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STATION-KEEPING FOR LATTICE-PRESERVING FLOWER CONSTELLATIONS

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2D-Lattice Flower Constellations present interesting dynamical features that allow us to explore a wide range of potential applications. Their particular initial distribution (lattice) and their symmetries disappear when some perturbations are considered, such as the J_2 effect. The new lattice-preserving Flower Constellations maintain over long periods of time the initial distribution and its symmetries under the J_2 perturbation, which is known as relative station-keeping. This paper deals with the study of the required velocity change that must be applied to the satellites of the constellation to have an absolute station-keeping.

INTRODUCTION

Flower Constellations present beautiful and interesting dynamical features that allow us to explore a wide range of potential applications, such as telecommunications, Earth and deep space observation, global positioning systems, distributed space systems, etc. The original theory of Flower Constellations[1,2,3] was developed in 2004 by Prof. D. Mortari in response to the need of including the eccentricity as another design parameter. Note that previous satellite constellations, such as Walker Constellations[4] around 1970s, consider all the satellites in circular orbits.

The original theory was substantially improved by the 2D-Lattice Flower Constellations[5] (2D-LFCs), making the theory independent of any reference frame (inertial or rotating), and making the theory works with minimal parametrization. In particular, these constellations can be described by three integer parameters and six continuous ones. The first three parameters are N_o, the number of orbital planes, N_{so}, the number of satellites per orbit, and $N_c \in [0, N_o - 1]$ a phasing parameter. These three integers completely determine the location of all the satellites in a lattice in the $(\Omega, M)-$space.[6] The other six parameters are the longitude of the ascending node and the mean anomaly of the reference satellite. The semi-major axis, the eccentricity, the inclination and the argument of perigee, which are the same for all the satellites in the constellation.

In the Keplerian model, 2D-LFCs remain 2D-LFCs in time. That is, the initial lattice of the constellation and its symmetries are maintained. However, when some perturbation is considered, such as the J_2 effect due to the non-spherical shape of the Earth, the initial structure is destroyed in a few days. The new theory of Lattice-preserving Flower Constellation[7] allows us to control the lattice and preserve its initial configuration providing a relative station-keeping, which consists of maintaining only the relative positions of the satellites of the constellation with respect to each other and not the absolute positions.[8] Thanks to the lattice-preserving Flower Constellations it is possible to maintain during several months the initial properties (observation, global coverage, etc.) of the

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constellation. The relative station-keeping is achieved following two procedures: the first consists of the modification of the semi-major axis slightly to control their orbital period, and second consists of the modification of the values of the eccentricity and inclination.

Nevertheless, even in the case where the initial lattice is preserved, some maneuvers must be applied if it is required an absolute station-keeping, which consists of maintaining each satellite of the constellation in a predefined mathematical box relative to the Earth or inertial space.[8]

This paper deals with the required change in velocity that must be applied to the satellites of the constellation to have an absolute station-keeping. As a first approach, we investigate the Δv required to keep the satellites according to the initial lattice when the main perturbation is the J_2 effect. As we will see, the Δv required for an absolute station-keeping is extremely high, making the relative station-keeping of the lattice-preserving Flower Constellation the most interesting property. This kind of constellations become attractive from an economical point of view (low fuel consumption) and from a practical point of view (fixed initial configuration).

This paper is organized as follows; first we introduce the 2D Lattice Flower Constellation Theory, and recall the satellite motion under the J_2 effect. Then, the effect of the J_2 perturbation on the (Ω, M)-space is presented, and we show the method applied to correct the lattice of the constellation to obtain the lattice-preserving Flower Constellation (relative station-keeping). After that, we present an example of design, in particular, the Galileo Constellation is built with the lattice-preserving Flower Constellation technique. Finally, the Δv concept is introduced and computed for the absolute station-keeping that requires the lattice-preserving Galileo Constellation. Furthermore, a numerical experiment is presented to show that the lattice-preserving technique is still valid even when the J_3 effect and Sun perturbation are considered.

PRELIMINARIES

In this section we describe the main tools used throughout the paper: the theory of Flower Constellations and the dynamics of a satellite orbiting the Earth.

2D-Lattice Flower Constellation Theory

The original theory of Flower Constellations was introduced around 2004 by professor D. Mortari.[1] The main purpose of this constellations was having all the satellites in the same (closed) trajectory with respect to a rotating frame.[2,3] The problem was that the design parameters did not have a clear physical meaning, making the theory quite difficult for the user. Consequently, it was improved by the 2D Lattice Flower Constellation theory[5] making it independent from any reference frame (inertial or rotating) and making the theory work with minimal parametrization.

A 2D Lattice Flower Constellation, hereinafter named 2D-LFC, is described by nine parameters: three integers and six continuous parameters. The first three parameters are the number of inertial orbits (N_o), the number of satellites per orbit (N_{so}) and the configuration number (N_c), which is a parameter that satisfies $N_c \in [0, N_o - 1]$ and governs the phasing of the constellation. In particular, the location of the satellites of a 2D-LFC corresponds to a lattice in the (Ω, M)-space.[6] This space can be regarded as a 2D torus (both axes, Ω and M, are modulo 2π), and coincides with the solutions of the following system of equations:

$$\begin{pmatrix} N_o & 0 \\ N_c & N_{so} \end{pmatrix} \begin{pmatrix} \Omega_{ij} - \Omega_{00} \\ M_{ij} - M_{00} \end{pmatrix} = 2\pi \begin{pmatrix} i \\ j \end{pmatrix}, \tag{1}$$

where $i = 0, \cdots, N_o - 1$, $j = 0, \cdots, N_{so} - 1$, and $N_c \in [0, N_o - 1]$. Indices (i, j) represent the j-th satellite on the i-th orbital plane.

Finally, the other six parameters are the semi-major axis (a), the eccentricity (e), the inclination (i) and the argument of perigee (ω), which are the same for all the satellites of the constellation, and the longitude of the ascending node and the mean anomaly of the reference satellite of the constellation i. e. Ω_{00} and M_{00}.

Satellite Motion

The dynamic of a satellite orbiting around the Earth is completely determined by the potential function in terms of the position vector, $V(\vec{r})$, which accounts for the gravitational interaction between the two bodies. Since the Earth is non-symmetric, the potential of an aspherical body must be obtained first. The potential function [9, p. 509] is usually split into two parts, the Keplerian part (spherical) and the perturbative part (aspherical):

$$V(\vec{r}) = V_{Kepler} + R. \tag{2}$$

If we only consider the J_2 effect, which is the coefficient corresponding to the first harmonic of the Earth's gravitational potential, the previous equation can be simplified into:

$$\begin{aligned} V(\vec{r}) &= V_{Kepler} + R_{J_2} \\ &= -\frac{\mu}{r} + J_2 \left(\frac{r_\oplus}{r}\right)^2 P_2\left(\sin(\phi_{sat})\right). \end{aligned} \tag{3}$$

where μ is the Earth gravitational constant, $r = ||\vec{r}||$, $r_\oplus$ is the Earth radius, $J_2 = -1.0826 \cdot 10^{-3}$ is the second zonal harmonic, P_2 is the 2^{nd}-order Legendre Polynomial and $\phi_{sat} \in [-\pi/2, \pi/2]$ represents the latitude of the satellite.

The motion of the satellite orbiting the Earth is described by the following first order system of equations that can be found in any book of astrodynamics:[10]

$$\begin{cases} \dot{\vec{r}}(t) &= \vec{v}(t), \\ \dot{\vec{v}}(t) &= -\nabla V(\vec{r}(t)), \end{cases} \tag{4}$$

where $\vec{r}(t)$ and $\vec{v}(t)$ represent the position and velocity of the satellite at time t, respectively. The system presented in Eq. (4) has six differential equations of order one. Given the initial position and velocity, the solution of the system at time t is presented through the state vector at time t, $\vec{r}(t)$ and $\vec{v}(t)$.

Once the state vector is known, the classical orbital elements $(a, e, i, \omega, \Omega, M)$ can be determined. In the Keplerian motion, a satellite has constant orbital elements except the mean anomaly M. Then, the evolution of them over time can be represented as a straight line, since M increases linearly: $M(t) = nt$. The perturbative effects (aspherical body, solar radiation pressure, etc.) cause deviations from the two-body (Keplerian) motion. Consequently, the orbital elements are no longer constant. However, the model can be considered instantaneously as a Keplerian model, i.e. at each instant of time it is possible to describe the movement as a Keplerian motion, using six orbital elements which depend on time. These parameters are named osculating elements; $a(t)$, $e(t)$, $i(t)$,

$\omega(t)$, $\Omega(t)$ and $M(t)$, whose evolution follow Lagrange Planetary Equations [10, p. 194], presented in Eq. (5):

$$
\begin{aligned}
\dot{a} &= -\frac{2}{na}\frac{\partial R}{\partial M}, \\
\dot{e} &= \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega} - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial M}, \\
\dot{i} &= \frac{1}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial \Omega} - \frac{\cos(i)}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial \omega}, \\
\dot{\omega} &= -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} + \frac{\cos(i)}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial i}, \\
\dot{\Omega} &= -\frac{1}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial i}, \\
\dot{M} &= n + \frac{2}{na}\frac{\partial R}{\partial a} + \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e}.
\end{aligned}
\tag{5}
$$

where R represents the perturbing part of the potential presented in Eq. (2). Analytical investigations of the oblateness effects of a central body on a satellite has shown that certain elements, such as ω, Ω, M, experience secular variations and periodic variations (short and long period variations). Other elements such as a, e, i are possessed of only periodic variations and, hence, they only vary about their mean values.

LATTICE-PRESERVING FLOWER CONSTELLATIONS

In this section we describe the problem found in a 2D-LFC when the J_2 effect is considered, which is that the lattice of the constellation is completely destroyed in a few days. However, the theory of lattice-preserving Flower constellations[7] describes a methodology to design a 2D-LFC in such a way that the initial lattice and symmetries are maintained over a long period of time i.e. relative station-keeping.

Effect of the Earth Oblateness on a 2D-LFC

In this work we consider only the J_2 effect since it is almost 1000 times larger than the next zonal harmonic coefficient J_3. Consequently, the potential function given in Eq. (3) governs the motion. The standard approach is to consider an averaged potential function $\overline{R_{J_2}}$ over an orbital period, instead of the full expression of R_{J_2}, to focus only on the non-periodic variation, i.e the secular terms of the orbital parameters that only affects to ω, Ω, M, as previously mentioned. In this particular case, all the satellites of the constellation experience the same rate of change and, consequently, they are perturbed in the same way preserving the lattice and the initial symmetric configuration of the constellation.

Nonetheless, when the propagation of satellites is done with the full expression of the potential function, i.e. considering the secular and non-secular terms, the osculating elements show a slightly different behaviour. In particular, each orbital element can be considered as the sum of its secular component and its non-secular component $a(t) = a_{sec}(t) + a_{non-sec}(t)$, $e(t) = e_{sec}(t) + e_{non-sec}(t)$, etc. (See details in Casanova et al. (2014)[7]). In this case, where the non-averaged expression of the potential function is considered, the initial lattice and symmetries are destroyed in just a few days.

Method of Lattice-preserving Flower Constellations

The first stage of the method consists of substituting the full expression of the potential function R_{J_2}, given in Eq. (3), into the Lagrange Planetary Equations, presented in Eq. (5). Notice that Lagrange Planetary Equations are strongly reduced and simplified ($\dot{a} = \dot{e} = \dot{i} = 0$) if an averaged potential function is considered ($\overline{R_{J_2}}$). However, in our particular case, where the full expression of the potential function is considered (R_{J_2}) Lagrange Planetary Equations become complex.

All the orbital elements are similar in a 2D-LFC except for the right ascension of the ascending node and the mean anomaly, which are defined following Eq. (1). We conclude, through different numerical experiments, that the slopes of the secular component of the osculating elements do not depend on the initial right ascension of the ascending node, which is different from each satellite. However, they depend on the initial mean anomaly of each satellite, which is a major problem since each satellite of a 2D-LFC have different mean anomaly, as defined in Eq. (1).

To overcome this problem we take into account that the secular motion of the mean anomaly and the semi-major axis are related by the following equation:

$$\dot{M}_{sec}(t) = n = \frac{2\pi}{T_p},$$

where, n is the mean motion and T_p is the orbital period, directly related to the semi-major axis. Then, the correction method states that, if we take the semi-major axis and the rate of change of the mean anomaly corresponding to the reference satellite of the constellation ($\dot{M}_{00}^{sec}$) as reference values, it is possible to obtain the same rate of change for the mean anomaly of all the satellites of the constellation by slightly modifying the semi-major axis of all the satellites as follow:[7]

$$a_{ij} = a \left(\frac{\dot{M}_{sec}^{ij}}{\dot{M}_{sec}^{00}} \right)^{\frac{2}{3}},$$

where ij represents the j-th satellite on the i-th orbital plane. A slightly modification of the semi-major axis is useful to control the secular variation of the satellites in order to experience all of them the same secular rate of change. Nevertheless, we should also control the non-secular motion of the satellites, but this control is unfeasible. Therefore, the solution proposed consists of selecting the initial conditions of the constellation in such a way that the non-secular variations of the osculating elements are minimized as much as possible.

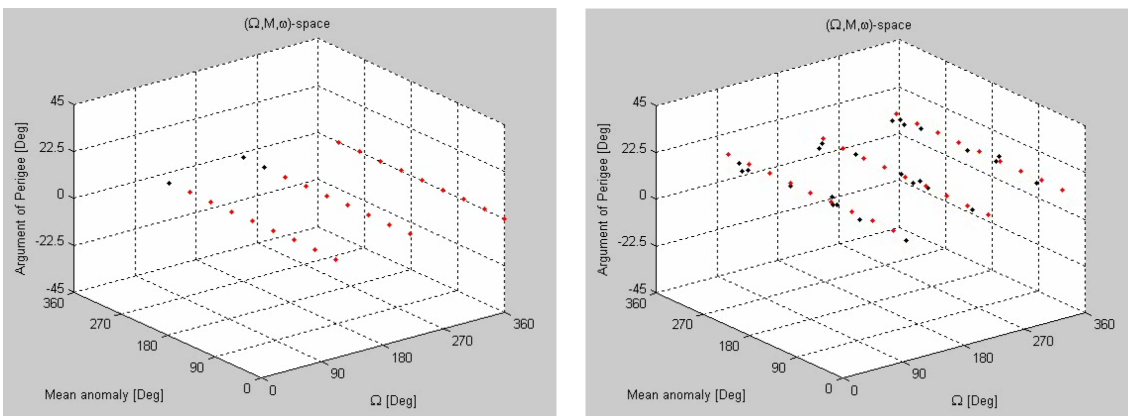
To sum up, we have a new procedure to design 2D-LFCs with stable structure even under the J_2 effect, named lattice-preserving Flower Constellations. It is achieved by two procedures, first by slightly modifying the semi-major axis of all satellites to have all of them the same slope of the secular part of their osculating elements (i.e. the secular perturbation is the same for all the satellites). The second procedure consists of selecting the values for the eccentricity and the inclination in such a way that they minimize the non-secular perturbation of the osculating elements as much as possible. Finally, we obtain a constellation where all the satellites are perturbed but in a similar way, preserving the initial configuration and the initial symmetries, and having a relative station-keeping motion.

Example of Design

We consider a Flower Constellation of 27 satellites. In particular, we take the parameters $N_o = 3$, $N_{so} = 9$, $N_c = 2$, $a = 29600.137$ [km], $e = 0$, $i = 0°$, $\omega = 0°$, $\Omega_{00} = 0°$ and $M_{00} = 0°$, which correspond to the parameters of Galileo constellation.[11]

We want to analyze the evolution of the initial lattice of the constellation i.e the (Ω, M)-space. For that purpose we illustrate in Figure 1(a) the initial lattice at time $t = 0$. Remark that, each point represents one satellite of the constellation. Figure 1(b) illustrates the position of the satellites after 1 year of propagation when the J_2 effect is considered, and it is also illustrated the lattice of the constellation computed by selecting the right ascension of the ascending node $\Omega_{00}(t)$ and the mean anomaly $M_{00}(t)$ of the reference satellite at time $t = 31536000$ [sec] following Eq. (1).

We observe that, at time $t = 0$ [sec] the lattice is perfectly distributed. However, after 1 year, the position of the satellites, if the J_2 is considered (black dots), is far away from the lattice computed following Eq. (1) and taking the data from the reference satellite, $\Omega_{00}(t), M_{00}(t)$ with $t = 31536000$ [sec] (red dots). We conclude that, if the full expression of the potential function is considered, the initial lattice and the initial symmetries are completely destroyed. Thus, the relative station-keeping that states in the Keplerian motion does not state when the J_2 effect is included.



(a) Initial distribution of satellites

(b) Distribution of satellites after 1 year propagation

Figure 1. Satellite distribution for Galileo Flower Constellation. The (Ω, M)-space computed from the reference satellite according to Eq. (1), at two different times $t = 0$ [sec] and $t = 31536000$ [sec] (1 year)

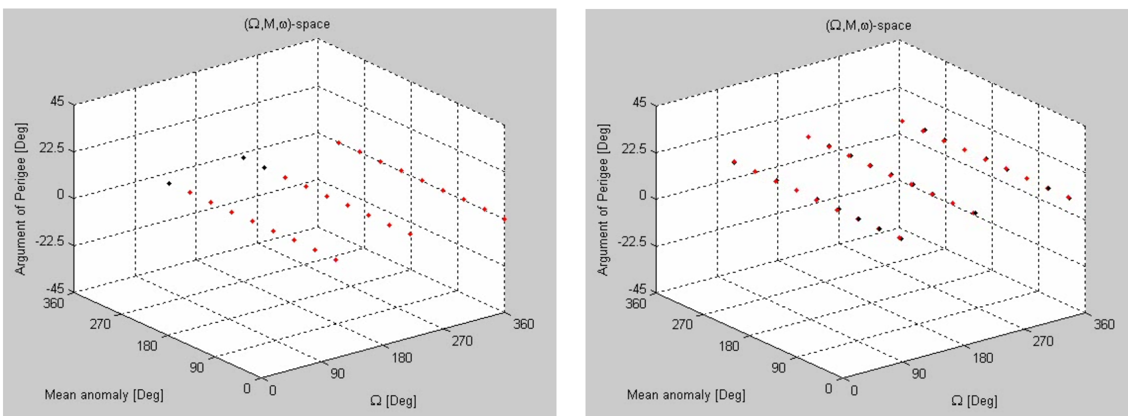
Therefore, we apply the correction method to this constellation of satellites. First, we correct the semi-major axis of the satellites to control their secular motion. Through this correction all the satellites will have the same slope for the osculating elements (secular perturbation), and consequently, they will be perturbed in the same way. Second, we compute the values for the eccentricity and the inclination that minimize the non-secular component of the osculating elements (non-secular perturbations) of the satellites in the constellation. According to Casanova et al. (2014),[7] the value for the eccentricity is $e = 0.01$, and for the inclination is $i = 56.0009°$.

Consequently, the new lattice-preserving Galileo constellation will have $e = 0.01$, $i = 56.0009°$, $\omega = 0°$ for all the satellites. The values for the right ascension of the ascending node and the mean anomaly are computed following Eq. (1), and finally, the values of the corrected semi-major axis are given in Table 1 where satellite (i, j) represents the j-th satellite on the i-th orbital plane.

Table 1. Semi-major axis of Lattice-preserving Galileo Flower Constellation.

Satellite (i,j)	a [km]	Satellite (i,j)	a [km]	Satellite (i,j)	a [km]
$(0,0)$	29600.137	$(1,0)$	29599.524	$(2,0)$	29598.167
$(0,1)$	29598.872	$(1,1)$	29599.976	$(2,1)$	29599.978
$(0,2)$	29597.165	$(1,2)$	29598.165	$(2,2)$	29599.522
$(0,3)$	29597.843	$(1,3)$	29597.085	$(2,3)$	29597.553
$(0,4)$	29599.783	$(1,4)$	29598.519	$(2,4)$	29597.329
$(0,5)$	29599.784	$(1,5)$	29600.101	$(2,5)$	29599.216
$(0,6)$	29597.845	$(1,6)$	29599.218	$(2,6)$	29600.101
$(0,7)$	29597.166	$(1,7)$	29597.329	$(2,7)$	29598.522
$(0,8)$	29598.874	$(1,8)$	29597.554	$(2,8)$	29597.085

Finally, Figure 2 illustrates the power of the correction method. We observe that the distribution of the satellites after 1 year of propagation, where the semi-major axis are corrected and the J_2 effect is considered, coincides with the lattice computed from the reference satellite according to Eq. (1) at time $t = 31536000$ [sec]. We conclude that the initial lattice and the initial symmetries are maintained for long periods of time. Thus, the relative station-keeping of the satellites in the constellation states.



(a) Initial distribution of satellites

(b) Distribution of satellites after 1 year

Figure 2. Satellite distribution for the *corrected* Galileo Flower Constellation. The (Ω, M)-space computed from the reference satellite according to Eq. (1), at two different times $t = 0$ [sec] and $t = 31536000$ [sec] (1 year)

ΔV REQUIRED FOR THE STATION-KEEPING

In the previous section was presented a correction method to maintain the symmetries and the structure of the 2D-Lattice Flower Constellations i.e. the satellites of the constellation present a relative station-keeping. The main idea of lattice-preserving Flower Constellations is that all the satellites in the constellation are perturbed but in a similar way, and consequently, the initial lattice is maintained over time.

However, some missions require absolute station-keeping, which consists of minimizing the natural drift experienced by a satellite due to the central body's shape, that in constellation design means that each satellite of the constellation remains in a predefined mathematical box relative to the Earth or inertial space. In this section we analyze if an absolute station-keeping is feasible in a lattice-preserving Flower Constellation. In particular, we study the required velocity change (Δv) to have an absolute station-keeping.

Velocity change or Δv

A space mission always requires a reconfiguration of satellites to maintain a specific structure. The reconfiguration may consists of putting the satellites in a parking orbit, transferring the satellites into a different orbit, going through a series of re-phasing, moving the satellites into a final orbit at the end of its useful life, having an absolute station-keeping, etc. The concept Δv is usually used to measure the energy required to make the mentioned changes in the orbit of the satellites of the constellation. This value, denoted by Δv, is the sum of the velocity changes that must be applied to the satellites of the constellation to obtain the desired reconfiguration of satellites.[8] In satellite constellation design we should minimize the Δv required to reduce the cost of the mission.

In a Flower Constellation, the required Δv can be computed from the following equation:

$$\Delta \vec{v} = \sum_{i=0}^{N_o-1} \sum_{j=0}^{N_{so}-1} \sum_{k=0}^{N} \Delta \vec{v}_k^{(i,j)}, \tag{6}$$

where $\Delta \vec{v}_k^{(i,j)}$ represents the k-th change of velocity applied to the satellite (i, j) in the direction given by $\vec{v}_k$. However, the usual expression is the norm of the previous quantity, i.e.:

$$||\Delta \vec{v}|| = \Delta v. \tag{7}$$

Time Evolution of the (Ω, M)-space

As we observed in the previous section, a Flower Constellation without a lattice-preserving design is destroyed in a few days when some perturbations are included, such as the J_2 effect. If we consider two satellites of the constellation, we numerically obtain that the slopes of the right ascension of the ascending node are similar for both satellites, and the slopes of the mean anomaly are also similar for both satellites, as the lattice-preserving technique states. Table 2 shows the secular variation of the osculating elements of each satellite of the corrected Lattice-preserving Galileo Flower Constellation.[7]

In this particular constellation, we compute the time evolution of $Sat_{(0,0)}$ and $Sat_{(0,1)}$ under the J_2 effect, and we observe that the secular variation of the right ascension of the ascending node is the same for both satellites, and it is equal to $-5.228{\cdot}10^{-9}$ [rad/sec]. Actually, Figure 3 shows the time evolution of this variable where we observe its secular variation as well as the overlay that occurs in the evolution of the right ascension of the ascending node for both satellites.

The idea is that all the satellites in this constellation are perturbed in a similar way under the J_2 effect and the initial lattice and symmetries are maintained, resulting into a perfect relative station-keeping constellation. However, some missions may require an absolute station-keeping, i.e. not only maintain the initial lattice in time (relative station-keeping) but also correct the drift experienced by each satellites of the constellation to remain in a predefined mathematical box relative to the Earth or inertial space. For this purpose several maneuvers must be performed.

If we want to keep the initial 2D-lattice we must perform different maneuvers to control the drift of the right ascension of the ascending node, but also phasing maneuvers,[12] which are used

Table 2. Slopes of the osculating elements corresponding to the satellites of the Lattice-preserving Galileo Flower Constellation. Satellite (i, j) represents the j-th satellite on the i-th orbital plane.

Sat. (i, j)	a [km] corrected	$\dot{a}_{sec}$ [km/sec]	$\dot{e}_{sec}$ [sec^{-1}]	$\dot{i}_{sec}$ [rad/sec]	$\dot{\omega}_{sec}$ [rad/sec]	$\dot{\Omega}_{sec}$ [rad/sec]	$\dot{M}_{sec}$ [rad/sec]
$(0,0)$	29600.137	$-2.833{\cdot}10^{-11}$	$-8.944{\cdot}10^{-17}$	$-3.227{\cdot}10^{-16}$	$2.661{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(0,1)$	29598.872	$8.298{\cdot}10^{-12}$	$-1.198{\cdot}10^{-17}$	$9.466{\cdot}10^{-17}$	$2.638{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(0,2)$	29597.165	$3.114{\cdot}10^{-11}$	$3.891{\cdot}10^{-16}$	$3.549{\cdot}10^{-16}$	$2.629{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(0,3)$	29597.843	$2.525{\cdot}10^{-12}$	$9.406{\cdot}10^{-16}$	$2.883{\cdot}10^{-17}$	$2.620{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398265{\cdot}10^{-4}$
$(0,4)$	29599.783	$-3.029{\cdot}10^{-11}$	$8.598{\cdot}10^{-16}$	$-3.450{\cdot}10^{-16}$	$2.622{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(0,5)$	29599.784	$-1.305{\cdot}10^{-11}$	$-3.276{\cdot}10^{-16}$	$-1.486{\cdot}10^{-16}$	$2.617{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(0,6)$	29597.845	$2.569{\cdot}10^{-11}$	$-1.847{\cdot}10^{-16}$	$2.928{\cdot}10^{-16}$	$2.612{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398265{\cdot}10^{-4}$
$(0,7)$	29597.166	$2.198{\cdot}10^{-11}$	$-4.364{\cdot}10^{-16}$	$2.506{\cdot}10^{-16}$	$2.620{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(0,8)$	29598.874	$-1.811{\cdot}10^{-11}$	$-1.044{\cdot}10^{-15}$	$-2.062{\cdot}10^{-16}$	$2.629{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,0)$	29599.524	$-2.764{\cdot}10^{-11}$	$-7.800{\cdot}10^{-16}$	$-3.148{\cdot}10^{-16}$	$2.628{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,1)$	29599.976	$-1.926{\cdot}10^{-11}$	$-3.502{\cdot}10^{-16}$	$-2.194{\cdot}10^{-16}$	$2.643{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,2)$	29598.165	$2.096{\cdot}10^{-11}$	$1.680{\cdot}10^{-16}$	$2.389{\cdot}10^{-16}$	$2.636{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,3)$	29597.085	$2.650{\cdot}10^{-11}$	$4.320{\cdot}10^{-16}$	$3.020{\cdot}10^{-16}$	$2.623{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,4)$	29598.519	$-1.175{\cdot}10^{-11}$	$9.635{\cdot}10^{-16}$	$-1.338{\cdot}10^{-16}$	$2.623{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,5)$	29600.101	$-3.061{\cdot}10^{-11}$	$5.169{\cdot}10^{-16}$	$-3.488{\cdot}10^{-16}$	$2.622{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,6)$	29599.218	$1.081{\cdot}10^{-12}$	$-5.280{\cdot}10^{-16}$	$1.245{\cdot}10^{-17}$	$2.612{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,7)$	29597.329	$3.094{\cdot}10^{-11}$	$1.278{\cdot}10^{-17}$	$3.526{\cdot}10^{-16}$	$2.607{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(1,8)$	29597.554	$9.650{\cdot}10^{-12}$	$-8.179{\cdot}10^{-16}$	$1.101{\cdot}10^{-16}$	$2.624{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,0)$	29598.167	$-4.737{\cdot}10^{-12}$	$-1.048{\cdot}10^{-15}$	$-5.379{\cdot}10^{-17}$	$2.627{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,1)$	29599.978	$-3.129{\cdot}10^{-11}$	$-2.438{\cdot}10^{-16}$	$-3.565{\cdot}10^{-16}$	$2.630{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,2)$	29599.522	$-6.135{\cdot}10^{-12}$	$-1.903{\cdot}10^{-16}$	$-6.979{\cdot}10^{-17}$	$2.637{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,3)$	29597.553	$2.915{\cdot}10^{-11}$	$3.121{\cdot}10^{-16}$	$3.322{\cdot}10^{-16}$	$2.634{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,4)$	29597.329	$1.624{\cdot}10^{-11}$	$6.098{\cdot}10^{-16}$	$1.851{\cdot}10^{-16}$	$2.614{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,5)$	29599.216	$-2.352{\cdot}10^{-11}$	$1.012{\cdot}10^{-15}$	$-2.680{\cdot}10^{-16}$	$2.622{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,6)$	29600.101	$-2.443{\cdot}10^{-11}$	$7.844{\cdot}10^{-17}$	$-2.783{\cdot}10^{-16}$	$2.620{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$
$(2,7)$	29598.522	$1.497{\cdot}10^{-11}$	$-3.629{\cdot}10^{-16}$	$1.707{\cdot}10^{-16}$	$2.609{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398265{\cdot}10^{-4}$
$(2,8)$	29597.085	$2.962{\cdot}10^{-11}$	$-6.970{\cdot}10^{-17}$	$3.376{\cdot}10^{-16}$	$2.614{\cdot}10^{-9}$	$-5.228{\cdot}10^{-9}$	$1.2398266{\cdot}10^{-4}$

to change the position of the satellite in its orbit (i.e change the mean anomaly) by speeding up or decelerating the satellite. However, if an absolute station-keeping is required we must control also the drift of the perigee or the drift in the inclination increasing the cost of the maneuvers.

Outperform phasing maneuvers require a small Δv since just a retrofire is required to speed the satellite up or decelerate it. The maneuvers become increasingly complicated when dealing with the right ascension of the ascending node. We analyze the variation of the right ascension of the ascending node for one satellite during one year,

$$\dot{\Omega} = \frac{\Delta\Omega}{\Delta t}. \quad \Rightarrow \quad \Delta\Omega = \dot{\Omega} \cdot \Delta t = 5.228{\cdot}10^{-9} \cdot 3600 \cdot 24 \cdot 365 = 0.16487\,[\text{rad}] = 9.4464^{\circ}. \quad (8)$$

The Δv required for such a planar change needs a big fuel consumption just for one satellite. Furthermore, this analysis can be generalized to all the satellites of the constellation, and consequently, each satellite of the constellation requires a similar Δv to correct the drift on the right ascension of the ascending node.

Then, the Δv required to control the right ascension of the ascending node plus the minimum Δv required for the phasing maneuvers are enough to control the 2D-lattice. However, we should apply other maneuvers to completely control the argument of perigee and the inclination. We can conclude

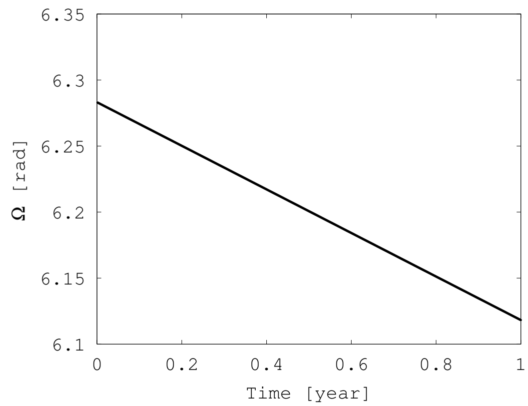


Figure 3. Evolution of the right ascension of the ascending node of $Sat_{(0,0)}$ and $Sat_{(0,1)}$ of the corrected Galileo Constellation when the J_2 effect is considered.

that an absolute station-keeping is feasible but it is a high fuel consuming process. Consequently, we disregard the absolute station-keeping in favor of the relative station-keeping, and we take advantage of the main characteristic of a lattice-preserving Flower Constellation, whose initial configuration and symmetries are maintained over time.

Case of Lattice-preserving when J_3 and Sun perturbation are considered

In this subsection we analyze if the lattice-preserving Flower constellation concept is still valid when other perturbations such as the zonal harmonic J_3, or Sun perturbation are included.

In particular, we compute numerically the time evolution of the right ascension of the ascending node of $Sat_{(0,0)}$ and $Sat_{(0,1)}$ of the corrected Galileo constellation under the J_2, J_3 effects and Sun perturbation. Figure 4 illustrates the time evolution of the right ascension of the ascending node of both satellites and we observe a similar evolution as well as the overlay that occurs in the evolution of the right ascension of the ascending node for both satellites.

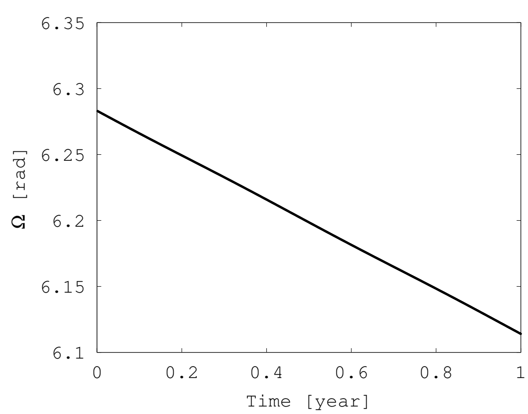


Figure 4. Evolution of the right ascension of the ascending node of $Sat_{(0,0)}$ and $Sat_{(0,1)}$ of the corrected Galileo Constellation when the zonal harmonic J_2 and J_3 plus Sun perturbation are considered.

We can observe a similar slope than the one illustrated in Figure 3. In particular the variation of the right ascension of the ascending node for one satellite in one year is $\Delta\Omega = 0.16886$ [rad]. Note that, when only the J_2 effect is considered the variation of the right ascension of the ascending node in one year is $\Delta\Omega = 0.16487$ rad.

We can expect a similar behavior for the remaining satellites of the constellation proving that the lattice-preserving Flower Constellation technique is still valid under the zonal harmonics J_2, J_3 and the Sun as a third body. This technique becomes into an efficient tool to design 2D-LFCs whose initial configuration and symmetries are maintained over time under the J_2, J_3 effect and Sun perturbation. However a complex analysis is required as a future work.

CONCLUSION

A novel way to design Flower Constellations that maintain the initial distribution of satellites and the initial symmetries over time i.e. relative station-keeping was presented. This kind of constellations are named 2D lattice-preserving Flower Constellations. The main characteristic is that all the satellites in the constellation are perturbed in a similar way, and consequently, the initial distribution of the satellites (initial lattice), and specially its symmetries are time-preserving. We showed that the Δv required to have an absolute station-keeping is unfeasible from an economical point of view. For that reason, the necessity of reconfiguration is disregarded and we should take advantage of the characteristics of the lattice-preserving Flower Constellations under the J_2 effect. Furthermore, numerical experiments showed that the lattice-preserving Flower Constellation technique is still valid under the J_3 effect and Sun perturbation.

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