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Trabajo Fin de Máster

Modelado estadístico de amplificadores no lineales
Statistical modeling of non-linear amplifiers

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1. Introducción

El creciente incremento en la demanda de datos móviles de nuestra sociedad está llevando a la tecnología a buscar nuevas soluciones para aumentar la eficiencia espectral de los canales de comunicaciones. De entre las posibles soluciones para abordar este problema, una de las más extendidas es la conocida como MIMO (del inglés *Multiple Input Multiple Output*), cuyo concepto consiste en el uso de múltiples antenas en transmisión y recepción para comunicación simultánea. La importancia de esta idea está avalada por su presencia en varios estándares recientes (IEEE802.11n y LTE entre otros).

Al mismo tiempo, las limitaciones en el hardware de los equipos de radio frecuencia usados para desplegar escenarios MIMO degradan las prestaciones de los enlaces radio. Algunas de las deficiencias más importantes reconocidas en dicho equipamiento son el offset en la frecuencia de portadora, el ruido de fase, el jitter en el instante de muestreo, el desbalance entre fase y cuadratura o, la que será objeto de estudio en este trabajo, la nolinealidad del amplificador de potencia (PA, del inglés *Power Amplifier*), cuyo impacto se puede observar en la Figura 1[1]. Todos estos efectos se están viendo incrementados debido al desarrollo de sistemas cuyos componentes operan cada vez más cerca de los límites de rendimiento. Debido al crecimiento en complejidad de las comunicaciones inalámbricas a medida que evolucionan, se observa una necesidad de caracterizar de forma precisa dichas no idealidades en el hardware tal y como se ha reconocido en la literatura. Dichos modelos serán de gran utilidad a la hora de diseñar algoritmos capaces de combatirlas.

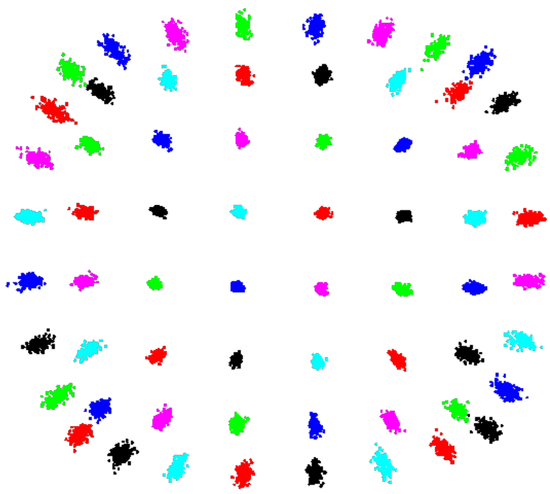

Figura 1: Una constelación 64-QAM distorsionada por un amplificador de potencia. Se puede observar tanto una escalación dependiente del símbolo de la constelación y una sutil rotación de la misma.

[1]Medida obtenida de un PA real con *RF WebLab*. `http://dpdcompetition.com/rfweblab/`

De esta forma, el objetivo principal de este Trabajo Fin de Master será el diseño de un modelo que sea capaz de caracterizar la no linealidad del amplificador en despliegues multiantena. Para ello, en este proyecto se seguirá la línea marcada por Thomas Eriksson y Nicolò Mazzali en su próximamente publicado artículo para un esquema SISO (del inglés *Single Input Single Output*).

2. Modelo SISO

El trabajo de los citados autores se basa en la asunción de que un modelo polinómico de tercer orden sin memoria es suficiente para representar el fenómeno a estudiar, siendo conscientes de que a pesar de la presencia de mayores órdenes en la naturaleza no-lineal del amplificador, el tercer orden es el dominante. Dicho modelo se muestra en la Ecuación 1, donde α es el coeficiente de no linealidad. El resto de señales involucradas en dicha ecuación pueden consultarse en la el diagrama de bloques del sistema en la Figura 2 .

$$p(t) = b(t) + \alpha b(t)|b(t)|^2 = \left(|b(t)| + \alpha|b(t)|^3\right) e^{j\angle b(t)} \tag{1}$$

Derivando las escuaciones del sistema, los autores identifican un término de distorsión d_n fruto de dicha no linealidad que puede escribirse como en la Ecuación 2, donde λ_{klm} es un término dependiente de los filtros del sistema.

$$\begin{aligned} d_n &= \int_{-\infty}^{\infty} \alpha \sum_{k=-\infty}^{\infty} x_k e(s-kT) \left| \sum_{l=-\infty}^{\infty} x_l e(s-lT) \right|^2 f(nT-s)ds = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x_{n-k} x_{n-l} x^*_{n-m} \underbrace{\int_{-\infty}^{\infty} e\left(\frac{s}{T}-k\right) e\left(\frac{s}{T}-l\right) e\left(\frac{s}{T}-m\right) f\left(\frac{s}{T}\right) ds}_{\lambda_{klm}} = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} x_{n-k} x_{n-l} x^*_{n-m} \end{aligned} \tag{2}$$

Este término d_n puede modelarse con la ayuda de cuatro coeficientes en función de si los símbolos que distorsionan al símbolo actual x_n son iguales o no al propio x_n en el instante de estudio n, Ecuación 3.

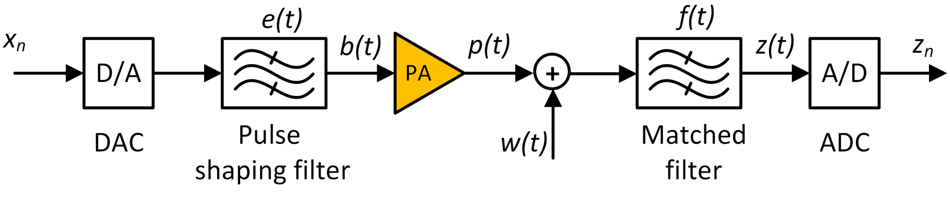

Figura 2: Diagrama de bloques del sistema que contiene un amplificador de potencia no lineal.

$$d_n = c_0 + c_1|x_n| + c_2|x_n|^2 + c_3|x_n|^3 \quad (3)$$

Mediante el estudio de la esperanza y varianza de dichos coeficientes, los autores concluyen que la señal de salida del sistema puede describirse como en el Ecuación 4. Donde $u^{(i)}$ es la esperanza del coeficiente c_i y $v^{(i)}$ son variables aleatorias con media cero y varianza fijada por la varianza del coeficiente c_i. De esta forma, los $u^{(i)}$ rotan y reescalan de forma no lineal la constelación mientras que los $v^{(i)}$ añaden rudio a la constelación generado por el amplificador .

$$z_n = x_n\left(1 + u^{(1)} + u^{(3)}\frac{|x_n|^2}{\sigma_x^2}\right) + v_n^{(0)} + v_n^{(1)}\frac{|x_n|}{\sigma_x} + v_n^{(2)}\frac{|x_n|^2}{\sigma_x^2} + w_n \quad (4)$$

Lo que esto signfica es que la señal de salida del sistema, afectada por el PA, puede ser artificialmente generada gracias a una ecuación dependiente de la no linealidad del PA α, de los filtros del sistema (λ_{klm}) y de los momentos estadísticos de la señal de entrada.

3. Modelo MIMO

Con el objetivo de conseguir un modelo general, se opta por llevar a cabo una estrategia paso a paso: desde el caso más sencillo, compuesto por dos antenas transmisoras y dos usuarios, hasta un caso genérico en el que se suponen M antenas en transmisión (nos referiremos a ellas como antenas por comodidad) y K antenas en recepción (usuarios). De esta forma, se trata de evitar pasar por alto cualquier detalle en el proceso de ecuaciones del escenario. Para ello, se mantiene la misma estructura de sistema propuesta por Eriksson y Mazzali añadiendo dos bloques muy comunes en sistemas de comunicaciones: precodificador y canal, tal y como se puede observar en el diagrama de la Figura 3. Ambos bloques se mantienen sin determinar y de forma genérica en las ecuaciones para facilitar el estudio de distintos esquemas en función del propósito. Dado que el objetivo de este trabajo es estudiar el efecto de la no linealidad del PA, se asume total conocimiento de las condiciones de canal

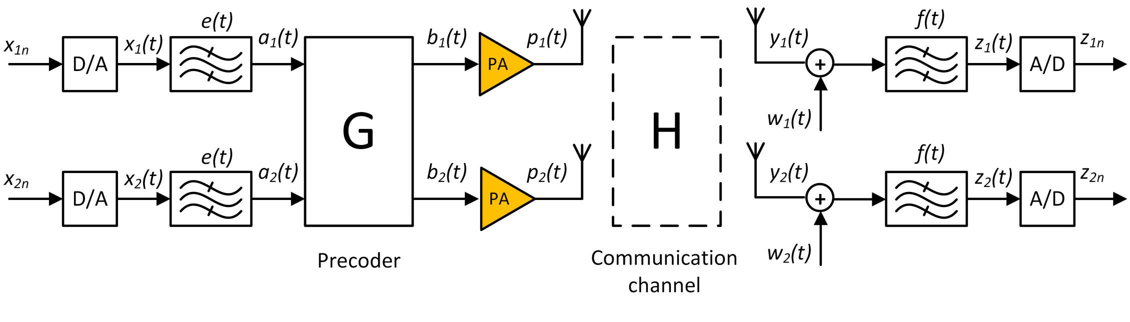

Figura 3: Diagrama de bloques del sistema con 2 antenas y 2 usuarios. Bloques de precoficación y canal son añadidos.

y se utilizará el Zero Forcer (ZF) para eliminar, en la mayor medida posible, la interferencia multiusuario.

3.1. Modelo 2x2

Llevando a cabo los mismos razonamientos que para el modelo SISO, se puede llegar a la conclusión de que en este caso aparece un término de distorsion para el Usuario 1, d_{1n}, (análogo al que experimentaría el Usuario 2) que puede ser descrito como en la Ecuación 5, donde en los términos L_i está contenida la información de canal y precodificador de cada producto cruzado.

$$
\begin{aligned}
d_{1n} = \frac{1}{2^2} \Bigg[& \underbrace{\lambda_{klm} L_1 x_1 x_1 x_1^*}_{d_{x_{1n}}} + \underbrace{\lambda_{klm} L_2 x_2 x_2 x_2^*}_{d_{x_{2n}}} + \underbrace{\lambda_{klm} L_3 x_1 x_2 x_2^*}_{d_{x_{1n} x_{2n} x_{2n}^*}} + \\
& + \underbrace{\lambda_{klm} L_4 x_1 x_1 x_2^*}_{d_{x_{1n} x_{1n} x_{2n}^*}} + \underbrace{\lambda_{klm} L_5 x_1 x_1^* x_2}_{d_{x_{1n} x_{1n}^* x_{2n}}} + \underbrace{\lambda_{klm} L_6 x_1^* x_2 x_2}_{d_{x_{1n}^* x_{2n} x_{2n}}} \Bigg]
\end{aligned} \tag{5}
$$

A continuación, cada uno de dichos productos cruzados que componen d_{1n} puede modelarse de nuevo mediante diferentes coeficientes cuyas propiedades estadísticas se estudian por separado. De esta forma, se llega a la Ecuación 6 con una estructura similar a la del modelo SISO en la que destaca la existencia de interferencia multiusuario, sin importar el sistema de precodificación utilizado debido precisamente a la no linealidad del PA.

$$
\begin{aligned}
z_{1n} = & \, x_{1n} \left(1 + u^{(1)} + u^{(9)} + u^{(3)} \frac{|x_{1n}|^2}{\sigma_{x_1}^2} \right) + \left(v_n^{(1)} + v_n^{(9)} + v_n^{(14)} + v_n^{(19)} + v_n^{(24)} \right) \frac{|x_{1n}|}{\sigma_{x_1}} + \\
& + \left(v_n^{(2)} + v_n^{(15)} + v_n^{(20)} \right) \frac{|x_{1n}|^2}{\sigma_{x_1}^2} + x_{2n} \left(u^{(5)} + u^{(21)} + u^{(7)} \frac{|x_{2n}|^2}{\sigma_{x_2}^2} \right) + \\
& + \left(v_n^{(5)} + v_n^{(10)} + v_n^{(16)} + v_n^{(21)} + v_n^{(25)} \right) \frac{|x_{2n}|}{\sigma_{x_2}} + \left(v_n^{(6)} + v_n^{(11)} + v_n^{(26)} \right) \frac{|x_{2n}|^2}{\sigma_{x_2}^2} + \\
& + \left(u^{(22)} + u^{(27)} \right) \frac{|x_{1n}|^2}{\sigma_{x_1}^2} \frac{|x_{2n}|}{\sigma_{x_2}} + \left(u^{(12)} + u^{(17)} \right) \frac{|x_{1n}|}{\sigma_{x_1}} \frac{|x_{2n}|^2}{\sigma_{x_2}^2} + \\
& + v_n^{(0)} + v_n^{(4)} + v_n^{(8)} + v_n^{(13)} + v_n^{(18)} + v_n^{(23)} + w_{1n}
\end{aligned} \tag{6}
$$

3.2. Modelo MxK

Tras un delicado proceso hasta llegar al modelo general, estudiando como un cambio en el número de antenas o de usuarios afecta a las ecuaciones, se obtiene el término de distorsión que experimenta un usuario genérico A y que se muestra en la Ecuación 7. Agrupando y analizando los términos que aparecen para este escenario, debido a su numerosidad y la extrema complejidad que supondría elaborar un método exacto, bajo consejo de los directores se toma la decisión de considerar únicamente los términos que contienen señal del usuario A. El resto de términos se agruparán en m_A tal y como se puede observar en la Ecuación 8.

$$
\begin{aligned}
d_A(t) = \frac{1}{M^2}\cdot\lambda_{rst}\Bigg[&\left(\sum_{k=1}^{K} x_k(t)x_k(t)x_k^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{mk}(t)|g_{mk}(t)|^2\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K} x_A(t)x_A(t)x_k^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * (g_{m1}(t))^2 * g_{mk}^*(t)\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K} 2\cdot x_A(t)x_A^*(t)x_k(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * |g_{m1}(t)|^2 * g_{mk}(t)\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K} 2\cdot x_A(t)x_k(t)x_k^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}(t)|g_{mk}(t)|^2\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K} x_A^*(t)x_k(t)x_k(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}^*(t) * (g_{mk}(t))^2\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K}\sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} 2\cdot x_A(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}(t) * g_{km}(t) * g_{mi}^*(t)\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K}\sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} 2\cdot x_A^*(t)x_k(t)x_i(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}^*(t) * g_{mk}(t) * g_{mi}(t)\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K}\sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} 2\cdot x_k(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * (g_{mk}(t))^2 * g_{mi}^*(t)\right) + \\
&+\left(\sum_{\substack{k=1\\k\neq A}}^{K}\sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} x_k(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * (g_{mk}(t))^2 * g_{mi}(t)\right)\Bigg]
\end{aligned}
\tag{7}
$$

$$
z_{An} = x_{An}\left[1 + U^{(1)} + U^{(2)}\frac{|x_{An}|}{\sigma_{x_A}} + U^{(3)}\frac{|x_{An}|^2}{\sigma_{x_A}^2}\right] + V^{(1)}\frac{|x_{An}|}{\sigma_{x_A}} + V^{(2)}\frac{|x_{An}|^2}{\sigma_{x_A}^2} + m_{An} + w_{An} \tag{8}
$$

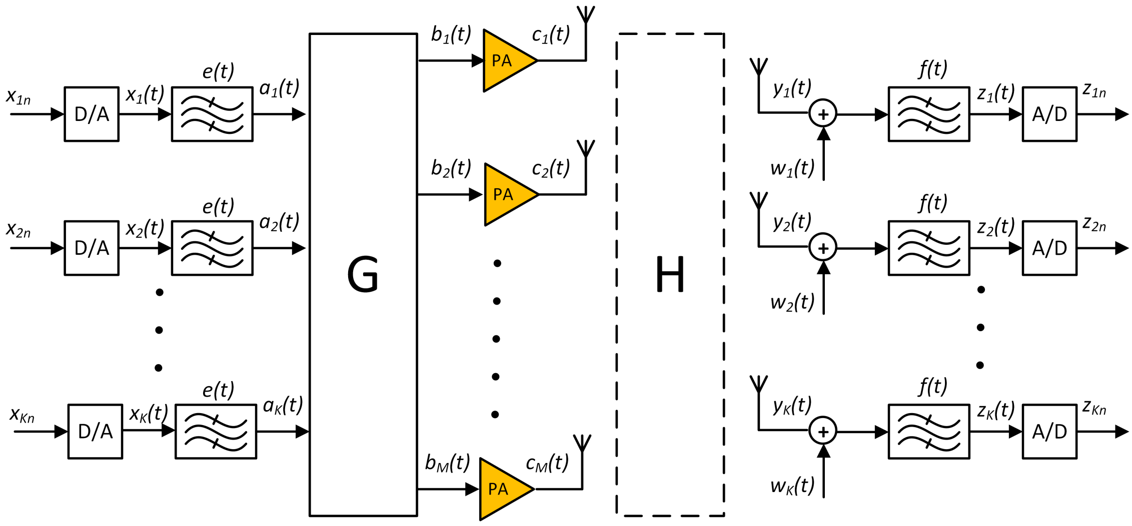

Figura 4: Diagrama de bloques del sistema final con M antenas y K usuarios.

4. Resultados

4.1. Simulación en escenario MIMO

Con el propósito de observar el comportamiento del fenómeno de estudio de este proyecto, se realiza simulaciones para distintas configuraciones. Para comenzar, en la Figura 5 se estudia visualmente la evolución con el número de antenas cuando el número de usuarios se mantiene fijo en 2. Se observa un decremento en la distorsión provocada por el amplificador. Tras reflexionar sobre este fenómeno, la conclusión alcanzada es que debido a la *Ley de los grandes números* la distorsión desaparece asintóticamente con el número de antenas M. Para cuantificar de alguna manera esta mejora, se realiza un estudio de la SNR que experimenta un usuario cuando se aumenta el número de antenas. Además, se estudia el mismo fenómeno para distinto número de usuarios. Los resultados del mismo se muestran en la Figura 6. Ahí se observa que para todos los casos de estudio la SNR aumenta con M como se ha justificado previamente. No obstante, el hecho más interesante que arroja está gráfica son los distintos comportamientos de los casos de estudio. El aumento en el número de usuarios a K=4 y K=8 produce una significativa mejora de la SNR, hecho que se atribuye al Teorema del límite central. Sin embargo, cuando se aumenta el número de usuarios a 16, se identifica un claro empeoramiento atribuido al crecimiento de la interferencia multiusuario.

4.2. Modelo 2x2

Para evaluar las prestaciones del método diseñado en este trabajo todas las ecuaciones desarrolladas en la sección 3.1 serán llevadas a MATLAB. Para ello es de extrema importancia ser metículoso manipulando las distintas variables presentes a la hora de generar las variables aleatorias que tratarán de predecir la salida del sistema. No obstante, debe tenerse en cuenta que el canal de comunicaciones, en

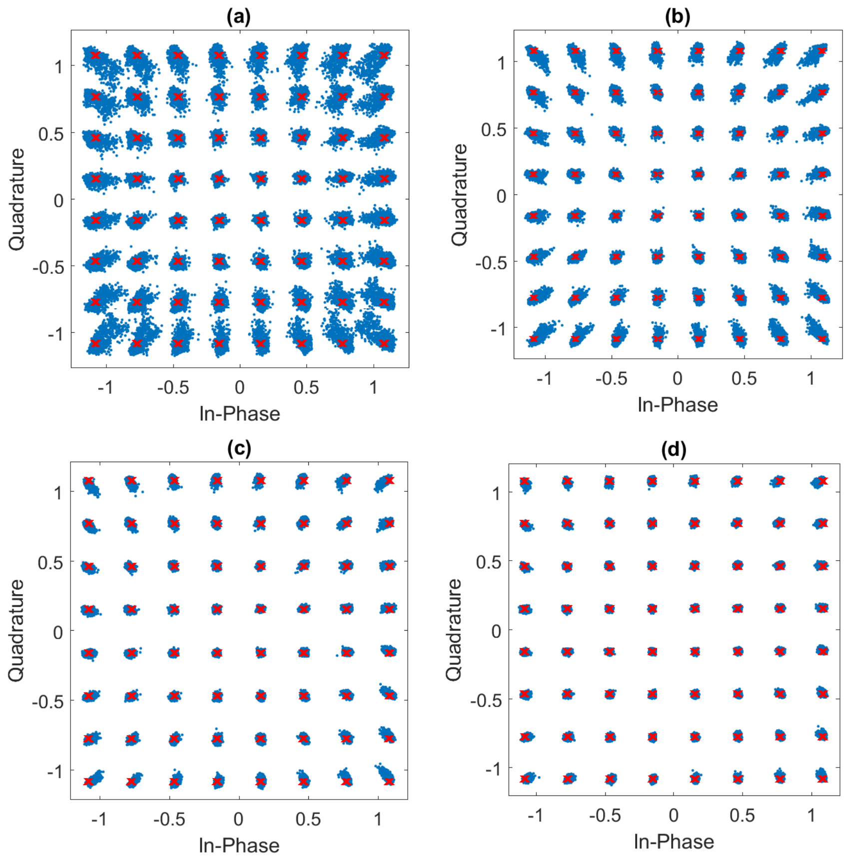

Figura 5: Constelación recibida en el Usuario 1 para los escenarios: (a) 3x2, (b) 10x2, (c) 25x2 and (d) 100x2.

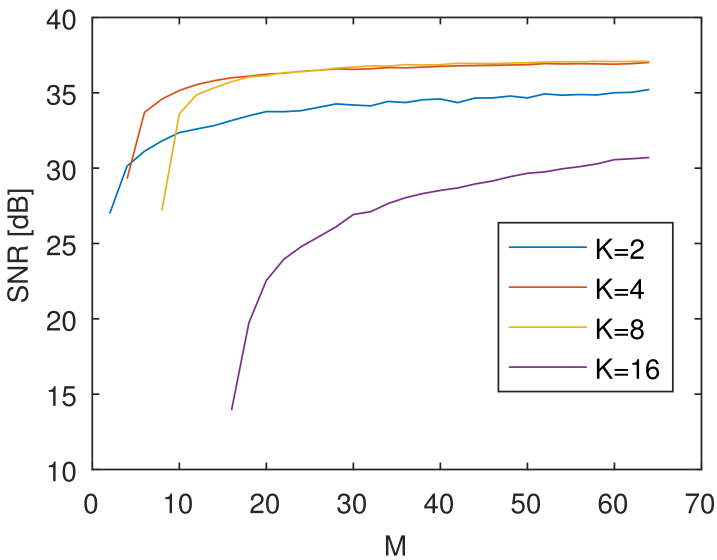

Figura 6: Evolution of the Signal to Noise Ratio for different number of antennas and users.

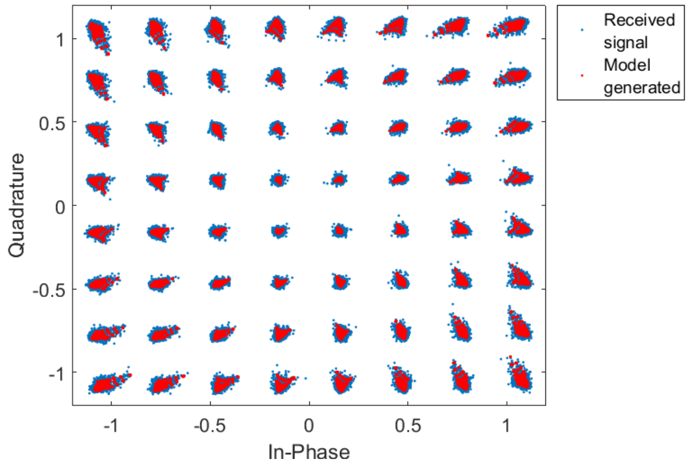

Figura 7: Ejemplo de las prestaciones del modelo 2x2 para unas condiciones de canal específicas.

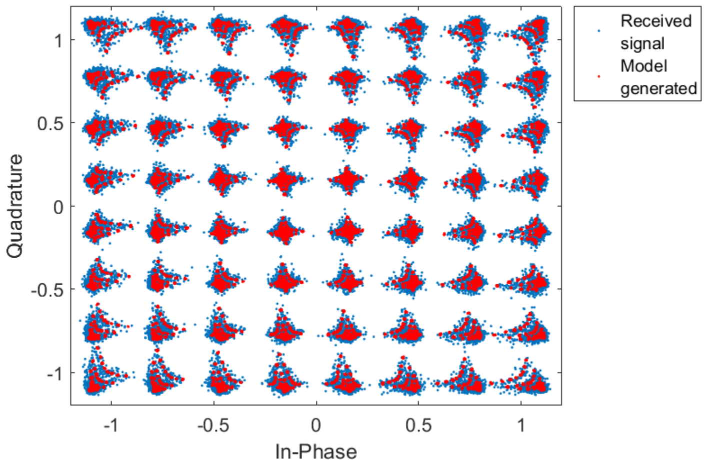

Figura 8: Otro ejemplo de las prestaciones del modelo 2x2 para unas condiciones de canal distintas.

función de su realización (simulado con distribución Rayleigh) afectará en mayor o menor grado a la señal distorsionada recibida. Por ello se muestran dos ejemplos de las prestaciones de dicho modelo en las Figuras 7 y 8. Lo representado en dichas gráficas es: en azul, la señal que recibe el Usuario 1, z_1, al final de la cadena de transmisión presentada en la figura 3 y, en rojo, la señal obtenida utilizando el modelo creado a base de variables aleatorias con ciertas esperanza y varianza. Se puede concluir a la vista de los resultados que en el caso 2x2 el método es capaz de predecir el comportamiento de la señal para unas condiciones de canal dadas.

4.3. Modelo MxK

Llevando a cabo el mismo proceso para el modelo general, los resultados se muestran en la Figura 9. Un comportamiento menos preciso es reconocible debido a la aproximación realizada.

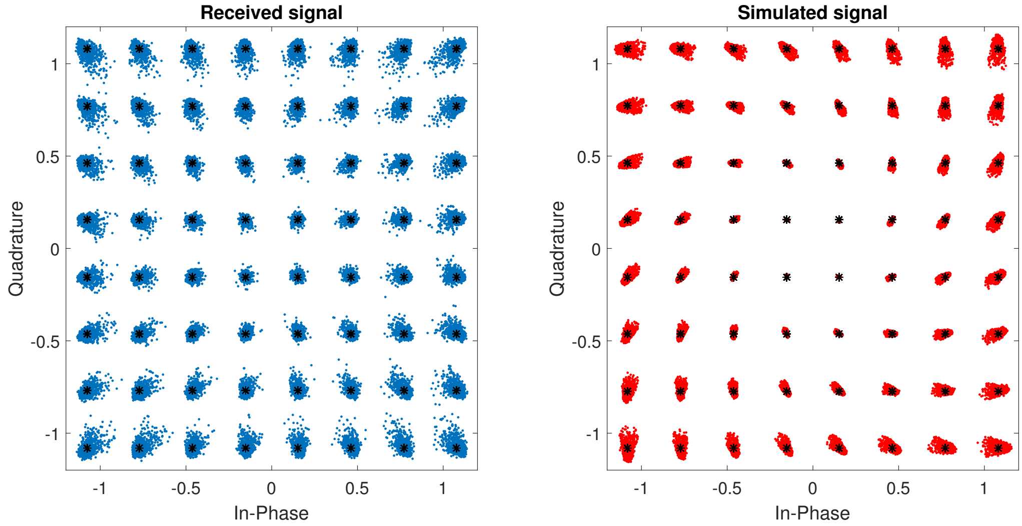

Figura 9: Ejemplo de las prestaciones del modelo MxK para unas condiciones de canal.

5. Conclusiones

En este Trabajo Fin de Máster, no sólo se ha llevado a cabo una revisión exhaustiva de un novedoso método matemático-probabilístico para caracterizar la distorsión no lineal del amplificador de potencia si no que, además, se ha llevado a cabo una extensión de dicho modelo.

La primera conclusión del estudio fue que la distorsión no lineal del PA se desvanece de forma asintótica con el número de antenas debido a la *Ley de los grandes números*. Respecto de la evolución con el número de usuarios, una primera zona dominada por la no linealidad del PA experimenta una mejora debido al *Teorema del límite central* que se ve truncada cuando el número de usuarios es lo suficientemente elevado como para que la interferencia multiusuario domine.

Respecto de los modelos elaborados y a la vista de los resultados de simulación, se reconoce su capacidad para caracterizar la no linealidad del amplificador de potencia en entornos multiantena. No obstante, la precisión de los mismos es dependiente de su exactitud, es decir, de las aproximaciones realizadas.

6. Líneas futuras

Para concluir, se propone el análisis de las prestaciones de los modelos desarrollados a lo largo del proyecto para otras configuraciones de canal y/o precodificador además del estudio de otras de las deficiencias del hardware en despligues MIMO.

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Statistical modeling of non-linear amplifiers
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Abstract

Complexity relaxations in wireless communication systems operating at the boundaries of components performance are foreseen. Due to this matter, a need to properly characterize hardware impairments degrading the performance of communications links has been recognized in previous studies. Statistical models might be the definitive tool for assessing the impact of these non-idealities.
Particularly, the non-linearity of power amplifiers in multi-antenna scenarios was examined in this thesis. To this matter, a full revision of a novel method capable of capturing this limitation for a single-antenna and thus, a single power amplifier, was carried out. Thereafter, the strategy was to commence with the simplest multi-antenna case and to increase the number of antennas and users until a general model was achieved.
Results of the developed models in the thesis are provided concluding that these kind of approaches are valid to model the non-linear nature of power amplifiers in MIMO systems. Another studies of the evolution of this distortion when varying the number of antennas and users are also presented and analyzed.
Finally, it must be emphasized the fact that this master thesis was carried out within the scope of a collaboration project between *Ericsson AB* and *Chalmers University of Technology.*

Keywords: hardware impairments, non-linearity, power amplifiers, statistical models, multi-antenna scenarios.

Contents

List of Figures

Acronyms

AM/AM	AMplitude-to-AMplitude conversion
AM/PM	AMplitude-to-Phase conversion
ACI	Adjacent Channel Interference
AWGN	Additive White Gaussian Noise
CLT	Central Limit Theorem
CSI	Channel State Information
EVM	Error Vector Magnitude
IEEE	Institute of Electrical and Electronics Engineers
IM3	Intermodulation Distortion
IQ	In-Phase Quadrature
ISI	InterSymbol Interference
LLN	Law of Large Numbers
LTE	Long Term Evolution
MIMO	Multiple Input Multiple Output
M-QAM	Multilevel Quadrature Amplitude Modulation
OFDM	Orthogonal Frequency Division Multiplexing
PA	Power Amplifier
PAPR	Peak to Average Power Ratio
PSK	Phase Shift Keying modulation
RF	Radio Frequency
RRC	Root-Raised-Cosine filter
RV	Random Variable
SEL	Soft-Envelope Limiter amplifier
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
SSPA	Solid State Power Amplifier
TWTA	Travelling Wave Tube Amplifier
ZF	Zero Forcing precoder

Symbols

$\boldsymbol{\alpha}$	Non-linear factor of the power amplifier.
$\boldsymbol{\beta}$	Roll-off factor of the pulse shaping filter.
$\boldsymbol{\lambda_{klm}}$	Function of the pulse shaping/matched filters.
$\lvert\boldsymbol{x}\rvert$	Absolute value of the random variable x.
$\boldsymbol{E}[\boldsymbol{x}]$	Expected value or mean of the random variable x.
$\boldsymbol{E}[\lvert\boldsymbol{x}\rvert^{\mathbf{2}}]$ or $\boldsymbol{\sigma_x^2}$	Variance/power of the random variable x.
$\boldsymbol{G}$	Precoder.
$\boldsymbol{H}$	Communication channel.
$\boldsymbol{K}$	Number of users.
$\boldsymbol{M}$	Number of antennas.
$\boldsymbol{T}$	Symbol period.
$\boldsymbol{a_i(t)}$	Pulse shaped signal for user i.
$\boldsymbol{b_i(t)}$	Precoded signal at power amplifier i.
$\boldsymbol{d_i(t)}$	Distortion signal for user i.
$\boldsymbol{e(t)}$	Pulse shaping filter.
$\boldsymbol{f(t)}$	Matched filter.
$\boldsymbol{p_i(t)}$	Output signal of the power amplifier i.
$\boldsymbol{x_i(t)}$	Input signal for user i.
$\boldsymbol{y_i(t)}$	Received signal at user i.
$\boldsymbol{z_i(t)}$	Output information signal at user i.
$\boldsymbol{w_i(t)}$	Additive White Gaussian Noise signal for user i.
$\boldsymbol{c_i}$	Distortion coefficient i.
$\boldsymbol{d_n}$	Distortion term for n-th transmitted symbol.
$\boldsymbol{L_i}$	Channel and precoder information for term i.
$\boldsymbol{m_n}$	Multi-user interference term in MxK model.
$\boldsymbol{u^{(i)}}$	Expected value of distortion coefficient i.
$\boldsymbol{U^{(i)}}$	Expected value of distortion term i in MxK model.
$\boldsymbol{v^{(i)}}$	Variance of distortion coefficient i.
$\boldsymbol{V^{(i)}}$	Variance of distortion term i in MxK model.

1

Introduction

Hardware impairments of radio-frequency transceivers and, specifically, the non-linear distortion of power amplifiers tends to degrade the performance of communications links [1]. These effects are increasing due to the development towards systems operating closer and closer to the limits of hardware capabilities and the pressure towards low-cost devices. The impact of some of these impairments can be compensated for at the transmitter through calibration or pre-distortion, or at the receiver using sophisticated post-compensation methods. As modern wireless communication systems grows in complexity, it becomes increasingly important to characterize the impact of hardware impairments in order to make better, informed decisions on the degree of necessary compensation. Classical linear link-budget tools may fail as they do not represent the temporal and spatial correlation of the signal and distortion components, therefore, statistical models that describe the effect of hardware impairments is crucial to obtain accurate system throughput estimates using improved information-theoretic methods. Such methods would further allow us to identify potential bottlenecks in a transceiver chain in order to partition the signal processing efforts in a more efficient manner.

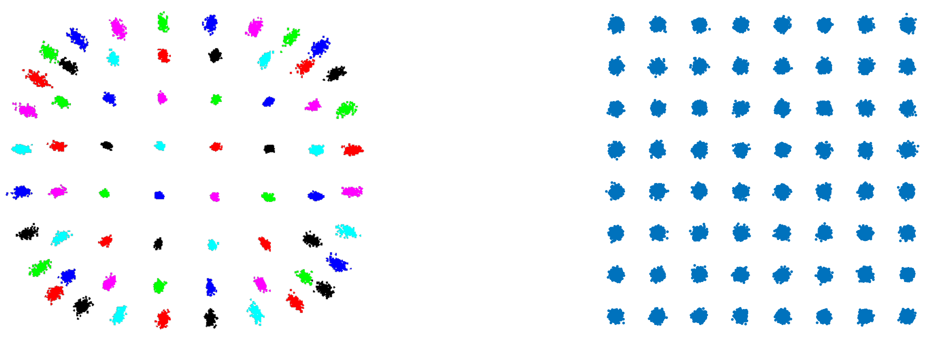
1.1 Background

The increasing demand for data traffic by our society is leading technology to find new solutions for increasing the spectral efficiency of communications channels [2]. One way to address this challenge is the use of multiple antennas at each radio-base station to transmit to different users, technique known as *MIMO* (Multiple Input-Multiple Output). The importance of this idea is illustrated by their presence in many recent standards, such as IEEE802.11n and Long Term Evolution (LTE). Interestingly, like many other brilliant solutions developed in these years in the field of digital communications, the MIMO concept is not a new idea but dated back to the 1970s and was studied as a model for multipair telephone cables [3]. MIMO techniques improve communications performance by exploiting multipath scattering in the communications channel between transmitter and receiver. MIMO techniques create spatial diversity which allows for using spatial multiplexing of users [4].

The spectral efficiency of a wireless MIMO system is limited by the theoretic capacity achievable, which depends on several factors such as the signal-to-noise ratio (SNR), the spatial correlation in the propagation environment, the channel estimation accuracy, the signal processing resources and the transceiver hardware impairments. It is of profound importance to increase the spectral efficiency of future networks, to keep up with the need for bigger data amounts. However, this is a challenging task and usually comes at the price of having stricter hardware and overhead requirements. MIMO techniques have the potential of both increasing the spectral efficiency and relaxing the aforementioned implementation issues.

In order to cope with the extreme demands, another technique using very large antenna arrays called massive MIMO is considered an enabling technology [5]. The amount of published research on *massive MIMO* is quite rampant, much like the interest in using low-precision hardware. However, most of these are lacking in terms of accurate hardware modeling, which so far has been addressed in a few studies [6]–[8]. These impairments degrade the quality of the signal beyond the impact of fading or thermal-noise [9]. Some of the most important sources of impairment are amplifier non-linearities, carrier-frequency offset, sampling-jitter, phase-noise and IQ-imbalance. In this thesis work we will focus in the non-linear distortion of the power amplifier. The non-linearity of commercial amplifiers is often specified through their *intermodulation distortion* (IM3), which is defined as the output power where the spectrum of the third order intermodulation product is equally strong as the spectrum of the corresponding linear component [10]. This is translated into a distortion of the constellation and, therefore, errors in the information received. An example of a constellation with normalized power obtained using *RF WebLab* [1] and affected by a real amplifier can be observed in Figure 1.1 (a). The main characteristics of this distortion are its non-Gaussian nature and the stronger compression and rotation of the constellation's outer points. It is observable that Gaussian models may fail trying to capture the non-linear behaviour of a real PA, Figure 1.1 (b).

The need to characterize the hardware in a statistical way has been recognized [11]–[13]. In fact, for the successful design of future radio access understanding of radio hardware characteristics is of tremendous significance. Furthermore, statistical abstractions will enable link and system level assessments of end user experience and network capacity to aid algorithm design and future standardization leading to



(a) Real amplifier distorion

(b) Gaussian model

Figure 1.1: Shortcomings of Gaussian models when modeling non-linear effects.

[1] RF WebLab. `http://dpdcompetition.com/rfweblab/`

adequate radio product requirements.

Some authors showed that a Gaussian model is a reasonable model for the sum of hardware impairments [14] while others developed this model towards Massive MIMO systems [9], [15]. However, these models do not seek any quantitative conclusions but limit the study to a particular case. In these thesis work a model that accounts for the non-linear distortion of the power amplifier and supported by numerical simulations will be looked into. In contrast to previous works, Thomas Eriksson and Nicolò Mazzali show in their soon published work that distortion cannot be accurately described simply as a stationary additive noise, but that there are also noise components that depends on the statistical properties of the transmit signal.

1.2 Goals

The investigation aims to model one of the limiting factors of any wireless multi-antenna communication system: non-linearity of power amplifiers, responding to the need for applying statistical accurate models that characterize hardware impairments. The research will specifically look into develop a single-antenna model to a multi-antenna environment. This will be done by statistical analysis and calculations in the same way it is done by Eriksson and Mazzali. This work seeks to identify how the distortion terms are affected by the signal power, the severity of the non-linearity, the pulse shaping/matched filters or the normalized moments of the constellation (e.g. PAPR). Simulations and validations will also be performed in order to evaluate the model.

Although it is not related to the outcome of this thesis, it is important to mention that a full revision and understanding of the yet not officially published work which this thesis is based on is accomplished. Fruit of this revision, a little contribution was made as long as an simulator from the ground to test model's performance.

1.3 Outline

The thesis report consists of seven chapters: Introduction, Prerequisites, Single-Antenna model, Multi-Antenna model, Results, Conclusions and Future work. Introduction gives an overview of the thesis work and introduces the necessity of developing this sort of models. Prerequisites is used to describe previous key concepts where the next chapters are based: first describes the non-linearity of the power amplifier, continues presenting some mathematical and probabilistic tools useful to understand subsequent reasoning in this thesis and finishes two new blocks included in the system. Multi-Antenna model presents the models developed from the simplest 2 by 2 model to a general scenario involving different number of antennas and users. Results shows the outcomes of the simulations and evaluations conducted in the thesis. Conclusions extract main ideas based on the results and, finally, Future work concludes the thesis proposing new problems to study.

2

Prerequisites

In this chapter, assumed theoretical used in the thesis are presented. First, non-linear behavior of power amplifier is examined. Among the different existing models of PAs a third polynomial model is used. Subsequently, some necessary mathematical tools to understand the development of the equations are introduced. Finally, system elements included in our project structure are described.

2.1 Non-linearity of power amplifiers

The power amplifier, which operates at the RF level, is assumed to operate in the linear region for simplicity of the performance analysis and system design. Nevertheless, this assumption is not valid in practical situations. Indeed, the power amplifier may operate in its nonlinear region, especially when it works at the medium and high-power signal levels. In such cases, the power amplifier nonlinearity will cause in-band and out-of-band distortions [16]. The in-band distortion will increase the error vector magintude (EVM) and reduce the system capacity. On the other hand, the out-of-band distortion appears as a spectral regrowth, hence resulting in adjacent channel interference (ACI).

The characterization of nonlinear power amplifiers can be demonstrated by two kinds of models: memoryless models with frequency-flat responses and memory models with frequency-selective responses. Memoryless power amplifier models are characterized by their amplitude-to-amplitude (AM/AM) and amplitude-to-phase (AM/PM) conversions. Some examples of this kind of models are [16]: travelling wave tube amplifier (TWTA) model, solid state power amplifier (SSPA) model, and soft-envelope limiter (SEL) model. Examples of memory models are the Volterra series (such as Wiener, Hammerstein, Wiener-Hammerstein and memory polynomial models). In this thesis, a simple third order memoryless polynomial model will be used.

As it is shown in [17] a baseband power amplifier can be modeled as:

$$p(t) = \sum_{k=0}^{K} \alpha_k |b(t)|^{2k} b(t), \tag{2.1}$$

where $b(t)$ is the baseband PA input signal, $y(t)$ is the baseband PA output signal, and α_k are complex-valued coefficients that can be extracted using standard linear regression methods [18]. The highest nonlinearity order is 2K+1 . The fact that only odd-order nonlinear terms appear in Eq. 2.1 is attributed to the bandpass nonlinear nature of the PA [17].

2.2 Probability

Next some of the probability basic terms required to understand this thesis will be shortly defined. Nearly all definitions and properties can be consulted in [19]. Nevertheless, it will be pointed out when another source is used.

2.2.1 Definitions

Definition 1 *Expected value or expectation.*

$$E(X) = \sum_{x \in \Omega} x \cdot m(x), \tag{2.2}$$

Definition 2 *Variance or power. Denoted by σ_X^2,*

$$\sigma_X^2 = E[|X - E(X)|^2], \tag{2.3}$$

2.2.2 Theorems

Theorem 1 *Moment theorem [20]. Suppose $x_n, = x(t_n)$ for (n = 1, 2, ... N) are samples from zero-mean, complex Gaussian process x(t).*

a) If $s \neq t$, then

$$E[x_1 x_2 \cdots x_s x_1^* x_2^* \cdots x_t^*] = 0 \tag{2.4}$$

b) If $s = t$, then

$$E[x_1 x_2 \cdots x_s \; x_1^* x_2^* \cdots x_t^*] = \sum_n \left(E[x_1 x_1^*]\right) \left(E[x_2 x_2^*]\right) \cdots \left(E[x_t x_t^*]\right) \tag{2.5}$$

Proof of this can be consulted in [20].

2.3 Channel and precoding

In order to accomplish a model as realistic as possible, *communication channel* and *precoding* concepts are included in the system diagram. The communication channel is merely the medium used to transmit the signal from a transmitter to a receiver. It may be a pair of wires, a coaxial cable, a band of radio frequencies, a beam of light, etc. [21]. It is important that it is an shared medium so it will cause interferences and other problems. Furthermore, MIMO techniques require orthogonal channels in order to work. This means, in the symmetrical MxM case, that rich scattering is needed, whereas in the asymmetrical case, it may be created using many antennas at the base-station [22]. There are different models of channel such as AWGN, Gaussian channels, Rayleigh channels, propagation models...

The principal complication in multi-antenna channels is the separation of the data streams which are sent in parallel. Precoding or pre-equalization, a key element of MIMO techniques, is one way to mitigate the interference between different data streams. It is a generalization of beamforming that can support transmission for multiple streams in multi-antenna communication systems. Beamforming is a powerful technique to increase the link signal-to-noise ratio (SNR) through focusing the energy into desired direction. It is achieved when the transmit antennas are appropriately weighted in gain and phase for each transmission stream. In the case of multiple data streams, precoding generally combines the streams in orthogonal directions using weighting matrices according to the channel distribution. This type of processing at the transmitter requires the Channel State Information (CSI) at the transmitter. In order to be able to obtain CSI at the transmitter, the channel should be fixed (non-mobile) or approximately constant over a reasonably large time period, known as *channel coherence time*. If CSI is available at the transmitter, the transmitted symbols, either for a single-user or for multiple users, can be partially separated by means of pre-equalization at the transmitter. In this thesis, perfect-channel knowledge is assumed.

3

Single-Antenna model

In this chapter the model that acts as starting point of our project is described. This single-antenna model that accounts for the distortion of the PA by probabilistic calculations is explained in detail since it is vital a complete understanding to comprehend the work carried out in this master thesis.

3.1 System description

As mentioned in section 2.1, a complex baseband model represented by a third order polynomial will be used, Eq. (3.1). Without loss of generality, the parameter for the linear term may be normalized to 1, leaving it as a simple single-parameter model.

$$p(t) = b(t) + \alpha b(t)|b(t)|^2 = \left(|b(t)| + \alpha |b(t)|^3\right) e^{j\angle b(t)} \tag{3.1}$$

For strong non-linearities or memory effects, one should consider using higher order terms and memory taps. However, for the purpose of this study, a simple third order model is good enough. The approximation of the amplifier with its 3rd order Taylor expansion is the only approximation the authors made; in the following, the equations are exact. The block diagram of the system is shown in Figure 3.1 where the channel response is considered as 1.

Being x_n the input symbol to the system, the output symbol z_n will be derived mathematically. First, the input signal is pulse shaped, crucial to limit the effective bandwidth of the transmission as well as keeping the intersymbol interference (ISI) in control [23]. The pulse shaping filter used in Eriksson and Mazzali's work and in this thesis is Root-Raised-Cosine filter (RRC), a double-parameter model: T which is the symbol period and β, so called roll-off factor, which is the excess bandwidth divided by half of the symbol rate. Filtering is shown in Eq. (3.2) where the pulse shaping filter is denoted by $e(t)$.

$$b(t) = \sum_{k=-\infty}^{\infty} x_k e(t - kT) \tag{3.2}$$

The output from the amplifier, Eq. (3.1), is contaminated with Additive White Gaussian Noise (AWGN) before the matched filter. This front-end filter with impulse

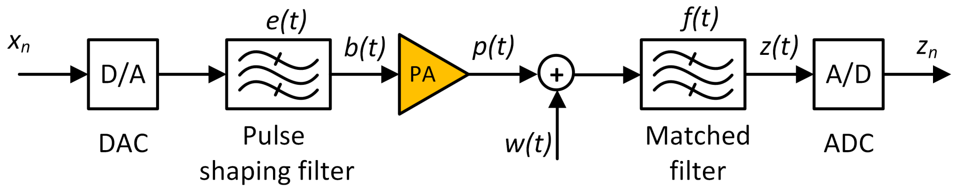


Figure 3.1: Block diagram of the transceiver which contains a non-linear RF power amplifier (PA).

response $f(t)$ and normalized power, Eq. (3.3), constitutes the last step before the sampling proccess, Eq. (3.4).

$$z(t) = (y(t) + w(t)) * f(t) = \int_{-\infty}^{\infty} (y(s) + w(s))\, f(t-s)ds \tag{3.3}$$

$$\begin{aligned}
z_n &= z(t)\Big|_{t=nT} = \\
&= \int_{-\infty}^{\infty} \Big(b(s) + \alpha b(s)|b(s)|^2 + w(s)\Big)\, f(nT-s)ds = \\
&= \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} x_k \int_{-\infty}^{\infty} e(s-kT)f(nT-s)ds + \\
&+ \int_{-\infty}^{\infty} \alpha \sum_{k=-\infty}^{\infty} x_k e(s-kT) \left| \sum_{l=-\infty}^{\infty} x_l e(s-lT) \right|^2 f(nT-s)ds + \\
&+ \int_{-\infty}^{\infty} w(s)f(nT-s)ds
\end{aligned} \tag{3.4}$$

Applying the property of the matched filter in Eq. (3.5), it is seen that in addition to the desired symbol x_n and the additive white noise w_n, there is a distortion term d_n due to the non-linearity. In the following section, the distortion term will be separately studied and derive its statistical properties derived.

$$z_n = x_n + w_n + \underbrace{\int_{-\infty}^{\infty} \alpha \sum_{k=-\infty}^{\infty} x_k e\left(\frac{s}{T} - k\right) \left| \sum_{l=-\infty}^{\infty} x_l e\left(\frac{s}{T} - l\right) \right|^2 f\left(n - \frac{s}{T}\right) ds}_{d_n} \tag{3.5}$$

For the calculations, some symmetry properties of the constellation of the input x_n will be assumed. First, it is assumed that the real and the imaginary parts are both symmetric around zero. Second, real and imaginary will be assumed symmetric, so that both parts have the same statistics. Both those symmetry aspects are fulfilled by common single-carrier constellations such as M-QAM and M-PSK, and by any OFDM modulation.

3.2 Statistical amplifier analysis

The authors identify the distortion term, d_n, in Eq. (3.5). The objective now will be to derive the statistical properties of this term and see how the non-linearity affects to its behaviour. Applying a time translation and doing some rearrangement gives Eq. (3.6).

$$\begin{aligned} d_n &= \int_{-\infty}^{\infty} \alpha \sum_{k=-\infty}^{\infty} x_k e(s-kT) \left| \sum_{l=-\infty}^{\infty} x_l e(s-lT) \right|^2 f(nT-s) ds = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x_k x_l x_m^* \int_{-\infty}^{\infty} e\left(\frac{s}{T}-k\right) e\left(\frac{s}{T}-l\right) e\left(\frac{s}{T}-m\right) f\left(\frac{s}{T}-n\right) ds = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x_{n-k} x_{n-l} x_{n-m}^* \underbrace{\int_{-\infty}^{\infty} e\left(\frac{s}{T}-k\right) e\left(\frac{s}{T}-l\right) e\left(\frac{s}{T}-m\right) f\left(\frac{s}{T}\right) ds}_{\lambda_{klm}} = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} x_{n-k} x_{n-l} x_{n-m}^* \end{aligned} \tag{3.6}$$

Studying the equation, it is observed that the distortion term is a linear function of α, and depends on the third order statistics of the transmitted symbol sequence. Particularly, the distortion term depends on the value of the current transmitted symbol x_n, which will be utilized in the following. The parameter λ_{klm} is entirely a function of the pulse shaping/matched filters. It has its highest values when all three parameters k, l, m are close to zero and it is invariant with respect to index permutations, i.e., so that $\lambda_{klm} = \lambda_{lmk}$ etc.. For a filter with long impulse response (such as root-raised cosine with low roll-off β), λ_{klm} decays slowly with k, l, m and many transmitted symbols x_{n-k} affects the current distortion term d_n. In the following, the statistics of d_n for a given transmitted symbol x_n will be studied, but where the rest of the transmitted symbols x_{n-k}, $k \neq 0$, are regarded as (independent) random variables with variance σ_x^2 and mean zero. This therm can be rewritten in polar notation as in Eq. (3.7).

$$d_n = \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} |x_{n-k}||x_{n-l}|\left|x_{n-m}^*\right| e^{j(\angle x_{n-k} + \angle x_{n-l} - \angle x_{n-m})} \tag{3.7}$$

Focusing on the n-th sample of the distortion term, d_n, the interest at this stage will be in determining how the other symbols of the constellation contribute to contaminate the current symbol, x_n. In order to isolate the terms depending on the magnitude of the current symbol x_n, d_n can be modeled with several coefficients, c_i, which are independent of x_n. Note that these coefficients are time-varying random processes $c_i(n)$, but the time index has been omitted to simplify the notation.

Coefficients c_0, c_1, c_2 and c_3 represent the four possibilities of this third-order expression: none of the symbols are equal to x_n (coefficient c_0), just one of the symbols are equal to x_n (coefficient c_1), two of them are equal to x_n (coefficient c_2) and, finally, the tree of them are x_n (coefficient c_3).

$$d_n = c_0 + c_1|x_n| + c_2|x_n|^2 + c_3|x_n|^3 \tag{3.8}$$

where,

$$c_0 = \alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k} x_{n-l} x_{n-m}^* \tag{3.9}$$

$$c_1 = \alpha e^{-j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k} x_{n-l} + 2\alpha e^{j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k} x_{n-l}^* \tag{3.10}$$

$$c_2 = \alpha e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^* + 2\alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k} \tag{3.11}$$

$$c_3 = \alpha \lambda_{000} e^{j\angle x_n} \tag{3.12}$$

In the following, coefficient properties are studied separately and calculations are explained to ease approaching this complex theoretical problem and, in this way, comprehend key concepts of the thesis. It is important to emphasize that from now on, in all the statistical analysis, the random variable is x_{n-k} (or x_{n-l}, x_{n-m}, ...) which is independent from the current symbol, x_n, and from there, the importance of knowing how closest symbols affect it.

Starting off with the first coefficient, c_0, calculation of its expectation and variance is tackled. This term is the sum of many independent components and can be approximated as Gaussian, with mean given by Eq. (3.13). Firstly, it was exploited the fact that symbols are zero mean and uncorrelated, which means that only when the symbols experiment the same delay their contribution to a expected value will be non-zero, and that $E\left[x_{n-k}|x_{n-k}|^2\right] = 0$ due to the symmetry properties according to Theorem 1.

$$\begin{aligned} E\left[c_0\right] &= \alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k} x_{n-l} x_{n-m}^*\right]}_{uncorrelated} = \\ &= \alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kkk} \underbrace{E\left[x_{n-k}|x_{n-k}|^2\right]}_{symmetry \Rightarrow 0} = 0 \end{aligned} \tag{3.13}$$

The variance is calculated in Eq. (3.14). It can be observed that it scales linearly with $|\alpha|^2$ (the severity of the nonlinearity) and with σ_x^6, depends on the pulse-shaping filter (through λ_{klm}, depends on the moments of order 4 and 6 of the magnitude of the normalized transmit constellation, $E\left[|x_{n-k}/\sigma_x|^4\right]$ and $E\left[|x_{n-k}/\sigma_x|^6\right]$.

The latter implies that a high peak-to-average-power-ratio (PAPR), leading to a high sixth order moment, cause a higher variance of this term.

$$\begin{aligned}
E\left[|c_0|^2\right] &= |\alpha|^2 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\sum_{\substack{q=-\infty\\q\neq 0}}^{\infty}\lambda_{klm}\lambda_{opq}\underbrace{E\left[x_{n-k}x_{n-l}x^*_{n-m}x^*_{n-o}x^*_{n-p}x_{n-q}\right]}_{uncorrelated} = \\
&= |\alpha|^2 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\lambda^2_{klm}E\left[|x_{n-k}|^2|x_{n-l}|^2|x_{n-m}|^2\right] = \\
&= |\alpha|^2\sigma_x^6 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\lambda^2_{klm}\underbrace{E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2\left|\frac{x_{n-l}}{\sigma_x}\right|^2\left|\frac{x_{n-m}}{\sigma_x}\right|^2\right]}_{\substack{\text{constellation-dependent parameter}\\\text{(independent of transmit power }\sigma_x^2)}}
\end{aligned} \tag{3.14}$$

Therefore, the analytical expression of the variance of c_0 enables optimization of transmit power, pulse-shaping filter and constellation with respect to nonlinear distortion. Furthermore, it gives us a better tool than peak-to-average-power-ratio to judge the severity of the nonlinear effects for a given constellation, which is generally useful.

Applying the same procedure to coefficient c_1, Eq. (3.15), it is discovered that it is not a zero-mean RV. 2^{nd} line in the equation is due to uncorrelated consecutive symbols, and the 3^{rd} line is due to symmetry requirements on the constellation. The power of c_1 can be computed as in Eq. (3.16).

$$\begin{aligned}
E\left[c_1\right] &= \alpha e^{-j\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}\underbrace{E\left[x_{n-k}x_{n-l}\right]}_{uncorrelated} + 2\alpha e^{j\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}\underbrace{E\left[x_{n-k}x^*_{n-l}\right]}_{uncorrelated} = \\
&= \alpha e^{-j\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{kk0}\underbrace{E\left[x^2_{n-k}\right]}_{symmetry\Rightarrow 0} + 2\alpha e^{j\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{kk0}E\left[|x_k|^2\right] = \\
&= 2\alpha\sigma_X^2 e^{j\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{kk0}
\end{aligned} \tag{3.15}$$

$$
\begin{aligned}
E\left[|c_1|^2\right] = E&\left[\left|\alpha e^{-j\angle x_n}\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\lambda_{kl0}x_{n-k}x_{n-l} + 2\alpha e^{j\angle x_n}\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\lambda_{kl0}x_{n-k}x^*_{n-l}\right|^2\right] = \\
&= |\alpha|^2\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}\left(x_{n-o}x_{n-p}\right)^*\right] + \\
&+ 4|\alpha|^2\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x^*_{n-l}\left(x_{n-o}x^*_{n-p}\right)^*\right] + \\
&+ 2|\alpha|^2e^{-j2\angle x_n}\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq0}}^{\infty}\lambda_{kl0}\lambda_{op0}\underbrace{E\left[x_{n-k}x_{n-l}\left(x_{n-o}x^*_{n-p}\right)^*\right]}_{symmetry\Rightarrow0} + \\
&+ 2|\alpha|^2e^{j2\angle x_n}\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq0}}^{\infty}\lambda_{kl0}\lambda_{op0}\underbrace{E\left[\left(x_{n-k}x_{n-l}\right)^*x_{n-o}x^*_{n-p}\right]}_{symmetry\Rightarrow0} = \\
&= 5|\alpha|^2\sigma_x^4\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\lambda_{kl0}^2E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2\left|\frac{x_{n-l}}{\sigma_x}\right|^2\right]
\end{aligned}
\tag{3.16}
$$

Since c_1 does not have mean zero, it is convenient to decompose it into a constant (i.e., the mean) and a zero-mean variable, according to Eq. (3.17). The new variable will have zero mean and a variance given by Eq. (3.19).

$$
\begin{aligned}
c_1' &= c_1 - E\left[c_1\right] = \\
&= \alpha e^{-j\angle x_n}\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\lambda_{kl0}x_{n-k}x_{n-l} + 2\alpha e^{j\angle x_n}\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\lambda_{kl0}x_{n-k}x^*_{n-l} - \\
&- \left[2\alpha\sigma_X^2e^{j\angle x_n}\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\lambda_{kk0}\right]
\end{aligned}
\tag{3.17}
$$

$$
E\left[c_1'\right] = 0 \tag{3.18}
$$

$$
E\left[|c_1'|^2\right] = 5|\alpha|^2\sigma_x^4\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq0}}^{\infty}\left(\lambda_{kl0}^2E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2\left|\frac{x_{n-l}}{\sigma_x}\right|^2\right] - \lambda_{kk0}\lambda_{ll0}\right) \tag{3.19}
$$

With the same procedure as before c_2 and c_3 expectations and variances can be extracted.

$$E\left[c_2\right] = \alpha e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x^*_{n-k}\right] + 2\alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}\right] = 0 \tag{3.20}$$

$$\begin{aligned} E\left[|c_2|^2\right] &= E\left[\left|\alpha e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x^*_{n-k} + 2\alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}\right|^2\right] = \\ &= 5|\alpha|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda^2_{k00} E\left[|x_{n-k}|^2\right] = 5|\alpha|^2 \sigma_x^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda^2_{k00} \end{aligned} \tag{3.21}$$

$$E\left[c_3\right] = \alpha \lambda_{000} e^{j\angle x_n} \tag{3.22}$$

$$E\left[|c_3|^2\right] = 0 \tag{3.23}$$

To conclude, a depiction of the four coefficients is made in Figure 3.2 to study their behaviour separately. On one hand, it is perceived that c_1, more strongly, and c_2 mark the non-linear shape in the constellation mainly visible in the outer points of the constellation. On the other, hand c_3 seems to contribute directly to a slight compression of the signal. Finally, c_0 present a totally linear appearence so it may be conclude that it does not produce non-linear effects.

3.3 Statistical model

After analyzing the different coefficients, the authors finalize their work proposing the model in Eq. (3.24).

$$z_n = x_n \left(1 + u^{(1)} + u^{(3)} \frac{|x_n|^2}{\sigma_x^2}\right) + v_n^{(0)} + v_n^{(1)} \frac{|x_n|}{\sigma_x} + v_n^{(2)} \frac{|x_n|^2}{\sigma_x^2} + w_n \tag{3.24}$$

Where $u^{(i)}$ terms are constants which non-linearly rescale and rotate the constellation, and $v^{(i)}$ are random variables with mean zero. The variances of $v^{(i)}$ terms are functions of σ_x^6, which indicates a very strong scaling with the signal power. Further, they are a function of the severity of the nonlinearity $|\alpha|^2$, of the pulse shaping/matched filters as given by λ_{klm}, and of the normalized moments of the constellation (e.g. PAPR).

$$u^{(1)} = 2\alpha\sigma_X^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \tag{3.25}$$

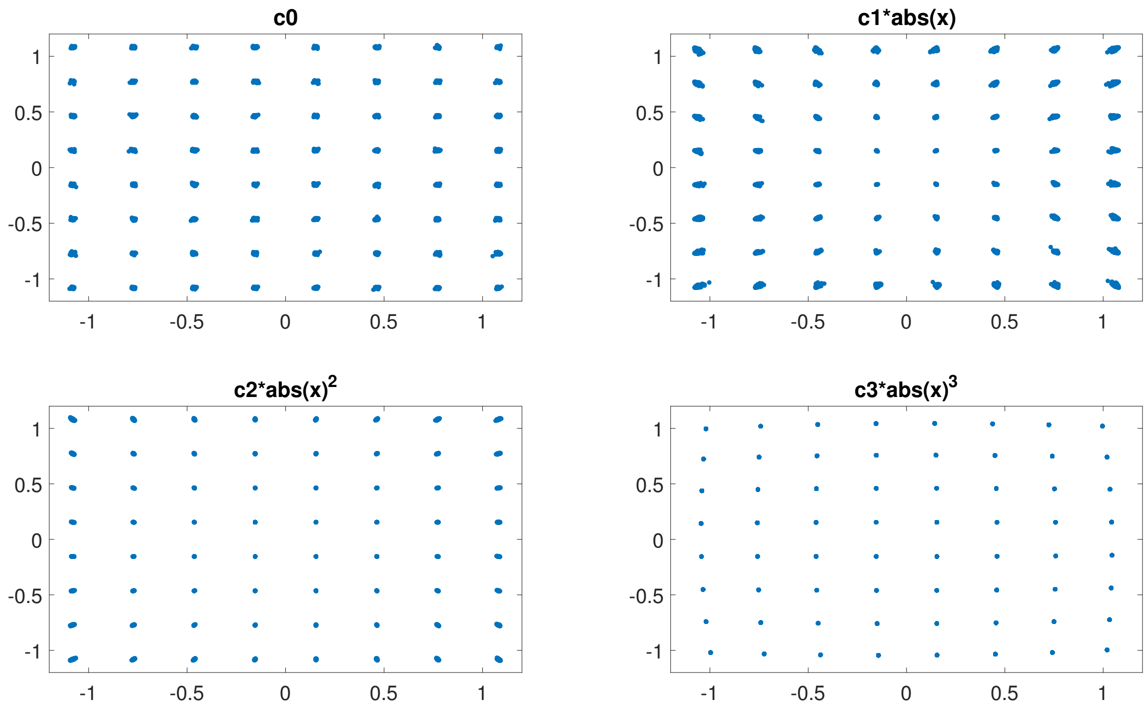


Figure 3.2: Simulation of the model coefficients over a 64-QAM constellation.

$$u^{(3)} = \alpha \sigma_x^2 \lambda_{000} \tag{3.26}$$

$$E\left[\left|v_n^{(0)}\right|^2\right] = |\alpha|^2 \sigma_x^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2 \left|\frac{x_{n-l}}{\sigma_x}\right|^2 \left|\frac{x_{n-m}}{\sigma_x}\right|^2\right] \tag{3.27}$$

$$E\left[\left|v_n^{(1)}\right|^2\right] = 5|\alpha|^2 \sigma_x^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2 \left|\frac{x_{n-l}}{\sigma_x}\right|^2\right] - \lambda_{kk0}\lambda_{ll0}\right) \tag{3.28}$$

$$E\left[\left|v_n^{(2)}\right|^2\right] = 5|\alpha|^2 \sigma_x^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{3.29}$$

The authors have shown that the pulse-shaping filters and transmit power can be optimized using the new statistical framework and that new optimal constellations can be designed. New analytical expressions have been found for how the distortion scales both with the (cube of the) average signal power, and with the instantaneous power. Furthermore, an analytical expression for nonlinear constellation rescaling is found. These expressions will be useful to understand the effects of amplifiers in various communication scenarios, and to optimize constellations, amplifiers and matched filters to minimize such effects.

4

Multi-Antenna model

In this chapter, the path from a single-antenna model to a multi-antenna model is drawn. The aim is to design a scenario that connects a multitude of transmission antennas to different users. The model structure designed is similar to the SISO (Single Input Single Output) case except for the channel and precoder blocks placed in order to get the most realistic model possible. Nevertheless, due a matter of size, not all the calculations are included in this chapter. Instead, they will be placed in different Appendixes, each one indicated when needed. In the following, a matrix version of the equations will be shown.

4.1 2x2-Antenna model

The goal of this section is to develop a statistical model capable of describing the received signal as a decomposition from different moments of x_n together with the filter contribution (as done in the SISO model) but adding channel and precoder participation. Hence, the model will start off approaching the simplest case: 2 transmit antennas and 2 users. At this point, the communication channel H defined as in Eq. (4.1) due to the necessity of immediate reference. The block diagram depicted in Figure 4.1 may be very helpful to comprehend the process described in the following. In the following equations of this chapter, both channel and precoding algorithms will be kept general, leaving to the choice of the simulator chose the ones preferred.

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{4.1}$$

In the same manner as in the single-antenna model, the input signal x_n is first lead to a pulse shaping filter e_n resulting in a new signal a_n, Eq. (4.2), where $\circledast$ is defined as element-wise convolution.

$$\mathbf{a} = \mathbf{x} \circledast e_n = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \circledast e_n = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{4.2}$$

Afterwards, the precoding is performed by multiplication with matrix G. The objective of the precoder is to help retrieving the original information which, due to presence of the channel, will be blended in the channel later on. There are several ways to approach this mission. Due to the simplicity of calculation of the inverse

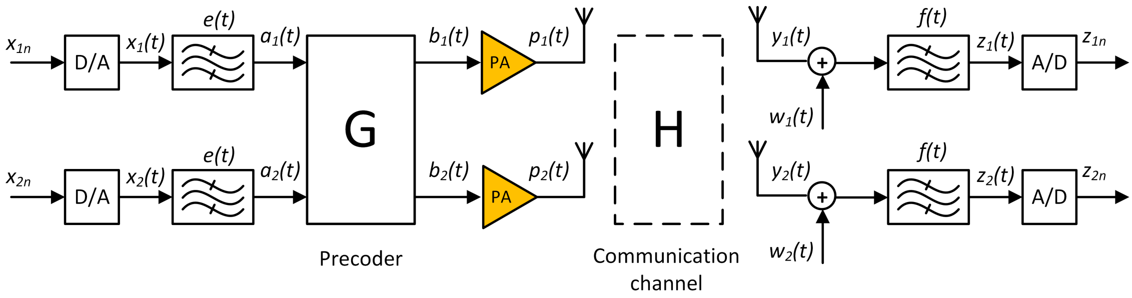


Figure 4.1: Block diagram of the system consisting of 2 antennas and 2 users. Precoder and channel blocks were added.

matrix of a square 2x2 matrix, which will guarantee a perfect recovery, the inverse matrix will be used as precoder, Eq. (4.3).

$$\mathbf{G} = \mathbf{H}^{-1} = \frac{1}{\sqrt{2}} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \frac{1}{\sqrt{2}} \cdot \frac{1}{h_{11}h_{22} - h_{12}h_{21}} \cdot \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix} \tag{4.3}$$

Signal precoding produces a new signal b_n, Eq. (4.4), which goes directly to the process of interest in this thesis, the non-linear amplification, Eq. (4.5), before passing through the channel, Eq. (4.6).

$$\mathbf{b} = \mathbf{G}\mathbf{a} = \frac{1}{\sqrt{2}} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{4.4}$$

$$\mathbf{p} = \begin{bmatrix} b_1 + \alpha_1 b_1 |b_1|^2 \\ b_2 + \alpha_2 b_2 |b_2|^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{4.5}$$

$$\mathbf{y} = \mathbf{H}\mathbf{p} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{4.6}$$

Finally, the received signal goes to the matched filter f producing the output signal z, Eq. (4.7).

$$\mathbf{z} = [\mathbf{y} + \mathbf{w}] \circledast f_n = \left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \right) \circledast f_n = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{4.7}$$

In the output signal z, an equal number of signals to the number of users will be obtained. From this moment on, the signal for User 1 z_1 will be the center of attention since calculation for other users can be achieved similarly. Deriving all the equations above, which can be consulted in Appendix B, the final output signal for User 1 can be written as in Eq. (4.8) where the distortion term has been shortened by d_{1n}. In Eq. (4.9) all the terms of the distortion that appear from the cross-products are shown.

$$z_{1n} = x_{1n} + w_{1n} + d_{1n} \tag{4.8}$$

$$\begin{aligned} d_{1n} = \frac{1}{2^2} \Bigg[& \underbrace{\lambda_{klm} L_1 x_1 x_1 x_1^*}_{d_{x_{1n}}} + \underbrace{\lambda_{klm} L_2 x_2 x_2 x_2^*}_{d_{x_{2n}}} + \underbrace{\lambda_{klm} L_3 x_1 x_2 x_2^*}_{d_{x_{1n} x_{2n} x_{2n}^*}} + \\ & + \underbrace{\lambda_{klm} L_4 x_1 x_1 x_2^*}_{d_{x_{1n} x_{1n} x_{2n}^*}} + \underbrace{\lambda_{klm} L_5 x_1 x_1^* x_2}_{d_{x_{1n} x_{1n}^* x_{2n}}} + \underbrace{\lambda_{klm} L_6 x_1^* x_2 x_2}_{d_{x_{1n}^* x_{2n} x_{2n}}} \Bigg] \end{aligned} \tag{4.9}$$

where L_4, as representative of L_i terms is shown in Eq. (4.10). The evolution of term L_4 with the number of antennas and users will be shown in next sections in order to compare.

$$L_4 = \alpha_1 * h_{11}(t) * g_{12}^*(t) * (g_{11}(t))^2 + \alpha_2 * h_{12}(t) * (g_{21}(t))^2 * g_{22}^*(t) \tag{4.10}$$

The next task will be to analyze each one of the terms in Eq. (4.9) as it was done with the only term in the single-antenna model. Each one of the terms must be treated separately knowing that two kind of terms are present: either terms where there is presence of signal from only one user (2 terms, one coming from User 1 and another from User 2) or terms where information coming from both users are combined. For the first kind of terms, the statistical analysis is already known and they will be modeled as in the SISO model. For the second kind, the statistical analysis is carried out. As an example of this, calculations on coefficient c_{13} are shown, Eqs. (4.11) to (4.13), where it can be noticed that due to independence between signals from different users third order statistic moments will never appear. Remainder calculations of this model can be found in Appendix B.

$$c_{13} = L_4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(1)} x_{n-l}^{(1)} x_{n-m}^{(2)^*} \tag{4.11}$$

$$\begin{aligned} E\left[c_{13}\right] &= L_4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(1)} x_{n-m}^{(2)^*}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= L_4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kkl} \underbrace{E\left[\left(x_{n-k}^{(1)}\right)^2\right]}_{0} \underbrace{E\left[x_{n-l}^{(2)^*}\right]}_{0} = 0 \end{aligned} \tag{4.12}$$

$$
\begin{aligned}
E\left[\left|c_{13}\right|^2\right] &= \left|L_4\right|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(1)}\right|^2 \left|x_{n-m}^{(2)}\right|^2\right] = \\
&= \left|L_4\right|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(1)}\right|^2\right] E\left[\left|x_{n-m}^{(2)}\right|^2\right] = \\
&= \left|L_4\right|^2 \sigma_{x^{(1)}}^4 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-m}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right]
\end{aligned} \tag{4.13}
$$

Finally, when the analysis of all terms is ready, it is possible to write an equation model alike to the single-antenna model of Eriksson and Mazzali as in Eq. (4.14). The strategy of arrangement of the coefficients obtained has been to put together those terms which are multiplying the same signal magnitude.

$$
\begin{aligned}
z_{1n} = {} & x_{1n}\left(1 + u^{(1)} + u^{(9)} + u^{(3)}\frac{\left|x_{1n}\right|^2}{\sigma_{x_1}^2}\right) + \left(v_n^{(1)} + v_n^{(9)} + v_n^{(14)} + v_n^{(19)} + v_n^{(24)}\right)\frac{\left|x_{1n}\right|}{\sigma_{x_1}} + \\
& + \left(v_n^{(2)} + v_n^{(15)} + v_n^{(20)}\right)\frac{\left|x_{1n}\right|^2}{\sigma_{x_1}^2} + x_{2n}\left(u^{(5)} + u^{(21)} + u^{(7)}\frac{\left|x_{2n}\right|^2}{\sigma_{x_2}^2}\right) + \\
& + \left(v_n^{(5)} + v_n^{(10)} + v_n^{(16)} + v_n^{(21)} + v_n^{(25)}\right)\frac{\left|x_{2n}\right|}{\sigma_{x_2}} + \left(v_n^{(6)} + v_n^{(11)} + v_n^{(26)}\right)\frac{\left|x_{2n}\right|^2}{\sigma_{x_2}^2} + \\
& + \left(u^{(22)} + u^{(27)}\right)\frac{\left|x_{1n}\right|^2}{\sigma_{x_1}^2}\frac{\left|x_{2n}\right|}{\sigma_{x_2}} + \left(u^{(12)} + u^{(17)}\right)\frac{\left|x_{1n}\right|}{\sigma_{x_1}}\frac{\left|x_{2n}\right|^2}{\sigma_{x_2}^2} + \\
& + v_n^{(0)} + v_n^{(4)} + v_n^{(8)} + v_n^{(13)} + v_n^{(18)} + v_n^{(23)} + w_{1n}
\end{aligned} \tag{4.14}
$$

The most notable aspect of the system equation obtained is the existence of Multi-User Interference (MUI) in the distortion term. This term cannot be eliminated with any precoder since it comes from the non-linearity of the power amplifiers.

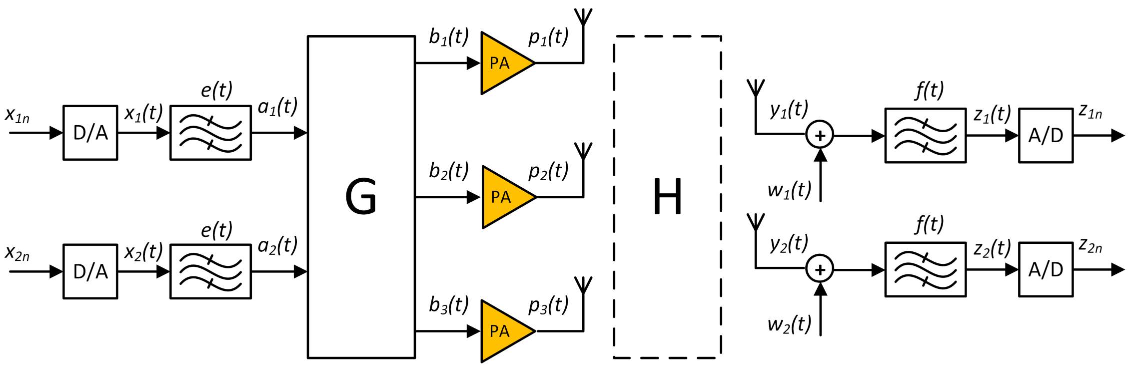


Figure 4.2: Block diagram of the system consisting of 3 antennas and 2 users.

4.2 3x2-Antenna analysis

Adding a new antenna to the scenario (Figure 4.2) will cause several variations in the equation process. Firstly, the precoded data matrix b will hold 3 different signals instead of 2. Therefore, from this moment until the very end of the equation procedure three signals will be combined instead of two.

These facts are translated in a distortion signal for the User 1 which has the same six terms as in the 2x2 model but the contribution of channel and precoder is greater due to the presence of the third power amplier, Eqs. (4.15) and (4.16). All the calculations derived for this model can be consulted in Appendix C.

$$
\begin{aligned}
d_{1n} = \frac{1}{3^2} \Bigg[& \underbrace{\lambda_{klm} L_1 x_1 x_1 x_1^*}_{d_{x_{1n}}} + \underbrace{\lambda_{klm} L_2 x_2 x_2 x_2^*}_{d_{x_{2n}}} + \underbrace{\lambda_{klm} L_3 x_1 x_2 x_2^*}_{d_{x_{1n} x_{2n} x_{2n}^*}} + \\
& + \underbrace{\lambda_{klm} L_4 x_1 x_1 x_2^*}_{d_{x_{1n} x_{1n} x_{2n}^*}} + \underbrace{\lambda_{klm} L_5 x_1 x_1^* x_2}_{d_{x_{1n} x_{1n}^* x_{2n}}} + \underbrace{\lambda_{klm} L_6 x_1^* x_2 x_2}_{d_{x_{1n}^* x_{2n} x_{2n}}} \Bigg]
\end{aligned}
\tag{4.15}
$$

where, for this case, L_4 is

$$
\begin{aligned}
L_4 = \alpha_1 h_{11}(t) * (g_{11}(t))^2 * g_{12}^*(t) + \alpha_2 h_{12}(t) * (g_{21}(t))^2 * g_{22}^*(t) + \\
+ \alpha_3 h_{13}(t) * (g_{31}(t))^2 * g_{32}^*(t)
\end{aligned}
\tag{4.16}
$$

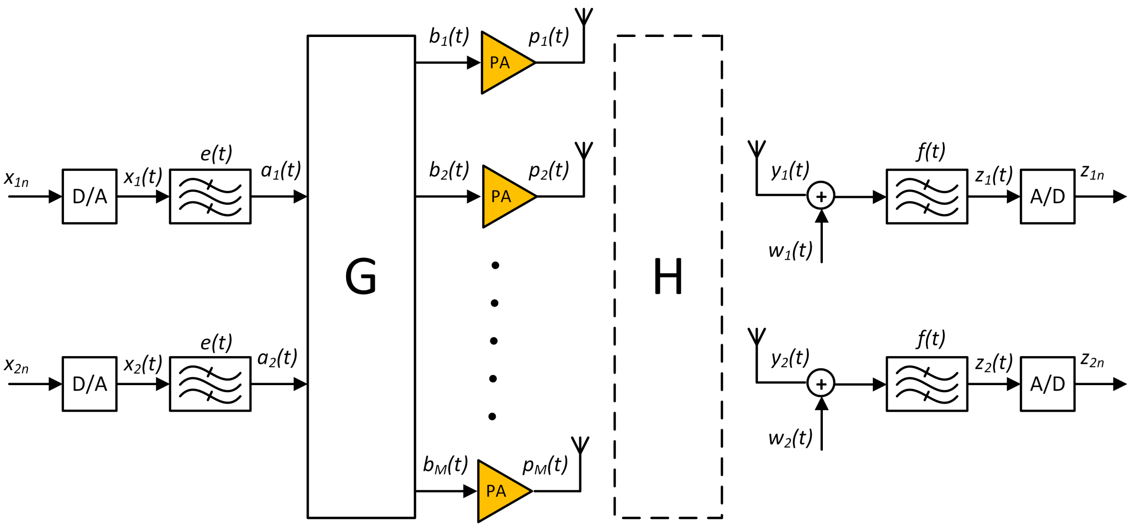


Figure 4.3: Block diagram of the system consisting of M antennas and 2 users.

4.3 Mx2-Antenna analysis

At this moment, it is known how the model is affected by an increment of transmit antennas. Extending this reasoning to a general case where the number of antennas is M and keeping the number of users in 2, it is seen that the number of signals involved in the process will be M (Figure 4.3). Following the same procedure as before, a distortion signal for the User 1 is reached, Eq. (4.17). Once again, the same number of products between signals emerges but contribution from channel and precoder is now a sum over M term for each product, Eq. (4.18). All the calculations derived for this model can be consulted in Appendix D.

$$d_n = \frac{1}{M^2}\Bigg[\underbrace{\lambda_{klm}L_1x_1x_1x_1^*}_{d_{x_{1n}}} + \underbrace{\lambda_{klm}L_2x_2x_2x_2^*}_{d_{x_{2n}}} + \underbrace{\lambda_{klm}L_3x_1x_2x_2^*}_{d_{x_{1n}x_{2n}x_{2n}^*}} + \\ + \underbrace{\lambda_{klm}L_4x_1x_1x_2^*}_{d_{x_{1n}x_{1n}x_{2n}^*}} + \underbrace{\lambda_{klm}L_5x_1x_1^*x_2}_{d_{x_{1n}x_{1n}^*x_{2n}}} + \underbrace{\lambda_{klm}L_6x_1^*x_2x_2}_{d_{x_{1n}^*x_{2n}x_{2n}}}\Bigg] \tag{4.17}$$

Where again L_4, as illustrative of L_i terms, is shown in Eq. (4.18).

$$L_4 = \sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{m1}(t))^2 * g_{m2}^*(t) \tag{4.18}$$

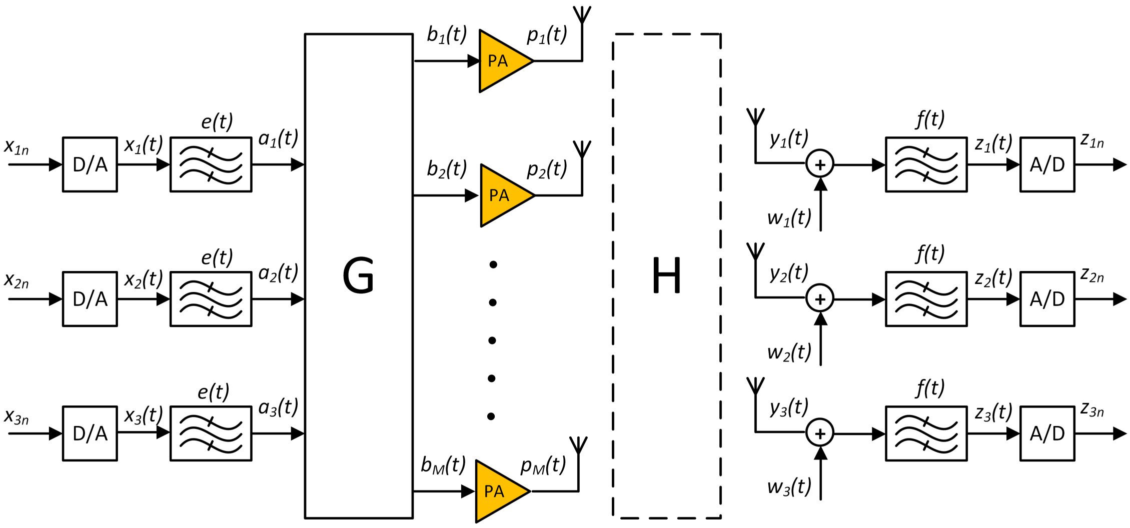


Figure 4.4: Block diagram of the system consisting of M antennas and 3 users.

4.4 Mx3-Antenna analysis

It is time now to discover how an increase in the number of users influences the system of equations built. This change is the most delicate in the equations since a new signal of information x_3 will take part in the process meaning an exponential growth of factors. This can be seen in the distorsion signal obtained Eq. (4.19). All the calculations derived for this model can be consulted in Appendix E.

$$
\begin{aligned}
d_1(t) = \frac{1}{M^2} \cdot \lambda_{rst} \Bigg[& \left(\sum_{k=1}^{3} x_k(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{mk}(t) |g_{mk}(t)|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} x_1(t) x_1(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{m1}(t))^2 * g_{mk}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} 2 \cdot x_1(t) x_1^*(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * |g_{m1}(t)|^2 * g_{mk}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} 2 \cdot x_1(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t) |g_{mk}(t)|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} x_1^*(t) x_k(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}^*(t) * (g_{mk}(t))^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} \sum_{\substack{i=1 \\ i\neq 1 \\ i\neq k}}^{3} 2 \cdot x_1(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t) * g_{km}(t) * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} \sum_{\substack{i=1 \\ i\neq 1 \\ i\neq k}}^{3} 2 \cdot x_1^*(t) x_k(t) x_i(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}^*(t) * g_{mk}(t) * g_{mi}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} \sum_{\substack{i=1 \\ i\neq 1 \\ i\neq k}}^{3} 2 \cdot x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{mk}(t))^2 * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k\neq 1}}^{3} \sum_{\substack{i=1 \\ i\neq 1 \\ i\neq k}}^{3} x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{mk}(t))^2 * g_{mi}(t) \right) \Bigg]
\end{aligned}
\tag{4.19}
$$

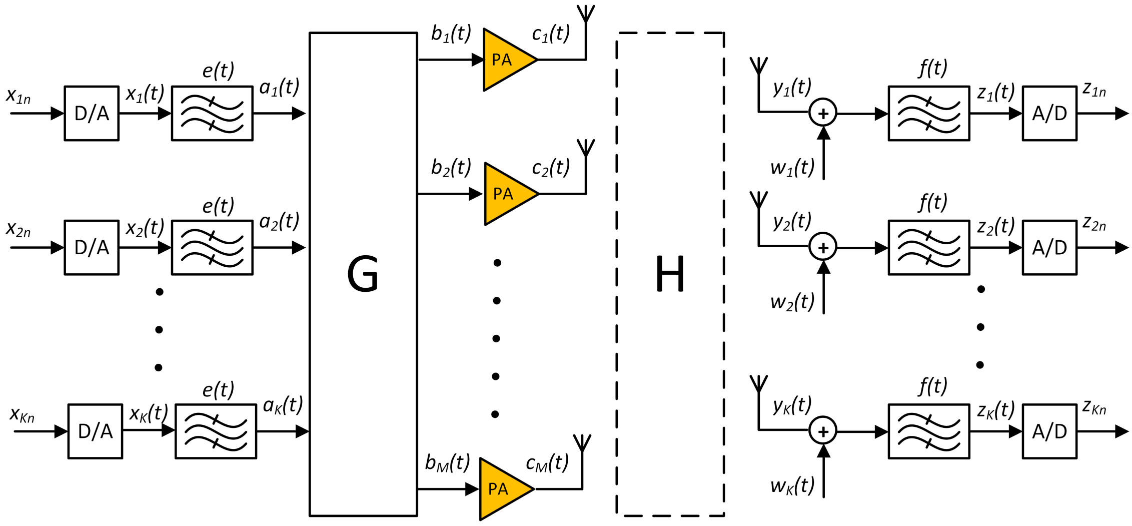


Figure 4.5: Final block diagram of the system consisting of M antennas and K users.

4.5 MxK-antenna model

Finally, in order to achieve a general model that describes a scenario formed by M transmit antennas and K users (always being $M \geq K$) the last step in the model is approached (Figure 4.5). With the information from the previous analysis, an equation of the distortion signal for the user of interest shown in Eq. (4.21) is reached. The user chosen for this from this moment on will be a generic User A. This means that all the users can be selected in the model in order to describe the distortion they are experiencing. All the calculations derived for this model can be consulted in Appendix F.

From Eq. (4.21) it is key to distinguish the number of different terms present. Due to the great difficulty in building an exact model containing the countless distortion terms from this scenario, an important decision was made: to consider only those who involve x_A as useful and the other as some kind of distortion, which can be seen as a big approximation. The latter terms will be gathered in a different signal as will be pointed out later on. To this matter, terms where there is any signal coming from User A will be separated from those where there is not presence of User A as it can be seen in Appendix F.

After a delicate process, analyzing and grouping the terms listed above separately, a final project equation is reached, Eq. (4.20). The remainder terms where User A is not involved are gathered in term m_{An}.

$$z_{An} = x_{An}\left[1 + U^{(1)} + U^{(2)}\frac{|x_{An}|}{\sigma_{x_A}} + U^{(3)}\frac{|x_{An}|^2}{\sigma_{x_A}^2}\right] + V^{(1)}\frac{|x_{An}|}{\sigma_{x_A}} + V^{(2)}\frac{|x_{An}|^2}{\sigma_{x_A}^2} + m_{An} + w_{An} \tag{4.20}$$

$$
\begin{aligned}
d_A(t) = \frac{1}{M^2} \cdot \lambda_{rst} \Bigg[& \left(\sum_{k=1}^{K} x_k(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{mk}(t) |g_{mk}(t)|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} x_A(t) x_A(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{m1}(t))^2 * g_{mk}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} 2 \cdot x_A(t) x_A^*(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * |g_{m1}(t)|^2 * g_{mk}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} 2 \cdot x_A(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t) |g_{mk}(t)|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} x_A^*(t) x_k(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}^*(t) * (g_{mk}(t))^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} 2 \cdot x_A(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t) * g_{km}(t) * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} 2 \cdot x_A^*(t) x_k(t) x_i(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}^*(t) * g_{mk}(t) * g_{mi}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} 2 \cdot x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{mk}(t))^2 * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{mk}(t))^2 * g_{mi}(t) \right) \Bigg]
\end{aligned}
\tag{4.21}
$$

Where $U^{(i)}$ are constants and $V^{(i)}$ are random variables with zero mean, Eqs. (4.22) to (4.26), result of grouping those coefficients which multiply the same magnitude of x_A.

$$U^{(1)} = 2 \cdot e^{j\angle x_n^{(A)}} \left[\sum_{k=1}^{K} \sigma_{x^{(k)}}^2 \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{ll0} + \lambda_{000} \left((K-1) + 2 \cdot \sum_{\substack{k=1 \\ k\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq k \\ j\neq A}}^{K} e^{j\left(\angle x^{(k)} - \angle x^{(j)}\right)} + \right.\right.$$
$$\left.\left. + \ e^{-j2\angle x^{(A)}} \sum_{\substack{k=1 \\ k\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq k \\ j\neq A}}^{K} e^{j\left(\angle x^{(k)} + \angle x^{(j)}\right)} \right)\right] \tag{4.22}$$

$$U^{(2)} = \lambda_{000} \left[e^{j2\angle x_n^{(A)}} \sum_{\substack{k=1 \\ k\neq A}}^{K} e^{-j\angle x_n^{(k)}} + e^{-j\angle x_n^{(A)}} \sum_{\substack{k=1 \\ k\neq A}}^{K} e^{j2\angle x_n^{(k)}} + 2 \cdot \sum_{\substack{k=1 \\ k\neq A}}^{K} e^{j\angle x_n^{(k)}} \right] \tag{4.23}$$

$$U_3 = \lambda_{000} e^{j\angle x_n^{(A)}} \tag{4.24}$$

$$E\left[\left|V^{(1)}\right|^2\right] = \sum_{k=1}^{K} \sigma_{x^{(k)}}^4 \sum_{\substack{r=-\infty \\ r\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(5 \cdot \lambda_{rl0}^2 E\left[\left| \frac{x_{n-r}^{(k)}}{\sigma_{x^{(k)}}} \right|^2 \left| \frac{x_{n-l}^{(k)}}{\sigma_{x^{(k)}}} \right|^2 \right] - 4 \cdot \lambda_{rr0}\lambda_{ll0} \right) + \sigma_{x^{(A)}}^4 \lambda_{rr0}\lambda_{ll0} +$$
$$+ \sum_{\substack{k=1 \\ k\neq A}}^{K} \sigma_{x^{(k)}}^2 \sum_{\substack{r=-\infty \\ r\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{rl0}^2 \left(8 \cdot \sigma_{x^{(A)}}^2 + 20 \cdot \sum_{\substack{j=1 \\ j\neq i \\ j\neq A}}^{K} \sigma_{x^{(j)}}^2 \right) + 40 \cdot \sum_{\substack{k=1 \\ k\neq A}}^{K} \sigma_{x^{(k)}}^2 \sum_{\substack{r=-\infty \\ r\neq 0}}^{\infty} \lambda_{r00}^2 \tag{4.25}$$

$$E\left[\left|V^{(2)}\right|^2\right] = 5 \cdot \sum_{k=1}^{K} \sigma_{x^{(k)}}^2 \sum_{\substack{r=-\infty \\ r\neq 0}}^{\infty} \lambda_{r00}^2 \tag{4.26}$$

5

Results

The time to verify the accuracy of the assumptions and ideas carried out along this master thesis has come. Aside from the theoretical model development, the remaining outcomes of this thesis are obtained by simulations using MATLAB, including tests over the mentioned models. First, different transmission scenarios are explored and simulated in order to study how non-linear distortion evolves when varying the number of antennas and users. Finally, results obtained for the different statistical models developed throughout the thesis are presented together with its deterministic simulation. All of this accompanied by analysis and explanation of the figures presented.

5.1 MIMO scenario simulation

From the beginning of this thesis it was believed that in a MIMO environment, the distortion from power amplifiers will become more linear if the number of transmit antennas is considerably larger than the number of users, i.e., a Massive MIMO scenario. In order to confirm this supposition, a multi-antenna simulation scenario is built. As indicated in previous chapters, power amplifiers are modelled on a third-order polynomial with a typical value of α.

Regarding the blocks inserted in the thesis, the communication channel H

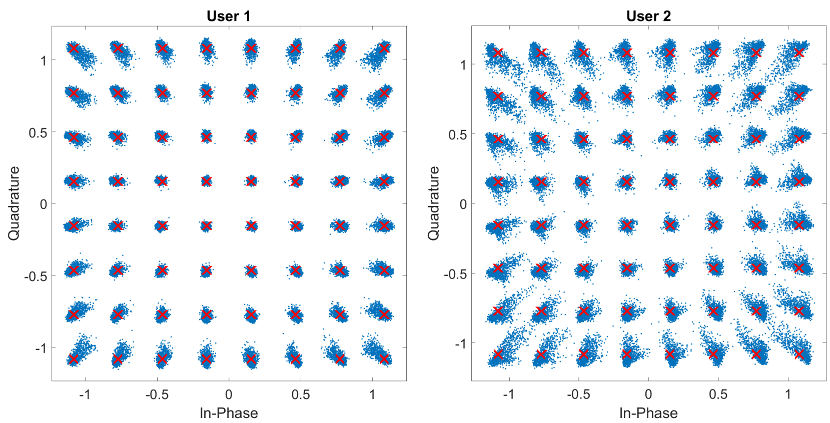


Figure 5.1: Received constellations at each user in a 2x2 scenario. A greater degree of non-linear distortion can be observed for User 2 due to channel conditions.

realization will be simulated with a Rayleigh distribution, whereas the precoder G will be modelled as the Zero Forcer (ZF), i.e., G will be the Moore-Penrose pseudoinverse of H, Eq. (5.1). This decision lies in avoiding the linear multi-user interference produced by the channel since it might alter the study goal of the thesis which is the non-linear distortion. For the same reason, the AWGN $w(n)$ is not included either in the simulations. Nevertheless, as mentioned before, the non-linearity from the power amplifiers will produce

$$\mathbf{G} = \mathbf{H}^{+} \tag{5.1}$$

Following the reasoning of this work, the starting point will be the simplest multi-antenna case, 2 transmitters and 2 users. Due to the random nature of the channel, the users may experiment different channel conditions which can be seen as users with different SNR (Figure 5.1). Increasing the number of transmit antennas in one do not produce significant changes (Figure 5.2 (a)). However, the more the number of antennas is increased, the more the distortion vanishes. This is more notable in the outer points of the constellation, particularly in the corners. In Figure 5.2, it can be observed how the distortion evolves with the number of transmit antennas. The explanation found behind this is that due to the *Law of Large Numbers* (LLN) non-linear distortion from power amplifiers vanishes. In another words, it

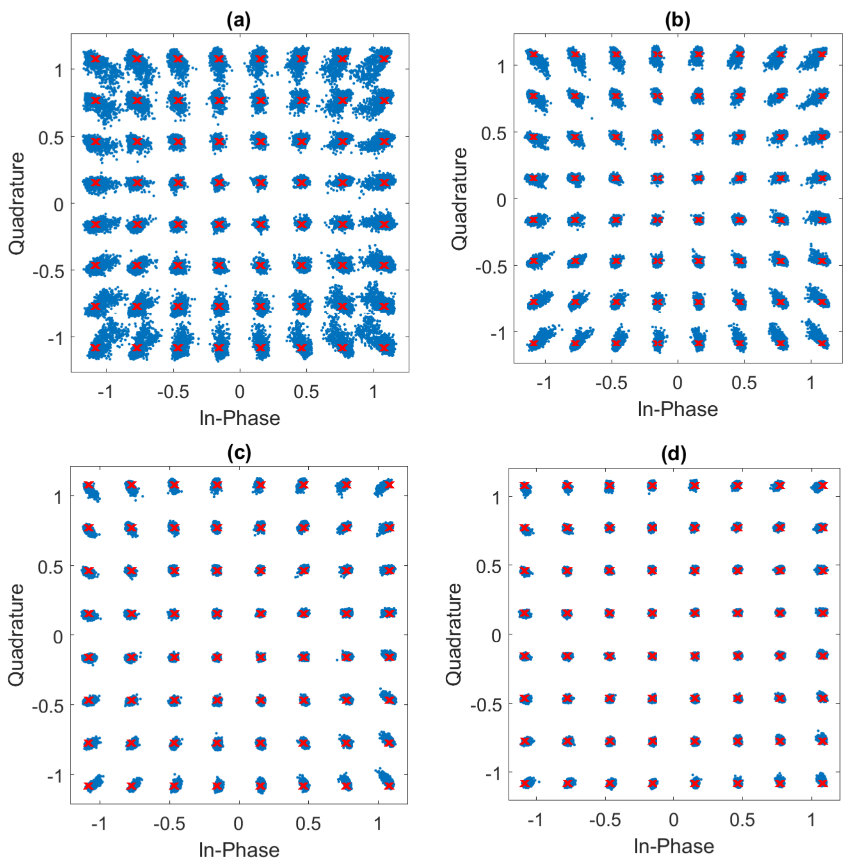


Figure 5.2: Constellation received at User 1 for: (a) 3x2, (b) 10x2, (c) 25x2 and (d) 100x2 scenarios.

can be proved that factors from the non-linear distortion term in Eq. (4.21) satisfy Eq. (5.2) when M, the number of transmit antennas, tends to infinity.

$$\frac{1}{M^4} \cdot [h_i(t) * h_j(t)] = 0 \text{ when } M \to \infty \text{ and } i \neq j \tag{5.2}$$

In order to determine this improvement in terms of distortion, a study of the SNR users experiment for different number of antennas is performed. This is done by averaging the obtained SNR over multiple simulations with random channel realizations. Besides, it was considered relevant to examine the variation for different number of users. This investigation is shown in Figure 5.3.

First, it is clearly recognizable that SNR increases when including new antennas in the scenario for all the cases studied. Second, an interesting phenomenon for different number of users. On one hand, it is seen that for $K = 4$ and $K = 8$ the SNR improves the case where $K = 2$. This was concluded related to the *Central Limit Theorem* (CLT): adding new users to the scenario can be seen as the inclusion of new random variables which makes the convergence to their expected values faster when new antennas are put in. Nevertheless, for $K = 16$ a relapse is clearly identified. The explanation found is that for that scenario the Multi-User Interference is what is limiting the performance by increasing the variance over the constellation points.

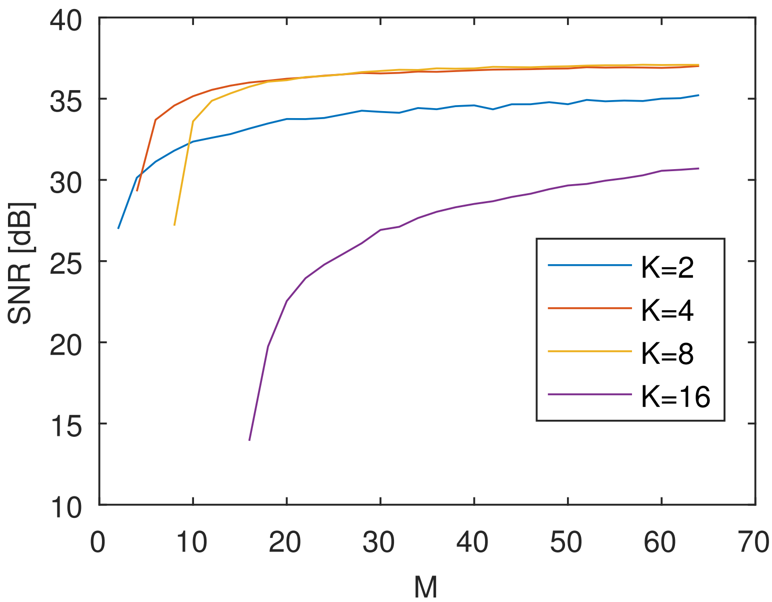


Figure 5.3: Evolution of the Signal to Noise Ratio for different number of antennas and users.

5.2 2x2-Antenna model

The next step to evaluate the performance of the work carried out in this thesis will be to simulate the model achieved for the 2x2 scenario described in section 4.1. To that end, all the equations are translated to MATLAB. It is thus very important to be thorough in this process.

Therefore, to recreate the mathematical conditions of the model there is a significant number of variables to handle: channel, precoder, non-linear severity, phase information, etc. A small change in the conditions can drastically modify the signal whose generation with the developed model is pursued. Following, several examples of the results obtained are put forward. The first in Figure 5.4, shows two different signals. In blue, the signal received for User 1 and generated with a third order polynomial. This signal constitutes the objective signal to obtain by model generation. In red, it is the outcome of the developed statistical model. Both signals, goal and result, are superimposed to facilitate its comparison. It can be observed that the shape of both signals match in a high degree.

In order to keep evaluating its performance, a new couple of examples are presented. First, in Figure 5.5, an example where the channel contribution produces a stronger rotation in the constellation compared to 5.4 is shown. Second, in Figure 5.6, a case where the the symbols are distorted changing clearly the expected shape of a normal PA distorted constellation. These different examples can be seen as cases where the SNR experienced by the studied user varies. However, to the sight of the results it can be concluded that the developed model based in statistical properties of the signals and with full knowledge of the channel state is capable of describing the power amplifier non-linearity for the simplest multi-antenna scenario.

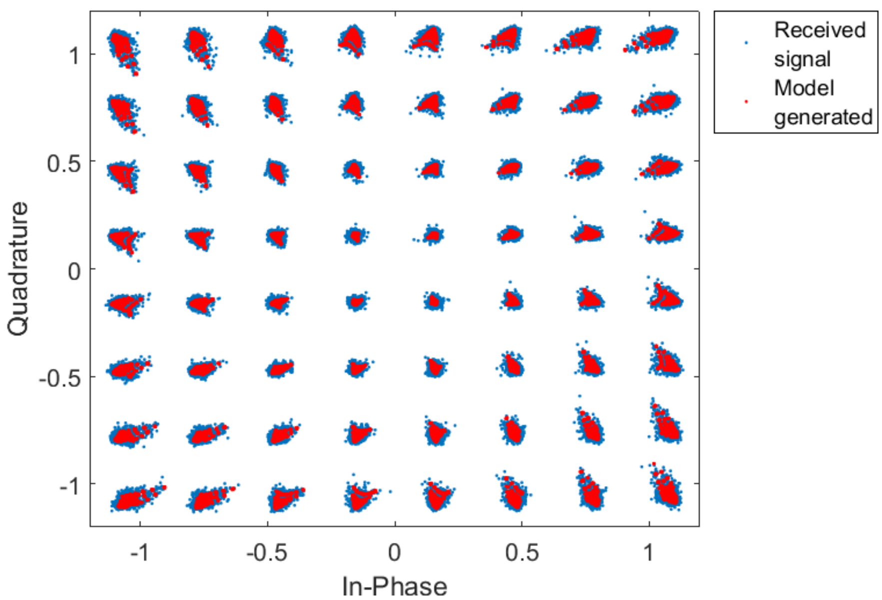


Figure 5.4: Example of the 2x2-Antenna model performance for a specific channel realization.

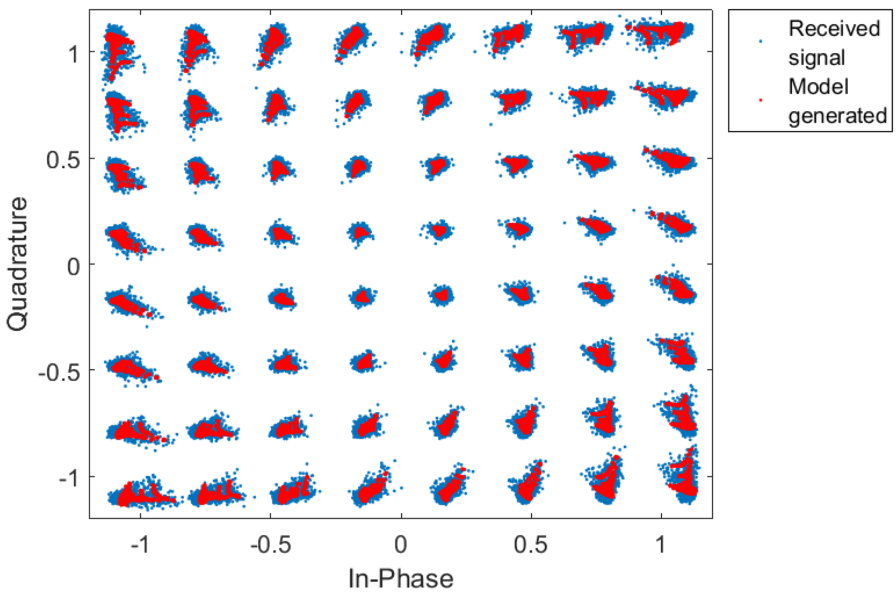


Figure 5.5: Example of the 2x2-Antenna model performance for a specific channel realization.

5.3 MxN-Antenna model

Finally, the general MxK model is tested. Insisting in the approximation done in section 4.5 fruit of the great complexity to handle all the terms present in the scenario, the results in Figure (5.7) must be commented accordingly. To that matter,

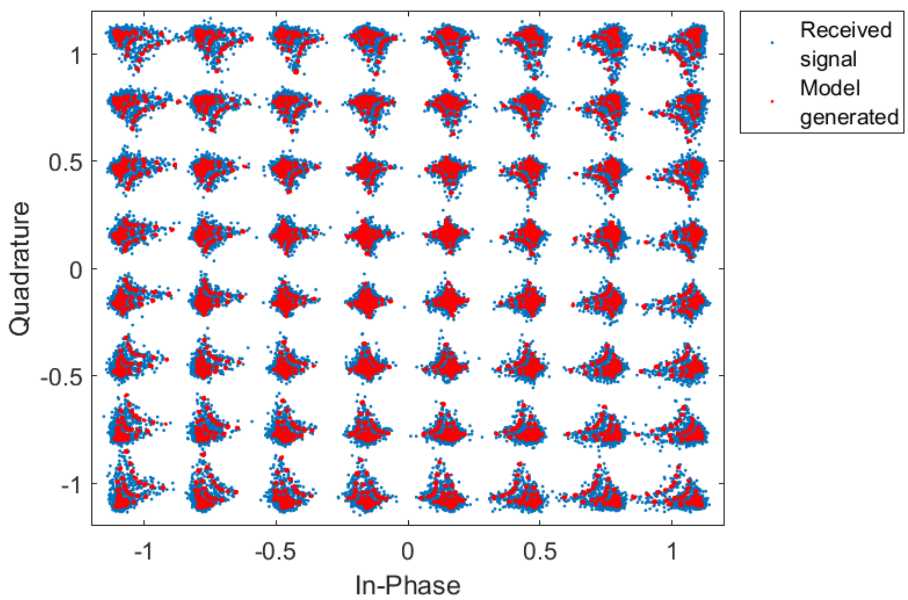


Figure 5.6: Example of the 2x2-Antenna model performance for a specific channel realization.

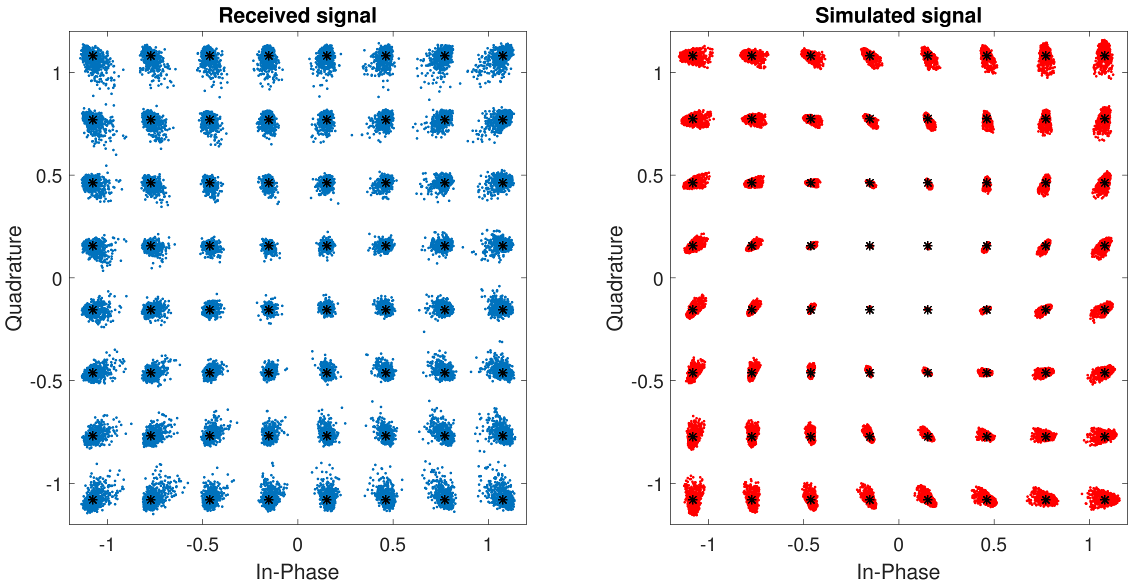


Figure 5.7: Example of the MxK-Antenna model performance for a specific channel realization.

the figure is divided in two sides: to the left, the third order polynomial generated signal and, to the right, the statistical model generated. It is observed that the accuracy of this model is not as high as in the 2 by 2 case due to the mentioned approximation. This fact insist on the idea that only through a precise model that considers all the variables present in the scenario will fully capture the nature real signal received at a final user.

6

Conclusions

Along this thesis, not only an exhaustive revision of a complex mathematical model capable of characterizing non-linear distortion produced by power amplifiers was accomplished but also an extension to a general communication scenario was carried through.

Firstly, it was thus vital to comprehend the intrinsic mechanism within the original single-antenna model. To that matter, its different coefficients were studied separately achieving a full characterization of their contribution to the distortion. Magnitude rescalation or rotation of the constellation and noisy contribution were observed.

Once a full understanding was reached, a methodical extension of the basis model was executed. In an attempt to construct a more realistic model, two key blocks were included: channel and precoder whose significance will be commented subsequently. The strategy followed was to add one element at a time in order not to miss any detail in this delicate procedure. In this way, a path from a 2 antennas and 2 users scenario to a general MxK deployment was drawn. Nevertheless, only the first and last stages of this process were considered relevant to describe statistically and thus statistical models were derived.

In order to validate the built models, simulations were performed. Nevertheless, first a study on the evolution of the non-linear distortion when the number of antennas and users is modified was carried out. The results show a decrease of non-linear behaviour when increasing the number of antennas and users due to the Law of Large Numbers due to the presence of the channel. Therefore, the channel plays a key role in multi-antenna scenarios and it will be of great importance to take it into account in the statistical models. Regarding simulations involving different number of users distortion SNR studies were performed discovering areas dominated by non-linear distortion or by Multi-User Interference. Finally, after testing the statistical models developed in this thesis and in the light of the simulation results, the assumptions and procedure fulfilled were proven to reasonable.

In conclusion, it has been shown just as in previous studies the ability to describe hardware impairments with a statistical approach. Contributions in kind will shape the characterization and study of the future of communications.

7

Future work

This project, framed within the statistical studies whose objective is to characterize hardware impairments, proposes new and future work lines. The following lines are suggested:

- Analyze the behaviour of the distortion and the developed models when other channels and/or precoders are disposed.
- Study other hardware impairments in MIMO scenarios following the reasoning underlying in this thesis.

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A

Appendix. Single-Antenna model

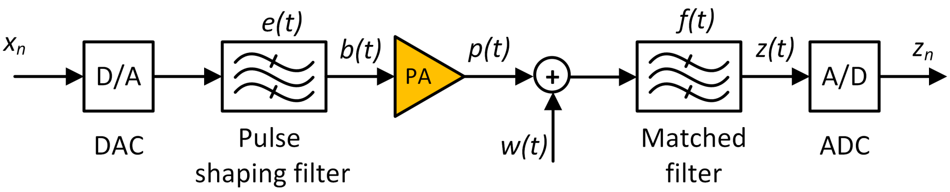

Figure A.1: Block diagram of the transceiver which contains a non-linear RF power amplifier (PA).

A.1 System model

$$p(t) = b(t) + \alpha b(t)|b(t)|^2 = \left(|b(t)| + \alpha|b(t)|^3\right) e^{j\angle b(t)} \tag{A.1}$$

$$b(t) = \sum_{k=-\infty}^{\infty} x_k e(t - kT) \tag{A.2}$$

$$z(t) = (p(t) + w(t)) * f(t) = \int_{-\infty}^{\infty} (p(s) + w(s))\, f(t - s) ds \tag{A.3}$$

$$\begin{aligned} z_n &= z(t)\Big|_{t=nT} = \\ &= \int_{-\infty}^{\infty} \left(b(s) + \alpha b(s)|b(s)|^2 + w(s)\right) f(nT - s) ds = \\ &= \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} x_k \int_{-\infty}^{\infty} e(s - kT) f(nT - s) ds + \\ &+ \int_{-\infty}^{\infty} \alpha \sum_{k=-\infty}^{\infty} x_k e(s - kT) \left| \sum_{l=-\infty}^{\infty} x_l e(s - lT) \right|^2 f(nT - s) ds + \\ &+ \int_{-\infty}^{\infty} w(s) f(nT - s) ds \end{aligned} \tag{A.4}$$

$$z_n = x_n + w_n + \underbrace{\int_{-\infty}^{\infty} \alpha \sum_{k=-\infty}^{\infty} x_k e\left(\frac{s}{T} - k\right) \left| \sum_{l=-\infty}^{\infty} x_l e\left(\frac{s}{T} - l\right) \right|^2 f\left(n - \frac{s}{T}\right) ds}_{d_n} \tag{A.5}$$

$$\begin{aligned} d_n &= \int_{-\infty}^{\infty} \alpha \sum_{k=-\infty}^{\infty} x_k e(s - kT) \left| \sum_{l=-\infty}^{\infty} x_l e(s - lT) \right|^2 f(nT - s) ds = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x_k x_l x_m^* \int_{-\infty}^{\infty} e\left(\frac{s}{T} - k\right) e\left(\frac{s}{T} - l\right) e\left(\frac{s}{T} - m\right) f\left(\frac{s}{T} - n\right) ds = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x_{n-k} x_{n-l} x_{n-m}^* \underbrace{\int_{-\infty}^{\infty} e\left(\frac{s}{T} - k\right) e\left(\frac{s}{T} - l\right) e\left(\frac{s}{T} - m\right) f\left(\frac{s}{T}\right) ds}_{\lambda_{klm}} = \\ &= \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} x_{n-k} x_{n-l} x_{n-m}^* \end{aligned} \tag{A.6}$$

$$d_n = \alpha \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} |x_{n-k}| |x_{n-l}| \left| x_{n-m}^* \right| e^{j(\angle x_{n-k} + \angle x_{n-l} - \angle x_{n-m})} \tag{A.7}$$

A.2 Distortion analysis

$$d_n = c_0 + c_1|x_n| + c_2|x_n|^2 + c_3|x_n|^3 \tag{A.8}$$

A.2.1 Coefficient c_0:

$$c_0 = \alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k} x_{n-l} x_{n-m}^* \tag{A.9}$$

$$\begin{aligned} E\left[c_0\right] &= \alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k} x_{n-l} x_{n-m}^*\right]}_{uncorrelated} = \\ &= \alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kkk} \underbrace{E\left[x_{n-k}|x_{n-k}|^2\right]}_{symmetry \Rightarrow 0} = 0 \end{aligned} \tag{A.10}$$

$$\begin{aligned} E\left[|c_0|^2\right] &= |\alpha|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \sum_{\substack{o=-\infty \\ o\neq 0}}^{\infty} \sum_{\substack{p=-\infty \\ p\neq 0}}^{\infty} \sum_{\substack{q=-\infty \\ q\neq 0}}^{\infty} \lambda_{klm}\lambda_{opq} \underbrace{E\left[x_{n-k} x_{n-l} x_{n-m}^* x_{n-o}^* x_{n-p}^* x_{n-q}\right]}_{uncorrelated} = \\ &= |\alpha|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[|x_{n-k}|^2 |x_{n-l}|^2 |x_{n-m}|^2\right] = \\ &= |\alpha|^2 \sigma_x^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 \underbrace{E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2 \left|\frac{x_{n-l}}{\sigma_x}\right|^2 \left|\frac{x_{n-m}}{\sigma_x}\right|^2\right]}_{\substack{\text{constellation-dependent parameter} \\ \text{(independent of transmit power } \sigma_x^2\text{)}}} \end{aligned} \tag{A.11}$$

A.2.2 Coefficient c_1:

$$c_1 = \alpha e^{-j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k} x_{n-l} + 2\alpha e^{j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k} x_{n-l}^* \tag{A.12}$$

$$\begin{aligned} E\left[c_1\right] &= \alpha e^{-j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k} x_{n-l}\right]}_{uncorrelated} + 2\alpha e^{j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k} x_{n-l}^*\right]}_{uncorrelated} = \\ &= \alpha e^{-j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \underbrace{E\left[x_{n-k}^2\right]}_{symmetry \Rightarrow 0} + 2\alpha e^{j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} E\left[|x_k|^2\right] = \\ &= 2\alpha \sigma_X^2 e^{j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \end{aligned} \tag{A.13}$$

$$
\begin{aligned}
E\left[|c_1|^2\right] &= E\left[\left|\alpha e^{-j\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}x_{n-l} + 2\alpha e^{j\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}x_{n-l}^{*}\right|^2\right] = \\
&= |\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}\left(x_{n-o}x_{n-p}\right)^{*}\right] + \\
&+ 4|\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}^{*}\left(x_{n-o}x_{n-p}^{*}\right)^{*}\right] + \\
&+ 2|\alpha|^2 e^{-j2\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}\underbrace{E\left[x_{n-k}x_{n-l}\left(x_{n-o}x_{n-p}^{*}\right)^{*}\right]}_{symmetry\Rightarrow 0} + \\
&+ 2|\alpha|^2 e^{j2\angle x_n}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}\underbrace{E\left[\left(x_{n-k}x_{n-l}\right)^{*}x_{n-o}x_{n-p}^{*}\right]}_{symmetry\Rightarrow 0} = \\
&= |\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}x_{n-o}^{*}x_{n-p}^{*}\right] + \\
&+ 4|\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}^{*}x_{n-o}^{*}x_{n-p}\right] = \\
&= |\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}x_{n-o}^{*}x_{n-p}^{*}\right] + \\
&+ 4|\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kp0}\lambda_{ol0}E\left[x_{n-k}x_{n-p}x_{n-l}^{*}x_{n-o}^{*}\right] = \\
&= |\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}x_{n-o}^{*}x_{n-p}^{*}\right] + \\
&+ 4|\alpha|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{o=-\infty\\o\neq 0}}^{\infty}\sum_{\substack{p=-\infty\\p\neq 0}}^{\infty}\lambda_{kl0}\lambda_{op0}E\left[x_{n-k}x_{n-l}x_{n-o}^{*}x_{n-p}^{*}\right] = \\
&= 5|\alpha|^2\sigma_x^4\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2\left|\frac{x_{n-l}}{\sigma_x}\right|^2\right]
\end{aligned}
\tag{A.14}
$$

A.2.2.1 Coefficient c_1':

$$\begin{aligned} c_1' &= c_1 - E\left[c_1\right] = \\ &= \alpha e^{-j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k} x_{n-l} + 2\alpha e^{j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k} x_{n-l}^{*} - \\ &- \left[2\alpha\sigma_X^2 e^{j\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \right] \end{aligned} \tag{A.15}$$

$$E\left[c_1'\right] = 0 \tag{A.16}$$

$$E\left[\left|c_1'\right|^2\right] = 5|\alpha|^2\sigma_x^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2 \left|\frac{x_{n-l}}{\sigma_x}\right|^2 \right] - \lambda_{kk0}\lambda_{ll0} \right) \tag{A.17}$$

A.2.3 Coefficient c_2:

$$c_2 = \alpha e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{*} + 2\alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k} \tag{A.18}$$

$$E\left[c_2\right] = \alpha e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{*}\right] + 2\alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}\right] = 0 \tag{A.19}$$

$$\begin{aligned} E\left[\left|c_2\right|^2\right] &= E\left[\left| \alpha e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{*} + 2\alpha \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k} \right|^2 \right] = \\ &= |\alpha|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00}\lambda_{l00} \underbrace{E\left[x_{n-k} x_{n-l}^{*}\right]}_{uncorrelated} + \\ &+ 4|\alpha|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00}\lambda_{l00} \underbrace{E\left[x_{n-k} x_{n-l}^{*}\right]}_{uncorrelated} + \\ &+ 2|\alpha|^2 e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00}\lambda_{l00} \underbrace{E\left[x_{n-k} \left(x_{n-l}^{*}\right)^{*}\right]}_{symmetry \Rightarrow 0} + \\ &+ 2|\alpha|^2 e^{j2\angle x_n} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00}\lambda_{l00} \underbrace{E\left[\left(x_{n-k}\right)^{*} x_{n-l}^{*}\right]}_{symmetry \Rightarrow 0} = \\ &= 5|\alpha|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 E\left[\left|x_{n-k}\right|^2\right] = 5|\alpha|^2\sigma_x^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \end{aligned} \tag{A.20}$$

A.2.4 Coefficient c_3:

$$c_3 = \alpha \lambda_{000} e^{j \angle x_n} \tag{A.21}$$

$$E\left[c_3\right] = \alpha \lambda_{000} e^{j \angle x_n} \tag{A.22}$$

$$E\left[\left|c_3\right|^2\right] = 0 \tag{A.23}$$

A.3 Statistical model

$$z_n = x_n \left(1 + u^{(1)} + u^{(3)} \frac{|x_n|^2}{\sigma_x^2}\right) + v_n^{(0)} + v_n^{(1)} \frac{|x_n|}{\sigma_x} + v_n^{(2)} \frac{|x_n|^2}{\sigma_x^2} + w_n \tag{A.24}$$

$$u^{(1)} = 2\alpha\sigma_X^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \tag{A.25}$$

$$u^{(3)} = \alpha\sigma_x^2 \lambda_{000} \tag{A.26}$$

$$E\left[\left|v_n^{(0)}\right|^2\right] = |\alpha|^2 \sigma_x^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2 \left|\frac{x_{n-l}}{\sigma_x}\right|^2 \left|\frac{x_{n-m}}{\sigma_x}\right|^2\right] \tag{A.27}$$

$$E\left[\left|v_n^{(1)}\right|^2\right] = 5|\alpha|^2 \sigma_x^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}}{\sigma_x}\right|^2 \left|\frac{x_{n-l}}{\sigma_x}\right|^2\right] - \lambda_{kk0}\lambda_{ll0}\right) \tag{A.28}$$

$$E\left[\left|v_n^{(2)}\right|^2\right] = 5|\alpha|^2 \sigma_x^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{A.29}$$

B

Appendix. 2x2-Antenna model

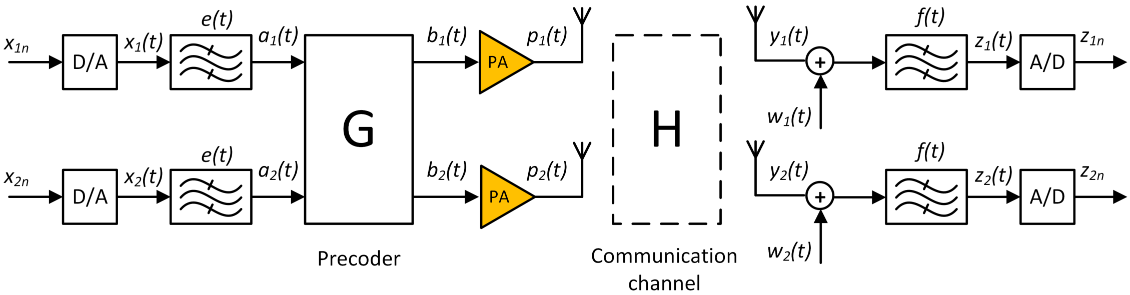

Figure B.1: Block diagram of the system consisting of 2 antennas and 2 users.

B.1 System model

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{B.1}$$

$$\mathbf{G} = \frac{1}{\sqrt{2}} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \tag{B.2}$$

$$\mathbf{a} = \mathbf{x} \circledast e_n = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \circledast e_n = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{B.3}$$

$$\mathbf{b} = \mathbf{Ga} = \frac{1}{\sqrt{2}} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{B.4}$$

$$\mathbf{p} = \begin{bmatrix} b_1 + \alpha_1 b_1 |b_1|^2 \\ b_2 + \alpha_2 b_2 |b_2|^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{B.5}$$

$$\mathbf{y} = \mathbf{Hp} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{B.6}$$

$$\mathbf{z} = [\mathbf{y} + \mathbf{w}] \circledast f_n = \left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \right) \circledast f_n = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{B.7}$$

$$a_1(t) = x_1(t) * e(t) \tag{B.8}$$

$$a_2(t) = x_2(t) * e(t) \tag{B.9}$$

$$b_1(t) = \frac{1}{\sqrt{2}} \left[x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) \right] \tag{B.10}$$

$$b_2(t) = \frac{1}{\sqrt{2}} \left[x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) \right] \tag{B.11}$$

$$\begin{aligned} p_1(t) &= b_1(t) + \alpha_1 b_1(t) |b_1(t)|^2 = \left(|b_1(t)| + \alpha_1 |b_1(t)|^3 \right) e^{j\angle b_1(t)} = \\ &= \left(\frac{1}{\sqrt{2}} |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{2^3}} \alpha_1 |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t)|^3 \right) e^{j\angle b_1(t)} \end{aligned} \tag{B.12}$$

$$\begin{aligned} p_2(t) &= b_2(t) + \alpha_2 b_2(t) |b_2(t)|^2 = \left(|b_2(t)| + \alpha_2 |b_2(t)|^3 \right) e^{j\angle b_2(t)} = \\ &= \left(\frac{1}{\sqrt{2}} |x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{2^3}} \alpha_2 |x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t)|^3 \right) e^{j\angle b_2(t)} \end{aligned} \tag{B.13}$$

$$y_1(t) = \frac{1}{\sqrt{2}} \left[p_1(t) * h_{11}(t) + p_2(t) * h_{12}(t) \right] \tag{B.14}$$

$$y_2(t) = \frac{1}{\sqrt{2}} \left[p_1(t) * h_{21}(t) + p_2(t) * h_{22}(t) \right] \tag{B.15}$$

$$e(t) * f(t) = 1 \tag{B.16}$$

$$
\begin{aligned}
z_1(t) = [y_1(t) + w_1(t)] * f(t) = \left[\frac{1}{\sqrt{2}}\left[p_1(t) * h_{11}(t) + p_2(t) * h_{12}(t)\right] + w_1(t)\right] * f(t) = \\
&= \frac{1}{2}\left[x_1(t) * (g_{11}(t) * h_{11}(t) + g_{21}(t) * h_{12}(t)) +\right. \\
&+ x_2(t) * (g_{12}(t) * h_{11}(t) + g_{22}(t) * h_{12}(t))] + \\
&+ w_1(t) * f(t) + \\
&+ \alpha_1 \cdot \frac{1}{2^2}\left[\left(x_1(t)x_1(t)x_1^*(t) * e(t)|e(t)|^2 * f(t) * h_{11}(t) * g_{11}(t)|g_{11}(t)|^2\right) +\right. \\
&+ 2 \cdot \left(x_1(t)x_2(t)x_2^*(t) * e(t)|e(t)|^2 * f(t) * h_{11}(t) * g_{11}(t) * |g_{12}(t)|^2\right) + \\
&+ \left(x_1(t)x_1(t)x_2^*(t) * e(t)|e(t)|^2 * f(t) * h_{11}(t) * g_{12}^*(t) * (g_{11}(t))^2\right) + \\
&+ 2 \cdot \left(x_1(t)x_1^*(t)x_2(t) * e(t)|e(t)|^2 * f(t) * h_{11}(t) * g_{12}(t) * |g_{11}(t)|^2\right) + \\
&+ \left(x_2(t)x_2(t)x_2^*(t) * e(t)|e(t)|^2 * f(t) * h_{11}(t) * g_{12}(t)|g_{12}(t)|^2\right) + \\
&+ \left.\left(x_1^*(t)x_2(t)x_2(t) * e(t)|e(t)|^2 * f(t) * h_{11}(t) * g_{11}^*(t) * (g_{12}(t))^2\right)\right] + \\
&+ \alpha_2 \cdot \frac{1}{2^2}\left[\left(x_1(t)x_1(t)x_1^*(t) * e(t)|e(t)|^2 * f(t) * h_{12}(t) * g_{21}(t)|g_{21}(t)|^2\right) +\right. \\
&+ 2 \cdot \left(x_1(t)x_2(t)x_2^*(t) * e(t)|e(t)|^2 * f(t) * h_{12}(t) * g_{21}(t) * |g_{22}(t)|^2\right) + \\
&+ \left(x_1(t)x_1(t)x_2^*(t) * e(t)|e(t)|^2 * f(t) * h_{12}(t) * (g_{21}(t))^2 * g_{22}^*(t)\right) + \\
&+ 2 \cdot \left(x_1(t)x_1^*(t)x_2(t) * e(t)|e(t)|^2 * f(t) * h_{12}(t) * g_{22}(t) * |g_{21}(t)|^2\right) + \\
&+ \left(x_2(t)x_2(t)x_2^*(t) * e(t)|e(t)|^2 * f(t) * h_{12}(t) * g_{22}(t)|g_{22}(t)|^2\right) + \\
&+ \left.\left(x_1^*(t)x_2(t)x_2(t) * e(t)|e(t)|^2 * f(t) * h_{12}(t) * (g_{22}(t))^2 * g_{21}^*(t)\right)\right]
\end{aligned}
\tag{B.17}
$$

$$
e(t)|e(t)|^2 * f(t) = \lambda_{klm} \tag{B.18}
$$

$$
\begin{aligned}
d_n = \frac{1}{2^2} \cdot \lambda_{klm} \Bigg[& x_1 x_1 x_1^* \underbrace{\Big(\alpha_1 * h_{11}(t) * g_{11}(t) |g_{11}(t)|^2 + \alpha_2 * h_{12}(t) * g_{21}(t) |g_{21}(t)|^2\Big)}_{L_1} + \\
& + x_2 x_2 x_2^* \underbrace{\Big(\alpha_1 * h_{11}(t) * g_{12}(t) |g_{12}(t)|^2 + \alpha_2 * h_{12}(t) * g_{22}(t) |g_{22}(t)|^2\Big)}_{L_2} + \\
& + x_1 x_2 x_2^* \underbrace{2 \cdot \Big(\alpha_1 * h_{11}(t) * g_{11}(t) * |g_{12}(t)|^2 + \alpha_2 * h_{12}(t) * g_{21}(t) * |g_{22}(t)|^2\Big)}_{L_3} + \\
& + x_1 x_1 x_2^* \underbrace{\Big(\alpha_1 * h_{11}(t) * g_{12}^*(t) * (g_{11}(t))^2 + \alpha_2 * h_{12}(t) * (g_{21}(t))^2 * g_{22}^*(t)\Big)}_{L_4} + \\
& + x_1 x_1^* x_2 \underbrace{2 \cdot \Big(\alpha_1 * h_{11}(t) * g_{12}(t) * |g_{11}(t)|^2 + \alpha_2 * h_{12}(t) * g_{22}(t) * |g_{21}(t)|^2\Big)}_{L_5} + \\
& + x_1^* x_2 x_2 \underbrace{\Big(\alpha_1 * h_{11}(t) * g_{11}^*(t) * (g_{12}(t))^2 + \alpha_2 * h_{12}(t) * (g_{22}(t))^2 * g_{21}^*(t)\Big)}_{L_6} \Bigg]
\end{aligned} \tag{B.19}
$$

$$
\begin{aligned}
d_n = \frac{1}{2^2} \Bigg[& \underbrace{\lambda_{klm} L_1 x_1 x_1 x_1^*}_{d_{x_{1n}}} + \underbrace{\lambda_{klm} L_2 x_2 x_2 x_2^*}_{d_{x_{2n}}} + \underbrace{\lambda_{klm} L_3 x_1 x_2 x_2^*}_{d_{x_{1n} x_{2n} x_{2n}^*}} + \\
& + \underbrace{\lambda_{klm} L_4 x_1 x_1 x_2^*}_{d_{x_{1n} x_{1n} x_{2n}^*}} + \underbrace{\lambda_{klm} L_5 x_1 x_1^* x_2}_{d_{x_{1n} x_{1n}^* x_{2n}}} + \underbrace{\lambda_{klm} L_6 x_1^* x_2 x_2}_{d_{x_{1n}^* x_{2n} x_{2n}}} \Bigg]
\end{aligned} \tag{B.20}
$$

$$
d_n = \frac{1}{2^2} \left[d_{x_{1n}} + d_{x_{2n}} + d_{x_{1n} x_{2n} x_{2n}^*} + d_{x_{1n} x_{1n} x_{2n}^*} + d_{x_{1n} x_{1n}^* x_{2n}} + d_{x_{1n}^* x_{2n} x_{2n}} \right] \tag{B.21}
$$

B.2 Distortion analysis

B.2.1 $d_{x_{1n}}$

$$d_{x_{1n}} = c_0 + c_1|x_{1n}| + c_2|x_{1n}|^2 + c_3|x_{1n}|^3 \tag{B.22}$$

$$d_{x_{1n}} = L_1 \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(1)}\right|\left|x_{n-l}^{(1)}\right|\left|x_{n-m}^{(1)^*}\right| e^{j\left(\angle x_{n-k}^{(1)} + \angle x_{n-l}^{(1)} - \angle x_{n-m}^{(1)}\right)} \tag{B.23}$$

B.2.1.1 Coefficient c_0:

$$c_0 = L_1 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(1)} x_{n-l}^{(1)} x_{n-m}^{(1)^*} \tag{B.24}$$

$$E\left[c_0\right] = 0 \tag{B.25}$$

$$E\left[|c_0|^2\right] = |L_1|^2 \sigma_{x^{(1)}}^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 \underbrace{E\left[\left|\frac{x_{n-k}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-m}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right]}_{\substack{\text{constellation-dependent parameter} \\ \text{(independent of transmit power } \sigma_{x^{(1)}}^2\text{)}}} \tag{B.26}$$

B.2.1.2 Coefficient c_1:

$$c_1 = L_1 \left[e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)} + 2e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)^*} \right] \tag{B.27}$$

$$E\left[c_1\right] = 2L_1 \sigma_{X^{(1)}}^2 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \tag{B.28}$$

$$E\left[|c_1|^2\right] = 5|L_1|^2 \sigma_{x^{(1)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right] \tag{B.29}$$

Coefficient c_1':

$$c_1' = c_1 - E\left[c_1\right] = \\ = L_1 \left[e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)} + 2e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)^*} - \\ - \left[2\sigma_{X^{(1)}}^2 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \right] \right] \tag{B.30}$$

$$E\left[c_1'\right] = 0 \tag{B.31}$$

$$E\left[\left|c_1'\right|^2\right] = 5|L_1|^2 \sigma_{x^{(1)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left| \frac{x_{n-k}^{(1)}}{\sigma_{x^{(1)}}} \right|^2 \left| \frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}} \right|^2 \right] - \lambda_{kk0}\lambda_{ll0} \right) \tag{B.32}$$

B.2.1.3 Coefficient c_2:

$$c_2 = L_1 \left[e^{j2\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(1)^*} + 2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(1)} \right] \tag{B.33}$$

$$E\left[c_2\right] = 0 \tag{B.34}$$

$$E\left[\left|c_2\right|^2\right] = 5|L_1|^2 \sigma_{x^{(1)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{B.35}$$

B.2.1.4 Coefficient c_3:

$$c_3 = L_1 \lambda_{000} e^{j\angle x_n^{(1)}} \tag{B.36}$$

$$E\left[c_3\right] = L_1 \lambda_{000} e^{j\angle x_n^{(1)}} \tag{B.37}$$

$$E\left[\left|c_3\right|^2\right] = 0 \tag{B.38}$$

B.2.2 $d_{x_{2n}}$

$$d_{x_{2n}} = c_4 + c_5|x_{2n}| + c_6|x_{2n}|^2 + c_7|x_{2n}|^3 \tag{B.39}$$

$$d_{x_{2n}} = L_2 \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(2)}\right| \left|x_{n-l}^{(2)}\right| \left|x_{n-m}^{(2)^*}\right| e^{j\left(\angle x_{n-k}^{(2)} + \angle x_{n-l}^{(2)} - \angle x_{n-m}^{(2)}\right)} \tag{B.40}$$

B.2.2.1 Coefficient c_4:

$$c_4 = L_2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(2)} x_{n-l}^{(2)} x_{n-m}^{(2)^*} \tag{B.41}$$

$$E\left[c_4\right] = 0 \tag{B.42}$$

$$E\left[|c_4|^2\right] = |L_2|^2 \sigma_{x^{(2)}}^6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 \underbrace{E\left[\left|\frac{x_{n-k}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-m}^{(2)}}{\sigma_{x^{(2)}}}\right|^2\right]}_{\substack{\text{constellation-dependent parameter} \\ \text{(independent of transmit power } \sigma_{x^{(2)}}^2\text{)}}} \tag{B.43}$$

B.2.2.2 Coefficient c_5:

$$c_5 = L_2 \left[e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)} + 2e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)^*} \right] \tag{B.44}$$

$$E\left[c_5\right] = 2L_2 \sigma_{X^{(2)}}^2 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \tag{B.45}$$

$$E\left[|c_5|^2\right] = 5|L_2|^2 \sigma_{x^{(2)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}}\right|^2\right] \tag{B.46}$$

Coefficient c'_5:

$$
\begin{aligned}
c'_5 &= c_5 - E\left[c_5\right] = \\
&= L_2 \left[e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)} + 2e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)^*} - \right. \\
&\left. - \left[2\sigma_{X^{(2)}}^2 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \right] \right]
\end{aligned} \tag{B.47}
$$

$$
E\left[c'_5\right] = 0 \tag{B.48}
$$

$$
E\left[\left|c'_5\right|^2\right] = 5|L_2|^2 \sigma_{x^{(2)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left| \frac{x_{n-k}^{(2)}}{\sigma_{x^{(2)}}} \right|^2 \left| \frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}} \right|^2 \right] - \lambda_{kk0}\lambda_{ll0} \right) \tag{B.49}
$$

B.2.2.3 Coefficient c_6:

$$
c_6 = L_2 \left[e^{j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(2)^*} + 2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(2)} \right] \tag{B.50}
$$

$$
E\left[c_6\right] = 0 \tag{B.51}
$$

$$
E\left[\left|c_6\right|^2\right] = 5|L_2|^2 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{B.52}
$$

B.2.2.4 Coefficient c_7:

$$
c_7 = L_2 \lambda_{000} e^{j\angle x_n^{(2)}} \tag{B.53}
$$

$$
E\left[c_7\right] = L_2 \lambda_{000} e^{j\angle x_n^{(2)}} \tag{B.54}
$$

$$
E\left[\left|c_7\right|^2\right] = 0 \tag{B.55}
$$

B.2.3 $d_{x_{1n}x_{2n}x_{2n}^*}$

$$d_{x_{1n}x_{2n}x_{2n}^*} = c_8 + c_9|x_{1n}| + c_{10}|x_{2n}| + c_{11}|x_{2n}|^2 + c_{12}|x_{1n}||x_{2n}|^2 \tag{B.56}$$

$$d_{x_{1n}x_{2n}x_{2n}^*} = L_3 \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(1)}\right| \left|x_{n-l}^{(2)}\right| \left|x_{n-m}^{(2)^*}\right| e^{j\left(\angle x_{n-k}^{(1)} + \angle x_{n-l}^{(2)} - \angle x_{n-m}^{(2)}\right)} \tag{B.57}$$

B.2.3.1 Coefficient c_8:

$$c_8 = L_3 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(1)} x_{n-l}^{(2)} x_{n-m}^{(2)^*} \tag{B.58}$$

$$\begin{aligned} E\left[c_8\right] &= L_3 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(2)} x_{n-m}^{(2)^*}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= L_3 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kll} \underbrace{E\left[x_{n-k}^{(1)}\right]}_{0} E\left[\left|x_{n-l}^{(2)}\right|^2\right] = 0 \end{aligned} \tag{B.59}$$

$$\begin{aligned} E\left[\left|c_8\right|^2\right] &= \left|L_3\right|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(2)}\right|^2 \left|x_{n-m}^{(2)}\right|^2\right] = \\ &= \left|L_3\right|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 \underbrace{E\left[\left|x_{n-k}^{(1)}\right|^2\right]}_{\sigma_{x^{(1)}}^2} E\left[\left|x_{n-l}^{(2)}\right|^2 \left|x_{n-m}^{(2)}\right|^2\right] = \\ &= \left|L_3\right|^2 \sigma_{x^{(1)}}^2 \sigma_{x^{(2)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-m}^{(2)}}{\sigma_{x^{(2)}}}\right|^2\right] \end{aligned} \tag{B.60}$$

B.2.3.2 Coefficient c_9:

$$c_9 = L_3 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)^*} \tag{B.61}$$

$$\begin{aligned} E\left[c_9\right] &= L_3 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(2)} x_{n-l}^{(2)^*}\right]}_{uncorrelated} = \\ &= L_3 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} E\left[\left|x_k^{(2)}\right|^2\right] = \\ &= L_3 \sigma_{X^{(2)}}^2 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \end{aligned} \tag{B.62}$$

$$\begin{aligned} E\left[\left|c_9\right|^2\right] &= E\left[\left|L_3 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)^*}\right|^2\right] = \\ &= \left|L_3\right|^2 \sigma_{x^{(2)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}}\right|^2\right] \end{aligned} \tag{B.63}$$

Coefficient c_9':

$$\begin{aligned} c_9' = c_9 - E\left[c_9\right] = L_3 &\left[e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)^*} - \right. \\ &\left. - \left[\sigma_{X^{(2)}}^2 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0}\right]\right] \end{aligned} \tag{B.64}$$

$$E\left[c_9'\right] = 0 \tag{B.65}$$

$$E\left[\left|c_9'\right|^2\right] = \left|L_3\right|^2 \sigma_{x^{(2)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}}\right|^2\right] - \lambda_{kk0}\lambda_{ll0}\right) \tag{B.66}$$

B.2.3.3 Coefficient c_{10}:

$$c_{10} = L_3 \left[e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(2)} + e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(2)^*}\right] \tag{B.67}$$

$$E\left[c_{10}\right] = L_3 \left[e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(2)}\right]}_{0} + \right.$$
$$\left. + \; e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(2)^*}\right]}_{0} \right] = 0 \tag{B.68}$$

$$E\left[\left|c_{10}\right|^2\right] = E\left[\left| L_3 \left(e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(2)} + e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(2)^*} \right) \right|^2\right] =$$
$$= \left|L_3\right|^2 \left(2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left| x_{n-k}^{(1)} x_{n-l}^{(2)} \right|^2\right] + \right.$$
$$+ \; e^{-j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\underbrace{x_{n-k}^{(1)} x_{n-l}^{(2)} \left(x_{n-k}^{(1)} x_{n-l}^{(2)^*} \right)^*}_{\left|x_{n-k}^{(1)}\right|^2 x_{n-l}^{(2)}} \right] +$$
$$\left. + \; e^{j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\underbrace{\left(x_{n-k}^{(1)} x_{n-l}^{(2)} \right)^* x_{n-k}^{(1)} x_{n-l}^{(2)^*}}_{\left|x_{n-k}^{(1)}\right|^2 x_{n-l}^{(2)}} \right] \right) =$$
$$= \left|L_3\right|^2 \left(2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left| x_{n-k}^{(1)} \right|^2\right] E\left[\left| x_{n-l}^{(2)} \right|^2\right] + \right.$$
$$+ \; e^{-j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left| x_{n-l}^{(1)} \right|^2\right] \underbrace{E\left[\left(x_{n-k}^{(2)} \right)^2\right]}_{0} +$$
$$\left. + \; e^{j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left| x_{n-k}^{(1)} \right|^2\right] \underbrace{E\left[\left(x_{n-l}^{(2)^*} \right)^2\right]}_{0} \right) =$$
$$= 2\left|L_3\right|^2 \sigma_{x^{(1)}}^2 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 \tag{B.69}$$

B.2.3.4 Coefficient c_{11}:

$$c_{11} = L_3 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(1)} \tag{B.70}$$

$$E\left[c_{11}\right] = L_3 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(1)}\right] = 0 \tag{B.71}$$

$$\begin{aligned} E\left[\left|c_{11}\right|^2\right] &= E\left[\left|L_3 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(1)}\right|^2\right] = \\ &= \left|L_3\right|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00}\lambda_{l00} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(1)^*}\right]}_{uncorrelated} = \\ &= \left|L_3\right|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 E\left[\left|x_{n-k}^{(1)}\right|^2\right] = \left|L_3\right|^2 \sigma_{x^{(1)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \end{aligned} \tag{B.72}$$

B.2.3.5 Coefficient c_{12}:

$$c_{12} = L_3 \lambda_{000} e^{j\angle x_n^{(1)}} \tag{B.73}$$

$$E\left[c_{12}\right] = L_3 \lambda_{000} e^{j\angle x_n^{(1)}} \tag{B.74}$$

$$E\left[\left|c_{12}\right|^2\right] = 0 \tag{B.75}$$

B.2.4 $d_{x_{1n}x_{1n}x_{2n}^*}$

$$d_{x_{1n}x_{1n}x_{2n}^*} = c_{13} + c_{14}|x_{1n}| + c_{15}|x_{1n}|^2 + c_{16}|x_{2n}| + c_{17}|x_{1n}|^2|x_{2n}| \tag{B.76}$$

$$d_{x_{1n}x_{2n}x_{2n}^*} = L_4 \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(1)}\right| \left|x_{n-l}^{(1)}\right| \left|x_{n-m}^{(2)^*}\right| e^{j\left(\angle x_{n-k}^{(1)} + \angle x_{n-l}^{(1)} - \angle x_{n-m}^{(2)}\right)} \tag{B.77}$$

B.2.4.1 Coefficient c_{13}:

$$c_{13} = L_4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(1)} x_{n-l}^{(1)} x_{n-m}^{(2)^*} \tag{B.78}$$

$$\begin{aligned} E\left[c_{13}\right] &= L_4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(1)} x_{n-m}^{(2)^*}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= L_4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kkl} \underbrace{E\left[\left(x_{n-k}^{(1)}\right)^2\right]}_{0} \underbrace{E\left[x_{n-l}^{(2)^*}\right]}_{0} = 0 \end{aligned} \tag{B.79}$$

$$\begin{aligned} E\left[\left|c_{13}\right|^2\right] &= |L_4|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(1)}\right|^2 \left|x_{n-m}^{(2)}\right|^2\right] = \\ &= |L_4|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(1)}\right|^2\right] E\left[\left|x_{n-m}^{(2)}\right|^2\right] = \\ &= |L_4|^2 \sigma_{x^{(1)}}^4 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-m}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right] \end{aligned} \tag{B.80}$$

B.2.4.2 Coefficient c_{14}:

$$c_{14} = 2 \cdot L_4 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(2)^*} \tag{B.81}$$

$$\begin{aligned} E\left[c_{14}\right] &= 2 \cdot L_4 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(2)^*}\right]}_{\textit{independent}} = \\ &= 2 \cdot L_4 e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} E\left[x_{n-k}^{(1)}\right] E\left[x_{n-l}^{(2)^*}\right] = 0 \end{aligned} \tag{B.82}$$

$$\begin{aligned}
E\left[|c_{14}|^2\right] &= E\left[\left|2\cdot L_4 e^{j\angle x_n^{(1)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(1)}x_{n-l}^{(2)^*}\right|^2\right] = \\
&= 4\cdot|L_4|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 E\left[\left|x_{n-k}^{(1)}\right|^2\right]E\left[\left|x_{n-l}^{(2)}\right|^2\right] = \\
&= 4\cdot|L_4|^2\sigma_{x^{(1)}}^2\sigma_{x^{(2)}}^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2
\end{aligned} \tag{B.83}$$

B.2.4.3 Coefficient c_{15}:

$$c_{15} = L_4 e^{j2\angle x_n^{(1)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}x_{n-k}^{(2)^*} \tag{B.84}$$

$$E\left[c_{15}\right] = L_4 e^{j2\angle x_n^{(1)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}E\left[x_{n-k}^{(2)^*}\right] = 0 \tag{B.85}$$

$$\begin{aligned}
E\left[|c_{15}|^2\right] &= E\left[\left|L_4 e^{j2\angle x_n^{(1)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}x_{n-k}^{(2)^*}\right|^2\right] = \\
&= |L_4|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{k00}\lambda_{l00}\underbrace{E\left[x_{n-k}^{(2)}x_{n-l}^{(2)^*}\right]}_{uncorrelated} = \\
&= |L_4|^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}^2 E\left[\left|x_{n-k}^{(2)}\right|^2\right] = \\
&= |L_4|^2\sigma_{x^{(2)}}^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}^2
\end{aligned} \tag{B.86}$$

B.2.4.4 Coefficient c_{16}:

$$c_{16} = L_4 e^{-j\angle x_n^{(2)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(1)}x_{n-l}^{(1)} \tag{B.87}$$

$$
\begin{aligned}
E\left[c_{16}\right] &= L_4 e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(1)}\right]}_{uncorrelated} = \\
&= L_4 e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \underbrace{E\left[\left(x_{n-k}^{(1)}\right)^2\right]}_{symmetry} = 0
\end{aligned} \tag{B.88}
$$

$$
\begin{aligned}
E\left[\left|c_{16}\right|^2\right] &= E\left[\left|L_4 e^{-j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)}\right|^2\right] = \\
&= \left|L_4\right|^2 \sigma_{x^{(1)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right]
\end{aligned} \tag{B.89}
$$

B.2.4.5 Coefficient c_{17}:

$$
c_{17} = L_4 \lambda_{000} e^{j\left(2\angle x_n^{(1)} - \angle x_n^{(2)}\right)} \tag{B.90}
$$

$$
E\left[c_{17}\right] = L_4 \lambda_{000} e^{j\left(2\angle x_n^{(1)} - \angle x_n^{2}\right)} \tag{B.91}
$$

$$
E\left[\left|c_{17}\right|^2\right] = 0 \tag{B.92}
$$

B.2.5 $d_{x_{1n}x_{1n}^*x_{2n}}$

$$d_{x_{1n}x_{1n}^*x_{2n}} = c_{18} + c_{19}|x_{1n}| + c_{20}|x_{1n}|^2 + c_{21}|x_{2n}| + c_{22}|x_{1n}|^2|x_{2n}| \tag{B.93}$$

$$d_{x_{1n}x_{1n}^*x_{2n}} = L_5 \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(1)}\right|\left|x_{n-l}^{(1)^*}\right|\left|x_{n-m}^{(2)}\right| e^{j\left(\angle x_{n-k}^{(1)} - \angle x_{n-l}^{(1)} + \angle x_{n-m}^{(2)}\right)} \tag{B.94}$$

B.2.5.1 Coefficient c_{18}:

$$c_{18} = L_5 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(1)} x_{n-l}^{(1)^*} x_{n-m}^{(2)} \tag{B.95}$$

$$\begin{aligned} E\left[c_{18}\right] &= L_5 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(1)^*} x_{n-m}^{(2)}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= L_5 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kkl} E\left[\left|x_{n-k}^{(1)}\right|^2\right] \underbrace{E\left[x_{n-l}^{(2)}\right]}_{0} = 0 \end{aligned} \tag{B.96}$$

$$\begin{aligned} E\left[\left|c_{18}\right|^2\right] &= |L_5|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(1)}\right|^2 \left|x_{n-m}^{(2)}\right|^2\right] = \\ &= |L_5|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(1)}\right|^2\right] \underbrace{E\left[\left|x_{n-m}^{(2)}\right|^2\right]}_{\sigma_{x^{(2)}}^2} = \\ &= |L_5|^2 \sigma_{x^{(1)}}^4 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-k}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right] \end{aligned} \tag{B.97}$$

B.2.5.2 Coefficient c_{19}:

$$c_{19} = L_5 \left[e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(2)} + e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)^*} x_{n-l}^{(2)} \right] \tag{B.98}$$

$$E\left[c_{19}\right] = L_5 \left[e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(2)}\right]}_{0} + \right. \\ \left. + e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)^*}\right]}_{0} \underbrace{E\left[x_{n-l}^{(2)}\right]}_{0} \right] = 0 \tag{B.99}$$

$$E\left[|c_{19}|^2\right] = E\left[\left| L_5 \left(e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(2)} + e^{j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)^*} x_{n-l}^{(2)} \right) \right|^2\right] = \\ = 2|L_5|^2 \sigma_{x^{(1)}}^2 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 \tag{B.100}$$

B.2.5.3 Coefficient c_{20}:

$$c_{20} = L_5 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(2)} \tag{B.101}$$

$$E\left[c_{20}\right] = L_5 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(2)}\right] = 0 \tag{B.102}$$

$$E\left[|c_{20}|^2\right] = E\left[\left| L_5 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(2)} \right|^2\right] = \\ = |L_5|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00} \lambda_{l00} \underbrace{E\left[x_{n-k}^{(2)} x_{n-l}^{(2)^*}\right]}_{uncorrelated} = \\ = |L_5|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 E\left[\left|x_{n-k}^{(2)}\right|^2\right] = |L_5|^2 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{B.103}$$

B.2.5.4 Coefficient c_{21}:

$$c_{21} = L_5 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)^*} \tag{B.104}$$

$$\begin{aligned} E\left[c_{21}\right] &= L_5 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(1)^*}\right]}_{uncorrelated} = \\ &= L_5 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \lambda_{kk0} E\left[\left|x_k^{(1)}\right|^2\right] = \\ &= L_5 \sigma_{X^{(1)}}^2 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \lambda_{kk0} \end{aligned} \quad (B.105)$$

$$\begin{aligned} E\left[\left|c_{21}\right|^2\right] &= E\left[\left|L_5 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)^*}\right|^2\right] = \\ &= \left|L_5\right|^2 \sigma_{x^{(1)}}^4 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right] \end{aligned} \quad (B.106)$$

Coefficient c'_{21}:

$$\begin{aligned} c'_{21} = c_{21} - E\left[c_{21}\right] = L_5 &\left[e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)} x_{n-l}^{(1)^*} - \right. \\ &- \left.\left[\sigma_{X^{(1)}}^2 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \lambda_{kk0}\right]\right] \end{aligned} \quad (B.107)$$

$$E\left[c'_{21}\right] = 0 \quad (B.108)$$

$$E\left[\left|c'_{21}\right|^2\right] = \left|L_5\right|^2 \sigma_{x^{(1)}}^4 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(1)}}{\sigma_{x^{(1)}}}\right|^2 \left|\frac{x_{n-l}^{(1)}}{\sigma_{x^{(1)}}}\right|^2\right] - \lambda_{kk0}\lambda_{ll0}\right) \quad (B.109)$$

B.2.5.5 Coefficient c_{22}:

$$c_{22} = L_5 \lambda_{000} e^{j\angle x_n^{(2)}} \quad (B.110)$$

$$E\left[c_{22}\right] = L_5 \lambda_{000} e^{j\angle x_n^{(2)}} \quad (B.111)$$

$$E\left[\left|c_{22}\right|^2\right] = 0 \quad (B.112)$$

B.2.6 $d_{x_{1n}^* x_{2n} x_{2n}}$

$$d_{x_{1n}^* x_{2n} x_{2n}} = c_{23} + c_{24}|x_{1n}| + c_{25}|x_{2n}| + c_{26}|x_{2n}|^2 + c_{27}|x_{1n}||x_{2n}|^2 \tag{B.113}$$

$$d_{x_{1n}^* x_{2n} x_{2n}} = L_6 \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left| x_{n-k}^{(1)^*} \right| \left| x_{n-l}^{(2)} \right| \left| x_{n-m}^{(2)} \right| e^{j\left(-\angle x_{n-k}^{(1)} + \angle x_{n-l}^{(2)} + \angle x_{n-m}^{(2)}\right)} \tag{B.114}$$

B.2.6.1 Coefficient c_{23}:

$$c_{23} = L_6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(1)^*} x_{n-l}^{(2)} x_{n-m}^{(2)} \tag{B.115}$$

$$\begin{aligned} E\left[c_{23}\right] &= L_6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(1)^*} x_{n-l}^{(2)} x_{n-m}^{(2)}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= L_6 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kll} \underbrace{E\left[x_{n-k}^{(1)^*}\right]}_{0} \underbrace{E\left[\left(x_{n-l}^{(2)}\right)^2\right]}_{0} = 0 \end{aligned} \tag{B.116}$$

$$\begin{aligned} E\left[\left|c_{23}\right|^2\right] &= |L_6|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2 \left|x_{n-l}^{(2)}\right|^2 \left|x_{n-m}^{(2)}\right|^2\right] = \\ &= |L_6|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(1)}\right|^2\right] E\left[\left|x_{n-l}^{(2)}\right|^2 \left|x_{n-m}^{(2)}\right|^2\right] = \\ &= |L_6|^2 \sigma_{x^{(1)}}^2 \sigma_{x^{(2)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-m}^{(2)}}{\sigma_{x^{(2)}}}\right|^2\right] \end{aligned} \tag{B.117}$$

B.2.6.2 Coefficient c_{24}:

$$c_{24} = L_6 e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)} \tag{B.118}$$

$$\begin{aligned} E\left[c_{24}\right] &= L_6 e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(2)} x_{n-l}^{(2)}\right]}_{\textit{uncorrelated}} = \\ &= L_6 e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \underbrace{E\left[\left(x_{n-k}^{(2)}\right)^2\right]}_{\textit{symmetry}} = 0 \end{aligned} \tag{B.119}$$

$$
\begin{aligned}
E\left[\left|c_{24}\right|^2\right] &= E\left[\left|L_6 e^{-j\angle x_n^{(1)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(2)} x_{n-l}^{(2)}\right|^2\right] = \\
&= \left|L_6\right|^2 \sigma_{x^{(2)}}^4 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(2)}}{\sigma_{x^{(2)}}}\right|^2 \left|\frac{x_{n-l}^{(2)}}{\sigma_{x^{(2)}}}\right|^2\right]
\end{aligned}
\tag{B.120}
$$

B.2.6.3 Coefficient c_{25}:

$$
c_{25} = 2 \cdot L_6 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)^*} x_{n-l}^{(2)}
\tag{B.121}
$$

$$
\begin{aligned}
E\left[c_{25}\right] &= 2 \cdot L_6 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(1)^*} x_{n-l}^{(2)}\right]}_{independent} = \\
&= 2 \cdot L_6 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} E\left[x_{n-k}^{(1)^*}\right] E\left[x_{n-l}^{(2)}\right] = 0
\end{aligned}
\tag{B.122}
$$

$$
\begin{aligned}
E\left[\left|c_{25}\right|^2\right] &= E\left[\left|2 \cdot L_6 e^{j\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(1)^*} x_{n-l}^{(2)}\right|^2\right] = \\
&= 4 \cdot \left|L_6\right|^2 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|x_{n-k}^{(1)}\right|^2\right] E\left[\left|x_{n-l}^{(2)}\right|^2\right] = \\
&= 4 \cdot \left|L_6\right|^2 \sigma_{x^{(1)}}^2 \sigma_{x^{(2)}}^2 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0}^2
\end{aligned}
\tag{B.123}
$$

B.2.6.4 Coefficient c_{26}:

$$
c_{26} = L_6 e^{j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(1)^*}
\tag{B.124}
$$

$$
E\left[c_{26}\right] = L_6 e^{j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(1)^*}\right] = 0
\tag{B.125}
$$

$$\begin{aligned} E\left[|c_{26}|^2\right] &= E\left[\left|L_6 e^{j2\angle x_n^{(2)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(1)^*}\right|^2\right] = \\ &= |L_6|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00}\lambda_{l00} \underbrace{E\left[x_{n-k}^{(1)} x_{n-l}^{(1)^*}\right]}_{uncorrelated} = \\ &= |L_6|^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 E\left[\left|x_{n-k}^{(1)}\right|^2\right] = \\ &= |L_6|^2 \sigma_{x^{(1)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \end{aligned} \tag{B.126}$$

B.2.6.5 Coefficient c_{27}:

$$c_{27} = L_6 \lambda_{000} e^{j\left(2\angle x_n^{(2)} - \angle x_n^{(1)}\right)} \tag{B.127}$$

$$E\left[c_{27}\right] = L_6 \lambda_{000} e^{j\left(2\angle x_n^{(2)} - \angle x_n^{(1)}\right)} \tag{B.128}$$

$$E\left[|c_{27}|^2\right] = 0 \tag{B.129}$$

B.3 Statistical model

$$\begin{aligned}
z_{1n} = {} & x_{1n}\left(1+u^{(1)}+u^{(9)}+u^{(3)}\frac{|x_{1n}|^2}{\sigma_{x_1}^2}\right)+\left(v_n^{(1)}+v_n^{(9)}+v_n^{(14)}+v_n^{(19)}+v_n^{(24)}\right)\frac{|x_{1n}|}{\sigma_{x_1}}+\\
&+\left(v_n^{(2)}+v_n^{(15)}+v_n^{(20)}\right)\frac{|x_{1n}|^2}{\sigma_{x_1}^2}+x_{2n}\left(u^{(5)}+u^{(21)}+u^{(7)}\frac{|x_{2n}|^2}{\sigma_{x_2}^2}\right)+\\
&+\left(v_n^{(5)}+v_n^{(10)}+v_n^{(16)}+v_n^{(21)}+v_n^{(25)}\right)\frac{|x_{2n}|}{\sigma_{x_2}}+\left(v_n^{(6)}+v_n^{(11)}+v_n^{(26)}\right)\frac{|x_{2n}|^2}{\sigma_{x_2}^2}+\\
&+\left(u^{(22)}+u^{(27)}\right)\frac{|x_{1n}|^2}{\sigma_{x_1}^2}\frac{|x_{2n}|}{\sigma_{x_2}}+\left(u^{(12)}+u^{(17)}\right)\frac{|x_{1n}|}{\sigma_{x_1}}\frac{|x_{2n}|^2}{\sigma_{x_2}^2}+\\
&+v_n^{(0)}+v_n^{(4)}+v_n^{(8)}+v_n^{(13)}+v_n^{(18)}+v_n^{(23)}+w_{1n}
\end{aligned} \tag{B.130}$$

C

Appendix. 3x2-Antenna model

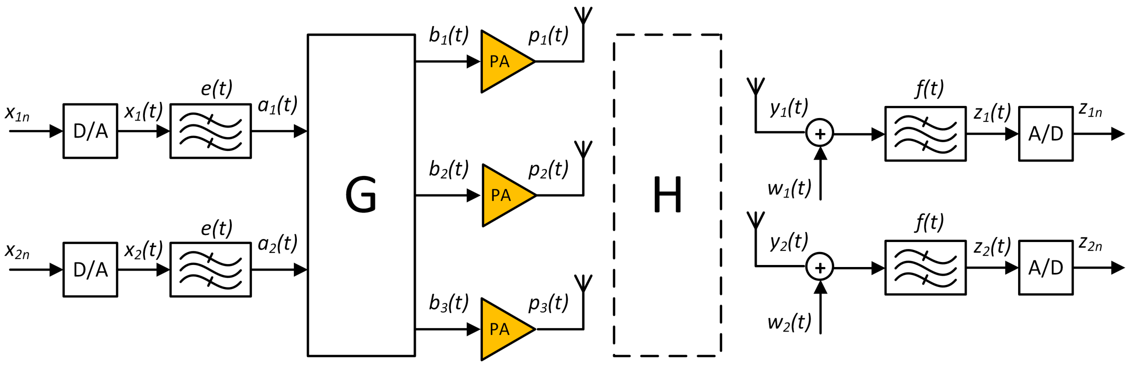

Figure C.1: Block diagram of the system consisting of 3 antennas and 2 users.

C.1 System model

$$\mathbf{H} = \frac{1}{\sqrt{3}} \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{bmatrix} \tag{C.1}$$

$$\mathbf{G} = \frac{1}{\sqrt{3}} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix} \tag{C.2}$$

$$\mathbf{a} = \mathbf{x} \circledast e_n = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{C.3}$$

$$\mathbf{b} = \mathbf{Ga} = \frac{1}{\sqrt{3}} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{C.4}$$

$$\mathbf{c} = \begin{bmatrix} b_1 + \alpha_1 b_1 |b_1|^2 \\ b_2 + \alpha_2 b_2 |b_2|^2 \\ b_3 + \alpha_3 b_3 |b_3|^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \tag{C.5}$$

$$\mathbf{y} = \mathbf{Hc} = \frac{1}{\sqrt{3}} \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{C.6}$$

$$\mathbf{z} = [\mathbf{y} + \mathbf{w}] \circledast f_n = \left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \right) \circledast f_n = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{C.7}$$

$$a_1(t) = x_1(t) * e(t) \tag{C.8}$$

$$a_2(t) = x_2(t) * e(t) \tag{C.9}$$

$$b_1(t) = \frac{1}{\sqrt{3}} \left[x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) \right] \tag{C.10}$$

$$b_2(t) = \frac{1}{\sqrt{3}} \left[x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) \right] \tag{C.11}$$

$$b_3(t) = \frac{1}{\sqrt{3}} \left[x_1(t) * e(t) * g_{31}(t) + x_2(t) * e(t) * g_{32}(t) \right] \tag{C.12}$$

$$\begin{aligned} p_1(t) &= b_1(t) + \alpha_1 b_1(t) |b_1(t)|^2 = \left(|b_1(t)| + \alpha_1 |b_1(t)|^3 \right) e^{j\angle b_1(t)} = \\ &= \left(\frac{1}{\sqrt{3}} |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{3^3}} \alpha_1 |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t)|^3 \right) e^{j\angle b_1(t)} \end{aligned} \tag{C.13}$$

$$\begin{aligned} p_2(t) &= b_2(t) + \alpha_2 b_2(t) |b_2(t)|^2 = \left(|b_2(t)| + \alpha_2 |b_2(t)|^3 \right) e^{j\angle b_2(t)} = \\ &= \left(\frac{1}{\sqrt{3}} |x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{3^3}} \alpha_2 |x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t)|^3 \right) e^{j\angle b_2(t)} \end{aligned} \tag{C.14}$$

$$\begin{aligned} p_3(t) &= b_3(t) + \alpha_3 b_3(t) |b_3(t)|^2 = \left(|b_3(t)| + \alpha_3 |b_3(t)|^3 \right) e^{j\angle b_3(t)} = \\ &= \left(\frac{1}{\sqrt{3}} |x_1(t) * e(t) * g_{31}(t) + x_2(t) * e(t) * g_{32}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{3^3}} \alpha_3 |x_1(t) * e(t) * g_{31}(t) + x_2(t) * e(t) * g_{32}(t)|^3 \right) e^{j\angle b_3(t)} \end{aligned} \tag{C.15}$$

$$y_1(t) = \frac{1}{\sqrt{3}} \left[p_1(t) * h_{11}(t) + p_2(t) * h_{12}(t) + p_3(t) * h_{13}(t) \right] \tag{C.16}$$

$$y_2(t) = \frac{1}{\sqrt{3}} \left[p_1(t) * h_{21}(t) + p_2(t) * h_{22}(t) + p_3(t) * h_{23}(t) \right] \tag{C.17}$$

$$
\begin{aligned}
z_1(t) &= \left[\frac{1}{\sqrt{3}}\left[p_1(t)*h_{11}(t)+p_2(t)*h_{12}(t)+p_3(t)*h_{13}(t)\right]+w_1(t)\right]*f(t)= \\
&= \frac{1}{3}\left[x_1(t)*(h_{11}(t)*g_{11}(t)+h_{12}(t)*g_{21}(t)+h_{13}(t)*g_{31}(t))\right. \\
&+ \left. x_2(t)*(h_{11}(t)*g_{12}(t)+h_{12}(t)*g_{22}(t)+h_{13}(t)*g_{32}(t))\right] \\
&+ w_1(t)*f(t)+ \\
&+ \frac{1}{3^2}\cdot\lambda_{klm}\left[x_1(t)x_1(t)x_1^*(t)*\underbrace{\left(\alpha_1 h_{11}(t)*g_{11}(t)|g_{11}(t)|^2+\alpha_2 h_{12}(t)*g_{21}(t)|g_{21}(t)|^2+\alpha_3 h_{13}(t)*g_{31}(t)|g_{31}(t)|^2\right)}_{L_1}+\right. \\
&+ x_2(t)x_2(t)x_2^*(t)*\underbrace{\left(\alpha_1 h_{11}(t)*g_{12}(t)|g_{12}(t)|^2+\alpha_2 h_{12}(t)*g_{22}(t)|g_{22}(t)|^2+\alpha_3 h_{13}(t)*g_{32}(t)|g_{32}(t)|^2\right)}_{L_2}+ \\
&+ x_1(t)x_2(t)x_2^*(t)*\underbrace{2\cdot\left(\alpha_1 h_{11}(t)*g_{11}(t)|g_{12}(t)|^2+\alpha_2 h_{12}(t)*g_{21}(t)*|g_{22}(t)|^2+\alpha_3 h_{13}(t)*g_{31}(t)*|g_{32}(t)|^2\right)}_{L_3}+ \\
&+ x_1(t)x_1(t)x_2^*(t)\underbrace{\left(\alpha_1 h_{11}(t)*(g_{11}(t))^2*g_{12}^*(t)+\alpha_2 h_{12}(t)*(g_{21}(t))^2*g_{22}^*(t)+\alpha_3 h_{13}(t)*(g_{31}(t))^2*g_{32}^*(t)\right)}_{L_4}+ \\
&+ x_1(t)x_1^*(t)x_2(t)*\underbrace{2\cdot\left(\alpha_1 h_{11}(t)*|g_{11}(t)|^2*g_{12}(t)+\alpha_2 h_{12}(t)*|g_{21}(t)|^2*g_{22}(t)+\alpha_3 h_{13}(t)*|g_{31}(t)|^2*g_{32}(t)\right)}_{L_5}+ \\
&+ \left. x_1^*(t)x_2(t)x_2(t)*\underbrace{\left(\alpha_1 h_{11}(t)*g_{11}^*(t)*(g_{12}(t))^2+\alpha_2 h_{12}(t)*g_{21}^*(t)*(g_{22}(t))^2+\alpha_3 h_{13}(t)*g_{31}^*(t)*(g_{32}(t))^2\right)}_{L_6}\right]+
\end{aligned}
\tag{C.18}
$$

$$d_n = \frac{1}{3^2}\Bigg[\underbrace{\lambda_{klm}L_1x_1x_1x_1^*}_{d_{x_{1n}}} + \underbrace{\lambda_{klm}L_2x_2x_2x_2^*}_{d_{x_{2n}}} + \underbrace{\lambda_{klm}L_3x_1x_2x_2^*}_{d_{x_{1n}x_{2n}x_{2n}^*}} + \\ + \underbrace{\lambda_{klm}L_4x_1x_1x_2^*}_{d_{x_{1n}x_{1n}x_{2n}^*}} + \underbrace{\lambda_{klm}L_5x_1x_1^*x_2}_{d_{x_{1n}x_{1n}^*x_{2n}}} + \underbrace{\lambda_{klm}L_6x_1^*x_2x_2}_{d_{x_{1n}^*x_{2n}x_{2n}}}\Bigg] \tag{C.19}$$

D

Appendix. Mx2-Antenna model

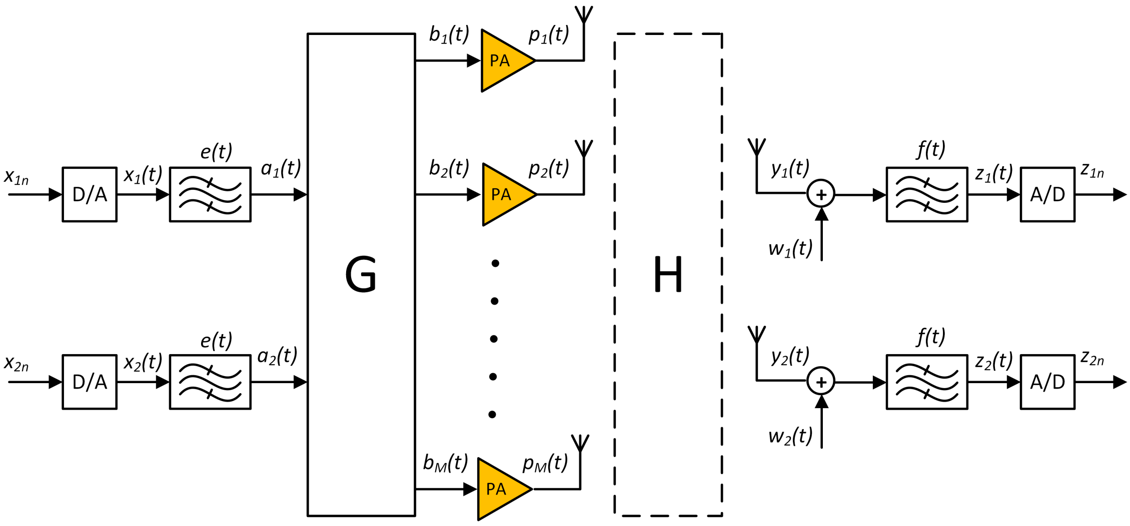

Figure D.1: Block diagram of the system consisting of M antennas and 2 users.

D.1 System model

$$\mathbf{H} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \end{bmatrix} \tag{D.1}$$

$$\mathbf{G} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ \vdots & \vdots \\ g_{M1} & g_{M2} \end{bmatrix} \tag{D.2}$$

$$\mathbf{a} = \mathbf{x} \circledast e_n = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{D.3}$$

$$\mathbf{b} = \mathbf{Ga} = \frac{1}{\sqrt{M}} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ \vdots & \vdots \\ g_{M1} & g_{M2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix} \tag{D.4}$$

$$\mathbf{p} = \begin{bmatrix} b_1 + \alpha_1 b_1 |b_1|^2 \\ b_2 + \alpha_2 b_2 |b_2|^2 \\ \vdots \\ b_M + \alpha_M b_M |b_M|^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} \tag{D.5}$$

$$\mathbf{y} = \mathbf{H}\mathbf{p} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{D.6}$$

$$\mathbf{z} = [\mathbf{y} + \mathbf{w}] \circledast f_n = \left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \right) \circledast f_n = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{D.7}$$

$$a_1(t) = x_1(t) * e(t) \tag{D.8}$$

$$a_2(t) = x_2(t) * e(t) \tag{D.9}$$

$$b_1(t) = \frac{1}{\sqrt{M}} \left[x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) \right] \tag{D.10}$$

$$b_2(t) = \frac{1}{\sqrt{M}} \left[x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) \right] \tag{D.11}$$

$$b_M(t) = \frac{1}{\sqrt{M}} \left[x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t) \right] \tag{D.12}$$

$$\begin{aligned} p_1(t) &= b_1(t) + \alpha_1 b_1(t) |b_1(t)|^2 = \left(|b_1(t)| + \alpha_1 |b_1(t)|^3 \right) e^{j\angle b_1(t)} = \\ &= \left(\frac{1}{\sqrt{M}} |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{M^3}} \alpha_1 |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t)|^3 \right) e^{j\angle b_1(t)} \end{aligned} \tag{D.13}$$

$$\begin{aligned} p_2(t) &= b_2(t) + \alpha_2 b_2(t) |b_2(t)|^2 = \left(|b_2(t)| + \alpha_2 |b_2(t)|^3 \right) e^{j\angle b_2(t)} = \\ &= \left(\frac{1}{\sqrt{M}} |x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{M^3}} \alpha_2 |x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t)|^3 \right) e^{j\angle b_2(t)} \end{aligned} \tag{D.14}$$

$$
\begin{aligned}
p_M(t) &= b_M(t) + \alpha_M b_M(t)|b_M(t)|^2 = \left(|b_M(t)| + \alpha_M |b_M(t)|^3\right) e^{j\angle b_M(t)} = \\
&= \left(\frac{1}{\sqrt{M}}|x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t)| + \right. \\
&\left. + \frac{1}{\sqrt{M^3}}\alpha_M |x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t)|^3\right) e^{j\angle b_M(t)}
\end{aligned}
\tag{D.15}
$$

$$
y_1(t) = \frac{1}{\sqrt{M}} \cdot [p_1(t) * h_{11}(t) + p_2(t) * h_{12}(t) + \ldots + p_M(t) * h_{1M}(t)] \tag{D.16}
$$

$$
y_2(t) = \frac{1}{\sqrt{M}} \cdot [p_1(t) * h_{21}(t) + p_2(t) * h_{22}(t) + \ldots + p_M(t) * h_{2M}(t)] \tag{D.17}
$$

$$
y_k(t) = \frac{1}{\sqrt{M}} \cdot \sum_{m=1}^{M} p_m(t) * h_{km}(t) \tag{D.18}
$$

$$
\begin{aligned}
z_1(t) &= \left[\frac{1}{\sqrt{M}} \cdot \sum_{m=1}^{M} p_m(t) * h_{1m}(t) + w_1(t)\right] * f(t) = \\
&= \frac{1}{M}\left[\sum_{k=1}^{2} x_i(t) * \left(\sum_{m=1}^{M} h_{km}(t) * g_{mk}(t)\right)\right] + w_1(t) * f(t) + \\
&+ \frac{1}{M^2} \cdot \lambda_{klm} \Bigg[x_1(t)x_1(t)x_1^*(t) * \underbrace{\left(\sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t)|g_{m1}(t)|^2\right)}_{L_1} + \\
&+ x_2(t)x_2(t)x_2^*(t) * \underbrace{\left(\sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m2}(t)|g_{m2}(t)|^2\right)}_{L_2} + \\
&+ x_1(t)x_2(t)x_2^*(t) * \underbrace{\left(2 \cdot \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t)|g_{m2}(t)|^2\right)}_{L_3} + \\
&+ x_1(t)x_1(t)x_2^*(t) \underbrace{\left(\sum_{m=1}^{M} \alpha_m h_{1m}(t) * (g_{m1}(t))^2 * g_{m2}^*(t)\right)}_{L_4} + \\
&+ x_1(t)x_1^*(t)x_2(t) * \underbrace{\left(2 \cdot \sum_{m=1}^{M} \alpha_m h_{1m}(t) * |g_{m1}(t)|^2 * g_{m2}(t)\right)}_{L_5} + \\
&+ x_1^*(t)x_2(t)x_2(t) * \underbrace{\left(\sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}^*(t) * (g_{m2}(t))^2\right)}_{L_6} \Bigg]
\end{aligned}
\tag{D.19}
$$

$$d_n = \frac{1}{M^2}\Bigg[\underbrace{\lambda_{klm}L_1x_1x_1x_1^*}_{d_{x_{1n}}} + \underbrace{\lambda_{klm}L_2x_2x_2x_2^*}_{d_{x_{2n}}} + \underbrace{\lambda_{klm}L_3x_1x_2x_2^*}_{d_{x_{1n}x_{2n}x_{2n}^*}} + \\ + \underbrace{\lambda_{klm}L_4x_1x_1x_2^*}_{d_{x_{1n}x_{1n}x_{2n}^*}} + \underbrace{\lambda_{klm}L_5x_1x_1^*x_2}_{d_{x_{1n}x_{1n}^*x_{2n}}} + \underbrace{\lambda_{klm}L_6x_1^*x_2x_2}_{d_{x_{1n}^*x_{2n}x_{2n}}}\Bigg] \quad \text{(D.20)}$$

E

Appendix. Mx3-Antenna model

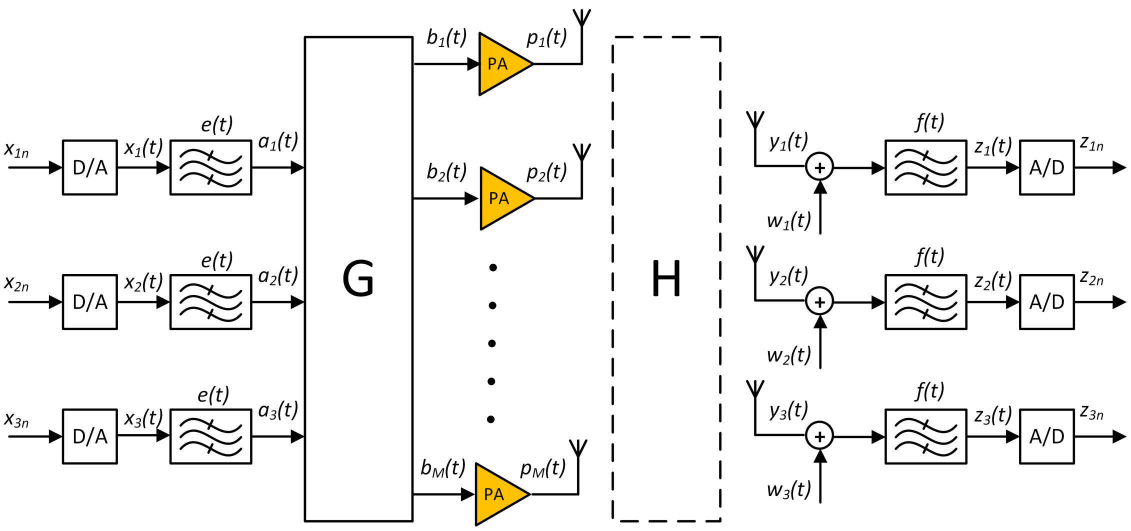

Figure E.1: Block diagram of the system consisting of M antennas and 3 users.

E.1 System model

$$\mathbf{H} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ h_{31} & h_{32} & \dots & h_{3M} \end{bmatrix} \tag{E.1}$$

$$\mathbf{G} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ \vdots & \vdots & \vdots \\ g_{M1} & g_{M2} & g_{M3} \end{bmatrix} \tag{E.2}$$

$$\mathbf{a} = \mathbf{x} \circledast e_n = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{E.3}$$

$$\mathbf{b} = \mathbf{G}\mathbf{a} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ \vdots & \vdots & \vdots \\ g_{M1} & g_{M2} & g_{M3} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix} \tag{E.4}$$

$$\mathbf{p} = \begin{bmatrix} b_1 + \alpha_1 b_1 |b_1|^2 \\ b_2 + \alpha_2 b_2 |b_2|^2 \\ \vdots \\ b_M + \alpha_M b_M |b_M|^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} \tag{E.5}$$

$$y = \mathbf{H}\mathbf{p} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ h_{31} & h_{32} & \dots & h_{3M} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{E.6}$$

$$\mathbf{z} = [\mathbf{y} + \mathbf{w}] \circledast f_n = \left(\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \right) \circledast f_n = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{E.7}$$

$$a_1(t) = x_1(t) * e(t) \tag{E.8}$$

$$a_2(t) = x_2(t) * e(t) \tag{E.9}$$

$$a_3(t) = x_3(t) * e(t) \tag{E.10}$$

$$b_1(t) = \frac{1}{\sqrt{M}} \left[x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) + x_3(t) * e(t) * g_{13}(t) \right] \tag{E.11}$$

$$b_2(t) = \frac{1}{\sqrt{M}} \left[x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) + x_3(t) * e(t) * g_{23}(t) \right] \tag{E.12}$$

$$b_M(t) = \frac{1}{\sqrt{M}} \left[x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t) + x_3(t) * e(t) * g_{M3}(t) \right] \tag{E.13}$$

$$\begin{aligned} p_1(t) &= b_1(t) + \alpha_1 b_1(t)|b_1(t)|^2 = \left(|b_1(t)| + \alpha_1 |b_1(t)|^3 \right) e^{j\angle b_1(t)} = \\ &= \left(\frac{1}{\sqrt{M}} |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) + x_3(t) * e(t) * g_{13}(t)| + \right. \\ &+ \left. \frac{1}{\sqrt{M^3}} \alpha_1 |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) + x_3(t) * e(t) * g_{13}(t)|^3 \right) e^{j\angle b_1(t)} \end{aligned} \tag{E.14}$$

$$p_2(t) = b_2(t) + \alpha_2 b_2(t)|b_2(t)|^2 = \left(|b_2(t)| + \alpha_2|b_2(t)|^3\right) e^{j\angle b_2(t)} =$$
$$= \left(\frac{1}{\sqrt{M}}|x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) + x_3(t) * e(t) * g_{23}(t)| + \right.$$
$$\left. + \frac{1}{\sqrt{M^3}}\alpha_2|x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) + x_3(t) * e(t) * g_{23}(t)|^3\right) e^{j\angle b_2(t)} \tag{E.15}$$

$$p_M(t) = b_M(t) + \alpha_M b_M(t)|b_M(t)|^2 = \left(|b_M(t)| + \alpha_M|b_M(t)|^3\right) e^{j\angle b_M(t)} =$$
$$= \left(\frac{1}{\sqrt{M}}|x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t) + x_3(t) * e(t) * g_{M3}(t)| + \right.$$
$$\left. + \frac{1}{\sqrt{M^3}}\alpha_M|x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t) + x_3(t) * e(t) * g_{M3}(t)|^3\right) e^{j\angle b_M(t)} \tag{E.16}$$

$$y_1(t) = \frac{1}{\sqrt{M}} \cdot (p_1(t) * h_{11}(t) + p_2(t) * h_{12}(t) + \ldots + p_M(t) * h_{1M}(t)] \tag{E.17}$$

$$y_2(t) = \frac{1}{\sqrt{M}} \cdot [p_1(t) * h_{21}(t) + p_2(t) * h_{22}(t) + \ldots + p_M(t) * h_{2M}(t)] \tag{E.18}$$

$$y_3(t) = \frac{1}{\sqrt{M}} \cdot [p_1(t) * h_{31}(t) + p_2(t) * h_{32}(t) + \ldots + p_M(t) * h_{3M}(t)] \tag{E.19}$$

$$y_k(t) = \frac{1}{\sqrt{M}} \cdot \sum_{m=1}^{M} p_m(t) * h_{km}(t) \tag{E.20}$$

$$
\begin{aligned}
z_1(t) = {} & \frac{1}{M}\left[\sum_{k=1}^{3} x_k(t) * \left(\sum_{m=1}^{M} h_{km}(t) * g_{mk}(t)\right)\right] + w_1(t) * f(t) + \\
& + \frac{1}{M^2}\cdot\lambda_{rst}\left[\left(\sum_{k=1}^{3} x_k(t)x_k(t)x_k^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{mk}(t)\left|g_{mk}(t)\right|^2\right) + \right. \\
& + \left(\sum_{\substack{k=1\\k\neq 1}}^{3} x_1(t)x_1(t)x_k^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * \left(g_{m1}(t)\right)^2 * g_{mk}^*(t)\right) + \\
& + \left(\sum_{\substack{k=1\\k\neq 1}}^{3} 2\cdot x_1(t)x_1^*(t)x_k(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * \left|g_{m1}(t)\right|^2 * g_{mk}(t)\right) + \\
& + \left(\sum_{\substack{k=1\\k\neq 1}}^{3} 2\cdot x_1(t)x_k(t)x_k^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}(t)\left|g_{mk}(t)\right|^2\right) + \\
& + \left(\sum_{\substack{k=1\\k\neq 1}}^{3} x_1^*(t)x_k(t)x_k(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}^*(t) * \left(g_{mk}(t)\right)^2\right) + \\
& + \left(\sum_{\substack{k=1\\k\neq 1}}^{3}\sum_{\substack{i=1\\i\neq 1\\i\neq k}}^{3} 2\cdot x_1(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}(t) * g_{km}(t) * g_{mi}^*(t)\right) + \\
& + \left(\sum_{\substack{k=1\\k\neq 1}}^{3}\sum_{\substack{i=1\\i\neq 1\\i\neq k}}^{3} 2\cdot x_1^*(t)x_k(t)x_i(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * g_{m1}^*(t) * g_{mk}(t) * g_{mi}(t)\right) + \\
& + \left(\sum_{\substack{k=1\\k\neq 1}}^{3}\sum_{\substack{i=1\\i\neq 1\\i\neq k}}^{3} 2\cdot x_k(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * \left(g_{mk}(t)\right)^2 * g_{mi}^*(t)\right) + \\
& \left. + \left(\sum_{\substack{k=1\\k\neq 1}}^{3}\sum_{\substack{i=1\\i\neq 1\\i\neq k}}^{3} x_k(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M}\alpha_m h_{1m}(t) * \left(g_{mk}(t)\right)^2 * g_{mi}(t)\right)\right]
\end{aligned}
\tag{E.21}
$$

$$
\begin{aligned}
d_1(t) = \frac{1}{M^2} \cdot \lambda_{rst} \Bigg[& \left(\sum_{k=1}^{3} x_k(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{mk}(t) \left|g_{mk}(t)\right|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} x_1(t) x_1(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * \left(g_{m1}(t)\right)^2 * g_{mk}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} 2 \cdot x_1(t) x_1^*(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * \left|g_{m1}(t)\right|^2 * g_{mk}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} 2 \cdot x_1(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t) \left|g_{mk}(t)\right|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} x_1^*(t) x_k(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}^*(t) * \left(g_{mk}(t)\right)^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} \sum_{\substack{i=1 \\ i \neq 1 \\ i \neq k}}^{3} 2 \cdot x_1(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}(t) * g_{km}(t) * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} \sum_{\substack{i=1 \\ i \neq 1 \\ i \neq k}}^{3} 2 \cdot x_1^*(t) x_k(t) x_i(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * g_{m1}^*(t) * g_{mk}(t) * g_{mi}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} \sum_{\substack{i=1 \\ i \neq 1 \\ i \neq k}}^{3} 2 \cdot x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * \left(g_{mk}(t)\right)^2 * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq 1}}^{3} \sum_{\substack{i=1 \\ i \neq 1 \\ i \neq k}}^{3} x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{1m}(t) * \left(g_{mk}(t)\right)^2 * g_{mi}(t) \right) \Bigg]
\end{aligned}
\tag{E.22}
$$

F

Appendix. MxK-Antenna model

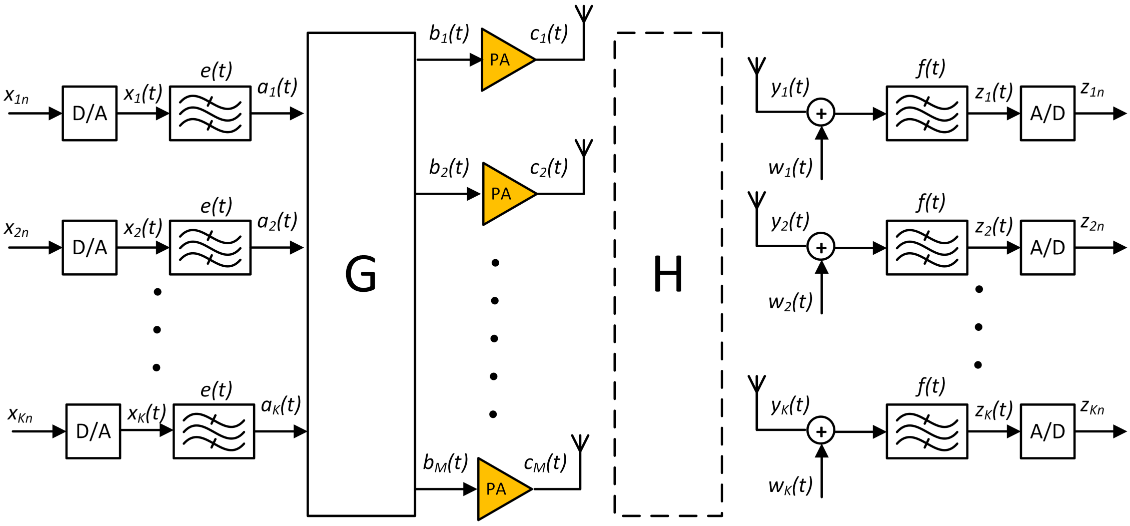

Figure F.1: Block diagram of the system consisting of M antennas and K users.

F.1 System model

$$\mathbf{H} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \dots & h_{KM} \end{bmatrix} \tag{F.1}$$

$$\mathbf{G} = \frac{1}{\sqrt{M}} \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1K} \\ g_{21} & g_{22} & \cdots & g_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M1} & g_{M2} & \cdots & g_{MK} \end{bmatrix} \tag{F.2}$$

$$\mathbf{a} = \mathbf{x} \circledast e_n = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \end{bmatrix} \tag{F.3}$$

$$\mathbf{b} = \mathbf{G}\mathbf{a} = \frac{1}{\sqrt{M}} \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1K} \\ g_{21} & g_{22} & \dots & g_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M1} & g_{M2} & \dots & g_{MK} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix} \tag{F.4}$$

$$\mathbf{p} = \begin{bmatrix} b_1 + \alpha_1 b_1 |b_1|^2 \\ b_2 + \alpha_2 b_2 |b_2|^2 \\ \vdots \\ b_M + \alpha_M b_M |b_M|^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} \tag{F.5}$$

$$\mathbf{y} = \mathbf{H}\mathbf{p} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \dots & h_{KM} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix} \tag{F.6}$$

$$\mathbf{z} = [\mathbf{y} + \mathbf{w}] \circledast f_n = \left(\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix} \right) \circledast f_n = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_M \end{bmatrix} \tag{F.7}$$

$$\begin{aligned} a_1(t) &= x_1(t) * e(t) \\ a_2(t) &= x_2(t) * e(t) \\ &\vdots \\ a_K(t) &= x_K(t) * e(t) \end{aligned} \tag{F.8}$$

$$\begin{aligned} b_1(t) &= x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) + \ldots + x_K(t) * e(t) * g_{1K}(t) \\ b_2(t) &= x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) + \ldots + x_K(t) * e(t) * g_{2K}(t) \\ &\vdots \\ b_M(t) &= x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t) + \ldots + x_K(t) * e(t) * g_{MK}(t) \end{aligned} \tag{F.9}$$

$$\begin{aligned}
p_1(t) &= b_1(t) + \alpha_1 b_1(t)|b_1(t)|^2 = \left(|b_1(t)| + \alpha_1|b_1(t)|^3\right) e^{j\angle b_1(t)} = \\
&= (|x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) + \ldots + x_K(t) * e(t) * g_{1K}(t)| + \\
&+ \alpha_1 |x_1(t) * e(t) * g_{11}(t) + x_2(t) * e(t) * g_{12}(t) + \ldots + x_K(t) * e(t) * g_{1K}(t)|^3\Big) e^{j\angle b_1(t)} \\
p_2(t) &= b_2(t) + \alpha_2 b_2(t)|b_2(t)|^2 = \left(|b_2(t)| + \alpha_2|b_2(t)|^3\right) e^{j\angle b_2(t)} = \\
&= (|x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) + \ldots + x_K(t) * e(t) * g_{2K}(t)| + \\
&+ \alpha_2 |x_1(t) * e(t) * g_{21}(t) + x_2(t) * e(t) * g_{22}(t) + \ldots + x_K(t) * e(t) * g_{2K}(t)|^3\Big) e^{j\angle b_2(t)} \\
&\vdots \\
p_M(t) &= b_M(t) + \alpha_M b_M(t)|b_M(t)|^2 = \left(|b_M(t)| + \alpha_M|b_M(t)|^3\right) e^{j\angle b_M(t)} = \\
&= (|x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t) + \ldots + x_K(t) * e(t) * g_{MK}(t)| + \\
&+ \alpha_M |x_1(t) * e(t) * g_{M1}(t) + x_2(t) * e(t) * g_{M2}(t) + \ldots + x_K(t) * e(t) * g_{MK}(t)|^3\Big) e^{j\angle b_M(t)}
\end{aligned} \tag{F.10}$$

$$\begin{aligned}
y_1(t) &= \frac{1}{\sqrt{M}} \cdot (p_1(t) * h_{11}(t) + p_2(t) * h_{12}(t) + \ldots + p_M(t) * h_{1M}(t)) \\
y_2(t) &= \frac{1}{\sqrt{M}} \cdot (p_1(t) * h_{21}(t) + p_2(t) * h_{22}(t) + \ldots + p_M(t) * h_{2M}(t)) \\
&\vdots \\
y_K(t) &= \frac{1}{\sqrt{M}} \cdot (p_1(t) * h_{K1}(t) + p_2(t) * h_{K2}(t) + \ldots + p_M(t) * h_{KM}(t))
\end{aligned} \tag{F.11}$$

$$
\begin{aligned}
z_A(t) = &\frac{1}{M}\left[\sum_{k=1}^{K} x_k(t) * \left(\sum_{m=1}^{M} h_{km}(t) * g_{mk}(t)\right)\right] + w_1(t) * f(t) + \\
&+ \frac{1}{M^2} \cdot \lambda_{rst}\left[\left(\sum_{k=1}^{K} x_k(t)x_k(t)x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mk}(t)|g_{mk}(t)|^2\right) + \right.\\
&+ \left(\sum_{\substack{k=1\\k\neq A}}^{K} x_A(t)x_A(t)x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * (g_{mA}(t))^2 * g_{mk}^*(t)\right) + \\
&+ \left(\sum_{\substack{k=1\\k\neq A}}^{K} 2 \cdot x_A(t)x_A^*(t)x_k(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * |g_{mA}(t)|^2 * g_{mk}(t)\right) + \\
&+ \left(\sum_{\substack{k=1\\k\neq A}}^{K} 2 \cdot x_A(t)x_k(t)x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}(t)|g_{mk}(t)|^2\right) + \\
&+ \left(\sum_{\substack{k=1\\k\neq A}}^{K} x_A^*(t)x_k(t)x_k(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}^*(t) * (g_{mk}(t))^2\right) + \\
&+ \left(\sum_{\substack{k=1\\k\neq A}}^{K} \sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} 2 \cdot x_A(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}(t) * g_{km}(t) * g_{mi}^*(t)\right) + \\
&+ \left(\sum_{\substack{k=1\\k\neq A}}^{K} \sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} 2 \cdot x_A^*(t)x_k(t)x_i(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}^*(t) * g_{mk}(t) * g_{mi}(t)\right) + \\
&+ \left(\sum_{\substack{k=1\\k\neq A}}^{K} \sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} 2 \cdot x_k(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * (g_{mk}(t))^2 * g_{mi}^*(t)\right) + \\
&+ \left.\left(\sum_{\substack{k=1\\k\neq A}}^{K} \sum_{\substack{i=1\\i\neq A\\i\neq k}}^{K} x_k(t)x_k(t)x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * (g_{mk}(t))^2 * g_{mi}(t)\right)\right]
\end{aligned}
\tag{F.12}
$$

$$
\begin{aligned}
d_A(t) = \frac{1}{M^2} \cdot \lambda_{rst} \Bigg[& \left(\sum_{k=1}^{K} x_k(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mk}(t) \left| g_{mk}(t) \right|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} x_A(t) x_A(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * \left(g_{mA}(t) \right)^2 * g_{mk}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} 2 \cdot x_A(t) x_A^*(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * \left| g_{mA}(t) \right|^2 * g_{mk}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} 2 \cdot x_A(t) x_k(t) x_k^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}(t) \left| g_{mk}(t) \right|^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} x_A^*(t) x_k(t) x_k(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}^*(t) * \left(g_{mk}(t) \right)^2 \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} 2 \cdot x_A(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}(t) * g_{km}(t) * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} 2 \cdot x_A^*(t) x_k(t) x_i(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * g_{mA}^*(t) * g_{mk}(t) * g_{mi}(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} 2 \cdot x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * \left(g_{mk}(t) \right)^2 * g_{mi}^*(t) \right) + \\
& + \left(\sum_{\substack{k=1 \\ k \neq A}}^{K} \sum_{\substack{i=1 \\ i \neq A \\ i \neq k}}^{K} x_k(t) x_k(t) x_i^*(t) * \sum_{m=1}^{M} \alpha_m h_{Am}(t) * \left(g_{mk}(t) \right)^2 * g_{mi}(t) \right) \Bigg]
\end{aligned} \tag{F.13}
$$

F.2 Term analysis

In a M-by-K (with $K > 2$ and $M > K$) environment we will get these different terms for each user:

1. $x_i(t)x_i(t)x_i^*(t)$. Term where the three signals are from the same source (one of them conjugate). We will get this term K times.
2. $x_i(t)x_i(t)x_j^*(t)$. Term where two signals are from the same source (none of them conjugate) and the other one from a different signal (and conjugate). We will get this term 2K times.
3. $2 \cdot x_i(t)x_i^*(t)x_j(t)$. Term where two signals are from the same source (one of them conjugate) and the other one from a different signal (and no-conjugate). We will get this term 2K times as well.
4. $2 \cdot x_i(t)x_j(t)x_k^*(t)$. Term where all three signals are from different sources (and one of them conjugate). We will get this term K times.

F.2.1 $x_i(t)x_i(t)x_i^*(t)$

1. User A.1 case

$$x_A(t)x_A(t)x_A^*(t) \tag{F.14}$$

2. Other distortion. (K-1) cases

$$\sum_{\substack{i=1 \\ i \neq A}}^{K} x_i(t)x_i(t)x_i^*(t) \tag{F.15}$$

F.2.2 $x_i(t)x_i(t)x_j^*(t)$

1. (K-1) cases

$$x_A(t)x_A(t)\sum_{\substack{i=1 \\ i \neq A}}^{K} x_i^*(t) \tag{F.16}$$

2. (K-1) cases

$$x_A^*(t)\sum_{\substack{i=1 \\ i \neq A}}^{K} x_i(t)x_i(t) \tag{F.17}$$

3. Other distortion. $(K-1)(K-2)$ cases

$$\sum_{\substack{i=1 \\ i \neq A}}^{K} \sum_{\substack{j=1 \\ j \neq i \\ j \neq A}}^{K} x_i(t)x_i(t)x_j^*(t) \tag{F.18}$$

F.2.3 $2 \cdot x_i(t)x_i^*(t)x_j(t)$

1. (K-1) cases

$$2 \cdot x_A(t)x_A^*(t)\sum_{\substack{i=1 \\ i \neq A}}^{K} x_i(t) \tag{F.19}$$

2. (K-1) cases

$$2 \cdot x_A(t)\sum_{\substack{i=1 \\ i \neq A}}^{K} x_i(t)x_i^*(t) \tag{F.20}$$

3. Other distortion. $(K-1)(K-2)$ cases

$$2 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sum_{\substack{j=1 \\ j \neq i \\ j \neq A}}^{K} x_i(t)x_i^*(t)x_j(t) \tag{F.21}$$

F.2.4 $2 \cdot x_i(t)x_j(t)x_k^*(t)$

1. $(K-1)(K-2)$ cases

$$4 \cdot x_A(t) \sum_{\substack{i=1 \\ i\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq i \\ j\neq A}}^{K} x_i(t)x_j^*(t) \tag{F.22}$$

2. $(K-1)(K-2)$ cases

$$2 \cdot x_A^*(t) \sum_{\substack{i=1 \\ i\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq i \\ j\neq A}}^{K} x_i(t)x_i(t)x_j(t) \tag{F.23}$$

3. Other distortion. $(K-1)(K-2)(K-3)$ cases

$$2 \cdot \sum_{\substack{i=1 \\ i\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq i \\ j\neq A}}^{K} \sum_{\substack{k=1 \\ k\neq i \\ k\neq j \\ k\neq A}}^{K} x_i(t)x_j(t)x_k^*(t) \tag{F.24}$$

F.3 Distortion analysis

F.3.1 $x_i(t)x_i(t)x_i^*(t)$

F.3.1.1 $x_A(t)x_A(t)x_A^*(t)$

$$d_{x_A(t)x_A(t)x_A^*(t)} = c_0 + c_1|x_{An}| + c_2|x_{An}|^2 + c_3|x_{An}|^3 \tag{F.25}$$

$$d_{x_A(t)x_A(t)x_A^*(t)} = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \lambda_{klm}\left|x_{n-k}^{(A)}\right|\left|x_{n-l}^{(A)}\right|\left|x_{n-m}^{(A)^*}\right| e^{j\left(\angle x_{n-k}^{(A)}+\angle x_{n-l}^{(A)}-\angle x_{n-m}^{(A)}\right)} \tag{F.26}$$

F.3.1.1.1 Coefficient c_0:

$$c_0 = \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(A)} x_{n-l}^{(A)} x_{n-m}^{(A)^*} \tag{F.27}$$

$$E\left[c_0\right] = 0 \tag{F.28}$$

$$E\left[\left|c_0\right|^2\right] = \sigma_{x^{(A)}}^6 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm}^2 \underbrace{E\left[\left|\frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\left|\frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\left|\frac{x_{n-m}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\right]}_{\substack{\text{constellation-dependent parameter}\\ \text{(independent of transmit power } \sigma_{x^{(A)}}^2)}} \tag{F.29}$$

F.3.1.1.2 Coefficient c_1:

$$c_1 = \left[e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)} + 2e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)^*}\right] \tag{F.30}$$

$$E\left[c_1\right] = 2\sigma_{X^{(A)}}^2 e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty} \lambda_{kk0} \tag{F.31}$$

$$E\left[\left|c_1\right|^2\right] = 5\sigma_{x^{(A)}}^4 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\left|\frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\right] \tag{F.32}$$

Coefficient c_1':

$$c_1' = c_1 - E\left[c_1\right] =$$
$$= \left[e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)} + 2e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)^*} - \right.$$
$$\left. - \left[2\sigma_{X^{(A)}}^2 e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \right] \right] \tag{F.33}$$

$$E\left[c_1'\right] = 0 \tag{F.34}$$

$$E\left[\left|c_1'\right|^2\right] = 5\sigma_{x^{(A)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left| \frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}} \right|^2 \left| \frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}} \right|^2 \right] - \lambda_{kk0}\lambda_{ll0} \right) \tag{F.35}$$

F.3.1.1.3 Coefficient c_2:

$$c_2 = \left[e^{j2\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(A)^*} + 2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(A)} \right] \tag{F.36}$$

$$E\left[c_2\right] = 0 \tag{F.37}$$

$$E\left[\left|c_2\right|^2\right] = 5\sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{F.38}$$

F.3.1.1.4 Coefficient c_3:

$$c_3 = \lambda_{000} e^{j\angle x_n^{(A)}} \tag{F.39}$$

$$E\left[c_3\right] = \lambda_{000} e^{j\angle x_n^{(A)}} \tag{F.40}$$

$$E\left[\left|c_3\right|^2\right] = 0 \tag{F.41}$$

F.3.1.2 Other distortion. (K-1) cases ⇒ DISTORTION

F.3.2 $x_i(t)x_i(t)x_j^*(t)$

F.3.2.1 $x_A(t)x_A(t)x_i^*(t)$

$$d_{x_A(t)x_A(t)x_i^*(t)} = c_4 + c_5|x_{An}| + c_6|x_{An}|^2 + c_7|x_{in}| + c_8|x_{An}|^2|x_{in}| \tag{F.42}$$

$$d_{x_A(t)x_A(t)x_i^*(t)} = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \lambda_{klm}\left|x_{n-k}^{(A)}\right|\left|x_{n-l}^{(A)}\right|\left|x_{n-m}^{(i)^*}\right|e^{j\left(\angle x_{n-k}^{(A)}+\angle x_{n-l}^{(A)}-\angle x_{n-m}^{(i)}\right)} \tag{F.43}$$

F.3.2.1.1 Coefficient c_4:

$$c_4 = \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(A)} x_{n-l}^{(A)} x_{n-m}^{(i)^*} \tag{F.44}$$

$$\begin{aligned} E\left[c_4\right] &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(A)} x_{n-l}^{(A)} x_{n-m}^{(i)^*}\right]}_{\substack{\text{independent}\\\text{and uncorrelated}}} = \\ &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kkl} \underbrace{E\left[\left(x_{n-k}^{(A)}\right)^2\right]}_{0} \underbrace{E\left[x_{n-l}^{(i)^*}\right]}_{0} = 0 \end{aligned} \tag{F.45}$$

$$\begin{aligned} E\left[\left|c_4\right|^2\right] &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(A)}\right|^2\left|x_{n-l}^{(A)}\right|^2\left|x_{n-m}^{(i)}\right|^2\right] = \\ &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(A)}\right|^2\left|x_{n-l}^{(A)}\right|^2\right] E\left[\left|x_{n-m}^{(i)}\right|^2\right] = \\ &= \sigma_{x^{(A)}}^4 \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\left|\frac{x_{n-m}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\right] \end{aligned} \tag{F.46}$$

F.3.2.1.2 Coefficient c_5:

$$c_5 = 2\cdot e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(i)^*} \tag{F.47}$$

$$\begin{aligned} E\left[c_5\right] &= 2\cdot e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(A)} x_{n-l}^{(i)^*}\right]}_{\textit{independent}} = \\ &= 2\cdot e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} E\left[x_{n-k}^{(A)}\right] E\left[x_{n-l}^{(i)^*}\right] = 0 \end{aligned} \tag{F.48}$$

$$
\begin{aligned}
E\left[|c_5|^2\right] &= E\left[\left|2 \cdot e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(i)^*}\right|^2\right] = \\
&= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|x_{n-k}^{(A)}\right|^2\right] E\left[\left|x_{n-l}^{(i)}\right|^2\right] = \\
&= 4 \cdot \sigma_{x^{(A)}}^2 \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2
\end{aligned}
\tag{F.49}
$$

F.3.2.1.3 Coefficient c_6:

$$
c_6 = e^{j2\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)^*} \tag{F.50}
$$

$$
E\left[c_6\right] = e^{j2\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(i)^*}\right] = 0 \tag{F.51}
$$

$$
\begin{aligned}
E\left[|c_6|^2\right] &= E\left[\left|e^{j2\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)^*}\right|^2\right] = \\
&= \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00}\lambda_{l00} \underbrace{E\left[x_{n-k}^{(i)} x_{n-l}^{(i)^*}\right]}_{uncorrelated} = \\
&= \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 E\left[\left|x_{n-k}^{(i)}\right|^2\right] = \\
&= \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2
\end{aligned}
\tag{F.52}
$$

F.3.2.1.4 Coefficient c_7:

$$
c_7 = e^{-j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)} \tag{F.53}
$$

$$
\begin{aligned}
E\left[c_7\right] &= e^{-j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(A)} x_{n-l}^{(A)}\right]}_{uncorrelated} = \\
&= e^{-j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \underbrace{E\left[\left(x_{n-k}^{(A)}\right)^2\right]}_{symmetry} = 0
\end{aligned}
\tag{F.54}
$$

$$E\left[|c_7|^2\right] = E\left[\left|e^{-j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)}\right|^2\right] = \\ = \sigma_{x^{(A)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}}\right|^2 \left|\frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\right] \tag{F.55}$$

F.3.2.1.5 Coefficient c_8:

$$c_8 = \lambda_{000} e^{j\left(2\angle x_n^{(A)} - \angle x_n^{(i)}\right)} \tag{F.56}$$

$$E[c_8] = \lambda_{000} e^{j\left(2\angle x_n^{(A)} - \angle x_n^{(i)}\right)} \tag{F.57}$$

$$E\left[|c_8|^2\right] = 0 \tag{F.58}$$

F.3.2.2 $x_i(t)x_i(t)x_A^*(t)$

$$d_{x_i(t)x_i(t)x_A^*(t)} = c_9 + c_{10}|x_{in}| + c_{11}|x_{in}|^2 + c_{12}|x_{An}| + c_{13}|x_{in}|^2|x_{An}| \tag{F.59}$$

$$d_{x_i(t)x_i(t)x_A^*(t)} = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \lambda_{klm}\left|x_{n-k}^{(i)}\right|\left|x_{n-l}^{(i)}\right|\left|x_{n-m}^{(A)^*}\right| e^{A\left(\angle x_{n-k}^{(i)}+\angle x_{n-l}^{(i)}-\angle x_{n-m}^{(A)}\right)} \tag{F.60}$$

F.3.2.2.1 Coefficient c_9:

$$c_9 = \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(i)} x_{n-l}^{(i)} x_{n-m}^{(A)^*} \tag{F.61}$$

$$\begin{aligned} E\left[c_9\right] &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(i)} x_{n-l}^{(i)} x_{n-m}^{(A)^*}\right]}_{\substack{\text{independent}\\\text{and uncorrelated}}} = \\ &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kkl} \underbrace{E\left[\left(x_{n-k}^{(i)}\right)^2\right]}_{0} \underbrace{E\left[x_{n-l}^{(A)^*}\right]}_{0} = 0 \end{aligned} \tag{F.62}$$

$$\begin{aligned} E\left[\left|c_9\right|^2\right] &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(i)}\right|^2\left|x_{n-l}^{(i)}\right|^2\left|x_{n-m}^{(A)}\right|^2\right] = \\ &= \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(i)}\right|^2\left|x_{n-l}^{(i)}\right|^2\right] E\left[\left|x_{n-m}^{(A)}\right|^2\right] = \\ &= \sigma_{x^{(i)}}^4 \sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-l}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\left|\frac{x_{n-m}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\right] \end{aligned} \tag{F.63}$$

F.3.2.2.2 Coefficient c_{10}:

$$c_{10} = 2\cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(i)} x_{n-l}^{(A)^*} \tag{F.64}$$

$$\begin{aligned} E\left[c_{10}\right] &= 2\cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(i)} x_{n-l}^{(A)^*}\right]}_{\textit{independent}} = \\ &= 2\cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty} \lambda_{kl0} E\left[x_{n-k}^{(i)}\right] E\left[x_{n-l}^{(A)^*}\right] = 0 \end{aligned} \tag{F.65}$$

$$
\begin{aligned}
E\left[\left|c_{10}\right|^2\right] &= E\left[\left|2 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(i)} x_{n-l}^{(A)^*}\right|^2\right] = \\
&= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|x_{n-k}^{(i)}\right|^2\right] E\left[\left|x_{n-l}^{(A)}\right|^2\right] = \\
&= 4 \cdot \sigma_{x^{(i)}}^2 \sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2
\end{aligned}
\tag{F.66}
$$

F.3.2.2.3 Coefficient c_{11}:

$$
c_{11} = e^{j2\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(A)^*} \tag{F.67}
$$

$$
E\left[c_{11}\right] = e^{j2\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(A)^*}\right] = 0 \tag{F.68}
$$

$$
\begin{aligned}
E\left[\left|c_{11}\right|^2\right] &= E\left[\left|e^{j2\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(A)^*}\right|^2\right] = \\
&= \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00} \lambda_{l00} \underbrace{E\left[x_{n-k}^{(A)} x_{n-l}^{(A)^*}\right]}_{uncorrelated} = \\
&= \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 E\left[\left|x_{n-k}^{(A)}\right|^2\right] = \\
&= \sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2
\end{aligned}
\tag{F.69}
$$

F.3.2.2.4 Coefficient c_{12}:

$$
c_{12} = e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(i)} x_{n-l}^{(i)} \tag{F.70}
$$

$$
\begin{aligned}
E\left[c_{12}\right] &= e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(i)} x_{n-l}^{(i)}\right]}_{uncorrelated} = \\
&= e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \underbrace{E\left[\left(x_{n-k}^{(i)}\right)^2\right]}_{symmetry} = 0
\end{aligned}
\tag{F.71}
$$

$$E\left[|c_{12}|^2\right] = E\left[\left|e^{-j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(i)}\right|^2\right] = \\ = \sigma_{x^{(i)}}^4\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\left|\frac{x_{n-l}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\right] \tag{F.72}$$

F.3.2.2.5 Coefficient c_{13}:

$$c_{13} = \lambda_{000}e^{j\left(2\angle x_n^{(i)} - \angle x_n^{(A)}\right)} \tag{F.73}$$

$$E\left[c_{13}\right] = \lambda_{000}e^{j\left(2\angle x_n^{(i)} - \angle x_n^{(A)}\right)} \tag{F.74}$$

$$E\left[|c_{13}|^2\right] = 0 \tag{F.75}$$

F.3.2.3 Other distortion. $(K-1)^2$ cases $\Rightarrow$ DISTORTION

F.3.3 $2 \cdot x_i(t)x_i^*(t)x_j(t)$

F.3.3.1 $2 \cdot x_A(t)x_A^*(t)x_i(t)$

$$d_{x_A(t)x_A^*(t)x_i(t)} = c_{14} + c_{15}|x_{An}| + c_{16}|x_{An}|^2 + c_{17}|x_{in}| + c_{18}|x_{An}|^2|x_{in}| \tag{F.76}$$

$$d_{x_A(t)x_A^*(t)x_i(t)} = 2 \cdot \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(A)}\right| \left|x_{n-l}^{(A)^*}\right| \left|x_{n-m}^{(i)}\right| e^{j\left(\angle x_{n-k}^{(A)} - \angle x_{n-l}^{(A)} + \angle x_{n-m}^{(i)}\right)} \tag{F.77}$$

F.3.3.1.1 Coefficient c_{14}:

$$c_{14} = 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(A)} x_{n-l}^{(A)^*} x_{n-m}^{(i)} \tag{F.78}$$

$$\begin{aligned} E\left[c_{14}\right] &= 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(A)} x_{n-l}^{(A)^*} x_{n-m}^{(i)}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kkl} E\left[\left|x_{n-k}^{(A)}\right|^2\right] \underbrace{E\left[x_{n-l}^{(i)}\right]}_{0} = 0 \end{aligned} \tag{F.79}$$

$$\begin{aligned} E\left[\left|c_{14}\right|^2\right] &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(A)}\right|^2 \left|x_{n-l}^{(A)}\right|^2 \left|x_{n-m}^{(i)}\right|^2\right] = \\ &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(A)}\right|^2 \left|x_{n-l}^{(A)}\right|^2\right] \underbrace{E\left[\left|x_{n-m}^{(i)}\right|^2\right]}_{\sigma_{x^{(i)}}^2} = \\ &= 4 \cdot \sigma_{x^{(A)}}^4 \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}}\right|^2 \left|\frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\right] \end{aligned} \tag{F.80}$$

F.3.3.1.2 Coefficient c_{15}:

$$c_{15} = 2 \cdot \left[e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(i)} + e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)^*} x_{n-l}^{(i)}\right] \tag{F.81}$$

$$E\left[c_{15}\right] = 2 \cdot \left[e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(A)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(i)}\right]}_{0} + \right.$$
$$\left. + \, e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(A)^*}\right]}_{0} \underbrace{E\left[x_{n-l}^{(i)}\right]}_{0} \right] = 0 \tag{F.82}$$

$$E\left[\left|c_{15}\right|^2\right] = E\left[\left|2 \cdot \left(e^{-j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(i)} + e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)^*} x_{n-l}^{(i)} \right)\right|^2\right] =$$
$$= 4 \cdot \sigma_{x^{(A)}}^2 \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 \tag{F.83}$$

F.3.3.1.3 Coefficient c_{16}:

$$c_{16} = 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)} \tag{F.84}$$

$$E\left[c_{16}\right] = 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(i)}\right] = 0 \tag{F.85}$$

$$E\left[\left|c_{16}\right|^2\right] = E\left[\left|2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)}\right|^2\right] =$$
$$= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{k00} \lambda_{l00} \underbrace{E\left[x_{n-k}^{(i)} x_{n-l}^{(i)^*}\right]}_{uncorrelated} = \tag{F.86}$$
$$= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 E\left[\left|x_{n-k}^{(i)}\right|^2\right] = 4 \cdot \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2$$

F.3.3.1.4 Coefficient c_{17}:

$$c_{17} = 2 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)^*} \tag{F.87}$$

$$E\left[c_{17}\right] = 2 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(A)} x_{n-l}^{(A)^*}\right]}_{uncorrelated} = \\
= 2 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} E\left[\left|x_k^{(A)}\right|^2\right] = \\
= 2 \cdot \sigma_{X^{(A)}}^2 e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \tag{F.88}$$

$$E\left[\left|c_{17}\right|^2\right] = E\left[\left|2 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)^*}\right|^2\right] = \\
= 4 \cdot \sigma_{x^{(A)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}}\right|^2 \left|\frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\right] \tag{F.89}$$

Coefficient c'_{17}:

$$c'_{17} = c_{17} - E\left[c_{17}\right] = 2 \cdot \left[e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(A)^*} - \right. \\
\left. - \left[\sigma_{X^{(A)}}^2 e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0}\right]\right] \tag{F.90}$$

$$E\left[c'_{17}\right] = 0 \tag{F.91}$$

$$E\left[\left|c'_{17}\right|^2\right] = 4 \cdot \sigma_{x^{(A)}}^4 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}}\right|^2 \left|\frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}}\right|^2\right] - \lambda_{kk0}\lambda_{ll0}\right) \tag{F.92}$$

F.3.3.1.5 Coefficient c_{18}:

$$c_{18} = 2 \cdot \lambda_{000} e^{j\angle x_n^{(i)}} \tag{F.93}$$

$$E\left[c_{18}\right] = 2 \cdot \lambda_{000} e^{j\angle x_n^{(i)}} \tag{F.94}$$

$$E\left[\left|c_{18}\right|^2\right] = 0 \tag{F.95}$$

F.3.3.2 $2 \cdot x_i(t)x_i^*(t)x_A(t)$

$$d_{x_i(t)x_i^*(t)x_A(t)} = c_{19} + c_{20}|x_{in}| + c_{21}|x_{in}|^2 + c_{22}|x_{An}| + c_{23}|x_{in}|^2|x_{An}| \tag{F.96}$$

$$d_{x_i(t)x_i^*(t)x_A(t)} = 2 \cdot \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(i)}\right| \left|x_{n-l}^{(i)*}\right| \left|x_{n-m}^{(A)}\right| e^{j\left(\angle x_{n-k}^{(i)} - \angle x_{n-l}^{(i)} + \angle x_{n-m}^{(A)}\right)} \tag{F.97}$$

F.3.3.2.1 Coefficient c_{19}:

$$c_{19} = 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(i)} x_{n-l}^{(i)*} x_{n-m}^{(A)} \tag{F.98}$$

$$\begin{aligned} E\left[c_{19}\right] &= 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(i)} x_{n-l}^{(i)*} x_{n-m}^{(A)}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kkl} E\left[\left|x_{n-k}^{(i)}\right|^2\right] \underbrace{E\left[x_{n-l}^{(A)}\right]}_{0} = 0 \end{aligned} \tag{F.99}$$

$$\begin{aligned} E\left[\left|c_{19}\right|^2\right] &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(i)}\right|^2 \left|x_{n-l}^{(i)}\right|^2 \left|x_{n-m}^{(A)}\right|^2\right] = \\ &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(i)}\right|^2 \left|x_{n-l}^{(i)}\right|^2\right] \underbrace{E\left[\left|x_{n-m}^{(A)}\right|^2\right]}_{\sigma_{x^{(A)}}^2} = \\ &= 4 \cdot \sigma_{x^{(i)}}^4 \sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|\frac{x_{n-k}^{(i)}}{\sigma_{x^{(i)}}}\right|^2 \left|\frac{x_{n-l}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\right] \end{aligned} \tag{F.100}$$

F.3.3.2.2 Coefficient c_{20}:

$$c_{20} = 2 \cdot \left[e^{-j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(i)} x_{n-l}^{(A)} + e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(i)*} x_{n-l}^{(A)} \right] \tag{F.101}$$

$$\begin{aligned} E\left[c_{20}\right] = 2 \cdot &\left[e^{-j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(i)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(A)}\right]}_{0} + \right. \\ &\left. + e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(i)*}\right]}_{0} \underbrace{E\left[x_{n-l}^{(A)}\right]}_{0} \right] = 0 \end{aligned} \tag{F.102}$$

$$E\left[|c_{20}|^2\right] = E\left[\left|2\cdot\left(e^{-j\angle x_n^{(i)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(A)} + e^{j\angle x_n^{(i)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)^*}x_{n-l}^{(A)}\right)\right|^2\right] =$$
$$= 4\cdot\sigma_{x^{(i)}}^2\sigma_{x^{(A)}}^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 \tag{F.103}$$

F.3.3.2.3 Coefficient c_{21}:

$$c_{21} = 2\cdot\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}x_{n-k}^{(A)} \tag{F.104}$$

$$E\left[c_{21}\right] = 2\cdot\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}E\left[x_{n-k}^{(A)}\right] = 0 \tag{F.105}$$

$$E\left[|c_{21}|^2\right] = E\left[\left|2\cdot\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}x_{n-k}^{(A)}\right|^2\right] =$$
$$= 4\cdot\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{k00}\lambda_{l00}\underbrace{E\left[x_{n-k}^{(A)}x_{n-l}^{(A)^*}\right]}_{uncorrelated} = \tag{F.106}$$
$$= 4\cdot\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}^2E\left[\left|x_{n-k}^{(A)}\right|^2\right] = 4\cdot\sigma_{x^{(A)}}^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}^2$$

F.3.3.2.4 Coefficient c_{22}:

$$c_{22} = 2\cdot e^{j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(i)^*} \tag{F.107}$$

$$E\left[c_{22}\right] = 2\cdot e^{j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}\underbrace{E\left[x_{n-k}^{(i)}x_{n-l}^{(i)^*}\right]}_{uncorrelated} =$$
$$= 2\cdot e^{j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{kk0}E\left[\left|x_k^{(i)}\right|^2\right] = \tag{F.108}$$
$$= 2\cdot\sigma_{X^{(i)}}^2e^{j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{kk0}$$

$$E\left[|c_{22}|^2\right] = E\left[\left|2\cdot e^{j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(i)^*}\right|^2\right] = \\ = 4\cdot\sigma_{x^{(i)}}^4\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\left|\frac{x_{n-l}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\right] \tag{F.109}$$

Coefficient c'_{22}:

$$c'_{22} = c_{22} - E\left[c_{22}\right] = 2\cdot\left[e^{j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(i)^*} - \right. \\ \left. - \left[\sigma_{X^{(i)}}^2 e^{j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{kk0}\right]\right] \tag{F.110}$$

$$E\left[c'_{22}\right] = 0 \tag{F.111}$$

$$E\left[|c'_{22}|^2\right] = 4\cdot\sigma_{x^{(i)}}^4\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\left(\lambda_{kl0}^2 E\left[\left|\frac{x_{n-k}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\left|\frac{x_{n-l}^{(i)}}{\sigma_{x^{(i)}}}\right|^2\right] - \lambda_{kk0}\lambda_{ll0}\right) \tag{F.112}$$

F.3.3.2.5 Coefficient c_{23}:

$$c_{23} = 2\cdot\lambda_{000}e^{j\angle x_n^{(A)}} \tag{F.113}$$

$$E\left[c_{23}\right] = 2\cdot\lambda_{000}e^{j\angle x_n^{(A)}} \tag{F.114}$$

$$E\left[|c_{23}|^2\right] = 0 \tag{F.115}$$

F.3.3.3 Other distortion. $(K-1)^2$ cases $\Rightarrow$ DISTORTION

F.3.4 $2 \cdot x_i(t)x_j(t)x_k^*(t)$

F.3.4.1 $4 \cdot x_A(t)x_i(t)x_j^*(t)$

$$d_{x_A(t)x_i(t)x_j^*(t)} = c_{24} + c_{25}|x_{An}| + c_{26}|x_{in}| + c_{27}|x_{jn}| + c_{28}|x_{An}||x_{in}| + c_{29}|x_{An}||x_{jn}| + \\ + c_{30}|x_{in}||x_{jn}| + c_{31}|x_{An}||x_{in}||x_{jn}| \tag{F.116}$$

$$d_{x_A(t)x_i(t)x_j^*(t)} = 4 \cdot \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(A)}\right|\left|x_{n-l}^{(i)}\right|\left|x_{n-m}^{(j)^*}\right| e^{j\left(\angle x_{n-k}^{(A)} + \angle x_{n-l}^{(i)} - \angle x_{n-m}^{(j)}\right)} \tag{F.117}$$

F.3.4.1.1 Coefficient c_{24}:

$$c_{24} = 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(A)} x_{n-l}^{(i)} x_{n-m}^{(j)^*} \tag{F.118}$$

$$\begin{aligned} E\left[c_{24}\right] &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(A)} x_{n-l}^{(i)} x_{n-m}^{(j)^*}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(A)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(i)}\right]}_{0} \underbrace{E\left[x_{n-m}^{(j)^*}\right]}_{0} = 0 \end{aligned} \tag{F.119}$$

$$\begin{aligned} E\left[\left|c_{24}\right|^2\right] &= 16 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(A)}\right|^2 \left|x_{n-l}^{(i)}\right|^2 \left|x_{n-m}^{(j)}\right|^2\right] = \\ &= 16 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(A)}\right|^2\right] E\left[\left|x_{n-l}^{(i)}\right|^2\right] E\left[\left|x_{n-m}^{(j)}\right|^2\right] = \\ &= 16 \cdot \sigma_{x^{(A)}}^2 \sigma_{x^{(i)}}^2 \sigma_{x^{(j)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 \end{aligned} \tag{F.120}$$

F.3.4.1.2 Coefficient c_{25}:

$$c_{25} = 4 \cdot e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(i)} x_{n-l}^{(k)^*} \tag{F.121}$$

$$E\left[c_{25}\right] = 4 \cdot e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(i)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(j)^*}\right]}_{0} = 0 \tag{F.122}$$

$$E\left[\left|c_{25}\right|^2\right] = E\left[\left|4 \cdot e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(i)} x_{n-l}^{(k)^*}\right|^2\right] = 16 \cdot \sigma_{x^{(i)}}^2 \sigma_{x^{(j)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 \tag{F.123}$$

F.3.4.1.3 Coefficient c_{26}:

$$c_{26} = 4 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(k)^*} \tag{F.124}$$

$$E\left[c_{26}\right] = 4 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(A)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(j)^*}\right]}_{0} = 0 \tag{F.125}$$

$$E\left[\left|c_{26}\right|^2\right] = E\left[\left|4 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(k)^*}\right|^2\right] = 16 \cdot \sigma_{x^{(A)}}^2 \sigma_{x^{(j)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 \tag{F.126}$$

F.3.4.1.4 Coefficient c_{27}:

$$c_{27} = 4 \cdot e^{-j\angle x_n^{(j)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(i)} \tag{F.127}$$

$$E\left[c_{27}\right] = 4 \cdot e^{-j\angle x_n^{(j)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(A)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(i)}\right]}_{0} = 0 \tag{F.128}$$

$$E\left[\left|c_{27}\right|^2\right] = E\left[\left|4 \cdot e^{-j\angle x_n^{(j)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(A)} x_{n-l}^{(i)}\right|^2\right] = 16 \cdot \sigma_{x^{(A)}}^2 \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0}^2 \tag{F.129}$$

F.3.4.1.5 Coefficient c_{28}:

$$c_{28} = 4 \cdot e^{j\left(\angle x_n^{(A)} + \angle x_n^{(i)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(k)^*} \tag{F.130}$$

$$E\left[c_{28}\right] = 4 \cdot e^{j\left(\angle x_n^{(A)} + \angle x_n^{(i)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(k)^*}\right] = 0 \tag{F.131}$$

$$E\left[\left|c_{28}\right|^2\right] = E\left[\left|4 \cdot e^{j\left(\angle x_n^{(A)} + \angle x_n^{(i)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(k)^*}\right|^2\right] = 16 \cdot \sigma_{x^{(j)}}^2 \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00}^2 \tag{F.132}$$

F.3.4.1.6 Coefficient c_{29}:

$$c_{29} = 4 \cdot e^{j\left(\angle x_n^{(A)} - \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)} \tag{F.133}$$

$$E\left[c_{29}\right] = 4 \cdot e^{j\left(\angle x_n^{(A)} - \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(i)}\right] = 0 \tag{F.134}$$

$$E\left[\left|c_{29}\right|^2\right] = E\left[\left|4 \cdot e^{j\left(\angle x_n^{(A)} - \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)}\right|^2\right] = 16 \cdot \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00}^2 \tag{F.135}$$

F.3.4.1.7 Coefficient c_{30}:

$$c_{30} = 4 \cdot e^{j\left(\angle x_n^{(i)} - \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(A)} \tag{F.136}$$

$$E\left[c_{30}\right] = 4 \cdot e^{j\left(\angle x_n^{(i)} - \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(A)}\right] = 0 \tag{F.137}$$

$$E\left[\left|c_{30}\right|^2\right] = E\left[\left|4 \cdot e^{j\left(\angle x_n^{(i)} - \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(A)}\right|^2\right] = 16 \cdot \sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00}^2 \tag{F.138}$$

F.3.4.1.8 Coefficient c_{31}:

$$c_{31} = 4 \cdot \lambda_{000} e^{j\left(\angle x^{(A)} + \angle x^{(i)} - \angle x^{(j)}\right)} \tag{F.139}$$

$$E\left[c_{31}\right] = 4 \cdot \lambda_{000} e^{j\left(\angle x^{(A)} + \angle x^{(i)} - \angle x^{(j)}\right)} \tag{F.140}$$

$$E\left[\left|c_{31}\right|^2\right] = 0 \tag{F.141}$$

F.3.4.2 $2 \cdot x_i(t)x_j(t)x_A^*(t)$

$$d_{x_i(t)x_j(t)x_A^*(t)} = c_{32} + c_{33}|x_{in}| + c_{34}|x_{jn}| + c_{35}|x_{An}| + c_{36}|x_{in}||x_{jn}| + c_{37}|x_{in}||x_{An}| + \\ + c_{38}|x_{jn}||x_{An}| + c_{39}|x_{in}||x_{jn}||x_{An}| \tag{F.142}$$

$$d_{x_i(t)x_j(t)x_A^*(t)} = 2 \cdot \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \lambda_{klm} \left|x_{n-k}^{(i)}\right| \left|x_{n-l}^{(j)}\right| \left|x_{n-m}^{(A)^*}\right| e^{j\left(\angle x_{n-k}^{(i)} + \angle x_{n-l}^{(j)} - \angle x_{n-m}^{(A)}\right)} \tag{F.143}$$

F.3.4.2.1 Coefficient c_{32}:

$$c_{32} = 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} x_{n-k}^{(i)} x_{n-l}^{(j)} x_{n-m}^{(A)^*} \tag{F.144}$$

$$\begin{aligned} E\left[c_{32}\right] &= 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(i)} x_{n-l}^{(j)} x_{n-m}^{(A)^*}\right]}_{\substack{\text{independent} \\ \text{and uncorrelated}}} = \\ &= 2 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm} \underbrace{E\left[x_{n-k}^{(i)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(j)}\right]}_{0} \underbrace{E\left[x_{n-m}^{(k)^*}\right]}_{0} = 0 \end{aligned} \tag{F.145}$$

$$\begin{aligned} E\left[\left|c_{32}\right|^2\right] &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(i)}\right|^2 \left|x_{n-l}^{(j)}\right|^2 \left|x_{n-m}^{(A)}\right|^2\right] = \\ &= 4 \cdot \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 E\left[\left|x_{n-k}^{(i)}\right|^2\right] E\left[\left|x_{n-l}^{(j)}\right|^2\right] E\left[\left|x_{n-m}^{(A)}\right|^2\right] = \\ &= 4 \cdot \sigma_{x^{(i)}}^2 \sigma_{x^{(j)}}^2 \sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \lambda_{klm}^2 \end{aligned} \tag{F.146}$$

F.3.4.2.2 Coefficient c_{33}:

$$c_{33} = 2 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} x_{n-k}^{(j)} x_{n-l}^{(k)^*} \tag{F.147}$$

$$E\left[c_{33}\right] = 2 \cdot e^{j\angle x_n^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{kl0} \underbrace{E\left[x_{n-k}^{(j)}\right]}_{0} \underbrace{E\left[x_{n-l}^{(A)^*}\right]}_{0} = 0 \tag{F.148}$$

$$E\left[|c_{33}|^2\right] = E\left[\left|2\cdot e^{j\angle x_n^{(i)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(j)}x_{n-l}^{(k)^*}\right|^2\right] = 4\cdot\sigma_{x^{(j)}}^2\sigma_{x^{(A)}}^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 \tag{F.149}$$

F.3.4.2.3 Coefficient c_{34}:

$$c_{34} = 2\cdot e^{j\angle x_n^{(j)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(k)^*} \tag{F.150}$$

$$E\left[c_{34}\right] = 2\cdot e^{j\angle x_n^{(j)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}\underbrace{E\left[x_{n-k}^{(i)}\right]}_{0}\underbrace{E\left[x_{n-l}^{(A)^*}\right]}_{0} = 0 \tag{F.151}$$

$$E\left[|c_{34}|^2\right] = E\left[\left|2\cdot e^{j\angle x_n^{(j)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(k)^*}\right|^2\right] = 4\cdot\sigma_{x^{(i)}}^2\sigma_{x^{(A)}}^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 \tag{F.152}$$

F.3.4.2.4 Coefficient c_{35}:

$$c_{35} = 2\cdot e^{-j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(j)} \tag{F.153}$$

$$E\left[c_{35}\right] = 2\cdot e^{-j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}\underbrace{E\left[x_{n-k}^{(i)}\right]}_{0}\underbrace{E\left[x_{n-l}^{(j)}\right]}_{0} = 0 \tag{F.154}$$

$$E\left[|c_{35}|^2\right] = E\left[\left|2\cdot e^{-j\angle x_n^{(A)}}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}x_{n-k}^{(i)}x_{n-l}^{(j)}\right|^2\right] = 4\cdot\sigma_{x^{(i)}}^2\sigma_{x^{(j)}}^2\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\sum_{\substack{l=-\infty\\l\neq 0}}^{\infty}\lambda_{kl0}^2 \tag{F.155}$$

F.3.4.2.5 Coefficient c_{36}:

$$c_{36} = 2\cdot e^{j\left(\angle x_n^{(i)}+\angle x_n^{(j)}\right)}\sum_{\substack{k=-\infty\\k\neq 0}}^{\infty}\lambda_{k00}x_{n-k}^{(k)^*} \tag{F.156}$$

$$E\left[c_{36}\right] = 2 \cdot e^{j\left(\angle x_n^{(i)} + \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(k)*}\right] = 0 \tag{F.157}$$

$$E\left[\left|c_{36}\right|^2\right] = E\left[\left|2 \cdot e^{j\left(\angle x_n^{(i)} + \angle x_n^{(j)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(k)*}\right|^2\right] = 4 \cdot \sigma_{x^{(A)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{F.158}$$

F.3.4.2.6 Coefficient c_{37}:

$$c_{37} = 2 \cdot e^{j\left(\angle x_n^{(i)} - \angle x_n^{(A)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(j)} \tag{F.159}$$

$$E\left[c_{37}\right] = 2 \cdot e^{j\left(\angle x_n^{(i)} - \angle x_n^{(A)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(j)}\right] = 0 \tag{F.160}$$

$$E\left[\left|c_{37}\right|^2\right] = E\left[\left|2 \cdot e^{j\left(\angle x_n^{(i)} - \angle x_n^{(A)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(j)}\right|^2\right] = 4 \cdot \sigma_{x^{(j)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{F.161}$$

F.3.4.2.7 Coefficient c_{38}:

$$c_{38} = 2 \cdot e^{j\left(\angle x_n^{(j)} - \angle x_n^{(A)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)} \tag{F.162}$$

$$E\left[c_{38}\right] = 2 \cdot e^{j\left(\angle x_n^{(j)} - \angle x_n^{(A)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} E\left[x_{n-k}^{(i)}\right] = 0 \tag{F.163}$$

$$E\left[\left|c_{38}\right|^2\right] = E\left[\left|2 \cdot e^{j\left(\angle x_n^{(j)} - \angle x_n^{(A)}\right)} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00} x_{n-k}^{(i)}\right|^2\right] = 4 \cdot \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{k00}^2 \tag{F.164}$$

F.3.4.2.8 Coefficient c_{39}:

$$c_{39} = 2 \cdot \lambda_{000} e^{j\left(\angle x^{(i)} + \angle x^{(j)} - \angle x^{(A)}\right)} \tag{F.165}$$

$$E\left[c_{39}\right] = 2 \cdot \lambda_{000} e^{j\left(\angle x^{(i)} + \angle x^{(j)} - \angle x^{(A)}\right)} \tag{F.166}$$

$$E\left[\left|c_{39}\right|^2\right] = 0 \tag{F.167}$$

F.3.4.3 Other distortion: $(K-1)^3$ cases $\Rightarrow$ DISTORTION

F.4 Statistical model

$$z_{An} = x_{An}\left[1 + \underbrace{\left(u^{(1)} + u^{(22)} + u^{(23)} + u^{(31)} + u^{(39)}\right)}_{u^{(1)}} + \underbrace{\left(u^{(8)} + u^{(13)} + u^{(18)}\right)}_{u^{(2)}} \frac{|x_{An}|}{\sigma_{x_A}} + u^{(3)}\frac{|x_{An}|^2}{\sigma_{x_A}^2}\right] +$$
$$+ \underbrace{\left(v_n^{(1)} + v_n^{(5)} + v_n^{(12)} + v_n^{(15)} + v_n^{(22)} + v_n^{(25)} + v_n^{(28)} + v_n^{(29)} + v_n^{(35)} + v_n^{(37)} + v_n^{(38)}\right)}_{V^{(1)}} \frac{|x_{An}|}{\sigma_{x_A}} +$$
$$+ \underbrace{\left(v_n^{(2)} + v_n^{(6)} + v_n^{(16)}\right)}_{V^{(2)}} \frac{|x_{An}|^2}{\sigma_{x_A}^2} + m_n + w_n =$$
$$= x_{An}\left[1 + U^{(1)} + U^{(2)}\frac{|x_{An}|}{\sigma_{x_A}} + U^{(3)}\frac{|x_{An}|^2}{\sigma_{x_A}^2}\right] + V^{(1)}\frac{|x_{An}|}{\sigma_{x_A}} + V^{(2)}\frac{|x_{An}|^2}{\sigma_{x_A}^2} + m_n + w_n \tag{F.168}$$

F.4.1 $U^{(1)}$

$$u^{(1)} = 2\sigma_{x^{(A)}}^2 e^{j\angle x_n^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \tag{F.169}$$

$$u^{(22)} = 2 \cdot e^{j\angle x_n^{(A)}} \sum_{\substack{i=1 \\ i\neq A}}^{K} \sigma_{x^{(i)}}^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \lambda_{kk0} \tag{F.170}$$

$$u^{(23)} = 2 \cdot (K-1) \cdot \lambda_{000} e^{j\angle x_n^{(A)}} \tag{F.171}$$

$$u^{(31)} = 4 \cdot e^{j\angle x^{(A)}} \lambda_{000} \sum_{\substack{j=1 \\ j\neq A}}^{K} \sum_{\substack{k=1 \\ k\neq j \\ k\neq A}}^{K} e^{j\left(\angle x^{(j)} - \angle x^{(k)}\right)} \tag{F.172}$$

$$u^{(39)} = 2 \cdot e^{-j\angle x^{(A)}} \lambda_{000} \sum_{\substack{i=1 \\ i\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq i \\ j\neq A}}^{K} e^{j\left(\angle x^{(i)} + \angle x^{(j)}\right)} \tag{F.173}$$

$$U^{(1)} = 2 \cdot e^{j\angle x_n^{(A)}} \left[\sum_{k=1}^{K} \sigma^2_{x^{(k)}} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda_{ll0} + \lambda_{000} \left((K-1) + 2 \cdot \sum_{\substack{k=1 \\ k\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq k \\ j\neq A}}^{K} e^{j\left(\angle x^{(k)} - \angle x^{(j)}\right)} + \right.\right.$$
$$\left.\left. + \; e^{-j2\angle x^{(A)}} \sum_{\substack{k=1 \\ k\neq A}}^{K} \sum_{\substack{j=1 \\ j\neq k \\ j\neq A}}^{K} e^{j\left(\angle x^{(k)} + \angle x^{(j)}\right)} \right) \right] \tag{F.174}$$

F.4.2 $U^{(2)}$

$$u^{(8)} = \lambda_{000} e^{j2\angle x_n^{(A)}} \sum_{\substack{i=1 \\ i\neq A}}^{K} e^{-j\angle x_n^{(i)}} \tag{F.175}$$

$$u^{(13)} = \lambda_{000} e^{-j\angle x_n^{(A)}} \sum_{\substack{i=1 \\ i\neq A}}^{K} e^{j2\angle x_n^{(i)}} \tag{F.176}$$

$$u^{(18)} = 2 \cdot \lambda_{000} \sum_{\substack{i=1 \\ i\neq A}}^{K} e^{j\angle x_n^{(i)}} \tag{F.177}$$

$$U^{(2)} = \lambda_{000} \left[e^{j2\angle x_n^{(A)}} \sum_{\substack{k=1 \\ k\neq A}}^{K} e^{-j\angle x_n^{(k)}} + e^{-j\angle x_n^{(A)}} \sum_{\substack{k=1 \\ k\neq A}}^{K} e^{j2\angle x_n^{(k)}} + 2 \cdot \sum_{\substack{k=1 \\ k\neq A}}^{K} e^{j\angle x_n^{(k)}} \right] \tag{F.178}$$

F.4.3 $V^{(1)}$

$$v^{(1)} = 5\sigma^4_{x^{(A)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left(\lambda^2_{kl0} E\left[\left| \frac{x_{n-k}^{(A)}}{\sigma_{x^{(A)}}} \right|^2 \left| \frac{x_{n-l}^{(A)}}{\sigma_{x^{(A)}}} \right|^2 \right] - \lambda_{kk0}\lambda_{ll0} \right) \tag{F.179}$$

$$v^{(5)} = 4 \cdot \sigma^2_{x^{(A)}} \sum_{\substack{i=1 \\ i\neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda^2_{kl0} \tag{F.180}$$

$$v^{(12)} = \sum_{\substack{i=1 \\ i\neq A}}^{K} \sigma^4_{x^{(i)}} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \lambda^2_{kl0} E\left[\left| \frac{x_{n-k}^{(i)}}{\sigma_{x^{(i)}}} \right|^2 \left| \frac{x_{n-l}^{(i)}}{\sigma_{x^{(i)}}} \right|^2 \right] \tag{F.181}$$

$$v^{(15)} = 4 \cdot \sigma^2_{x^{(A)}} \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \lambda^2_{kl0} \tag{F.182}$$

$$v^{(22)} = 4 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^4_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \left(\lambda^2_{kl0} E\left[\left| \frac{x^{(i)}_{n-k}}{\sigma_{x^{(i)}}} \right|^2 \left| \frac{x^{(i)}_{n-l}}{\sigma_{x^{(i)}}} \right|^2 \right] - \lambda_{kk0}\lambda_{ll0} \right) \tag{F.183}$$

$$v^{(25)} = 16 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sum_{\substack{j=1 \\ j \neq i \\ j \neq A}}^{K} \sigma^2_{x^{(i)}} \sigma^2_{x^{(j)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \lambda^2_{kl0} \tag{F.184}$$

$$v^{(28)} = 16 \cdot \sum_{\substack{j=1 \\ j \neq A}}^{K} \sigma^2_{x^{(j)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda^2_{k00} \tag{F.185}$$

$$v^{(29)} = 16 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda^2_{k00} \tag{F.186}$$

$$v^{(35)} = 4 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sum_{\substack{j=1 \\ j \neq i \\ j \neq A}}^{K} \sigma^2_{x^{(i)}} \sigma^2_{x^{(j)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \lambda^2_{kl0} \tag{F.187}$$

$$v^{(37)} = 4 \cdot \sum_{\substack{j=1 \\ j \neq A}}^{K} \sigma^2_{x^{(j)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda^2_{k00} \tag{F.188}$$

$$v^{(38)} = 4 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda^2_{k00} \tag{F.189}$$

$$v^{(5)} + v^{(15)} = 8 \cdot \sigma^2_{x^{(A)}} \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \lambda^2_{kl0} \tag{F.190}$$

$$v^{(25)} + v^{(35)} = 20 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sum_{\substack{j=1 \\ j \neq i \\ j \neq A}}^{K} \sigma^2_{x^{(i)}} \sigma^2_{x^{(j)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \lambda^2_{kl0} \tag{F.191}$$

$$v^{(5)} + v^{(15)} + v^{(25)} + v^{(35)} = \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \lambda^2_{kl0} \left(8 \cdot \sigma^2_{x^{(A)}} + 20 \cdot \sum_{\substack{j=1 \\ j \neq i \\ j \neq A}}^{K} \sigma^2_{x^{(j)}} \right) \tag{F.192}$$

$$v^{(28)} + v^{(29)} + v^{(37)} + v^{(38)} = 40 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda^2_{k00} \tag{F.193}$$

$$\begin{aligned} & v^{(1)} + v^{(12)} + v^{(22)} = \\ & = \sum_{i=1}^{K} \sigma^4_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \left(5 \cdot \lambda^2_{kl0} E\left[\left| \frac{x^{(i)}_{n-k}}{\sigma_{x^{(i)}}} \right|^2 \left| \frac{x^{(i)}_{n-l}}{\sigma_{x^{(i)}}} \right|^2 \right] - 4 \cdot \lambda_{kk0} \lambda_{ll0} \right) + \sigma^4_{x^{(A)}} \lambda_{kk0} \lambda_{ll0} \end{aligned} \tag{F.194}$$

$$\begin{aligned} E\left[\left| V^{(1)} \right|^2 \right] = & \sum_{k=1}^{K} \sigma^4_{x^{(k)}} \sum_{\substack{r=-\infty \\ r \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \left(5 \cdot \lambda^2_{rl0} E\left[\left| \frac{x^{(k)}_{n-r}}{\sigma_{x^{(k)}}} \right|^2 \left| \frac{x^{(k)}_{n-l}}{\sigma_{x^{(k)}}} \right|^2 \right] - 4 \cdot \lambda_{rr0} \lambda_{ll0} \right) + \sigma^4_{x^{(A)}} \lambda_{rr0} \lambda_{ll0} + \\ & + \sum_{\substack{k=1 \\ k \neq A}}^{K} \sigma^2_{x^{(k)}} \sum_{\substack{r=-\infty \\ r \neq 0}}^{\infty} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \lambda^2_{rl0} \left(8 \cdot \sigma^2_{x^{(A)}} + 20 \cdot \sum_{\substack{j=1 \\ j \neq i \\ j \neq A}}^{K} \sigma^2_{x^{(j)}} \right) + 40 \cdot \sum_{\substack{k=1 \\ k \neq A}}^{K} \sigma^2_{x^{(k)}} \sum_{\substack{r=-\infty \\ r \neq 0}}^{\infty} \lambda^2_{r00} \end{aligned} \tag{F.195}$$

F.4.4 $V^{(2)}$

$$v^{(2)} = 5\sigma^2_{x^{(A)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda^2_{k00} \tag{F.196}$$

$$v^{(6)} = \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma^2_{x^{(i)}} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda^2_{k00} \tag{F.197}$$

$$v^{(16)} = 4 \cdot \sum_{\substack{i=1 \\ i \neq A}}^{K} \sigma_{x^{(i)}}^{2} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \lambda_{k00}^{2} \tag{F.198}$$

$$E\left[\left|V^{(2)}\right|^{2}\right] = 5 \cdot \sum_{k=1}^{K} \sigma_{x^{(k)}}^{2} \sum_{\substack{r=-\infty \\ r \neq 0}}^{\infty} \lambda_{r00}^{2} \tag{F.199}$$