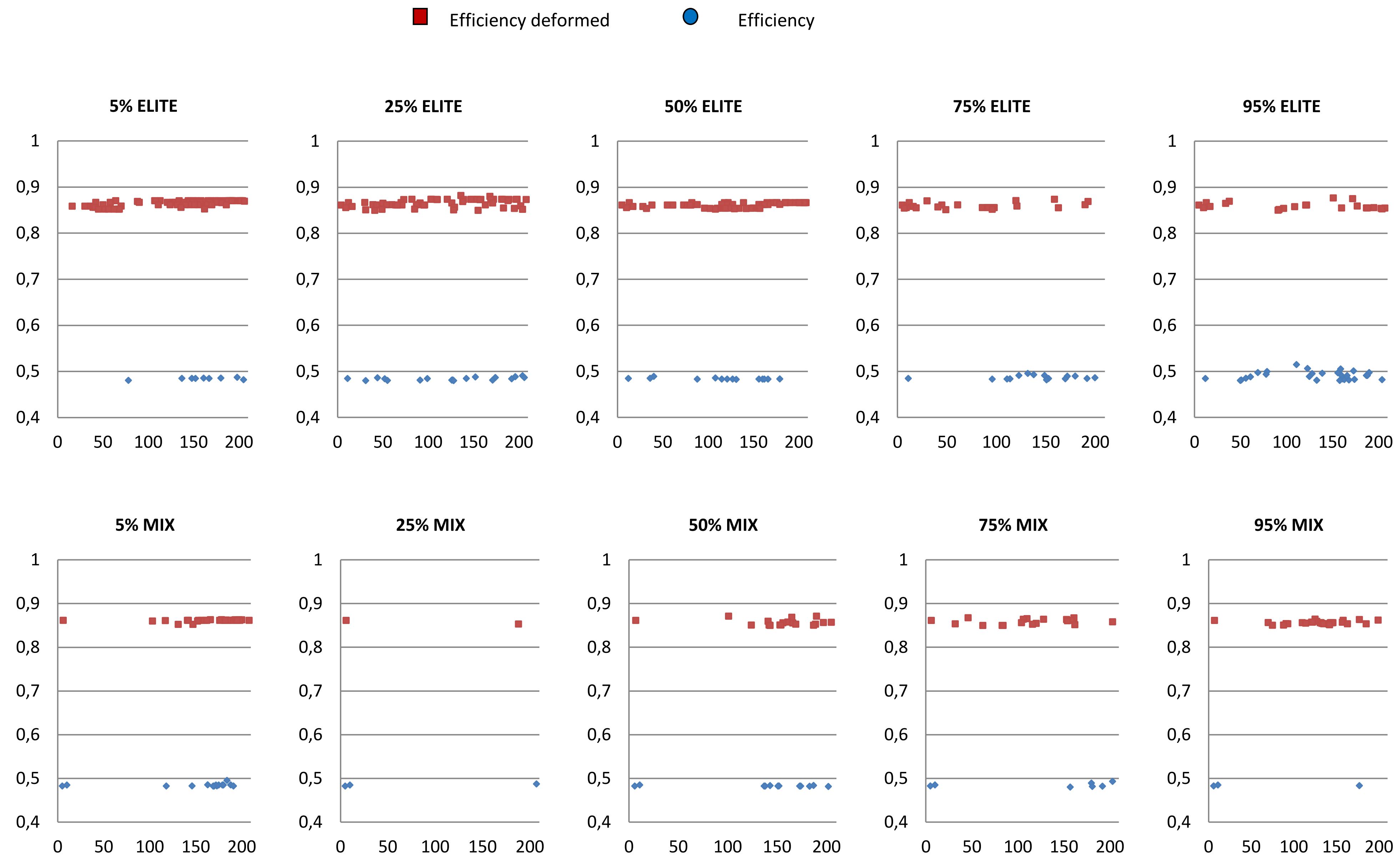


Figure 5.8: Good results compared by crossover rate and initial population

5.5.3 Alternative crossover methods/Hybrid

Hybrid combines diverse crossover operators on the parent chromosomes in order to take advantage of their distinct offspring generation mechanism. OptiSLang provides the selection of two crossover operators to keep the number of control parameter as low as possible (J. Will, 2009)

This operator was used in order to check if the most suitable designs achieved by normal crossover are improved. Exploitation should be more accurate due to the parameter control.
Multipoint and Simulated Binary was the crossover settings selected to observe the effect of hybrid method in crash box optimization. [Multipoint (7 number of points) crossover calls for seven points to be selected on the parent organism strings. Everything between the seven points is swapped between the parent organisms, rendering seven child organisms]. Crossover (Wikipedia).
Simulations for elite start design were made again and their results are in Figure 5.9 compared with those obtained by applying only Simulated Binary operator.

For rates from 5% to 50% “Efficiency” objective is very softly improved, reaching values that exceed 0.5, however surprisingly the number of good results achieved by hybrid methodology is lower than using only Simulated Binary, especially if “Efficiency deformed” objective is compared.
Rates from 75% to 95% show a similar behavior, so hybrid did not get an improvement f the objectives, are even slightly worsened.

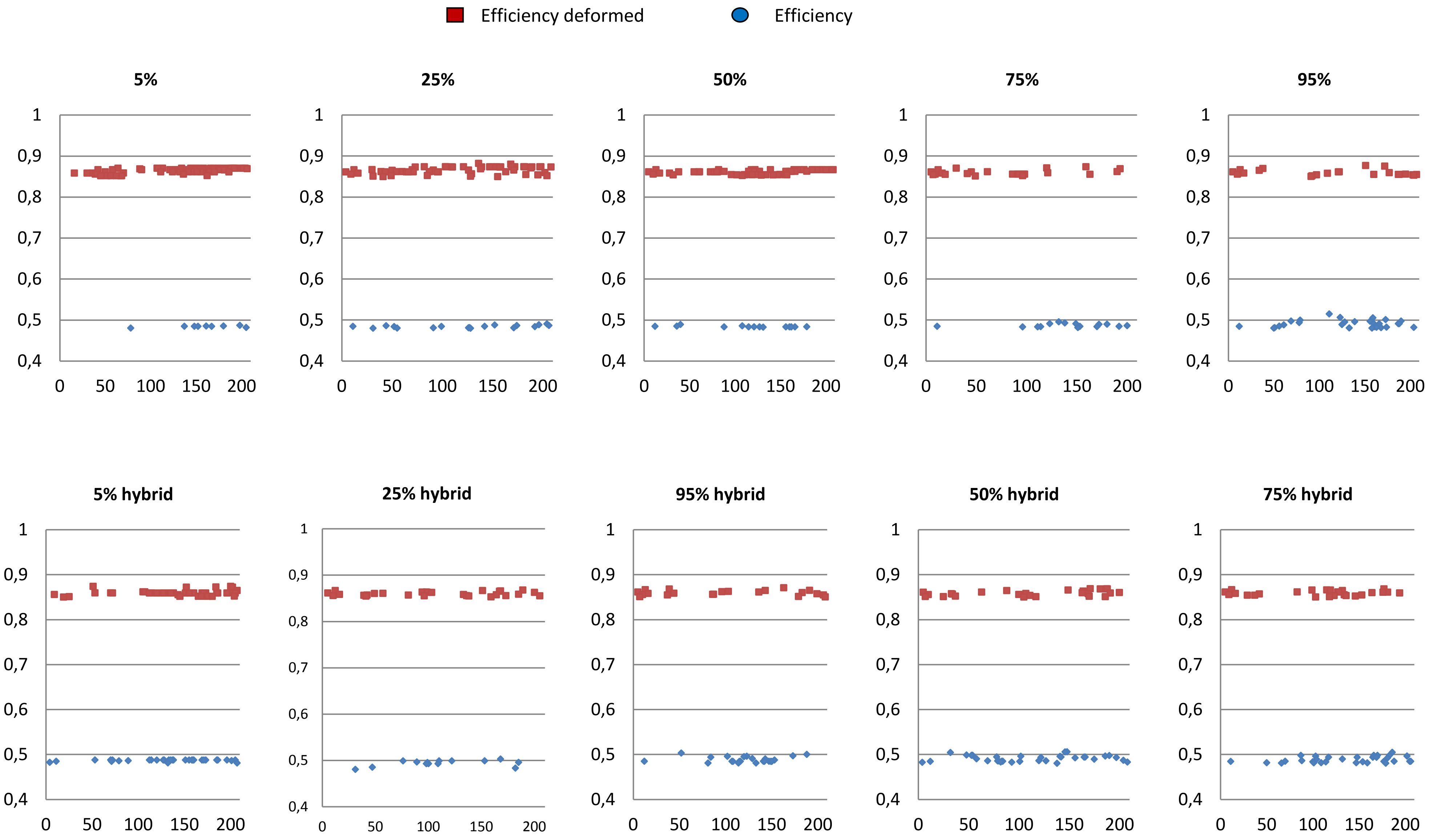


Figure 5.9: Good results compared by crossover operator and rate

5.5.4 Searching the best optimal solver and design

Once the crossover optimization process has been analyzed, it is possible to focus the investigation to find the best design. In Appendix A.6, the best objectives results obtained during the application of crossover rates are indicated according to different criteria, in the same way it was done for mutation best design searching. Those criteria are best efficiency, best efficiency deformed and Pareto.

It can be claimed that the optimal crossover solver for the crash box depends on the strategy the investigation is following. If what is desired to achieve is higher values on one specific objective to use elite start design and crossover rates from 50% to 95% is the correct way to proceed. However, if reducing the difference between objectives, as long as both objectives keep still a reasonable value, this inquiry concludes that mix start designs and crossover values from 5% to 50% is the correct choice.

It is time now to determine which designs provides the best crash box characteristics, this will be done throughout Pareto’s front analysis. In Appendix A.7 every Pareto front collected from each one of the crossover methods employed are illustrated.

Several designs were selected, from each plot according to their space position which depends on the objective values, the procedure to achieve the most suitable design was to analyzed all of them and compare.

Finally in Table 5.10 information related to the possible best solutions is collected. Two filters were applied to decide which designs were the most suitable. Only those that satisfied both are listed: “Efficiency”>0.48 and “Efficiency deformed”>0.85. The third objective “Difference” is used to demarcate the best one.

Table 5.10; Most promising designs obtained by crossover

	Design	Crasbox Efficiency	Crashbox Efficiency Deformed	Difference
Elite 50	179	0,4838234	0,8630614	0,379238
Elite 50	121	0,4835437	0,8628388	0,3792669
Elite 50	156	0,4835245	0,8627914	0,3792951
Elite 50	127	0,483519	0,8628079	0,3792889
Mix 75	203	0,4931969	0,8583844	0,3651875
Elite 95	187	0,4913842	0,8552771	0,3638929
Elite 95	188	0,491361	0,8552368	0,3638758
Elite 95	160	0,4913533	0,8552644	0,3639111

Most of them belong to Elite start designs, except for Mix 75 – 203. Any design from Elite 95 could be chosen as the most suitable because of their results are practically the same. Although they have a lower value of “Efficiency deformed” than those produced by Elite 50, the difference between efficiencies is slightly lower. This fact benefits the crash box response, as it was explained when “Difference” objective was defined.

In Figure 5.11 promising designs shapes are illustrated and as can be observed they are almost identical. However they are different of the pattern found using mutation operator.

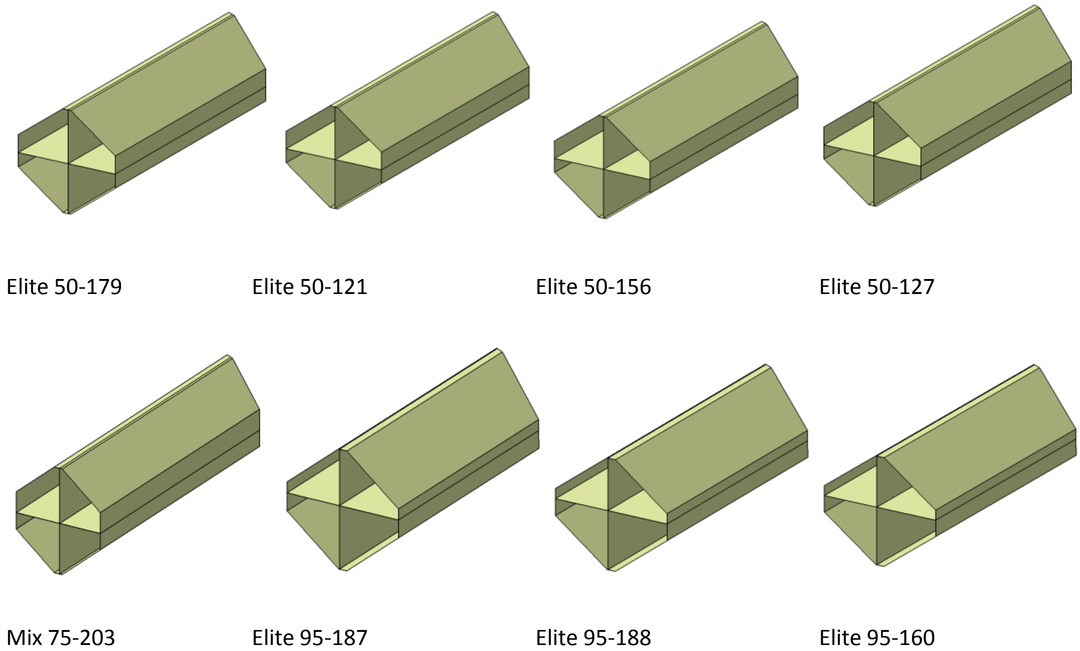


Figure 5.10: Most suitable designs compared by shape obtained by crossover

5.5.5 Optimal optimization solver related to influence parameters

Due to the lack of concordance between designs patterns presented by mutation and crossover operator, it is following analyzed how significant parameters are distributed on crossover designs. So that it might be corroborated that important parameters are located around the same space region.

For that, in Figure 5.12 and Figure 5.13 boxplots in regarding to “Blechdicke” and “Verkürzung” are illustrated.

“Blechdicke” values are extremely located in the third quartile for Elite start designs, even more than when mutation Elite was applied where the symmetry was more considerable. It is a fact that any optimization operator does not vary widely the parameters values when they are previously distributed in a specific area.

By contrast Mix start designs search in randomly way the optimal region, and the result is not successful. As can be checked in the plots, ranges are wider and in particular “Verkürzung” parameter is distant from the values which produced the highest efficiencies.

The difference between mutation and crossover best designs shapes is not located on the relevant parameters, but on “Zwischenblech” and “Zwischenblech vertical”. Following investigations might delve into this fact.

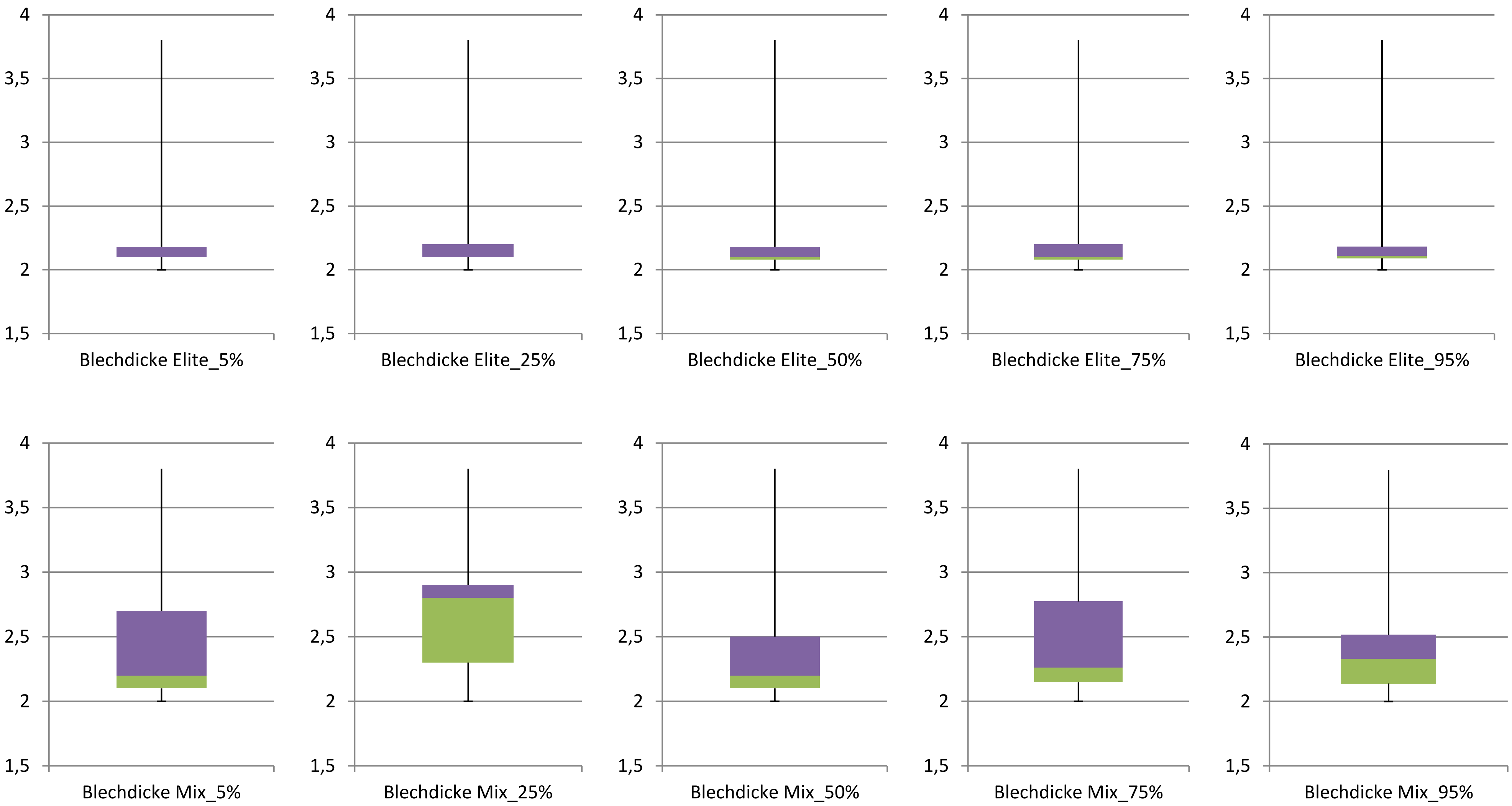


Figure 5.11: Blechdicke Boxplots for every crossover rate, and initial population

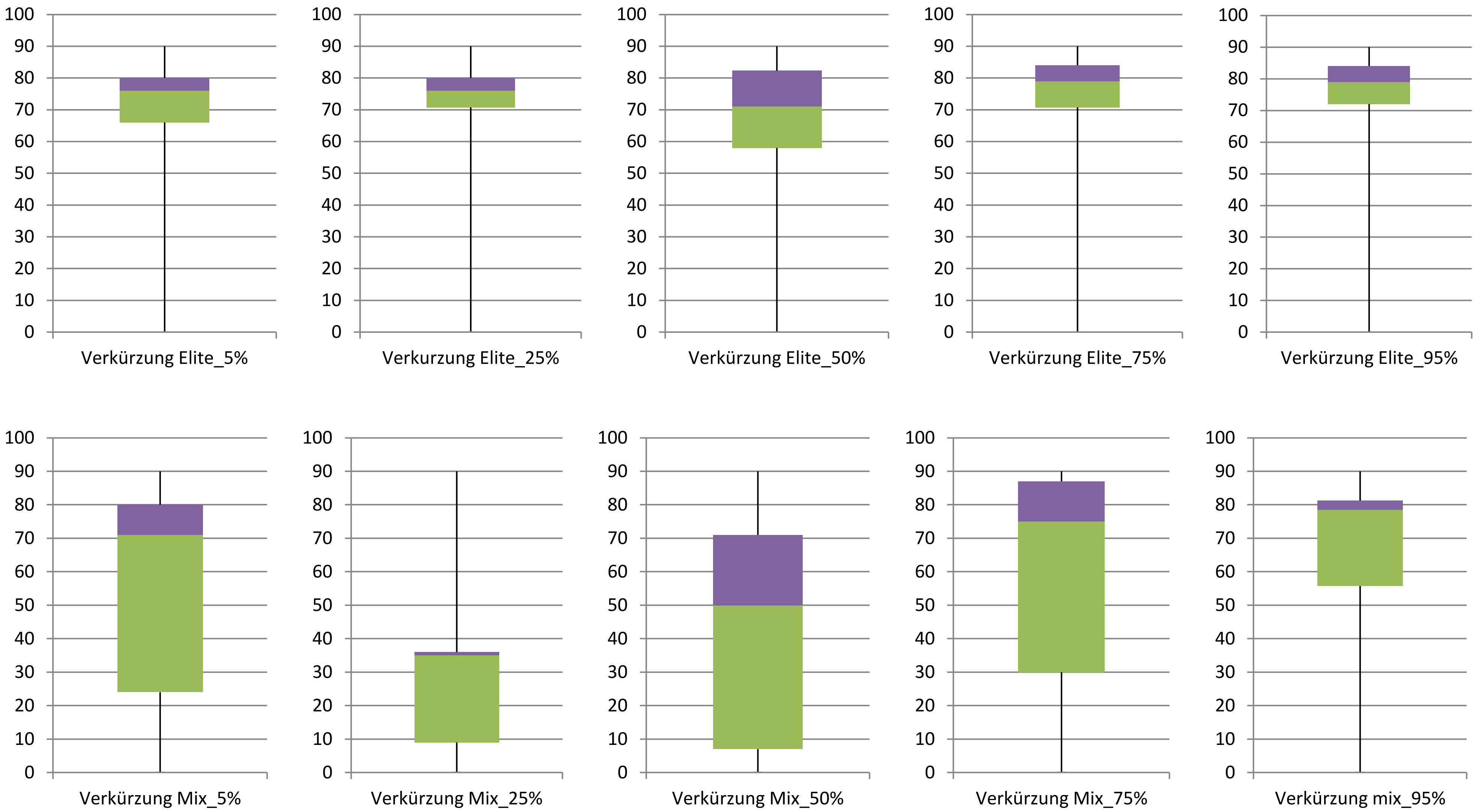


Figure 5.12: Verkürzung Boxplots for every crossover rate and initial population

5.6 Results of applying mutation on crossover offspring

Previous sections have concluded that application of mutation operators produce a better optimization of the crash box. Crossover operator was not as effective, but it is interesting to figure what happen if crossover offspring are altered by low mutation rates. That is the goal of this section. In order to compare the improvement for a wide range, three crossover rates (5%, 50% and 95%) for both initial populations (Elite and Mix) were selected.

The reason to carry this methodology out is to vary the gene code of the offspring produced by crossover. Crossover offspring always keep the genetic information of their parents, contrary to mutation due to its random bits recombination what produces a population genetic change. So the exploration area is expanded.

Twelve new groups of results were obtained when mutation 1% and 5% was used on crossover offspring. Mutation type employed was Self-adaptive, because of their slightly advantages in regarding to Normal distribution, as it was demonstrated along the mutation section.

5.6.1 Effect on Elite Start designs

In Figure 5.14 the mutation impact on crossover offspring for Elite start designs is shown. Round dispersion points indicate those good results obtained by the application of only crossover operator, square and triangle points show the results reached by applying 1% and 5% mutation rates on crossover offspring respectively.

Analyzing the development of “Efficiency” can be observed how for 5% crossover rate, adding 5% mutation operator improves notably the calculation times of good results. For crossover 50% the improvement is due to the objective value, although the velocity of optimization is practically the same. However for 95% crossover rate, there is not any amelioration.

If “Efficiency deformed” objective is analyzed, the assessment is similar. For low crossover rates the improvement is bigger than for high crossover rates.

So it is claimed that mutation affects positively the optimization process for low-medium crossover rates, but the performance for high crossover rates is not worthy. Calculation time is wasted and the results improvement is deficient.

5.6.2 Effect on Mix start designs

On the basis that designs calculated by Mix start designs are normally less effective than those calculated by Elite. It is interesting to evaluate if despite starting with lower objectives values, Mix manages to reach Elite's yield. In Figure 5.15 dispersion plots for established crossover and mutation rates are illustrated.

The same shape criteria distinguish the results. Definitely mutation improves the optimization, especially for 5% crossover rate. It is obvious that the number of good results is thoroughly increased. Both objectives are also grown. But they could not reached objective values as good as Elite did and the number of good results is also considerably lower, especially for "Efficiency". Anyway, on the same line as the influence elite start designs, mutation works better on low crossover rates offspring.

However, "Efficiency" results cannot still compete with the best designs calculated by only mutation operator, none achieved values over 0.5. On the other hand "Efficiency deformed" got several values around 0.87.

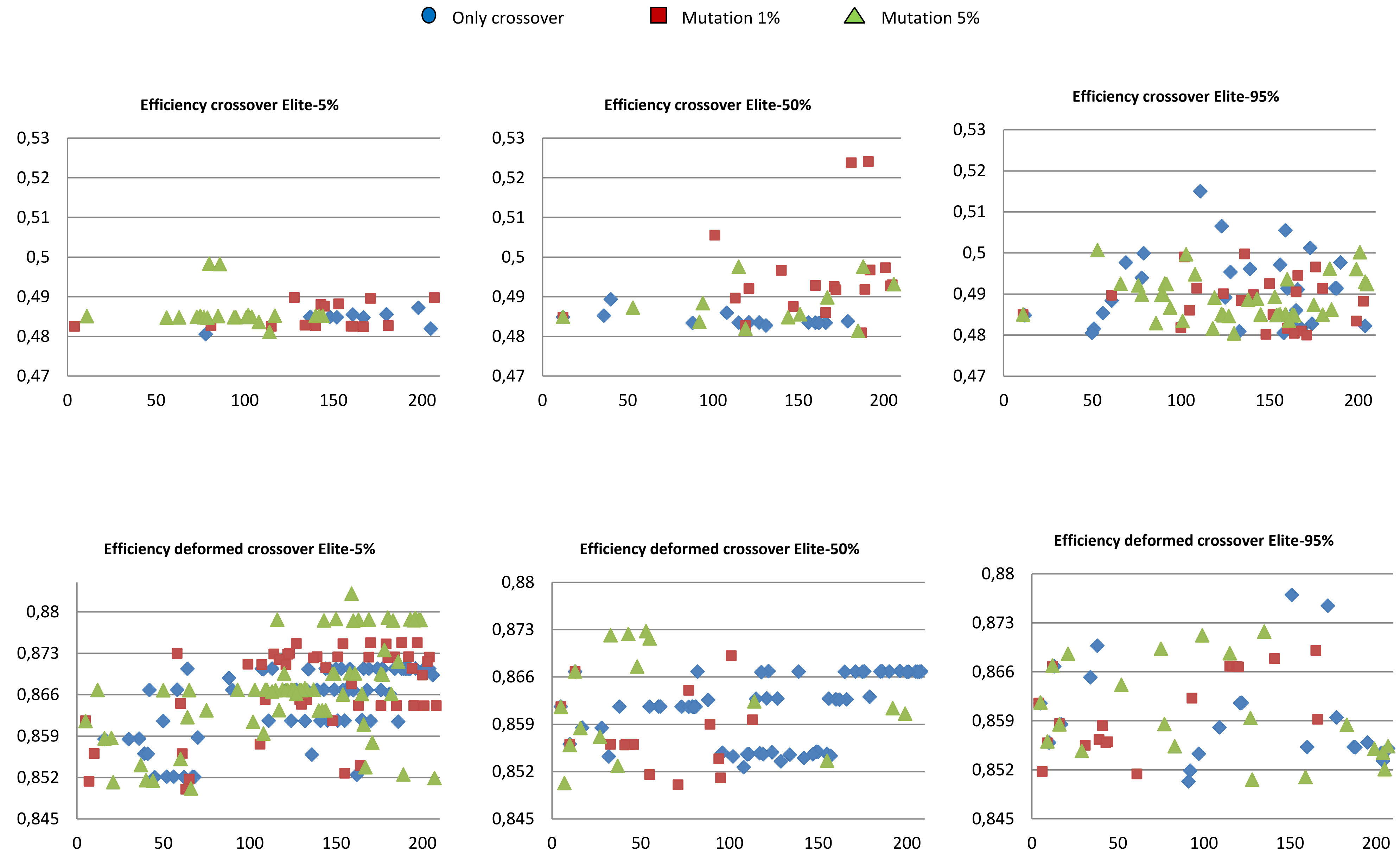


Figure 5.13: 1% and 5% mutation rates influence on Elite crossover

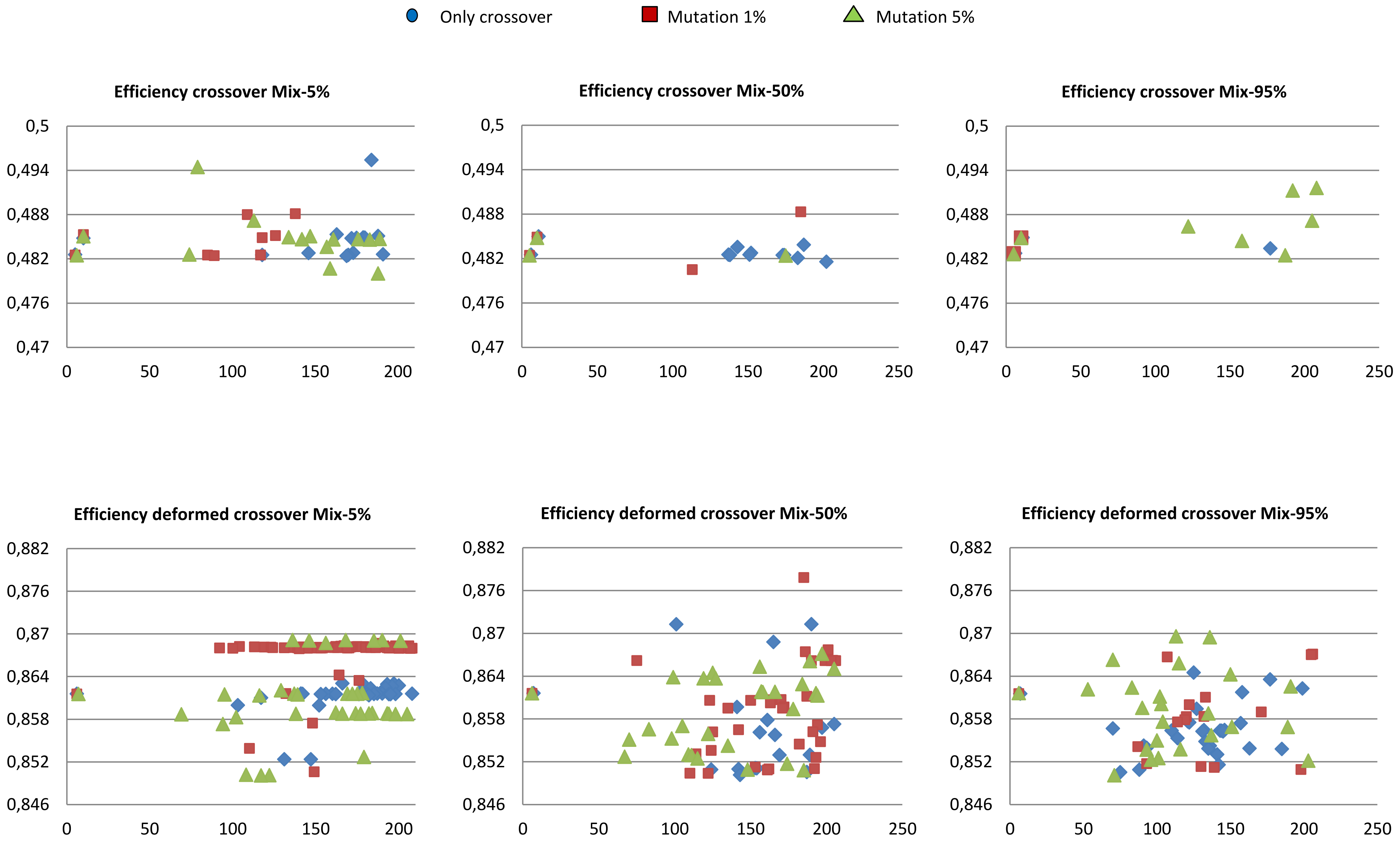


Figure 5.14 1% and 5% mutation rate influence on Mix crossover

6. COMPARISON OF OPTIMIZATION SOLVERS BY AN ALTERNATIVE SENSITIVITY ANALYSIS

As the designs were calculated, it was realized that despite the crash box efficiencies were being optimized in a good way, the increasing such values were not as high as expected. This fact was due to some really good pre-optimized designs had been obtained during the sensitivity analysis. So the question might be contemplated is; what is the reason for doing an accurate sensitivity analysis?

Sensitivity analysis accuracy not only depends on the sampling type, but also the number of samples. In order to analyze how the optimization processes worked, it was selected 600 samples to look for the reference designs that were used as initial population for optimization. Now four solvers will be reset, using this time new reference designs. New Sensitivity analysis was set up with 100 samples under the same criteria it was done before for Sensitivity Analysis (600), then elite and mix designs were collected. So that, calculations for new Elite and Mix start designs, mutation 30% and 70% were simulated, to be compared with results have already been calculated by Sensitivity Analysis (600).

Table 5.11 provides objectives ranges of both Sensitivity analyses to understand from which initial point the optimization starts.

Table 5.11: Sensitivity analysis results compared by number of samples

		Sensitivity Analysis (100)	**Sensitivity Analysis (600)**
Elite	Efficiency	0.3-0.37	0.43-0.48
	Efficiency Deformed	0.79-0.85	0.67-0.86
Mix	Efficiency	0.12-0.37	0.12-0.48
	Efficiency Deformed	0.71-0.85	0.46-0.86

It can be observed how SA[4] (600) reaches “Efficiency” values around 10% higher than SA (100) while “Efficiency Deformed” is practically the same, despite the number of samples. This fact lets deduced that for small parametrical models optimization, a Sensitivity Analysis with a low number of samples ought to be enough as long as only one objective is desired to be optimized. Nevertheless, results after optimization must be analyzed to make a clearer idea. Table 5.12 shows a summary of the results obtained, indicating the number of good results that have exceed a determinate objective value according to the sensitivity analysis used as pre-optimizer, as in the percentage of improvement after optimizing, and the number in brackets indicates the highest objective value reached by the pertinent method.

Table 5.12 Optimization yield according to number of samples of Sensitivity Analysis

	Sensitivity Analysis (100)	Sensitivity Analysis (600)
Elite Mutation 30%	1>0.46 9% 13>0.86 1%	80>0.46 (0.52) 4% 38>0.86 (0.88) 2%
Mix Mutation 30%	2>0.42 5% 11>0.86 1%	42>0.42 (0.51) 3% 3>0.86 (0.86) 0%
Elite Mutation 70%	1>0.46 9% 7>0.86 1%	93>0.46 (0.54) 6% 35>0.86 (0.88) 2%
Mix Mutation 70%	2>0.42 3% 4>0.86 1%	59>0.42 (0.48) 0% 7>0.86 (0.87) 1%

It is visible how the number of results that exceed the limits established is much bigger when start designs belong to SA (600) and also “Efficiency” objective is better optimized, surpassing normally 0.5. However, “Efficiency deformed” remains practically constant, and the difference between use SA (100) or SA (600) is unsubstantial.

It is claimed that for an accurate multi-objective optimization more than 100 samples should be considered for the sensitivity analysis. But it is also certain that 600 samples might be an excessive number of samples, due to sometimes the following optimization processes are worthless.

However, if a DoE of 100 samples is selected due to the particular features of the problem to be analyzed, then Elite start design provides again a better exploration path. Results obtained by solvers that used SA (100), do not indicate a clear settings configuration. It is certain that mutation 30% reached a slightly bigger number of good results than mutation 70%, but that does not assure anything regarding to the correct configuration. In order to make a reliable statement about the settings to optimize a DoE of 100 samples, new model calculations applying crossover before mutation operator ought to be done. Additionally rates taken into account must be more than two, as it was set up during the SA (600) investigation. Expectations are that working line will keep the same setting’s pattern as SA (600), where mutation high rates are the most effective configuration to find the best objective optimizations. But anything can be claimed without full information about SA (100).

7. SUMMARY

At the beginning of this thesis was pointed out that the crash box model used during the simulations was a parametrically reduced one. However its shape and the optimization development keep an accurate resemblance and behavior.

If thesis goals are looked back, all the information that was desired to know has been analyzed. Several common patterns have been identified into the optimizers used. Nevertheless, firstly must be mentioned that without any doubt, the crash box thickness ("Blechdicke") and its length ("Verkürzung") are the crucial parameters which influence the most to increase the crash box efficiencies.

Moreover, it is a fact that Sensitivity Analysis set to 600 samples makes a precise pre-optimization of the model. Reducing range improvements when the optimization process is carried out. Although the Sensitivity Analysis number of samples depends on what sort of optimization is desired. If the goal is to search the most efficient crash box, maximizing objectives as much as possible, regardless the investigation time it could take, then 500 or 600 samples are suggested before optimization. In addition, when more than one objective is considered, optimizer yield is higher if a precise pre-optimization has been carried out before. By contrast, if what is looking for is quick information about the parameter space distribution, few solutions that make a real objective improvement and no necessities to get the best crash box. Then a faster 100 samples Sensitivity Analysis should be made. It helps to be informed about several patterns that can provide an adequate response.

Paying attention to the optimization processes, results of initial population, which was the most used investigation criteria, must be considered. Definitely, Elite start designs provide better objective values and achieves a bigger number of good results according to the filters applied during the thesis. This fact was reproduced for every mutation and crossover optimization methodology. So, in order to reach the best objectives optimization, the highest objectives values from the Sensitivity Analysis must be collected and used as reference initial population. At the end using Mix start designs is only interesting to notice that the worse designs are used as reference, the bigger the optimization of the objectives is. But that is due to the fact that the initial values are lower and how it was corroborated throughout the parameters boxplots, the space distribution is much wider than using Elite. At the end what is desired is to make shorter the parameters ranges and locate where those parameters values that optimize the objectives are. Boxplots also informed about the relevant parameters location after and before the optimization. The point is that at the end optimization does not find a different parameters distribution than the Sensitivity Analysis. So the optimal is located on the same space region. In addition “Blechdicke” distribution varies for good results between 2 and 2.3 so far. The election of the crash box thickness will depend on manufacturing conditions and machines accuracy.

On the other hand, it is concluded that the mutation type is not as significant as the start designs. Results obtained by Self-adaptive mutation type were slightly better than those calculated by Normal Distribution. That is the reason for suggesting that non-linear behavior might work better using Self-adaptive mutation type, due to the distribution is not pre-established for a specific curve. A similar conclusion is reached when hybrid operator was used on crossover. Combination of two crossover type did not achieve the effectiveness that was expected, Multipoint and Simulated binary combination did not contribute to reach better results than applying only one.

It is interesting to underline that mutation works better for high rates; indeed the most suitable designs are found when 70% and 90% rate were set. Contrary to crossover, that best results were found for low rates. This inquiry has demarcated two possible best crash box solutions according to the criteria is followed, in section 5.4.4 those designs were mentioned. “Efficiency” was optimized exceeding 0.54 and “Efficiency deformed” reached values around 0.88. These designs were expected to be defeated by mutation and crossover offspring. Surprisingly designs never reached so high objectives values, but what is absolutely certain is that crossover optimization was improved when mutation was applied on their offspring, especially for low crossover rates.

Table 7.1 summarizes the best five designs according to the objectives. Two designs obtained by mutation (mut.), two more by crossover (cross.) and the last one by applying mutation 1% on crossover offspring (cross+mut.). The interesting point of this comparison is to realize if best designs follow a similar parameter pattern. Such parameter comparison is shown in Table 7.2

Table 7.1: Best global designs

Design	Efficiency	Efficiency Deformed	Difference
Elite 70% - 205(mut.)	0.5441	0.8817	0.3375
Elite 70% - 183(mut.)	0.5442	0.8590	0.3151
Elite 90% - 201(mut.)	0.5039	0.8844	0.3805
Mix 75 – 203 (cross.)	0.4931	0.8583	0.3651
Elite 50% - 191 (cross+mut.)	0.5241	0.8233	0.2992

Table 7.2: Comparison between best designs parameters

Design	P_B	dicke	Verkürzung	Fase	P_H	Zwischen blech	Zwischen Blech Vertical
Elite 70% - 205(mut.)	28	2.04	90	0.27	28	0.18	0.99
Elite 70% - 183(mut.)	28	2	90	0.26	28	0.22	0.99
Elite 90% - 201(mut.)	28	2.2	90	0.24	26	0.2	0.99
Mix 75 – 203 (cross.)	20	2.1	71	0.49	28	0.56	0.31
Elite 50% - 191 (cross+mut.)	20	2.1	79	0.45	25	0.38	0.39

As it was mentioned in section 5.4.4, those designs obtained when only mutation operator is applied, tend to a regular hexagonal shape. Their parameters are exactly the same as can be checked in Table 7.2, except for “Blechdicke”that varies between 2 and 2.2, “Fase” between 0.24 and 0.27, “P_H” between 26 and 28 and “Zwischenblech” between 0.18 and 0.22.

The other two designs, where crossover operators have taken importance, the shape are similar between them, but different from the others explained before. This time the hexagonal shape is not regular due to “Zwischenblech” values are higher and “Zwischenblech vertical” values are two times smaller.

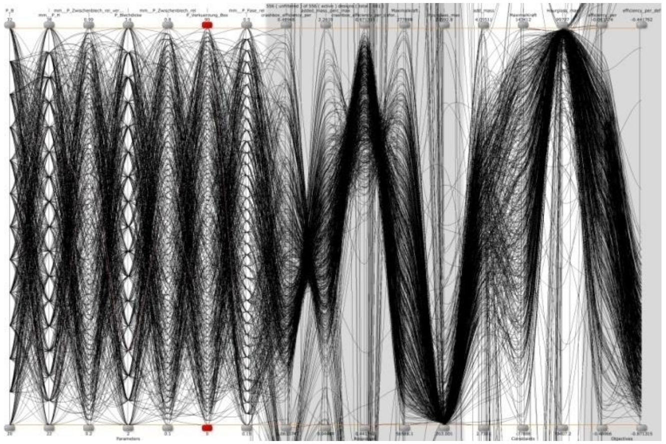
In order to conclude this thesis the following recommendation are suggested.

- Wider mutation rates on crossover offspring should be analyzed to find where the optimal space-design is.
- The model has been analyzed for a specific sort of impact. It would be interesting to check the results when the impact angle or the vehicle velocity is changed. Otherwise it can happen that the crash box was not useful in real life.
- As the highest objective values are looked for, Elite start design work definitely more effective than Mix. This fact ought to be verified on the full parametric crash box.

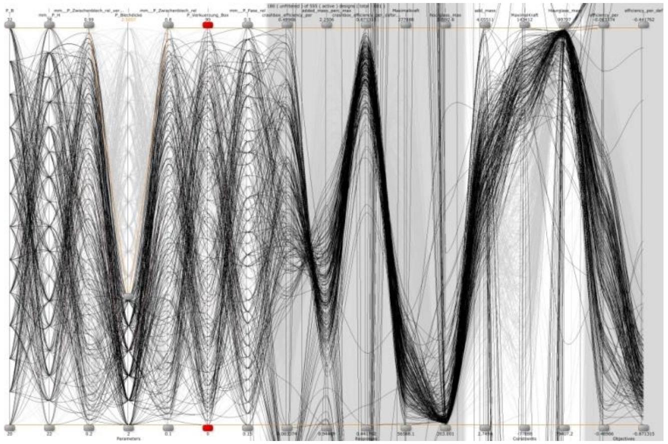
- It must be checked that model can be manufactured according to the parameters given. Manufacturing costs, feasibility of its production regarding to tolerances and welding capacity.
- If it is desired to analyze a model based on a large number of parameters, an accurate Sensitivity Analysis should be carried out, because not only the number of relevant parameters would be reduced, but also the space-design. Making an exploration of a vast space-design would be a tedious and slow process that is worthless in terms of time.
- There are more mutation and crossover types to be set. Check it on OptiSLang manual.
- The optimization process of the model has been aboard just from the Evolutionary Algorithms point of view. Downhill simplex Method, explained in section 5.2.2 could be an interesting alternative.
- Deeper investigation of regular hexagonal shape, according to the parameter values on this thesis expounded.
- Deeper investigation of irregular hexagonal shape, according to the parameter values on this thesis expounded.

APPENDIX

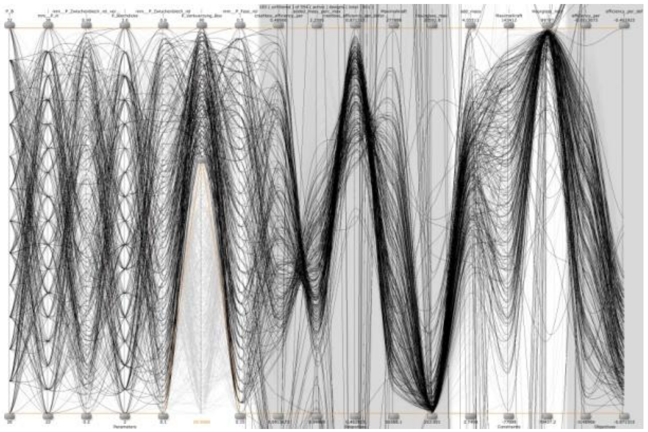
Global



Blechdicke



Verkurzung



Fase

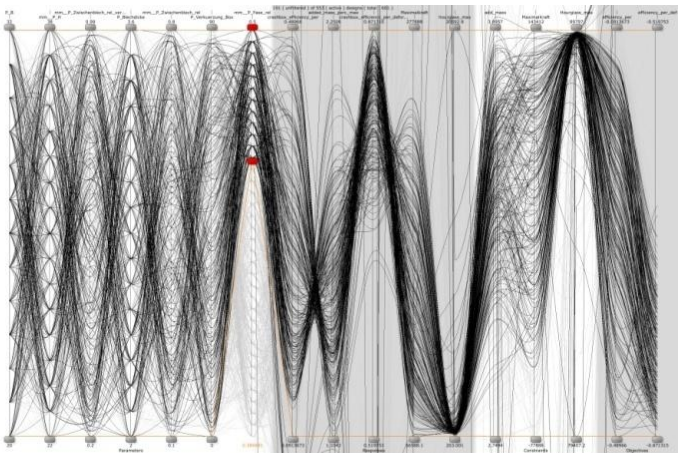


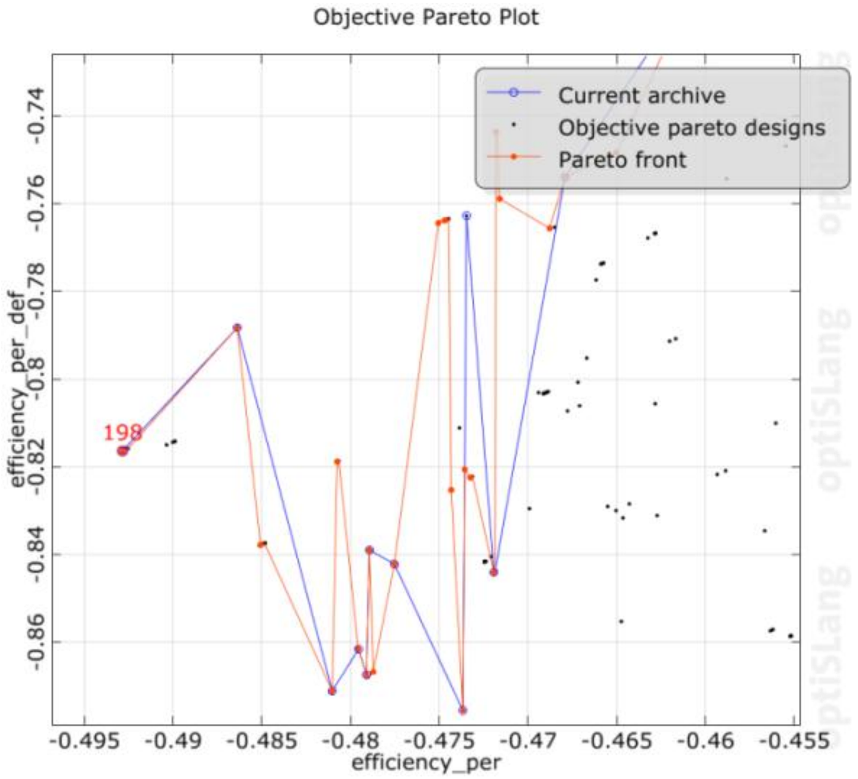
Figure A.!: Relevant parameters ang global Parallel plots

Table A.2: Suitable mutation designs according to "Efficiency", "Efficiency deformed" and Pareto criteria

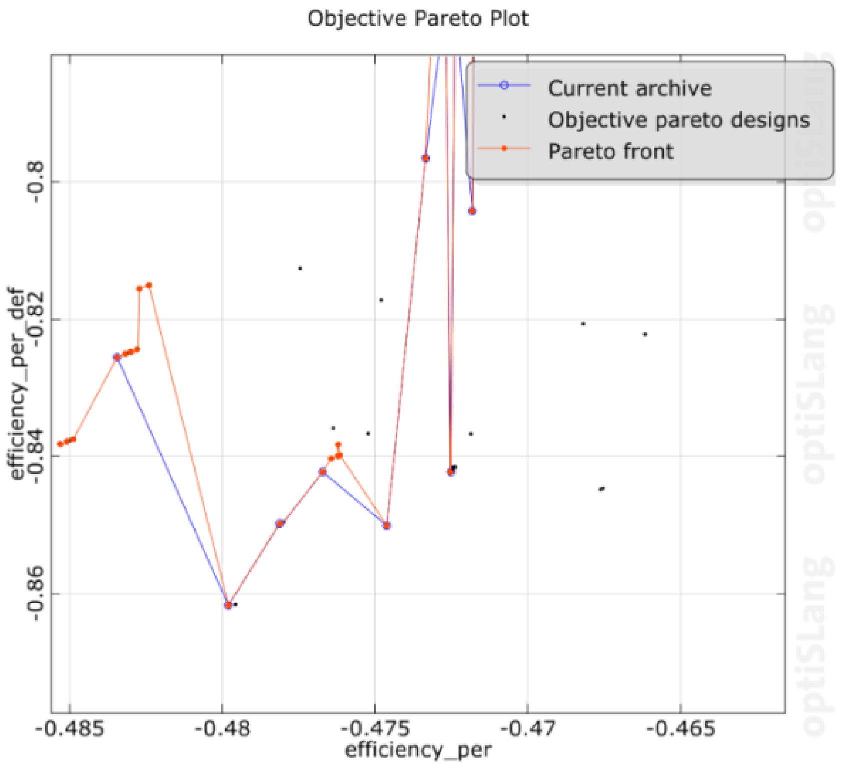
Mutatio type	Criteria	Design	P_B	Blechdicke	Verkuerzung	Fase_ rel	P_H	Zwischenblech rel	Zwischenblech rel_vertikal	Maximalkraft	added mass	crashbox efficiency	crashbox efficiency deformed
ELITE_10%	**Best efficiency**	**198**	**21**	**2.05**	**85**	**0.39**	**34**	**0.64**	**0.54**	**91185.4**	**1.1604**	**0.492841**	**0.81634**
	Best efficiency deformed	**171**	**21**	**2.1**	**57**	**0.45**	**27**	**0.23**	**0.41**	**62967.1**	**1.2015**	**0.473567**	**0.875491**
	Best pareto	**198**	**21**	**2.05**	**85**	**0.39**	**34**	**0.64**	**0.54**	**91185.4**	**1.1604**	**0.492841**	**0.81634**
MIX_SA_10%	**Best efficiency**	**71**	**26**	**2.1**	**78**	**0.49**	**33**	**0.34**	**0.97**	**82075**	**1.2065**	**0.485303**	**0.838226**
	Best efficiency deformed	**7**	**24**	**2.1**	**71**	**0.49**	**32**	**0.56**	**0.32**	**77550.6**	**1.161**	**0.479796**	**0.861684**
	Best pareto	**71**	**26**	**2.1**	**78**	**0.49**	**33**	**0.34**	**0.97**	**82075**	**1.2065**	**0.485303**	**0.838226**
MIX_ND_10%	**Best efficiency**	**130**	**26**	**2.06**	**78**	**0.49**	**33**	**0.34**	**0.97**	**79756**	**1.1985**	**0.501872**	**0.84371**
	Best efficiency deformed	**190**	**24**	**2.1**	**71**	**0.48**	**32**	**0.55**	**0.32**	**74454.8**	**1.249**	**0.475295**	**0.869451**
	Best pareto	**130**	**26**	**2.06**	**78**	**0.49**	**33**	**0.34**	**0.97**	**79756**	**1.1985**	**0.501872**	**0.84371**
ELITE-30%	**Best efficiency**	182	20	2	87	0,4	34	0,63	0,57	85793,5	11.783	0,5257112	0,8018489
	Best efficiency deformed	160	29	2,1	76	0,21	24	0,15	0,95	68351,7	12.319	0,452604	0,8829759
	Best pareto	182	20	2	87	0,4	34	0,63	0,57	85793,5	11.783	0,5257112	0,8018489
MIX_SA-30%	**Best efficiency**	189	26	2	80	0,5	33	0,34	0,96	82168,5	11.416	0,5143492	0,8115014
	Best efficiency deformed	203	24	2,2	71	0,49	32	0,52	0,34	82203,5	12.813	0,4365897	0,8653688
	Best pareto	189	26	2	80	0,5	33	0,34	0,96	82168,5	11.416	0,5143492	0,8115014
MIX_ND-30%	**Best efficiency**	10	26	2,1	78	0,49	33	0,34	0,97	82155,9	12.065	0,48487	0,837466

	Best efficiency deformed	7	24	2,1	71	0,49	32	0,56	0,32	77550,5	11.607	0,4795416	0,8615313
	Best pareto	6	23	2	51	0,44	23	0,14	0,22	61160,4	11.374	0,4826875	0,8155114
ELITE-50%	**Best efficiency**	136	28	2	68	0,44	25	0,76	0,73	64092,9	11.711	0,5065654	0,8330983
	Best efficiency deformed	87	21	2	49	0,45	28	0,18	0,85	58609,2	12.007	0,4807383	0,8800904
	Best pareto	136	28	2	68	0,44	25	0,76	0,73	64092,9	11.711	0,5065654	0,8330983
MIX_SA-50%	**Best efficiency**	192	24	2	75	0,48	32	0,55	0,42	76718,4	12.171	0,5026647	0,8296251
	Best efficiency deformed	197	25	2,2	72	0,47	23	0,58	0,95	74850,4	12.258	0,493125	0,8762637
	Best pareto	192	24	2	75	0,48	32	0,55	0,42	76718,4	12.171	0,5026647	0,8296251
MIX_ND-50%	**Best efficiency**	10	26	2,1	78	0,49	33	0,34	0,97	82132,3	12.065	0,484894	0,8374811
	Best efficiency deformed	6	24	2,1	71	0,49	32	0,56	0,32	77550,6	11.607	0,4795565	0,8615449
	Best pareto	10	26	2,1	78	0,49	33	0,34	0,97	82132,3	12.065	0,484894	0,8374811
ELITE-70%	**Best efficiency**	205	28	2,04	90	0,27	28	0,18	0,99	76128,7	11.747	0,5441415	0,8817399
	Best efficiency deformed	169	29	2,09	78	0,21	28	0,12	0,99	74557	12.592	0,4504534	0,8842508
	Best pareto	183	28	2	90	0,26	28	0,22	0,99	77212,9	11.778	0,5439215	0,859054
MIX_SA-70%	**Best efficiency**	181	23	2	48	0,44	23	0,14	0,22	58020,7	11.617	0,4881759	0,8375465
	Best efficiency deformed	145	20	2,5	84	0,15	38	0,33	0,2	102704	13.346	0,3529604	0,8771298
	Best pareto	181	23	2	48	0,44	23	0,14	0,22	58020,7	11.617	0,4881759	0,8375465
MIX_ND-70%	**Best efficiency**	130	22	2	46	0,42	23	0,14	0,27	55909,2	11.948	0,4952739	0,8638109

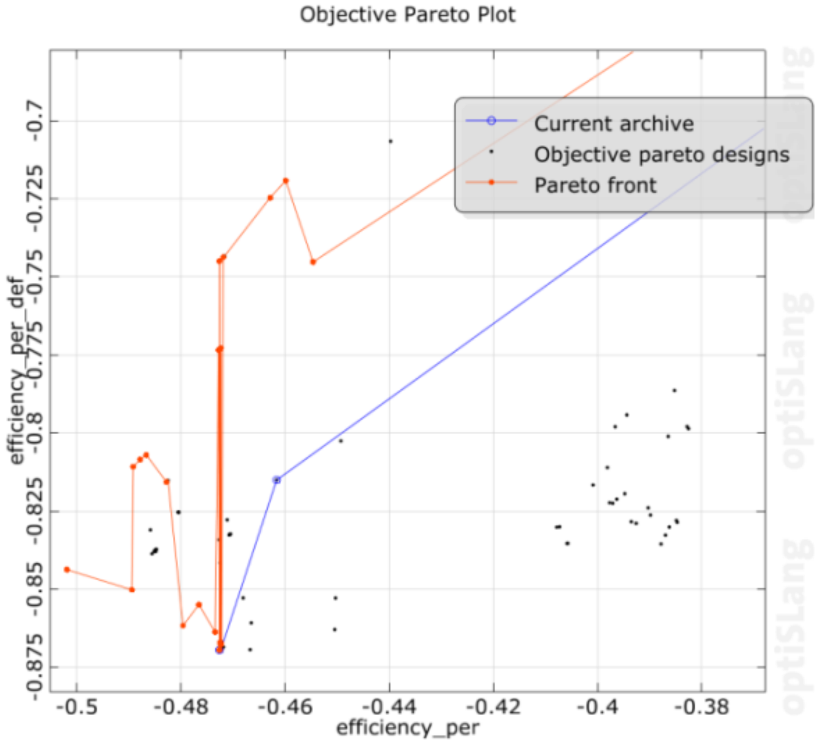
	Best efficiency deformed	149	21	2	41	0,42	22	0,18	0,24	56880,7	11.852	0,4859851	0,875327
	Best pareto	193	21	2	75	0,44	30	0,22	0,42	84556,1	11.873	0,4911423	0,7763303
ELITE-90%	**Best efficiency**	123	26	2,13	90	0,24	25	0,16	0,99	82160,3	11.639	0,5207974	0,87 III
	Best efficiency deformed	185	30	2,19	82	0,24	26	0,2	0,97	78662,3	12.489	0,4428215	0,8895536
	Best pareto	123	26	2,13	90	0,24	25	0,16	0,99	82160,3	11.639	0,5207974	0,8731634
MIX_SA-90%	**Best efficiency**	207	23	2	72	0,49	32	0,56	0,39	67751,8	11.622	0,5058799	0,8560476
	Best efficiency deformed	87	25	2,31	69	0,5	31	0,61	0,52	81660	12.511	0,4140674	0,8622027
	Best pareto	207	23	2	72	0,49	32	0,56	0,39	67751,8	11.622	0,5058799	0,8560476
MIX_ND-90%	**Best efficiency**	207	21	2,22	87	0,5	27	0,75	0,36	97122,3	11.673	0,4966007	0,7934425
	Best efficiency deformed	144	21	2,18	59	0,47	25	0,3	0,37	67910,4	12.195	0,4726265	0,8727286
	Best pareto	207	21	2,22	87	0,5	27	0,75	0,36	97122,3	11.673	0,4966007	0,7934425



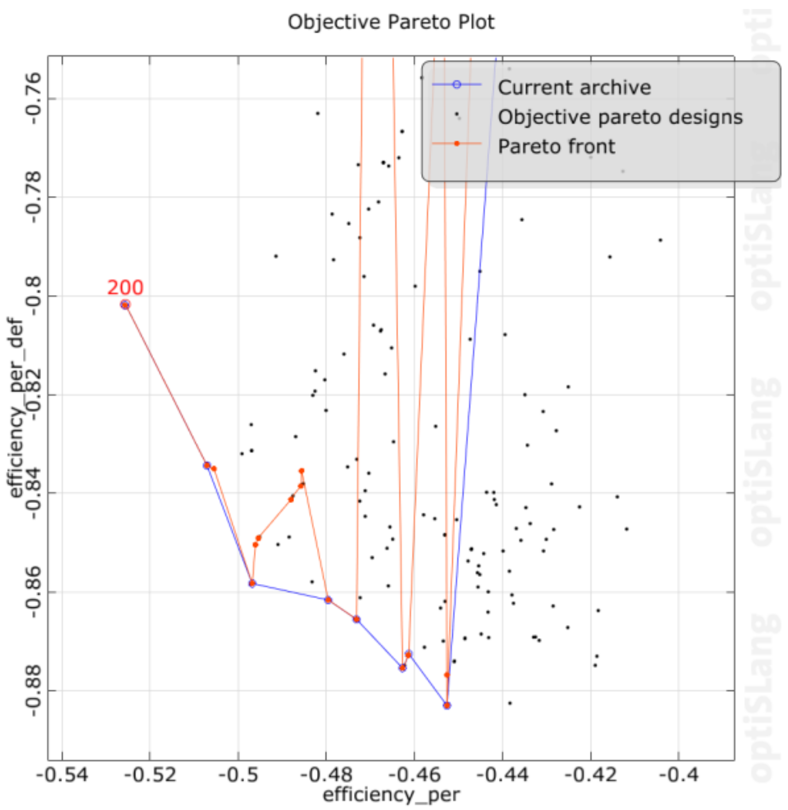
Elite 10%



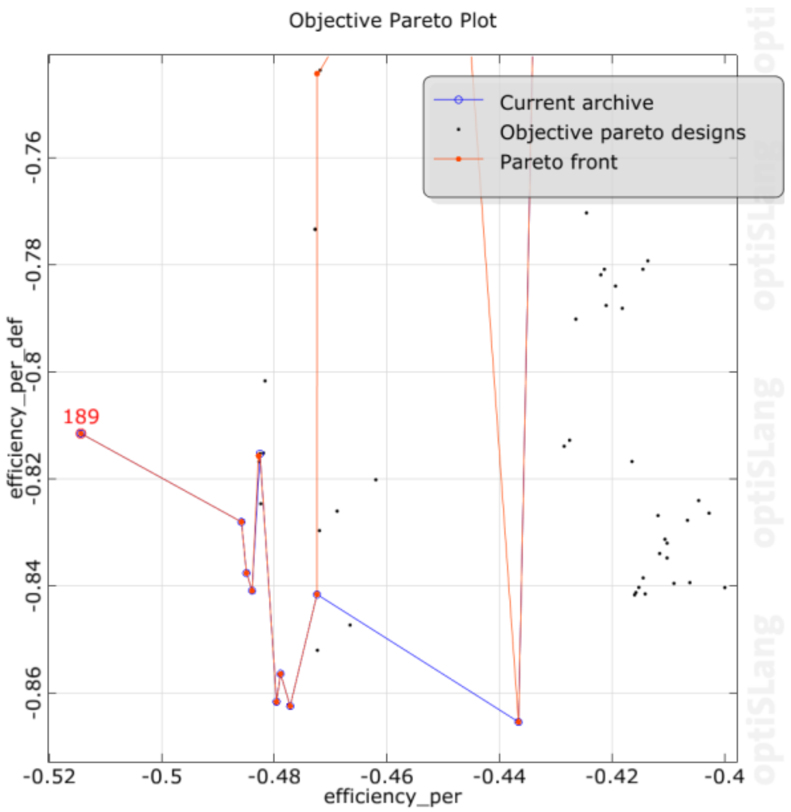
Mix-SA 10%



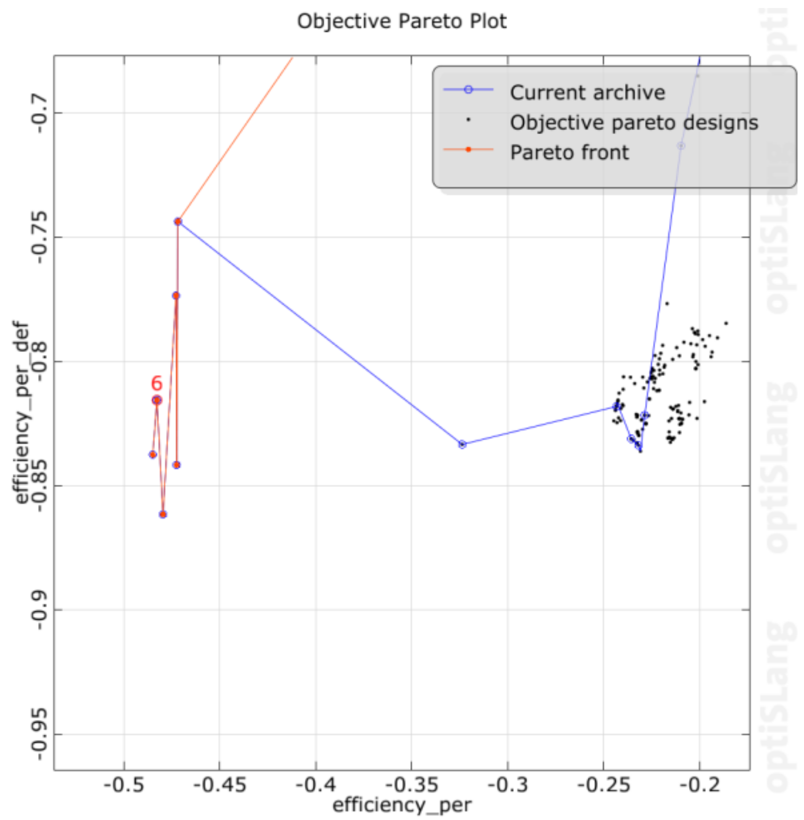
Mix-ND 10%



Elite 30%



Mix-SA 30%



Mix-ND 30%

Figure A.3: Mutation 2-D Pareto fronts

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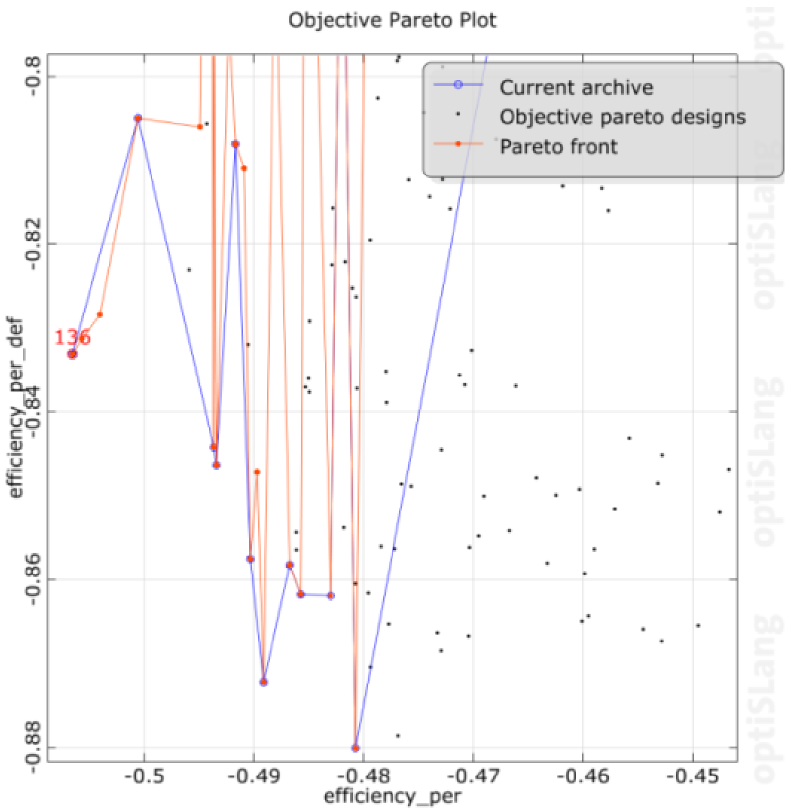
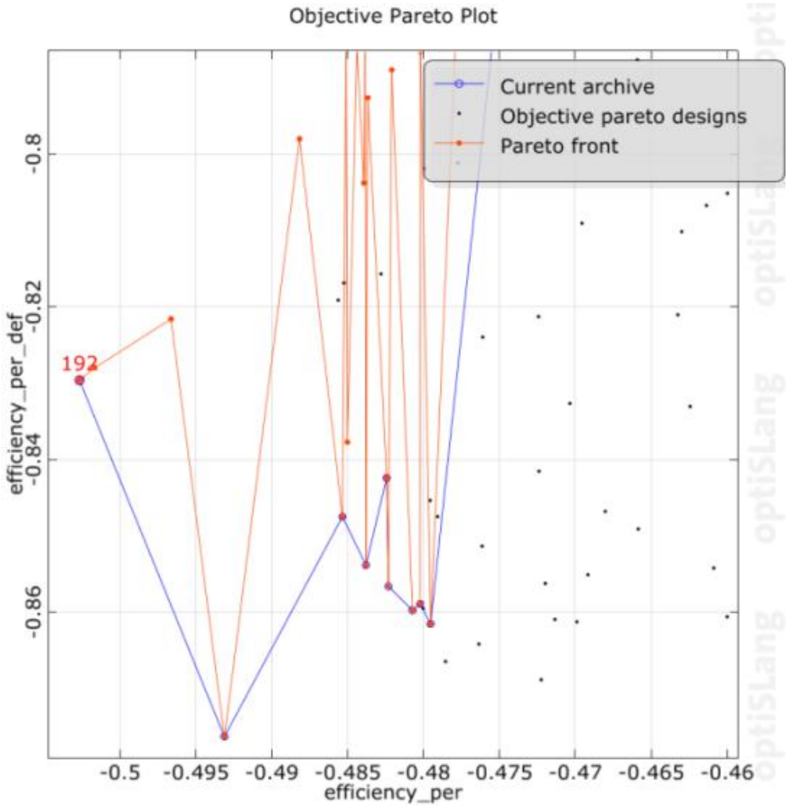
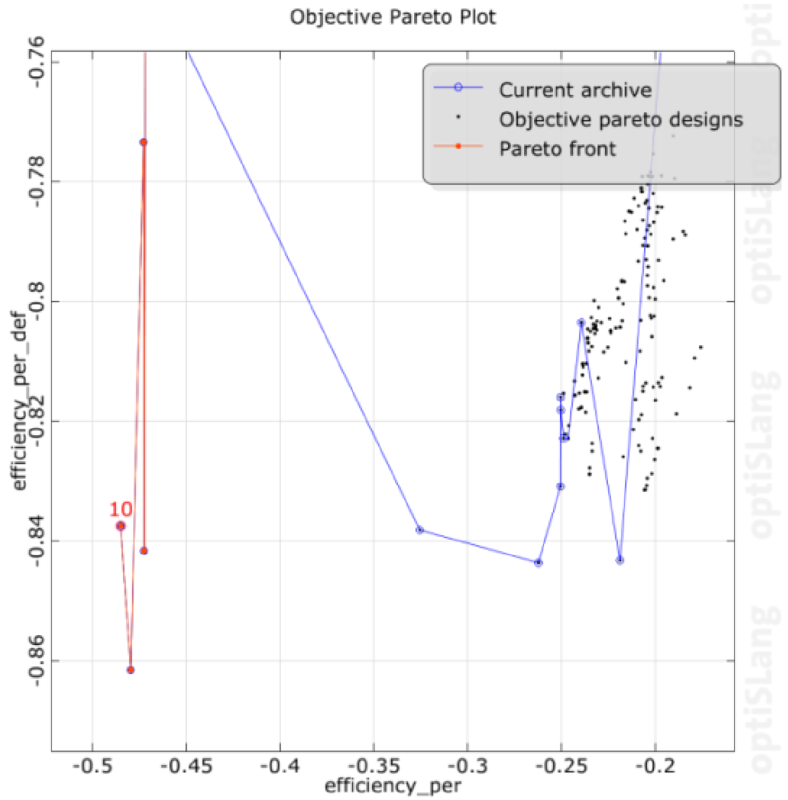


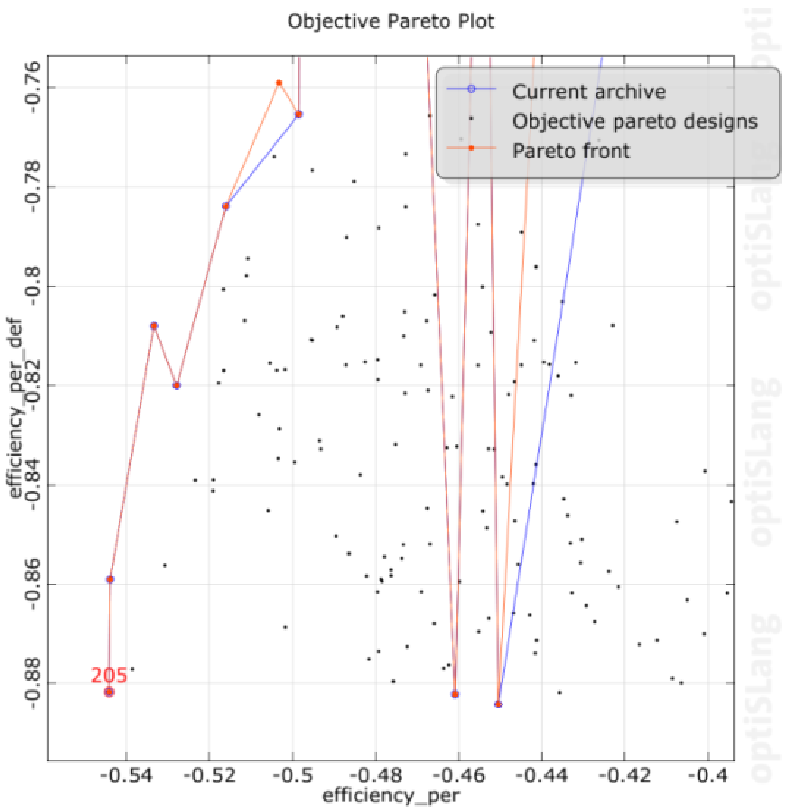
Elite 50%



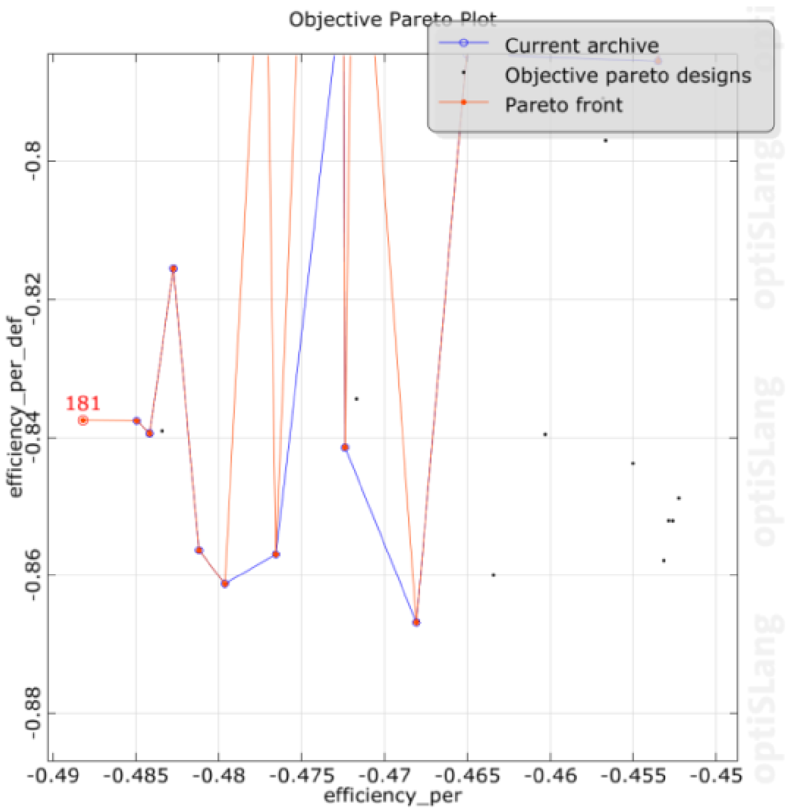
Mix-SA 50%



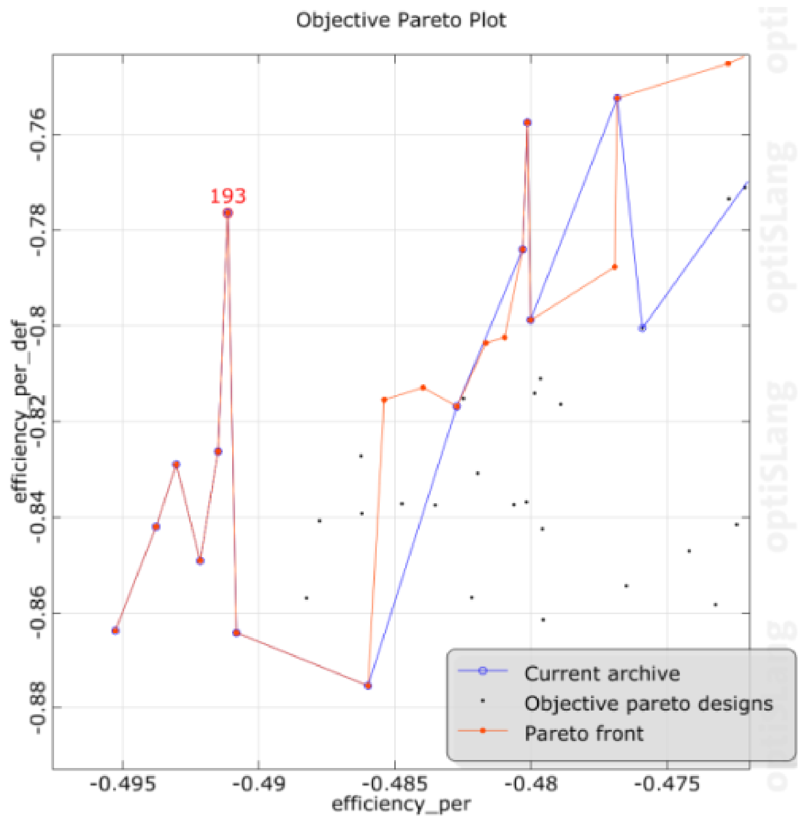
Mix-ND 50%



Elite 70%

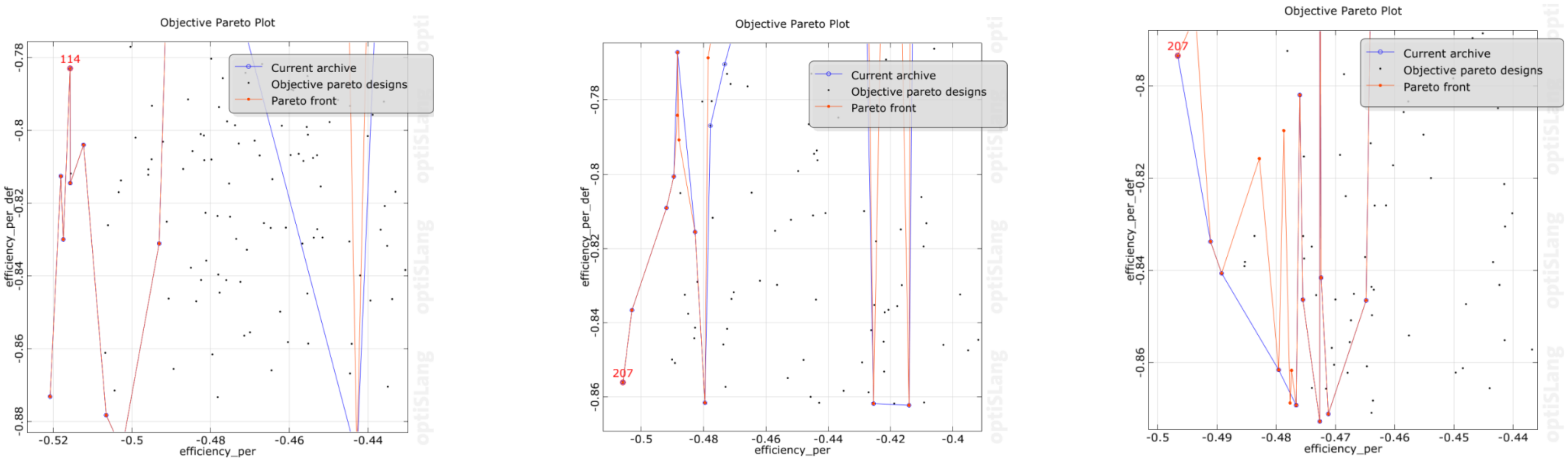


Mix-SA 70%



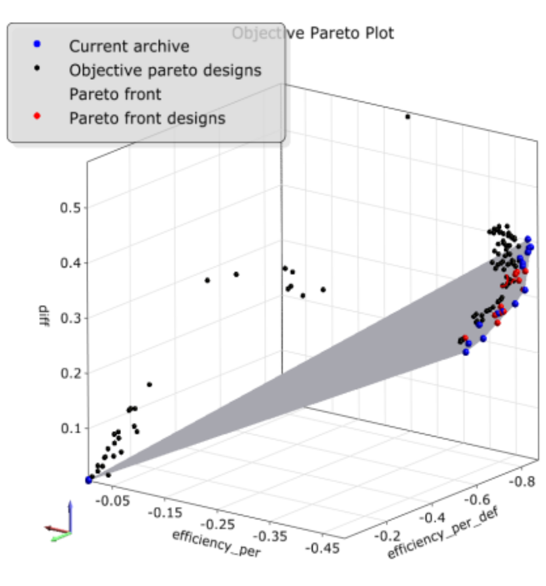
Mix-ND 70%

Figure A.3: Mutation 2-D Pareto fronts

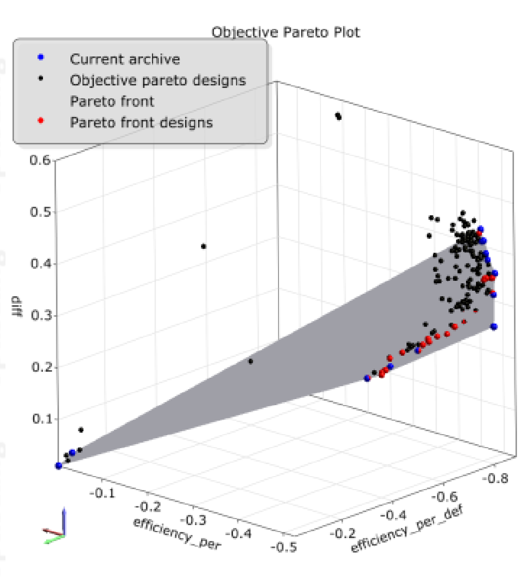


Elite 90% Mix-SA 90% Mix-ND 90%

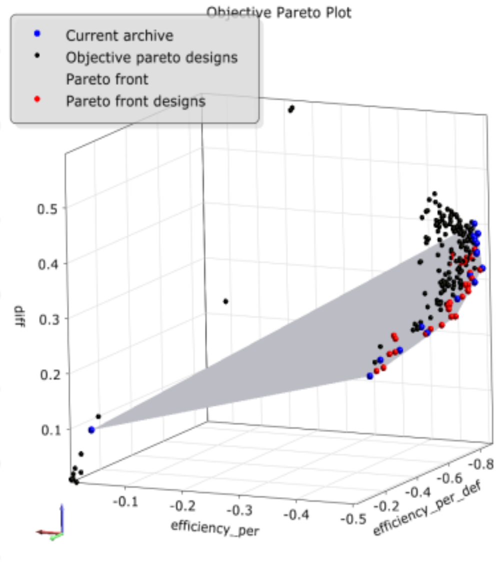
Figure A.3: Mutation 2-D Pareto fronts



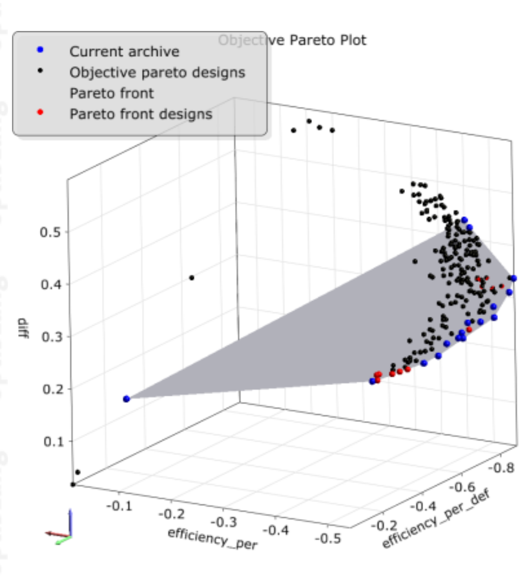
Elite 10%



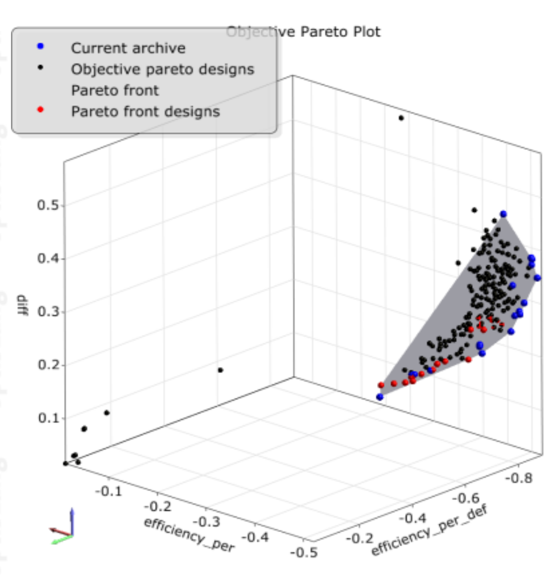
Elite 30%



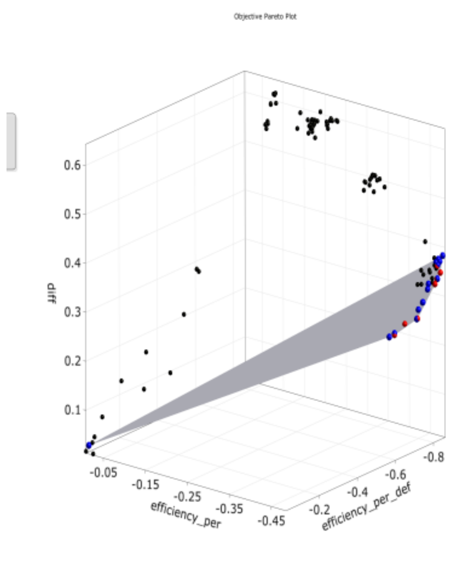
Elite 50%



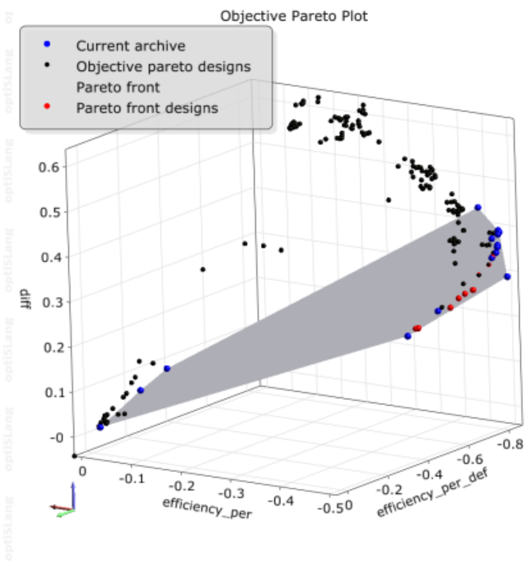
Elite 70%



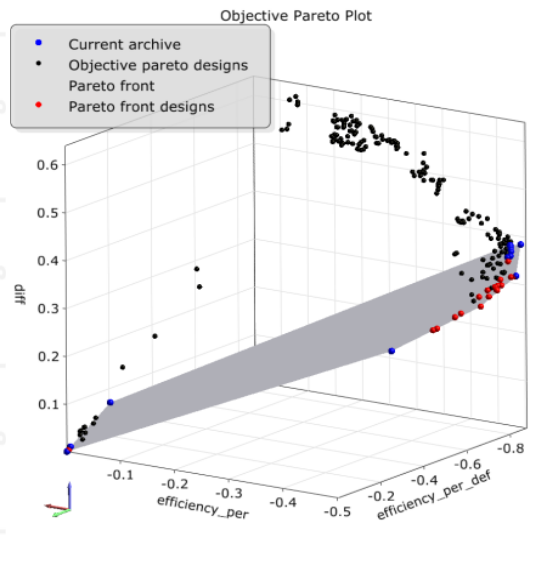
Elite 90%



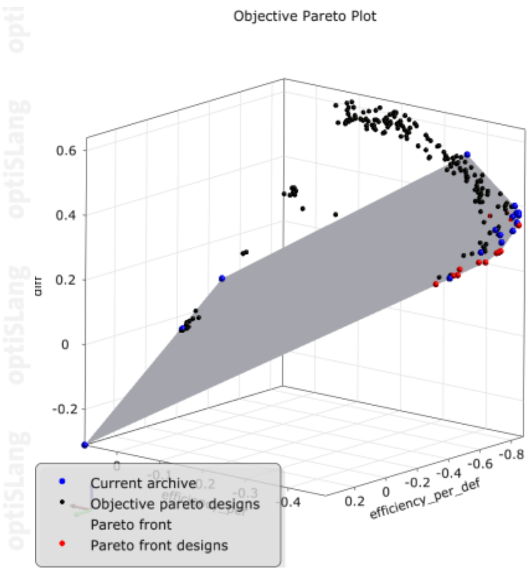
Mix-SA 10%



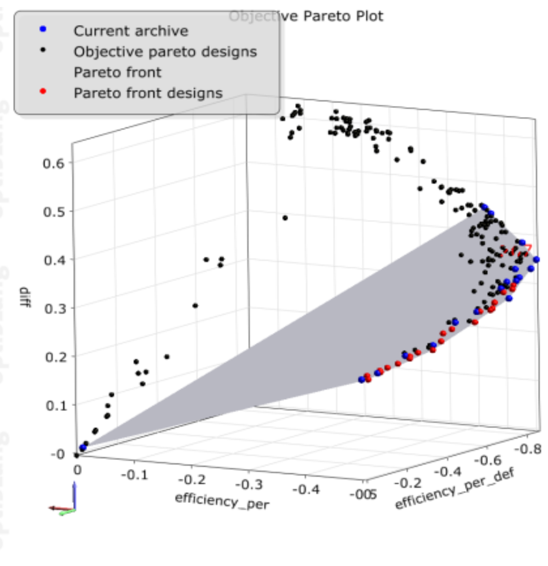
Mix-SA 30%



Mix-SA 50%

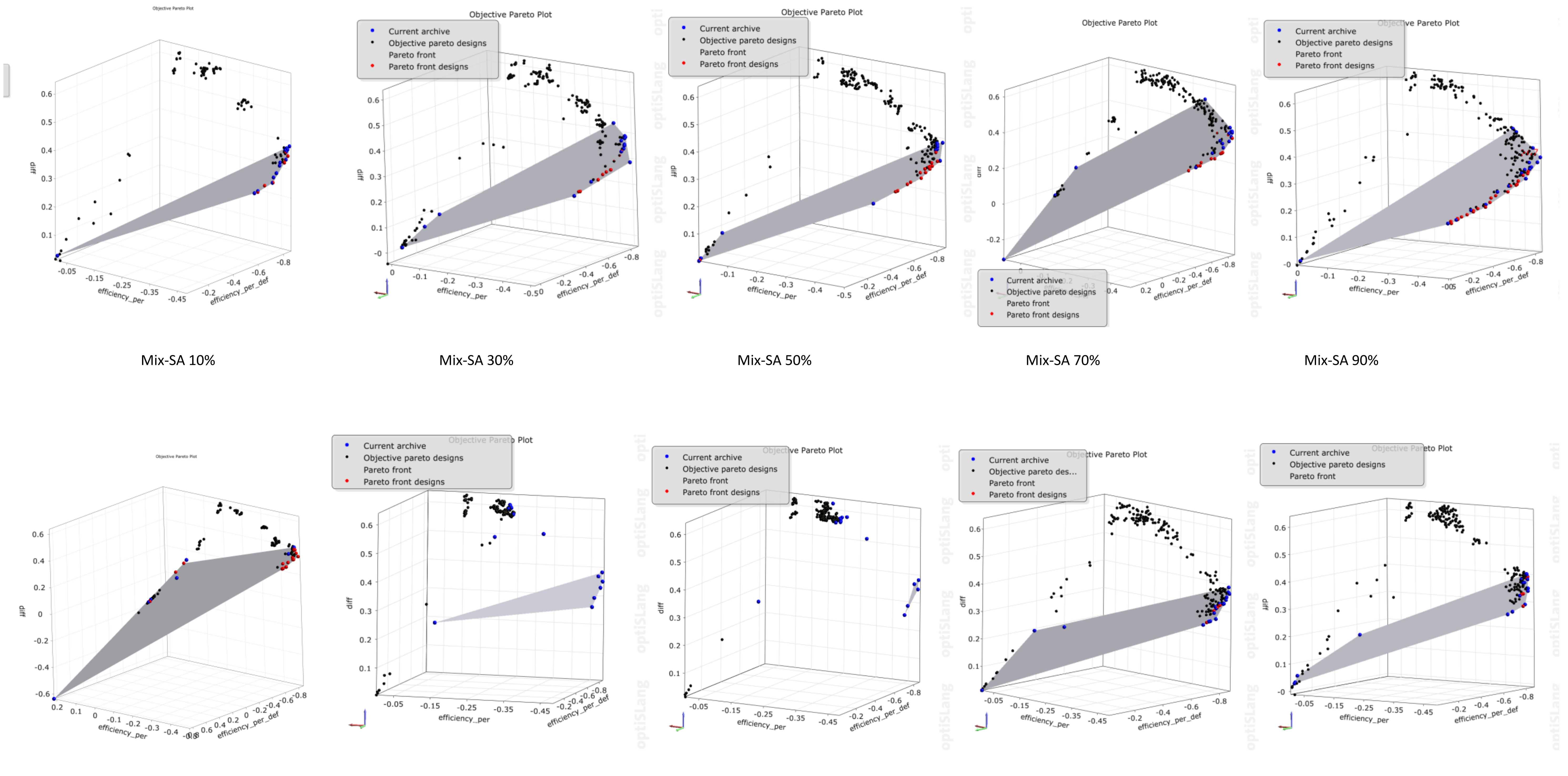


Mix-SA 70%



Mix-SA 90%

Figure A.4: 3-D Pareto fronts according mutation rate and initial population



Mix-SA 10%

Mix-SA 30%

Mix-SA 50%

Mix-SA 70%

Mix-SA 90%

Mix ND 10%

Mix ND 30%

Mix ND 50%

Mix ND 70%

Mix ND 90%

Figure A.5: 3-D Pareto fronts according to mutation rate and type

Table A.6: Suitable crossover designs according to “Efficiency”, “Efficiency deformed” and Pareto criteria

Crossover type	Criteria	Design	P_B	Blechdicke		Fase real	P_H	Zwischenblech real	Zwischenblech rel_vertikal	Maximalkraft	added mass	crashbox efficiency deformed	**crashbox efficiency deformed6**
ELITE_5%	**Best efficiency**	198	29	2	72	0,44	32	0,14	0,94	77216,6	12.376	0,4872203	0,8300297
	Best efficiency deformed	196	29	01,02,2018	76	0,25	25	0,15	0,95	73481,5	12.314	0,4189494	0,8704311
	Best pareto	198	29	2	72	0,44	32	0,14	0,94	77216,6	12.376	0,4872203	0,8300297
MIX_5%	**Best efficiency**	184	26	02,01,2015	79	0,49	33	0,34	0,97	81799,2	12.235	0,4954218	0,8481918
	Best efficiency deformed	178	24	02,02,2015	71	0,49	32	0,56	0,32	82243,7	1.182	0,44729	0,863067
	Best pareto	184	26	02,01,2015	79	0,49	33	0,34	0,97	81799,2	12.235	0,4954218	0,8481918
ELITE-25%	**Best efficiency**	204	26	01,02,2019	75	0,48	25	0,51	0,94	78662,8	12.073	0,490831	0,8523859
	Best efficiency deformed	136	26	02,01,2015	76	0,25	25	0,6	0,95	72818,5	12.631	0,449778	0,8823502
	Best pareto	204	26	01,02,2019	75	0,48	25	0,51	0,94	78662,8	12.073	0,490831	0,8523859
MIX_25%	**Best efficiency**	207	26	02,01,2015	78	0,49	33	0,33	0,97	81757,8	12.091	0,4873603	0,8406321
	Best efficiency deformed	6	24	02,01,2015	71	0,49	32	0,56	0,32	77550,6	11.607	0,4795745	0,8615904
	Best pareto	207	26	02,01,2015	78	0,49	33	0,33	0,97	81757,8	12.091	0,4873603	0,8406321
ELITE-50%	**Best efficiency**	40	29	2	72	0,49	32	0,14	0,24	76809	11.763	0,4893825	0,7905026
	Best efficiency deformed	196	28	02,01,2015	76	0,25	25	0,15	0,95	69476,9	1.198	0,4446425	0,8668853
	Best pareto	40	29	2	72	0,49	32	0,14	0,24	76809	11.763	0,4893825	0,7905026
MIX_50%	**Best efficiency**	11	26	02,01,2015	78	0,49	33	0,34	0,97	82143,1	12.066	0,4849858	0,8376398
	Best efficiency deformed	190	21	02,01,2015	65	0,42	28	0,71	0,36	69002,2	12.174	0,4721639	0,8712949
	Best pareto	11	26	02,01,2015	78	0,49	33	0,34	0,97	82143,1	12.066	0,4849858	0,8376398
ELITE-75%	**Best efficiency**	132	26	02,01,2015	82	0,49	32	0,27	0,99	88642,6	11.795	0,4958405	0,8087756
	Best efficiency deformed	159	21	02,01,2015	57	0,46	27	0,41	0,82	64371,6	1.204	0,4668644	0,8741654
	Best pareto	123	21	02,09,2015	82	0,45	27	0,37	0,23	90817,5	11.727	0,4914553	0,7856406
MIX_75%	**Best efficiency**	203	20	02,01,2015	71	0,49	28	0,56	0,31	70082,7	12.114	0,4931969	0,8583844
	Best efficiency deformed	46	31	02,03,2015	20	0,4	33	0,79	0,58	78481	1.514	0,2907704	0,8676784

	Best pareto	180	21	02,02,2015	87	0,5	27	0,74	0,34	99289,6	11.653	0,4893192	0,771821
ELITE-95%	**Best efficiency**	111	21	02,05,2015	79	0,49	27	0,38	0,33	87904,8	11.781	0,5150454	0,802779
	Best efficiency deformed	151	26	02,09,2015	64	0,4	25	0,36	0,95	66460,1	11.804	0,4656703	0,8769951
	Best pareto	111	21	02,05,2015	79	0,49	27	0,38	0,33	87904,8	11.781	0,5150454	0,802779
MIX_95%	**Best efficiency**	11	26	02,01,2015	78	0,49	33	0,34	0,97	82143,3	12.065	0,4848941	0,8375076
	Best efficiency deformed	125	22	02,08,2015	55	0,44	27	0,17	0,34	59774	11.701	0,465522	0,8645093
	Best pareto	11	26	02,01,2015	78	0,49	33	0,34	0,97	82143,3	12.065	0,4848941	0,8375076

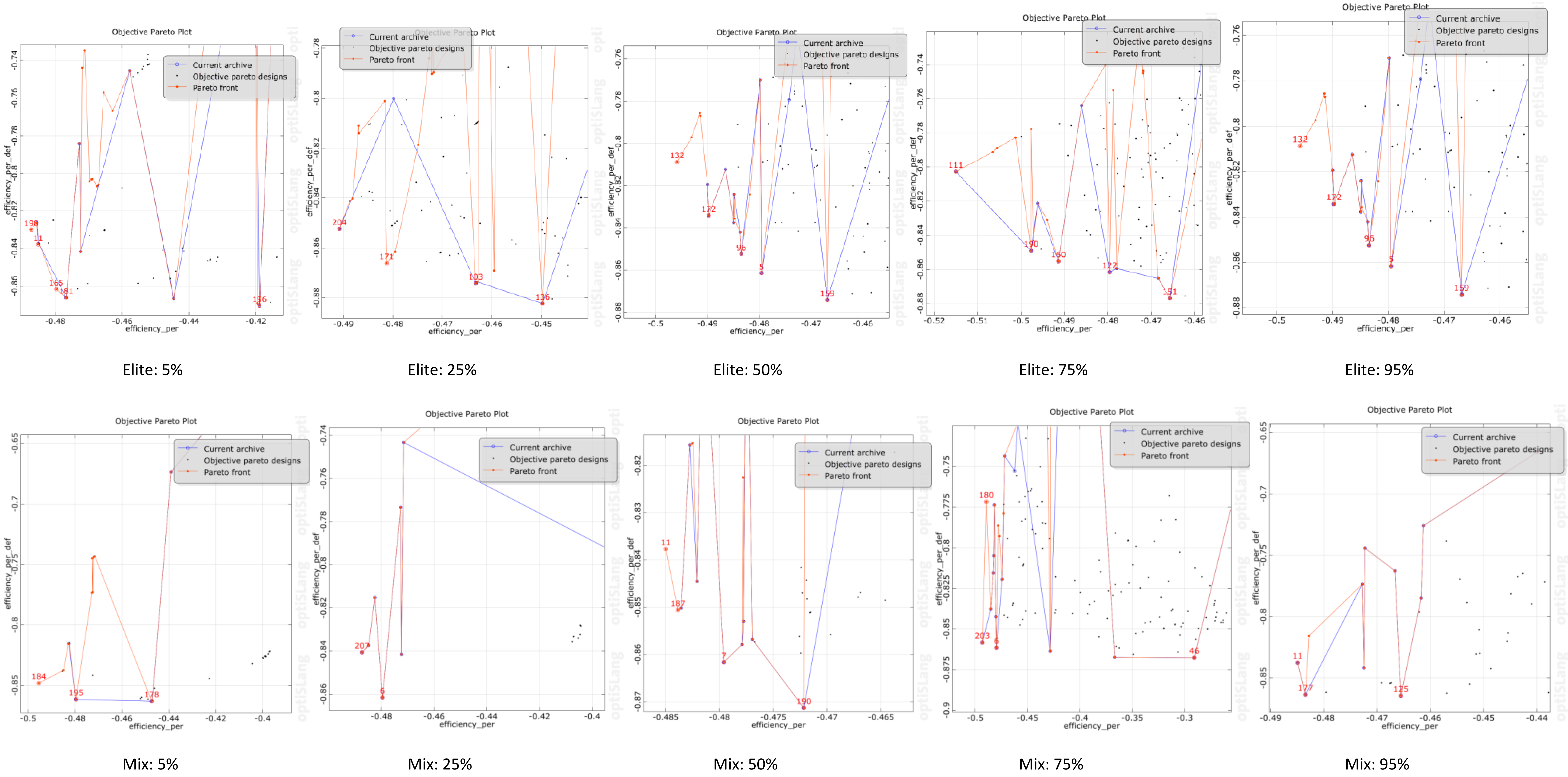
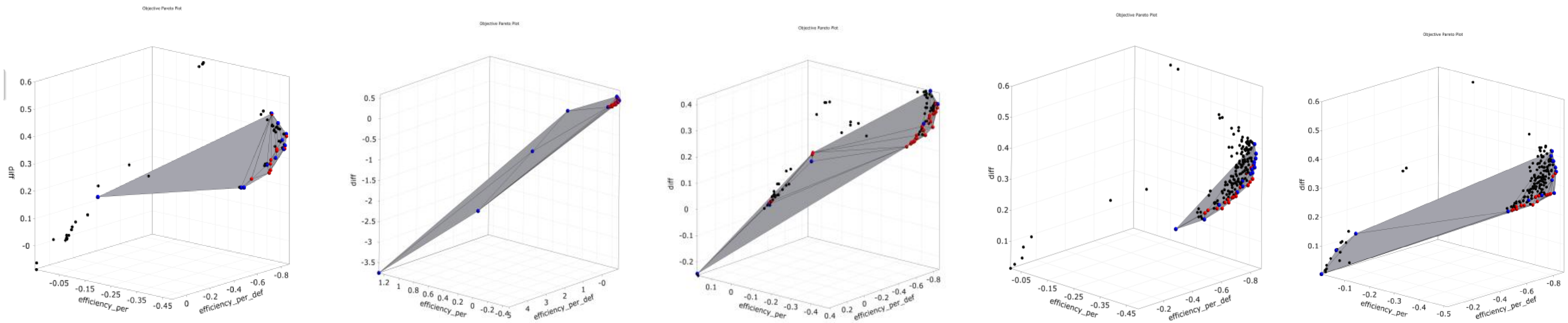
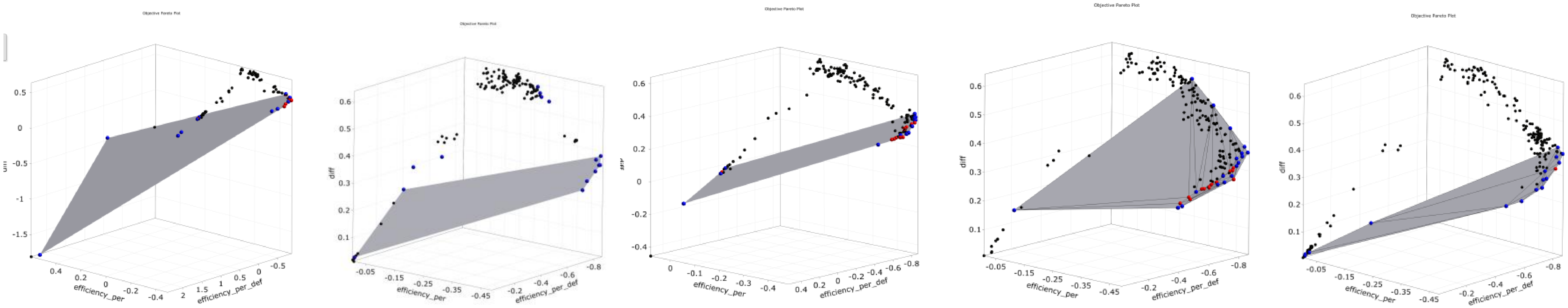


Figure A.7: Pareto according to crossover rate and initial population



Elite 5% Elite 25% Elite 50% Elite 75% Elite 95%



Mix ND 5% Mix ND 25% Mix ND 50% Mix ND 75% Mix ND 95%

Figure A.8 Crossover 3-D Pareto fronts

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